I09 - Comparing means

STAT 587 (Engineering)
Iowa State University

October 4, 2020

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Now, we will consider what happens when you have multiple μs.

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where $y = (y_{1,1}, \ldots, y_{1,n_1}, y_{2,1}, \ldots, y_{2,n_2})$.

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And you have the following summary statistics:

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# A tibble: 2 x 4
  process      n  mean    sd
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1 P1          22  7.74  1.87
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p-values and confidence intervals

Because there is no indication that you have any expectation regarding the sensitivities of process 1 compared to process 2, we will conduct a two-sided two-sample t-test assuming the variances are not equal,

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```
t.test(sensitivity ~ process, data = d2)

Welch Two Sample t-test

data:  sensitivity by process
t = -2.6932, df = 50.649, p-value = 0.009571
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -2.610398 -0.380530
sample estimates:
mean in group P1 mean in group P2 
        7.743761         9.239224 
```

Posterior for μ_1, μ_2

Assume

$$Y_{g,i} \stackrel{ind}{\sim} N(\mu_g, \sigma_g^2) \quad \text{and} \quad p(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2) \propto \frac{1}{\sigma_1^2}\frac{1}{\sigma_2^2}.$$

Then

$$\mu_g | y \stackrel{ind}{\sim} t_{n_g - 1}(\overline{y}_g, s_g^2/n_g)$$

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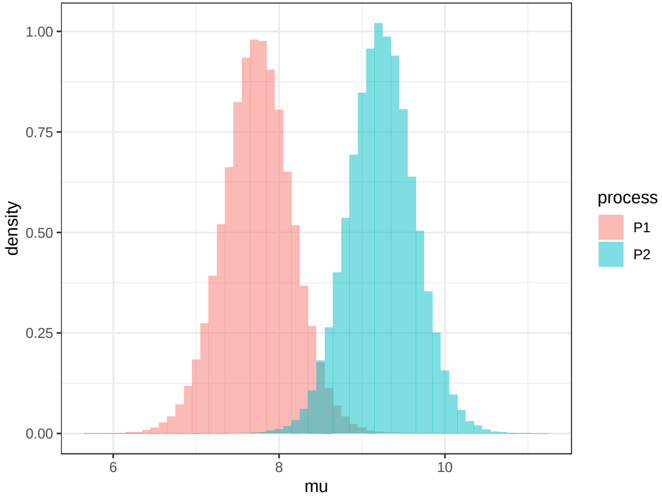
Simulations:

We can use these draws to compare the posteriors



Credible interval for the difference

To obtain statistical inference on the difference, we use the samples and take the difference

```
d3 <- sims %>%
  spread(process, mu) %>%
  mutate(diff = P1-P2)

# Bayes estimate for the difference
mean(d3$diff)

[1] -1.493267

# Estimated 95% equal-tail credible interval
quantile(d3$diff, c(.025,.975))

      2.5%      97.5%
-2.6339752 -0.3483025

# Estimate of the probability that mu1 is larger than mu2
mean(d3$diff > 0)

[1] 0.00591
```

Three or more means

Now, let's consider the more general problem of

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```
oneway.test(sensitivity ~ process, data = d)

One-way analysis of means (not assuming equal variances)

data:  sensitivity and process
F = 7.6287, num df = 2.000, denom df = 17.418, p-value = 0.004174
```

Pairwise differences

Then we typically look at pairwise differences:

```
pairwise.t.test(d$sensitivity,
                d$process,
                pool.sd = FALSE,
                p.adjust.method = "none")

Pairwise comparisons using t tests with non-pooled SD

data:  d$sensitivity and d$process

   P1     P2
P2 0.0096 -
P3 0.0045 0.0870

P value adjustment method: none
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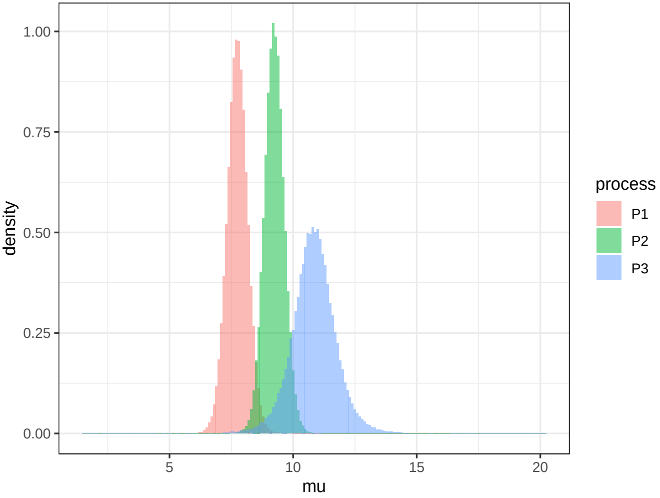
Compare posteriors



Credible intervals for differences

Use the simulations to calculate posterior probabilities and credible intervals for differences.

```
# Estimate of the probability that one mean is larger than another
sims %>%
  spread(process, mu) %>%
  mutate(`mu1-mu2` = P1-P2,
         `mu1-mu3` = P1-P3,
         `mu2-mu3` = P2-P3) %>%
  select(`mu1-mu2`,`mu1-mu3`,`mu2-mu3`) %>%
  gather(comparison, diff) %>%
  group_by(comparison) %>%
  summarize(probability = mean(diff>0) %>% round(4),
            lower = quantile(diff, .025) %>% round(2),
            upper = quantile(diff, .975) %>% round(2)) %>%
  mutate(credible_interval = paste("(",lower,",",upper,")", sep="")) %>%
  select(comparison, probability, credible_interval)

# A tibble: 3 x 3
  comparison probability credible_interval
  <chr>            <dbl> <chr>
1 mu1-mu2         0.0059 (-2.63,-0.35)
2 mu1-mu3         0.0037 (-5.06,-1.11)
3 mu2-mu3         0.0493 (-3.56,0.37)
```

Common variance model

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we can calculate a p-value for the following null hypothesis:

$$H_0 : \sigma_g = \sigma \quad \text{for all} \quad g$$

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Bartlett test of homogeneity of variances

data:  sensitivity by process
Bartlett's K-squared = 0.90949, df = 2, p-value = 0.6346
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This assumption is common when the number of observations in the groups is small.

Comparing means when the variances are equal

Assuming $Y_{g,i} \overset{ind}{\sim} N(\mu_g, \sigma^2)$, we can test

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oneway.test(sensitivity ~ process, data = d, var.equal = TRUE)


One-way analysis of means

data:  sensitivity and process
F = 6.7543, num df = 2, denom df = 60, p-value = 0.002261
```

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One-way analysis of means

data:  sensitivity and process
F = 6.7543, num df = 2, denom df = 60, p-value = 0.002261
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Then we typically look at pairwise differences, i.e. $H_0 : \mu_g = \mu_{g'}$.

```
pairwise.t.test(d$sensitivity, d$process, p.adjust.method = "none")


Pairwise comparisons using t tests with pooled SD

data:  d$sensitivity and d$process

   P1     P2
P2 0.0116 -
P3 0.0012 0.0720
```

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$$\mu_g|y, \sigma^2 \stackrel{ind}{\sim} N(\overline{y}_g, \sigma^2/n_g) \quad \sigma^2|y \sim IG\left(\frac{n-G}{2}, \frac{1}{2}\sum_{g=1}^{G}\sum_{i=1}^{n_g}(y_{g,i} - \overline{y}_g)^2\right)$$

where $n = \sum_{g=1}^{G} n_g$.

Posteriors for μ

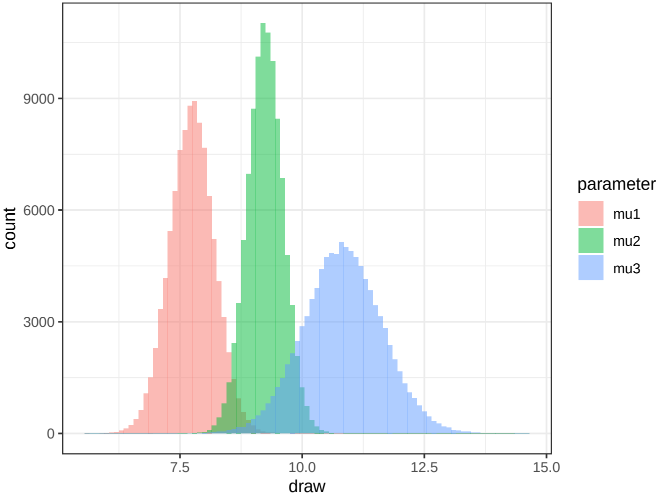
If $Y_{g,i} \stackrel{ind}{\sim} N(\mu_g, \sigma^2)$ and we use the prior $p(\mu_1, \ldots, \mu_G, \sigma^2) \propto 1/\sigma^2$, then

$$\mu_g|y, \sigma^2 \stackrel{ind}{\sim} N(\overline{y}_g, \sigma^2/n_g) \quad \sigma^2|y \sim IG\left(\frac{n-G}{2}, \frac{1}{2}\sum_{g=1}^{G}\sum_{i=1}^{n_g}(y_{g,i} - \overline{y}_g)^2\right)$$

where $n = \sum_{g=1}^{G} n_g$. and thus, we obtain joint samples for μ by performing the following

1. $\sigma^{2(m)} \sim p(\sigma^2|y)$
2. For $g = 1, \ldots, G$, $\mu_g \sim p(\mu_g|y, \sigma^{2(m)})$.

Compare posteriors



Credible interval for the differences

To compare the means, we compare the samples drawn from the posterior.

```
sims %>%
  mutate(`mu1-mu2` = mu1-mu2,
         `mu1-mu3` = mu1-mu3,
         `mu2-mu3` = mu2-mu3) %>%
  select(`mu1-mu2`,`mu1-mu3`,`mu2-mu3`) %>%
  gather(comparison, diff) %>%
  group_by(comparison) %>%
  summarize(probability = mean(diff>0) %>% round(4),
            lower = quantile(diff, .025) %>% round(2),
            upper = quantile(diff, .975) %>% round(2)) %>%
  mutate(credible_interval = paste("(",lower,",",upper,")", sep="")) %>%
  select(comparison, probability, credible_interval)

# A tibble: 3 x 3
  comparison probability credible_interval
  <chr>            <dbl> <chr>
1 mu1-mu2         0.0059 (-2.65,-0.35)
2 mu1-mu3         0.0007 (-4.92,-1.26)
3 mu2-mu3         0.036  (-3.34,0.15)
```

Summary

Multiple (independent) normal means

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Multiple (independent) normal means

- p-values
- confidence intervals
- posterior densities
- credible intervals
- posterior probabilities