STAT 401A - Statistical Methods for Research Workers

Two-way ANOVA

Jarad Niemi (Dr. J)

Iowa State University

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- 3 different varieties (A,B,C)

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This is also referred to as a full factorial or fully crossed design.

Hypotheses

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- is there any effect and
- if yes, what is the nature of the effect.

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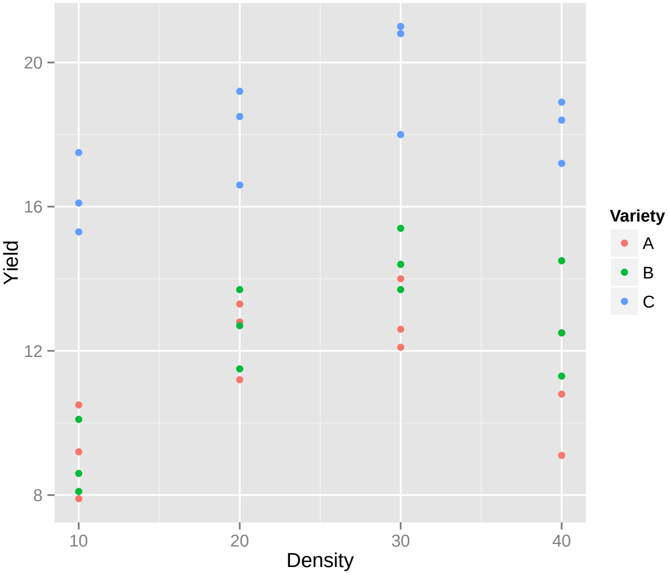
Confidence intervals can answer these questions.


Yield
Density
20
16
12
8
10
20
30
40
Variety
A
B
C

Summary statistics

Number of replicates

```
  Variety 10 20 30 40
1       A  3  3  3  3
2       B  3  3  3  3
3       C  3  3  3  3
```

Mean Yield

```
  Variety        10       20       30       40
1       A  9.200000 12.43333 12.90000 10.80000
2       B  8.933333 12.63333 14.50000 12.76667
3       C 16.300000 18.10000 19.93333 18.16667
```

Standard deviation of yield

```
  Variety       10       20        30        40
1       A 1.300000 1.096966 0.9848858 1.7000000
2       B 1.040833 1.101514 0.8544004 1.6165808
3       C 1.113553 1.345362 1.6772994 0.8736895
```

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Consider the models:

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  Consider the models:
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	10	20	30	40
A	μ_{11}	μ_{12}	μ_{13}	μ_{14}
B	μ_{21}	μ_{22}	μ_{23}	μ_{24}
C	μ_{31}	μ_{32}	μ_{33}	μ_{34}

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4. The additive model:

$$\begin{aligned}\mu_i = \quad & \beta_0 + \beta_1 \mathrm{I}(V_i = A) + \beta_2 \mathrm{I}(V_i = B) \\ & + \beta_3 \mathrm{I}(D_i = 10) + \beta_4 \mathrm{I}(D_i = 20) + \beta_5 \mathrm{I}(D_i = 30).\end{aligned}$$

β_1 is the expected difference in yield between varieties A and C at any fixed density

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5. The cell-means model:

$$\begin{aligned}\mu_i = \; & \beta_0 + \beta_1 \mathrm{I}(V_i = A) + \beta_2 \mathrm{I}(V_i = B) \\ & + \beta_3 \mathrm{I}(D_i = 10) + \beta_4 \mathrm{I}(D_i = 20) + \beta_5 \mathrm{I}(D_i = 30) \\ \\ & + \beta_6 \mathrm{I}(V_i = A)\mathrm{I}(D_i = 10) + \beta_7 \mathrm{I}(V_i = A)\mathrm{I}(D_i = 20) + \beta_8 \mathrm{I}(V_i = A)\mathrm{I}(D_i = 30) \\ & + \beta_9 \mathrm{I}(V_i = B)\mathrm{I}(D_i = 10) + \beta_{10} \mathrm{I}(V_i = B)\mathrm{I}(D_i = 20) + \beta_{11} \mathrm{I}(V_i = B)\mathrm{I}(D_i = 30)\end{aligned}$$

   β_1 is the expected difference in yield between varieties A and C at a density of 40

ANOVA Table

ANOVA Table - Additive model

Source	SS	df	MS	F
Factor A	SSA	(I-1)	SSA/(I-1)	MSA/MSE
Factor B	SSB	(J-1)	SSB/(J-1)	MSB/MSE
Error	SSE	n-I-J+1	SSE/(n-I-J+1)	
Total	SST	n-1		

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Factor B	SSB	(J-1)	SSB/(J-1)	MSB/MSE
Error	SSE	n-I-J+1	SSE/(n-I-J+1)	
Total	SST	n-1		

ANOVA Table - Cell-means model

Source	SS	df	MS	
Factor A	SSA	I-1	SSA/(I-1)	MSA/MSE
Factor B	SSB	J-1	SSB/(J-1)	MSB/MSE
Interaction AB	SSAB	(I-1)(J-1)	SSAB /(I-1)(J-1)	MSAB/MSE
Error	SSE	n-IJ	SSE/(n-IJ)	
Total	SST	n-1		

Additive vs cell-means

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Estimate of σ^2	Biased	Unbiased

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We will continue using the cell-means model to answer the scientific questions of interest.

Two-way ANOVA using PROC GLM

```
DATA tomato;
  INFILE 'Ch13-tomato.csv' DSD FIRSTOBS=2;
  INPUT variety $ density yield;

PROC GLM DATA=tomato PLOTS=all;
  CLASS variety density;
  MODEL yield = variety|density / SOLUTION;
  LSMEANS variety / cl adjust=tukey;
  LSMEANS density / cl adjust=tukey;
  LSMEANS variety*density / cl adjust=tukey;
  RUN;
```

Two-way ANOVA using PROC GLM

The GLM Procedure

Dependent Variable: yield

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	422.3155556	38.3923232	24.22	<.0001
Error	24	38.0400000	1.5850000		
Corrected Total	35	460.3555556			

R-Square	Coeff Var	Root MSE	yield Mean
0.917368	9.064568	1.258968	13.88889

Source	DF	Type I SS	Mean Square	F Value	Pr > F
variety	2	327.5972222	163.7986111	103.34	<.0001
density	3	86.6866667	28.8955556	18.23	<.0001
variety*density	6	8.0316667	1.3386111	0.84	0.5484

Source	DF	Type III SS	Mean Square	F Value	Pr > F
variety	2	327.5972222	163.7986111	103.34	<.0001
density	3	86.6866667	28.8955556	18.23	<.0001
variety*density	6	8.0316667	1.3386111	0.84	0.5484

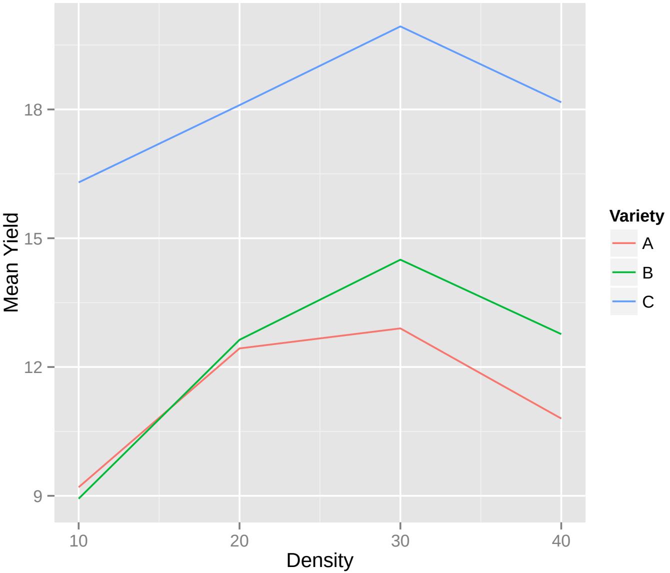
The Type I and Type III SS are equal because the design is balanced.

Two-way ANOVA using PROC GLM

```
MODEL yield = variety|density / SOLUTION;
                                    The GLM Procedure

                                                        Standard
        Parameter                     Estimate             Error    t Value    Pr > |t|

        Intercept                  18.16666667 B      0.72686542      24.99      <.0001
        variety         A          -7.36666667 B      1.02794293      -7.17      <.0001
        variety         B          -5.40000000 B      1.02794293      -5.25      <.0001
        variety         C           0.00000000 B       .                .           .
        density         10         -1.86666667 B      1.02794293      -1.82      0.0819
        density         20         -0.06666667 B      1.02794293      -0.06      0.9488
        density         30          1.76666667 B      1.02794293       1.72      0.0986
        density         40          0.00000000 B       .                .           .
        variety*density A 10        0.26666667 B      1.45373083       0.18      0.8560
        variety*density A 20        1.70000000 B      1.45373083       1.17      0.2537
        variety*density A 30        0.33333333 B      1.45373083       0.23      0.8206
        variety*density A 40        0.00000000 B       .                .           .
        variety*density B 10       -1.96666667 B      1.45373083      -1.35      0.1887
        variety*density B 20       -0.06666667 B      1.45373083      -0.05      0.9638
        variety*density B 30       -0.03333333 B      1.45373083      -0.02      0.9819
        variety*density B 40        0.00000000 B       .                .           .
        variety*density C 10        0.00000000 B       .                .           .
```


Mean Yield
18
15
12
9
10
20
30
40
Density
Variety
A
B
C

Is the mean yield for variety A different from B on average?

```
LSMEANS variety / cl adjust=tukey;
                                Least Squares Means
                    Adjustment for Multiple Comparisons: Tukey

...

                       Least Squares Means for effect variety
                        Pr > |t| for H0: LSMean(i)=LSMean(j)

                             Dependent Variable: yield

                 i/j             1             2             3
                    1                     0.2249        <.0001
                    2       0.2249                      <.0001
                    3       <.0001        <.0001

               variety     yield LSMEAN      95% Confidence Limits
               A              11.333333       10.583245     12.083422
               B              12.208333       11.458245     12.958422
               C              18.125000       17.374912     18.875088

                        Least Squares Means for Effect variety
                             Difference          Simultaneous 95%
                                Between       Confidence Limits for
                i    j            Means        LSMean(i)-LSMean(j)
                1    2        -0.875000       -2.158534      0.408534
                1    3        -6.791667       -8.075201     -5.508132
                2    3        -5.916667       -7.200201     -4.633132
```

Is the mean yield at density 10 different from density 20 on average?

```
LSMEANS density / cl adjust=tukey;
                              Least Squares Means
                     Adjustment for Multiple Comparisons: Tukey
                                     ...

                 density     yield LSMEAN      95% Confidence Limits
                 10             11.477778       10.611650    12.343905
                 20             14.388889       13.522762    15.255016
                 30             15.777778       14.911650    16.643905
                 40             13.911111       13.044984    14.777238

                        Least Squares Means for Effect density
                            Difference           Simultaneous 95%
                               Between        Confidence Limits for
                  i    j         Means         LSMean(i)-LSMean(j)

                  1    2     -2.911111       -4.548299    -1.273923
                  1    3     -4.300000       -5.937188    -2.662812
                  1    4     -2.433333       -4.070521    -0.796145
                  2    3     -1.388889       -3.026077     0.248299
                  2    4      0.477778       -1.159410     2.114966
                  3    4      1.866667        0.229479     3.503855
```

Is mean yield different for particular combinations?

```
LSMEANS variety*density / cl adjust=tukey;
```

variety	density	yield LSMEAN	95% Confidence Limits	
A	10	9.200000	7.699824	10.700176
A	20	12.433333	10.933157	13.933510
A	30	12.900000	11.399824	14.400176
A	40	10.800000	9.299824	12.300176
B	10	8.933333	7.433157	10.433510
B	20	12.633333	11.133157	14.133510
B	30	14.500000	12.999824	16.000176
B	40	12.766667	11.266490	14.266843
C	10	16.300000	14.799824	17.800176
C	20	18.100000	16.599824	19.600176
C	30	19.933333	18.433157	21.433510
C	40	18.166667	16.666490	19.666843

Is mean yield different for particular combinations?

```
LSMEANS variety*density / cl adjust=tukey;
```

```
                    Least Squares Means for Effect variety*density

                      Difference        Simultaneous 95%
                         Between     Confidence Limits for
      i      j             Means       LSMean(i)-LSMean(j)
      1      2         -3.233333        -6.939704     0.473037
      1      3         -3.700000        -7.406371     0.006371
      1      4         -1.600000        -5.306371     2.106371
      1      5          0.266667        -3.439704     3.973037
      1      6         -3.433333        -7.139704     0.273037
      1      7         -5.300000        -9.006371    -1.593629
      1      8         -3.566667        -7.273037     0.139704
      1      9         -7.100000       -10.806371    -3.393629
      1     10         -8.900000       -12.606371    -5.193629
      1     11        -10.733333       -14.439704    -7.026963
      1     12         -8.966667       -12.673037    -5.260296
      2      3         -0.466667        -4.173037     3.239704
      2      4          1.633333        -2.073037     5.339704
      2      5          3.500000        -0.206371     7.206371
      2      6         -0.200000        -3.906371     3.506371
      2      7         -2.066667        -5.773037     1.639704
      2      8         -0.333333        -4.039704     3.373037
      2      9         -3.866667        -7.573037    -0.160296
      2     10         -5.666667        -9.373037    -1.960296
      2     11         -7.500000       -11.206371    -3.793629
      2     12         -5.733333        -9.439704    -2.026963
      3      4          2.100000        -1.606371     5.806371
      3      5          3.966667         0.260296     7.673037
      3      6          0.266667        -3.439704     3.973037
```

```
tomato = read.csv("Ch13-tomato.csv")
tomato$Density = factor(tomato$Density)
m = lm(Yield~Variety*Density, tomato)
anova(m)


Analysis of Variance Table

Response: Yield
                Df Sum Sq Mean Sq  F value    Pr(>F)
Variety          2 327.60 163.799 103.3430 1.608e-12 ***
Density          3  86.69  28.896  18.2306 2.212e-06 ***
Variety:Density  6   8.03   1.339   0.8445    0.5484
Residuals       24  38.04   1.585
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
library(lsmeans)
lsmeans(m, pairwise~Variety)


$lsmeans
 Variety   lsmean        SE df lower.CL upper.CL
 A       11.33333 0.3634327 24 10.58325 12.08342
 B       12.20833 0.3634327 24 11.45825 12.95842
 C       18.12500 0.3634327 24 17.37491 18.87509

Results are averaged over the levels of: Density
Confidence level used: 0.95

$contrasts
 contrast  estimate        SE df t.ratio p.value
 A - B    -0.875000 0.5139715 24  -1.702  0.2249
 A - C    -6.791667 0.5139715 24 -13.214  <.0001
 B - C    -5.916667 0.5139715 24 -11.512  <.0001

Results are averaged over the levels of: Density
P value adjustment: tukey method for a family of 3 means
```

```
lsmeans(m, pairwise~Density)


$lsmeans
 Density   lsmean        SE df lower.CL upper.CL
 10      11.47778 0.4196559 24 10.61165 12.34391
 20      14.38889 0.4196559 24 13.52276 15.25502
 30      15.77778 0.4196559 24 14.91165 16.64391
 40      13.91111 0.4196559 24 13.04498 14.77724

Results are averaged over the levels of: Variety
Confidence level used: 0.95

$contrasts
 contrast   estimate        SE df t.ratio p.value
 10 - 20  -2.9111111 0.5934831 24  -4.905  0.0003
 10 - 30  -4.3000000 0.5934831 24  -7.245  <.0001
 10 - 40  -2.4333333 0.5934831 24  -4.100  0.0022
 20 - 30  -1.3888889 0.5934831 24  -2.340  0.1169
 20 - 40   0.4777778 0.5934831 24   0.805  0.8514
 30 - 40   1.8666667 0.5934831 24   3.145  0.0213

Results are averaged over the levels of: Variety
P value adjustment: tukey method for a family of 4 means
```

```
lsmeans(m, pairwise~Variety*Density)


$lsmeans
 Variety Density    lsmean        SE df  lower.CL upper.CL
 A       10       9.200000 0.7268654 24  7.699824 10.70018
 B       10       8.933333 0.7268654 24  7.433157 10.43351
 C       10      16.300000 0.7268654 24 14.799824 17.80018
 A       20      12.433333 0.7268654 24 10.933157 13.93351
 B       20      12.633333 0.7268654 24 11.133157 14.13351
 C       20      18.100000 0.7268654 24 16.599824 19.60018
 A       30      12.900000 0.7268654 24 11.399824 14.40018
 B       30      14.500000 0.7268654 24 12.999824 16.00018
 C       30      19.933333 0.7268654 24 18.433157 21.43351
 A       40      10.800000 0.7268654 24  9.299824 12.30018
 B       40      12.766667 0.7268654 24 11.266490 14.26684
 C       40      18.166667 0.7268654 24 16.666490 19.66684

Confidence level used: 0.95

$contrasts
 contrast        estimate       SE df t.ratio p.value
 A,10 - B,10   0.26666667 1.027943 24   0.259  1.0000
 A,10 - C,10  -7.10000000 1.027943 24  -6.907  <.0001
 A,10 - A,20  -3.23333333 1.027943 24  -3.145  0.1284
 A,10 - B,20  -3.43333333 1.027943 24  -3.340  0.0873
 A,10 - C,20  -8.90000000 1.027943 24  -8.658  <.0001
 A,10 - A,30  -3.70000000 1.027943 24  -3.599  0.0507
 A,10 - B,30  -5.30000000 1.027943 24  -5.156  0.0013
 A,10 - C,30 -10.73333333 1.027943 24 -10.442  <.0001
 A,10 - A,40  -1.60000000 1.027943 24  -1.557  0.9085
 A,10 - B,40  -3.56666667 1.027943 24  -3.470  0.0668
 A,10 - C,40  -8.96666667 1.027943 24  -8.723  <.0001
 B,10 - C,10  -7.36666667 1.027943 24  -7.166  <.0001
 B,10 - A,20  -3.50000000 1.027943 24  -3.405  0.0764
 B,10 - B,20  -3.70000000 1.027943 24  -3.599  0.0507
```

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- Check model assumptions
- Consider alternative models, e.g. treating density as continuous