STAT 401A - Statistical Methods for Research Workers

Regression diagnostics

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>
> *This isn't just true in statistics! Maps are a type of model; they are wrong. But good maps are very useful.*

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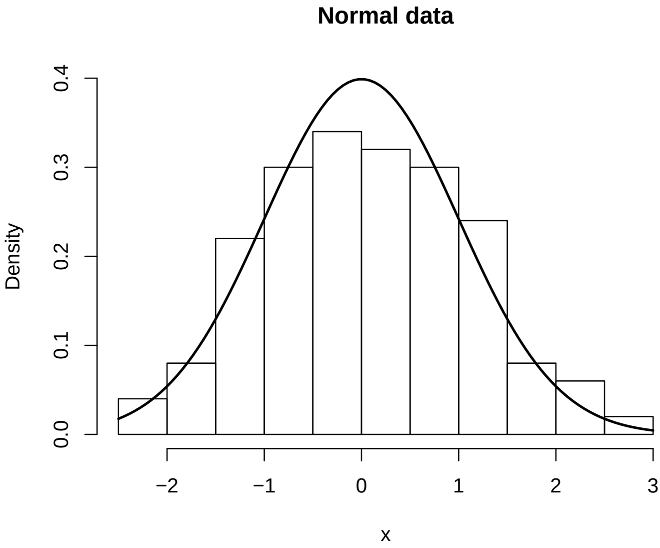
- Normality of the errors
- Constant variance of the errors
- Independence of the errors
- Linearity between mean response and explanatory variable

Histograms with best fitting bell curves



Normal QQ-plot

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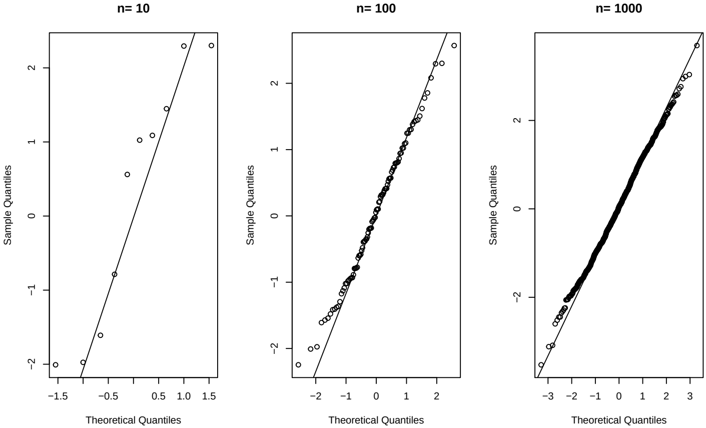
A normal qq-plot graphs the theoretical quantiles from a normal distribution versus the observed quantiles.

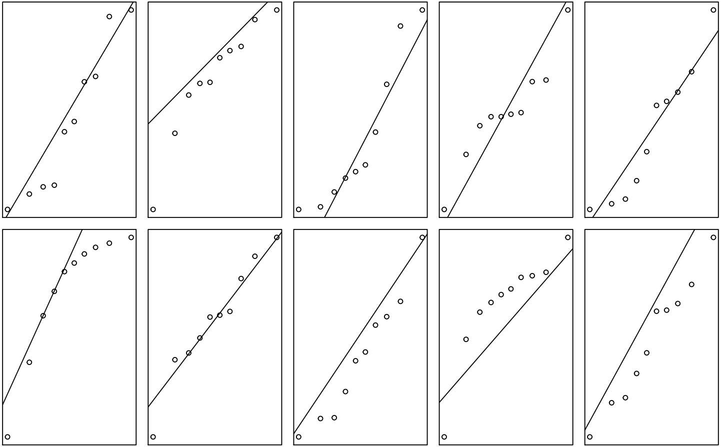
Remark The bottom line is that, if the distribution assumption is satisfied, the points should fall roughly along the y=x line.

Normal



SAS swaps the x and y axes

Normal (n=10)



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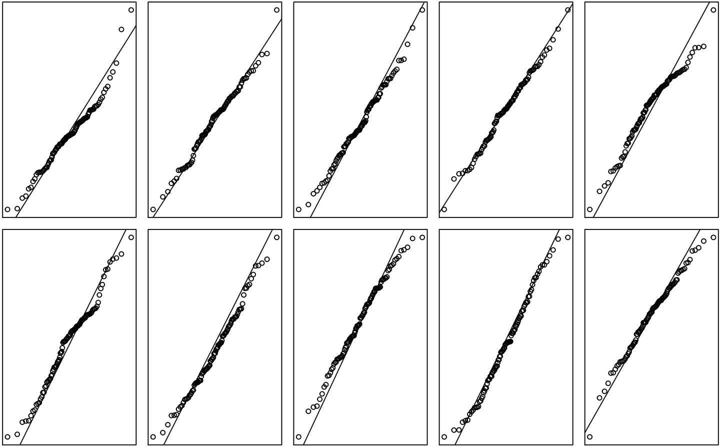

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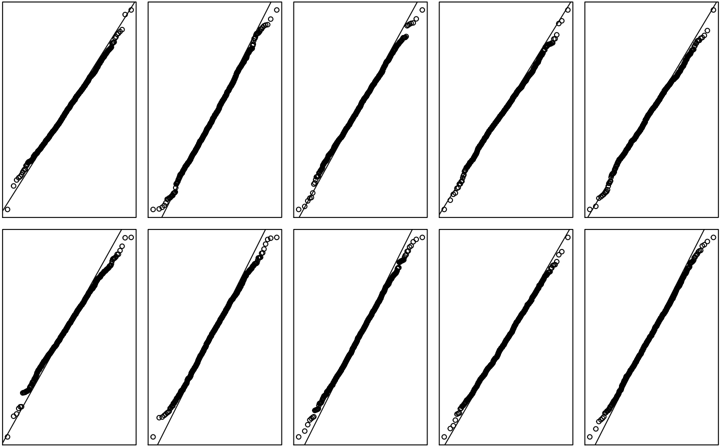
Normal (n=1000)



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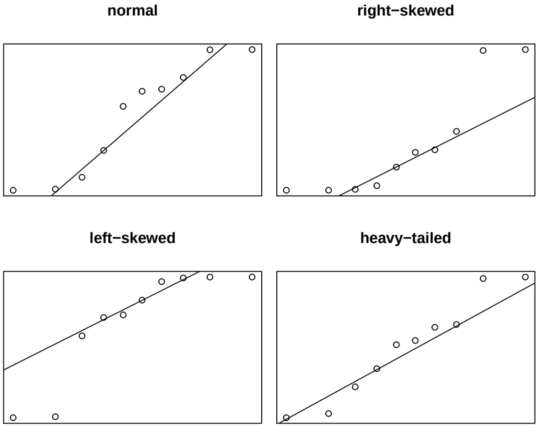
Not normal (n=10)



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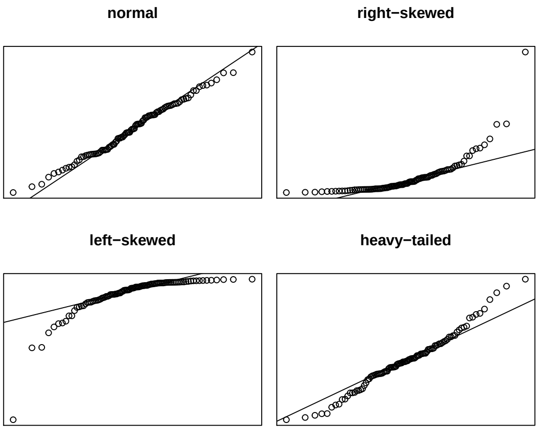

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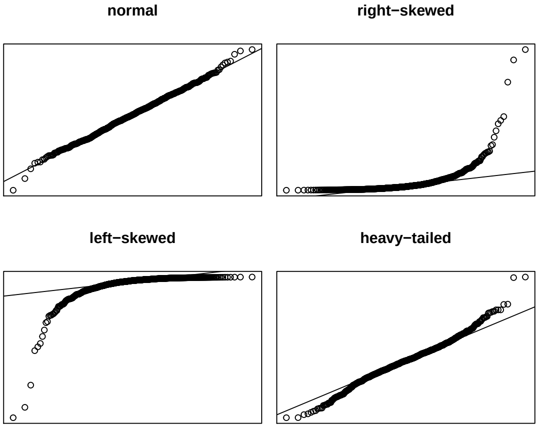

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Constant variance

Recall the model

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so the variance for the e_i is constant.

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The most common way this assumption is violated is by having increasing variance with increasing mean, thus we often look at a residuals vs predicted (fitted) mean plot.

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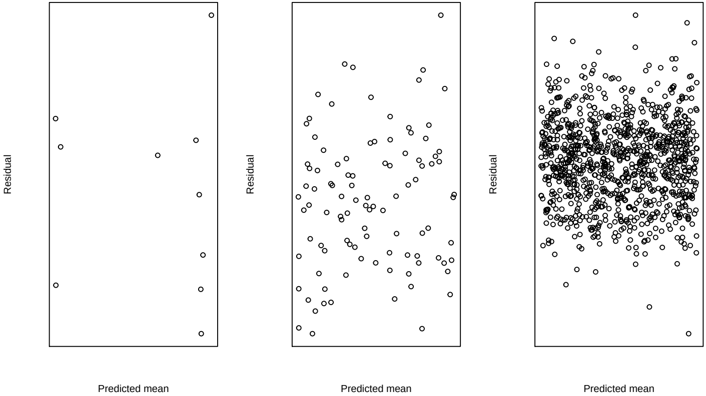

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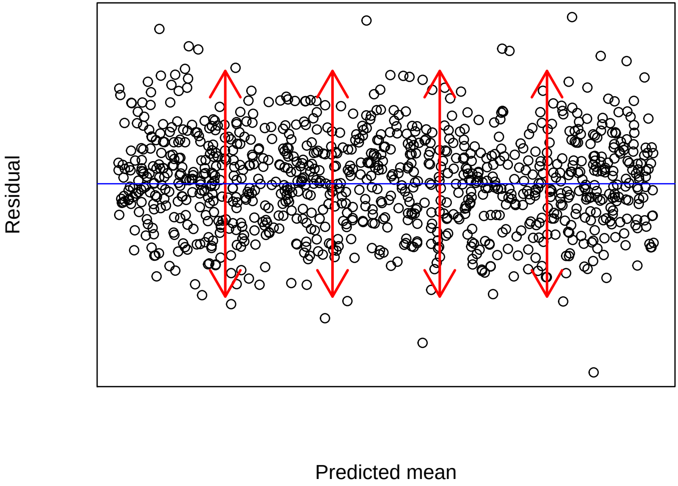
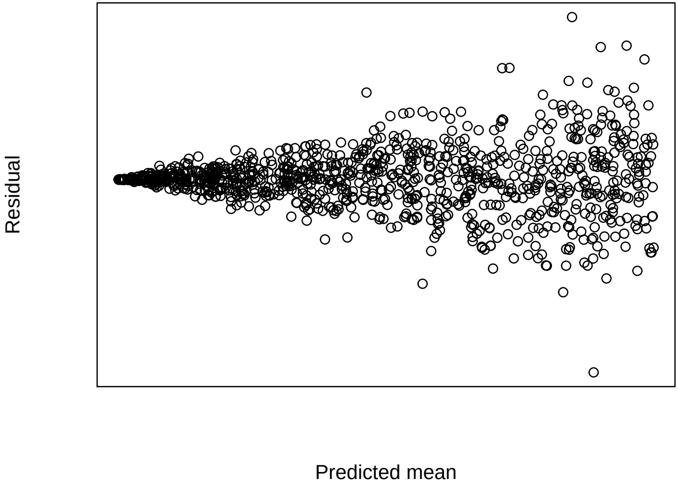


Extreme non-constant variance (funnel)



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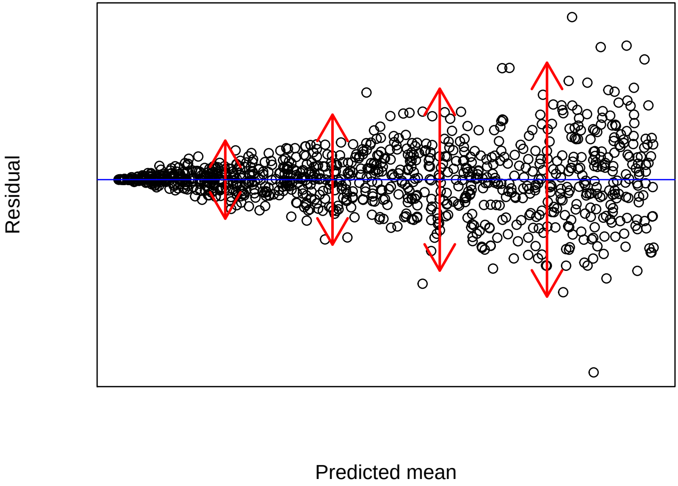
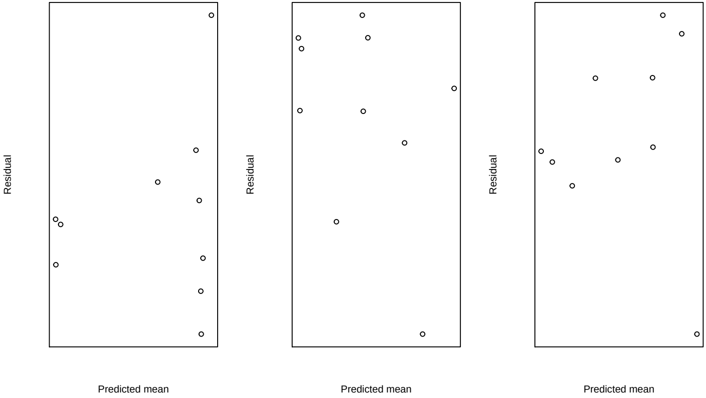
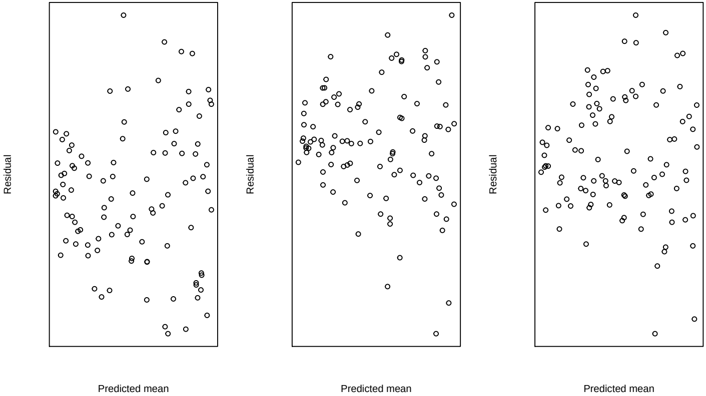


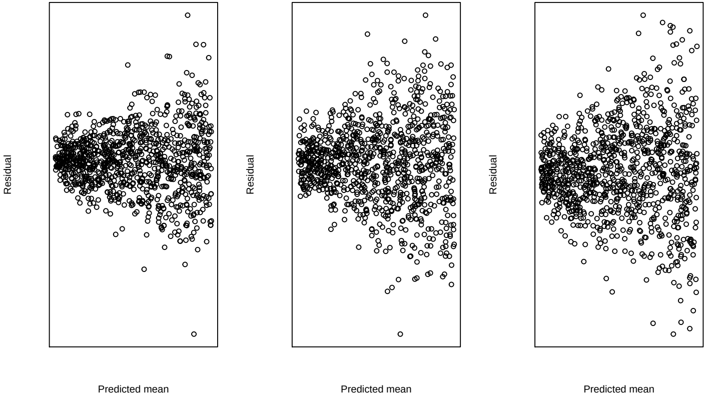
Non-constant variance (n=10, $\sigma_2/\sigma_1 = 4$)



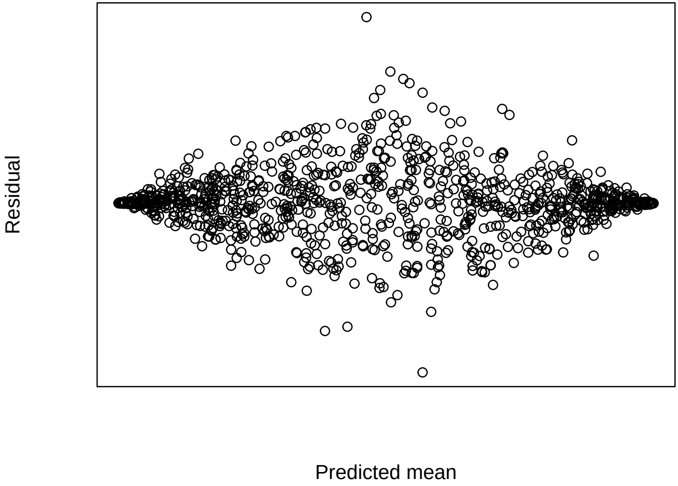
Non-constant variance (n=100, $\sigma_2/\sigma_1 = 4$)



Non-constant variance (n=1000, $\sigma_2/\sigma_1 = 4$)



Extreme non-constant variance (football)



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Summary

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- transformed response vs transformed explanatory
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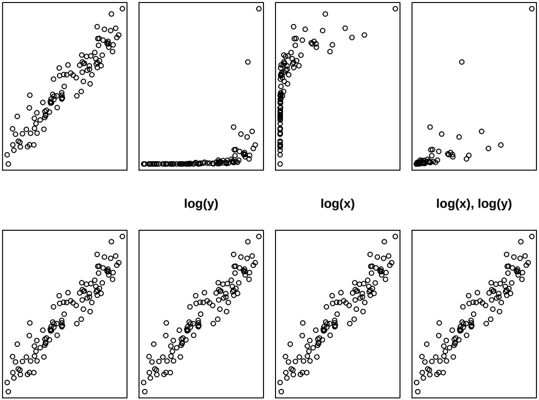
- qqplot of residuals
- residual vs fitted value
- residual vs explanatory
- residual vs observation number
- residual vs any other variable

Linearity

Assess using scatterplots of (transformed) response vs (transformed) explanatory variable:



Testing Composite hypotheses

Comparing two models

- H_0 : (reduced)
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Do the following

1. Calculate extra sum of squares.
2. Calculate extra degrees of freedom
3. Calculate

$$\text{F-statistic} = \frac{\text{Extra sum of squares / Extra degrees of freedom}}{\hat{\sigma}^2_{full}}$$

4. Compare this to an F-distribution with
   - numerator degrees of freedom = extra degrees of freedom
   - denominator degrees of freedom = degrees of freedom in estimating $\hat{\sigma}^2_{full}$

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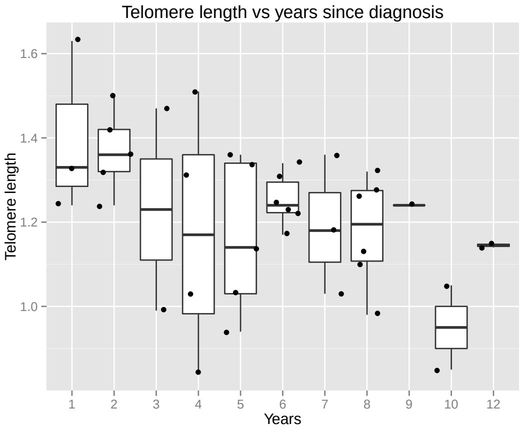
- Lack-of-fit F-test requires multiple observations at a few X_j values!

Telomere length



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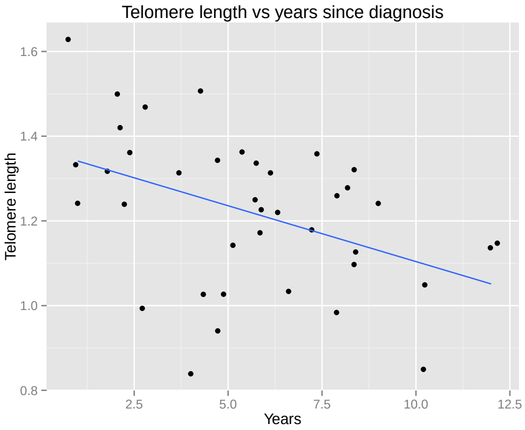

SAS code

```
DATA t;
  INFILE 'telomeres.csv' DSD FIRSTOBS=2;
  INPUT years length;

PROC REG DATA=t;
  MODEL length = years / CLB LACKFIT;
  RUN;
```

```
                         The REG Procedure
                          Model: MODEL1
                   Dependent Variable: length

              Number of Observations Read          39
              Number of Observations Used          39


                        Analysis of Variance

                                   Sum of          Mean
Source                  DF        Squares        Square    F Value    Pr > F

Model                    1        0.22777       0.22777       8.42    0.0062
Error                   37        1.00033       0.02704
  Lack of Fit            9        0.18223       0.02025       0.69    0.7093
  Pure Error            28        0.81810       0.02922
Corrected Total         38        1.22810
```

```
# Use as.factor to turn a continuous variable into a categorical variable
m_anova = lm(telomere.length ~ as.factor(years), Telomeres)
m_reg   = lm(telomere.length ~           years , Telomeres)
anova(m_reg, m_anova)


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No evidence of a lack of fit.

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  - Transform explanatory variable
  - Add other explanatory variables

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- when both are logged.

Neither response nor explanatory variable are logged

If

$$E[Y|X] = \beta_0 + \beta_1 X$$

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- β_0 is the expected response when X is zero and

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