FINAL EXAM PRACTICE

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The questions on the final exam will be roughly evenly split between Chapter 2 (see Midterm 1 practice), Chapter 4 (see Midterm 2 practice), and material from Chapters 9 and 10. Reviewing the problems from Midterm 1 and Midterm 2 will be helpful. Here are additional practice problems for Chapters 9 and 10. You will not be tested on computing eigenvalues and eigenvectors, they will be given as part of a problem when needed. A former final exam for this course can be found here: https://www.math.ucla.edu/~wconley/33b.3.07f/FinalExam.pdf (you can disregard 5,6,7,8.)

Suggested textbook problems from Chapters 9 and 10:

- Section 9.1: Linear Systems with Constant Coefficients, Exercises 1-56 (feel free to use a computer to find eigenvalues/vectors and make sure you can solve the rest of the problem).
- Section 9.2: Planar Systems, Exercises 1-56.
- Section 9.3: Phase Portraits, Exercises 1-23.
- Section 9.4: Trace-Determinant Plane, Exercises 1-24.
- Section 10.1: Linearization of a nonlinear system, Exercises 1-27.
- Section 10.2: Long-term behavior of solutions, Exercises 1-18.

1. Consider the system of linear differential equations

$$\vec{y}\,' = \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \end{pmatrix} \vec{y}.$$

Eigenvalues are $1, -1, -2$; Eigenvectors (in same order): $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}.$

(a) Express the general solution.

(b) What is the property of the eigenvectors that allows you to find constants to solve any initial value problem?

(c) Solve with the initial condition $\vec{y}(0) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$

2. Consider the linear, homogeneous, constant-coefficient system of differential equations:

$$\vec{y}\,' = \begin{pmatrix} 0 & 2 \\ -1 & -2 \end{pmatrix} \vec{y}.$$

Eigenvalues: $-1+i$, $-1-i$; Eigenvectors (in same order): $\begin{pmatrix} 2 \\ -1+i \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1-i \end{pmatrix}.$

(a) Find two linearly independent real solutions to the system of differential equations.

(b) For each initial condition $\vec{y}_0$ determine what is $\lim_{t\to\infty} \vec{y}(t)$.

3. Consider the nonlinear autonomous system of differential equations:

$$x' = x(1 - x - y)$$
$$y' = y(1 - y - \frac{1}{2}x^2)$$

(a) Find all the equilibrium points.

(b) For each equilibrium point <u>where neither x nor y is negative</u>, classify as:
[asymptotically stable / unstable / undetermined],
and as a:
[nodal source / nodal sink / saddle / spiral source / spiral sink / other].
Hint: For triangular matrices, the eigenvalues are along the diagonal.