Math Cookbook

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1 Linear Algebra

Sherman–Morrison formula:

$$\left(A + uv^\top\right)^{-1} = A^{-1} - \frac{A^{-1}uv^\top A^{-1}}{1 + v^\top A^{-1}u} \tag{1}$$

$$(A + UCV)^{-1} = A^{-1} - A^{-1}U\left(C^{-1} + VA^{-1}U\right)^{-1}VA^{-1} \tag{2}$$

2 Caculus

Leibniz integral rule:

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)dt\right) = f(x,b(x))\cdot\frac{d}{dx}b(x) - f(x,a(x))\cdot\frac{d}{dx}a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x}f(x,t)dt \tag{3}$$

Grönwall's inequality from two useful references [1, 2].

$$f(x) \geq a(x) + b(x)\int_{\underline{x}}^{x} f(u)du, x \in [\underline{x}, \bar{x}] \tag{4}$$

$$\Longrightarrow f(x) \geq a(x) + b(x)\int_{\underline{x}}^{x} a(u)\exp\left(\int_{u}^{x} b(s)ds\right)du, \quad x \in [\underline{x}, \bar{x}] \tag{5}$$

Grönwall's inequality in differential form:

$$f'(x) \leq b(x)f(x) \tag{6}$$

$$\Longrightarrow f(x) \leq f(\underline{x})\exp\left(\int_{\underline{x}}^{x} b(s)ds\right) \tag{7}$$

References

[1] G. Stephen Jones. Fundamental Inequalities for Discrete and Discontinuous Functional Equations. *Journal of the Society for Industrial and Applied Mathematics*, 12(1):43–57, 1964.

[2] Dragoslav S. Mitrinovic, Josip Pecaric, and Arlington M. Fink. *Inequalities Involving Functions and Their Integrals and Derivatives*, volume 53. Springer Science & Business Media, 2012.