SI 211: Numerical Analysis Homework 1

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Deadline: Sep 30, 2020

1. *Floating point numbers.* What is the bit representations of the floating point number 5.25 using the IEEE standard for double precision numbers? Solve this problem with pen and paper first. Use a computer program to verify your result.

2. *Numerical evaluation error.* Evaluate the function

$$f(x) = \frac{1 - \cos(x)}{x^2} \tag{1}$$

   with a compute program of your choice using the standard IEEE double precision floating point format. Plot the numerical result on a logarithmic scale for $x \in \left[10^{-15}, 10^{-1}\right]$.

3. *Taylor expansion.* Consider again the function

$$f(x) = \frac{1 - \cos(x)}{x^2} \tag{2}$$

   from the first homework problem. Can you approximate f by using a Taylor approximation? Does this help you to evaluate f with higher accuracy?

4. *Numerical differentiation.*

   Consider the five-point differentiation formula

$$f'(x) \approx \frac{1}{12h}\left[-25f\left(x_0\right) + 48f\left(x_0 + h\right) - 36f\left(x_0 + 2h\right) + 16f\left(x_0 + 3h\right) - 3f\left(x_0 + 4h\right)\right] .$$

   (a) What is the mathematical approximation error of this formula?

   (b) For which values of h would you expect that this formula leads to a minimum approximation error taking both the mathematical as well as the numerical approximation error into account?

   (c) Implement the above differentiation formula in a compute program of your choice and use it to find the derivative of the test function $f(x) = e^x$ at $x = 1$. Plot the total derivative evaluation error as a function of h and intepret your results.