Set R07 - Contrasts

STAT 401 (Engineering) - Iowa State University

April 5, 2017

Simple hypothesis

Consider the one-way ANOVA model: $Y_{ij} \sim N(\mu_j, \sigma^2)$ where $j = 1, \ldots, J$.

Here are a few simple alternative hypotheses:

1. Mean lifetimes for N/R50 and R/R50 diet are different.
2. Mean lifetimes for N/R40 is different than for N/R50 and R/R50 combined.
3. Mean lifetimes for high calorie (NP and N/N85) diets is different than for low calorie diets combined.

$H_0 : \gamma = 0 \qquad H_1 : \gamma \neq 0 :$

$$
\begin{aligned}
\gamma_1 &= \mu_{R/R50} - \mu_{N/R50} \\
\gamma_2 &= \mu_{N/R40} - \frac{1}{2}(\mu_{N/R50} + \mu_{R/R50}) \\
\gamma_3 &= \frac{1}{4}(\mu_{N/R50} + \mu_{R/R50} + \mu_{N/R40} + \mu_{lopro}) - \frac{1}{2}(\mu_{NP} + \mu_{N/N85})
\end{aligned}
$$

Contrasts

Definition

A linear combination of group means has the form

$$\gamma = C_1\mu_1 + C_2\mu_2 + \ldots + C_J\mu_J$$

where C_j are known coefficients and μ_j are the unknown population means.

Definition

A linear combination with $C_1 + C_2 + \cdots + C_J = 0$ is a contrast.

Remark Contrast interpretation is usually best if $|C_1| + |C_2| + \cdots + |C_J| = 2$, i.e. the positive coefficients sum to 1 and the negative coefficients sum to -1.

Inference on contrasts

$$\gamma = C_1\mu_1 + C_2\mu_2 + \cdots + C_J\mu_J$$

Estimated by

$$g = C_1\overline{Y}_1 + C_2\overline{Y}_2 + \cdots + C_J\overline{Y}_J$$

with standard error

$$SE(g) = s_p\sqrt{\frac{C_1^2}{n_1} + \frac{C_2^2}{n_2} + \cdots + \frac{C_J^2}{n_J}}$$

t-statistic (compare to t_{n-J}) and CI:

$$t = \frac{g}{SE(g)} \qquad g \pm t_{n-J,1-\alpha/2}SE(g)$$

Contrasts for mice lifetime dataset

For these contrasts:

1. Mean lifetimes for N/R50 and R/R50 diet are different.
2. Mean lifetimes for N/R40 is different than for N/R50 and R/R50 combined.
3. Mean lifetimes for high calorie (NP and N/N85) diets is different than for low calorie diets combined.

$H_0 : \gamma = 0 \qquad H_1 : \gamma \neq 0 :$

$$
\begin{aligned}
\gamma_1 &= \mu_{R/R50} - \mu_{N/R50} \\
\gamma_2 &= \mu_{N/R40} - \tfrac{1}{2}(\mu_{N/R50} + \mu_{R/R50}) \\
\gamma_3 &= \tfrac{1}{4}(\mu_{N/R50} + \mu_{R/R50} + \mu_{N/R40} + \mu_{lopro}) - \tfrac{1}{2}(\mu_{NP} + \mu_{N/N85})
\end{aligned}
$$

	N/N85	N/R40	N/R50	NP	R/R50	lopro
early rest - none @ 50kcal	0.00	0.00	-1.00	0.00	1.00	0.00
40kcal/week - 50kcal/week	0.00	1.00	-0.50	0.00	-0.50	0.00
lo cal - hi cal	-0.50	0.25	0.25	-0.50	0.25	0.25

Mice lifetime examples

	Diet	n	mean	sd
1	N/N85	57	32.69	5.13
2	N/R40	60	45.12	6.70
3	N/R50	71	42.30	7.77
4	NP	49	27.40	6.13
5	R/R50	56	42.89	6.68
6	lopro	56	39.69	6.99

Contrasts:

	g	SE(g)	t	p	L	U
early rest - none @ 50kcal	0.59	1.19	0.49	0.62	-1.76	2.94
40kcal/week - 50kcal/week	2.53	1.05	2.41	0.02	0.46	4.59
lo cal - hi cal	12.45	0.78	15.96	0.00	10.92	13.98

```
library(multcomp)
m = lm(Lifetime~Diet-1, case0501) # The -1 indicates no intercept (see Ch 7)
summary(m)


Call:
lm(formula = Lifetime ~ Diet - 1, data = case0501)

Residuals:
     Min       1Q   Median       3Q      Max
-25.5167  -3.3857   0.8143   5.1833  10.0143

Coefficients:
          Estimate Std. Error t value Pr(>|t|)
DietN/N85  32.6912     0.8846   36.96   <2e-16 ***
DietN/R40  45.1167     0.8622   52.33   <2e-16 ***
DietN/R50  42.2972     0.7926   53.37   <2e-16 ***
DietNP     27.4020     0.9540   28.72   <2e-16 ***
DietR/R50  42.8857     0.8924   48.06   <2e-16 ***
Dietlopro  39.6857     0.8924   44.47   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.678 on 343 degrees of freedom
Multiple R-squared:  0.9724, Adjusted R-squared:  0.9719
F-statistic:  2011 on 6 and 343 DF,  p-value: < 2.2e-16


K


                           N/N85 N/R40 N/R50   NP R/R50 lopro
early rest - none @ 50kcal   0.0  0.00 -1.00  0.0  1.00  0.00
40kcal/week - 50kcal/week    0.0  1.00 -0.50  0.0 -0.50  0.00
lo cal - hi cal             -0.5  0.25  0.25 -0.5  0.25  0.25
```

```r
t = glht(m, linfct=K)
summary(t)
```

```
 Simultaneous Tests for General Linear Hypotheses

Fit: lm(formula = Lifetime ~ Diet - 1, data = case0501)

Linear Hypotheses:
                                Estimate Std. Error t value Pr(>|t|)
early rest - none @ 50kcal == 0   0.5885     1.1936   0.493   0.9457
40kcal/week - 50kcal/week == 0    2.5252     1.0485   2.408   0.0488 *
lo cal - hi cal == 0             12.4497     0.7800  15.961   <1e-04 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Adjusted p values reported -- single-step method)
```

```r
confint(t, calpha=univariate_calpha())
```

```
 Simultaneous Confidence Intervals

Fit: lm(formula = Lifetime ~ Diet - 1, data = case0501)

Quantile = 1.9669
95% confidence level


Linear Hypotheses:
                                Estimate lwr     upr
early rest - none @ 50kcal == 0  0.5885  -1.7591  2.9361
40kcal/week - 50kcal/week == 0   2.5252   0.4628  4.5876
lo cal - hi cal == 0            12.4497  10.9155 13.9839
```

Summary

- Contrasts are linear combinations that sum to zero
- t-test tools are used to calculate pvalues and confidence intervals

Sulfur effect on scab disease in potatoes

The experiment was conducted to investigate the effect of sulfur on controlling scab disease in potatoes. There were seven treatments: control, plus spring and fall application of 300, 600, 1200 lbs/acre of sulfur. The response variable was percentage of the potato surface area covered with scab averaged over 100 random selected potatoes. A completely randomized design was used with 8 replications of the control and 4 replications of the other treatments.

Cochran and Cox. (1957) Experimental Design (2nd ed). pg96 and Agron. J. 80:712-718 (1988)

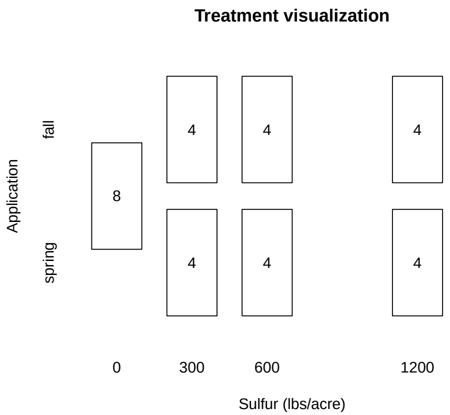
Scientific question:

- Does sulfur have any impact at all?
- Is there a difference between spring and fall?
- Is there an effect of increased sulfur (expect more sulfur causes less scab)?

Data

```
   inf trt row col
1    9  F3   4   1
2   12   0   4   2
3   18  S6   4   3
4   10 F12   4   4
5   24  S6   4   5
6   17 S12   4   6
7   30  S3   4   7
8   16  F6   4   8
9   10   0   3   1
10   7  S3   3   2
11   4 F12   3   3
12  10  F6   3   4
13  21  S3   3   5
14  24   0   3   6
15  29   0   3   7
16  12  S6   3   8
17   9  F3   2   1
18   7 S12   2   2
19  18  F6   2   3
20  30   0   2   4
21  18  F6   2   5
22  16 S12   2   6
23  16  F3   2   7
24   4 F12   2   8
25   9  S3   1   1
26  18   0   1   2
27  17 S12   1   3
28  19  S6   1   4
29  32   0   1   5
30   5 F12   1   6
```

Design

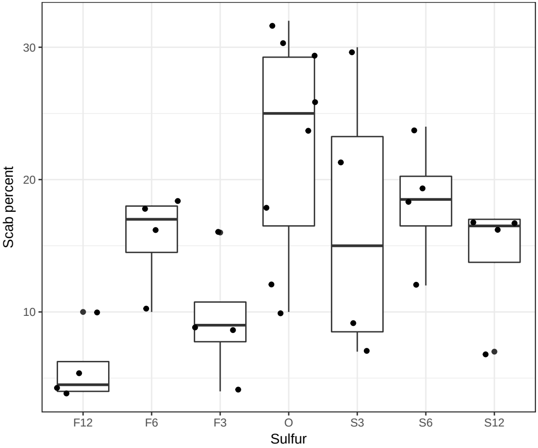

Treatment visualization
Application
fall
spring
8
4
4
4
4
4
4
0
300
600
1200
Sulfur (lbs/acre)

Design

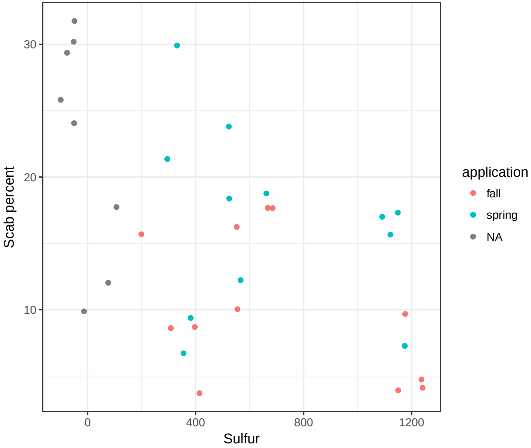
Completely randomized design
potato scab experiment

row \ col	1	2	3	4	5	6	7	8
4	F3	O	S6	F12	S6	S12	S3	F6
3	O	S3	F12	F6	S3	O	O	S6
2	F3	S12	F6	O	F6	S12	F3	F12
1	S3	O	S12	S6	O	F12	O	F3

Data

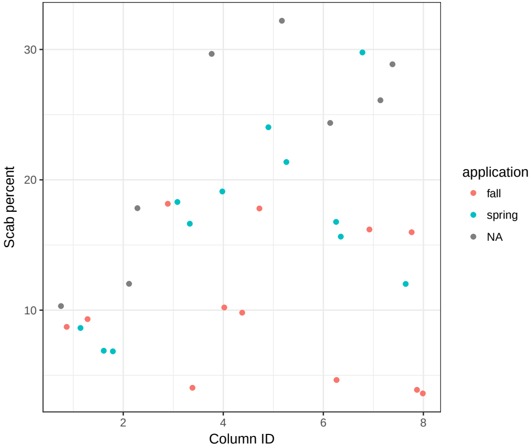


Data

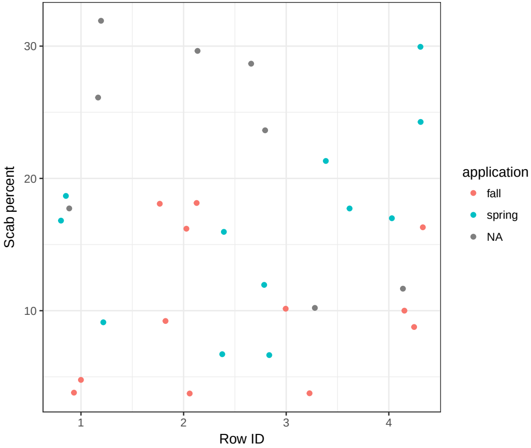


Scab percent
30
20
10
0
400
800
1200
Sulfur
application
fall
spring
NA

Data


Scab percent
30
20
10
2
4
6
8
Column ID
application
fall
spring
NA

Data



Model

Y_{ij}: avg % of surface area covered with scab for plot i in treatment j for $j = 1, \ldots, 7$.

Assume $Y_{ij} \stackrel{ind}{\sim} N(\mu_j, \sigma^2)$.

Hypotheses:

- Difference amongst any means: One-way ANOVA F-test
- *Any effect*: Control vs sulfur
- *Fall vs spring*: Contrast comparing fall vs spring applications
- *Sulfur level*: Linear trend contrast

Control vs sulfur

$$
\begin{aligned}
\gamma &= \frac{1}{6}(\mu_{F12} + \mu_{F6} + \mu_{F3} + \mu_{S3} + \mu_{S6} + \mu_{S12}) - \mu_O \\
&= \frac{1}{6}(\mu_{F12} + \mu_{F6} + \mu_{F3} + \mu_{S3} + \mu_{S6} + \mu_{S12} - 6\mu_O)
\end{aligned}
$$

Fall vs spring contrast

- *Fall vs spring*: Contrast comparing fall vs spring applications

$$
\begin{aligned}
\gamma &= \tfrac{1}{3}(\mu_{F12} + \mu_{F6} + \mu_{F3}) + 0\mu_O - \tfrac{1}{3}(\mu_{S3} + \mu_{S6} + \mu_{S12}) \\
&= \tfrac{1}{3}\mu_{F12} + \tfrac{1}{3}\mu_{F6} + \tfrac{1}{3}\mu_{F3} + 0\mu_O - \tfrac{1}{3}\mu_{S3} - \tfrac{1}{3}\mu_{S6} - \tfrac{1}{3}\mu_{S12} \\
&= \tfrac{1}{3}\left[\mu_{F12} + \mu_{F6} + \mu_{F3} + 0\mu_O - 1\mu_{S3} - 1\mu_{S6} - 1\mu_{S12}\right]
\end{aligned}
$$

Sulfur level: linear trend contrasts

- The unique sulfur levels (X_i) are 0, 3, 6, and 12.
- So the linear trend contrast ($X_i - \overline{X}$) is

X_i	0	3	6	12
$X_i - \overline{X}$	$-\frac{21}{4}$	$-\frac{9}{4}$	$\frac{3}{4}$	$\frac{27}{4}$

- But 3, 6, and 12 are duplicated, so we need the average of the groups

$$\begin{aligned}
\gamma &= -\tfrac{21}{4}\mu_0 - \tfrac{9}{4}\mu_3 + \tfrac{3}{4}\mu_6 + \tfrac{27}{4}\mu_{12} \\
&= -\tfrac{21}{4}\mu_0 - \tfrac{9}{4}\left(\tfrac{\mu_{S3}+\mu_{F3}}{2}\right) + \tfrac{3}{4}\left(\tfrac{\mu_{S6}+\mu_{F6}}{2}\right) + \tfrac{27}{4}\left(\tfrac{\mu_{S12}+\mu_{F12}}{2}\right) \\
&= \tfrac{1}{8}\left[-42\mu_0 - 9\mu_{S3} - 9\mu_{F3} + 3\mu_{S6} + 3\mu_{F6} + 27\mu_{S12} + 27\mu_{F12}\right]
\end{aligned}$$

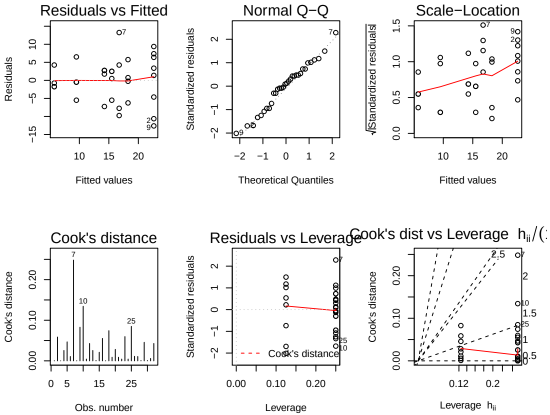
Contrasts

Trt	F12	F6	F3	O	S3	S6	S12	Div
Sulfur v control	1	1	1	-6	1	1	1	6
Fall v Spring	1	1	1	0	-1	-1	-1	3
Linear Trend	27	3	-9	-42	-9	3	27	8

```r
library(multcomp)
K = rbind("sulfur - control" = c(1, 1, 1, -6, 1, 1, 1)/6,
          "fall - spring"    = c(1,1,1,0,-1,-1,-1)/3,
          "linear trend"     = c(27,3,-9,-42,-9,3,27)/8)
m = lm(inf~trt,d)
anova(m)
```

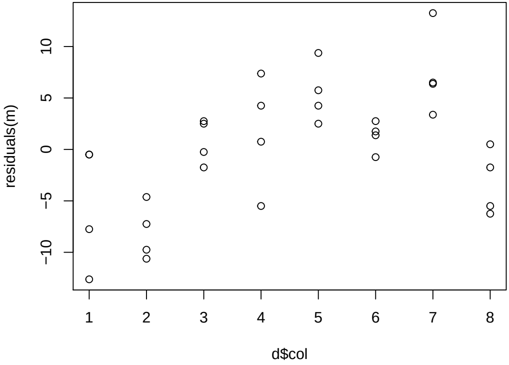
```
Analysis of Variance Table

Response: inf
          Df  Sum Sq Mean Sq F value  Pr(>F)
trt        6  972.34 162.057  3.6081 0.01026 *
Residuals 25 1122.88  44.915
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
par(mfrow=c(2,3))
plot(m,1:6)
```



```
g = glht(lm(inf~trt-1,d), linfct=K) # notice the -1 in the model
summary(g, test=adjusted(type="none")) # unadjusted pvalues


 Simultaneous Tests for General Linear Hypotheses

Fit: lm(formula = inf ~ trt - 1, data = d)

Linear Hypotheses:
                      Estimate Std. Error t value Pr(>|t|)
sulfur - control == 0   -9.292      2.736  -3.396  0.00229 **
fall - spring == 0      -6.167      2.736  -2.254  0.03322 *
linear trend == 0      -68.156     21.027  -3.241  0.00336 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Adjusted p values reported -- none method)


confint(g, calpha=univariate_calpha()) # unadjusted confidence intervals


 Simultaneous Confidence Intervals

Fit: lm(formula = inf ~ trt - 1, data = d)

Quantile = 2.0595
95% confidence level


Linear Hypotheses:
                      Estimate  lwr       upr
sulfur - control == 0   -9.2917  -14.9266   -3.6567
fall - spring == 0      -6.1667  -11.8016   -0.5317
linear trend == 0      -68.1562 -111.4620  -24.8505
```

```
plot(d$col,residuals(m))
```



Summary

For this particular data analysis

- Significant differences in means between the groups (ANOVA $F_{6,25} = 3.61$ p=0.01)
- Sulfur had a significant impact on scab (p=0.002)
- Fall was better than spring (p=0.03, 95% CI (0.53, 11.8))
- Linear trend in sulfur was significant(p=0.003)

- Concerned about spatial correlation among columns
- Consider a transformation of the response
  - CI for F12 (-1.2, 12.7)
  - Non-constant variance (residuals vs predicted, sulfur, application)