Math 33B Fall 2023

## MIDTERM 2 PRACTICE

## Aaron Palmer

Suggested textbook problems: (practice these until you can solve them routinely and quickly)

- Section 4.1 Second Order Equations, 1-30.
- Section 4.3 Linear, Homogeneous, Constant Coefficients, 1-38.
- Section 4.4 Harmonic Motion, 1-25 (excluding circuits).
- Section 4.5 Undetermined Coefficients, 1-39.
- Section 4.7 Forced Harmonic Motion, 1-10.

**Problem 1.** Consider the following second order ODE:

$$\begin{cases} x'' + 4x' + 4x = 0. \end{cases} \tag{1}$$

- (a) Find the general solution.
- (b) Solve the ODE with the initial condition x(0) = 1 and x'(0) = v.
- (c) For what values of v is x(t) > 0 for all  $t \ge 0$ ?

**Problem 2.** A spring has mass m = 2kg and spring constant  $k = 6kg/s^2$ . A force is applied of  $5t^2$  N.

- (a) Write down the nonhomogeneous undamped spring equation and find a particular solution.
- (b) Instead, a force is applied of  $10\cos(\omega t)$ . Find a particular solution in the case that  $\omega$  is not the natural spring frequency.
- (c) Solve with x(0) = 5 and x'(0) = 0 with  $\omega$  equal to the natural spring frequency.

**Problem 3.** (a) For what values of b does the equation

$$y'' + by' + 2y = 0$$

have linearly independent solutions of the form  $y_1 = e^{\lambda_1 x}$  and  $y_2 = e^{\lambda_2 x}$  for real numbers  $\lambda_1$  and  $\lambda_2$ ?

(b) When b = 3, solve the equation with y(0) = 1 and y'(0) = 0.

**Problem 4.** Solve the initial value problem

$$y'' - y' - 2y = te^{t},$$
  
 $y(0) = 0,$   
 $y'(0) = 1.$