COMPUTATIONAL HW 1

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Release date: **Wednesday, October 4**
Due date: **Wednesday, October 25, 11:59PM**

Instructions for the computational homework: Completing this assignment is optional but can earn bonus points for your course grade. Please submit your homework via Gradescope (link on Bruinlearn page) in pdf format. If you are using Google Colab, where you can combine typed solution and code, I recommend choosing the print option and "save as pdf". You may also submit written solutions along with the computationally generated plots requested. You must show all your work to receive full credit. Using any resource is allowed so long as you reference it and you are the one answering the question.

Problem 1. Consider the first order autonomous differential equation

$$y' = -2 + 5y^2 - y^5.$$

(a) Plot the right side of the equation,

$$f(y) = -2 + 5y^2 - y^5,$$

on the interval $-2 \leqslant y \leqslant 2$ (with y on horizontal axis).
Graphically determine how many roots f has and their approximate values.

(b) Plot a direction field and the solutions to the ODE for each of three initial conditions: $y(0) = 0$, $y(0) = 1$, and $y(0) = 2$.

(c) Let $y(t)$ be the solution satisfying one of the initial conditions above. Determine the limit

$$\lim_{t\to\infty} y(t)$$

for each initial condition.

Problem 2. Suppose that a population $y(t)$ satisfies the initial value problem

$$\frac{dy}{dt} = r(t)y - k \ , \quad y(0) = y_0$$

where the growth rate $r(t)$ is given by

$$r(t) = \frac{1}{5}(1 + \sin(t))$$

and k is a constant (called the rate of predation).

(a) Let $k = 0.2$. Use a numeric ode solver to find an approximation of $y(t)$ on the interval $0 \leqslant t \leqslant 30$ for each of the initial conditions

$$y(0) = 0.81, 0.82, 0.83, 0.84, 0.85.$$

Plot the results in the same figure and include a legend and title.

(b) Estimate (to 2 decimal places) the critical initial population y_c below which the population will become extinct. In other words, find the smallest initial value $y(0)$ such that the corresponding solution $y(t)$ never reaches 0.

Problem 3. Euler's method approximates the solution to the initial value problem:

$$y' = f(t, y)$$
$$y(t_0) = y_0,$$

with a step size h at the points $t_n = t_0 + n\,h$ by the incremental update

$$y_{n+1} = y_n + h\,f(t_n, y_n).$$

This way, y_n approximates the solution y at time t_n.

Consider the initial value problem

$$y' = ty - y, \quad y(0) = 0.5.$$

Implement Euler's method with each step size, $h = 0.2, 0.1, 0.05$, for $0 \leqslant t \leqslant 1$ and plot the results in a single figure. Solve the ODE (by hand) and plot the exact solution in the same figure. Include a title and a legend.