R08 - Experimental design

STAT 587 (Engineering) - Iowa State University

April 23, 2019

Random samples and random treatment assignment

Recall that the objective of data analysis is often to make an inference about a population based on a sample. For the inference to be statistically valid, we need a random sample fromt the population.

Often we also want to make a causal statement about the relationship between explanatory variables (X) and a response (Y). In order to make a causal statment, the levels of the explanatory variables need to be randomly assigned to the experimental units. If levels are randomly assigned, we often refer to the explanatory variables as treatments and refer to the data collection as a randomized experiment. If the levels are not (randomly) assigned, we refer to the data collection as an observational study.

Data collection

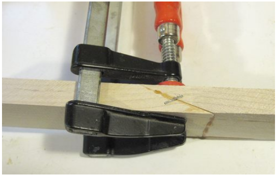
	Treatment randomly assigned?	
	No	Yes
Sample	Observational study	Randomized experiment
Not random	No cause-and-effect No inference to population	Yes cause-and-effect No inference to population
Random	No cause-and-effect Yes inference to population	Yes cause-and-effect Yes inference to population

Strength of wood glue

You are interested in testing two different wood glues:

- Gorilla Wood Glue
- Titebond 1413 Wood Glue

On a scarf joint:



So you collect up some wood, glue the pieces together, and determine the weight required to break the joint. (There are lots of details missing here.)

Inspiration: `https://woodgears.ca/joint_strength/glue.html`

Completely Randomized Design (CRD)

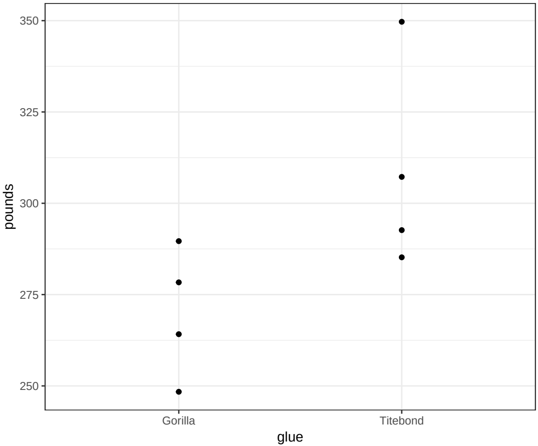
Suppose I have 8 pieces of wood laying around. I cut each piece and randomly use either Gorilla or Titebond glue to recombine the pieces. I do the randomization in such a way that I have exactly 4 Gorilla and 4 Titebond results, e.g.

```
# A tibble: 8 x 2
  woodID glue
  <fct>  <chr>
1 wood1  Gorilla
2 wood2  Titebond
3 wood3  Gorilla
4 wood4  Titebond
5 wood5  Titebond
6 wood6  Titebond
7 wood7  Gorilla
8 wood8  Gorilla
```

This is called a completely randomized design (CRD).

Visualize the data

```r
ggplot(d, aes(glue, pounds)) + geom_point() + theme_bw()
```



Model

Let

- P_w be the weight (pounds) needed to break wood w,
- T_w be an indicator that the Titebond glue was used on wood w, i.e.

$$T_w = \mathrm{I}(\text{glue}_w = \text{Titebond}).$$

Then a regression model for these data is

$$P_w \stackrel{ind}{\sim} N(\beta_0 + \beta_1 T_w, \sigma^2)$$

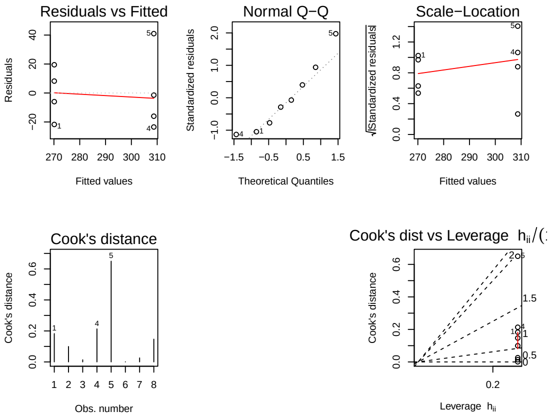
where

- β_1 is the expected difference in weight when using Titebond glue compared to using Gorilla glue.

Check model assumptions

```
m <- lm(pounds ~ glue, data = d)
opar = par(mfrow=c(2,3)); plot(m, 1:6, ask=FALSE); par(opar)


hat values (leverages) are all = 0.25
and there are no factor predictors; no plot no.  5
```



Obtain statistics

```
coefficients(m)
```

```
 (Intercept) glueTitebond 
   270.13553     38.55651 
```

```
summary(m)$r.squared
```

```
[1] 0.4630249
```

```
confint(m)
```

```
                  2.5 %    97.5 %
(Intercept)  240.806326 299.46474
glueTitebond  -2.921249  80.03428
```

```
emmeans(m, ~glue)
```

```
 glue     emmean SE df lower.CL upper.CL
 Gorilla     270 12  6      241      299
 Titebond    309 12  6      279      338

Confidence level used: 0.95 
```

Interpret results

A randomized experiment was designed to evaluate the effectiveness of Gorilla and Titebond in preventing failures in scarf joints cut at a 20 degree angle through 1" $\times$ 2" spruce with 4 replicates for each glue type. The mean break weight (pounds) was 270 with a 95% CI of (241,299) for Gorilla and 309 (279, 338) for Titebond. Titebond glue caused an increase in break weight of 39 (-3,80) compared to Gorilla Glue type accounted for 46% of the variability in break weight.

Randomized complete block design (RCBD)

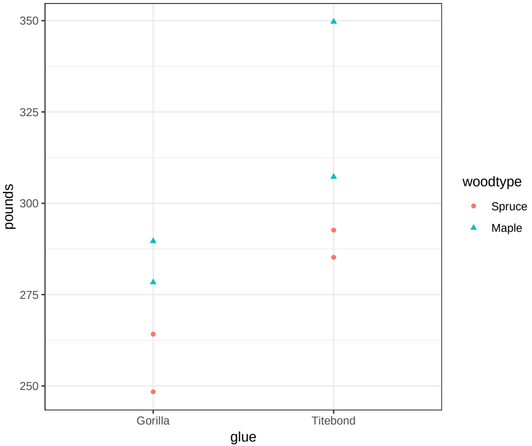
Suppose the wood actually came from two different types: Maple and Spruce. And perhaps you have reason to believe the glue will work differently depending on the type of wood. In this case, you would want to block by wood type and perform the randomization within each block, i.e.

```
# A tibble: 8 x 3
  woodID woodtype glue
  <fct>  <fct>    <chr>
1 wood1  Spruce   Gorilla
2 wood2  Spruce   Titebond
3 wood3  Spruce   Gorilla
4 wood4  Spruce   Titebond
5 wood5  Maple    Titebond
6 wood6  Maple    Titebond
7 wood7  Maple    Gorilla
8 wood8  Maple    Gorilla
```

This is called a randomized complete block design (RCBD).

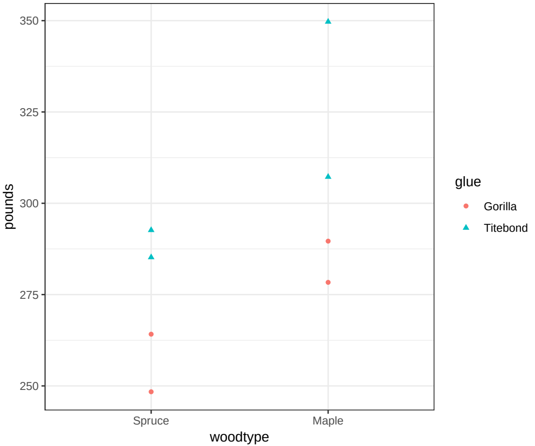
Visualize the data

```
ggplot(d, aes(glue, pounds, color=woodtype, shape=woodtype)) + geom_point() + theme_bw()
```



Visualize the data - a more direct comparison

```
ggplot(d, aes(woodtype, pounds, color=glue, shape=glue)) + geom_point() + theme_bw()
```



Main effects model

Let

- P_w be the weight (pounds) needed to break wood w
- T_w be an indicator that Titebond glue was used on wood w, and
- M_w be an indicator that wood w was Maple.

Then a regression model for these data is

$$P_w \stackrel{ind}{\sim} N(\beta_0 + \beta_1 T_w + \beta_2 M_w, \sigma^2)$$

where

- β_1 is the expected difference in weight when using Titebond glue compared to using Gorilla glue when adjusted for type of wood, i.e. the type of wood is held constant, and
- β_2 is the expected difference in weight when using Spruce compared to Maple when adjusted for type of glue, i.e. the glue is held constant.

Perform analysis

```
m <- lm(pounds ~ woodtype + glue, data = d)
summary(m)


Call:
lm(formula = pounds ~ woodtype + glue, data = d)

Residuals:
      1       2       3       4       5       6       7       8 
 -4.929   0.768  10.835  -6.674  24.186 -18.279  -8.594   2.688 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)    253.324      9.435  26.848 1.34e-06 ***
woodtypeMaple   33.623     10.895   3.086   0.0273 *  
glueTitebond    38.557     10.895   3.539   0.0166 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.41 on 5 degrees of freedom
Multiple R-squared:  0.8151,Adjusted R-squared:  0.7412 
F-statistic: 11.02 on 2 and 5 DF,  p-value: 0.01469


confint(m)


                  2.5 %    97.5 %
(Intercept)   229.069570 277.57817
woodtypeMaple   5.616873  61.62978
glueTitebond   10.550061  66.56297
```

Replication

Since there are more than one observation for each woodtype-glue combination, the design is replicated:

```
d %>% group_by(woodtype, glue) %>% summarize(n = n())
```

```
# A tibble: 4 x 3
# Groups:   woodtype [?]
  woodtype glue         n
  <fct>    <chr>    <int>
1 Spruce   Gorilla      2
2 Spruce   Titebond     2
3 Maple    Gorilla      2
4 Maple    Titebond     2
```

When the design is replicated, we can consider assessing an interaction. In this example, an interaction between glue and woodtype would indicate that the effect of glue depends on the woodtype, i.e. the difference in expected weight between the two glues depends on woodtype. At an extreme, it could be that Gorilla works better on Spruce and Titebond works better on Maple.

Interaction model

Let

- P_w be the weight (pounds) needed to break wood w
- T_w be an indicator that Titebond glue was used on wood w, and
- M_w be an indicator that wood w was Maple.

Then a regression model for these data is

$$P_w \stackrel{ind}{\sim} N(\beta_0 + \beta_1 T_w + \beta_2 M_w + \beta_3 T_w M_w, \sigma^2)$$

where

- β_1 is the expected difference in weight when moving from Gorilla to Titebond glue for Spruce,
- β_2 is the expected difference in weight when moving from Spruce to Maple for Gorilla glue, and
- β_3 is more complicated.

Assessing an interaction using a t-test

```
m <- lm(pounds ~ woodtype * glue, data = d)
summary(m)


Call:
lm(formula = pounds ~ woodtype * glue, data = d)

Residuals:
      1       2       3       4       5       6       7       8 
 -7.882   3.721   7.882  -3.721  21.233 -21.233  -5.641   5.641 

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)    
(Intercept)                  256.28      11.82  21.686 2.67e-05 ***
woodtypeMaple                 27.72      16.71   1.658    0.173    
glueTitebond                  32.65      16.71   1.954    0.122    
woodtypeMaple:glueTitebond    11.81      23.64   0.500    0.643    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 16.71 on 4 degrees of freedom
Multiple R-squared:  0.826,Adjusted R-squared:  0.6955 
F-statistic:  6.33 on 3 and 4 DF,  p-value: 0.05335
```

Assessing an interaction using an F-test

```
anova(m)
```

```
Analysis of Variance Table

Response: pounds
              Df  Sum Sq Mean Sq F value  Pr(>F)
woodtype       1 2261.06 2261.06  8.0952 0.04662 *
glue           1 2973.21 2973.21 10.6449 0.03100 *
woodtype:glue  1   69.77   69.77  0.2498 0.64346
Residuals      4 1117.24  279.31
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

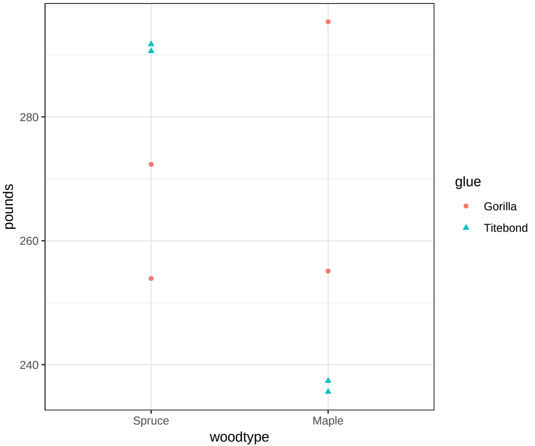
```
drop1(m, test='F')
```

```
Single term deletions

Model:
pounds ~ woodtype * glue
              Df Sum of Sq    RSS    AIC F value Pr(>F)
<none>                     1117.2 47.513
woodtype:glue  1    69.769 1187.0 45.998  0.2498 0.6435
```

What if this had been your data?

```
ggplot(d, aes(woodtype, pounds, color=glue, shape=glue)) + geom_point() + theme_bw()
```



Assessing an interaction using a t-test

```
m <- lm(pounds ~ woodtype * glue, data = d)
summary(m)


Call:
lm(formula = pounds ~ woodtype * glue, data = d)

Residuals:
      1        2        3        4        5        6        7        8 
 -9.2083  -0.5529   0.5529   9.2083  -0.8764  20.1215 -20.1215   0.8764 

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)    
(Intercept)                  263.12      11.08  23.755 1.86e-05 ***
woodtypeMaple                 12.10      15.66   0.773   0.4829    
glueTitebond                  28.03      15.66   1.790   0.1480    
woodtypeMaple:glueTitebond   -66.76      22.15  -3.014   0.0394 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.66 on 4 degrees of freedom
Multiple R-squared:  0.7648,Adjusted R-squared:  0.5883 
F-statistic: 4.335 on 3 and 4 DF,  p-value: 0.09522
```

Unreplicated study

Suppose you now have

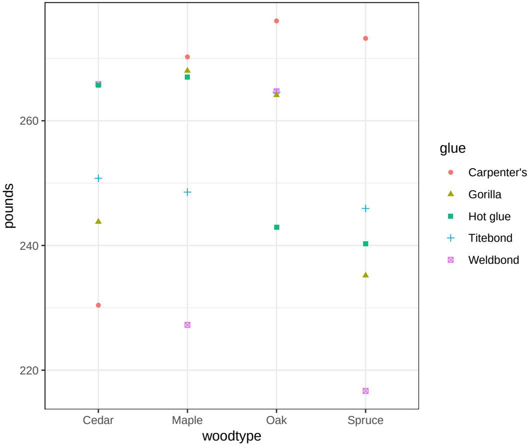
- 5 glue choices
- 4 different types of wood with
- 5 samples of each type of wood.

Thus you can only run each glue choice once on each type of wood.

Then you can run an unreplicated RCBD.

Visualize

```
ggplot(d, aes(woodtype, pounds, color=glue, shape=glue)) +
  geom_point() + theme_bw()
```



Fit the main effects (or additive) model

```
m <- lm(pounds ~ woodtype + glue, data = d)
anova(m)

Analysis of Variance Table

Response: pounds
          Df Sum Sq Mean Sq F value Pr(>F)
woodtype   3 1091.4  363.80  1.1474 0.3697
glue       4  714.8  178.71  0.5636 0.6937
Residuals 12 3804.9  317.07
```

Fit the main effects (or additive) model

```
summary(m)


Call:
lm(formula = pounds ~ woodtype + glue, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-30.302  -7.093   2.316  10.326  23.992 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)     260.717     11.262  23.150 2.51e-11 ***
woodtypeMaple     4.907     11.262   0.436    0.671    
woodtypeOak      11.157     11.262   0.991    0.341    
woodtypeSpruce   -9.056     11.262  -0.804    0.437    
glueGorilla      -9.696     12.591  -0.770    0.456    
glueHot glue     -8.460     12.591  -0.672    0.514    
glueTitebond    -10.018     12.591  -0.796    0.442    
glueWeldbond    -18.834     12.591  -1.496    0.161    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 17.81 on 12 degrees of freedom
Multiple R-squared:  0.3219,Adjusted R-squared:  -0.07366 
F-statistic: 0.8138 on 7 and 12 DF,  p-value: 0.5931
```

Fit the full (with interaction) model

```
m <- lm(pounds ~ woodtype * glue, data = d)
anova(m)

Warning in anova.lm(m):  ANOVA F-tests on an essentially perfect fit are unreliable

Analysis of Variance Table

Response: pounds
              Df Sum Sq Mean Sq F value Pr(>F)
woodtype       3 1091.4  363.80
glue           4  714.8  178.71
woodtype:glue 12 3804.9  317.07
Residuals      0    0.0
```

Fit the full (with interaction) model

```
summary(m)


Call:
lm(formula = pounds ~ woodtype * glue, data = d)

Residuals:
ALL 20 residuals are 0: no residual degrees of freedom!

Coefficients:
                            Estimate Std. Error t value Pr(>|t|)
(Intercept)                   230.41         NA      NA       NA
woodtypeMaple                  39.83         NA      NA       NA
woodtypeOak                    45.59         NA      NA       NA
woodtypeSpruce                 42.80         NA      NA       NA
glueGorilla                    13.38         NA      NA       NA
glueHot glue                   35.32         NA      NA       NA
glueTitebond                   20.35         NA      NA       NA
glueWeldbond                   35.46         NA      NA       NA
woodtypeMaple:glueGorilla     -15.61         NA      NA       NA
woodtypeOak:glueGorilla       -25.27         NA      NA       NA
woodtypeSpruce:glueGorilla    -51.41         NA      NA       NA
woodtypeMaple:glueHot glue    -38.52         NA      NA       NA
woodtypeOak:glueHot glue      -68.37         NA      NA       NA
woodtypeSpruce:glueHot glue   -68.22         NA      NA       NA
woodtypeMaple:glueTitebond    -42.03         NA      NA       NA
woodtypeOak:glueTitebond      -31.80         NA      NA       NA
woodtypeSpruce:glueTitebond   -47.64         NA      NA       NA
woodtypeMaple:glueWeldbond    -78.44         NA      NA       NA
woodtypeOak:glueWeldbond      -46.74         NA      NA       NA
woodtypeSpruce:glueWeldbond   -92.00         NA      NA       NA
```