STAT 401A - Statistical Methods for Research Workers

Simple linear regression

Jarad Niemi (Dr. J)

Iowa State University

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Simple Linear Regression

Recall the one-way ANOVA model:

$$Y_{ij} \stackrel{ind}{\sim} N(\mu_j, \sigma^2)$$

where Y_{ij} is the observation for individual i in group j.

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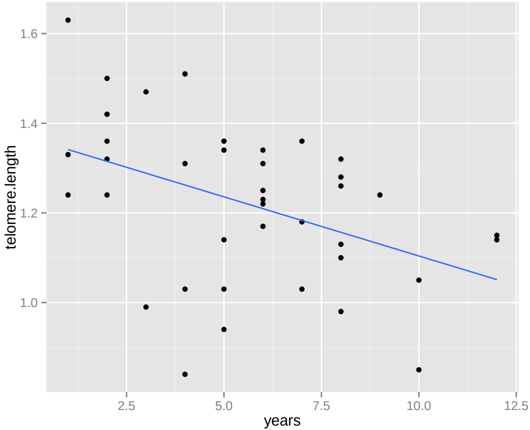
Terminology (all of these are equivalent):

response	explanatory
outcome	covariate
dependent	independent
endogenous	exogenous


1.6
1.4
1.2
1.0
telomere.length
2.5
5.0
7.5
10.0
12.5
years

Telomere length

http://www.pnas.org/content/101/49/17312

People who are stressed over long periods tend to look haggard, and it is commonly thought that psychological stress leads to premature aging and the earlier onset of diseases of aging.

...

This design allowed us to examine the importance of perceived stress and measures of objective stress (caregiving status and chronicity of caregiving stress based on the number of years since a child's diagnosis).

...

Telomere length values were measured from DNA by a quantitative PCR assay that determines the relative ratio of telomere repeat copy number to single-copy gene copy number (T/S ratio) in experimental samples as compared with a reference DNA sample.

Interpretation

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  β_1 is the expected increase in the response for each unit increase in the explanatory variable.

- σ is the standard deviation of the response for a fixed value of the explanatory variable.

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$$\begin{aligned}
\hat{\beta}_1 &= SXY/SXX \\
\hat{\beta}_0 &= \overline{Y} - \hat{\beta}_1 \overline{X} \\
\hat{\sigma}^2 &= SSE/(n-2) \qquad \text{df} = n-2
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\overline{X} & = \frac{1}{n}\sum_{i=1}^n X_i \\
\overline{Y} & = \frac{1}{n}\sum_{i=1}^n Y_i \\
\\
SXY & = \sum_{i=1}^n (X_i - \overline{X})(Y_i - \overline{Y}) \\
SXX & = \sum_{i=1}^n (X_i - \overline{X})(X_i - \overline{X}) = \sum_{i=1}^n (X_i - \overline{X})^2 \\
SSE & = \sum_{i=1}^n r_i^2
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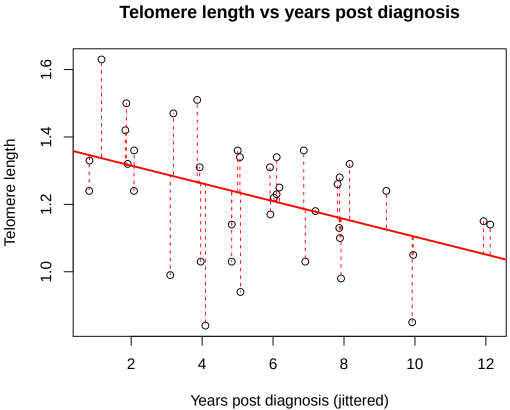
The coefficient of determination (R^2) is the proportion of the total response variation explained by the explanatory variable(s).


Telomere length vs years post diagnosis
Telomere length
1.0
1.2
1.4
1.6
2
4
6
8
10
12
Years post diagnosis (jittered)

Pvalues and confidence interval

We can compute two-sided pvalues via

$$2P\left(t_{n-2} < -\left|\frac{\hat{\beta}_0}{SE(\beta_0)}\right|\right) \qquad \text{and} \qquad 2P\left(t_{n-2} < -\left|\frac{\hat{\beta}_1}{SE(\beta_1)}\right|\right)$$

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These test the null hypothesis that the corresponding parameter is zero.

We can construct $100(1-\alpha)\%$ two-sided confidence intervals via

$$\hat{\beta}_0 \pm t_{n-2}(1-\alpha/2)SE(\beta_0) \qquad \text{and} \qquad \hat{\beta}_1 \pm t_{n-2}(1-\alpha/2)SE(\beta_1)$$

These provide ranges of the parameters consistent with the data.

```
DATA t;
  INFILE 'telomeres.csv' DSD FIRSTOBS=2;
  INPUT years length;

PROC CORR DATA=t;
  VAR length;
  WITH years;
  RUN;
```

```
                                    The CORR Procedure

                                1 With Variables:    years
                                1      Variables:    length


                                    Simple Statistics

   Variable           N          Mean       Std Dev           Sum       Minimum       Maximum

   years             39       5.58974       2.93543     218.00000       1.00000      12.00000
   length            39       1.22026       0.17977      47.59000       0.84000       1.63000


                          Pearson Correlation Coefficients, N = 39
                                Prob > |r| under H0: Rho=0

                                                 length

                                    years      -0.43065
                                                 0.0062
```

```
PROC GLM DATA=t;
  MODEL length = years / SOLUTION CLPARM;
  RUN;

                              The GLM Procedure
                   Number of Observations Read          39
                   Number of Observations Used          39

Dependent Variable: length

                                            Sum of
       Source                      DF      Squares     Mean Square    F Value    Pr > F
       Model                        1   0.22776588      0.22776588       8.42    0.0062
       Error                       37   1.00033156      0.02703599
       Corrected Total             38   1.22809744

                    R-Square     Coeff Var      Root MSE    length Mean
                    0.185462      13.47473      0.164426       1.220256

       Source                      DF    Type I SS     Mean Square    F Value    Pr > F
       years                        1   0.22776588      0.22776588       8.42    0.0062

       Source                      DF  Type III SS     Mean Square    F Value    Pr > F
       years                        1   0.22776588      0.22776588       8.42    0.0062

                                       Standard
 Parameter          Estimate              Error    t Value    Pr > |t|      95% Confidence Limits
 Intercept       1.367682067         0.05721112      23.91      <.0001     1.251761335  1.483602799
 years          -0.026374315         0.00908674      -2.90      0.0062    -0.044785794 -0.007962836
```

Regression in R

```
m = lm(telomere.length~years, Telomeres)
with(Telomeres, cor(telomere.length,years))
```

```
[1] -0.4307
```

```
anova(m)
```

```
Analysis of Variance Table

Response: telomere.length
          Df Sum Sq Mean Sq F value Pr(>F)
years      1  0.228   0.228    8.42 0.0062 **
Residuals 37  1.000   0.027
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Regression in R

```
m = lm(telomere.length~years, Telomeres)
summary(m)



Call:
lm(formula = telomere.length ~ years, data = Telomeres)

Residuals:
    Min      1Q  Median      3Q     Max
-0.4222 -0.0854  0.0206  0.1074  0.2887

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.36768    0.05721    23.9   <2e-16 ***
years       -0.02637    0.00909    -2.9   0.0062 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.164 on 37 degrees of freedom
Multiple R-squared:  0.185, Adjusted R-squared:  0.163
F-statistic: 8.42 on 1 and 37 DF,  p-value: 0.0062


confint(m)


               2.5 %    97.5 %
(Intercept)  1.25176  1.483603
years       -0.04479 -0.007963
```

Conclusion

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> *The zero-order correlation between chronicity of caregiving [years] and mean telomere length, r,is 0.445 (P <0.01).*
> *[$R^2 = 0.198$ was shown in the plot.]*

Remark I'm guessing our analysis and that reported in the paper don't match exactly due to a discrepancy in the data.

Summary

- The simple linear regression model is

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  - The coefficient of determination (R^2) is the percentage of the total response variation explained by the explanatory variable(s).