# Sampling distribution

STAT 587 (Engineering)
Iowa State University

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# Sampling distribution

The sampling distribution of a statistic is the distribution of the statistic *over different realizations of the data*.

Find the following sampling distributions:

- If $Y_i \overset{ind}{\sim} N(\mu, \sigma^2)$,

$$\overline{Y} \qquad \text{and} \qquad \frac{\overline{Y} - \mu}{S/\sqrt{n}}.$$

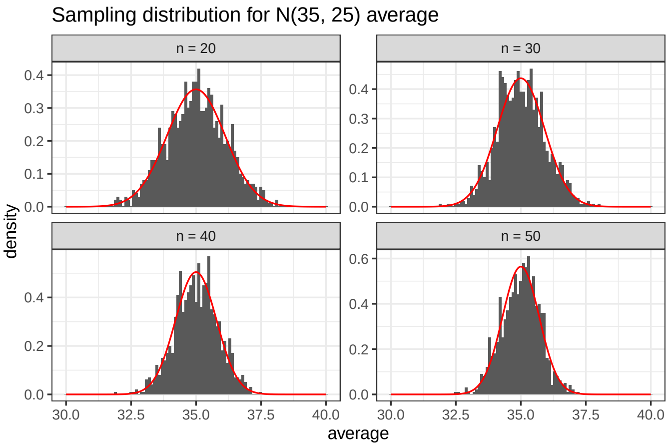
- If $Y \sim Bin(n, p)$,

$$\frac{Y}{n}.$$

# Normal model

Let $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$, then

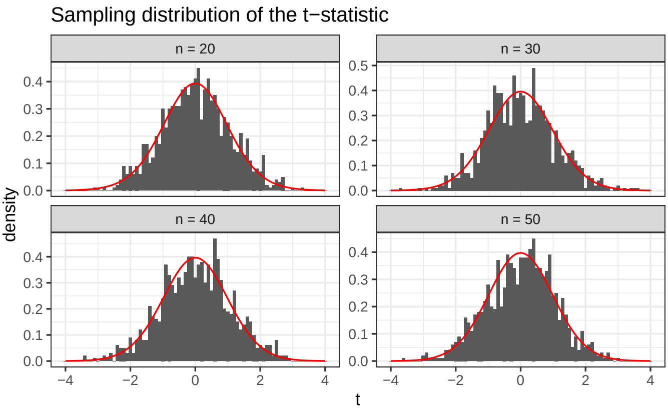
$$\overline{Y} \sim N(\mu, \sigma^2/n).$$



# Normal model

Let $Y_i \overset{ind}{\sim} N(\mu, \sigma^2)$, then the t-statistic

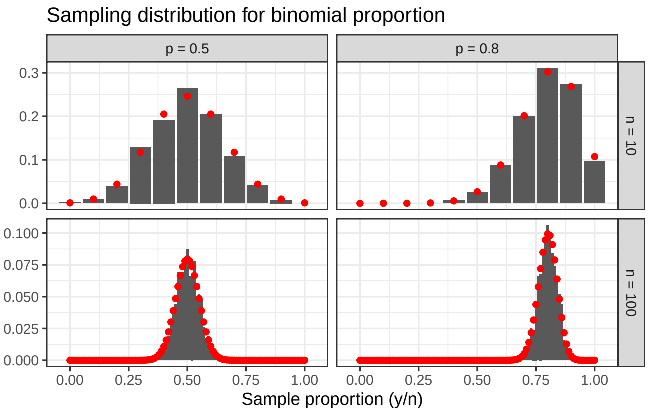
$$T = \frac{\overline{Y} - \mu}{S/\sqrt{n}} \sim t_{n-1}.$$



# Binomial model

Let $Y \sim Bin(n, p)$, then

$$P\left(\frac{Y}{n} = p\right) = P(Y = np), \qquad p = 0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, 1.$$



# Approximate sampling distributions

Recall that from the Central Limit Theorem (CLT):

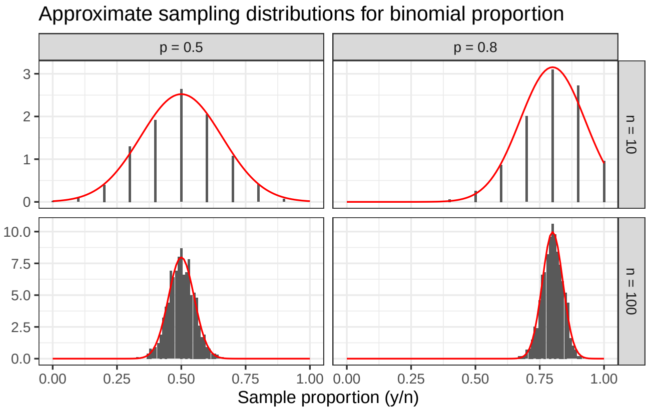
$$S = \sum_{i=1}^{n} X_i \,\dot\sim\, N(n\mu, n\sigma^2) \qquad \text{and} \qquad \overline{X} = S/n \,\dot\sim\, N(\mu, \sigma^2/n)$$

for independent $X_i$ with $E[X_i] = \mu$ and $Var[X_i] = \sigma^2$.

# Approximate sampling distribution for binomial proportion

If $Y = \sum_{i=1}^{n} X_i$ with $X_i \stackrel{ind}{\sim} Ber(p)$, then

$$\frac{Y}{n} \dot{\sim} N\left(p, \frac{p[1-p]}{n}\right).$$



# Summary

Sampling distributions:

- If $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$,
  - $\overline{Y} \sim N(\mu, \sigma^2/n)$ and
  - $\frac{\overline{Y}-\mu}{S/\sqrt{n}} \sim t_{n-1}$.
- If $Y \sim Bin(n, p)$,
  - $P\left(\frac{Y}{n} = p\right) = P(Y = np)$ and
  - $\frac{Y}{n} \dot{\sim} N\left(p, \frac{p[1-p]}{n}\right)$.
- If $X_i$ independent with $E[X_i] = \mu$ and $Var[X_i] = \sigma^2$, then

  $$S = \sum_{i=1}^{n} X_i \dot{\sim} N(n\mu, n\sigma^2)$$

  and

  $$\overline{X} = S/n \dot{\sim} N(\mu, \sigma^2/n)$$