

Set 17 - Multiple comparisons

STAT 401 (Engineering) - Iowa State University

March 3, 2017

Mice diet effect on lifetimes

Female mice were randomly assigned to six treatment groups to investigate whether restricting dietary intake increases life expectancy.

Diet treatments were:

- NP - mice ate unlimited amount of nonpurified, standard diet
- N/N85 - mice fed normally before and after weaning. After weaning, ration was controlled at 85 kcal/wk
- N/R50 - normal diet before weaning and reduced calorie diet (50 kcal/wk) after weaning
- R/R50 - reduced calorie diet of 50 kcal/wk both before and after weaning
- N/R50 lopro - normal diet before weaning, restricted diet (50 kcal/wk) after weaning and dietary protein content decreased with advancing age
- N/R40 - normal diet before weaning and reduced diet (40 Kcal/wk) after weaning.

Exploratory analysis

```
library("Sleuth3")
# head(case0501)
summary(case0501)
```

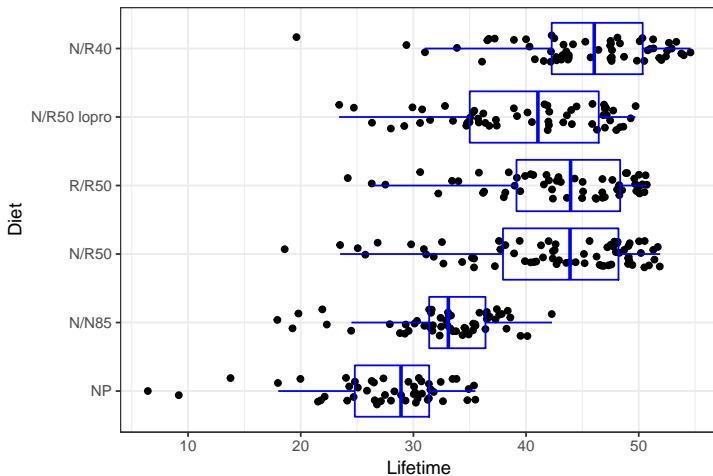
	Lifetime	Diet
Min.	: 6.4	N/N85:57
1st Qu.	:31.8	N/R40:60
Median	:39.5	N/R50:71
Mean	:38.8	NP :49
3rd Qu.	:46.9	R/R50:56
Max.	:54.6	lopro:56

```
case0501 <- case0501 %>%
  mutate(Diet = factor(Diet, c("NP", "N/N85", "N/R50", "R/R50", "lopro", "N/R40")),
         Diet = recode(Diet, lopro = "N/R50 lopro"))
case0501 %>% group_by(Diet) %>% summarize(n=n(), mean = mean(Lifetime), sd = sd(Lifetime))
```

```
# A tibble: 6  4
```

	Diet	n	mean	sd
	<fctr>	<int>	<dbl>	<dbl>
1	NP	49	27.40204	6.133701
2	N/N85	57	32.69123	5.125297
3	N/R50	71	42.29718	7.768195
4	R/R50	56	42.88571	6.683152
5	N/R50 lopro	56	39.68571	6.991695
6	N/R40	60	45.11667	6.703406

```
ggplot(case0501, aes(x=Diet, y=Lifetime)) +
  geom_jitter(width=0.2) +
  geom_boxplot(fill=NA, color='blue', outlier.color = NA) +
  coord_flip() +
  theme_bw()
```



Are the data compatible with a common mean?

Let Y_{ij} represent the lifetime of mouse j in diet i for $i = 1, \dots, I$ and $j = 1, \dots, n_i$.

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```
bartlett.test(Lifetime ~ Diet, data = case0501)
```

Bartlett test of homogeneity of variances

```
data: Lifetime by Diet
Bartlett's K-squared = 10.996, df = 5, p-value = 0.05146
```

```
oneway.test(Lifetime ~ Diet, data = case0501, var.equal = TRUE)
```

One-way analysis of means

```
data: Lifetime and Diet
F = 57.104, num df = 5, denom df = 343, p-value < 2.2e-16
```

```
oneway.test(Lifetime ~ Diet, data = case0501, var.equal = FALSE)
```

One-way analysis of means (not assuming equal variances)

```
data: Lifetime and Diet
F = 64.726, num df = 5.00, denom df = 157.84, p-value < 2.2e-16
```

Statistical testing errors

Definition

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We set our significance level α to control the type I error probability. If we set $\alpha = 0.05$, then we will incorrectly reject a true null hypothesis 5% of the time.

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Decision	Truth	
	H_0 true	H_0 false
H_0 not true	Type I error	Correct (power)
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The **familywise error rate** is the probability of rejecting at least one true null hypothesis.

Type I error for all pairwise comparisons of J groups

How many combinations when choosing 2 items out of J ?

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So there is a greater than 50% probability of falsely rejecting at least one true null hypothesis!

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If we do m tests and want the familywise error rate to be α , the **Bonferroni correction** uses α/m for each individual test.

Bonferroni correction

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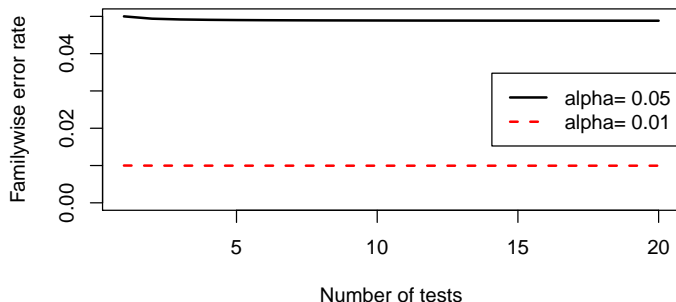
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Bonferroni familywise error rate



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```
pairwise.t.test(case0501$Lifetime, case0501$Diet, p.adjust.method = "none")
```

Pairwise comparisons using t tests with pooled SD

data: case0501\$Lifetime and case0501\$Diet

	NP	N/N85	N/R50	R/R50	N/R50	lopro
N/N85	5.9e-05	-	-	-	-	-
N/R50	< 2e-16	1.1e-14	-	-	-	-
R/R50	< 2e-16	8.9e-15	0.622	-	-	-
N/R50 lopro	< 2e-16	5.2e-08	0.029	0.012	-	-
N/R40	< 2e-16	< 2e-16	0.017	0.073	1.6e-05	-

P value adjustment method: none

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```
pairwise.t.test(case0501$Lifetime, case0501$Diet, p.adjust.method = "bonferroni")
```

Pairwise comparisons using t tests with pooled SD

data: case0501\$Lifetime and case0501\$Diet

	NP	N/N85	N/R50	R/R50	N/R50 lopro
N/N85	0.00089	-	-	-	-
N/R50	< 2e-16	1.6e-13	-	-	-
R/R50	< 2e-16	1.3e-13	1.00000	-	-
N/R50 lopro	< 2e-16	7.9e-07	0.44018	0.17507	-
N/R40	< 2e-16	< 2e-16	0.24881	1.00000	0.00024

P value adjustment method: bonferroni

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The Bonferroni correction is (in general) the **most** conservative multiple comparison adjustment, i.e. it will lead to the least null hypothesis rejections.

Constructing multiple confidence intervals

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LSD	$t_{n-J}(1 - \alpha/2)$	After significant F -test (no adjustment)

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Tukey-Kramer	$q_{J,n-J}(1 - \alpha)/\sqrt{2}$	All pairwise comparisons
Scheffé	$\sqrt{(J-1)F_{(J-1,n-J)}(1 - \alpha)}$	All contrasts
Bonferroni	$t_{n-J}(1 - (\alpha/m)/2)$	m tests (most generic)

Tukey for all pairwise comparisons

```
TukeyHSD(aov(Lifetime ~ Diet, data = case0501))
```

```
Tukey multiple comparisons of means
95% family-wise confidence level
```

```
Fit: aov(formula = Lifetime ~ Diet, data = case0501)
```

```
$Diet
```

	diff	lwr	upr	p adj
N/N85-NP	5.2891873	1.5606269	9.0177476	0.0008380
N/R50-NP	14.8951423	11.3405719	18.4497127	0.0000000
R/R50-NP	15.4836735	11.7397556	19.2275913	0.0000000
N/R50 lopro-NP	12.2836735	8.5397556	16.0275913	0.0000000
N/R40-NP	17.7146259	14.0294069	21.3998448	0.0000000
N/R50-N/N85	9.6059550	6.2021702	13.0097399	0.0000000
R/R50-N/N85	10.1944862	6.5934168	13.7955556	0.0000000
N/R50 lopro-N/N85	6.9944862	3.3934168	10.5955556	0.0000008
N/R40-N/N85	12.4254386	8.8854359	15.9654413	0.0000000
R/R50-N/R50	0.5885312	-2.8320696	4.0091319	0.9963976
N/R50 lopro-N/R50	-2.6114688	-6.0320696	0.8091319	0.2460200
N/R40-N/R50	2.8194836	-0.5367684	6.1757356	0.1564608
N/R50 lopro-R/R50	-3.2000000	-6.8169683	0.4169683	0.1167873
N/R40-R/R50	2.2309524	-1.3252222	5.7871269	0.4684413
N/R40-N/R50 lopro	5.4309524	1.8747778	8.9871269	0.0002306

False Discovery Rate

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Definition

False discovery rate procedures try to control the expected proportion of incorrectly rejected null hypotheses.

How to incorporate multiple comparison adjustments

1. Determine what tests are going to be run (before looking at the data) or what confidence intervals are going to be constructed.
2. Determine which multiple comparison adjustment is the most relevant
3. Use/state that adjustment and interpret your results