STAT 401A - Statistical Methods for Research Workers

Modeling assumptions

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last updated: September 10, 2014

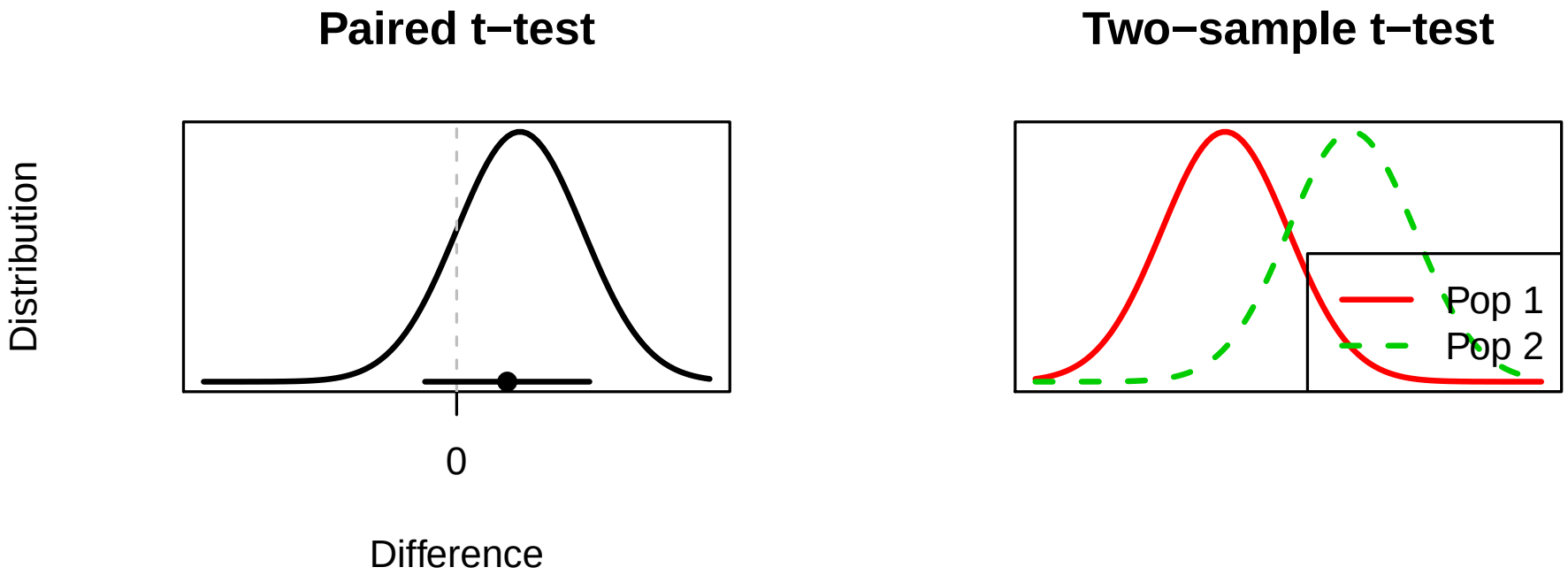
Normality assumptions

In the paired t-test, we assume

$$D_i \overset{iid}{\sim} N(\mu, \sigma^2).$$

In the two-sample t-test, we assume

$$Y_{ij} \overset{ind}{\sim} N(\mu_j, \sigma^2).$$



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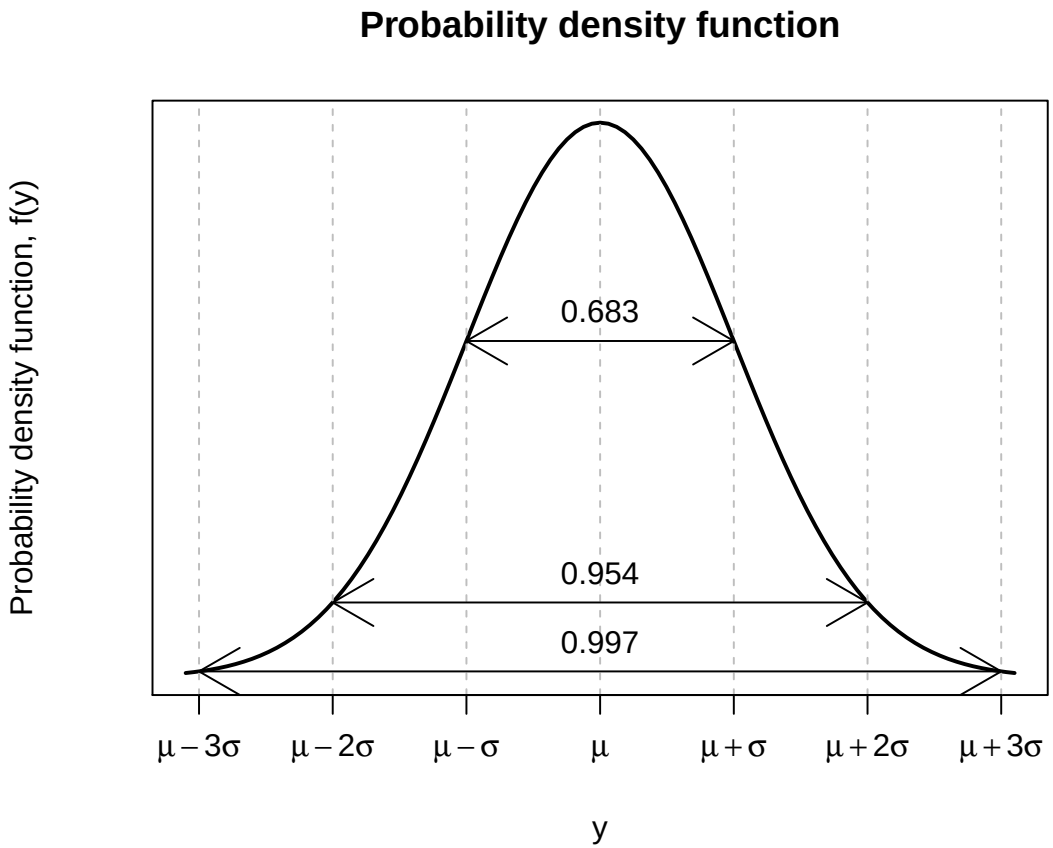
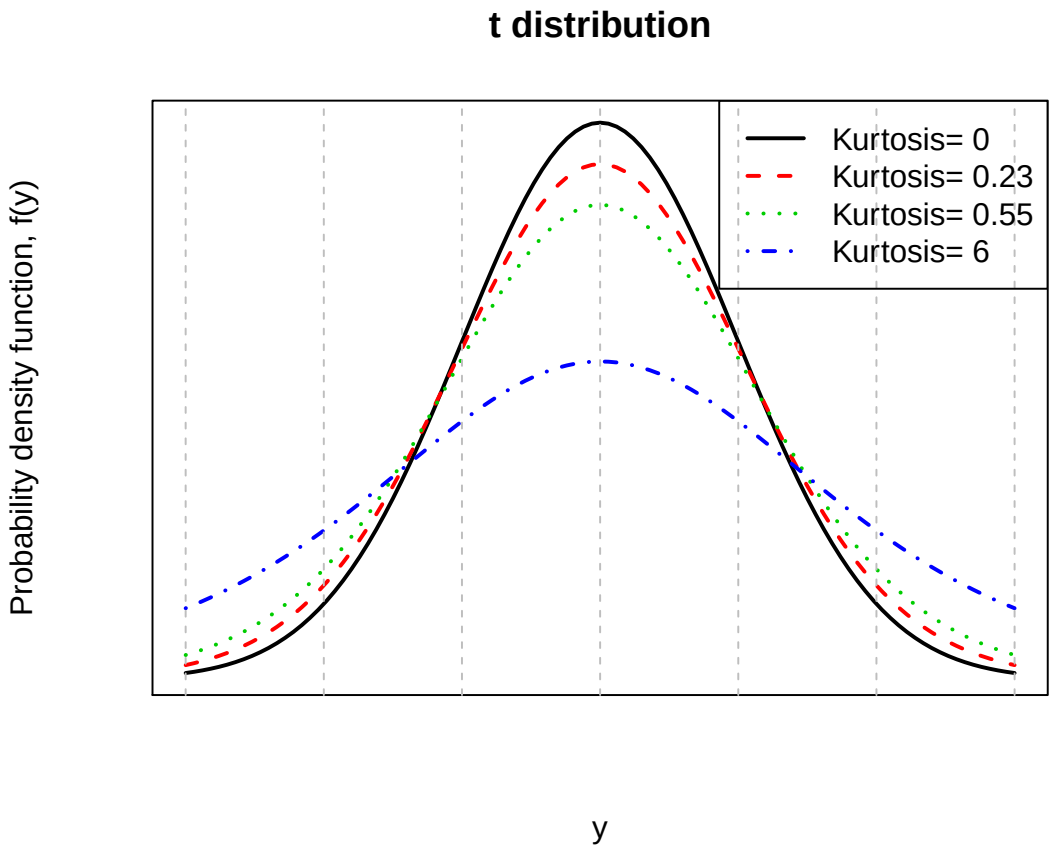
Key features of the normal distribution assumption:

- Centered at the mean (expectation) μ
- Standard deviation describes the spread
- Symmetric around μ (no skewness)
- Non-heavy tails, i.e. outliers are rare (no kurtosis)

Normality assumptions



Kurtosis (heavy-tailedness)



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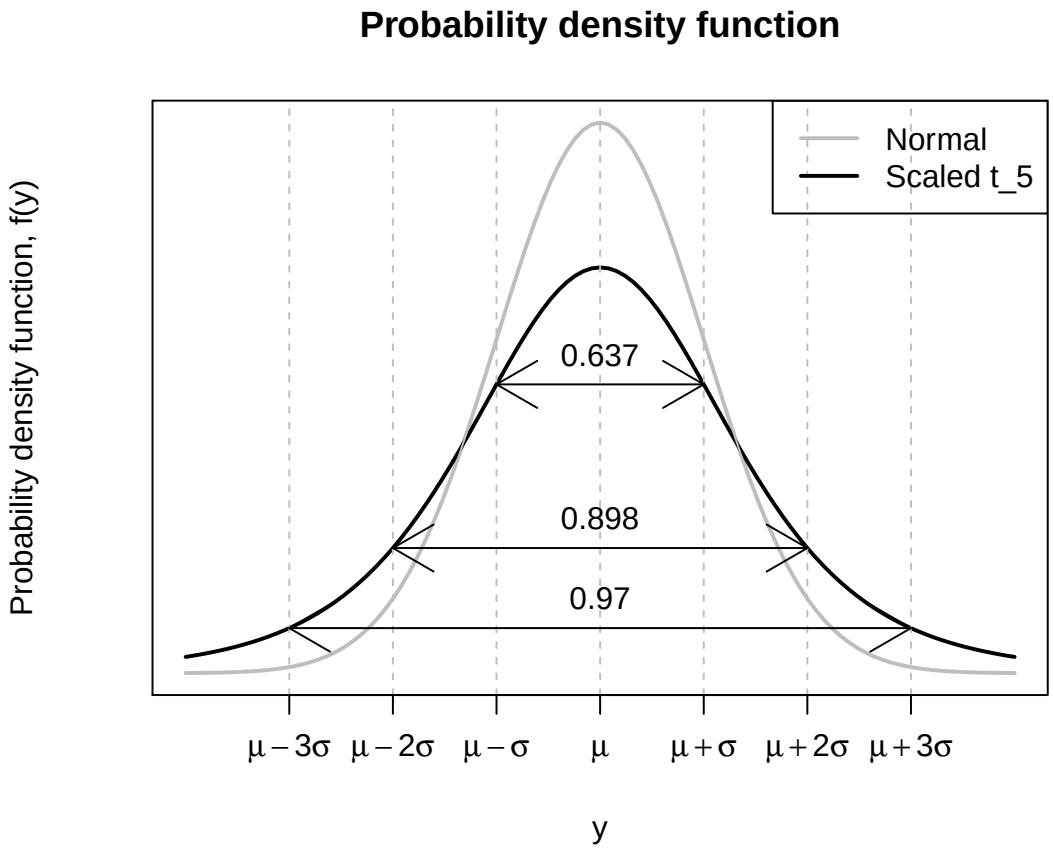

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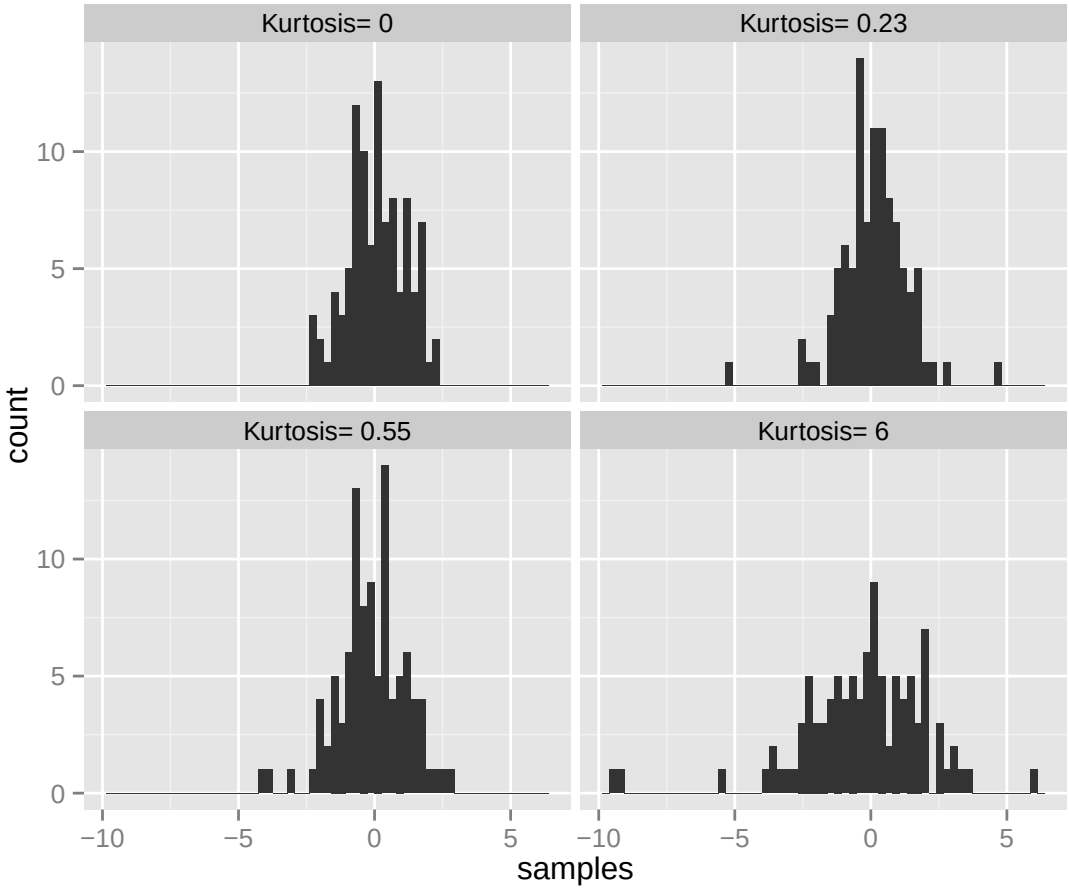

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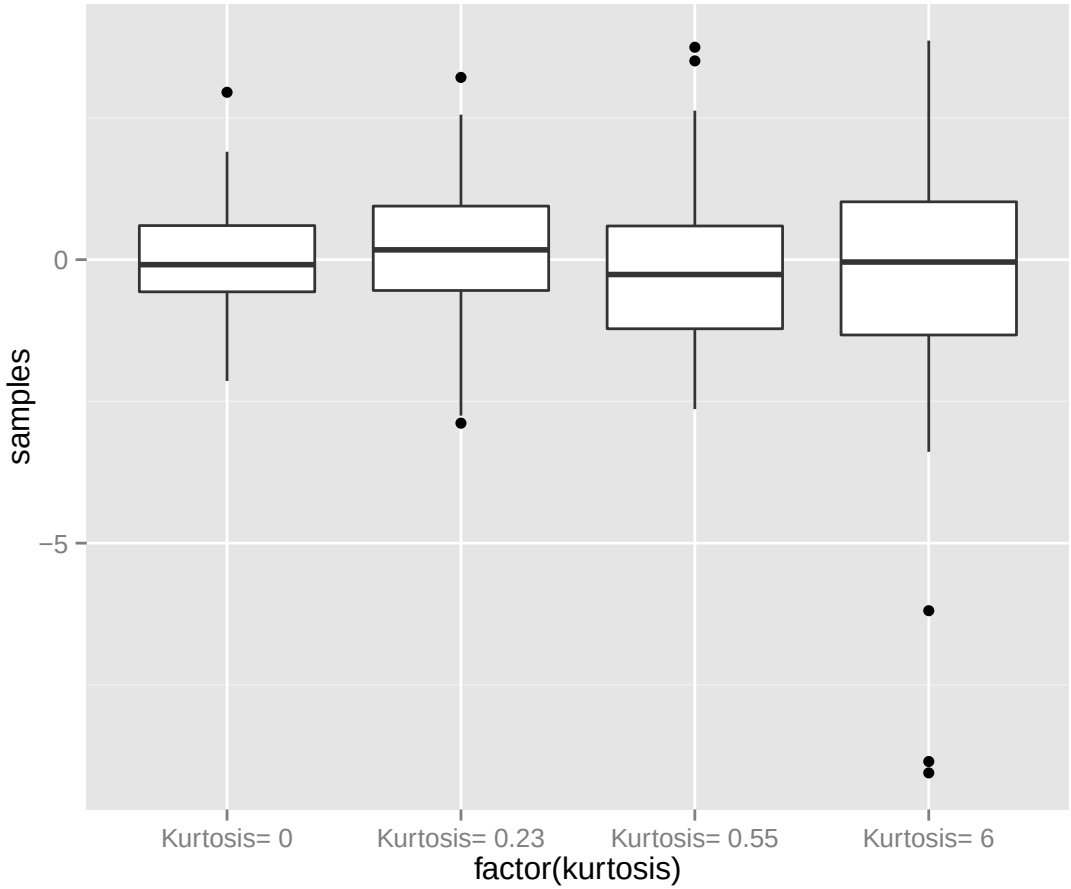
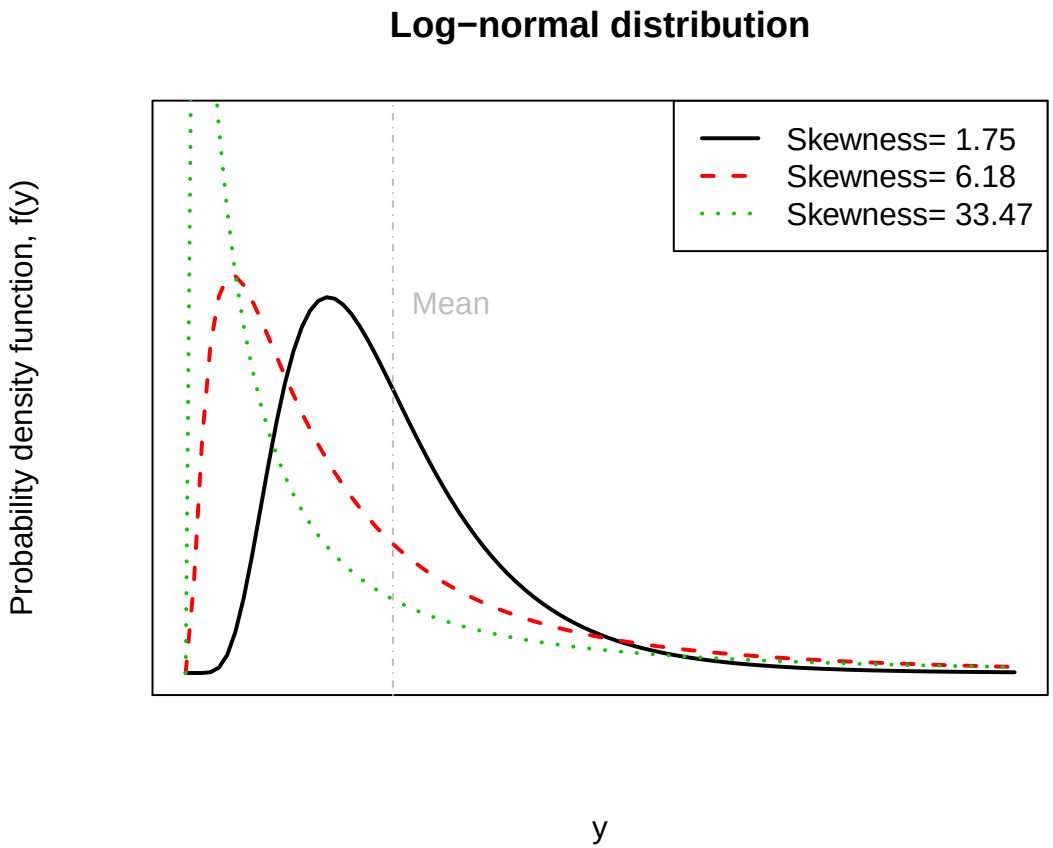
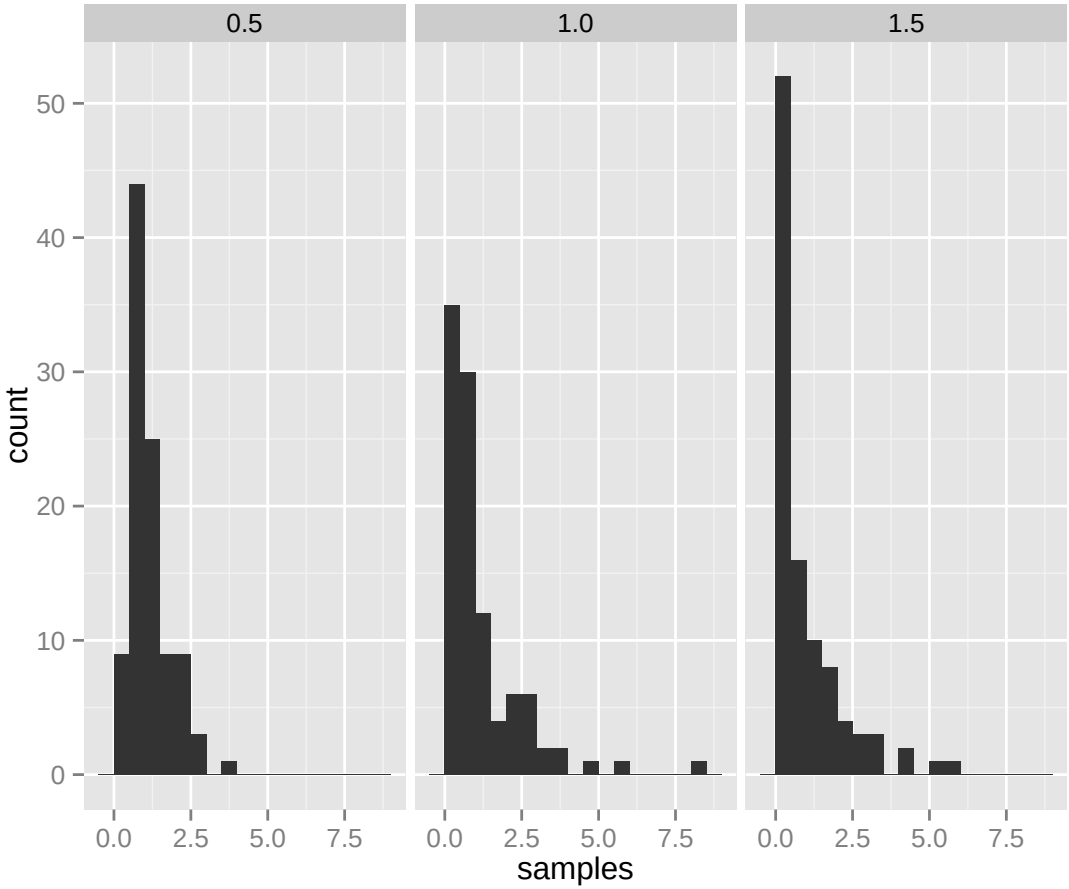


Skewness


Log−normal distribution
Skewness= 1.75
Skewness= 6.18
Skewness= 33.47
Mean
Probability density function, f(y)
y

Samples from skewed distributions



Robustness

Definition

A statistical procedure is robust to departures from a particular assumption if it is valid even when the assumption is not met.

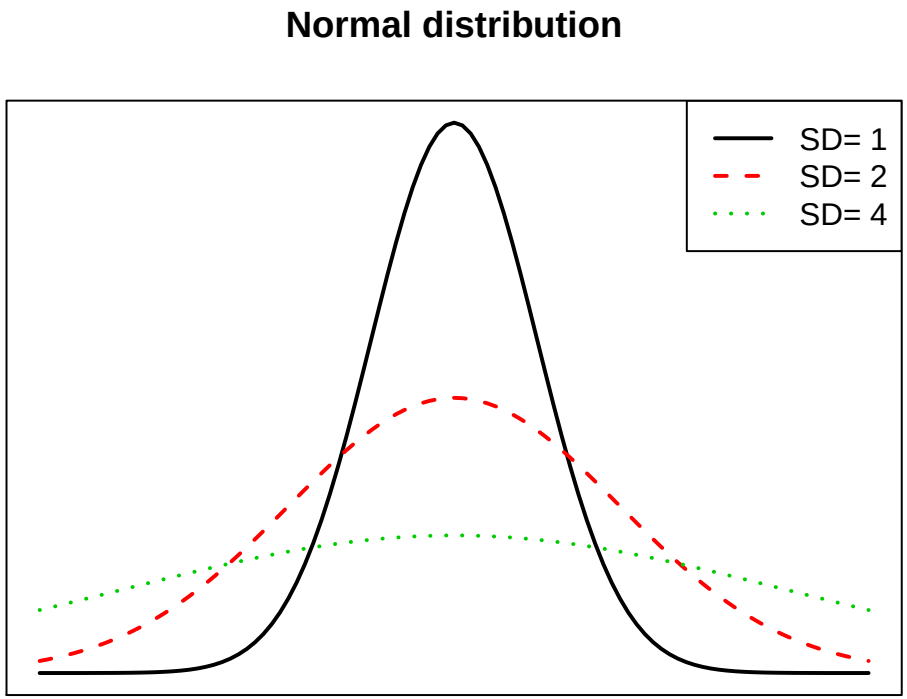
Remark If a 95% confidence interval is robust to departures from a particular assumption, the confidence interval should cover the true value about 95% of the time.

Robustness to skewness and kurtosis

Percentage of 95% confidence intervals that cover the true difference in means in an equal-sample two-sample t-test with non-normal populations (where the distributions are the same other than their means).

	sample.size	strongly.skewed	moderately.skewed	mildly.skewed	heavy.tailed	short.tailed
1	5	95.5	95.4	95.2	98.3	94.5
2	10	95.5	95.4	95.2	98.3	94.6
3	25	95.3	95.3	95.1	98.2	94.9
4	50	95.1	95.3	95.1	98.1	95.2
5	100	94.8	95.3	95.0	98.0	95.6

Differences in variances



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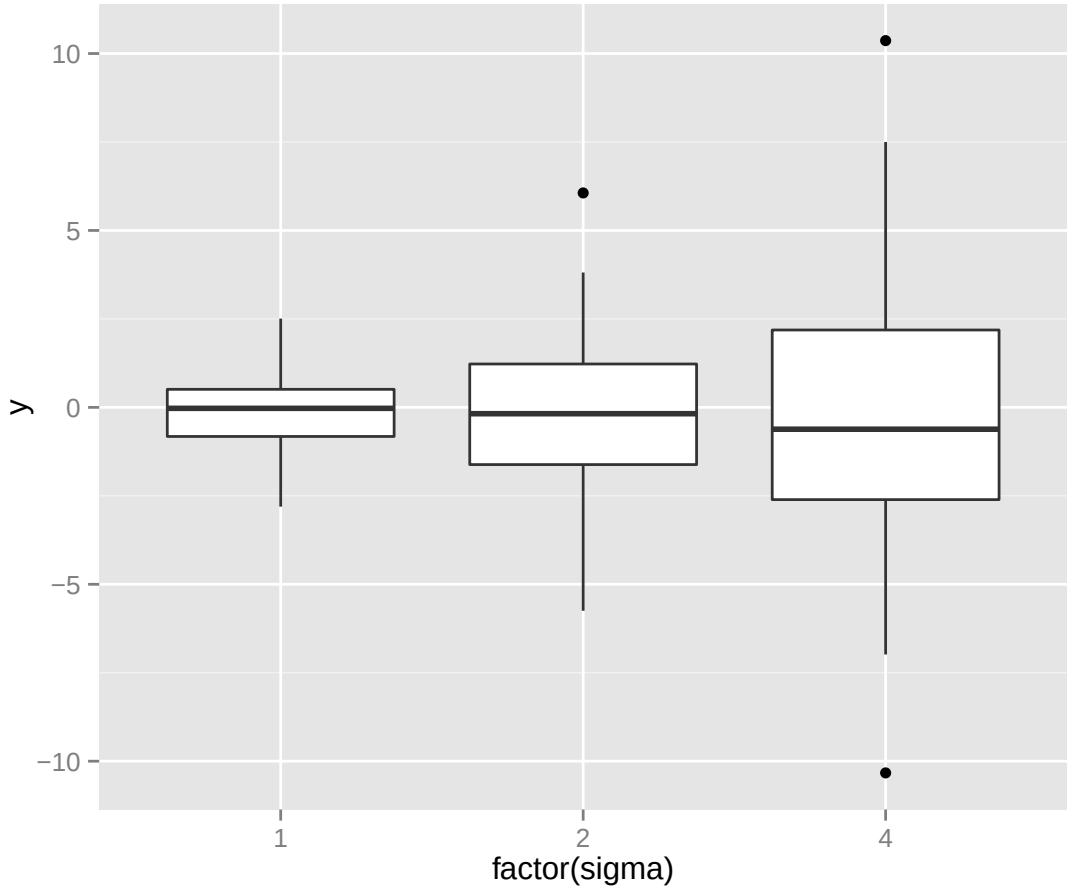

Robustness to differences in variances

Percentage of 95% confidence intervals that cover the true difference in means in an equal-sample two-sample t-test (r is the ratio of the standard deviations in the two populations).

	n1	n2	r0.25x	r0.5x	r1x	r2x	r4x
1	10	10	95.2	94.2	94.7	95.2	94.5
2	10	20	83.0	89.3	94.4	98.7	99.1
3	10	40	71.0	82.6	95.2	99.5	99.9
4	100	100	94.8	96.2	95.4	95.3	95.1
5	100	200	86.5	88.3	94.8	98.8	99.4
6	100	400	71.6	81.5	95.0	99.5	99.9

Outliers

Definition

A statistical procedure is resistant if it does not change very much when a small part of the data changes, perhaps drastically.

Identify outliers:

1. If recording errors, fix.
2. If outlier comes from a different population, remove and report.
3. If results are the same with and without outliers, report with outliers.
4. If results are different, use resistant analysis or report both analyses.

Common ways for independence to be violated

- Cluster effect
  - e.g. pigs in a pen
- Correlation effect
  - e.g. measurements in time with drifting scale
- Spatial effect
  - e.g. corn yield plots (drainage)