SI 211: Numerical Analysis Homework 6

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Deadline: Nov 27, 2020

1. Provide a formal proof of the fact that if $\|\cdot\|$ is a matrix norm (assume $\|I\| = 1$) and $A \in \mathbb{R}^{n \times n}$ a given matrix whose norm satisfies $\|A\| < 1$, then the matrix $I - A$ is invertible and

$$\left\|(I-A)^{-1}\right\| \leq \frac{1}{1 - \|A\|} \; .$$

2. Solve the linear equation

$$\begin{bmatrix} 2.0002 & 1.9998 \\ 1.9998 & 2.0002 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \; .$$

Next, add a disturbance to the right, $\delta b = (2 \cdot 10^{-4}, -2 \cdot 10^{-4})$, and solve the new linear equation:

$$\begin{bmatrix} 2.0002 & 1.9998 \\ 1.9998 & 2.0002 \end{bmatrix} \begin{bmatrix} \bar{x_1} \\ \bar{x_2} \end{bmatrix} = \begin{bmatrix} 4.002 \\ 3.998 \end{bmatrix} \; .$$

Compare both solutions and compute $\mathrm{cond}_\infty(A)$, $\frac{\|\delta x\|_\infty}{\|x\|_\infty}$, $\frac{\|\delta b\|_\infty}{\|b\|_\infty}$. Interpret your results.

3. Implement an LR-decomposition code (slides7-36) by a programming language of your choice and solve the equation system

$$\begin{aligned} x - 2y + z &= 0 \\ 2x + y - 3z &= 5 \\ 4x - 7y + z &= -1 \end{aligned}$$

4. If A is a symmetric positive band-structured matrix with bandwidth $2n+1$, then its Cholesky decomposition factor L is also a band-structured matrix. What is the bandwidth of L?