P1. In this algorithm, this people, use b banks to save coins.

Then we want to prove that, the ~~opt~~ opt solution is around $[\frac{b}{2}, b)$

which ~~which~~ means this Alg is at least ~~...~~ 2-approximation.

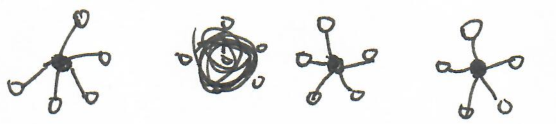
Therefore, we need to prove $\sum_{i=1}^{k} C_i = \sum_{i=1}^{b} P_i > \frac{1}{2} b \cdot V$

$\Longleftrightarrow$ prove $\sum_{i=1}^{b} \frac{P_i}{b} > \frac{1}{2} V \quad \cdots ①$

we prove ① by contradiction

if $\sum_{i=1}^{b} P_i/b \leq \frac{1}{2} V$, then there must exist some P_x and P_y, $1 \leq x, y \leq b$, make $P_x + P_y \leq \frac{1}{2} V$, and P_x, P_y can merge to 1 bank. However, this situation cannot be reached because the algorithm try each coin to all the existing banks, which will avoid this situation, therefore, $\sum_{i=1}^{b} P_i/b > \frac{1}{2} V$ is true, i.e. this algorithm is at least 2-approximation.

P2. Suppose that the result T contains n vertices, and the maximum degree of this graph is d ($d \neq 0$), then, the worst condition of this algorithm is show below, here we let $d=5, n>3$



, the black points are the vertices ~~dosen~~ chosen by this Alg.

Therefore, the best case will have at most $d \cdot n$ vertices

$\therefore$ the Alg is an $\frac{n}{dn} = \frac{1}{d}$ - approximation Alg

However, the degree of this ~~Alg~~ graph can be 0, so,

when $d=0$ this Alg is OPT Alg $\Rightarrow \frac{1}{1}$ - approximation Alg

$\Rightarrow \frac{1}{d+1}$ ......

$\because \frac{1}{d} > \frac{1}{d+1}$ so here we take the smaller one

this Alg is $\frac{1}{d+1}$ - approximation Alg

Alg: let $T = \{\emptyset\}$, let the value in all vertices be 0.

for v_j in V:

for all neighbor vertices of v_j:

if all the neighbor vertices' value is 0:

add v_j to T, and set v_j's value be 1.

return T.

P3. ~~suppose that the maximum Tm~~

Our Alg is that, for any (i,j) we send secrets by the shortest path, which means that we chose the smaller arc

Then, suppose that the T_{max} of our Alg is D, at child $\frac{1}{2}n$

we want to prove that T_{opt} of OPT Alg is $\geq \frac{1}{2}D$

we prove it by contradiction. suppose that T_{opt} is $< \frac{1}{2}D$

then, we want to reduce the notes number at child i to smaller than $\frac{1}{2}D$

then there must be more than $\frac{1}{2}D$ notes transfer by the longer arc.

as shown in this figure. We can find that, at our Alg, the

~~begin of~~ earliest child that send secrets through $\frac{1}{2}n^{th}$ child is 1^{st} child, and the latest child is n^{th}

then, if this ($> \frac{1}{2}D$) notes transfer by longer arc they must pass through $n^{th} \to 1^{st}$, therefore, the largest T in the longest arc is $> \frac{1}{2}D$, contradiction

$\therefore$ T_{opt} is $\geq \frac{1}{2}D$,