Facility Location Problem Statement

A company has 5 candidate locations where facilities can be installed to produce some product, and has to transport the product from the facilities to 4 customers. Transportation cost is proportional to the amount of product transported and the distance. There are 3 different sizes of that facility. At most one facility can be installed at one place. Once a facility is installed, the investment cost and operational cost will be paid. Customers' demands are of uncertainty. If the demand is not satisfied, an additional penalty will be paid. The objective is to minimize the annul cost, and we have to decide where to install those facilities.

Sets:

I – set of candidate sources, J – set of customers, T – set of facilities, S – set of scenarios

Parameters:

Locations of candidate sources and customers => $D_{\{i,j\}}$ - Distances between nodes $i, j, i \in I, j \in J$

$\bar{C}_t,\ I_t, O_t$ - Capacity, investment cost and operational cost of each facility $t, t \in T$

$d_{\{j,s\}}$ - Demand of customer j in each scenario $s, j \in J, s \in S$

p_s - Probability of scenario $s, s \in S$

θ - Transportation cost per unit of product transported per unit of transporting distance

λ - Penalty per unit of unsatisfied demand

Variables:

$y_{\{i,t\}}$ − binary variable: $y_{\{i,t\}} = 1 \Leftrightarrow$ facility t is installed at node i (first-stage variable), $i \in I, t \in T$

$f_{\{i,j\}}$ − flow of product from source i to customer j (second-stage variable), $i \in I, j \in J$

$z_{\{j\}}$ − unsatisfied demand of customer j (second-stage variable), $j \in J$

Parameters:

#Source	X(km)	Y(km)
A	0	0
B	10	40
C	45	35
D	60	95
E	100	30

		Pr	**0.33**	**0.5**	**0.17**
#Customer	X(km)	Y(km)	d1(t/yr)	d2(t/yr)	d3(t/yr)
I	25	55	120	100	160
II	50	65	65	45	90
III	70	15	480	390	600
IV	90	85	275	380	100

#Facility	Capacity (ton/yr)	Investment ($)	Operational ($/year)
t1	100	10000	1000
t2	200	16000	1600
t3	300	21000	2100

Model:

$$\min_{y} \quad \frac{1}{20}\sum_{i}\sum_{t} I_t y_{\{i,t\}} + \sum_{i}\sum_{t} O_t y_{\{i,t\}} + \sum_{s} p_s Q_s(y)$$

s.t.

$$\sum_{t} y_{\{i,t\}} \leq 1, \forall\, i \in I$$

$$y_{\{i,t\}} \in \{0,1\}, \forall\, i \in I, \forall\, t \in T$$

where

$$Q_s(y) = \min_{z,f} \sum_{i}\sum_{j} \theta f_{\{i,j\}} D_{\{i,j\}} + \sum_{j} \lambda z_j$$

s.t.

$$\sum_{i} f_{\{i,j\}} + z_j \geq d_{\{j,s\}}, \forall\, j \in J$$

$$\sum_{j} f_{\{i,j\}} \leq \sum_{t} y_{\{i,t\}}\, \bar{C}_t, \forall\, i \in I$$

$$f_{\{i,j\}} \geq 0, z_j \geq 0, \forall i \in I, \forall\, j \in J$$