Set R08 - Experimental design

STAT 401 (Engineering) - Iowa State University

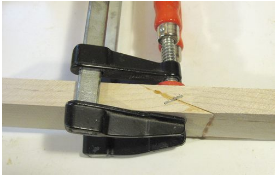
April 7, 2017

Strength of wood glue

You are interested in testing two different wood glues:

- Gorilla Wood Glue
- Titebond 1413 Wood glue

On a scarf joint:



So you collect up some wood, glue the pieces together, and determine the force required to break the joint. (There are lots of details missing here.)

Inspiration: `https://woodgears.ca/joint_strength/glue.html`

Completely Randomized Design (CRD)

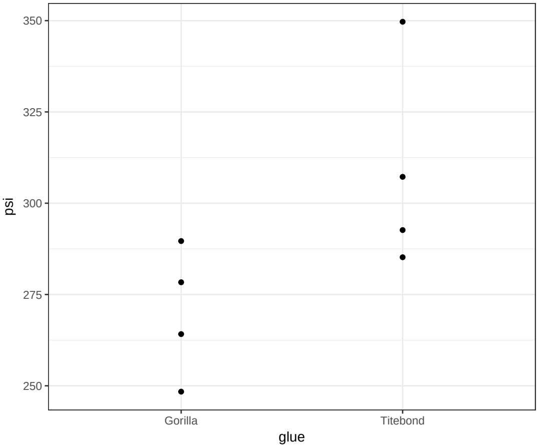
Suppose I have 8 pieces of wood laying around. I cut each piece and randomly use either Gorilla or Titebond glue to recombine the pieces. I do the randomization in such a way that I have exactly 4 Gorilla and 4 Titebond results, e.g.

```
# A tibble: 8  2
  woodID     glue
  <fctr>    <chr>
1  wood1  Gorilla
2  wood2 Titebond
3  wood3  Gorilla
4  wood4 Titebond
5  wood5 Titebond
6  wood6 Titebond
7  wood7  Gorilla
8  wood8  Gorilla
```

This is called a completely randomized design (CRD).

Visualize the data

```
ggplot(d, aes(glue, psi)) + geom_point() + theme_bw()
```



Model

Let

- F_w be the force needed to break wood w,
- T_w be an indicator that the Titebond glue was used on wood w, i.e.

$$T_w = \mathrm{I}(\text{glue}_w = \text{Titebond}).$$

Then a regression model for these data is

$$F_w \stackrel{ind}{\sim} N(\beta_0 + \beta_1 T_w, \sigma)$$

where

- β_1 is the expected difference in force when using Titebond glue compared to using Gorilla glue.

Perform analysis

```
m <- lm(psi ~ glue, data = d)
summary(m)


Call:
lm(formula = psi ~ glue, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-23.486 -17.468  -3.722  11.038  40.997 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)    270.14      11.99  22.537 4.99e-07 ***
glueTitebond    38.56      16.95   2.275   0.0633 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 23.97 on 6 degrees of freedom
Multiple R-squared:  0.463,	Adjusted R-squared:  0.3735 
F-statistic: 5.174 on 1 and 6 DF,  p-value: 0.06327


confint(m)


                  2.5 %    97.5 %
(Intercept)  240.806326 299.46474
glueTitebond  -2.921249  80.03428
```

Randomized complete block design (RCBD)

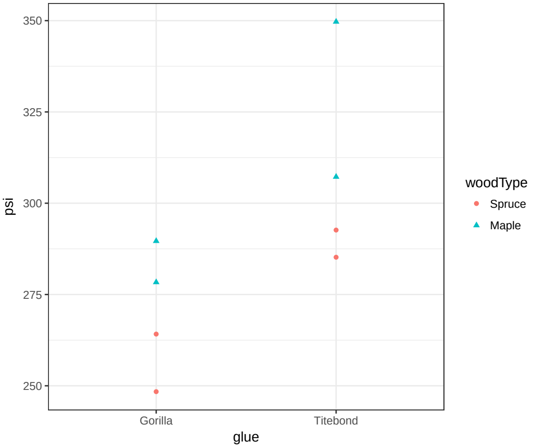
Suppose the wood actually came from two different types: Maple and Spruce. And perhaps you have reason to believe the glue will work differently depending on the type of wood. In this case, you would want to block by wood type and perform the randomization within each block, i.e.

```
# A tibble: 8  3
  woodID woodType     glue
  <fctr>   <fctr>    <chr>
1  wood1   Spruce  Gorilla
2  wood2   Spruce Titebond
3  wood3   Spruce  Gorilla
4  wood4   Spruce Titebond
5  wood5    Maple Titebond
6  wood6    Maple Titebond
7  wood7    Maple  Gorilla
8  wood8    Maple  Gorilla
```

This is called a randomized complete block design (RCBD).

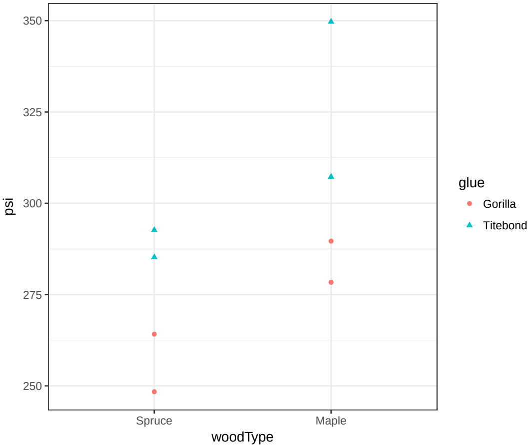
Visualize the data

```
ggplot(d, aes(glue, psi, color=woodType, shape=woodType)) + geom_point() + theme_bw()
```



Visualize the data - a more direct comparison

```
ggplot(d, aes(woodType, psi, color=glue, shape=glue)) + geom_point() + theme_bw()
```



Model

Let

- F_w be the force needed to break wood w
- T_w be an indicator that Titebond glue was used on wood w, and
- M_w be an indicator that wood w was Maple.

Then a regression model for these data is

$$F_w \stackrel{ind}{\sim} N(\beta_0 + \beta_1 T_w + \beta_2 M_w, \sigma)$$

where

- β_1 is the expected difference in force when using Titebond glue compared to using Gorilla glue when adjusted for type of wood, i.e. the type of wood is held constant, and
- β_2 is the expected difference in force when using Spruce compared to Maple when adjusted for type of glue, i.e. the glue is held constant.

Perform analysis

```
m <- lm(psi ~ glue + woodType, data = d)
summary(m)


Call:
lm(formula = psi ~ glue + woodType, data = d)

Residuals:
      1       2       3       4       5       6       7       8
 -4.929   0.768  10.835  -6.674  24.186 -18.279  -8.594   2.688

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    253.324      9.435  26.848 1.34e-06 ***
glueTitebond    38.557     10.895   3.539   0.0166 *
woodTypeMaple   33.623     10.895   3.086   0.0273 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.41 on 5 degrees of freedom
Multiple R-squared:  0.8151, Adjusted R-squared:  0.7412
F-statistic: 11.02 on 2 and 5 DF,  p-value: 0.01469


confint(m)


                  2.5 %    97.5 %
(Intercept)   229.069570 277.57817
glueTitebond   10.550061  66.56297
woodTypeMaple   5.616873  61.62978
```

Replication

Since there are more than one observation for each woodType-glue combination, the design is replicated:

```
d %>% group_by(woodType, glue) %>% summarize(n = n())


Source: local data frame [4 x 3]
Groups: woodType [?]

  woodType     glue     n
    <fctr>    <chr> <int>
1   Spruce  Gorilla     2
2   Spruce Titebond     2
3    Maple  Gorilla     2
4    Maple Titebond     2
```

When the design is replicated, we can consider assessing an interaction. In this example, an interaction between glue and woodType would indicate that the effect of glue depends on the woodType, i.e. the difference in expected force between the two glues depends on woodType. At an extreme, it could be that Gorilla works better on Spruce and Titebond works better on Maple.

Assessing an interaction using a t-test

```
m <- lm(psi ~ glue*woodType, data = d)
summary(m)


Call:
lm(formula = psi ~ glue * woodType, data = d)

Residuals:
      1       2       3       4       5       6       7       8
 -7.882   3.721   7.882  -3.721  21.233 -21.233  -5.641   5.641

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
(Intercept)                  256.28      11.82  21.686 2.67e-05 ***
glueTitebond                  32.65      16.71   1.954    0.122
woodTypeMaple                 27.72      16.71   1.658    0.173
glueTitebond:woodTypeMaple    11.81      23.64   0.500    0.643
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 16.71 on 4 degrees of freedom
Multiple R-squared:  0.826, Adjusted R-squared:  0.6955
F-statistic:  6.33 on 3 and 4 DF,  p-value: 0.05335
```

Assessing an interaction using an F-test

```
anova(m)
```

```
Analysis of Variance Table

Response: psi
              Df  Sum Sq Mean Sq F value  Pr(>F)
glue           1 2973.21 2973.21 10.6449 0.03100 *
woodType       1 2261.06 2261.06  8.0952 0.04662 *
glue:woodType  1   69.77   69.77  0.2498 0.64346
Residuals      4 1117.24  279.31
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

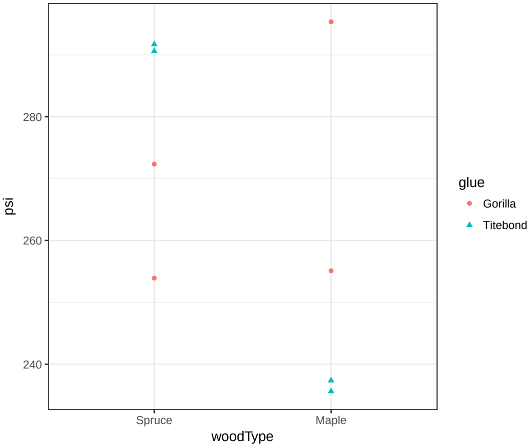
```
drop1(m, test='F')
```

```
Single term deletions

Model:
psi ~ glue * woodType
              Df Sum of Sq    RSS    AIC F value Pr(>F)
<none>                     1117.2 47.513
glue:woodType  1    69.769 1187.0 45.998  0.2498 0.6435
```

What if this had been your data?

```
ggplot(d, aes(woodType, psi, color=glue, shape=glue)) + geom_point() + theme_bw()
```



Assessing an interaction using a t-test

```
m <- lm(psi ~ glue*woodType, data = d)
summary(m)


Call:
lm(formula = psi ~ glue * woodType, data = d)

Residuals:
       1        2        3        4        5        6        7        8
 -9.2083  -0.5529   0.5529   9.2083  -0.8764  20.1215 -20.1215   0.8764

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
(Intercept)                  263.12      11.08  23.755 1.86e-05 ***
glueTitebond                  28.03      15.66   1.790   0.1480
woodTypeMaple                 12.10      15.66   0.773   0.4829
glueTitebond:woodTypeMaple   -66.76      22.15  -3.014   0.0394 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.66 on 4 degrees of freedom
Multiple R-squared:  0.7648, Adjusted R-squared:  0.5883
F-statistic: 4.335 on 3 and 4 DF,  p-value: 0.09522
```