

Name _____

Spring 2018

STAT 401C

Exam I
(100 points)

Instructions:

- Full credit will be given only if you show your work.
- The questions are not necessarily ordered from easiest to hardest.
- You are allowed to use any resource except aid from another individual.
- Aid from another individual, will automatically earn you a 0.

1. A diagnostic test for disease D has a sensitivity of 0.95 and a specificity of 0.9. The prevalence of the disease is 0.02. (20 points)
 - (a) Define notation for the following events (1 point each).
 - having the disease
 - not having the disease
 - testing positive
 - testing negative
 - (b) Use the notation in the previous step to define the following probabilities (2 points each).
 - sensitivity
 - specificity
 - prevalence
 - (c) If an individual tests positive for the disease, what is the probability they actually have the disease? (10 points)

2. Let X be a random variable with the following probability mass function:

x	-10	-5	0	5	10
P(X=x)	0.3	0.2	0.1	0.2	0.2

(a) Is $P(X = x)$ a valid probability mass function? Why or why not? (5 points)

(b) Calculate $E[X]$. (5 points)

(c) Let $Y = |X|$ what is the probability mass function for Y ? (5 points)

(d) Find $E[|X|]$. (5 points)

3. Answer the following questions based on this joint distribution for the random variables X and Y .

X	Y		
	1	2	3
-1	0.1	0.2	0.1
0	0.1	0.1	0.1
1	0.1	0.1	0.1

- (a) What is the image for the random variable Y ? (2 points)
- (b) Find the marginal probability mass function for X . (6 points)
- (c) Find $P(Y > X)$. (6 points)
- (d) Are X and Y independent? Why or why not? (6 points)

4. A warehouse has 46 high-intensity light bulbs and over the coming year the probability of each light failing is 5%. Assume light bulb failures are independent.
- (a) What is the probability that no light bulbs fail? (6 points)
- (b) What is the probability that more than 2 light bulbs fail? (6 points)
- (c) If each light bulb costs \$500 to replace, what is the expected expense due to light bulb replacement over the next year? (6 points)
- (d) Name one reason light bulb failures would not be independent. (2 points)

5. A positive displacement pump is used to fill an ethanol tanker. The pump measures 1 gallon of ethanol at a time with a mean of 1 gallon and a standard deviation of 0.01 gallons and independently of all other measurements. The pump repeats this process 30,001 times.
- (a) What is the approximate probability that the true amount of ethanol in the tanker is less than 30,000 gallons? (10 points)

- (b) The company wants to ensure the amount in the tanker is greater than 30,000 gallons with 99% probability. How many gallons above 30,000 should the pump measure to ensure with 99% probability that the true amount is greater than 30,000 gallons. (10 points)