SI 211: Numerical Analysis Homework 2

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1. *Jacobi matrix.* Let us consider the multivariate function

$$f(x) = \begin{pmatrix} x_1^2 x_2 + x_1 x_2^2 \\ x_1^2 + x_1 x_2 \end{pmatrix}$$

Compute the Jacobi matrix of f at $x = (1,2)^\intercal$.

2. *Polynomial interpolation.* Assume that a function $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(-1) = 6, f(2) = 12$ and $f(4) = 66$. Construct a polynomial function of the form $p(x) = a_0 + a_1 x + a_2 x^3$ such that p interpolates f at $x \in \{-1, 2, 4\}$. Find the a_0, a_1 and a_2.

3. *Interpolation with rational functions.* Let us assume that we have given points

$$\begin{aligned} (x_0, y_0) &= (-2, -6) \\ (x_1, y_1) &= (-1, -3) \\ (x_2, y_2) &= (1, 5) \\ (x_3, y_3) &= (2, 10) \end{aligned}$$

Construct a function $q : \mathbb{R} \to \mathbb{R}$ of the form

$$q(x) = \frac{a_{-1}}{x} + a_0 + a_1 x + a_2 x^2 \tag{1}$$

such that $q(x_i) = y_i$ for all $i \in \{0, 1, 2, 3\}$. Find the scalar coefficients a_{-1}, a_0, a_1, a_2.

4. *Hermite interpolation.* Construct a polynomial function p of degree 3 that satisfies

$$p(x_1) = 1,\ p(x_2) = 2,\ p'(x_1) = 2,\ p'(x_2) = 4$$

where $x_1 = 1, x_2 = 2$.

5. Polynomial Approximation Error

   (1) Implement a computer program that interpolates a function $f(x)$ at the points

$$x_0 = -5, x_2 = -4, x_3 = -3, \cdots, x_9 = 4, x_{10} = 5$$

   with a polynomial p of order 10. Test your program for

   (a) the function $f(x) = \sin(x)$

   (b) the function $f(x) = \frac{1}{1+x^2}$

   Plot the functions as well as their interpolating polynomials and analyze the approximation errors.