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2019
MCM/ICM
Summary Sheet

"Pyramid Exodus" Project

Summary

Maybe you have not watched the movie "The Da Vinci Code", but you must know Mona Lisa. Every year people from all over the world come to the Louvre to witness its charm. However, an Islamic terrorist attack happened in 2017 beneath the Louvre proves the significance of having an optimal evacuation plan.
Our goal is to find out adaptable approaches to evacuate for not only the Louvre, but also other large and crowded buildings with minimum time. We should also identify the potential bottlenecks which limits the movement speed. Based on these bottlenecks, more optimized evacuation routes will be proposed with the specific consideration of emergency personnel.
In the analysis section, we *enumerate influential factors* and talk about the behavior models of visitors when they are evacuating using **game theory**. When it comes to the modeling section, we designed a model called **Minimum Sum of Time Segments(MSTS) Model**. Its general model is to *obtain the minimum sum of the time slices* of each routes. Our innovation point is that we combined the **Dijkstra algorithm** with an *analogy between optical path with the relative path segment*. Then we used three definitions of bottlenecks to *identify the potential bottleneck.*
In the validation section, we use some data to *verify the correctness and flexibility of our model*. We also compare our approach with the reality, finding that it is fit for the Louvre and *other large buildings*.
In the conclusion section, we *give out some policy recommendations* such as introducing high-tech equipments to monitor real-time population and instruct the visitors automatically with a color-changeable signal board. We also state some policy regarding the disabled and the old particularly.
Finally, our MSTS model has more *strengths* than *weaknesses*. Although it is not very feasible when the building is too high or too complex, it provides an innovative and quick way to get the evacuation time and routes which helps the decision-makers through evacuation.

Keywords: Game Theory; Dijkstra Algorithm; Potential Bottleneck Identification

Contents

1 Introduction

1.1 Background

Terrorism has spread to all parts of the world at a high speed in recent years. Nowadays, Middle East, South Asia and Africa have become the main source of international terrorism. Though the targets of terrorist attacks are spread all over the world, Europe and the United States are becoming the new focus of the terrorist attacks.

From the news report, we could know that there have been at least 12 major terror-related incidents happened in France since 2012. [1] A terrorism attack occurred in 2017 when a man armed with a machete and shouting 'Allahu Akbar' as he tried to enter the shopping mall beneath Louvre. The man was going to attack the soldier by stabbing him with a knife.

These terrible terrorist attacks will hopefully be wake-up calls for the evacuation plans, especially for large-sized structures like Louvre in Paris. As a consequence, a well-designed evacuation plan is quite significant and meaningful for both the administers in the museum and the visitors to escape from danger as safely and quickly as possible.

1.2 Restatement of the Problem

"Pyramid Project" was conducted in 2014 to release the inconvenience of long waiting lines and noise problem. With the success of this project, Louvre increases its degree of satisfaction.[2]Thus, more and more visitors have come to visit the Louvre since 2016. The total amount of visitors of 2017 has become 8.1 million, while this number increased to 10.2 million in 2018.[3]

If we come across a terrorist attack when we were visiting the Louvre, how to leave it with safety and at high speed would be an important problem that need to be said twice. Additionally, a reserved way for emergency personnel to get into the Louvre as quickly as possible cannot be omitted. Using the service doors, employee entrances, VIP entrances, emergency exits and old secret exit has the potential to lead extra overhead. Only when the benefits overweigh the overhead could we use these exits.

Now, to clarify the problem we should consider the following factors:

- The languages which the visitors speak are not the same.
- Gender and age might influence the speed at which the visitors leave the Louvre.
- If a visitor has come to the Louvre earlier this year, he or she will be more familiar with the indoor structure of this building. So it will cost he or she

less time than those haven't come.

- People tend to go with their friends or family members even though when they are in danger, which means that we should not separate the groups apart.
- In France, people tend to walk on their right side. However, there exists some regions which are more used to walking on the left side. This small difference may cause collisions when escaping from the Louvre.

1.3 Overview of Our Work

To help the museum leader to find out an optimal way to evacuate the crowds, we divide the process of solving this problem in the following steps, as shown in the figure below:

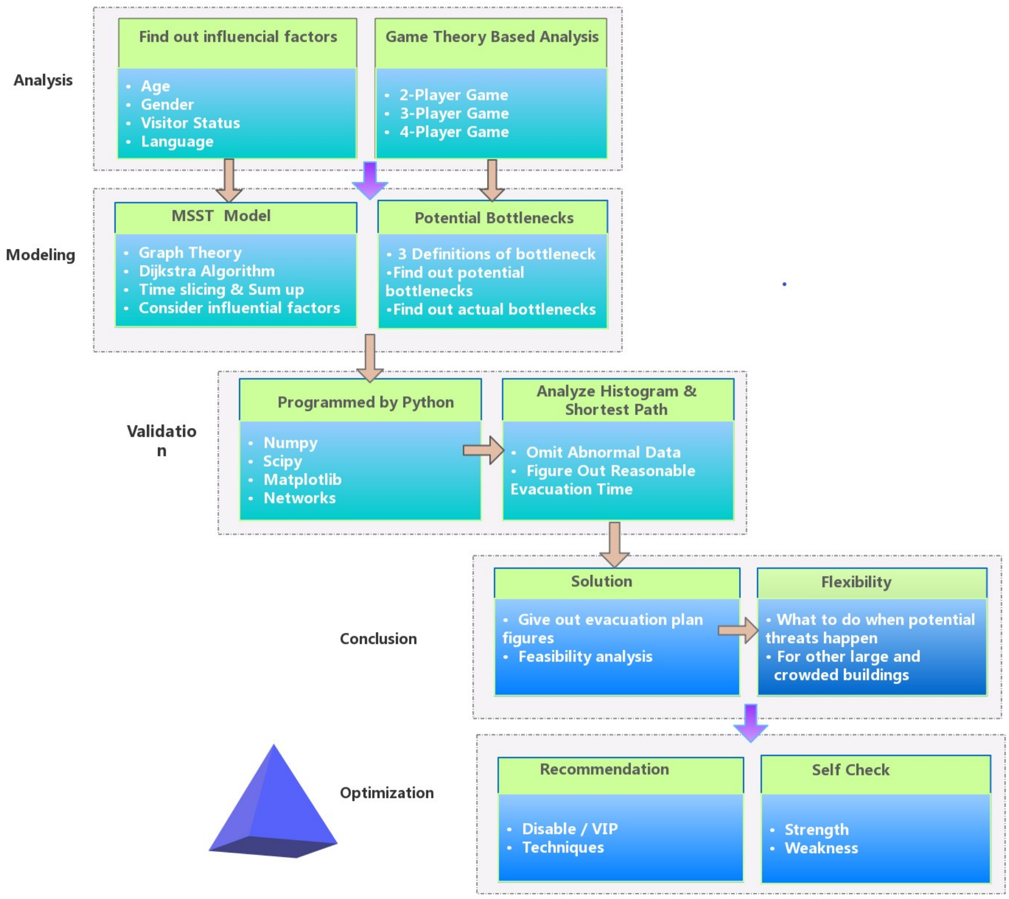


Figure 1: Mind Flow Chart

2 Assumptions & Notations

2.1 Assumptions

To simplify the given problem and made it more appropriate for the real life situation, we make the following basic assumptions, each of which is properly justified:

- The museum has mastered the total number of visitors in the museum in real time, and the museum can organize the evacuation of visitors immediately after the emergency;
- The population density of different exhibition halls is directly proportional to the relative popularity, and the population in each exhibition hall is evenly distributed;
- The speed of movement and the density of people during evacuation are negatively correlated. The ratio of the population density between the exhibition halls with the same exit remains the same, regardless of the density wave, the "fast is slow" effect, the bottleneck swing, etc;
- During the evacuation process, tourists pursue maximum individual profits, while the museum seeks to maximize the overall benefits;
- Tourists flee according to the rules of the museum, and the people who have already left the museum will not interfere with the evacuation of those are not;
- Disabled visitors and VIP visitors evacuate along the dedicated channels and do not occupy other visitors' evacuation space.

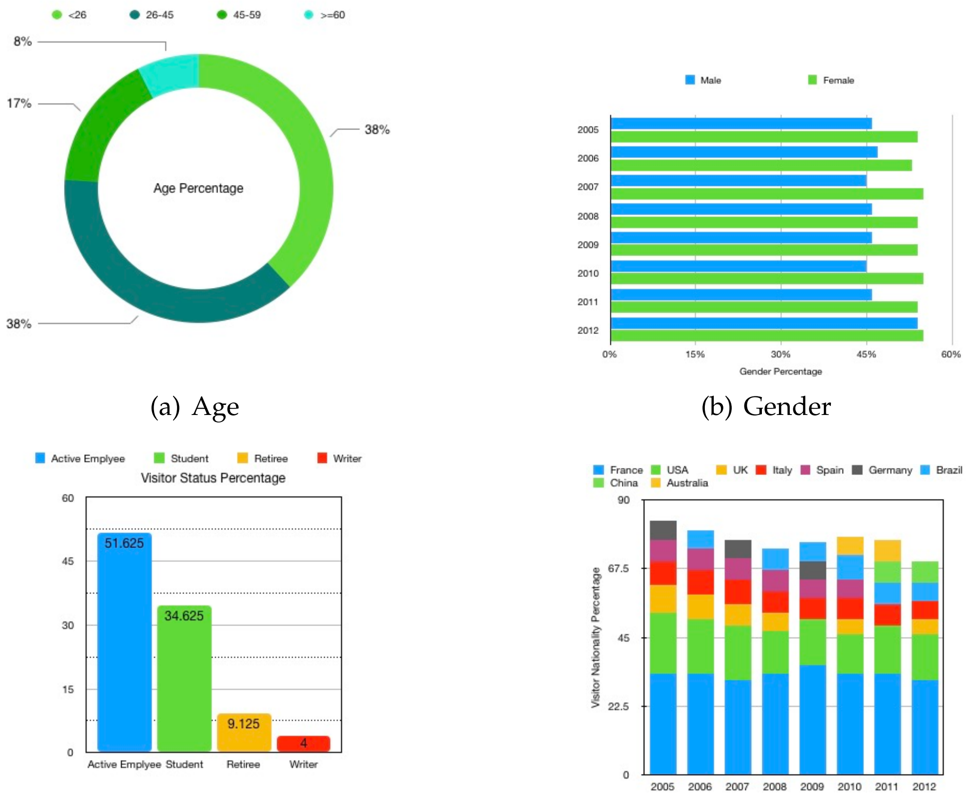
2.2 Notations

Notations	Descriptions
A_i	the area of block *i*
B	the potential bottleneck
e	the number of people evacuating from the Louvre per time unit
L_i	the distance of the segment *i* of the shortest path between one block and one exit
$L_i{}'$	the relative distance of the segment *i* of the shortest path between one block and one exit which is decided by the popularity of that block
N_0	the original number of the people in the Louvre
$N_{(t)}$	the total number of people in the Louvre at time *t*
$N_{i(t)}$	the number of people passing through block i from time 0 to time t
P_i	the population distribution probability of block *i*, which is related to its popularity
$\overline{P}$	the average population distribution probablity of all the blocks
v_0	free flow speed, which is a constant as 1.34 m/s
v_i	the average movment speed of the people in block *i*
T_P	the minimum value of the minimum evacuation time from block *P* to all the exits
T_{Pq}	the minimum time for all the people in block *P* to reach exit *q*
T_{min}	the minimum time for people in all the blocks to evacuate
ρ_0	the boundary population density, which is a constant as 0.2 $person/m^2$
$\rho_{i(t)}$	the population density of block *i* at time *t*
η_i	the utilization rate of node i
η_0	the real transport situation measurement index

3 Problem Analysis

3.1 Enumerate Influential Factors

From the website of Louvre, we have read the annual reports from 2005 to 2012 and got some information about the visitors[4]. We list four related factors of visitors evacuation, which are age, gender, status and language. It is intuitive for you to learn about these four factors in the following figures.



(a) Age (b) Gender (c) Visitor Status (d) Language

Figure 2: Influential Factors

(a) Age Factor
By analyzing the data, we find that 38% of the visitors are less than 26 years old, which is nearly the same with the ones between 26 and 45 years old. Those who are older than 45 and younger than 59 years old take the account of 17 percentage. While the visitors older than 60 years old is approximately 8%. We should consider the differences among these age segments. The average movement speed of the young and the old is slower than that of the middle ages.

(b) Gender Factor
For the eight years from 2005 to 2012, female visitors account for about 53%, while male visitors accounts for 43%. Researches showed that young people at signal-free intersections have a speed of 1.72 meters per second, middle-aged people have a speed of 1.47 meters per second, and elderly people have a speed

of 1.16 meters per second. It is studied that male speed is 1.41 meters per second and female speed was 1.28 meters per second. The above study shows that the composition of pedestrians will directly affect the speed of them.

(c) Visitor Status Factor
This factor is related to the age factor. About 51% of the visitors are still at work, so their age range tends to be $26 - 59$, close to the sum of the percentage of these age ranges. Students are often younger than 26 years old, which take the percentage of 34%. And the amount of retirees is 9% of the visitors, which is just the same ratio of the ones older than 60 years old. This factor also influences the time they visit the Louvre. Since employee works from Monday to Friday, they can only pay a visit on weekends. However, students are able to go exhibition both on weekends and on vacation. And the visiting time for retirees are quite flexible.

(d)Language Factor
From the official website, we can learn the main source countries where the visitors are from. Visitors from France speak French. Visitors from USA, UK, Brazil-Germany and Australia speak English. And visitors from Italy speak Italian. And visitors from Germany also speak German. Chinese visitors speak Chinese. During evacuation, visitors are under the instruction of the staff in the Louvre. Since about 33% of the visitors are from France and most European countries speak English. We consider that most of the staff in the Louvre is native, and they are able to speak both French and English. Only a small amount of people cannot understand the instruction of the staff. The solution to this factor will be proposed in the below section.

3.2 Problem Analysis Based on Game Theory

In general, the evacuation process of a group is a movement phenomenon exhibited by a group, which is formed by interaction between individuals and the surrounding environment, or among individuals.[4] The individual's psychology and behavior during evacuation have a magnificent influence on the selective strategy of evacuation route and the time required for evacuation. Therefore, understanding the individual's psychology and behavior plays an important role in improving evacuation efficiency. The typical characteristics of evacuation behavior are shown as follows:

1.Herd phenomenon[**?**]
In the process of group evacuation, the individual changes his or her walking strategy due to the pressure or influence of groups and turns into the same walking strategy as the others, that is, the herd phenomenon, also known as the herd behavior. To some extent, this blind behavior is irrational.

2.Bottleneck arching phenomenon[7]
If the crowd has the same destination, when the evacuation begins, the individual movement direction of the crowd goes straight to the exit, which increases

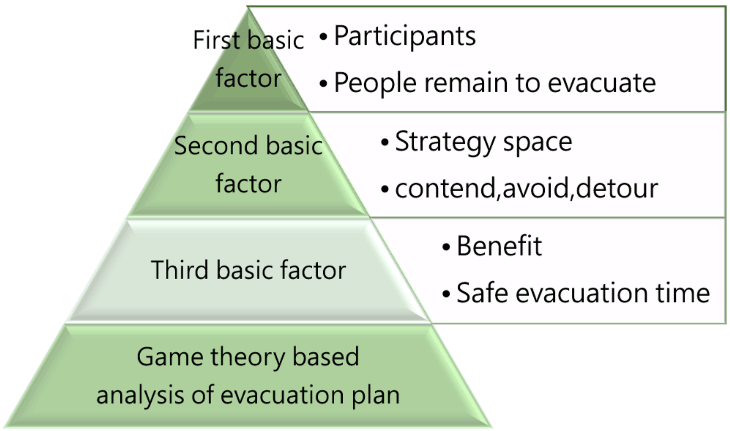


Figure 3: Three Basic Factors of Game Theory

the density of personnel at the exit gradually. As this process continues, the ability of people at the exit to flow is saturated. At the same time, the individual needs to find other alternative directions according to the real situation. In the process of selecting a new direction, the individual still hopes to reach the position as close to the exit as possible. Since every point on the circle has the same distance to the center of the circle. There also exists the limitation of the capacity at the exit. Consequently, when more individuals expect to be closest to the exit, the crowds who gathered at the exit tends to form a similar Round arrangement.

3."Fast is slow" effect
When the population density at the exit is high, the group evacuation time will first decreases and then increases as the individual speed increases. This effect is due to the inconsistent behavior among individuals. If some individuals accelerate their speed, the mutual friction and interference among them will increase at the exit, resulting in an increase in the overall resistance. The reduction of the evacuation efficiency of the people at the exit is called "fast is slow" effect. Therefore, individuals should proceed in an orderly manner to ensure stable evacuation efficiency rather than speed up blindly.

The game theory is introduced to study the decision-making and its balance of each participant when the behavior of the decision-making object is under competition situation. And the process of evacuation is a process of having the shortest time (distances) to the destination when the people round them have different choices of contending, avoiding, and detouring in the strategy space. These choices may affect the decision of themselves and the optimization process. Due to the limitation of evacuation space, the process of individuals approaching the destination is a competitive stage. Game theory can fully display the decision-making process, which explains the psychological process of their choice of routes. To analyze this problem by game theory, three basic factors must be satisfied, as shown in Figure[8].

According to ergonomic theory [9], the human body can be regarded as an ellipsoid with a long axis of $61cm$ and a short axis of $45.6cm$. Under crowded conditions, we can simplify the projection area of the human body in the direction perpendicular to the ground to a circle with a radius of $r = 0.45m$. The most likely game situations are two-player game, three-player game and four-player game, as shown in Figure 2. Now, we made the following assumptions:

- Each circle only represents one player. The dotted circle indicates that it is not occupied yet, while it can be occupied by adjacent circle through detouring. The solid circle indicates the competition object of each player, and the distance between its center and the destination is d.
- Each player has at most three strategies of contending, avoiding and detouring. The circles are closely adjacent and tangent to each circle.
- Each player moves at the same speed. And the maximum distance he can move during one game is twice the length of the radius.
- The individual profit is equal to the difference between the distance to the destination before and after gaming.

  $$p_i = d_{i\_after} - d_{i\_before}$$

  i indicates the sequence number of the player. p_i stands for the individual profit of player i. Group profit p is the sum of all the individual profits p_i.

  $$p = \sum_{i=1}^{n} p_i$$

Table 1: Two-Player Game

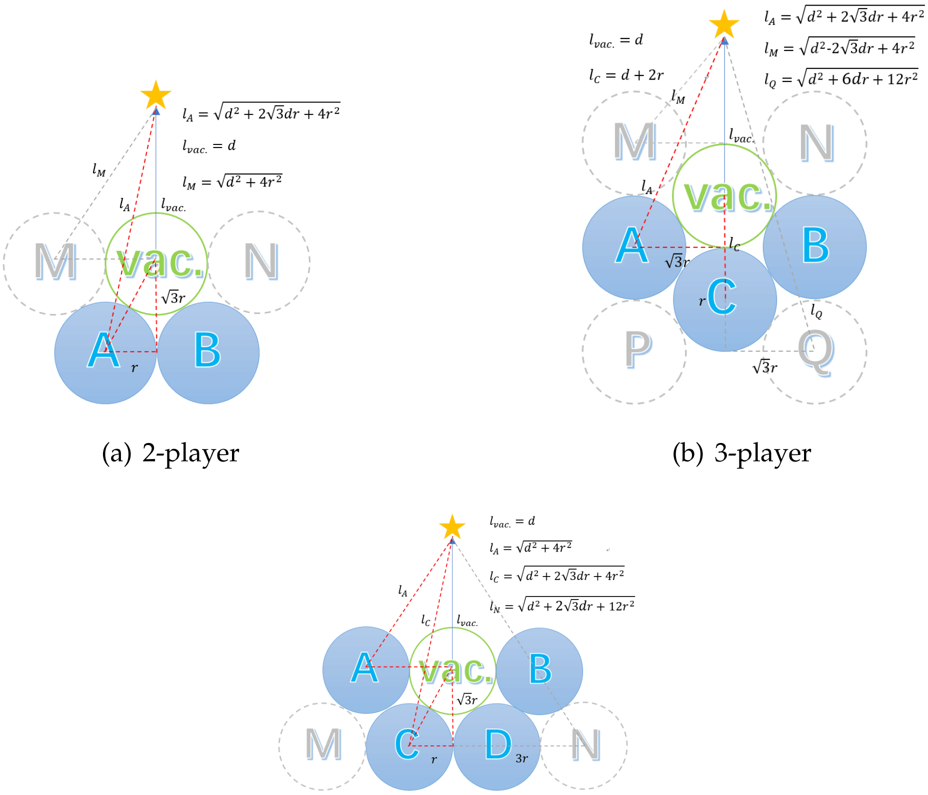
Participant	Profit Table of Two-Player Game			
A		Contend	Avoid	Detour
B	Contend	0	0	0.29
		0	0.66	0.66
		0	0.66	0.95
	Avoid	0.66	0	0.29
		0	0	0
		0.66	0	0.29
	Detour	0.66	0	0.29
		0.29	0.29	0.29
		0.95	0.29	0.58

Table 2: Three-Player Game

Par		Profit Table of Three-Player Game								
A		Contend			Avoid			Detour		
B		Con	Avo	Det	Con	Avo	Det	Con	Avo	Det
C	Con	0	0	0	0	0	0	0.66	0.66	0.66
		0	0	0.66	0	0	0.66	0	0	0.66
		0	0	0	0	0.90	0.90	0	0.90	0.90
		0	0	0.66	0	0.9	1.56	0.66	1.56	2.22
	Avo	0	0.66	0.66	0	0	0	0.66	0.66	0.66
		0	0	0.66	0.66	0	0.66	0.66	0	0.66
		0	0	0	0	0	0	0	0	0
		0	0.66	1.32	0.66	0	0.66	1.32	0.66	1.32
	Det	0	0.66	0.66	0	0	0	0.66	0.66	0.66
		0	0	0	0.66	0	0.66	0.66	0	0.66
		-0.58	-0.58	0.24	0.24	0.24	0.24	0.24	0.24	0.24
		-0.58	0.08	0.90	0.90	0.24	0.90	1.56	0.90	1.56

Table 3: Four-Player Game

Par		Profit Table of Four Player Game											
A		Contend						Avoiod					
B		Contend			Avoid			Contend			Avoid		
C		Co	Av	De	Co	Av	De	Co	Av	De	Co	Av	De
D	Co	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0
		0	0	-0.58	0	0	-0.58	0	0	-0.58	0	0	0
		0	0	0	0	0	0	0	0	0	0	0.90	0.90
		0	0	-0.58	0	0	-0.58	0	0	-0.58	0	0.90	0.90
	Av	0	0	0	0	0.66	0.66	0	0	0	0	0	0
		0	0	0	0	0	0	0	0.66	0.66	0	0	0
		0	0	-0.58	0	0	-0.58	0	0	-0.58	0.90	0	-0.58
		0	0	0	0	0	0	0	0	0	0	0	0
		0	0	-0.58	0	0.66	0.08	0	0.66	0.08	0.90	0	-0.58
	De	0	0	0	0	0.66	0.66	0	0	0	0	0	0
		0	0	0	0	0	0	0	0.66	0.66	0	0	0
		0	0	-0.58	0	0	-0.58	0	0	-0.58	0.90	0	-0.58
		-0.58	-0.58	-0.58	-0.58	-0.58	-0.58	-0.58	-0.58	-0.58	0	-0.58	-0.58
		-0.58	-0.58	-1.16	-0.58	0.08	-0.5	-0.58	0.08	-0.5	0.90	-0.58	-1.16



(a) 2-player (b) 3-player

(c) 4-player

Figure 4: 2-player, 3-player & 4-player models

From table 1, we can find: when two participants are gaming, the maximum group benefit happens when one is detouring and the other is contending. And the minimum benefit happens when both of them are contending or avoiding.

From table 2, we can find : when three participants are gaming, to achieve maximum group benefit, two of them are detouring and the other one is contending, and the one who is contending has the largest group profit. The minimum group profit happens when two of them are contending while the other one is detouring.

From table 3, we can find: when four participants are gaming, if three of them are avoiding and the other one is detouring or contending, the group benefit will be maximum. If two of them are detouring and the others are detouring or avoiding, the group benefit will be minimum. This minimum group benefit is even less than the above ones with two participants or three participants.

When we think of the individual benefit, the strategy to get maximum interest is contending when there are only two participants. And this strategy will be contending or detouring when there are three participants. When four participants are gaming, the optimal strategy will become contending or avoiding. When one is presuming the maximum individual profit, it is impossible to get the maximum group profit at the same time. Through this theory, we can explain why individuals tend to go after maximum individual profit by contending. This also come along with the herd phenomenon we talked above.

We suggest individuals go to the closest exit by detouring or even avoiding to get maximum overall profit, meanwhile the average group benefit will decease as the number of participants increases. This will cause bottleneck arching effect and "fast is slow" effect.

Through modifying the value of $\frac{d}{r}$, we come to the conclusion that when $\frac{d}{r}$ is rather big, the individual benefit of detouring and contending is nearly the same. (We don't give out the illustration process due to the page limit.) It is easy to explain why visitors can achieve maximum profit by detouring when they are far away from the exits relatively which have nothing to do with the number of participants.

4 Model Construction

4.1 MSTS Model

This is a model designed by ourselves. We name it as Minimum Sum of Time Segments(**MSTS)** Model. We use this model to calculate the minimum time for all the visitors and staff in the Louvre to leave.

Firstly, we see each room in the Louvre as a block. And then we use Dijkstra algorithm to find out the shortest path from the block where the visitor is located to all the exits. We keep the record of the these shortest paths of each block. The shortest path from one block to one exit is made up of different path segments from one block to another. After that, we slice the total evacuation time into several time segments according to the shortest path. Each time segment correspond with a path segment on the shortest path. Then sum up the time segments to get a minimum evacuation time from a block to an exit. With the same method, we are able to get different minimum evacuation time from this block to all exits. Regard the minimum one of those values calculated above as the real shortest evacuation time. Finally, the shortest path we used to reach our goal of finding the evacuation time of that block is defined as the optimal evacuation route.

Based on the above statements, let us begin to calculate the minimum evacuation time of one block.

First of all, T_{Pq} stands for the minimum time for all the people in block P to reach exit q.

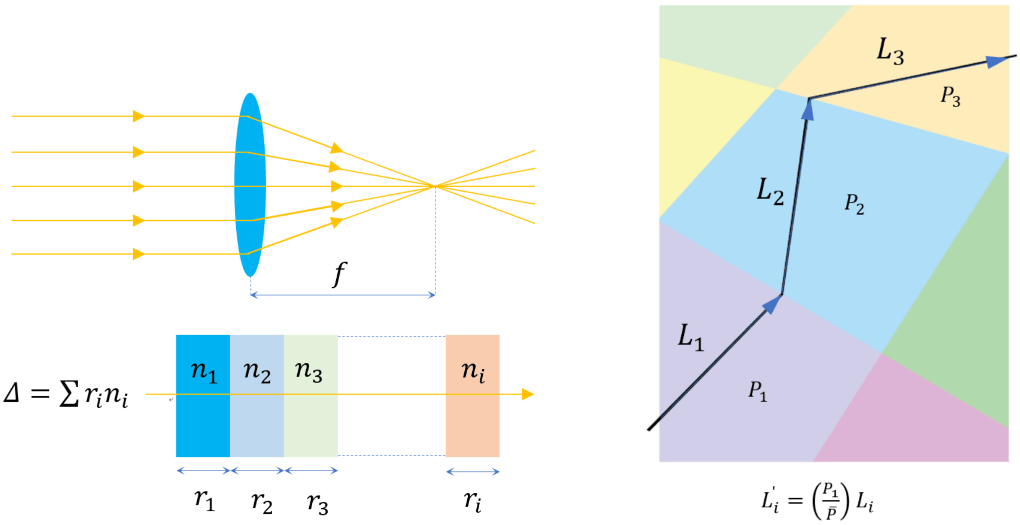
$$T_{Pq} = \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} \frac{{L_i}'}{v_i} \tag{1}$$

In equation (1), t_i stands for the evacuation time of segment i on the shortest path from block P to exit q. And n is the number of path segments.

T_P stands for the minimum value of the minimum evacuation time from block P to all the exits.

$$T_P = min \{T_{P_1}, T_{P_2}, \ldots, T_{P_s}\} \tag{2}$$

Secondly, we use Dijkstra algorithm to identify the shortest path from block P to exit 1, exit 2, ..., exit$s$. We define the distance of the segment i of the shortest path between one block and one exit as L_i. Now we make an analogy between ${L_i}'$ and the definition of optical path. In physics, the product of the geometric path of light propagating in the medium r and its refractive index n is defined as the optical path, that is $\Delta = nr$. Similarly, we introduce $k = \frac{P_i}{\overline{P}}$ to determine the popularity index of block i. P_i is the population distribution probability of block i, which is related to its popularity. And $\overline{P}$ is the average population distribution probabity of all the blocks. The similarity of them is shown in the following figure 7.



(a) optical path (b) $L_i{}'$

Figure 5: make an analogy

Thus, we can get the expression of $L_i{}'$.

$$L_i{}' = kL_i = \left(\frac{P_i}{\overline{P}}\right) L_i \tag{3}$$

To get $L_i{}'$, we should first calculate $\overline{P}$.

$$\overline{P} = \frac{\sum_{i=1}^{m} A_i P_i}{\sum_{i=1}^{m} A_i} \tag{4}$$

In order to simplify the following equations, we define the total area of the Louvre as A.

$$A = \sum_{i=1}^{m} A_i P_i \tag{5}$$

Thirdly, we should also know the the average movment speed of the people in block *i* which is known as v_i.

$$v_i = f(\rho_i) = \begin{cases} c & 0 < \rho_i \leq \rho_0 \\ a\rho_i + b & \rho_i > \rho_0 \end{cases} \tag{6}$$

In equation (6), v_i is a function of ρ_i. ρ_i is the population density of block *i*. And ρ_0 is the boundary population density, which is a constant as 0.2 $person/m^2$.

$$\rho_{i(t)} = \frac{N_{(t)}P_i}{A\overline{P}} \tag{7}$$

In equation(7), $\rho_{i(t)}$ is the population density of block *i* at time *t*. $N_{(t)}$ stands for the total number of people in the Louvre at time *t*.

$$N_{(t)} = N_0 - et \tag{8}$$

In equation (8), e is the number of people evacuate from the Louvre per time unit.

$$e = \frac{N_0}{T_{min}} \tag{9}$$

In equation9, T_{min} means the minimum time for people in all the blocks to evacuate.

$$T_{min} = min\{T_1, T_2, \ldots, T_n\} \tag{10}$$

By using equation(3) to equation (8), we get the derivation of $L_i{}'$ and v_i.

$$L_i{}' = \frac{P_i \sum_{i=1}^{m} A_i}{\sum_{i=1}^{m} A_i P_i} L_i \tag{11}$$

$$v_i = f(\rho_i) = \begin{cases} c & 0 < \rho_i \leq \rho_0 \\ (\frac{aN_0P_i}{\overline{P}} + b) - \frac{aeP_i}{\overline{P}A}t & \rho_i > \rho_0 \end{cases} \tag{12}$$

We can get the following equation by equation (11) and equation(12)

$$\int_{t_{i-1}}^{t_i} e^x \, dt = L_i{}' \tag{13}$$

Through iterative method, we can get the value of e via equation (1) (2) (9) (10) (13). We choose a constant as the original value of e. And when the value of e tends to remain stable, we consider it as the real value. Then we could work out t_i so as to get the minimum evacuation time.

4.2 Potential Bottleneck

4.2.1 Definitions of Bottleneck

Generally, a bottleneck refers to a resource or device which severely constrains the output of the system. According to different application requirement and different production and operation modes, definitions describe the bottlenecks of the production system and the location as well as state of the bottleneck at a certain time from different perspectives. The existing production system bottleneck definition can be roughly classified into the following three categories[9]:

1. Definition based on average waiting time
In the definition based on average waiting time, the machine with the longest average waiting time in the production system is defined as the bottleneck.

$$B = \{i|W_i = max(W_1, W_2, \ldots, W_n)\}$$

W_i is the average waiting time of the product in the *i-th* machine.

Theorem 4.1. *In a stable queuing system, the average number of customers L is equal to the product of the customer's effective arrival rate X and the average time spent by each customer in the system, that is $L = X < W$.*

Therefore the definition of bottleneck is also equivalent to the definition based on the average queue length. This method is suitable for analyzing queuing networks with infinite buffers (whose latency can be infinitely long), but not for systems that contain only a limited number of buffers or no buffers.

2. Definition based on average utilization
Based on the definition of average utilization, the machine with the highest average utilization rate in the production system is defined as the bottleneck.

$$B = \{i|\rho_i = max(\rho_1, \rho_2, \ldots, \rho_n)\}$$

ρ_i is the average utilization of the *i-th* machine.$\rho_i = \frac{\lambda_i}{\mu_i}$, among which λ_i and μ_i are the product arrival rate and service rate of the *i-th* machine. Since there may be more than one machine with the similar workload, the difference in utilization between these machines can be rather small. Although this method is very easy to implement, it might lead to the identification of multiple bottlenecks.

3. Sensitivity-based definition
The sensitivity-based method of defining the bottleneck is to find out the machine whose output has the greatest impact on the output of the whole system. It define the machine whose own parameter disturbs most to the overall productivity sensitivity value of the system as the bottleneck. For a production line in a production system, the productivity is determined by the average number of machines produced by the last machine. It is a function of all machine and buffer parameters:

$$\overrightarrow{PR} = \overrightarrow{PR}\,(p_1, \ldots, p_m, N_1, \ldots, N_m)$$

N_i is the buffer before the *i-th* machine, and p_i is the productivity of the *i-th* machine. At this time, the bottleneck of the system is:

$$B = \left\{ i| \left| \frac{\partial \overrightarrow{PR}}{\partial_{p_i}} \right| > \left| \frac{\partial \overrightarrow{PR}}{\partial_{p_j}} \right|, \forall i \neq j \right\}$$

The sensitivity-based bottleneck definition fully considers the impact of the machine's own capacity change on the overall capacity of the system. By calculating the sensitivity values of each machine and comparing them, the only bottleneck that restricts the production capacity of the production system is found. The only drawback of this method is that the overall capacity of the system is not easy to obtain with respect to the function of each machine parameter, which makes the calculation of the sensitivity value difficult.

4.2.2 Spot the Bottleneck of Our Model

According to the theory of constraints, any system has constraints. To increase the output of the system, the constraints of the system must be broken. Any system can be imagined as a wooden barrel made up of a series of wooden boards, and the amount of water in the barrel depends only on the shortest piece of wood. At the same time, taking into account the mobility of the crowd, we first calculate the utilization rate of each node. If the utilization rate is higher, we identify it as a **potential bottleneck**.

Then, calculate the ratio between the actual number of people passing per time unit and the maximum number that is allowed to pass per time unit. The node whose ratio is larger than one is the **actual bottleneck**. Under this circumstance, we are able to optimize the evacuation plan. The specific calculations are shown as follows:

- After calculating the optimal path from block i to the exit, calculate the coincidence nodes (or nodes within a range of less than 0.45 m which is the average distance between bodies) and the intersection nodes.
- The utilization rate is easy to get by dividing the number of people passing through node i from time 0 to time t $N_{i(t)}$ by the total number of people N_0.

$$\eta = \frac{N_{i(t)}}{N_0} \times 100\%$$

- When $\eta > \eta_0$, we consider this node as a potential bottleneck. η_0 is determined by the real transport situation. Based on the above equation, we can figure out the potential bottleneck and optimize the evacuation route by minimizing the number of potential bottlenecks.
- We look the nodes with low ability to pass through as an actual bottleneck. More specifically, D_i is the amount of people arriving at node i. Under ideal conditions (not considering the congestion), D_i is equal to the number of people passing through block i. We define M_i as the maximum number of people allowed to pass through node i.

If $\frac{D_i}{M_i} > 1.0$, this node i is an actual bottleneck.

5 Validation of Our Model

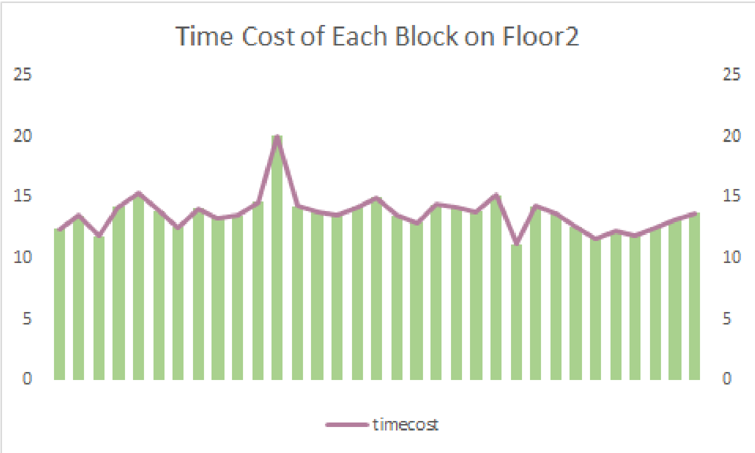
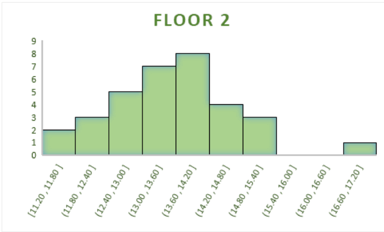


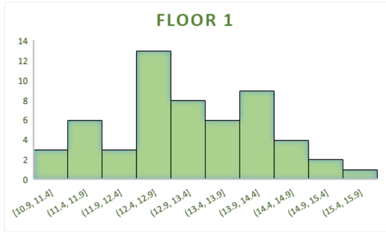
Figure 6: Time Cost of Floor 2

The test results show that the evacuation time of visitors in different blocks on the second floor is about 14 minutes. There is a set of data of 20, which is quite different from other groups of data, so we have dropped this value. According to GK Still's entry statistics for various football matches held at Wembley Stadium[11]: During the sporting event (accommodating 82,000 people), the audience arrived in a short time before the start of the game. From one and a half hours to half an hour, the maximum flow rate of imports can reach 120 140 people/minute., an average of 2,050 people passing the entrance per minute. In the model we built, there were a total of 27,412 people in the building. The time required was about 14 minutes, and the average evacuation was 1958 people per minute with an error of less than 10%. This illustrates that the model we built is realistic and feasible.

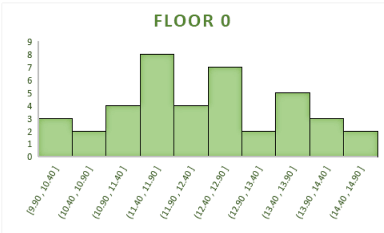
The histograms of different floors are shown as below:



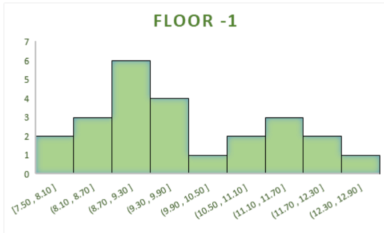
(a) Floor 2



(b) Floor 1



(c) Floor 0



(d) Floor -1

Figure 7: Histograms of different floors

6 Optional Solutions

6.1 Evacuation Plan Figures

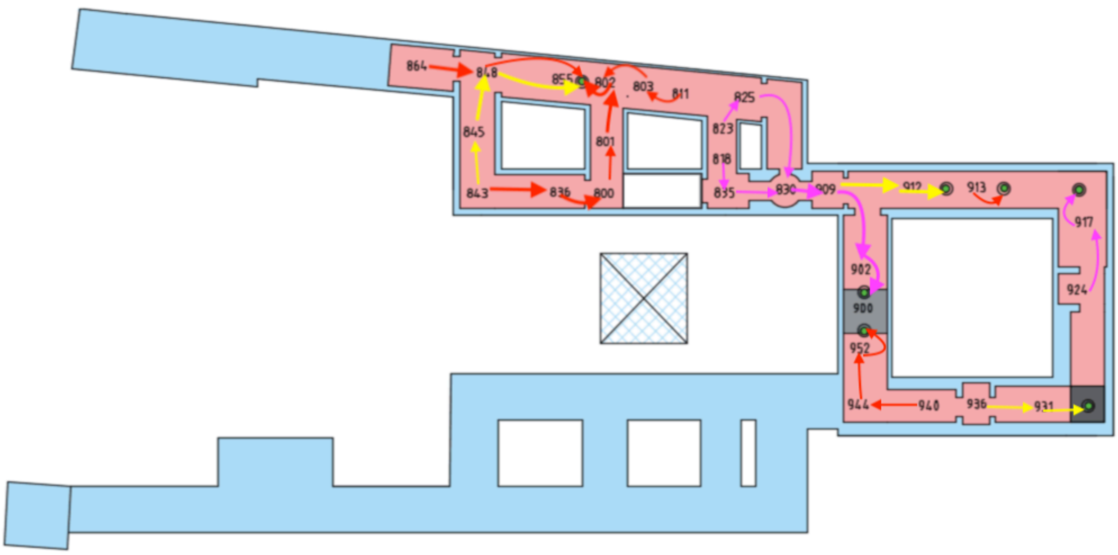


Figure 8: Evacuation Plan for Floor 2

Using our models proposed above, we can get the evacuation routes of each floor.We give out all the evacuation plan figures from the second floor to the lower ground floor. But we only analysis the second floor as an example.

In figure 8, if there are two or more optional routes to go in one block, it will be marked in different colors. The size of the arrows indicates the population density among that block. However, we cannot see the real-time change of the population from this figure.Before we finally decide the evacuation plan, we had better consider the practical factors instead of giving out a plan only based on our models.

When emergency occurs, first of all, the disabled, pregnant women, the elderly, children and other people with mobility difficulties are evacuated through the special channels near block 913, 917, 931, 936, 940 and 944. While VIP and group tourists can evacuate through the dedicated channels near block 900 and 913. Other tourists are not allowed to pass through them. Emergency personnels will enter the building through the above channels. if necessary, they may also pass through the channel near block 800. Other evacuation routes for tourists are shown in figure 8. For example, the tourists in block 818 and 835 evacuate to block 830 along the same route, and the tourists in block 823 and 825 were evacuated to block 830 along another route. Thus they are separated into two routes, evacuating to the stairs from block 902 or 912 respectively.

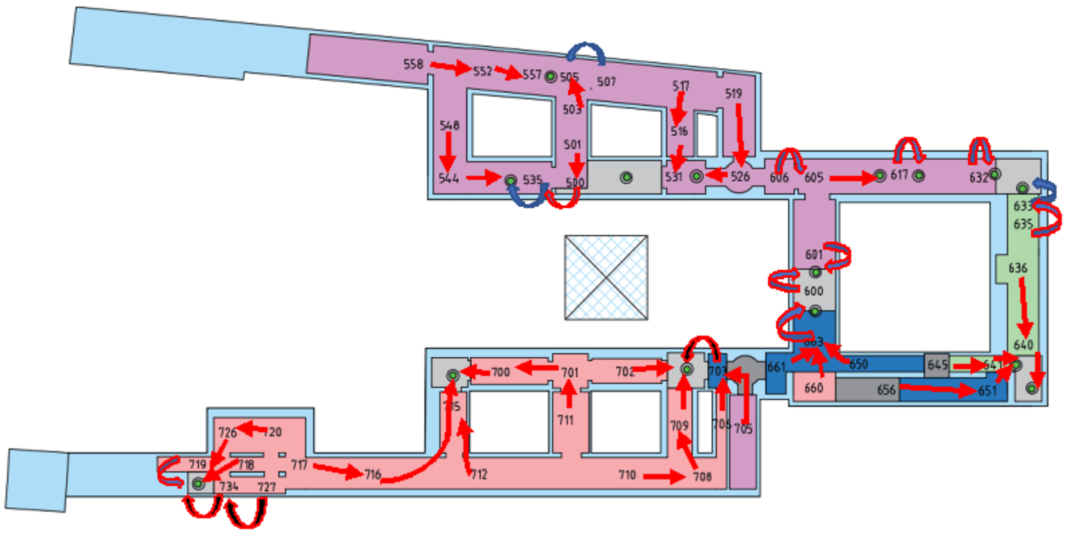


Figure 9: Evacuation Plan for Floor 1

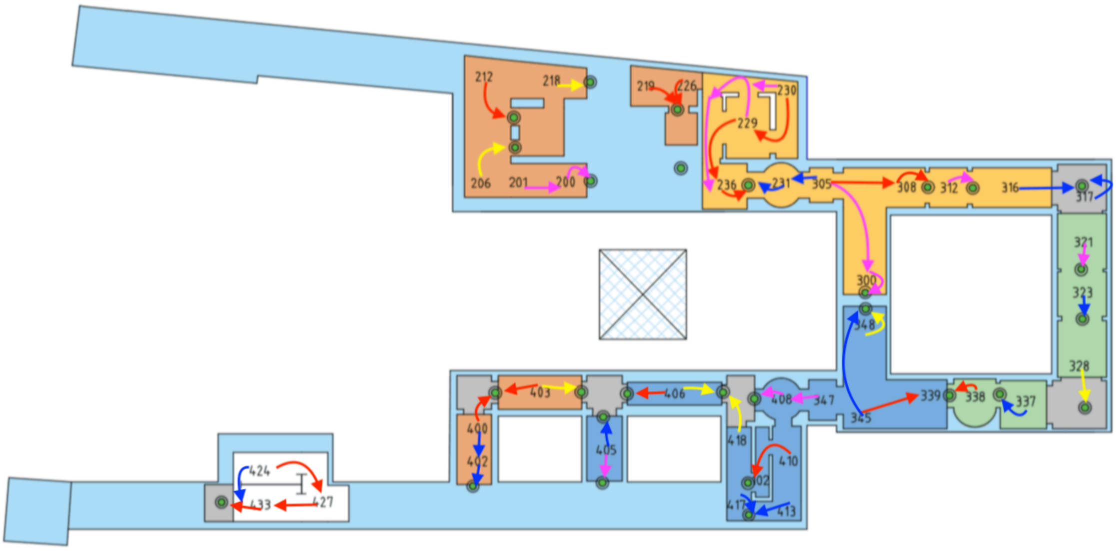


Figure 10: Evacuation Plan for Floor 0

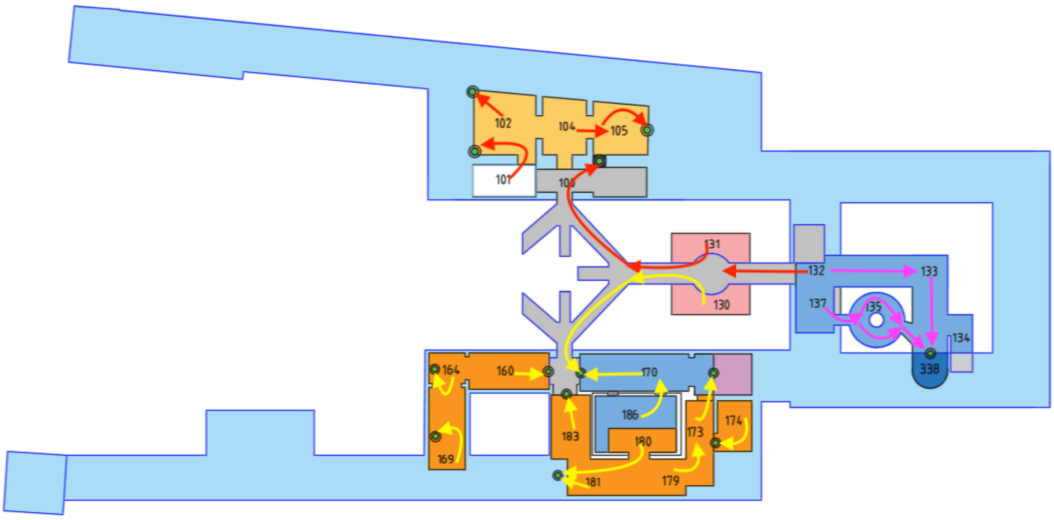


Figure 11: Evacuation Plan for Floor -1

6.2 More Adaptable Methods

In this section, we are devoted to proposing some more adaptable methods to help the museum leaders to face various situations. First, we also use the evacuation plan we suggested for the second floor as an example.

When an evacuation route is not available, it can be substituted by adjacent exits. For example, when the path via block 909 is removed, the tourists in blocks 902, 912 and 913 still evacuate along the original route, while the tourists in block 818, 823, 825, 830 and 835 may evacuate to the exit next to block 800.

Second, we divided the severity of accidents into four levels, as shown in the following table 4.

If the severity level is less than or equal to 2, for fear of introducing extra overhead of the potential safety risk, we do not suggest the leader allow to open the old secret entrances built by the monarchy. If the severity level is higher than 2, we should consider to open all the available entrances which are ensured safe. More staff are used to instruct the visitors to evacuate quickly and safely according to the severity level.

The Severity of Accidents				
Type of Accident	crowded stampede	power outages	earthquake (<=4.5)	earthquake (>4.5)
Level	1	1	2	3
Type of Accident	explosion	fire	poisonous gases	terrorist attack
Level	3	3	3	4

Table 4: The Severity of Accidents

7 Discussion & Conclusion

7.1 Policy Recommendations

- Considering that disabled visitors have limited movement speed, it costs them more time to evacuate. Therefore, it is necessary to arrange relatively few tourists to escape from the disable-privileged channels, so as to avoid conflicts between the other tourists and them.
- To ensure the VIP visitors leave the Louvre safely and quickly, arrange some visitors to detour to the exits next to the VIP exits.
- Set the staff elevator near the popular exhibition hall, so as to provide the staff with fast access to the accident spot to evacuate the crowd as soon as possible.
- Introducing some real-time monitoring equipment to keep records of the real-time population which can be used to estimate the evacuation time by the museum leaders.
- Use a color-changeable signal board to instruct the visitors just like the traffic light. The color red represents for congestion, while the color green represents the path is unobstructed.
- Add a language translator to the electronic interpreter, so that the visitors are able to follow the instruction of the interpreter according to the GPS location. And the interpreter solves the problem of the difference between languages.
- Advertise visitors using applications like Affluences and Louvre to get information about the Louvre before their visit. These applications also give out the waiting time which may help the visitors to master their time reasonably.

7.2 For Other Large Buildings

Our model has a high portability if we have known the real-time population inside the building, or estimating this value through knowing annual distribution of population and daily distribution of population over recent years. Meanwhile, we ought to master the distribution of population density and its movement characteristic to make the model more precise. With the information we listed above, we can figure out the optimal evacuation routes and the theoretical evacuation time using our model. We only need to modify the distribution function of population density and and the geometric features of this building according to different situations to make a rough plan.

It is also significant to optimize the original plan by finding the potential bottlenecks with the method we offered in our model. All the entrances, exits, stairs and channels are all possible potential bottlenecks. The structures listed above have various evacuation abilities. it is helpful if we could get the graphic design of this building. Since knowing the real size of structures like exits or stairs, a more optimized plan will be made.

Keep in mind that:

- We are supposed to avoid cross routes or the routes via many potential bottlenecks.
- Isolation and guidance equipment can be set according to the designed evacuation route. Additionally, staff are arranged in the bottleneck area to enable the personnel to evacuate in an orderly manner according to the design evacuation route.
- Special channels should be arranged for emergency personnel to avoid cross and collision with evacuated personnel.
- When an evacuation channel is not available, the personnel can be temporarily placed in a concealed safe area before evacuation. The personnel can also be dispersed to the adjacent exit to evacuate. If the dedicated channel is not available, the access to the emergency personnel should be isolated at other exits.

8 Strengths & Weaknesses

8.1 Strengths

This model has the strengths of wide adaptability, simplicity and practicality. The evacuation route in different buildings and different states can be determined by modifying the number of people in real time and the geometric features of the buildings. The model also provides the shortest-time-evacuation routes through flexible emergency personnel assignment and real-time evacuation guidance facilities. The innovation is to use game theory to analyze the individual behavior characteristics in the evacuation process and give the corresponding strategies. Meanwhile, it will discover the potential bottlenecks and optimize the original designed routes to minimize the intersection and collision of the crowd based on the average utilization rate and the relative evacuation ability of each block.

8.2 Weaknesses

- One of the weaknesses of this model is that the reliability of the calculation results. It depends on the number of people in real time and the accuracy of the architectural geometry. It is difficult to apply to buildings with complex structures.
- For buildings with a large number of access modes, entrances and exits, the complexity of calculation will increase to great extent. As a result, the real-time design of evacuation routes will lose its worth.
- The problems of counting the number of people in real-time accurately, applying evacuation routes guidance equipment, and arranging emergency personnels in time remains to be solved as well.

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Appendices

Appendix A First appendix

some more text **Input Python source:**

```
import networkx as nx
import matplotlib.pyplot as plt
import numpy as np
import math

#reserve the escape path of each block
node_list = []

#popularity of each block
p= np.array([1,1,1,1,1,1,1,2,1,1,2,1,1,1,
             1,1,1,1,2,1,2,2,1,1,2,1,1,0,0,
             0,0,0,0,0])
#area of each block
S=495
s= np.array([S,S,S,S,S,S,S,S,S,S,S,S,S,S,S,S,S,S,S,S
            ,S,S,S,S,S,S,S,0,0,0,0,0,0,0])

n0 = 27413        # total number of tourists
a=50   #customized parameter in model
b=20   #customized parameter in model
e=2400 #common escaping efficiency (individuals/minute)

s_sum = sum(s)
PS = p*s
p_sum = sum(PS) #sum up the popularity
p_avg = p_sum/s_sum
print("p_sum=",p_sum)

#popularity
popu1=1
popu2=2

#reserve escaping time of each block
TIME_Array = []

def quadratic_equation(a, b, c):
    t = math.sqrt(pow(b, 2) - 4 * a * c)
    if(pow(b, 2) - 4 * a * c) > 0:
        return (-b + t) / (2 * a), (-b - t) / (2 * a)
    elif (pow(b, 2) - 4 * a * c) == 0:
        return (-b + t) / (2 * a)
    else:
```

```
        return None

def Dijkstra(G,start,end):
    RG = G.reverse(); dist = {}; previous = {}
    for v in RG.nodes():
        dist[v] = float('inf')
        previous[v] = 'none'
    dist[end] = 0
    u = end
    while u!=start:
        u = min(dist, key=dist.get)
        distu = dist[u]
        del dist[u]
        for u,v in RG.edges(u):
            if v in dist:
                alt = distu + RG[u][v]['weight']
                if alt < dist[v]:
                    dist[v] = alt
                    previous[v] = u
    path=(start,)
    last= start
    while last != end:
        nxt = previous[last]
        path += (nxt,)
        last = nxt
    return path

L= np.array([30,22,22,22,30,30,30,8,0,30,22,22,38,30,
             22,30,30,38,0,0,0,22,22,30,0,8,0,0]) # route length
PC=np.array([1,1,1,1,1,1,2,1,1,1,2,1,1,1,1,1,1,1,1,1,1,1,
             2,2,1,1,1,2,1]) #reflect popularity
LC=(L*PC)/p_avg               #"optical path"
print(LC)

G=nx.DiGraph()

G.add_edge(0,1,weight=LC[0])
G.add_edge(1,4,weight=LC[1])
G.add_edge(2,1,weight=LC[2])
G.add_edge(3,5,weight=LC[3])
G.add_edge(5,8,weight=LC[4])
G.add_edge(8,7,weight=LC[5])
G.add_edge(7,6,weight=LC[6])
G.add_edge(6,27,weight=LC[7])
G.add_edge(4,27,weight=LC[8])
G.add_edge(9,6,weight=LC[9])
G.add_edge(10,9,weight=LC[10])
G.add_edge(12,11,weight=LC[11])
G.add_edge(11,15,weight=LC[12])
G.add_edge(15,16,weight=LC[13])
G.add_edge(13,14,weight=LC[14])
G.add_edge(14,15,weight=LC[15])
G.add_edge(16,20,weight=LC[16])
G.add_edge(16,17,weight=LC[17])
G.add_edge(20,30,weight=LC[18])
```

```
G.add_edge(17,28,weight=LC[19])
G.add_edge(18,29,weight=LC[20])
G.add_edge(19,18,weight=LC[21])
G.add_edge(21,19,weight=LC[22])
G.add_edge(23,26,weight=LC[23])
G.add_edge(26,33,weight=LC[24])
G.add_edge(25,33,weight=LC[25])
G.add_edge(24,32,weight=LC[26])
G.add_edge(22,31,weight=LC[27])


Dij_path_11_28 = Dijkstra(G,11,28)
Dij_path_11_30 = Dijkstra(G,11,30)

Dij_path_12_28 = Dijkstra(G,12,28)
Dij_path_12_30 = Dijkstra(G,12,30)

Dij_path_13_28 = Dijkstra(G,13,28)
Dij_path_13_30 = Dijkstra(G,13,30)

Dij_path_14_28 = Dijkstra(G,14,28)
Dij_path_14_30 = Dijkstra(G,14,30)

Dij_path_15_28 = Dijkstra(G,15,28)
Dij_path_15_30 = Dijkstra(G,15,30)

Dij_path_16_28 = Dijkstra(G,16,28)
Dij_path_16_30 = Dijkstra(G,16,30)

print(Dij_path_13_28)
print(Dij_path_13_30)


#Node_1
A0 = (-1)*a*e*popu1*0.5/p_sum
B0 = b+a*n0*popu1/p_sum
C0 = LC[1]*(-1)
t1_4 = abs(max(quadratic_equation(A0, B0, C0)))

A1=(-1)*a*e*popu1*0.5/p_sum
B1=b+a*n0*popu1/p_sum-a*e*popu1*t1_4*0.5/p_sum
C1=LC[8]*(-1)
t4_27 = abs(max(quadratic_equation(A1, B1, C1)))

t1_27=t1_4+t4_27
node_list.append(t1_27)
print("t1_27=",t1_27)

#Node_2

A0 = (-1)*a*e*popu1*0.5/p_sum
B0 = b+a*n0*popu1/p_sum
C0 = LC[2]*(-1)
t2_1 = max(quadratic_equation(A0, B0, C0))
```

```
A1=(-1)*a*e*popu1*0.5/p_sum
B1=b+a*n0*popu1/p_sum-a*e*popu1*t2_1/p_sum
C1=LC[1]*(-1)
t1_4 = abs(max(quadratic_equation(A1, B1, C1)))

A2=(-1)*a*e*popu1*0.5/p_sum
B2=b+a*n0*popu1/p_sum-a*e*popu1*(t2_1+t1_4)/p_sum
C2=LC[8]*(-1)
t4_27 = abs(max(quadratic_equation(A2, B2, C2)))

t2_27 = t2_1+t1_4+t4_27
node_list.append(t2_27)

print("t2_27=",t2_27)


#Node_3

A0 = (-1)*a*e*popu1*0.5/p_sum
B0 = b+a*n0*popu1/p_sum
C0 = LC[3]*(-1)
t3_5 = abs(max(quadratic_equation(A0, B0, C0)))

A1=(-1)*a*e*popu1*0.5/p_sum
B1=b+a*(n0-e*t3_5)*popu1/p_sum
C1=LC[4]*(-1)
t5_8 = abs(max(quadratic_equation(A1, B1, C1)))

A2=(-1)*a*e*popu1*0.5/p_sum
B2=b+a*(n0-e*(t3_5+t5_8))*popu1/p_sum
C2=LC[5]*(-1)
t8_7 = abs(max(quadratic_equation(A2, B2, C2)))

A3=(-1)*a*e*popu1*0.5/p_sum
B3=b+a*(n0-e*(t3_5+t5_8+t8_7))*popu1/p_sum
C3=LC[6]*(-1)
t7_6 = abs(max(quadratic_equation(A3, B3, C3)))

A4=(-1)*a*e*popu1*0.5/p_sum
B4=b+a*(n0-e*(t3_5+t5_8+t8_7+t7_6))*popu1/p_sum
C4=LC[7]*(-1)
t6_27 = abs(max(quadratic_equation(A4, B4, C4)))

t3_27 = t3_5+t5_8+t8_7+t7_6+t6_27
node_list.append(t3_27)
print("t3_27=",t3_27)


#Node#Node_16_28

A3=(-1)*a*e*popu1*0.5/p_sum
B3=b+a*n0*popu1/p_sum
C3=LC[17]*(-1)
t16_17 = max(quadratic_equation(A3, B3, C3))
```

```
A4=(-1)*a*e*popu1*0.5/p_sum
B4=b+a*n0*popu1/p_sum-a*e*popu1*(t16_17)*0.5/p_sum
C4=LC[19]*(-1)
t17_28 = abs(max(quadratic_equation(A4, B4, C4)))

t16_28 = t16_17+t17_28
node_list.append(t16_28)
print("t16_28=",t16_28)

#Node_16_30

A3=(-1)*a*e*popu1*0.5/p_sum
B3=b+a*n0*popu1/p_sum
C3=LC[16]*(-1)
t16_20 = max(quadratic_equation(A3, B3, C3))

A4=(-1)*a*e*popu1*0.5/p_sum
B4=b+a*n0*popu1/p_sum-a*e*popu1*(t16_20)*0.5/p_sum
C4=LC[18]*(-1)
t20_30 = abs(max(quadratic_equation(A4, B4, C4)))

t16_30 = t16_20+t20_30
node_list.append(t16_30)
print("t16_30=",t16_30)


plt.bar(range(len(node_list)), node_list, color = 'lightsteelblue')

plt.xticks(range(len(node_list)), node_list)
plt.xlabel('node')
plt.ylabel("escaping time")
plt.legend()
plt.show()
```