CS/ISyE 719: Stochastic Programming Modeling

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September 7, 2016

Module Outline

Stochastic Programming Modeling

- A little about stochastic modeling
- Crop planning example
- (Expected) Value of stochastic solution
- Expected value of perfect information
- Facility location example

Stocastic Programming Modeling

Optimization Modeling + Stochastic Modeling

- In this course, we tend to focus more on optimization modeling

Stochastic Modeling

- Deriving the models of the random variables in the model
- Heavily problem/context dependent

Data-Driven Stochastic Modeling

In short to medium-term problems, might be possible to use past data to construct stochastic model

- Forecasting, predictive modeling
- Ideally, build point and distribution estimates

Data-Driven Stochastic Modeling

Many (most) stochastic progamming methods only require ability to sample random variables appearing in model

- Used to construct a scenario approximation that is what is actually solved
- Or integrated into a stochastic algorithm
- Surprisingly "few" scenarios needed (relative to dimension of random vector)

$\Rightarrow$ Random variables can be generated from (possibly complicated) simulation models

- Key assumption: Simulation needs to be independent of decisions
- E.g., Weather simulation yields (correlated) wind power scenarios

Example: Linear Regression

Example model for random demand of doughnuts:

- Assume you have estimated a linear regression model:
$$X = \sum_{i=1}^{p} w_i Y_i + \epsilon$$
- Assume that at time decision is made, independent variables $Y_i, i = 1, \ldots, t$ are known as $\hat{Y}_i$ (e.g., current temperature, etc.)
- Other independent variables are unknown, but distribution estimates can readily be constructed from data (e.g., number of students who will attend the lecture)
- Obtaining random observation of X:
  - Sample $\tilde{Y}_i$ from estimated distrition of Y_i, $i = t+1, \ldots, p$ and $\tilde{\epsilon}$ from the error distribution
  - $\tilde{X} = \sum_{i=1}^{t} w_i \hat{Y}_i + \sum_{i=t+1}^{p} w_i \tilde{Y}_i + \tilde{\epsilon}$

What you should (usually) NOT do

Some people use the term Sample Average Approximation for the following approach:

- Use past observations of demand directly as the scenarios in your stochastic model

This is not (generally) Sample Average Approximation!

- SAA uses scenarios generated from a stochastic model (which ideally is based on all available data)
- Key difference is the ability to use information that is known when making decision to build a better stochastic model (e.g., current weather conditions, most recent demand, etc.)

"Expert"/case driven stochastic modeling

In some (many?) situations (e.g., long-term models), there may be no reasonable data to justify a stochastic model

What to do?

- Robust optimization: Just assume uncertain data lies in an "uncertainty set", and optimize the worst-case
  - Better than assuming deterministic, but may be overly conservative
  - Also, two-stage robust models are generally intractable
- Constructed scenarios with subjective probability estimates:
  - Demand for a product is "low, medium, or high".
  - Weather is "dry" or "wet".
  - The market will go "up" or "down"

George Box (Yes, again!)

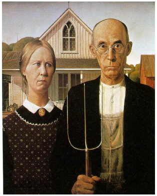
"All models are wrong. Some models are useful."

- Surely a subjectively constructed set of scenarios is WRONG
- But, the resulting stochastic programming model might be USEFUL in making better decisions

Key insight (again)

Comparing solution expected values is easier than estimating each solution's individual expected value.

Back to a Concrete Example



Farmer Ted

- In this example, the farmer has (real) *recourse* that is, he can do after observing random outcome. Not just sell his newspapers.
- Farmer Ted can grow Wheat, Corn, or Beans on his 500 acres.
- Farmer Ted requires 200 tons of wheat and 240 tons of corn to feed his cattle
- These can be grown on his land or bought from a wholesaler.

More Constraints

- Any excess production can be sold for \$170/ton (wheat) and \$150/ton (corn)
- Any shortfall must be bought from the wholesaler at a cost of \$238/ton (wheat) and \$210/ton (corn).
- Farmer Ted can also grow beans
  - Beans sell at \$36/ton for the first 6000 tons
  - Due to economic quotas on bean production, beans in excess of 6000 tons can only be sold at \$10/ton

The Data

- 500 acres available for planting

	Wheat	Corn	Beans
Yield (T/acre)	2.5	3	20
Planting Cost ($/acre)	150	230	260
Selling Price	170	150	36 ($\leq$ 6000T)
			10 ($>$6000T)
Purchase Price	238	210	N/A
Minimum Requirement	200	240	N/A

Formulate the LP – Decision Variables

- $x_{W,C,B}$ Acres of Wheat, Corn, Beans Planted
- $w_{W,C,B}$ Tons of Wheat, Corn, Beans sold (at favorable price).
- e_B Tons of beans sold at lower price
- $y_{W,C}$ Tons of Wheat, Corn purchased.

- Note that Farmer Ted has *recourse*. After he observes the weather event, he can decide how much of each crop to sell or purchase!

Formulation

$$\begin{aligned}\max -150x_W - 230x_C - 260x_B - 238y_W + 170w_W \\ -210y_C + 150w_C + 36w_B + 10e_B\end{aligned}$$

subject to

$$\begin{aligned}
x_W + x_C + x_B &\leq 500 \\
2.5x_W + y_W - w_W &= 200 \\
3x_C + y_C - w_C &= 240 \\
20x_B - w_B - e_B &= 0 \\
w_B &\leq 6000 \\
x_W, x_C, x_B, y_W, y_C, e_B, w_W, w_C, w_B &\geq 0
\end{aligned}$$

Solution with (expected) yields

	Wheat	Corn	Beans
Plant (acres)	120	80	300
Production	300	240	6000
Sales	100	0	6000
Purchase	0	0	0

- Profit: $118,600

It's the Weather, Stupid!

- Farmer Ted knows well enough to know that his yields aren't always precisely $Y = (2.5, 3, 20)$. He decides to run two more scenarios
- Good weather: $1.2Y$
- Bad weather: $0.8Y$.

Creating a Stochastic Model

Here is a general procedure for making a (scenario-based) 2-stage stochastic optimization problem

- For a "nominal" state of nature (scenario), formulate an appropriate (deterministic) optimization model
- Decide which decisions are made before uncertainty is revealed, and which are decided after
- All second stage variables get "scenario" index
- Constraints with scenario indices must hold for all scenarios
- Second stage variables in the objective function should be summed and weighted by the probability of the scenario occurring

What does this mean in our case?

- First stage variables are the x (or planting variables)
- Second stage variables are the y, w, e (purchase and sale variables)
- We have one copy of the y, w, e for each scenario!
- Attach a scenario subscript $s = 1, 2, 3$ to each of the purchase and sale variables.
    - 1: Good, 2: Average, 3: Bad
- w_{C2} : Tons of corn sold at favorable price in scenario 2
- e_{B3} : Tons of beans sold at unfavorable price in scenario 3.

Expected Profit

- The second stage cost for each submodel appears in the overall objective function weighted by the probability that scenario will happen

$$
\begin{aligned}
& -150x_W - 230x_C - 260x_B \\
& +1/3(-238y_{W1} + 170w_{W1} - 210y_{C1} + 150w_{C1} + 36w_{B1} + 10e_{B1}) \\
& +1/3(-238y_{W2} + 170w_{W2} - 210y_{C2} + 150w_{C2} + 36w_{B2} + 10e_{B2}) \\
& +1/3(-238y_{W3} + 170w_{W3} - 210y_{C3} + 150w_{C3} + 36w_{B3} + 10e_{B3})
\end{aligned}
$$

Constraints

$$\begin{aligned}
x_W + x_C + x_B &\leq 500 \\
3x_W + y_{W1} - w_{W1} &= 200 \\
3.6x_C + y_{C1} - w_{C1} &= 240 \\
24x_B - w_{B1} - e_{B1} &= 0
\end{aligned}$$

Constraints (cont.)

$$\begin{aligned}
2.5x_W + y_{W2} - w_{W2} &= 200 \\
3x_C + y_{C2} - w_{C2} &= 240 \\
20x_B - w_{B2} - e_{B2} &= 0 \\
2x_W + y_{W3} - w_{W3} &= 200 \\
2.4x_C + y_{C3} - w_{C3} &= 240 \\
16x_B - w_{B3} - e_{B3} &= 0 \\
w_{B1}, w_{B2}, w_{B3} &\leq 6000 \\
\text{All vars} &\geq 0
\end{aligned}$$

Optimal Solution

		Wheat	Corn	Beans
s	Plant (acres)	170	80	250
1	Production	510	288	6000
1	Sales	310	48	6000
1	Purchase	0	0	0
2	Production	425	240	5000
2	Sales	225	0	5000
2	Purchase	0	0	0
3	Production	340	192	4000
3	Sales	140	0	4000
3	Purchase	0	48	0

The Value of the Stochastic Solution (VSS)

- Suppose we just replaced the "random" quantities (the yields) by their mean values and solved that problem.
- Would we get the same expected value for the Farmer's profit?
- How can we check?
  - Solve the "mean-value" problem to get a first stage solution x.
  - Fix the first stage solution at that value x, and solve all the scenarios to see Farmer Ted's profit in each.
  - Take the weighted (by probability) average of the optimal objective value for each scenario
- Alternatively (and simpler), we can fix the x variables and solve the stochastic programming problem we created.

Computing FT's VSS

- Mean yields $Y = (2.5, 3, 20)$
- (We already solved this problem).
- $x_W = 120, x_C = 80, x_B = 300$

Fixed Policy – Average Yield Scenario

maximize

$$-150x_W - 230x_C - 260x_B - 238y_W + 170w_W - 210y_C + 150y_C + 36w_B + 10e_B$$

subject to

$$\begin{array}{rcl}
x_W & = & 120 \\
x_C & = & 80 \\
x_B & = & 300 \\
x_W + x_C + x_B & \leq & 500 \\
2.5x_W + y_W - w_W & = & 200 \\
3x_C + y_C - w_C & = & 240 \\
20x_B - w_B - e_B & = & 0 \\
w_B & \leq & 6000 \\
x_W, x_C, x_B, y_W, y_C, e_B, w_W, w_C, w_B & \geq & 0
\end{array}$$

Fixed Policy – Average Yield Scenario Solution

	Wheat	Corn	Beans
Plant (acres)	120	80	300
Production	300	240	6000
Sales	100	0	6000
Purchase	0	0	0

- Profit: $118,600

Fixed Policy – Bad Yield Scenario

maximize

$$-150x_W - 230x_C - 260x_B - 238y_W + 170w_W - 210y_C + 150y_C + 36w_B + 10e_B$$

subject to

$$\begin{aligned}
x_W &= 120 \\
x_C &= 80 \\
x_B &= 300 \\
x_W + x_C + x_B &\leq 500 \\
2x_W + y_W - w_W &= 200 \\
2.4x_C + y_C - w_C &= 240 \\
16x_B - w_B - e_B &= 0 \\
w_B &\leq 6000
\end{aligned}$$

- Objective Value: $55,120

Fixed Policy – Good Yield Scenario

maximize

$$-150x_W - 230x_C - 260x_B - 238y_W + 170w_W - 210y_C + 150y_C + 36w_B + 10e_B$$

subject to

$$\begin{aligned}
x_W &= 120 \\
x_C &= 80 \\
x_B &= 300 \\
x_W + x_C + x_B &\leq 500 \\
3x_W + y_W - w_W &= 200 \\
3.6x_C + y_C - w_C &= 240 \\
24x_B - w_B - e_B &= 0 \\
w_B &\leq 6000
\end{aligned}$$

- Objective Value: $148,000

What's it Worth to Model Randomness?

- If Farmer Ted implemented the policy based on using only "average" yields, he would plant $x_W = 120, x_C = 80, x_B = 300$
- He would expect in the long run to make an average profit of...
  - $1/3(118600) + 1/3(55120) + 1/3(148000) = 107240$
- If Farmer Ted implemented the policy based on the solution to the stochastic programming problem, he would plant $x_W = 170, x_C = 80, x_B = 250$.
  - From this he would expect to make 108390

VSS

- The difference of the values 108390-107240 is the Value of the Stochastic Solution : \$1150.

- On average, it would pay off \$1150 per growing season for Farmer Ted to use the "stochastic" solution rather than the "mean value" solution.
- \$1150 is precisely the (expected) "value" of implementing a planting policy based on the "stochastic solution", rather than the mean-value solution.

EVPI – Expected Value of Perfect Information

- The EVPI measures the maximum amount a decision maker would be willing to pay in return for complete and accurate information about the future.

- How much could Farmer Ted make if he could "wait and see" what the weather was going to be before deciding on how to plant?

Formulation – Good yields

$$\max -150x_W - 230x_C - 260x_B - 238y_W + 170w_W \\ - 210y_C + 150w_C + 36w_B + 10e_B$$

subject to

$$\begin{aligned} x_W + x_C + x_B &\leq 500 \\ 3x_W + y_W - w_W &= 200 \\ 3.6x_C + y_C - w_C &= 240 \\ 24x_B - w_B - e_B &= 0 \\ w_B &\leq 6000 \\ x_W, x_C, x_B, y_W, y_C, e_B, w_W, w_C, w_B &\geq 0 \end{aligned}$$

Solution with good yields

	Wheat	Corn	Beans
Plant (acres)	183.33	66.67	250
Production	550	240	6000
Sales	350	0	6000
Purchase	0	0	0

- Profit: $167,667

Formulation – Bad Yields

$$\max -150x_W - 230x_C - 260x_B - 238y_W + 170w_W \\ - 210y_C + 150w_C + 36w_B + 10e_B$$

subject to

$$\begin{aligned} x_W + x_C + x_B &\le 500 \\ 2x_W + y_W - w_W &= 200 \\ 2.4x_C + y_C - w_C &= 240 \\ 16x_B - w_B - e_B &= 0 \\ w_B &\le 6000 \\ x_W, x_C, x_B, y_W, y_C, e_B, w_W, w_C, w_B &\ge 0 \end{aligned}$$

Solution – Bad Yields

	Wheat	Corn	Beans
Plant (acres)	100	25	375
Production	200	60	6000
Sales	0	0	6000
Purchase	0	180	0

- Profit: $59,950

Computing Farmer Ted's EVPI

- With perfect information, Farmer Ted's Long Run Profit/Year would be:
  - $(1/3)(167667) + (1/3)(118600) + (1/3)(59950) = 115406$
  - The numbers 167667, 118600, 59950 are the optimal amounts of money he would make if he knew the yields beforehand.
- Without perfect information, using the stochastic programming solution, Farmer Ted can expected to make 108390.
- EVPI = 115406 - 108390 = 7016.

Perfect information solution value is sometimes a useful bound on optimal stochastic programming value

Farmer Ted: Final Thoughts

Model was obviously simple for illustrative purposes

- How might model of random yields be improved?
- What other types of uncertainty might Farmer Ted need to add to his model?
- Would you expect correlation between the different uncertain parameters?

(General) Stochastic Programming

General Stochastic Program

$$\min_{x \in X} \mathbb{E}_\xi[F(x, \xi)]$$

- ξ: Random r-vector defined on some probability space $(\Omega, \mathbb{P}, \Sigma)$, i.e., $\xi : \Omega \to \mathbb{R}^r$
- Cumulative distribution function (cdf) of ξ: $H_\xi(y) = \mathbb{P}[\xi \leq y]$
- Expected value[1]: $\mathbb{E}_\xi[F(x, \xi)] = \int_\Omega F(x, \xi(\omega)) d\mathbb{P}$
- X: Given set of deterministic constraints

[1] Can just think of it as a weighted sum

Two-stage Stochastic Programming

Two-Stage Stochastic Program w/Recourse

$$\min_{x \in X} c^\top x + \mathbb{E}_\xi[Q(x,\xi)]$$

where:

$$\begin{aligned} Q(x,\xi) := \min \ & q(\xi)^\top y \\ \text{s.t. } & W(\xi)y = h(\xi) - T(\xi)x \\ & y \geq 0 \end{aligned}$$

- x: First-stage decisions (most important output of model)
- y: Recourse decisions (implicitly depend on ξ)
- $Q(x,\xi)$: Recourse function (AKA "value function")

Fits general SP model with $F(x,\xi) = c^\top x + Q(x,\xi)$

Some Challenges in Stochastic Programming

Two-Stage Stochastic Program w/Recourse

$$\min_{x \in X} c^\top x + \mathbb{E}_\xi[Q(x,\xi)] \qquad \text{(here } F(x,\xi) = c^\top x + Q(x,\xi)\text{)}$$

where:

$$\begin{aligned} Q(x,\xi) := \min \; & q(\xi)^\top y \\ \text{s.t. } & W(\xi)y = h(\xi) - T(\xi)x \\ & y \geq 0 \end{aligned}$$

- Evaluating $\mathbb{E}_\xi[Q(x,\xi)] \Rightarrow$ Potentially high-dimensional integral
- Working with "implicit" definition of $Q(x,\xi)$

Discrete r.v.'s: The easy life!

- Assume $\xi \in \{\xi^1, \dots, \xi^K\} \subseteq \mathbb{R}^r$ with $\mathsf{P}(\xi = \xi^k) = p_k, k = 1, \dots, K$
- $T_k \stackrel{\text{def}}{=} T(\xi^k), h_s \stackrel{\text{def}}{=} h(\xi^k), q_k \stackrel{\text{def}}{=} q(\xi^k), W_k = W(\xi^k)$

$$\begin{aligned} \min\ & c^\top x + \sum_{k=1}^{K} p_k Q_k(x) \\ \text{s.t. } & Ax = b \\ & x \in \mathbb{R}^{n_1}_+ \end{aligned}$$

where for $k = 1, \dots, K$

$$Q_k(x) \stackrel{\text{def}}{=} Q(x, \xi^k) = \begin{aligned}[t] \min\ & q_k^\top y \\ \text{s.t. } & W_k y = h_k - T_k x \\ & y \in \mathbb{R}^{n_2}_+ \end{aligned}$$

Extensive Form

- When we have a finite number of scenarios, or if we approximate the problem with a finite number of scenarios[2], we can write an equivalent extensive form linear program:

$$
\begin{aligned}
\min_{x,y} \; & c^\top x + \sum_{k=1}^{K} p_k q_k^\top y_k \\
\text{s.t.} \; & Ax = b \\
& T_k x + W_k y_k = h_k, \quad \forall k = 1, \ldots, K \\
& x \geq 0, \; y_k \geq 0, \quad \forall k = 1, \ldots, K
\end{aligned}
$$

Note: Model works same with integer restrictions on any of the variables

[2] Stay Tuned for Sample Approximation Module

Extensive Form: Structure

$$
\begin{array}{ccccccccccc}
c^\top x & + & p_1 q_1^\top y_1 & + & p_2 q_2^\top y_2 & + & \cdots & + & p_K q_K^\top y_K & & \\
\text{s.t.} & & & & & & & & & & \\
Ax & & & & & & & & & = & b \\
T_1 x & + & W_1 y_1 & & & & & & & = & h_1 \\
T_2 x & & & + & W_2 y_2 & & & & & = & h_2 \\
\vdots & & & & & & \ddots & & & & \vdots \\
T_K x & & & & & & & + & W_K y_K & = & h_K \\
x \geq 0 & & y_1 \geq 0 & & y_2 \geq 0 & & & & y_K \geq 0 & &
\end{array}
$$

The Upshot

- This is just a larger linear program (or linear integer program)
- Might be VERY large, but it has special structure
- Stochastic programming algorithms exploit this structure

$$
\begin{array}{ccccccccccc}
c^T x & + & p_1 q_1^T y_1 & + & p_2 q_2^T y_2 & + & \cdots & + & p_K q_K^T y_K & & \\
\text{s.t.} & & & & & & & & & & \\
Ax & & & & & & & & & = & b \\
T_1 x & + & W_1 y_1 & & & & & & & = & h_1 \\
T_2 x & & & + & W_2 y_2 & & & & & = & h_2 \\
\vdots & & & & & & \ddots & & & & \vdots \\
T_K x & & & & & & & + & W_K y_K & = & h_K \\
x \geq 0 & & y_1 \geq 0 & & y_2 \geq 0 & & & & y_K \geq 0 & &
\end{array}
$$

Building the Extensive Form

1. Write a nominal (one scenario) model
2. Decide which variables are first stage, and second stage
3. Give s scenario index to all second stage variables and random parameters
4. "Give context" to all scenarios

Facility Location and Distribution

- Facilities: I
- Customers: J
- Fixed cost f_i, capacity u_i for facility $i \in I$
- Demand d_j: for $j \in J$
- Per unit Delivery cost: c_{ij} $\forall i \in J, j \in J$

$$\min \sum_{i \in I} f_i x_i + \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ij}$$

$$\sum_{i \in I} y_{ij} \geq d_j \quad \forall j \in J$$

$$\sum_{j \in J} y_{ij} - u_i x_i \leq 0 \quad \forall i \in I$$

$$x_i \in \{0, 1\}, y_{ij} \geq 0 \qquad \forall i \in I,\ j \in J$$

Evolution of Information

1. Build facilities now
2. Demand becomes known. One of the scenarios $d^k, k = 1, \ldots, K$ happens
3. Meet demand from open facilities

- First stage variables: x_i
- Second stage variables: y_{ijk}

The Extensive Form

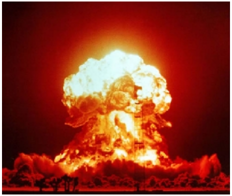
$$\min \sum_{i \in I} f_i x_i + \sum_{k=1}^{K} p_k \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ijk}$$

$$\sum_{i \in I} y_{ijk} \geq d_{jk} \quad \forall j \in J \ k = 1, \ldots, K$$

$$\sum_{j \in J} y_{ijk} - u_i x_i \leq 0 \quad \forall i \in I, \ k = 1, \ldots, K$$

$$x_i \in \{0, 1\}, y_{ijk} \geq 0 \qquad \forall i \in I, j \in J, k = 1, \ldots, K$$

Modeling Discussion



- Do we always need to meet demand?
  - Regardless of the outcome d^k?
- What happens on the off chance that our product is so popular that we can't possibly meet demand, even if we opened all of the facilities?
  - Does the world end? (Recourse problem is infeasible $\rightarrow$ infinite cost!)

Two Ideas

1. We could penalize not meeting demand of customers.
2. We only want to meet demand "most of the time". (Chance constraint)

SP Definitions

Let Ξ be the support[3] of random vector ξ

- A 2-stage stochastic optimization problem has complete recourse if for every possible outcome, there always exists a feasible recourse solution:

$$Q(x, \xi) < +\infty \ \forall x \in \mathbb{R}^n, \ \xi \in \Xi$$

- A 2-stage stochastic optimization problem has relatively complete recourse if for every outcome, *and for every feasible first stage solution*, there is always feasible recourse:

$$Q(x, \xi) < +\infty \ \forall x \in X, \ \xi \in \Xi$$

Good modeling practice: Include sufficiently flexible recourse to ensure model has relatively complete recourse.

[3]Support of ξ: smallest set Ξ such that $\mathbb{P}[\xi \in \Xi] = 1$

Penalize Shortfall: A Recourse Formulation

New parameter: λ_j = Per unit "cost" of not meeting demand of customer type j

$$\min \sum_{i \in I} f_i x_i + \sum_{k=1}^{K} p_k \left[\sum_{i \in I} \sum_{j \in J} c_{ij} y_{ijk} + \sum_{j \in J} \lambda_j z_{jk} \right]$$

$$\sum_{i \in I} y_{ijk} + z_{jk} \geq d_{jk} \quad \forall j \in J, \ k = 1, \ldots, K$$

$$\sum_{j \in J} y_{ijk} - u_i x_i \leq 0 \quad \forall i \in I, \ k = 1, \ldots, K$$

$$x_i \in \{0, 1\}, y_{ijk} \geq 0 \qquad \forall i \in I, j \in J, \ k = 1, \ldots, K$$

VSS: Value of the Stochastic Solution

- Let z_{S} be the optimal solution value to

$$z_{\text{S}} \stackrel{\text{def}}{=} \min_{x \in X} \mathbb{E}[F(x, \xi)]$$

- Let x_{MV} be an optimal solution to the "mean-value" problem:

$$x_{\text{MV}} \in \arg\min_{x \in X} F(x, \mathbb{E}[\xi])$$

- Let z_{MV} be the long run cost if you plan based on the policy obtained from the 'average' scenario:

$$z_{\text{MV}} \stackrel{\text{def}}{=} \mathbb{E}[F(x_{\text{MV}}, \xi)]$$

Value of Stochastic Solution

$$\text{VSS} \stackrel{\text{def}}{=} z_{\text{MV}} - z_{\text{S}}$$

- Simple HW: Prove $\text{VSS} \geq 0$

EVPI

- Let z_{S} be the optimal solution value to a stochastic program

$$z_{\text{S}} = \min_{x \in X} \mathbb{E}[F(x, \xi))]$$

- Let z_{PI} be the expected value of the optimal "wait and see" solution(s)

$$z_{\text{PI}} = \mathbb{E}[\min_{x \in X} F(x, \xi))]$$

Expected Value of Perfect Information

$$\text{EVPI} = z_{\text{S}} - z_{\text{PI}} \geq 0$$

- Simple HW: Prove $\text{EVPI} \geq 0$