

1. A company wants to understand what proportion (p) of customer's opt to give their change to a local charity. The plan is to take a random sample of the customers and record whether or not the customer gave their change to the charity. Define the random variable X_i for the response of the i th customer where a 0 represents that they did not give and a 1 represents that they did.

(a) What is $E[X_i]$?

(b) What is $E[\bar{X}]$?

(c) Is $E[\bar{X}]$ unbiased for the proportion p ?

(d) What is the variance of X_i ?

(e) What is the standard deviation of X_i ?

(f) What is the standard error of \bar{X} ?

2. Continue the previous example and suppose $p = 0.5$.

(a) What sample size is needed so that with (approximately) 68% probability we are within 0.1 of the true population proportion?

(b) What sample size is needed so that with (approximately) 95% probability we are within 0.1 of the true population proportion?

(c) What sample size is needed so that with (approximately) 99.7% probability we are within 0.1 of the true population proportion?

(d) How will these sample sizes change if p is closer to 0 or 1?