SI 211: Numerical Analysis Homework 5

Prof. Boris Houska

Due on Nov 4, 2020, 23:59

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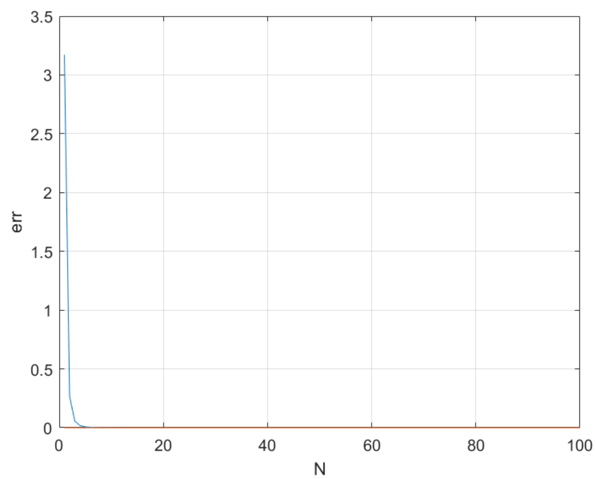
1. P1

   Ans:

   The code is shown in '**HW5_1.m**'

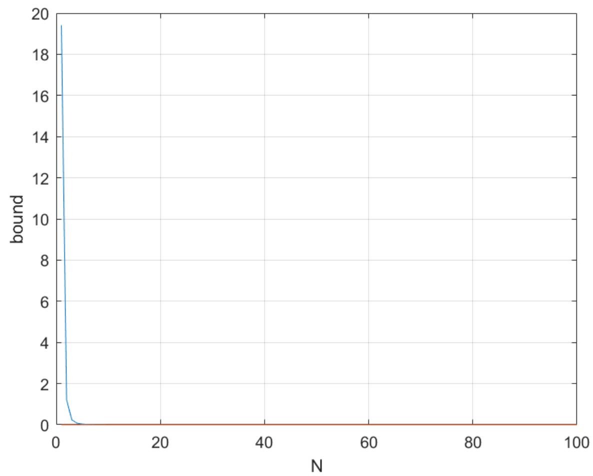


since, for simpson's method,

error upper bound is $\frac{(b-a)^5}{2880} \max |f^{(4)}(x)|$

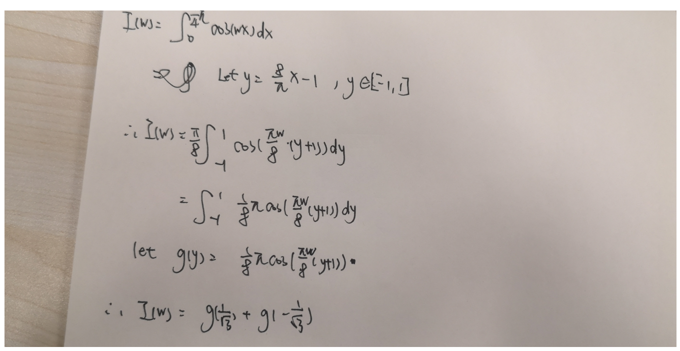
$\therefore$ for this case we got $\sum_{i=0}^{N-1} \frac{(\frac{4}{N})^5}{2880} \cdot \max_{x\in(a,b)} |f^{(4)}(x)|$ for each (a,b)

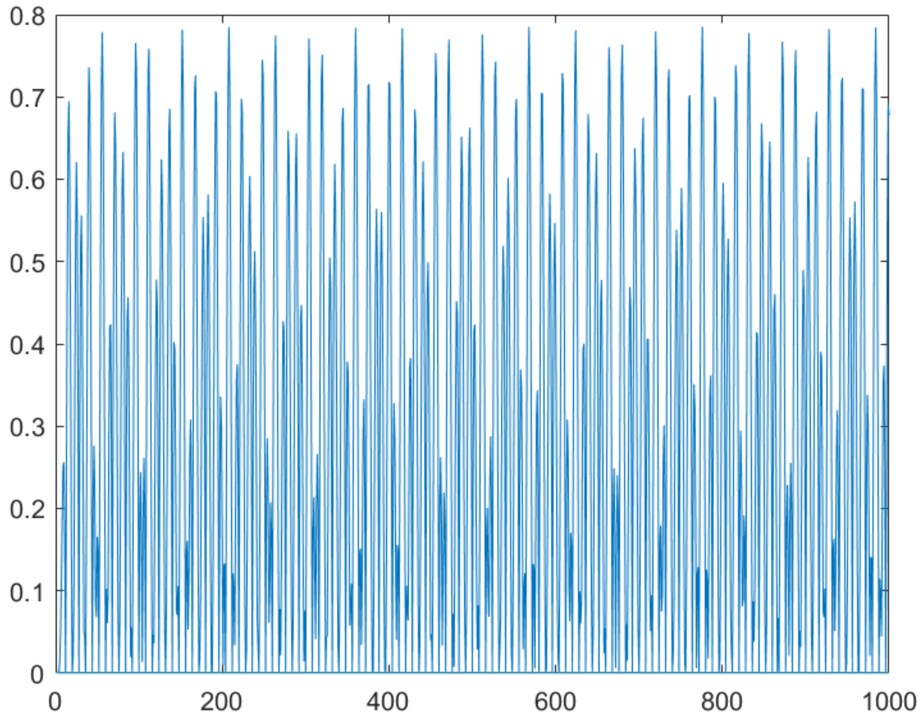
$\therefore$ ~~the~~ $\sum_{i=0}^{N-1} \frac{(\frac{4}{N})^5}{2880} \cdot \max_{x\in(a,b)} |f^{(4)}(x)| \le \sum_{i=0}^{N-1} \frac{(\frac{4}{N})^5}{2880} \cdot \max_{x\in[0,4]} e^x = \frac{4^5 \cdot e^4}{2880 N^4}$



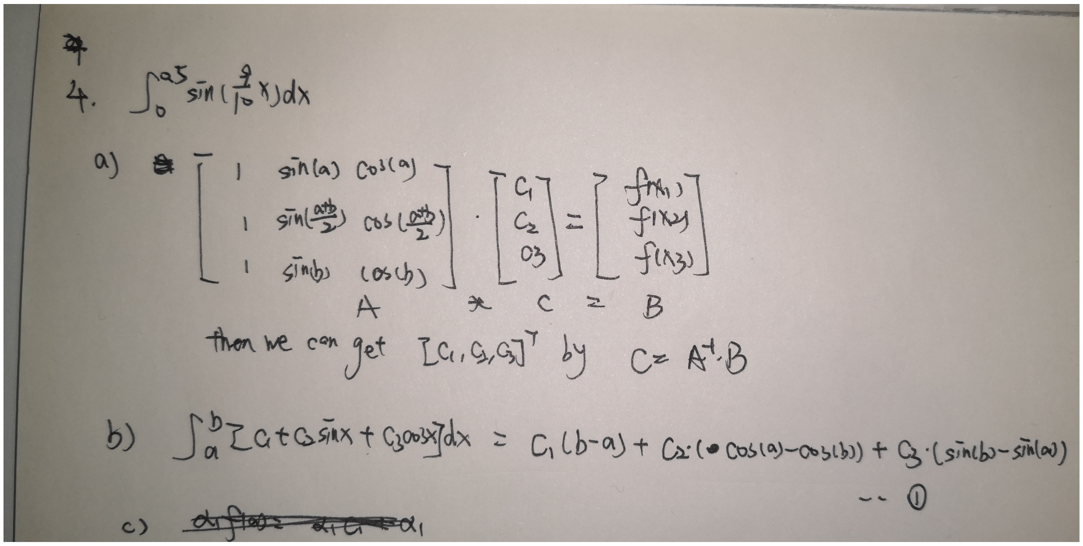
2. P2
   Ans:
   The code is shown in '**HW5_2.m**'

3. P3
   Ans:
   The code is shown in '**HW5_3.m**' The approximation error of $|I(1) - I_1(1)| = 6.352168618661835e - 05 \approx O(10^{-5})$ shown in matlab. When N increase, the error will be:





4. P4
   Ans:



4. $\int_0^{0.5} \sin(\frac{9}{10}x)dx$

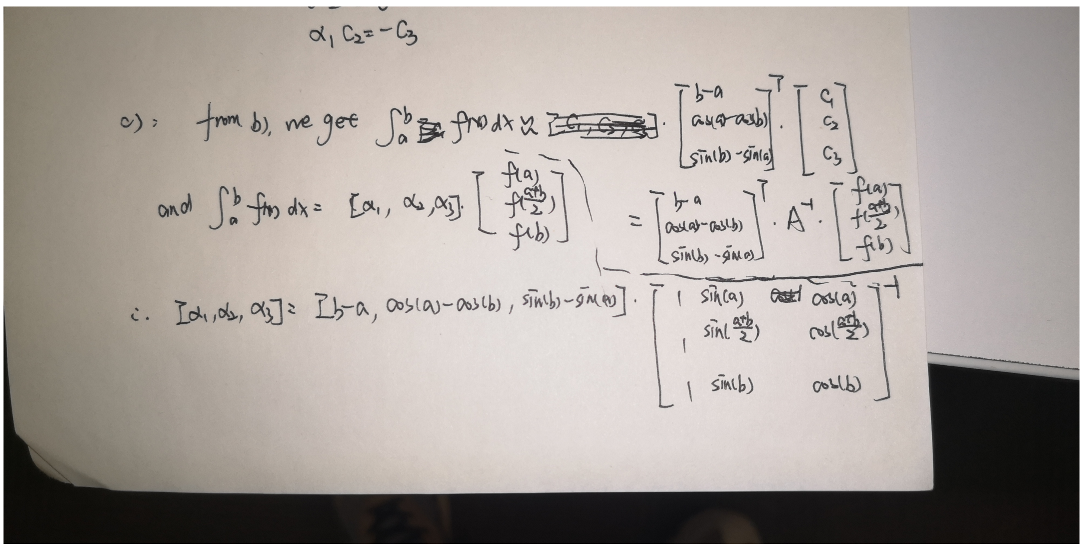
a) $\begin{bmatrix} 1 & \sin(a) & \cos(a) \\ 1 & \sin(\frac{a+b}{2}) & \cos(\frac{a+b}{2}) \\ 1 & \sin(b) & \cos(b) \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix}$

A * C = B

then we can get $[c_1, c_2, c_3]^T$ by $C = A^{-1} \cdot B$

b) $\int_a^b [c_1 + c_2 \sin x + c_3 \cos x]dx = c_1(b-a) + c_2 \cdot (\cos(a) - \cos(b)) + c_3 \cdot (\sin(b) - \sin(a))$ -- ①

c) ~~$\alpha_1 f(a)$ = $\alpha_1 c_1$~~ α_1



$\alpha_1 \; C_2 = -C_3$

c): from b), we get $\int_a^b f(x)dx \approx \begin{bmatrix} b-a \\ \cos(a)-\cos(b) \\ \sin(b)-\sin(a) \end{bmatrix}^T \cdot \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

and $\int_a^b f(x)dx = [\alpha_1, \alpha_2, \alpha_3] \cdot \begin{bmatrix} f(a) \\ f(\frac{a+b}{2}) \\ f(b) \end{bmatrix} = \begin{bmatrix} b-a \\ \cos(a)-\cos(b) \\ \sin(b)-\sin(a) \end{bmatrix}^T \cdot A^{-1} \cdot \begin{bmatrix} f(a) \\ f(\frac{a+b}{2}) \\ f(b) \end{bmatrix}$

$\therefore [\alpha_1, \alpha_2, \alpha_3] = [b-a, \cos(a)-\cos(b), \sin(b)-\sin(a)] \cdot \begin{bmatrix} 1 & \sin(a) & \cos(a) \\ 1 & \sin(\frac{a+b}{2}) & \cos(\frac{a+b}{2}) \\ 1 & \sin(b) & \cos(b) \end{bmatrix}^{-1}$

The result for each function is shown below. I find that this integration is good for trigonometric function. I think the reason is that, since all the f can be represented by fourier expansion, and the function like x^3 need more terms to have a better accuracy, therefore, in here this new integration method is too simple to represent a polynomial by *sin* and *cos*, we need more terms.

```
************  P1  ***********
This new numerical integration error: 3.725042e-07
Simpson error: 1.584501e-06
************  P2  ***********
This new numerical integration error: 1.051045e-03
Simpson error: 2.775558e-17
************  P3  ***********
This new numerical integration error: 0.000000e+00
Simpson error: 3.011074e-04
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