# STAT 401A - Statistical Methods for Research Workers

## Modeling assumptions

Jarad Niemi (Dr. J)

Iowa State University

last updated: September 10, 2014

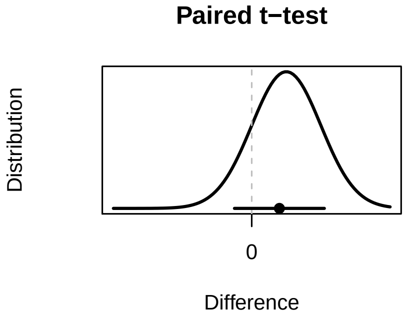
# Normality assumptions

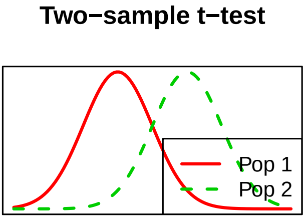
In the paired t-test, we assume

$$D_i \stackrel{iid}{\sim} N(\mu, \sigma^2).$$

In the two-sample t-test, we assume

$$Y_{ij} \stackrel{ind}{\sim} N(\mu_j, \sigma^2).$$



# Normality assumptions

In the paired t-test, we assume

$$D_i \stackrel{iid}{\sim} N(\mu, \sigma^2).$$

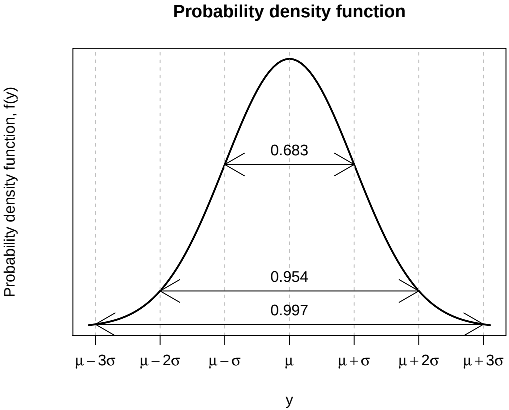
In the two-sample t-test, we assume

$$Y_{ij} \stackrel{ind}{\sim} N(\mu_j, \sigma^2).$$

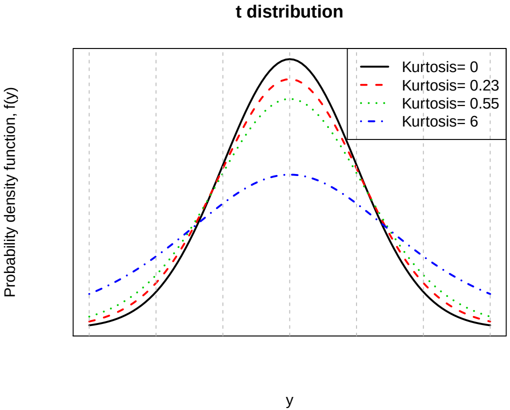
Key features of the normal distribution assumption:

- Centered at the mean (expectation) $\mu$
- Standard deviation describes the spread
- Symmetric around $\mu$ (no skewness)
- Non-heavy tails, i.e. outliers are rare (no kurtosis)

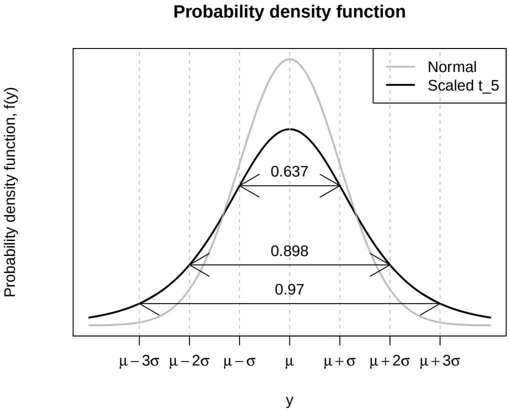
# Normality assumptions



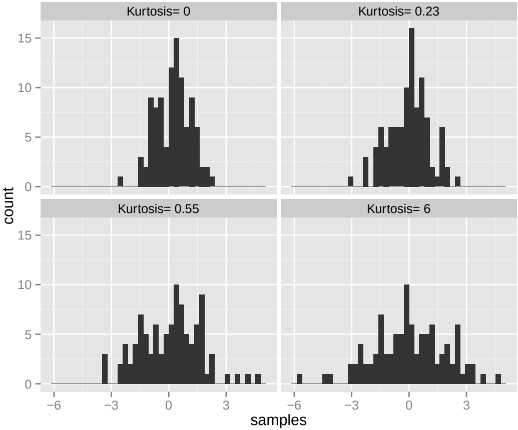
# Kurtosis (heavy-tailedness)



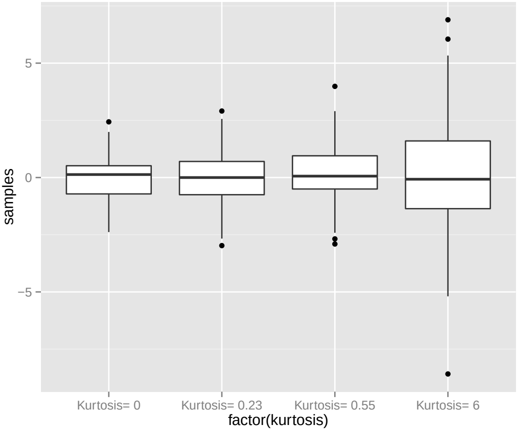
# Kurtosis (heavy-tailedness)



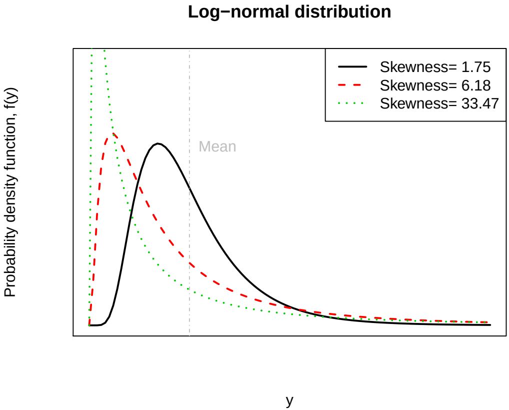
# Kurtosis (heavy-tailedness)



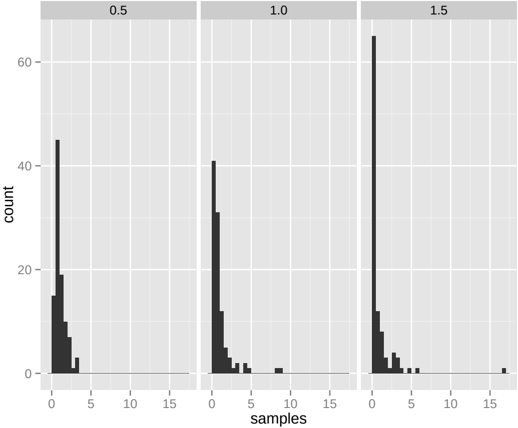
# Kurtosis (heavy-tailedness)



# Skewness


Log−normal distribution
Skewness= 1.75
Skewness= 6.18
Skewness= 33.47
Mean
Probability density function, f(y)
y

# Samples from skewed distributions



# Robustness

**Definition**

A statistical procedure is robust to departures from a particular assumption if it is valid even when the assumption is not met.

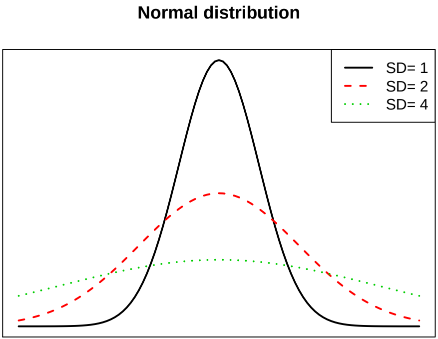
**Remark** If a 95% confidence interval is robust to departures from a particular assumption, the confidence interval should cover the true value about 95% of the time.

# Robustness to skewness and kurtosis

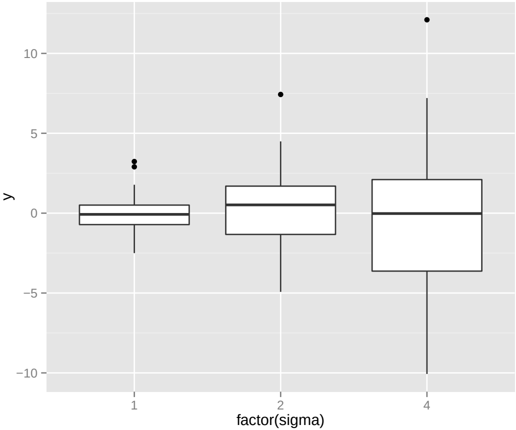
Percentage of 95% confidence intervals that cover the true difference in means in an equal-sample two-sample t-test with non-normal populations (where the distributions are the same other than their means).

| sample size | strongly skewed | moderately skewed | mildly skewed | heavy-tailed | short-tailed |
|---|---|---|---|---|---|
| 5 | 95.5 | 95.4 | 95.2 | 98.3 | 94.5 |
| 10 | 95.5 | 95.4 | 95.2 | 98.3 | 94.6 |
| 25 | 95.3 | 95.3 | 95.1 | 98.2 | 94.9 |
| 50 | 95.1 | 95.3 | 95.1 | 98.1 | 95.2 |
| 100 | 94.8 | 95.3 | 95.0 | 98.0 | 95.6 |

# Differences in variances



# Differences in variances



# Robustness to differences in variances

Percentage of 95% confidence intervals that cover the true difference in means in an equal-sample two-sample t-test ($r = \sigma_1/\sigma_2$).

| n1 | n2 | r=1/4 | r=1/2 | r=1 | r=2 | r=4 |
|---|---|---|---|---|---|---|
| 10 | 10 | 95.2 | 94.2 | 94.7 | 95.2 | 94.5 |
| 10 | 20 | 83.0 | 89.3 | 94.4 | 98.7 | 99.1 |
| 10 | 40 | 71.0 | 82.6 | 95.2 | 99.5 | 99.9 |
| 100 | 100 | 94.8 | 96.2 | 95.4 | 95.3 | 95.1 |
| 100 | 200 | 86.5 | 88.3 | 94.8 | 98.8 | 99.4 |
| 100 | 400 | 71.6 | 81.5 | 95.0 | 99.5 | 99.9 |

# Outliers

**Definition**

A statistical procedure is resistant if it does not change very much when a small part of the data changes, perhaps drastically.

Identify outliers:

1. If recording errors, fix.
2. If outlier comes from a different population, remove and report.
3. If results are the same with and without outliers, report with outliers.
4. If results are different, use resistant analysis or report both analyses.

# Common ways for independence to be violated

- Cluster effect
  - e.g. pigs in a pen
- Correlation effect
  - e.g. measurements in time with drifting scale
- Spatial effect
  - e.g. corn yield plots (drainage)