SI 211: Numerical Analysis Homework 3

Prof. Boris Houska

Due on Oct 23, 2020, 23:59

- You can write the homework using the latex template we provide or you can write by handwriting and submit your scan pdf(please make sure your scan is clear enough).
- You are required to write down all the major steps towards making your conclusions; otherwise you may obtain limited points ($\leq 20\%$) of the problem.
- Homework including many code assignment, so you need attach your code and result(table or plot), you can use any kind of programming language(c,c++,python,matlab), make sure your code can be run by TAs when checking your homwork.
- About submit your homework: You are required to submit your homework as a PDF on time; otherwise you will get no points of this homework.
- You are suggested to write your homework in English;
- Do your homework by yourself. Any form of plagiarism will lead to 0 point of this homework. If more than one plagiarisms during the semester are identified, we will prosecute all violations to the fullest extent of the university regulations, including but not limited to failing this course, academic probation, or expulsion from the university.
- If you have any doubts regarding the grading, you need to contact the instructor or the TAs within two days since the grade is announced.

1. Comparison of Interpolation and Natural Spline

   Solution: Using the code in HW2 and problem 1, we can get the Figure 1.

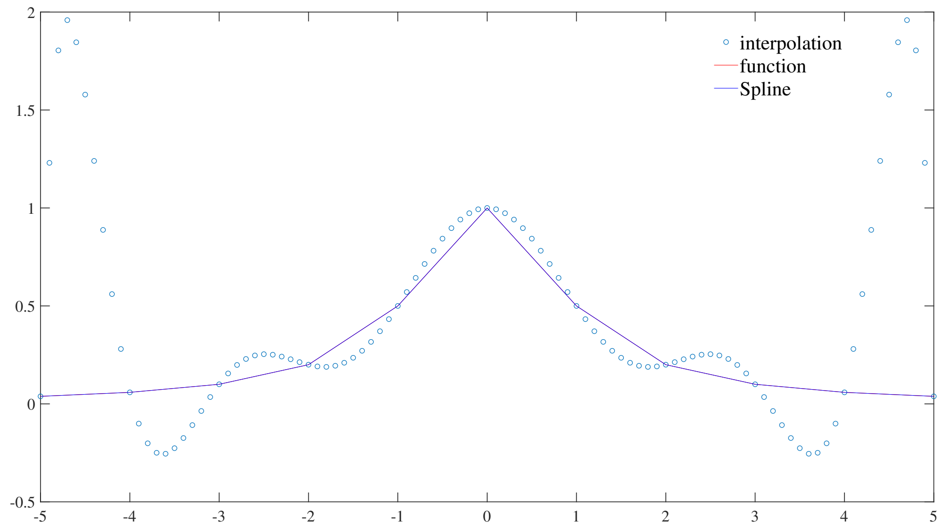

Figure 1: Interpolation Figure 1

2. Comparision of Interpolation and Natural Spline (continued)
   Solution: 1) We all know $f(x) = \frac{1}{(1+x^2)}$, thus

$$f'(x) = -\frac{2x}{(1+x^2)^2}, f''(x) = \frac{8x^2 - 2(1+x^2)}{(1+x^2)^3}$$

$$\int_{-5}^{5} [f''(x)]^2 dx = \int_{-5}^{5} \left[\frac{8x^2 - 2(1+x^2)}{(1+x^2)^3}\right]^2 dx \approx 2.3561$$

2) By using $diff$ and int in MATLAB, the results is

$$\int_{-5}^{5} [p''(x)]^2 dx \approx 2007.70$$

3) In every subinterval, we have

$$s_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, i = 1, \ldots, 10$$

Thus,

$$\begin{aligned} &\textstyle\int_{-5}^{5} [s''(x)]^2 dx = \sum\limits_{i=1}^{10} \int_{-6+i}^{-5+i} \left[s_i''(x)\right]^2 dx \\ &= \sum\limits_{i=1}^{10} \textstyle\int_{-6+i}^{-5+i} [2c_i + 6d_i(x - x_i)]^2 dx = 4\sum\limits_{i=1}^{10} c_i^2 + 12\sum\limits_{i=1}^{10} c_i d_i + 12\sum\limits_{i=1}^{10} d_i^2 \approx 2.2161 \end{aligned}$$

From the 1) and 3), we can find that the cubic spline s never oscillates more than the function f, since

$$\int_{-5}^{5} [s''(x)]^2 dx \le \int_{-5}^{5} [f''(x)]^2 dx$$

From the 1) and 2), we can find that the polynomial interpolation tends to oscillate drasticly.

$$\int_{-5}^{5} [f''(x)]^2 dx \ll \int_{-5}^{5} [p''(x)]^2 dx$$