

SI 211: Numerical Analysis Homework 3

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Due on Oct 23, 2020

1. *Natural Spline.*

Write a computer code in JULIA, Matlab, Python, C++, (or a similar programming language), which returns a natural spline that interpolates the function $f : [x_0, x_N] \rightarrow \mathbb{R}$ at the **equidistant** points

$$\forall i \in \{0, 1, 2, \dots, N\}, \quad x_i = x_0 + hi$$

$$\text{with } h = \frac{x_N - x_0}{N}.$$

2. *Comparison of Interpolation and Natural Spline*

We have constructed a computer program that interpolates a function $f(x) = \frac{1}{1+x^2}$ with a polynomial of order 10. Use your code from the first exercise to interpolate this function

$$f(x) = \frac{1}{1+x^2}$$

at the points $x_1 = -5, x_2 = -4, x_3 = -3, \dots, x_{10} = 4, x_{11} = 5$. Plot the function f as well as the natural spline that interpolates f and the interpolating polynomials (HW2).

3. *Comparison of Interpolation and Natural Spline (continued)*

For the function f , the interpolating polynomial p , and the interpolating spline s from the previous exercise, compute

(a) the value of the integral

$$\int_{-5}^5 [f''(x)]^2 dx$$

(b) the value of the integral

$$\int_{-5}^5 [p''(x)]^2 dx$$

for the interpolating polynomial p , and

(c) the value of the integral

$$\int_{-5}^5 [s''(x)]^2 dx$$

for the interpolating spline s .

Interpret your results.