t-tests

STAT 587 (Engineering)
Iowa State University

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Statistical hypothesis testing

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which make a claim about parameters in a model and a decision to either

- reject the null hypothesis or
- fail to reject the null hypothesis.

t-tests

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t-statistic

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$$H_A : \mu \neq \mu_0 \implies |T| \geq |t|$$

where $T \sim t_{n-1}$.

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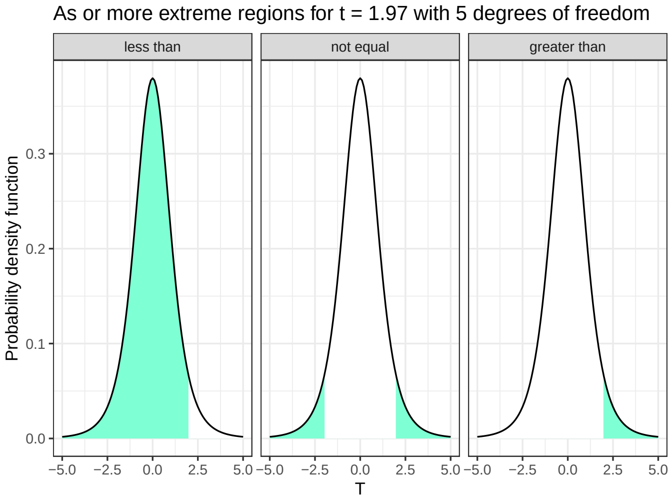
$$n = 6,\ \overline{y} = 6.3,\ \text{and } s = 4.1.$$

Then we can calculate

$$t = 1.97$$

which has a t_5 distribution if the null hypothesis is true.

as or more extreme regions



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t.test(y, mu = mu0, alternative = "less")$p.value

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$$H_A : \mu \neq 3$$

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$$H_A : \mu \neq 3$$

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t.test(y, mu = mu0, alternative = "two.sided")$p.value

[1] 0.1076051
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If you fail to reject H_0,then there is insufficient evidence to say that the data are incompatible with the null model.

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One Sample t-test

data:  y
t = 2.4213, df = 11, p-value = 0.03393
alternative hypothesis: true mean is not equal to 12
95 percent confidence interval:
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sample estimates:
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The small p-value suggests the data may be incompatible with the model $Y_i \stackrel{ind}{\sim} N(12, \sigma^2)$.

Summary

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- Use p-values to determine whether to
  - reject the null hypothesis or
  - fail to reject the null hypothesis.
- More assessment is required to determine if other model assumptions hold.