Amazon Reviews

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February 23, 2017

Amazon Reviews - Upright, bagless, cyclonic vacuum cleaners

	Number of ratings							
product_id	n1	n2	n3	n4	n5	n_total	mean	sd
B000REMVGK	21	17	2	8	7	55	2.33	1.44
B001EFMD8W	40	34	28	77	347	526	4.25	1.26
B001PB51GQ	14	12	13	31	69	139	3.93	1.36
B002DGSJVG	22	8	3	6	10	49	2.47	1.63
B002G9UQZC	8	0	1	1	1	11	1.82	1.47
B002GHBRX4	18	8	9	14	27	76	3.32	1.61
B002HF66BI	9	5	2	2	3	21	2.29	1.49
B003OA77MC	15	7	8	24	42	96	3.74	1.47
B003OAD24Y	7	7	4	9	19	46	3.57	1.53
B003Y3AA3C	20	3	1	2	2	28	1.68	1.28
B0043EW354	40	25	25	60	163	313	3.90	1.44
B00440EO8G	2	1	1	1	7	12	3.83	1.64
B004R9197I	9	1	1	9	26	46	3.91	1.58
B008L5F4H0	3	1	2	12	7	25	3.76	1.27

Model for Amazon Reviews

Let y_{ij} be the jth review for the ith product. Assume

$$y_{ij} \stackrel{ind}{\sim} N(\theta_i, \sigma^2)$$

and

$$\theta_i \stackrel{ind}{\sim} N(\mu, \tau^2)$$

and

$$p(\mu, \tau, \sigma) \propto Ca^+(\sigma; 0, 1) Ca^+(\tau; 0, 1)$$

Normal hierarchical model in Stan

```
normal_model = "
data {
  int <lower=1> n;
  int <lower=1> n_products;
  int <lower=1,upper=5> stars[n];
  int <lower=1,upper=n_products> id[n];
}

parameters {
  real mu;                // implied uniform prior
  real<lower=0> sigma;
  real<lower=0> tau;
  real theta[n_products];
}

model {
  // Prior
  sigma ~ cauchy(0,1);
  tau   ~ cauchy(0,1);

  // Hierarchial model
  theta ~ normal(mu,tau);

  // Data model
  for (i in 1:n) stars[i] ~ normal(theta[id[i]], sigma);
}
"
```

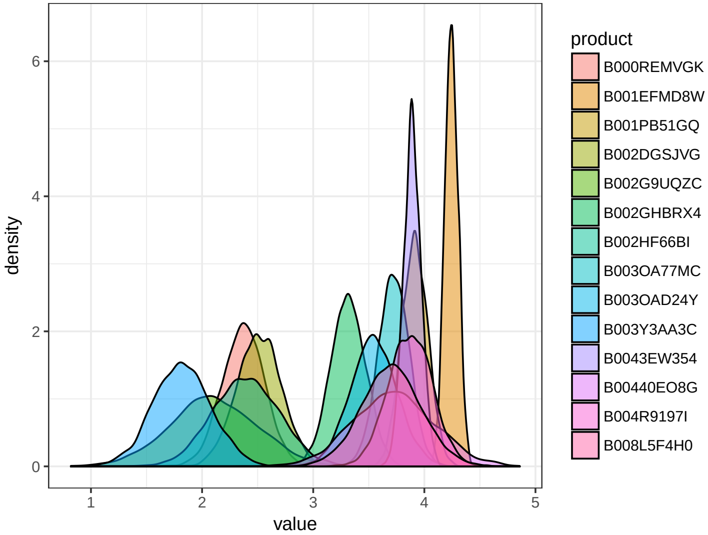
Fit model

```
m = stan_model(model_code = normal_model)


In file included from file238968dddd8.cpp:8:
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/src/st
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/BH/include/boost/math/tool
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/BH/include/boost/config.hp
/Library/Frameworks/R.framework/Versions/3.3/Resources/library/BH/include/boost/config/compiler/clang.hpp:196:1
#  define BOOST_NO_CXX11_RVALUE_REFERENCES
          ^
<command line>:6:9: note: previous definition is here
#define BOOST_NO_CXX11_RVALUE_REFERENCES 1
        ^
1 warning generated.


dat = list(n = nrow(d),
           n_products = nlevels(d$product_id),
           stars = d$stars,
           id = as.numeric(d$product_id))
r = sampling(m, dat)



SAMPLING FOR MODEL 'ce44497edda7358de70e693408d6c43d' NOW (CHAIN 1).

Chain 1, Iteration:    1 / 2000 [  0%]  (Warmup)
```

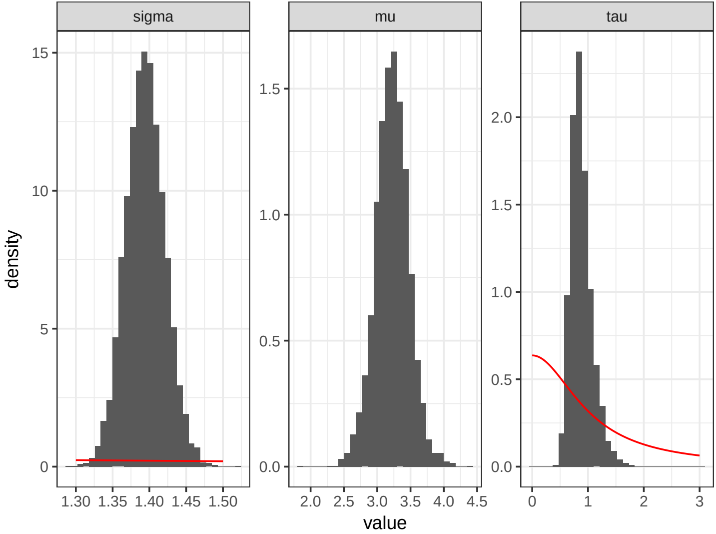
Tabular summary

```
Inference for Stan model: ce44497edda7358de70e693408d6c43d.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

              mean se_mean   sd     2.5%      25%      50%      75%    97.5% n_eff Rhat
mu            3.22    0.00 0.25     2.71     3.06     3.23     3.39     3.72  4000 1.00
sigma         1.39    0.00 0.03     1.34     1.38     1.39     1.41     1.45  4000 1.00
tau           0.88    0.00 0.20     0.59     0.75     0.85     0.99     1.36  4000 1.00
theta[1]      2.37    0.00 0.19     2.00     2.24     2.37     2.50     2.74  4000 1.00
theta[2]      4.24    0.00 0.06     4.13     4.20     4.24     4.29     4.36  4000 1.00
theta[3]      3.91    0.00 0.12     3.69     3.83     3.91     3.99     4.14  4000 1.00
theta[4]      2.51    0.00 0.20     2.12     2.38     2.51     2.65     2.90  4000 1.00
theta[5]      2.10    0.01 0.39     1.34     1.84     2.09     2.37     2.87  4000 1.00
theta[6]      3.31    0.00 0.16     3.00     3.21     3.31     3.42     3.62  4000 1.00
theta[7]      2.40    0.00 0.30     1.82     2.19     2.40     2.60     2.97  4000 1.00
theta[8]      3.72    0.00 0.14     3.45     3.63     3.72     3.82     3.99  4000 1.00
theta[9]      3.54    0.00 0.21     3.14     3.41     3.54     3.68     3.96  4000 1.00
theta[10]     1.82    0.00 0.26     1.31     1.64     1.82     1.99     2.32  4000 1.00
theta[11]     3.89    0.00 0.08     3.73     3.84     3.89     3.94     4.05  4000 1.00
theta[12]     3.72    0.01 0.36     3.01     3.47     3.72     3.97     4.43  4000 1.00
theta[13]     3.87    0.00 0.20     3.48     3.74     3.88     4.01     4.26  4000 1.00
theta[14]     3.70    0.00 0.27     3.17     3.53     3.70     3.88     4.24  4000 1.00
lp__      -1207.45    0.08 3.07 -1214.23 -1209.18 -1207.10 -1205.26 -1202.40  1344 1.01

Samples were drawn using NUTS(diag_e) at Thu Feb 23 11:30:59 2017.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

Vacuum cleaner mean posteriors (θ_i)



Other parameter posteriors



A quick rating

Suppose a new vacuum cleaner comes on the market and there are two Amazon reviews both with 5 stars. What do you think the average star rating will be (in the future) for this new product?

Let n^* be the number of new ratings and $\overline{y}^*$ be the average of those ratings, then

$$\begin{aligned} E[\theta^*|\overline{y}^*, n^*, \sigma, \mu, \tau] &= \frac{\frac{n^*}{\sigma^2}}{\frac{n^*}{\sigma^2}+\frac{1}{\tau^2}}\overline{y}^* + \frac{\frac{1}{\tau^2}}{\frac{n^*}{\sigma^2}+\frac{1}{\tau^2}}\mu \\ &= \frac{n^*}{n^*+\frac{\sigma^2}{\tau^2}}\overline{y}^* + \frac{\frac{\sigma^2}{\tau^2}}{n^*+\frac{\sigma^2}{\tau^2}}\mu \\ &= \frac{n^*}{n^*+m}\overline{y}^* + \frac{m}{n^*+m}\mu \end{aligned}$$

where $m = \sigma^2/\tau^2$ is a measure of how many *prior* samples there are.

IMDB rating

From `http://www.imdb.com/chart/top.html`:

```
weighted rating (WR) = (v / (v+m))  R + (m / (v+m))  C

Where:

R = average for the movie (mean) = (Rating)
v = number of votes for the movie = (votes)
m = minimum votes required to be listed in the Top 250
    (currently 25000)
C = the mean vote across the whole report (currently 7.1)
```

Thus IMDB uses a Bayesian estimate for the rating for each movie where $m = \sigma^2/\tau^2 = 25,000$. IMDB has enough data that the uncertainty in $\mu(C)$, σ^2, and τ^2 is pretty minimal.

Clearly incorrect model

We assumed

$$y_{ij} \stackrel{ind}{\sim} N(\theta_i, \sigma^2)$$

for the jth star rating of product i. Clearly this model is incorrect since $y_{ij} \in \{1, 2, 3, 4, 5\}$.

An alternative model is

$$z_{ij} \stackrel{ind}{\sim} Bin(4, \theta_i)$$

where $z_{ij} = y_{ij} - 1$ is the jth star rating minus 1 of product i and

$$\theta_i \sim Be(\alpha, \beta) \qquad \text{and} \qquad p(\alpha, \beta) \propto (\alpha + \beta)^{-5/2}.$$

The idea behind this model would be that product i the probability of earning each star is θ_i and each star is independent.

Binomial hierarchical model in Stan

```
binomial_model = "
data {
  int <lower=1> n;
  int <lower=1> n_products;
  int <lower=1,upper=5> stars[n];
  int <lower=1,upper=n_products> id[n];
}

transformed data {
  int <lower=0, upper=4> z[n];
  for (i in 1:n) z[i] = stars[i]-1;
}

parameters {
  real<lower=0> alpha;
  real<lower=0> beta;
  real<lower=0,upper=1> theta[n_products];
}

model {
  // Prior
  target += -5*log(alpha+beta)/2; // improper prior

  // Hierarchical model
  theta ~ beta(alpha,beta);

  // Data model
  for (i in 1:n) z[i] ~ binomial(4, theta[id[i]]);
}
"
```

Fit model

```
m = stan_model(model_code = binomial_model)
```

```
In file included from file238971aab19c.cpp:8:
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/src/st
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
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In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/BH/include/boost/math/tool
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/BH/include/boost/config.hp
/Library/Frameworks/R.framework/Versions/3.3/Resources/library/BH/include/boost/config/compiler/clang.hpp:196:1
#  define BOOST_NO_CXX11_RVALUE_REFERENCES
          ^
<command line>:6:9: note: previous definition is here
#define BOOST_NO_CXX11_RVALUE_REFERENCES 1
        ^
1 warning generated.
```

```
dat = list(n = nrow(d),
           n_products = nlevels(d$product_id),
           stars = d$stars,
           id = as.numeric(d$product_id))
r = sampling(m, dat)
```

```
SAMPLING FOR MODEL 'df26cacd884bfc422398b77c024fcc16' NOW (CHAIN 1).

Chain 1, Iteration:    1 / 2000 [  0%]  (Warmup)
```

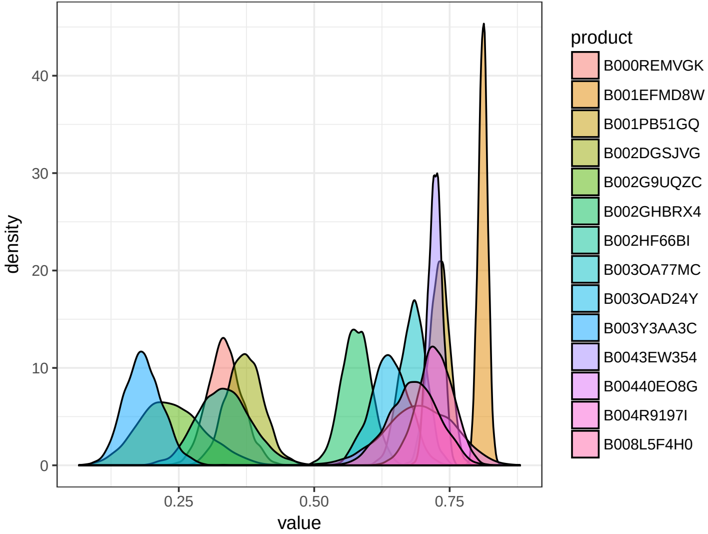
Tabular summary

```
Inference for Stan model: df26cacd884bfc422398b77c024fcc16.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

              mean se_mean   sd     2.5%      25%      50%      75%    97.5% n_eff Rhat
alpha         2.69    0.02 1.09     1.07     1.90     2.52     3.29     5.28  2777    1
beta          2.25    0.02 0.86     0.94     1.62     2.13     2.73     4.20  2931    1
theta[1]      0.34    0.00 0.03     0.27     0.31     0.33     0.36     0.40  4000    1
theta[2]      0.81    0.00 0.01     0.79     0.81     0.81     0.82     0.83  4000    1
theta[3]      0.73    0.00 0.02     0.69     0.72     0.73     0.74     0.77  4000    1
theta[4]      0.37    0.00 0.03     0.31     0.35     0.37     0.39     0.44  4000    1
theta[5]      0.24    0.00 0.06     0.13     0.20     0.23     0.28     0.36  4000    1
theta[6]      0.58    0.00 0.03     0.52     0.56     0.58     0.60     0.63  4000    1
theta[7]      0.33    0.00 0.05     0.24     0.30     0.33     0.37     0.44  4000    1
theta[8]      0.68    0.00 0.02     0.64     0.67     0.68     0.70     0.73  4000    1
theta[9]      0.64    0.00 0.03     0.57     0.62     0.64     0.66     0.70  4000    1
theta[10]     0.18    0.00 0.04     0.12     0.16     0.18     0.21     0.26  4000    1
theta[11]     0.72    0.00 0.01     0.70     0.71     0.72     0.73     0.75  4000    1
theta[12]     0.69    0.00 0.06     0.56     0.65     0.70     0.74     0.81  4000    1
theta[13]     0.72    0.00 0.03     0.66     0.70     0.72     0.75     0.78  4000    1
theta[14]     0.68    0.00 0.05     0.59     0.65     0.68     0.72     0.77  4000    1
lp__      -3265.19    0.06 2.83 -3271.77 -3266.89 -3264.90 -3263.17 -3260.56  2129    1

Samples were drawn using NUTS(diag_e) at Thu Feb 23 11:32:30 2017.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

Review mean posteriors (θ_i)



Other parameter posteriors

Recall that

- α is the prior success
- β is the prior failures

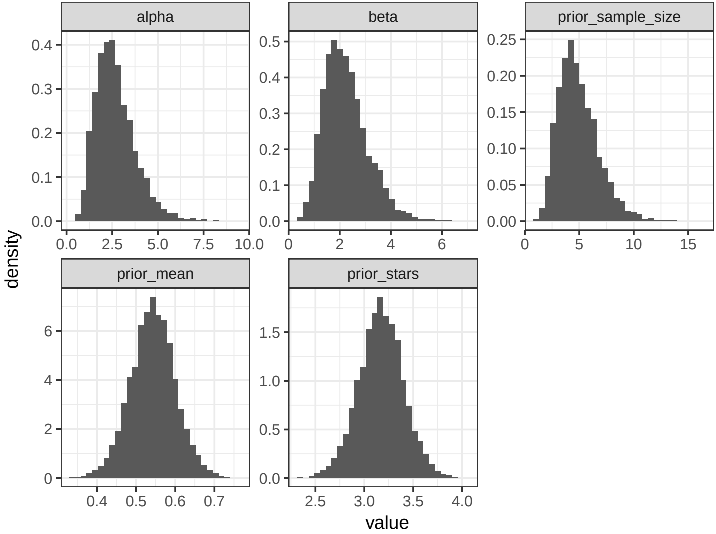
So

- $\alpha + \beta$ is the prior sample size
- $E[\theta_i|\alpha,\beta] = \frac{\alpha}{\alpha+\beta}$ is the prior expectation for the probability

But we might want to show results on the original scale (stars), so the expected number of stars for a new product is

$$\begin{aligned} E[\text{stars}_{*j}|\alpha,\beta] &= E[z_{*j}+1|\alpha,\beta] = E[z_{*j}|\alpha,\beta]+1 \\ &= E[E[z_{*j}|\theta^*]|\alpha,\beta]+1 = E[4\theta^*|\alpha,\beta]+1 \\ &= 4\frac{\alpha}{\alpha+\beta}+1 \end{aligned}$$

Other parameter posteriors



Uniform use of star ratings

This binomial model has the proper support $\{0, 1, 2, 3, 4\}$ for stars minus 1, but does it have the correct proportion of observations in each star category?

As an example, $\hat{\theta}_2 = 0.81$. Thus, we would expect if we used $\hat{\theta}_2$

stars	theoretical	observed
1	0.001	0.076
2	0.022	0.065
3	0.142	0.053
4	0.404	0.146
5	0.430	0.660

But this ignores the uncertainty in θ_2 (95% CI is (0.79, 0.83)), so perhaps this difference is due to this uncertainty.

Posterior predictive pvalue

To assess this model fit, we will simulate posterior predictive star ratings for product 2 and compare to the observed ratings:

product_id	n1	n2	n3	n4	n5	n_total
B001EFMD8W	40	34	28	77	347	526

Let $\tilde{z}_2$ be all the predictive data for product 2, i.e. $\tilde{z}_2 = (\tilde{z}_{21}, \ldots, \tilde{z}_{2J})$ with $J = 526$ where $\tilde{z}_{2j}$ is the jth predictive star rating minus 1 for review j of product 2. Then

$$p(\tilde{z}_2|z) = \int \left[\prod_{j=1}^{J} p(\tilde{z}_{2j}|\theta_2)\right] p(\theta_2|z) d\theta_2$$

Thus the following procedure will simulation from the joint distribution for the predictive ratings:

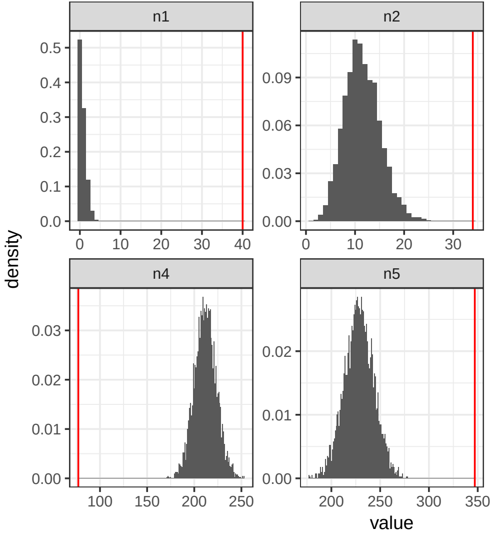
1. $\theta_2 \sim p(\theta_2|z)$,
2. For $j = 1, \ldots, 526$, $z_{2j} \stackrel{ind}{\sim} Bin(4, \theta_2)$, and
3. $\text{star}_{2j} = z_{2j} + 1$.

Posterior predictive distribution in R

```
theta2 = as.numeric(draws$theta[,2])
ztilde2 = adply(theta2, 1, function(x) {
  ztilde = rbinom(526, 4, x) + 1
  data.frame(n1 = sum(ztilde==1),
             n2 = sum(ztilde==2),
             n3 = sum(ztilde==3),
             n4 = sum(ztilde==4),
             n5 = sum(ztilde==5))
})
head(ztilde2)
```

```
  X1 n1 n2 n3  n4  n5
1  1  0 17 79 216 214
2  2  0  9 65 205 247
3  3  0 14 72 206 234
4  4  0  7 78 188 253
5  5  0  8 92 220 206
6  6  0 11 68 222 225
```

Posterior predictive distribution in R



Ordinal data model

Let $s_i = (s_{i1}, \ldots, s_{i5})$ be the vector of the number of 1-star to 5-star ratings for product i, assume

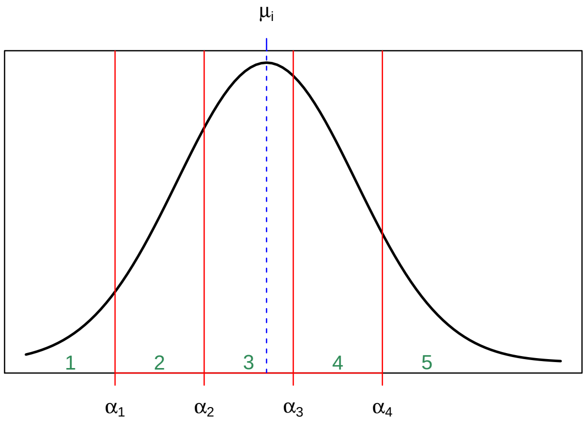
$$S_i \stackrel{ind}{\sim} Mult(n_i, \theta_i)$$

where θ_i is a probability vector

$$\theta_{ik} = \int_{\alpha_{k-1}}^{\alpha_k} N(x|\mu_i, 1)dx = \Phi(\alpha_k - \mu_i) - \Phi(\alpha_{k-1} - \mu_i)$$

where $\alpha_0 = -\infty$, $\alpha_1 = 0$, and $\alpha_5 = \infty$, and Φ is the standard normal cumulative distribution function (cdf).

Visualizing the model



Hierarchical model

So each product has its own mean μ_i. The larger μ_i is the more 5-star ratings the product will receive and the fewer 1-star ratings the product will review.

In order to borrow information across different products, we might assume a hierarchical model for the μ_i, e.g.

$$\mu_i \stackrel{ind}{\sim} N(\eta, \tau^2)$$

with a prior

$$p(\eta, \tau) \propto Ca(\tau; 0, 1).$$

```
ordinal_model = "
data {
  int <lower=1> n_products;
  int <lower=0> s[n_products,5]; // summarized count by product
}

parameters {
  real<lower=0> alpha_diff[3];
  real mu[n_products];
  real eta;
  real<lower=0> tau;
}

transformed parameters {
  ordered[4] alpha;              // cut points
  simplex[5] theta[n_products]; // each theta vector sums to 1

  alpha[1] = 0; for (i in 1:3) alpha[i+1] = alpha[i] + alpha_diff[i];

  for (p in 1:n_products) {
    theta[p,1] = Phi(-mu[p]);
    for (j in 2:4)
      theta[p,j] = Phi(alpha[j]-mu[p]) - Phi(alpha[j-1]-mu[p]);
    theta[p,5] = 1-Phi(alpha[4]-mu[p]);
  }
}

model {
  tau ~ cauchy(0,1);
  mu ~ normal(eta, tau);
  for (p in 1:n_products) s[p] ~ multinomial(theta[p]); // n_reviews[p] is implicit
}
"
```

Fit model

```
m = stan_model(model_code = ordinal_model)


In file included from file23897f9b2fb4.cpp:8:
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/src/st
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
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In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/StanHeaders/include/stan/m
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/BH/include/boost/math/tool
In file included from /Library/Frameworks/R.framework/Versions/3.3/Resources/library/BH/include/boost/config.hp
/Library/Frameworks/R.framework/Versions/3.3/Resources/library/BH/include/boost/config/compiler/clang.hpp:196:1
#  define BOOST_NO_CXX11_RVALUE_REFERENCES
          ^
<command line>:6:9: note: previous definition is here
#define BOOST_NO_CXX11_RVALUE_REFERENCES 1
        ^
1 warning generated.


dat = list(n_products = nrow(for_table),
           s = as.matrix(for_table[,2:6]))
r = sampling(m, dat, pars = c("alpha","eta","tau","mu"))



SAMPLING FOR MODEL 'cfd399bb3e758fc22eaf105a07c2068f' NOW (CHAIN 1).

Chain 1, Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 1, Iteration:  200 / 2000 [ 10%]  (Warmup)
Chain 1, Iteration:  400 / 2000 [ 20%]  (Warmup)
```
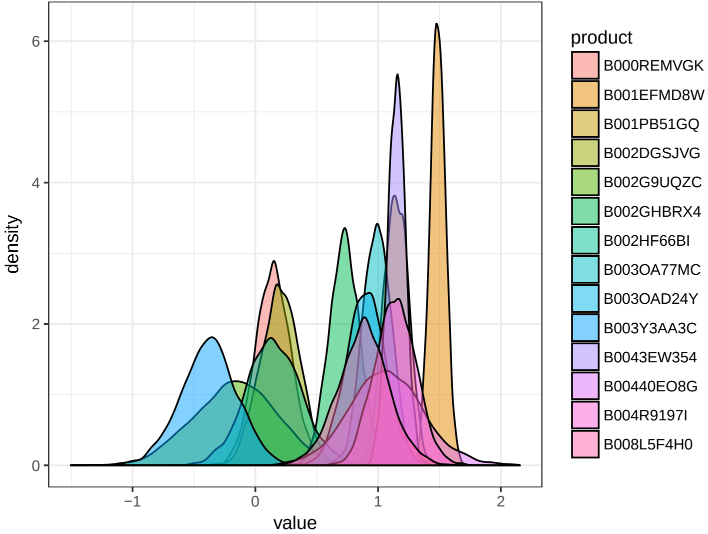
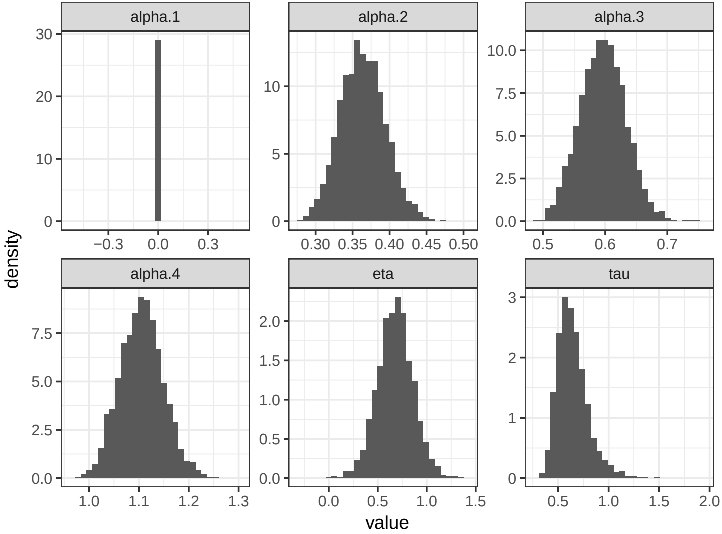

Fit model

```
r


Inference for Stan model: cfd399bb3e758fc22eaf105a07c2068f.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

             mean se_mean   sd     2.5%      25%      50%      75%    97.5% n_eff Rhat
alpha[1]     0.00    0.00 0.00     0.00     0.00     0.00     0.00     0.00  4000  NaN
alpha[2]     0.36    0.00 0.03     0.31     0.34     0.36     0.38     0.43  4000    1
alpha[3]     0.60    0.00 0.04     0.53     0.57     0.60     0.62     0.67  4000    1
alpha[4]     1.11    0.00 0.04     1.03     1.08     1.11     1.13     1.19  4000    1
eta          0.68    0.00 0.18     0.31     0.56     0.68     0.80     1.04  4000    1
tau          0.65    0.00 0.15     0.42     0.54     0.62     0.73     1.02  4000    1
mu[1]        0.15    0.00 0.14    -0.13     0.05     0.15     0.24     0.43  4000    1
mu[2]        1.49    0.00 0.06     1.36     1.45     1.49     1.53     1.61  4000    1
mu[3]        1.15    0.00 0.10     0.95     1.08     1.15     1.22     1.33  4000    1
mu[4]        0.20    0.00 0.16    -0.10     0.10     0.20     0.31     0.50  4000    1
mu[5]       -0.17    0.01 0.33    -0.81    -0.39    -0.16     0.06     0.46  4000    1
mu[6]        0.73    0.00 0.12     0.48     0.64     0.73     0.81     0.97  4000    1
mu[7]        0.15    0.00 0.22    -0.30     0.00     0.14     0.30     0.59  4000    1
mu[8]        0.99    0.00 0.11     0.77     0.91     0.99     1.07     1.21  4000    1
mu[9]        0.90    0.00 0.16     0.59     0.79     0.90     1.00     1.22  4000    1
mu[10]      -0.38    0.00 0.23    -0.84    -0.53    -0.37    -0.23     0.06  4000    1
mu[11]       1.15    0.00 0.07     1.01     1.10     1.15     1.20     1.29  4000    1
mu[12]       1.07    0.00 0.30     0.48     0.87     1.06     1.26     1.69  4000    1
mu[13]       1.14    0.00 0.17     0.83     1.03     1.14     1.26     1.47  4000    1
mu[14]       0.88    0.00 0.21     0.47     0.75     0.89     1.02     1.30  4000    1
lp__     -1835.69    0.09 3.13 -1842.60 -1837.59 -1835.37 -1833.46 -1830.53  1206    1

Samples were drawn using NUTS(diag_e) at Thu Feb 23 11:33:47 2017.
```

Review mean posteriors (θ_i)



Other parameter posteriors



Visualizing the model