

SI 211: Numerical Analysis Homework 6

Prof. Boris Houska

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1. P1

Ans:

1. Since $\|A\| = \|A - B + B\| \leq \|A - B\| + \|B\| \Rightarrow \|A\| - \|B\| \leq \|A - B\|$

$\therefore \|x\| = \|(I - A)x + Ax\| \leq \|(I - A)x\| + \|Ax\| \leq \|(I - A)x\| + \|A\|\|x\|$

$\therefore \|(I - A)x\| \geq (1 - \|A\|)\|x\| \therefore (I - A)x \neq 0 \text{ for all vectors } x \neq 0, \therefore I - A \text{ is invertible}$

$\therefore (I - A)^{-1}(I - A) = I \Rightarrow (I - A)^{-1} - A(I - A)^{-1} = I \Rightarrow (I - A)^{-1} = I + A(I - A)^{-1}$

$\therefore \|(I - A)^{-1}\| = \|(I + A(I - A)^{-1})\| \leq 1 + \|A\|\|(I - A)^{-1}\|$

$\therefore \|(I - A)^{-1}\| \leq \frac{1}{1 - \|A\|}$

2. P2

Ans:

$$2. \quad 1) \quad x_1 = x_2 = 1$$

$$\Rightarrow \bar{x}_1 = 1.5, \bar{x}_2 = 2.5$$

$$\therefore \text{cond}_{\infty}(A) = \|A\|_{\infty} \cdot \|A^{-1}\|_{\infty} = 10000$$

$$\frac{\|\bar{x}\|_{\infty}}{\|x\|_{\infty}} = 0.5, \quad \frac{\|\bar{b}\|_{\infty}}{\|b\|_{\infty}} = 5 \times 10^{-5}$$

\therefore we can find that, since $\text{cond}_{\infty}(A)$ is much larger than 1

\therefore the effect caused by \bar{b} is very large, and satisfies

$$\frac{\|\bar{x}\|}{\|x\|} \approx \text{cond}(A) \left(\frac{\|\bar{b}\|}{\|b\|} + \frac{\|A\|}{\|A\|} \right) = 10000 \times 5 \times 10^{-5} = 0.5$$

$$\text{and} \quad \frac{1}{\text{cond}_{\infty}(A)} \frac{\|\bar{b}\|_{\infty}}{\|b\|_{\infty}} \leq \frac{\|\bar{x}\|_{\infty}}{\|x\|_{\infty}} \leq \text{cond}_{\infty}(A) \frac{\|\bar{b}\|_{\infty}}{\|b\|_{\infty}}$$

3. P3

Ans:

The code is shown in **HW6.m**, and the result of this linear system is $x = [3, 2, 1]^T$

4. P4

Ans:

