Exponential distribution

STAT 587 (Engineering)
Iowa State University

September 17, 2020

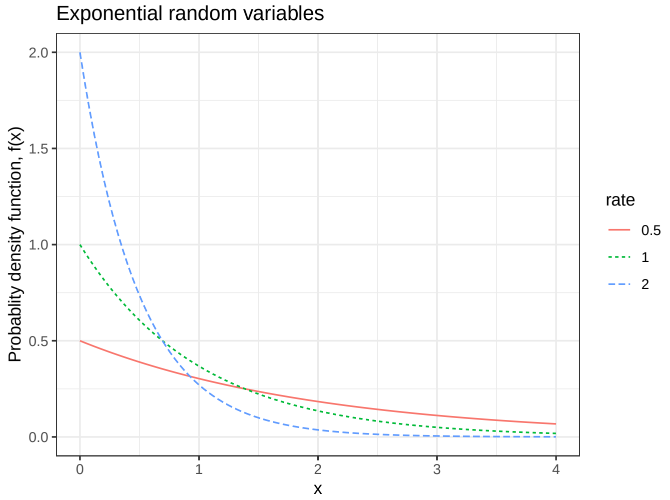
Exponential distribution

The random variable X has an exponential distribution with rate parameter $\lambda > 0$ if its probability density function is

$$p(x|\lambda) = \lambda e^{-\lambda x}\,\mathrm{I}(x > 0).$$

We write $X \sim Exp(\lambda)$.

Exponential probability density function



Exponential mean and variance

If $X \sim Exp(\lambda)$, then

$$E[X] = \int_0^\infty x\, \lambda e^{-\lambda x} dx = \cdots = \frac{1}{\lambda}$$

and

$$Var[X] = \int_0^\infty \left(x - \frac{1}{\lambda}\right)^2 \lambda e^{-\lambda x} dx = \cdots = \frac{1}{\lambda^2}.$$

Exponential cumulative distribution function

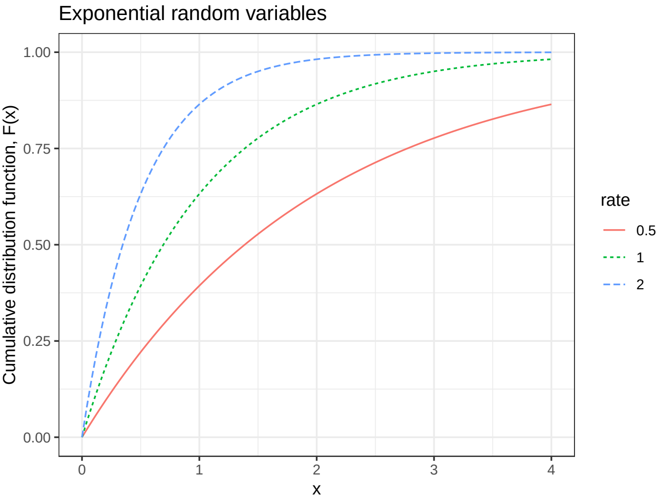
If $X \sim Exp(\lambda)$, then its cumulative distribution function is

$$F(x) = \int_0^x \lambda e^{-\lambda t} dt = \cdots = 1 - e^{-\lambda x}.$$

The inverse cumulative distribution function is

$$F^{-1}(p) = \frac{-\log(1-p)}{\lambda}.$$

Exponential cumulative distribution function - graphically



Memoryless property

Let $X \sim Exp(\lambda)$, then

$$P(X > x + c | X > c) = P(X > x).$$

Parameterization by the scale

A common alternative parameterization of the exponential distribution uses the scale $\beta = \frac{1}{\lambda}$. In this parameterization, we have

$$f(x) = \frac{1}{\beta} e^{-x/\beta} \mathrm{I}(x > 0)$$

and

$$E[X] = \beta \qquad \text{and} \qquad Var[X] = \beta^2.$$

Summary

Exponential random variable

- $X \sim Exp(\lambda)$, $\lambda > 0$
- $f(x) = \lambda e^{-\lambda x}$, $x > 0$
- $F(x) = 1 - e^{-\lambda x}$
- $F^{-1}(p) = \frac{-\log(1-p)}{\lambda}$
- $E[X] = \frac{1}{\lambda}$
- $Var[X] = \frac{1}{\lambda^2}$