MIDTERM 2 PRACTICE

Aaron Palmer

Suggested textbook problems: (practice these until you can solve them routinely and quickly)

- Section 4.1 Second Order Equations, 1-30.
- Section 4.3 Linear, Homogeneous, Constant Coefficients, 1-38.
- Section 4.4 Harmonic Motion, 1-25 (excluding circuits).
- Section 4.5 Undetermined Coefficients, 1-39.
- Section 4.7 Forced Harmonic Motion, 1-10.

Problem 1. Consider the following second order ODE:

$$\begin{cases} x'' + 4x' + 4x = 0. \end{cases} \tag{1}$$

(a) Find the general solution.

(b) Solve the ODE with the initial condition $x(0) = 1$ and $x'(0) = v$.

(c) For what values of v is $x(t) > 0$ for all $t \geqslant 0$?

Problem 2. A spring has mass $m = 2$kg and spring constant $k = 6kg/s^2$. A force is applied of $5t^2$ N.

(a) Write down the nonhomogeneous undamped spring equation and find a particular solution.

(b) Instead, a force is applied of $10\cos(\omega t)$. Find a particular solution in the case that ω is not the natural spring frequency.

(c) Solve with $x(0) = 5$ and $x'(0) = 0$ with ω equal to the natural spring frequency.

Problem 3. (a) For what values of b does the equation

$$y'' + by' + 2y = 0$$

have linearly independent solutions of the form $y_1 = e^{\lambda_1 x}$ and $y_2 = e^{\lambda_2 x}$ for real numbers λ_1 and λ_2?

(b) When $b = 3$, solve the equation with $y(0) = 1$ and $y'(0) = 0$.

Problem 4. Solve the initial value problem

$$\begin{aligned} y'' - y' - 2\,y &= t\,e^t, \\ y(0) &= 0, \\ y'(0) &= 1. \end{aligned}$$