MIDTERM 1 PRACTICE

Aaron Palmer

Suggested textbook problems: (practice these until you can solve them routinely and quickly)

- Section 2.2 Separable, 1-22.
- Section 2.4 Linear, 1-26.
- Section 2.5 Mixing, 1-14.
- Section 2.6 Exact, 1-30.
- Section 2.7 Existence and Uniqueness, 1-16.
- Section 2.9 Autonomous, 1-31.

Problem 1. Solve the following initial value problems for $y(t)$:

(a) $-xy + \frac{1}{x} - \frac{x^2}{2}\frac{dy}{dx} = 0, \quad y(1) = -2.$

(b) $y' = \cos(t)y + \cos(t), \quad y(0) = 2.$

Problem 2. Consider the initial value problem $y' = \cos(y) - 1, \quad y(0) = y_0.$

(a) Find all possible equilibrium solutions and classify them as I) stable, II) unstable, or III) semi-stable.

(b) For what range of y_0 values will $\lim_{t\to\infty} y(t) = 0$?

Problem 3. Consider the initial value problem (IVP): $ty' = y, \quad y(t_0) = y_0.$

(a) Show that if $t_0 = 0$ and $y_0 \neq 0$, there is no solution.

(b) Show that for all c, $y(t) := ct$ is a solution and so if $t_0 = 0$ and $y_0 = 0$, there is more than one solution.

Problem 4. Consider the following initial value problem:

$$\begin{cases} y' + xy = x^3, \\ y(0) = 1. \end{cases} \tag{1}$$

(a) Use the integrating factor or variation of parameters method to express the solution the problem (1) in terms of an indefinite integral with a constant of integration.

(b) Evaluate the integral and the constant to find an explicit solution to the problem (1).

(c) What is the maximum interval of existence for the solution of part (a)?