# SI 211: Numerical Analysis Homework 3

Prof. Boris Houska

Due on Oct 23, 2020, 23:59

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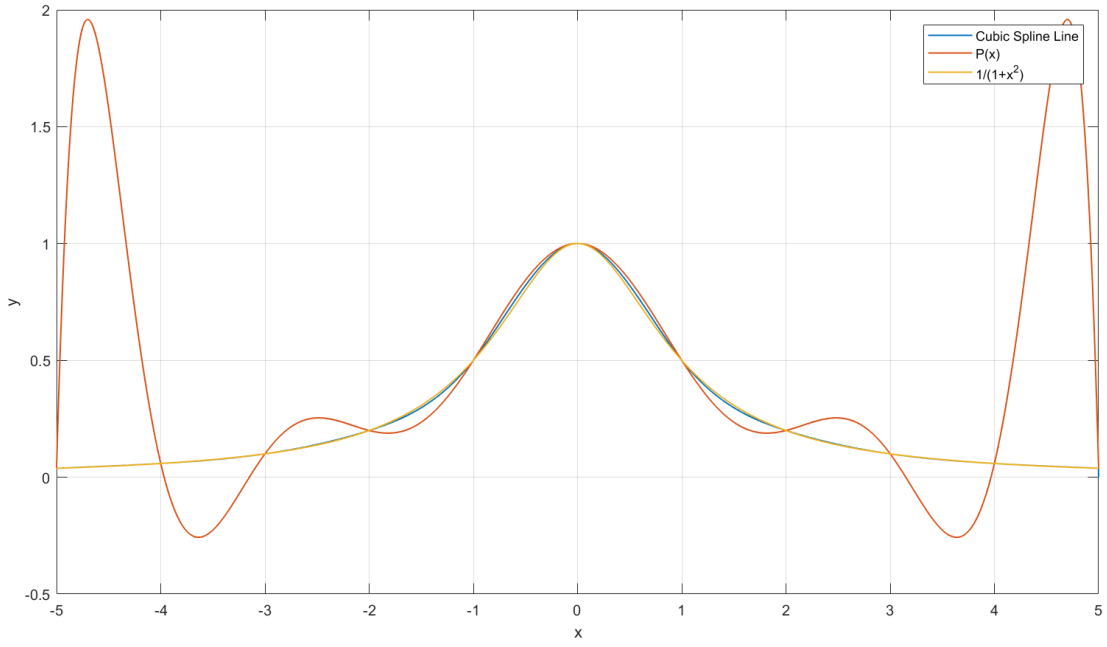
1. Natural Spline
   *Ans*:
   The code is shown in '**HW3_1.m**'

2. Comparision of Interpolation and Natural Spline
   *Ans*:



3. Comparision of Interpolation and Natural Spline (continued)
*Ans*:
In this part, I use infinitesimal method to get the solution of these integrals. For the problem 1) and 2), I first compute the value $y_f, y_p$ of function $f$ and polynomial $p$ between $[-5, 5]$ with the interval of $t = 1e-5$. Then I use $y_f, y_p$ to compute $y'_f, y'_p$ by finite differences at step $t$. Similarly, I can compute the $y''_f, y''_p$. Then I can compute the integral by $\sum(y''_f)^2 * t, \sum(y''_p)^2 * t$. For the problem 3), since the section are divided into 10 regions, I treat the integral of function $s$ as 10 separate parts and use the same method to find the integral of each part. Then, I combine the integral of each part as a whole, which is the integral I want. The code is shown in '**HW3_2.m**' and the answer is shown below.
1) 2.3561; 2) 2007.6; 3) 2.2161;
For the result we can find that since the polynomial function is overfitting compared to the original function, therefore, the polynomial function oscillates more than the original function, and the simulation result show that the difference between these integrals is very large. However, since the natural cubic spline satisfy the inequality shown below, the simulation result demonstrates it.

The natural cubic splines satisfy the inequality

$$\int_{x_{\min}}^{x_{\max}} |p''(x)|^2 \, \mathrm{d}x \leq \int_{x_{\min}}^{x_{\max}} |f''(x)|^2 \, \mathrm{d}x \; .$$