I06 - p-values

STAT 587 (Engineering)
Iowa State University

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p-value

A p-value is the probability of observing a statistic as or more extreme than observed if the model is true.

A p-value is the probability of observing a statistic as or more extreme than *the one you* observed if the model is true *when the data are considered random*.

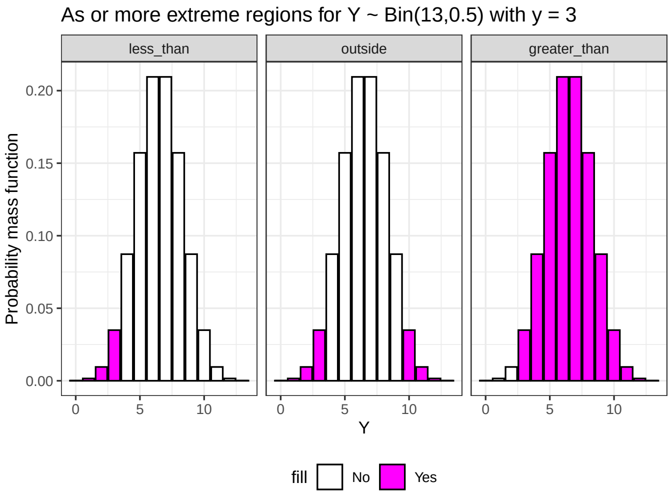
Binomial model

Let $H_0 : Y \sim Bin(13, 0.5)$ and observe $y = 3$.

Choos

- statistic is 3,
- its sampling distribution *when the model is true is* $Y \sim Bin(13, 0.5)$, and
- there are three *as or more extreme regions*:
    - $Y \leq 3$
    - $Y \geq 3$
    - $|Y - 13 \cdot 0.5| \geq |3 - 13 \cdot 0.5|$

as or more extreme regions



R Calculation

One-sided p-values:

- $P(Y \leq y)$:

```r
pbinom(y, size = n, prob = p)
[1] 0.04614258
```

- $P(Y \geq y) = 1 - P(Y < y) = 1 - P(Y \leq y - 1)$:

```r
1-pbinom(y-1, size = n, prob = p)
[1] 0.9887695
```

Two-sided p-value:

$$P(|Y - n\theta| \leq |y - n\theta|) = 2P(Y \leq y)$$

```r
2*pbinom(y, size = n, prob = p)
[1] 0.09228516
```

Normal model

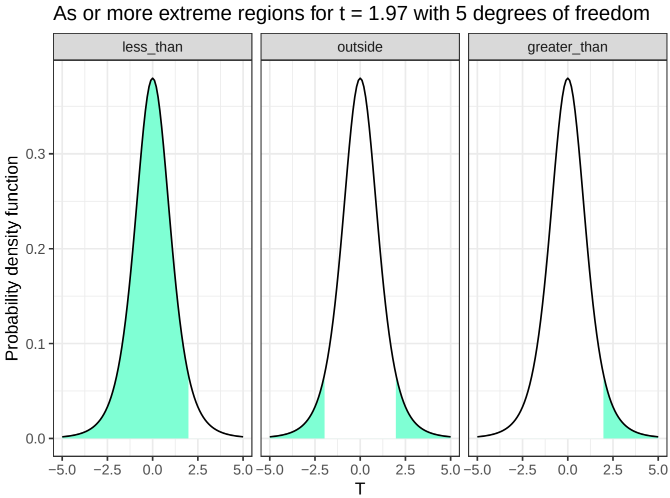
Let $H_0 : Y_i \sim N(3, 4^2)$ for $i = 1, \ldots, 6$ and you observe $\overline{y} = 6.3$, $s = 4.1$, and

$$t = \frac{\overline{y} - 3}{s/\sqrt{n}} = \frac{6.3 - 3}{4.1/\sqrt{6}} = 1.97.$$

Choose

- t-statistic $t = 1.97$,
- its sampling distribution *when the model is true is* $T_5 \sim t_5$, and
- there are three *as or more extreme regions*:
    - $T_5 \leq 1.97$
    - $T_5 \geq 1.97$
    - $|T_5| \geq |1.97|$

as or more extreme regions



R Calculation

- One-sided p-values:
  - $P(T_5 \leq t)$:

```r
pt(t, df = n-1)
```

```
[1] 0.9471422
```

  - $P(T_5 \geq t) = 1 - P(T_5 < t) = 1 - P(T_5 \leq t)$:

```r
1-pt(t, df = n-1)
```

```
[1] 0.05285775
```

- Two-sided p-value:

$$P(|T_5| \geq |t|) = 2P(T_5 \geq t)$$

```r
2*(1-pt(t, df = n-1))
```

```
[1] 0.1057155
```

Interpretation

Small p-values provide evidence that the data are incompatible with the model.

Recall

$$Y_i \overset{ind}{\sim} N(\mu, \sigma^2)$$

indicates the data

- are independent,
- are normally distributed,
- have a common mean, and
- have a common variance.

Summary

- p-value: the probability of observing a statistic as or more extreme than observed if the model is true
- small p-values provide evidence that the data are incompatible with the model