

R08 - Experimental design

STAT 401 (Engineering) - Iowa State University

April 11, 2018

Random samples and random treatment assignment

Recall that the objective of data analysis is often to make an inference about a population based on a sample. For the inference to be statistically valid, we need a **random** sample from the population.

Often we also want to make a **causal** statement about the relationship between explanatory variables (X) and a response (Y). In order to make a **causal** statement, the levels of the explanatory variables need to be **randomly** assigned to the **experimental units**. If levels are randomly assigned, we often refer to the explanatory variables as **treatments** and refer to the data collection as a **randomized experiment**. If the levels are not (randomly) assigned, we refer to the data collection as an **observational study**.

Data collection

Sample	Treatment randomly assigned?	
	No Observational study	Yes Randomized experiment
Not random	No cause-and-effect No inference to population	Yes cause-and-effect No inference to population
Random	No cause-and-effect Yes inference to population	Yes cause-and-effect Yes inference to population

Strength of wood glue

You are interested in testing two different wood glues:

- Gorilla Wood Glue
- Titebond 1413 Wood Glue

On a scarf joint:



So you collect up some wood, glue the pieces together, and determine the force required to break the joint. (There are lots of details missing here.)

Inspiration: https://woodgears.ca/joint_strength/glue.html

Completely Randomized Design (CRD)

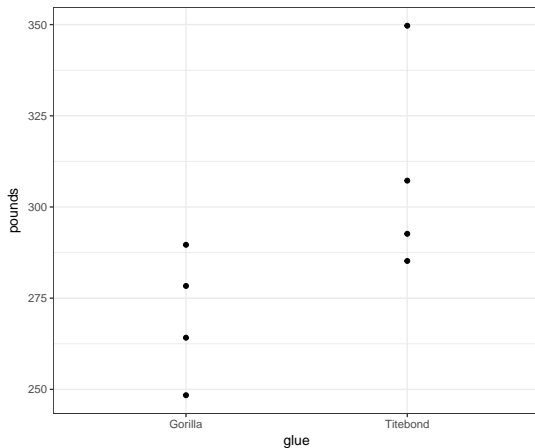
Suppose I have 8 pieces of wood laying around. I cut each piece and **randomly** use either Gorilla or Titebond glue to recombine the pieces. I do the randomization in such a way that I have exactly 4 Gorilla and 4 Titebond results, e.g.

```
# A tibble: 8 x 2
  woodID glue
<fctr> <chr>
1 wood1 Gorilla
2 wood2 Titebond
3 wood3 Gorilla
4 wood4 Titebond
5 wood5 Titebond
6 wood6 Titebond
7 wood7 Gorilla
8 wood8 Gorilla
```

This is called a **completely randomized design (CRD)**.

Visualize the data

```
ggplot(d, aes(glue, pounds)) + geom_point() + theme_bw()
```



Model

Let

- F_w be the force needed to break wood w ,
- T_w be an indicator that the Titebond glue was used on wood w , i.e.

$$T_w = I(\text{glue}_w = \text{Titebond}).$$

Then a regression model for these data is

$$F_w \stackrel{\text{ind}}{\sim} N(\beta_0 + \beta_1 T_w, \sigma^2)$$

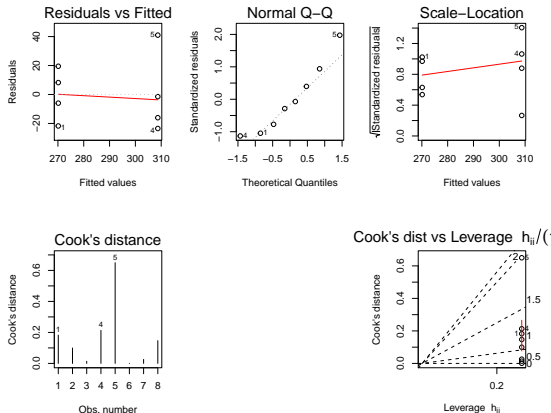
where

- β_1 is the expected difference in force when using Titebond glue compared to using Gorilla glue.

Check model assumptions

```
m <- lm(pounds ~ glue, data = d)
opar = par(mfrow=c(2,3)); plot(m, 1:6, ask=FALSE); par(opar)
```

*hat values (leverages) are all = 0.25
and there are no factor predictors; no plot no. 5*



Obtain statistics

```
coefficients(m)
```

```
(Intercept)  glueTitebond
      270.13553      38.55651
```

```
summary(m)$r.squared
```

```
[1] 0.4630249
```

```
confint(m)
```

```

                2.5 %    97.5 %
(Intercept)  240.806326 299.46474
glueTitebond  -2.921249  80.03428
```

```
emmeans(m, ~glue)
```

glue	emmean	SE	df	lower.CL	upper.CL
Gorilla	270.1355	11.98621	6	240.8063	299.4647
Titebond	308.6920	11.98621	6	279.3628	338.0213

```
Confidence level used: 0.95
```

Interpret results

The mean break pressure (pounds) was 270 (241,299) for Gorilla and 309 (279, 338) for Titebond. Moving from Gorilla to Titebond glue caused an increase in break pressure of 39 (-3,80). Glue type accounted for 46% of the variability in in break pressure.

Randomized complete block design (RCBD)

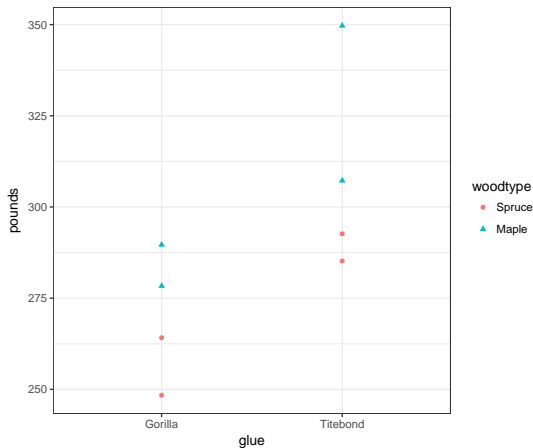
Suppose the wood actually came from two different types: Maple and Spruce. And perhaps you have reason to believe the glue will work differently depending on the type of wood. In this case, you would want to **block** by wood type and perform the randomization within each block, i.e.

```
# A tibble: 8 x 3
  woodID woodtype glue
  <fctr> <fctr>   <chr>
1 wood1  Spruce    Gorilla
2 wood2  Spruce    Titebond
3 wood3  Spruce    Gorilla
4 wood4  Spruce    Titebond
5 wood5  Maple     Titebond
6 wood6  Maple     Titebond
7 wood7  Maple     Gorilla
8 wood8  Maple     Gorilla
```

This is called a **randomized complete block design (RCBD)**.

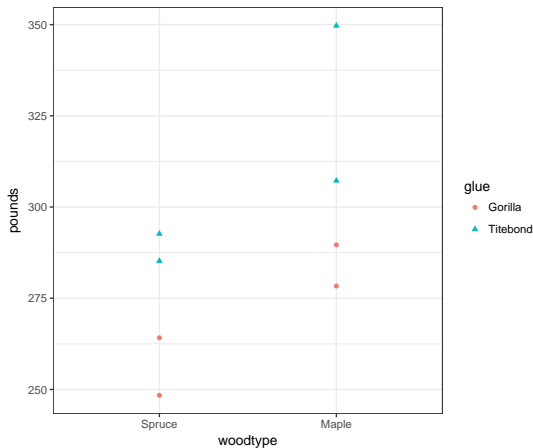
Visualize the data

```
ggplot(d, aes(glue, pounds, color=woodtype, shape=woodtype)) + geom_point() + theme_bw()
```



Visualize the data - a more direct comparison

```
ggplot(d, aes(woodtype, pounds, color=glue, shape=glue)) + geom_point() + theme_bw()
```



Main effects model

Let

- F_w be the force needed to break wood w
- T_w be an indicator that Titebond glue was used on wood w , and
- M_w be an indicator that wood w was Maple.

Then a regression model for these data is

$$F_w \stackrel{ind}{\sim} N(\beta_0 + \beta_1 T_w + \beta_2 M_w, \sigma^2)$$

where

- β_1 is the expected difference in force when using Titebond glue compared to using Gorilla glue when adjusted for type of wood, i.e. the type of wood is held constant, and
- β_2 is the expected difference in force when using Spruce compared to Maple when adjusted for type of glue, i.e. the glue is held constant.

Perform analysis

```
m <- lm(pounds ~ woodtype + glue, data = d)
summary(m)
```

```
Call:
lm(formula = pounds ~ woodtype + glue, data = d)
```

```
Residuals:
    1      2      3      4      5      6      7      8
-4.929  0.768 10.835 -6.674 24.186 -18.279 -8.594  2.688
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    253.324      9.435   26.848 1.34e-06 ***
woodtypeMaple    33.623     10.895    3.086  0.0273 *
glueTitebond     38.557     10.895    3.539  0.0166 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 15.41 on 5 degrees of freedom
Multiple R-squared:  0.8151, Adjusted R-squared:  0.7412
F-statistic: 11.02 on 2 and 5 DF, p-value: 0.01469
```

```
confint(m)
```

```
              2.5 %    97.5 %
(Intercept) 229.069570 277.57817
woodtypeMaple  5.616873 61.62978
glueTitebond  10.550061 66.56297
```

Replication

Since there are more than one observation for each woodtype-glue combination, the design is **replicated**:

```
d %>% group_by(woodtype, glue) %>% summarize(n = n())
```

```
# A tibble: 4 x 3
# Groups:   woodtype [?]
  woodtype glue      n
  <fctr>   <chr>  <int>
1 Spruce  Gorilla    2
2 Spruce  Titebond   2
3 Maple   Gorilla    2
4 Maple   Titebond   2
```

When the design is replicated, we can consider assessing an interaction. In this example, an interaction between glue and woodtype would indicate that the effect of glue depends on the woodtype, i.e. the difference in expected force between the two glues depends on woodtype. At an extreme, it could be that Gorilla works better on Spruce and Titebond works better on Maple.

Interaction model

Let

- F_w be the force needed to break wood w
- T_w be an indicator that Titebond glue was used on wood w , and
- M_w be an indicator that wood w was Maple.

Then a regression model for these data is

$$F_w \stackrel{ind}{\sim} N(\beta_0 + \beta_1 T_w + \beta_2 M_w + \beta_3 T_w M_w, \sigma^2)$$

where

- β_1 is the expected difference in force when moving from Gorilla to Titebond glue for Spruce,
- β_2 is the expected difference in force when moving from Spruce to Maple for Gorilla glue, and
- β_3 is more complicated.

Assessing an interaction using a t-test

```
m <- lm(pounds ~ woodtype * glue, data = d)
summary(m)
```

```
Call:
lm(formula = pounds ~ woodtype * glue, data = d)
```

Residuals:

1	2	3	4	5	6	7	8
-7.882	3.721	7.882	-3.721	21.233	-21.233	-5.641	5.641

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	256.28	11.82	21.686	2.67e-05 ***
woodtypeMaple	27.72	16.71	1.658	0.173
glueTitebond	32.65	16.71	1.954	0.122
woodtypeMaple:glueTitebond	11.81	23.64	0.500	0.643

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 16.71 on 4 degrees of freedom

Multiple R-squared: 0.826, Adjusted R-squared: 0.6955

F-statistic: 6.33 on 3 and 4 DF, p-value: 0.05335

Assessing an interaction using an F-test

```
anova(m)
```

Analysis of Variance Table

Response: pounds

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
woodtype	1	2261.06	2261.06	8.0952	0.04662 *
glue	1	2973.21	2973.21	10.6449	0.03100 *
woodtype:glue	1	69.77	69.77	0.2498	0.64346
Residuals	4	1117.24	279.31		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
drop1(m, test='F')
```

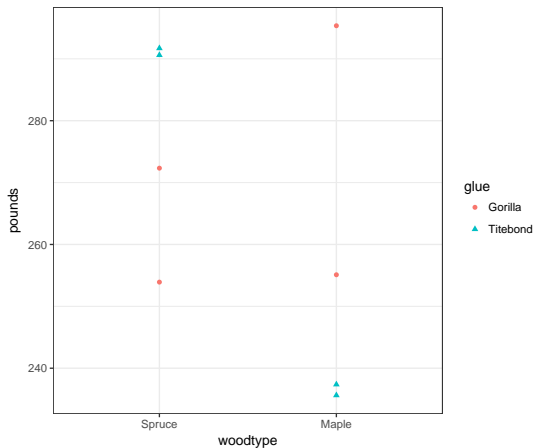
Single term deletions

```
Model:
pounds ~ woodtype * glue
```

	Df	Sum of Sq	RSS	AIC	F value	Pr(>F)
<none>			1117.2	47.513		
woodtype:glue	1	69.769	1187.0	45.998	0.2498	0.6435

What if this had been your data?

```
ggplot(d, aes(woodtype, pounds, color=glue, shape=glue)) + geom_point() + theme_bw()
```



Assessing an interaction using a t-test

```
m <- lm(pounds ~ woodtype * glue, data = d)
summary(m)
```

```
Call:
lm(formula = pounds ~ woodtype * glue, data = d)
```

Residuals:

1	2	3	4	5	6	7	8
-9.2083	-0.5529	0.5529	9.2083	-0.8764	20.1215	-20.1215	0.8764

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	263.12	11.08	23.755	1.86e-05	***
woodtypeMaple	12.10	15.66	0.773	0.4829	
glueTitebond	28.03	15.66	1.790	0.1480	
woodtypeMaple:glueTitebond	-66.76	22.15	-3.014	0.0394	*

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.66 on 4 degrees of freedom

Multiple R-squared: 0.7648, Adjusted R-squared: 0.5883

F-statistic: 4.335 on 3 and 4 DF, p-value: 0.09522

Unreplicated study

Suppose you now have

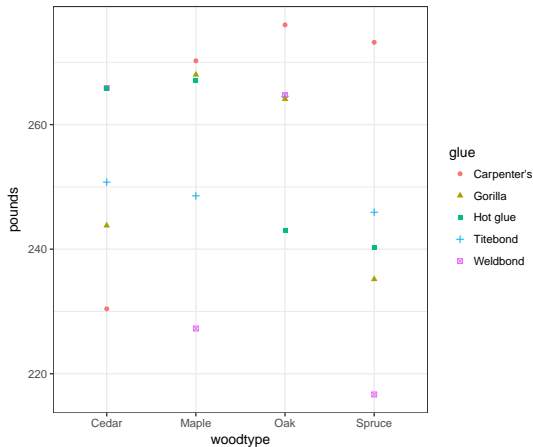
- 5 glue choices
- 4 different types of wood with
- 5 samples of each type of wood.

Thus you can only run each glue choice once on each type of wood.

Then you can run an unreplicated RCBD.

Visualize

```
ggplot(d, aes(woodtype, pounds, color=glue, shape=glue)) +  
  geom_point() + theme_bw()
```



Fit the main effects (or additive) model

```
m <- lm(pounds ~ woodtype + glue, data = d)
anova(m)
```

Analysis of Variance Table

Response: pounds

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
woodtype	3	1091.4	363.80	1.1474	0.3697
glue	4	714.8	178.71	0.5636	0.6937
Residuals	12	3804.9	317.07		

Fit the main effects (or additive) model

```
summary(m)
```

```
Call:
```

```
lm(formula = pounds ~ woodtype + glue, data = d)
```

```
Residuals:
```

	Min	1Q	Median	3Q	Max
	-30.302	-7.093	2.316	10.326	23.992

```
Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	260.717	11.262	23.150	2.51e-11 ***
woodtypeMaple	4.907	11.262	0.436	0.671
woodtypeOak	11.157	11.262	0.991	0.341
woodtypeSpruce	-9.056	11.262	-0.804	0.437
glueGorilla	-9.696	12.591	-0.770	0.456
glueHot glue	-8.460	12.591	-0.672	0.514
glueTitebond	-10.018	12.591	-0.796	0.442
glueWeldbond	-18.834	12.591	-1.496	0.161

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 17.81 on 12 degrees of freedom
```

```
Multiple R-squared:  0.3219, Adjusted R-squared:  -0.07366
```

```
F-statistic: 0.8138 on 7 and 12 DF,  p-value: 0.5931
```

Fit the full (with interaction) model

```
m <- lm(pounds ~ woodtype * glue, data = d)
anova(m)
```

Warning in anova.lm(m): ANOVA F-tests on an essentially perfect fit are unreliable

Analysis of Variance Table

Response: pounds

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
woodtype	3	1091.4	363.80		
glue	4	714.8	178.71		
woodtype:glue	12	3804.9	317.07		
Residuals	0	0.0			

Fit the full (with interaction) model

```
summary(m)
```

Call:

```
lm(formula = pounds ~ woodtype * glue, data = d)
```

Residuals:

ALL 20 residuals are 0: no residual degrees of freedom!

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	230.41	NA	NA	NA
woodtypeMaple	39.83	NA	NA	NA
woodtypeOak	45.59	NA	NA	NA
woodtypeSpruce	42.80	NA	NA	NA
glueGorilla	13.38	NA	NA	NA
glueHot glue	35.32	NA	NA	NA
glueTitebond	20.35	NA	NA	NA
glueWeldbond	35.46	NA	NA	NA
woodtypeMaple:glueGorilla	-15.61	NA	NA	NA
woodtypeOak:glueGorilla	-25.27	NA	NA	NA
woodtypeSpruce:glueGorilla	-51.41	NA	NA	NA
woodtypeMaple:glueHot glue	-38.52	NA	NA	NA
woodtypeOak:glueHot glue	-68.37	NA	NA	NA
woodtypeSpruce:glueHot glue	-68.22	NA	NA	NA
woodtypeMaple:glueTitebond	-42.03	NA	NA	NA
woodtypeOak:glueTitebond	-31.80	NA	NA	NA
woodtypeSpruce:glueTitebond	-47.64	NA	NA	NA
woodtypeMaple:glueWeldbond	-78.44	NA	NA	NA
woodtypeOak:glueWeldbond	-46.74	NA	NA	NA
woodtypeSpruce:glueWeldbond	-92.00	NA	NA	NA