STAT 401A - Statistical Methods for Research Workers

Multiple regression models

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Multiple regression

Recall the simple linear regression model is

$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 X_i, \sigma^2)$$

The multiple regression model is

$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 X_{i,1} + \cdots + \beta_p X_{i,p}, \sigma^2)$$

where

- Y_i is the response for observation i and
- $X_{i,p}$ is the p^{th} explanatory variable for observation i.

We may also write

$$Y_i \stackrel{ind}{\sim} N(\mu_i, \sigma^2) \quad \text{or} \quad Y_i = \mu_i + e_i, e_i \stackrel{iid}{\sim} N(0, \sigma^2)$$

where

$$\mu_i = \beta_0 + \beta_1 X_{i,1} + \cdots + \beta_p X_{i,p}.$$

Explanatory variables

There is a lot of flexibility in the mean

$$\mu_i = E[Y_i|X_{i,1}, \ldots, X_{i,p}] = \beta_0 + \beta_1 X_{i,1} + \cdots + \beta_p X_{i,p}$$

as there are many possibilities for the explanatory variables $X_{i,1}, \ldots, X_{i,p}$:

- Higher order terms (X^2)
- Additional explanatory variables (X_1 and X_2)
- Dummy/indicator variables for categorical variables ($X_1 = \mathrm{I}()$)
- Interactions ($X_1 X_2$)
    - Continuous-continuous
    - Continuous-categorical
    - Categorical-categorical

Interpretation

Model:

$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 X_{i,1} + \cdots + \beta_p X_{i,p}, \sigma^2)$$

The interpretation is

- β_0 is the expected value of the response Y_i when all explanatory variables are zero.
- β_p, $p \neq 0$ is the expected increase in the response for a one-unit increase in the p^{th} explanatory variable when all other explanatory variables are held constant.
- R^2 is the proportion of the variance in the response explained by the model

Higher order terms (X^2)

Let

- Y_i be the distance for the i^{th} run of the experiment and
- H_i be the height for the i^{th} run of the experiment.

Simple linear regression assumes

$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 H_i \qquad\qquad\qquad\qquad , \sigma^2)$$

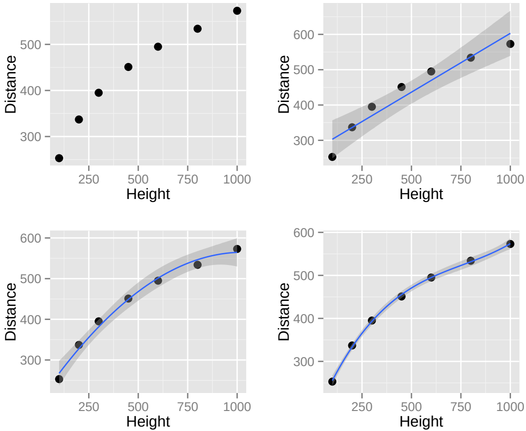
The quadratic multiple regression assumes

$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 H_i + \beta_2 H_i^2 \qquad\quad , \sigma^2)$$

The cubic multiple regression assumes

$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 H_i + \beta_2 H_i^2 + \beta_3 H_i^3, \sigma^2)$$

Case1001



SAS code and output

```
DATA case1001;
  INFILE 'case1001.csv' DSD FIRSTOBS=2;
  INPUT distance height;
  height2 = height*height;
  height3 = height*height2;

# PROC REG allows multiple MODEL statements
PROC REG DATA=case1001;
  MODEL distance = height;
  MODEL distance = height height2;
  MODEL distance = height height2 height3;
  RUN;
```

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	269.71246	24.31239	11.09	0.0001
height	1	0.33334	0.04203	7.93	0.0005
Intercept	1	199.91282	16.75945	11.93	0.0003
height	1	0.70832	0.07482	9.47	0.0007
height2	1	-0.00034369	0.00006678	-5.15	0.0068
Intercept	1	155.77551	8.32579	18.71	0.0003
height	1	1.11530	0.06567	16.98	0.0004
height2	1	-0.00124	0.00013842	-8.99	0.0029
height3	1	5.477104E-7	8.327329E-8	6.58	0.0072

SAS code and output

```
DATA case1001;
  INFILE 'case1001.csv' DSD FIRSTOBS=2;
  INPUT distance height;
  height2 = height ** 2;
  height3 = height ** 3;

PROC GLM DATA=case1001;
  MODEL distance = height height2 height3;
```

```
/* PROC GLM allows the variable construction within the MODEL statement
   and provides nicer output (not shown here) */
DATA case1001;
  INFILE 'case1001.csv' DSD FIRSTOBS=2;
  INPUT distance height;

/* This shorthand puts in H, H^2, and H^3 */
PROC GLM DATA=case1001;
  MODEL distance = height|height|height;

/* This only puts H^3 */
PROC GLM DATA=case1001;
  MODEL distance = height*height*height;
```

R code and output

```
# Construct the variables by hand
case1001$Height2 = case1001$Height^2
case1001$Height3 = case1001$Height^3

m1 = lm(Distance~Height,                 case1001)
m2 = lm(Distance~Height+Height2,         case1001)
m3 = lm(Distance~Height+Height2+Height3, case1001)

coefficients(m1)


(Intercept)      Height
   269.7125      0.3333


coefficients(m2)


(Intercept)      Height     Height2
  1.999e+02   7.083e-01  -3.437e-04


coefficients(m3)


(Intercept)      Height     Height2     Height3
  1.558e+02   1.115e+00  -1.245e-03   5.477e-07
```

R code and output

```
# Let R construct the variables for you
m = lm(Distance~poly(Height, 3, raw=TRUE), case1001)
summary(m)



Call:
lm(formula = Distance ~ poly(Height, 3, raw = TRUE), data = case1001)

Residuals:
      1       2       3       4       5       6       7
-2.4036  3.5809  1.8917 -4.4688 -0.0804  2.3216 -0.8414

Coefficients:
                              Estimate Std. Error t value Pr(>|t|)
(Intercept)                   1.56e+02   8.33e+00   18.71  0.00033 ***
poly(Height, 3, raw = TRUE)1  1.12e+00   6.57e-02   16.98  0.00044 ***
poly(Height, 3, raw = TRUE)2 -1.24e-03   1.38e-04   -8.99  0.00290 **
poly(Height, 3, raw = TRUE)3  5.48e-07   8.33e-08    6.58  0.00715 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.01 on 3 degrees of freedom
Multiple R-squared:  0.999, Adjusted R-squared:  0.999
F-statistic: 1.6e+03 on 3 and 3 DF,  p-value: 2.66e-05
```

Longnose Dace Abundance

From `http://udel.edu/~mcdonald/statmultreg.html`:

> *I extracted some data from the Maryland Biological Stream Survey. ... The dependent variable is the number of Longnose Dace (Rhinichthys cataractae) per 75-meter section of [a] stream. The independent variables are the area (in acres) drained by the stream; the dissolved oxygen (in mg/liter); the maximum depth (in cm) of the 75-meter segment of stream; nitrate concentration (mg/liter); sulfate concentration (mg/liter); and the water temperature on the sampling date (in degrees C).*

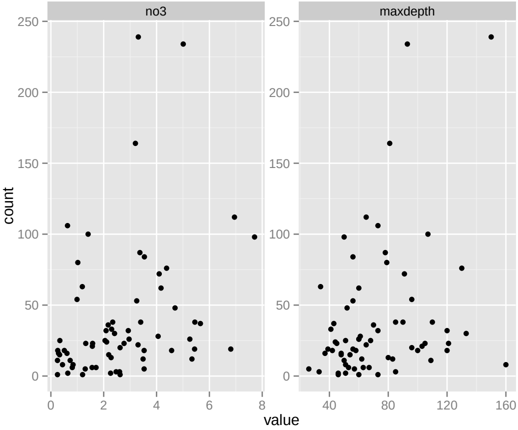
Consider the model

$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2}, \sigma^2)$$

where

- Y_i: count of Longnose Dace in stream i
- $X_{i,1}$: maximum depth (in cm) of stream i
- $X_{i,2}$: nitrate concentration (mg/liter) of stream i

Exploratory



```
DATA dace;
  INFILE 'Longnose Dace.csv' DSD FIRSTOBS=2;
  INPUT stream $ count acreage do2 maxdepth no3 so4 temp;

PROC REG DATA=dace;
  MODEL count = maxdepth no3;
  RUN;
```

```
                              The REG Procedure
                                Model: MODEL1
                          Dependent Variable: count

                    Number of Observations Read          67
                    Number of Observations Used          67


                             Analysis of Variance
                                    Sum of           Mean
      Source                DF     Squares         Square    F Value    Pr > F
      Model                  2       28930          14465       7.68    0.0010
      Error                 64      120503     1882.85220
      Corrected Total       66      149432

             Root MSE             43.39184    R-Square     0.1936
             Dependent Mean       39.10448    Adj R-Sq     0.1684
             Coeff Var           110.96388

                             Parameter Estimates
                           Parameter       Standard
         Variable     DF    Estimate          Error    t Value    Pr > |t|
         Intercept     1   -17.55503       15.95865      -1.10      0.2754
         maxdepth      1     0.48106        0.18111       2.66      0.0100
         no3           1     8.28473        2.95659       2.80      0.0067
```

R code and output

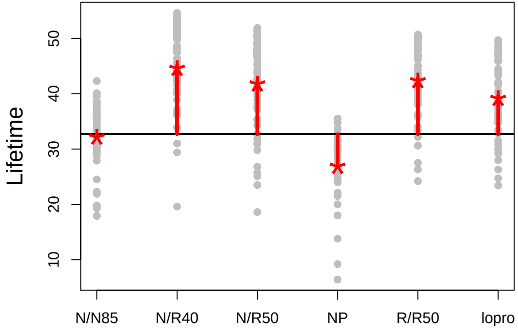
```
d = read.csv("longnosedace.csv")
m = lm(count~no3+maxdepth,d)
summary(m)
```

```
Call:
lm(formula = count ~ no3 + maxdepth, data = d)

Residuals:
   Min     1Q Median     3Q    Max
-55.06 -27.70  -8.68  11.79 165.31

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -17.555     15.959   -1.10   0.2754
no3            8.285      2.957    2.80   0.0067 **
maxdepth       0.481      0.181    2.66   0.0100 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 43.4 on 64 degrees of freedom
Multiple R-squared:  0.194, Adjusted R-squared:  0.168
F-statistic: 7.68 on 2 and 64 DF,  p-value: 0.00102
```

Interpretation

- Intercept (β_0): The expected count of Longnose Dace when maximum depth and nitrate concentration are both zero is -18.
- Coefficient for maxdepth (β_1): Holding nitrate concentration constant, each cm increase in maximum depth is associated with an additional 0.48 Longnose Dace counted on average.
- Coefficient for no3 (β_2): Holding maximum depth constant, each mg/liter increase in nitrate concentration is associated with an addition 8.3 Longnose Dace counted on average.
- Coefficient of determination (R^2): The model explains 19% of the variability in the count of Longnose Dace.

Using a categorical variable as an explanatory variable.



Regression with a categorical variable

- Choose one of the levels as the reference level, e.g. N/N85
- Construct dummy variables using indicator functions for the other levels, e.g.

$$
\begin{aligned}
X_{i,1} &= \mathrm{I}(\text{diet for observation } i \text{ is NP}) \\
X_{i,2} &= \mathrm{I}(\text{diet for observation } i \text{ is N/R50 lopro}) \\
X_{i,3} &= \mathrm{I}(\text{diet for observation } i \text{ is N/R50}) \\
X_{i,4} &= \mathrm{I}(\text{diet for observation } i \text{ is R/R50}) \\
X_{i,5} &= \mathrm{I}(\text{diet for observation } i \text{ is N/R40})
\end{aligned}
$$

- Run a multiple linear regression using these dummy variables.

An indicator function is

$$
\mathrm{I}(A) = \begin{cases} 1 & A \text{ is TRUE} \\ 0 & A \text{ is FALSE} \end{cases}
$$

```
DATA case0501;
  INFILE 'case0501.csv' DSD FIRSTOBS=2;
  INPUT lifetime diet $;
  IF diet ='NP'    THEN x1=1; ELSE x1=0;
  IF diet ='lopro' THEN x2=1; ELSE x2=0;
  IF diet ='N/R50' THEN x3=1; ELSE x3=0;
  IF diet ='R/R50' THEN x4=1; ELSE x4=0;
  IF diet ='N/R40' THEN x5=1; ELSE x5=0;
  RUN;

PROC GLM DATA=case0501;
  MODEL lifetime = x1 x2 x3 x4 x5;
  RUN;
  ods listing close;
```

```
                              The GLM Procedure

Dependent Variable: lifetime

                                      Sum of
       Source                DF      Squares     Mean Square    F Value    Pr > F
       Model                  5  12733.94181      2546.78836      57.10    <.0001
       Error                343  15297.41532        44.59888
       Corrected Total      348  28031.35713

               R-Square     Coeff Var      Root MSE    lifetime Mean
               0.454275      17.21323      6.678239         38.79713

                                            Standard
            Parameter        Estimate          Error    t Value    Pr > |t|
            Intercept     32.69122807     0.88455439      36.96      <.0001
            x1            -5.28918725     1.30100640      -4.07      <.0001
            x2             6.99448622     1.25652099       5.57      <.0001
            x3             9.60595503     1.18768248       8.09      <.0001
            x4            10.19448622     1.25652099       8.11      <.0001
            x5            12.42543860     1.23521298      10.06      <.0001
```

SAS code and output

```
DATA case0501;
  INFILE 'case0501.csv' DSD FIRSTOBS=2;
  INPUT lifetime diet $;

PROC GLM DATA=case0501;
  CLASS diet(REF='N/N85'); /* by default, SAS uses the alphabetically last group as the reference level */
  MODEL lifetime=diet / SOLUTION;
  RUN;
                                        The GLM Procedure

Dependent Variable: lifetime
                                               Sum of
       Source                      DF         Squares     Mean Square    F Value    Pr > F
       Model                        5     12733.94181      2546.78836      57.10    <.0001
       Error                      343     15297.41532        44.59888
       Corrected Total            348     28031.35713

                        R-Square     Coeff Var      Root MSE    lifetime Mean
                        0.454275      17.21323      6.678239         38.79713

                                                      Standard
           Parameter                Estimate             Error    t Value    Pr > |t|
           Intercept             32.69122807 B      0.88455439      36.96      <.0001
           diet      N/R40       12.42543860 B      1.23521298      10.06      <.0001
           diet      N/R50        9.60595503 B      1.18768248       8.09      <.0001
           diet      NP          -5.28918725 B      1.30100640      -4.07      <.0001
           diet      R/R50       10.19448622 B      1.25652099       8.11      <.0001
           diet      lopro        6.99448622 B      1.25652099       5.57      <.0001
           diet      N/N85        0.00000000 B       .                .           .

NOTE: The X'X matrix has been found to be singular, and a generalized inverse was used to solve the
```

R code and output

```
# by default, R uses the alphabetically first group as the reference level
case0501$Diet = relevel(case0501$Diet, ref='N/N85')

m = lm(Lifetime~Diet, case0501)
summary(m)



Call:
lm(formula = Lifetime ~ Diet, data = case0501)

Residuals:
    Min      1Q  Median      3Q     Max
-25.517  -3.386   0.814   5.183  10.014

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   32.691      0.885   36.96  < 2e-16 ***
DietN/R40     12.425      1.235   10.06  < 2e-16 ***
DietN/R50      9.606      1.188    8.09  1.1e-14 ***
DietNP        -5.289      1.301   -4.07  5.9e-05 ***
DietR/R50     10.194      1.257    8.11  8.9e-15 ***
Dietlopro      6.994      1.257    5.57  5.2e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.68 on 343 degrees of freedom
Multiple R-squared:  0.454, Adjusted R-squared:  0.446
F-statistic: 57.1 on 5 and 343 DF,  p-value: <2e-16
```

Interpretation

- $\beta_0 = E[Y_i|\text{reference level}]$, i.e. expected response for the reference level
  Note: the only way $X_{i,1} = \cdots = X_{i,p} = 0$ is if all indicators are zero, i.e. at the reference level.
- $\beta_p, p > 0$: expected change in the response moving from the reference level to the level associated with the p^{th} dummy variable
  Note: the only way for $X_{i,p}$ to increase by one and all other indicators to stay constant is if initially $X_{i,1} = \cdots = X_{i,p} = 0$ and now $X_{i,p} = 1$

For example,

- The expected lifetime for mice on the N/N85 diet is 37 weeks.
- The expected increase in lifetime for mice on the N/R40 diet compared to the N/N85 diet is 12.4 weeks.
- The model explains 45% of the variability in mice lifetimes.

Why an interaction?

> *Two explanatory variables are said to interact if the effect that one of them has on the mean response depends on the value of the other.*

For example,

- Longnose dace: The effect of no3 on longnose dace count depends on the maxdepth. (Continuous-continuous)
- Case1002: The effect of mass on energy depends on the species type. (Continuous-categorical)
- Yield: the effect of fertilizer depends on the block (Categorical-categorical)

Continuous-continuous interaction

For observation i, let

- Y_i be the response
- $X_{i,1}$ be the first explanatory variable and
- $X_{i,2}$ be the second explanatory variable.

The mean containing only main effects is

$$\mu_i = \beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2}.$$

The mean with the interaction is

$$\mu_i = \beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2} + \beta_3 X_{i,1} X_{i,2}.$$

Intepretation

- β_0: expected response when explanatory variables are zero
- β_1: expected change in the response for each change in $X_{i,1}$ when $X_{i,2}$ is zero
- β_2: expected change in the response for each change in $X_{i,2}$ when $X_{i,1}$ is zero
- $\beta_3 X_{i,2}$: expected change in the effect of $X_{i,1}$ on the response when $X_{i,2}$ is not zero
- $\beta_3 X_{i,1}$: expected change in the effect of $X_{i,2}$ on the response when $X_{i,1}$ is not zero

For example,

$$\begin{array}{rl}
E[Y_i|X_{i,1}=x_1+1, X_{i,2}=x_2] = & \beta_0+\beta_1(x_1+1)+\beta_2 x_2+\beta_3(x_1+1)x_2 \\
E[Y_i|X_{i,1}=x_1, X_{i,2}=x_2] = & \beta_0+\beta_1\ x_1+\beta_2 x_2+\beta_3\ x_1x_2 \\
\hline
= & \beta_1 +\beta_3 x_2
\end{array}$$

R code and output

```
d = read.csv("longnosedace.csv")
m = lm(count ~ maxdepth + no3 + maxdepth:no3, d)
summary(m)



Call:
lm(formula = count ~ maxdepth + no3 + maxdepth:no3, data = d)

Residuals:
   Min     1Q Median     3Q    Max 
-65.11 -21.40  -9.56   5.95 151.07 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)  
(Intercept)  13.32104   23.45571    0.57    0.572  
maxdepth     -0.00934    0.32918   -0.03    0.977  
no3          -4.64627    7.85693   -0.59    0.556  
maxdepth:no3  0.20122    0.11358    1.77    0.081 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 42.7 on 63 degrees of freedom
Multiple R-squared:  0.232, Adjusted R-squared:  0.195 
F-statistic: 6.34 on 3 and 63 DF,  p-value: 0.000797
```

Continuous-categorical interaction

Let category A be the reference level. Then observation i, let

- Y_i be the response
- $X_{i,1}$ be the continuous explanatory variable,
- B_i be a dummy variable for category B, and
- C_i be a dummy variable for category C.

The mean containing only main effects is

$$\mu_i = \beta_0 + \beta_1 X_{i,1} + \beta_2 B_i + \beta_3 C_i.$$

The mean with the interaction is

$$\mu_i = \beta_0 + \beta_1 X_{i,1} + \beta_2 B_i + \beta_3 C_i + \beta_4 X_{i,1} B_i + \beta_5 X_{i,1} C_i.$$

Interpretation for the main effect model

The mean containing only main effects is

$$\mu_i = \beta_0 + \beta_1 X_{i,1} + \beta_2 B_i + \beta_3 C_i.$$

- β_1 is the change in the response when the continuous explanatory variable increases by one for any category
- when the continuous explanatory variable is zero
  - β_0 is the expected response for category A
  - $\beta_0 + \beta_2$ is the expected response for category B
  - $\beta_0 + \beta_3$ is the expected response for category C

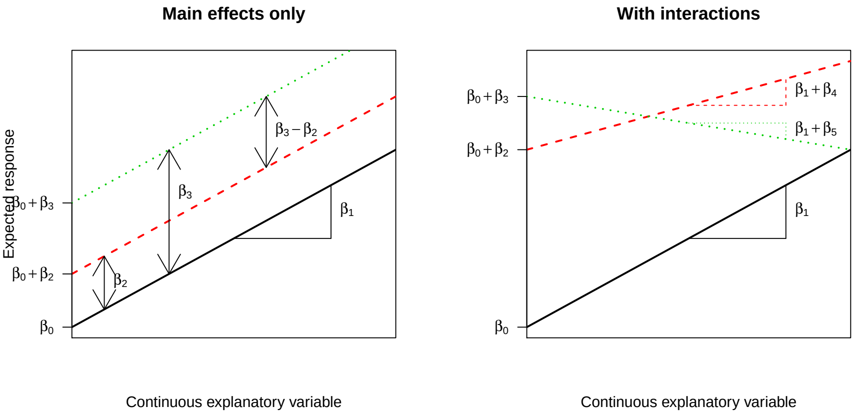
Interpretation for the model with an interaction

The model with an interaction is

$$\mu_i = \beta_0 + \beta_1 X_{i,1} + \beta_2 B_i + \beta_3 C_i + \beta_4 X_{i,1} B_i + \beta_5 X_{i,1} C_i$$

- β_0 is the expected response when the continuous explanatory variable is zero and we are in category A
- β_1 is the change in the response when the continuous explanatory variable increases by one for category A
- When the continuous explanatory variable ($X_{i,1}$) is zero,
  - β_2 is the change in the expected response when moving from category A to category B
  - β_3 is the change in the expected response when moving from category A to category C
- $\beta_1 + \beta_4$ is the change in the response when the continuous explanatory variable increases by one for category B
- $\beta_1 + \beta_5$ is the change in the response when the continuous explanatory variable increases by one for category C

Visualizing the models



R code and output - Main effects only

```
summary(lm(log(Energy)~log(Mass)+Type, case1002))
```

```

Call:
lm(formula = log(Energy) ~ log(Mass) + Type, data = case1002)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.2322 -0.1220 -0.0364  0.1257  0.3446 

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)    
(Intercept)                 -1.4977     0.1499   -9.99  2.8e-08 ***
log(Mass)                    0.8150     0.0445   18.30  3.8e-12 ***
Typenon-echolocating bats   -0.0787     0.2027   -0.39     0.70    
Typenon-echolocating birds   0.0236     0.1576    0.15     0.88    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.186 on 16 degrees of freedom
Multiple R-squared:  0.982, Adjusted R-squared:  0.978 
F-statistic:  284 on 3 and 16 DF,  p-value: 4.46e-14
```

R code and output - with interaction

```
summary(lm(log(Energy)~log(Mass)*Type, case1002))
```

```
Call:
lm(formula = log(Energy) ~ log(Mass) * Type, data = case1002)

Residuals:
    Min      1Q  Median      3Q     Max
-0.2515 -0.1264 -0.0095  0.0812  0.3284

Coefficients:
                                     Estimate Std. Error t value Pr(>|t|)
(Intercept)                           -1.4705     0.2477   -5.94  3.6e-05 ***
log(Mass)                              0.8047     0.0867    9.28  2.3e-07 ***
Typenon-echolocating bats              1.2681     1.2854    0.99     0.34
Typenon-echolocating birds            -0.1103     0.3847   -0.29     0.78
log(Mass):Typenon-echolocating bats   -0.2149     0.2236   -0.96     0.35
log(Mass):Typenon-echolocating birds   0.0307     0.1028    0.30     0.77
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.19 on 14 degrees of freedom
Multiple R-squared:  0.983, Adjusted R-squared:  0.977
F-statistic:  163 on 5 and 14 DF,  p-value: 6.7e-12
```

Categorical-categorical

Let category A and type 0 be the reference level. For observation i, let

- Y_i be the response,
- 1_i be a dummy variable for type 1,
- B_i be a dummy variable for category B, and
- C_i be a dummy variable for category C.

The mean containing only main effects is

$$\mu_i = \beta_0 + \beta_1 1_i + \beta_2 B_i + \beta_3 C_i.$$

The mean with an interaction is

$$\mu_i = \beta_0 + \beta_1 1_i + \beta_2 B_i + \beta_3 C_i + \beta_4 1_i B_i + \beta_5 1_i C_i.$$

Interpretation for the main effects model

The mean containing only main effects is

$$\mu_i = \beta_0 + \beta_1 1_i + \beta_2 B_i + \beta_3 C_i.$$

- β_0 is the expected response for category A and type 0
- β_1 is the change in response for moving from type 0 to type 1
- β_2 is the change in response for moving from category A to category B
- β_3 is the change in response for moving from category A to category C

The means are then

	Category		
Type	A	B	C
0	β_0	$\beta_0 \qquad + \beta_2$	$\beta_0 \qquad + \beta_3$
1	$\beta_0 + \beta_1$	$\beta_0 + \beta_1 + \beta_2$	$\beta_0 + \beta_1 + \beta_3$

Interpretation for the model with an interaction

The mean with an interaction is

$$\mu_i = \beta_0 + \beta_1 1_i + \beta_2 B_i + \beta_3 C_i + \beta_4 1_i B_i + \beta_5 1_i C_i.$$

- β_0 is the expected response for category A and type 0
- β_1 is the change in response for moving from type 0 to type 1 for category A
- β_2 is the change in response for moving from category A to category B for type 0
- β_3 is the change in response for moving from category A to category C for type 0
- β_4 is the difference in change in response for moving from category A to category B for type 1 compared to type 0
- β_5 is the difference in change in response for moving from category A to category C for type 1 compared to type 0

The means are then

	Category		
Type	A	B	C
0	β_0	$\beta_0 \quad + \beta_2$	$\beta_0 \quad + \beta_3$
1	$\beta_0 + \beta_1$	$\beta_0 + \beta_1 + \beta_2 + \beta_4$	$\beta_0 + \beta_1 + \beta_3 + \beta_5$

This is referred to as the cell-means model.

R code and output - main effects only

```
summary(lm(Cover~Block+Treat, case1301, subset=Block %in% c("B1","B2") & Treat %in% c("L","Lf","LfF")))
```

```

Call:
lm(formula = Cover ~ Block + Treat, data = case1301, subset = Block %in%
    c("B1", "B2") & Treat %in% c("L", "Lf", "LfF"))

Residuals:
   Min     1Q Median     3Q    Max
-2.333 -0.667  0.000  0.792  1.833

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)    4.667      0.768    6.07   0.0003 ***
BlockB2        2.167      0.768    2.82   0.0225 *
TreatLf       -1.500      0.941   -1.59   0.1496
TreatLfF      -3.000      0.941   -3.19   0.0128 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.33 on 8 degrees of freedom
Multiple R-squared:  0.694, Adjusted R-squared:  0.579
F-statistic: 6.04 on 3 and 8 DF,  p-value: 0.0188
```

R code and output - with interaction

```
summary(lm(Cover~Block*Treat, case1301, subset=Block %in% c("B1","B2") & Treat %in% c("L","Lf","LfF")))
```

```
Call:
lm(formula = Cover ~ Block * Treat, data = case1301, subset = Block %in%
    c("B1", "B2") & Treat %in% c("L", "Lf", "LfF"))

Residuals:
   Min     1Q Median     3Q    Max
-1.500 -0.625  0.000  0.625  1.500

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       4.00e+00   8.90e-01    4.50   0.0041 **
BlockB2           3.50e+00   1.26e+00    2.78   0.0319 *
TreatLf          -2.72e-16   1.26e+00    0.00   1.0000
TreatLfF         -2.50e+00   1.26e+00   -1.99   0.0941 .
BlockB2:TreatLf  -3.00e+00   1.78e+00   -1.69   0.1428
BlockB2:TreatLfF -1.00e+00   1.78e+00   -0.56   0.5945
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.26 on 6 degrees of freedom
Multiple R-squared:  0.795, Adjusted R-squared:  0.623
F-statistic: 4.64 on 5 and 6 DF,  p-value: 0.0443
```

When to include interaction terms

From _The Statistical Sleuth (3rd ed)_ page 250:

- when a question of interest pertains to an interaction
- when good reason exists to suspect interaction or
- when interactions are proposed as a more general model for the purpose of examining the goodness of fit of a model without interaction