STAT 401A - Statistical Methods for Research Workers

One-way ANOVA (contrasts and multiple comparisons)

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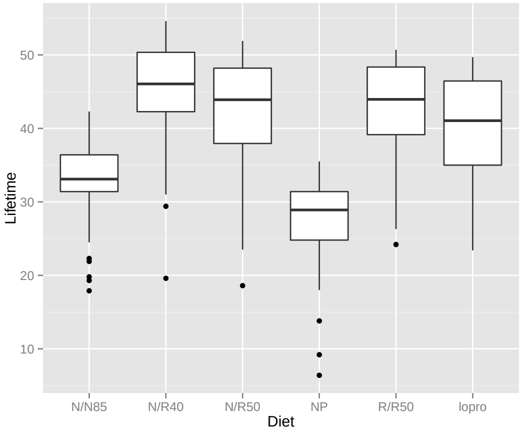
Mice lifetimes



Simple hypothesis

Consider the one-way ANOVA model: $Y_{ij} \sim N(\mu_j, \sigma^2)$ where $j = 1, \ldots, J$.

Here are a few simple alternative hypotheses:

1. Mean lifetimes for N/R50 and R/R50 diet are different.
2. Mean lifetimes for N/R40 is different than for N/R50 and R/R50 combined.
3. Mean lifetimes for high calorie (NP and N/N85) diets is different than for low calorie diets combined.

$H_0 : \gamma = 0 \qquad H_1 : \gamma \neq 0 :$

$$
\begin{aligned}
\gamma_1 &= \mu_{R/R50} - \mu_{N/R50} \\
\gamma_2 &= \mu_{N/R40} - \frac{1}{2}(\mu_{N/R50} + \mu_{R/R50}) \\
\gamma_3 &= \frac{1}{4}(\mu_{N/R50} + \mu_{R/R50} + \mu_{N/R40} + \mu_{lopro}) - \frac{1}{2}(\mu_{NP} + \mu_{N/N85})
\end{aligned}
$$

Contrasts

Definition

A linear combination of group means has the form

$$\gamma = C_1\mu_1 + C_2\mu_2 + \ldots + C_J\mu_J$$

where C_j are known coefficients and μ_j are the unknown population means.

Definition

A linear combination with $C_1 + C_2 + \cdots + C_J = 0$ is a contrast.

Remark Contrast interpretation is usually best if $|C_1| + |C_2| + \cdots + |C_J| = 2$, i.e. the positive sum to 1 and the negative coefficients sum to -1.

Inference on contrasts

$$\gamma = C_1\mu_1 + C_2\mu_2 + \cdots + C_J\mu_J$$

Estimated by

$$g = C_1\overline{Y}_1 + C_2\overline{Y}_2 + \cdots + C_J\overline{Y}_J$$

with standard error

$$SE(g) = s_p\sqrt{\frac{C_1^2}{n_1} + \frac{C_2^2}{n_2} + \cdots + \frac{C_J^2}{n_J}}$$

t-statistic (compare to t_{n-J}) and CI:

$$t = \frac{g}{SE(g)} \qquad g \pm t_{n-J}(1-\alpha/2)SE(g)$$

Contrasts for mice lifetime dataset

For these contrasts:

1. Mean lifetimes for N/R50 and R/R50 diet are different.
2. Mean lifetimes for N/R40 is different than for N/R50 and R/R50 combined.
3. Mean lifetimes for high calorie (NP and N/N85) diets is different than for low calorie diets combined.

$H_0 : \gamma = 0 \qquad H_1 : \gamma \neq 0 :$

$$\begin{aligned}
\gamma_1 &= \mu_{R/R50} - \mu_{N/R50} \\
\gamma_2 &= \mu_{N/R40} - \tfrac{1}{2}(\mu_{N/R50} + \mu_{R/R50}) \\
\gamma_3 &= \tfrac{1}{4}(\mu_{N/R50} + \mu_{R/R50} + \mu_{N/R40} + \mu_{lopro}) - \tfrac{1}{2}(\mu_{NP} + \mu_{N/N85})
\end{aligned}$$

	N/N85	N/R40	N/R50	NP	R/R50	lopro
early rest - none @ 50kcal	0.00	0.00	-1.00	0.00	1.00	0.00
40kcal/week - 50kcal/week	0.00	1.00	-0.50	0.00	-0.50	0.00
lo cal - hi cal	-0.50	0.25	0.25	-0.50	0.25	0.25

Mice liftime examples

	Diet	n	mean	sd
1	N/N85	57	32.69	5.13
2	N/R40	60	45.12	6.70
3	N/R50	71	42.30	7.77
4	NP	49	27.40	6.13
5	R/R50	56	42.89	6.68
6	lopro	56	39.69	6.99

Contrasts:

	g	SE(g)	t	p	L	U
early rest - none @ 50kcal	0.59	1.19	0.49	0.62	-1.76	2.94
40kcal/week - 50kcal/week	2.53	1.05	2.41	0.02	0.46	4.59
lo cal - hi cal	12.45	0.78	15.96	0.00	10.92	13.98

```
DATA case0501;
  INFILE 'case0501.csv' DSD FIRSTOBS=2;
  INPUT lifetime diet $ ;

PROC MEANS DATA=case0501;
  CLASS diet;
  VAR lifetime;
  RUN;
```

```
                                   The MEANS Procedure
                              Analysis Variable : lifetime

              N
 diet       Obs      N          Mean       Std Dev       Minimum       Maximum
 ------------------------------------------------------------------------------
 N/N85       57     57    32.6912281     5.1252972    17.9000000    42.3000000
 N/R40       60     60    45.1166667     6.7034058    19.6000000    54.6000000
 N/R50       71     71    42.2971831     7.7681947    18.6000000    51.9000000
 NP          49     49    27.4020408     6.1337010     6.4000000    35.5000000
 R/R50       56     56    42.8857143     6.6831519    24.2000000    50.7000000
 lopro       56     56    39.6857143     6.9916945    23.4000000    49.7000000
 ------------------------------------------------------------------------------
```

```
PROC GLM;
  CLASS diet;
  MODEL lifetime = diet / CLPARM;
  ESTIMATE 'early rest - none @ 50kcal' diet  0  1 -1  0  0  0 ;
  ESTIMATE '40kcal/week - 50kcal/week'  diet  0  2 -1  0 -1  0 / DIVISOR = 2;
  ESTIMATE 'lo cal - hi cal'            diet -2  1  1 -2  1  1 / DIVISOR = 4 ;
  RUN;
  QUIT;
```

```
                                      The GLM Procedure

                                              Sum of
     Source                      DF          Squares     Mean Square    F Value    Pr > F
     Model                        5      12733.94181      2546.78836      57.10    <.0001
     Error                      343      15297.41532        44.59888
     Corrected Total            348      28031.35713

                                                          Standard
      Parameter                            Estimate          Error    t Value    Pr > |t|
      early rest - none @ 50kcal          0.5885312     1.19355007       0.49      0.6223
      40kcal/week - 50kcal/week           2.5252180     1.04854904       2.41      0.0166
      lo cal - hi cal                    12.4496851     0.78001425      15.96      <.0001

                 Parameter                          95% Confidence Limits
                 early rest - none @ 50kcal        -1.7590676    2.9361299
                 40kcal/week - 50kcal/week          0.4628224    4.5876136
                 lo cal - hi cal                   10.9154718   13.9838985
```

```
library(multcomp)
m = lm(Lifetime~Diet-1, case0501) # The -1 indicates no intercept (see Ch 7)
summary(m)


Call:
lm(formula = Lifetime ~ Diet - 1, data = case0501)

Residuals:
    Min      1Q  Median      3Q     Max
-25.517  -3.386   0.814   5.183  10.014

Coefficients:
          Estimate Std. Error t value Pr(>|t|)
DietN/N85   32.691      0.885    37.0   <2e-16 ***
DietN/R40   45.117      0.862    52.3   <2e-16 ***
DietN/R50   42.297      0.793    53.4   <2e-16 ***
DietNP      27.402      0.954    28.7   <2e-16 ***
DietR/R50   42.886      0.892    48.1   <2e-16 ***
Dietlopro   39.686      0.892    44.5   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.68 on 343 degrees of freedom
Multiple R-squared:  0.972, Adjusted R-squared:  0.972
F-statistic: 2.01e+03 on 6 and 343 DF,  p-value: <2e-16


K


                           N/N85 N/R40 N/R50   NP R/R50 lopro
early rest - none @ 50kcal   0.0  0.00 -1.00  0.0  1.00  0.00
40kcal/week - 50kcal/week    0.0  1.00 -0.50  0.0 -0.50  0.00
lo cal - hi cal             -0.5  0.25  0.25 -0.5  0.25  0.25
```

```
t = glht(m, linfct=K)
summary(t)



 Simultaneous Tests for General Linear Hypotheses

Fit: lm(formula = Lifetime ~ Diet - 1, data = case0501)

Linear Hypotheses:
                                Estimate Std. Error t value Pr(>|t|)
early rest - none @ 50kcal == 0    0.589      1.194    0.49    0.946
40kcal/week - 50kcal/week == 0     2.525      1.049    2.41    0.049 *
lo cal - hi cal == 0              12.450      0.780   15.96   <1e-04 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Adjusted p values reported -- single-step method)


confint(t, calpha=univariate_calpha())



 Simultaneous Confidence Intervals

Fit: lm(formula = Lifetime ~ Diet - 1, data = case0501)

Quantile = 1.967
95% confidence level


Linear Hypotheses:
                                Estimate lwr    upr
early rest - none @ 50kcal == 0  0.589   -1.759  2.936
40kcal/week - 50kcal/week == 0   2.525    0.463  4.588
lo cal - hi cal == 0            12.450   10.915 13.984
```

Summary

- Contrasts are linear combinations that sum to zero
- t-test tools are used to calculate pvalues and confidence intervals