# Computing A Near-Maximum Independent Set In Dynamic Graphs

Weiguo Zheng[1, 2], Chengzhi Piao[1], Hong Cheng[1], Jeffrey Xu Yu[1]

[1]The Chinese University of Hong Kong, HK SAR, China

[2]Fudan University, China

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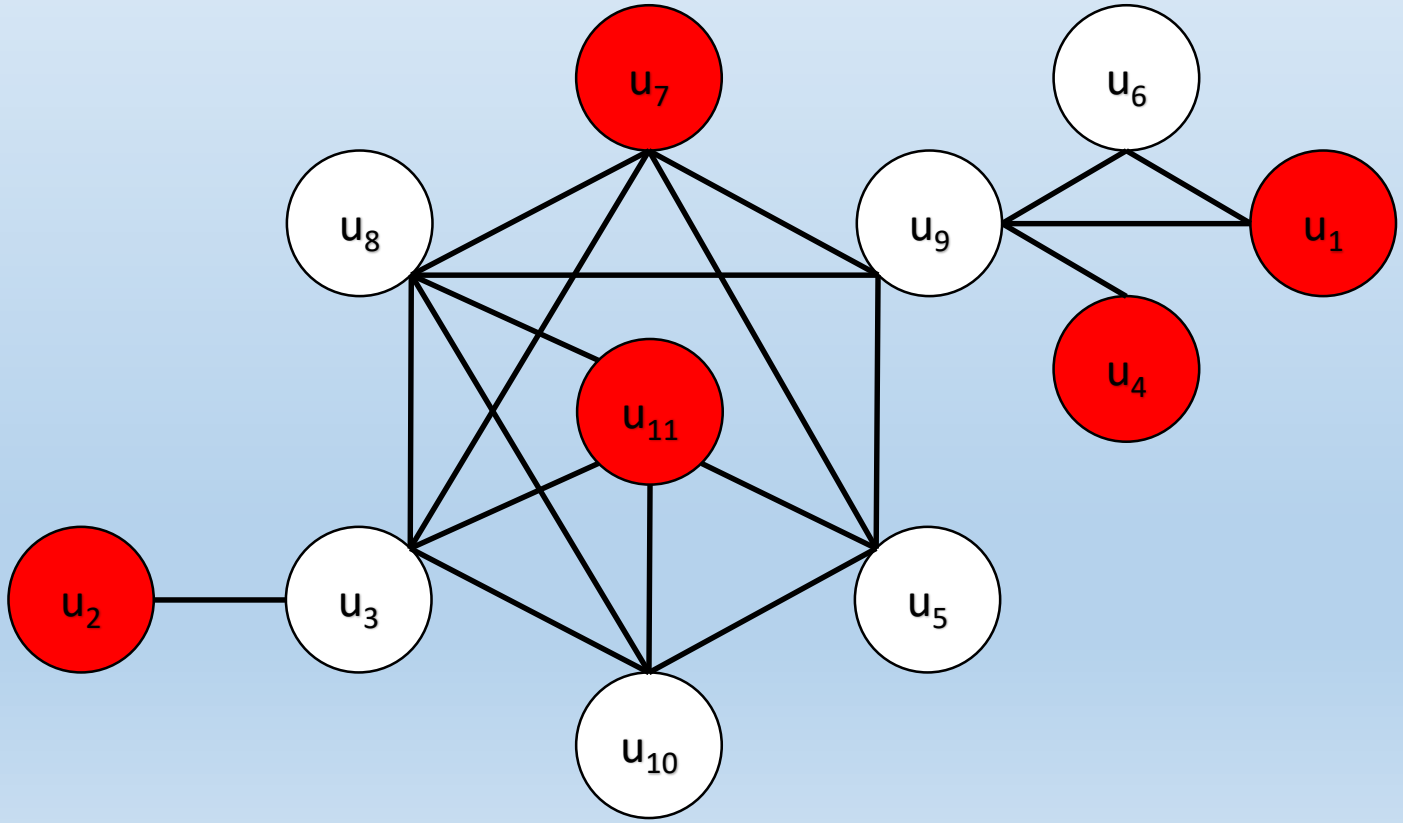
# Background

# Independent Set

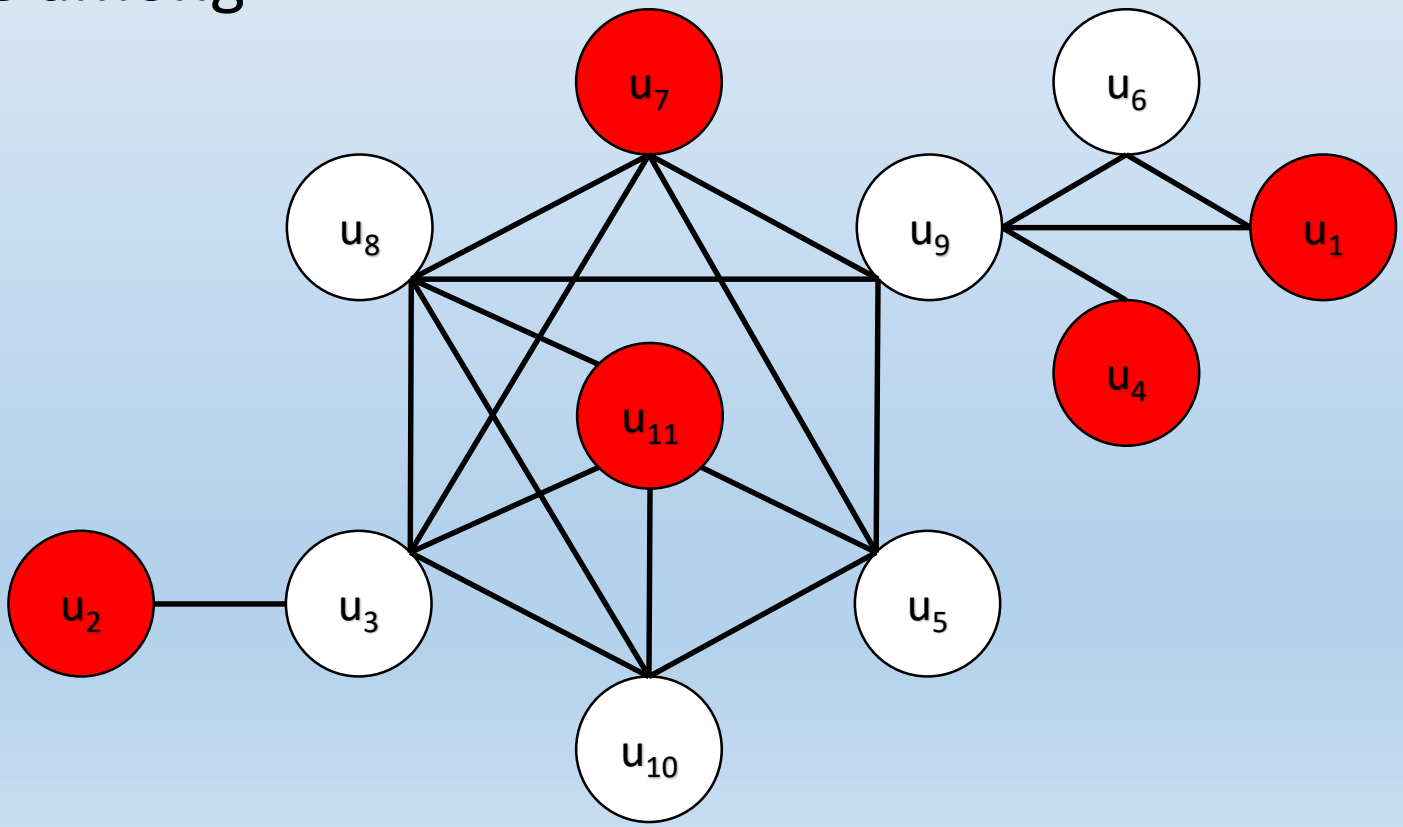
Given a set of vertices in graph G, we call it an independent set of G if and only if there are **no neighborhoods** among them.



# Independent Set

Maximum independent set(MIS)

- A MIS has the largest size among all independent sets.
- Maybe not unique.



# Independent Set

Finding a MIS is a fundamental NP-hard problem and has many applications.

- association rule mining
- automated map labeling
- road network routing

# Independent Set

However, it is too time-consuming to mine a MIS in large networks. Efficiently mining a near-maximum one is also quite reasonable.

Near-Maximum Independent Set(Near-MIS)

- |Near-MIS| is close to |MIS|
- the larger the better

# Dynamic Graph

In real-world, the graph structure is changing all the time.

For example, in a social network:

- register/cancel an account -> add/delete a node
- like/dislike another user -> add/delete an edge

Online dealing with the frequent changing operations needs much efficiency.

# Problem Definition

Target: maintain a Near-MIS in dynamic graphs

Input

- An undirected graph $G_0$ and the initial Near-$MIS_0$
- A sequence of updates on the graph: { $update_i$ }

Output

- online maintain the Near-MIS after each update: { Near-$MIS_i$ }

# Evaluation

$(G_i, \text{Near-MIS}_i) \rightarrow \text{update}_i \rightarrow (G_{i+1}, \text{Near-MIS}_{i+1})$

Effectiveness

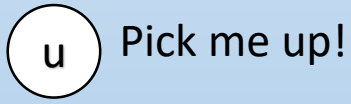
- $\text{Gap}_i = |\text{MIS}_i| - |\text{Near-MIS}_i|$
  - $\text{Accuracy}_i = |\text{Near-MIS}_i| / |\text{MIS}_i| <= 1$
  - $\text{Gap}_0$ depends on the input($G_0$ and $\text{Near-MIS}_0$)
- Increasing Rate of Gap
  - $IR(i) = (\text{Gap}_i - \text{Gap}_0) / i$
  - the lower the better

# Existing Algorithms

# Reduction Rule

Degree-Zero reduction: degree(u) = 0, no neighborhood

- The node u must be in the MIS.
- Directly put u into the solution and delete it from the graph.

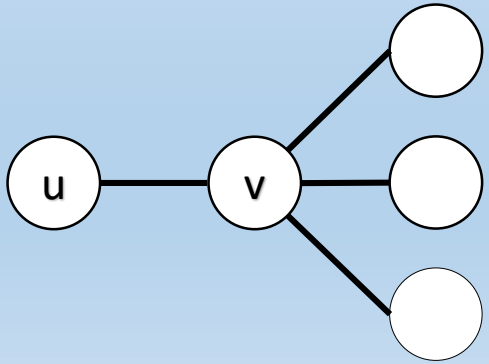


# Reduction Rule

Degree-One reduction: degree(u) = 1, (u, v)

- There must be a MIS containing u.
- Of course there may exist a MIS containing v but not u. However, u is obviously a better choice as it does not affect other nodes.
- So we can add u into the solution, and then delete u and v from the graph.

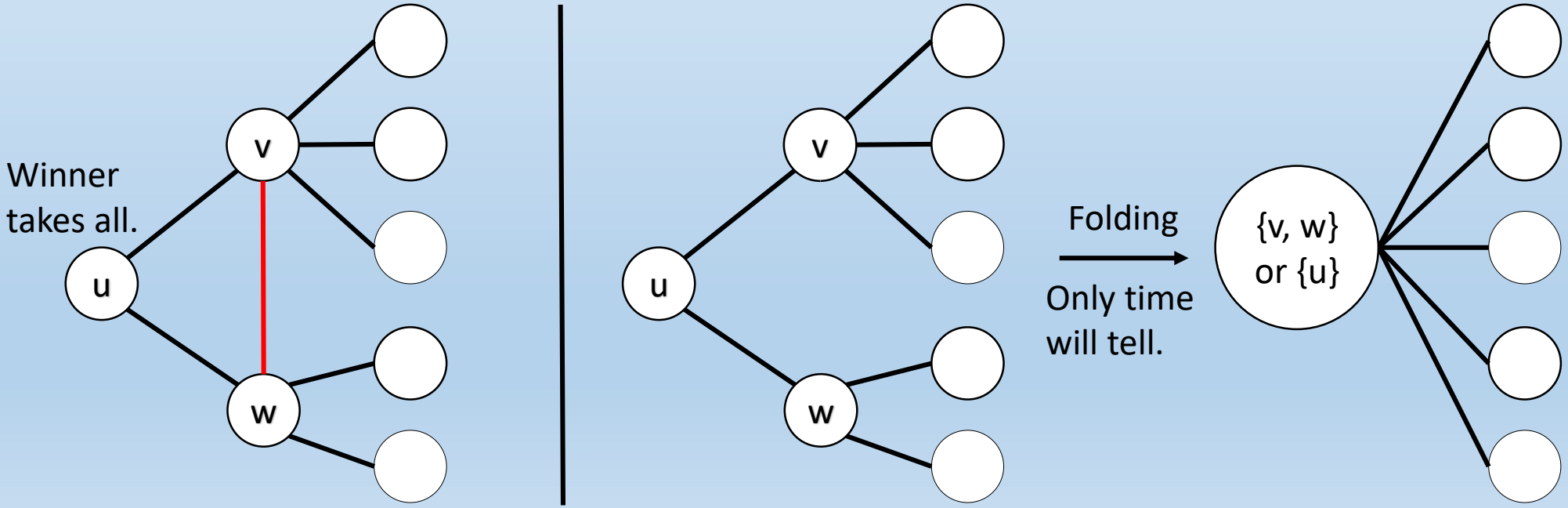
u or v?
That is not a question.



# Reduction Rule

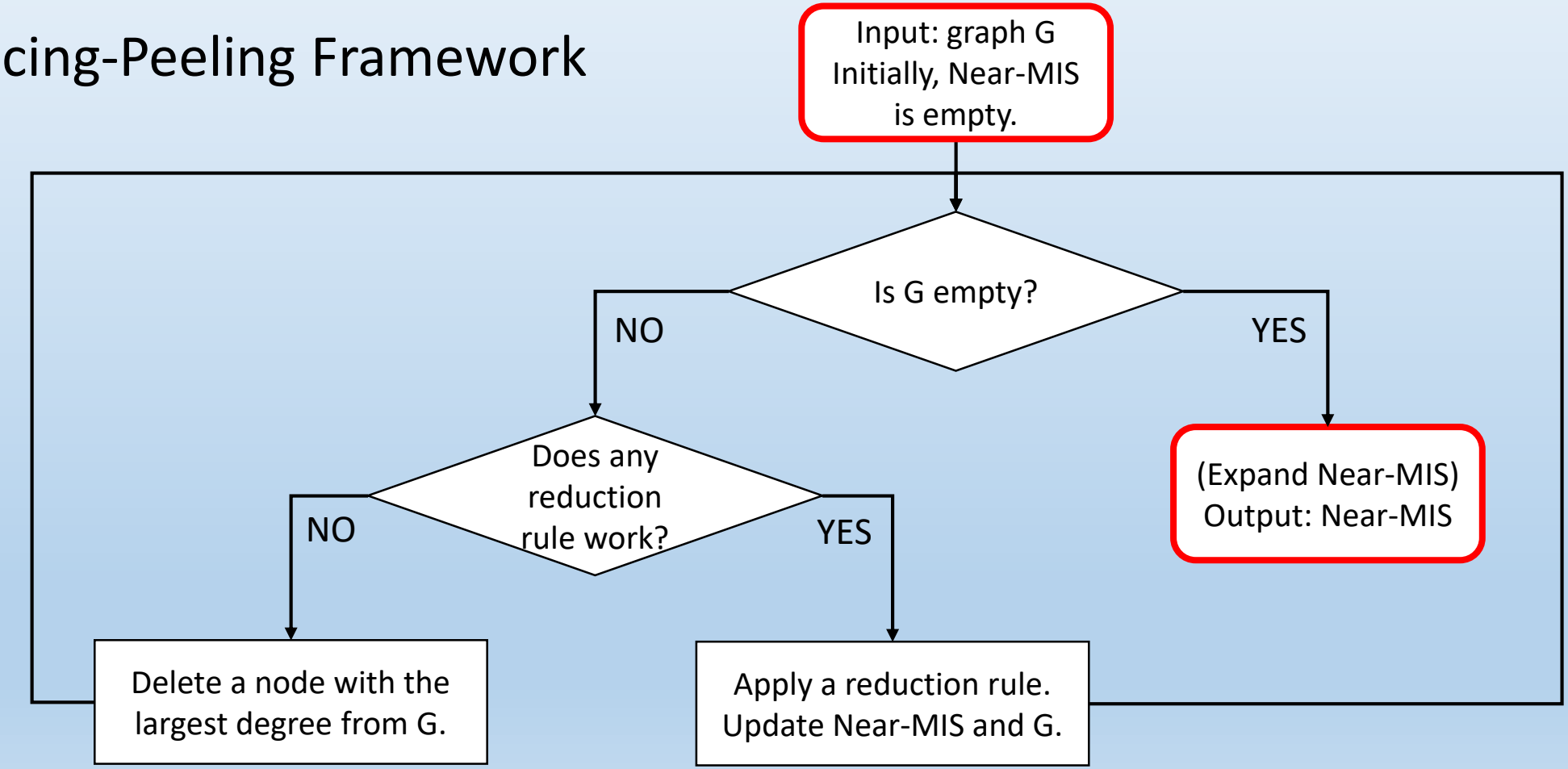
Degree two reduction: degree(u) = 2, (u, v) and (u, w)

- Isolation: (v, w) exists -> choose u and delete {u, v, w}
- Folding: (v, w) does not exist -> merge {u, v, w}



# Greedy Algorithm

## Reducing-Peeling Framework



# Greedy Algorithm

## Reducing-Peeling Framework

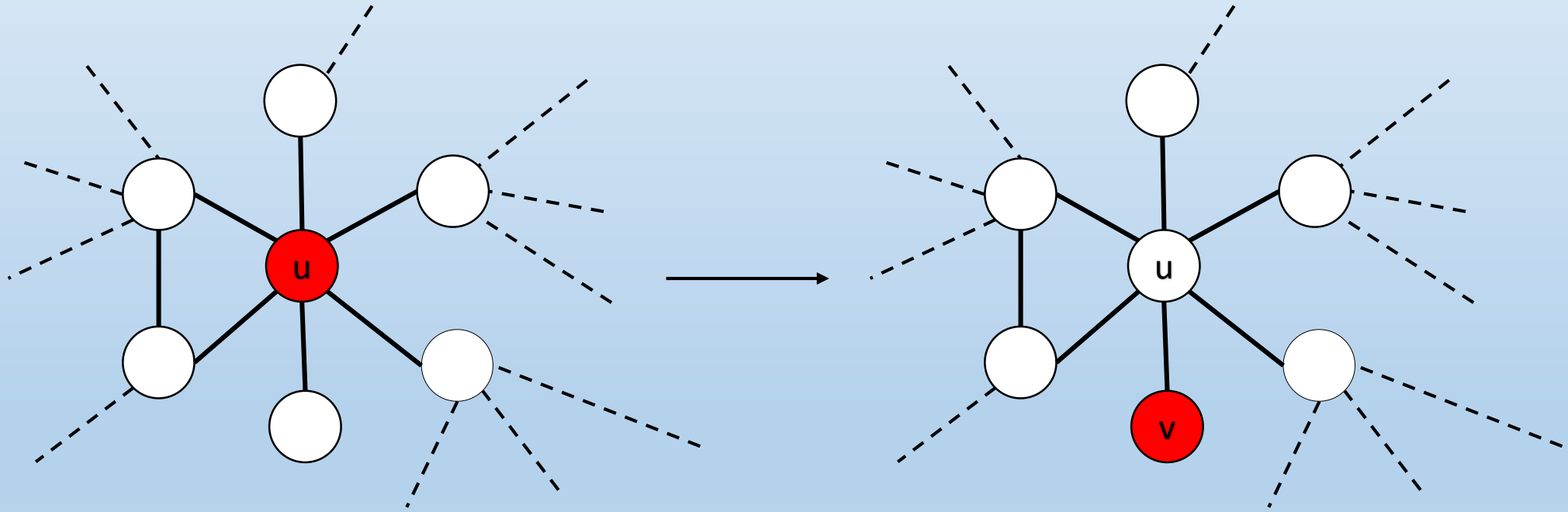
Delete a node with the largest degree from G.

- This step does an inexact reduction.
- Greedy Idea: The largest degree node is unlikely to be in a MIS.
- Gap = |MIS| - |Near-MIS| <= times of inexact reductions

# Dependency Graph based Framework

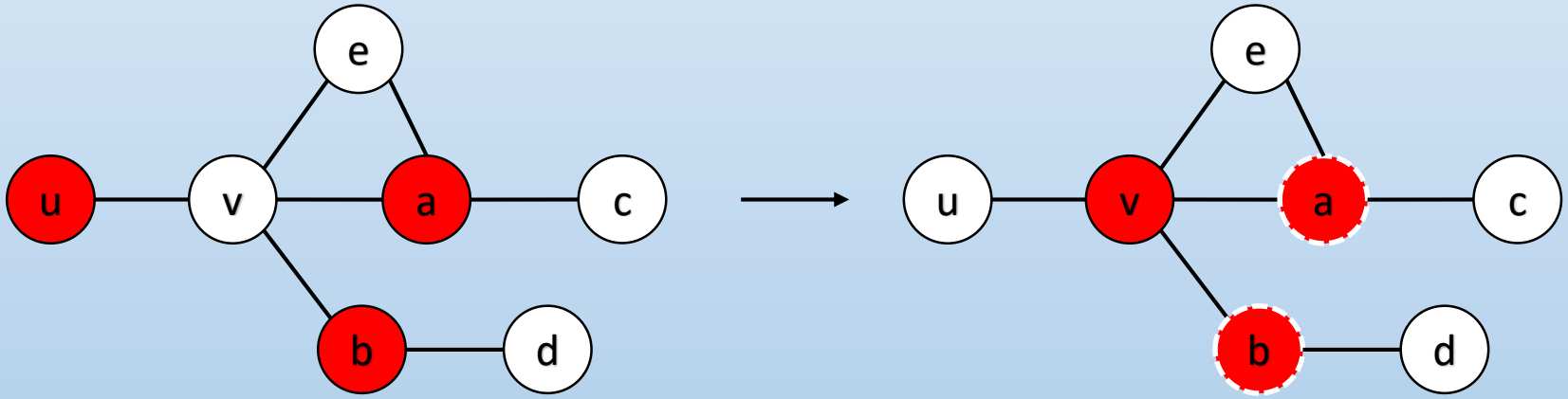
# Move Out u from the MIS

To maintain a MIS, we must choose a neighbor to replace u.



# Move Out u from the MIS

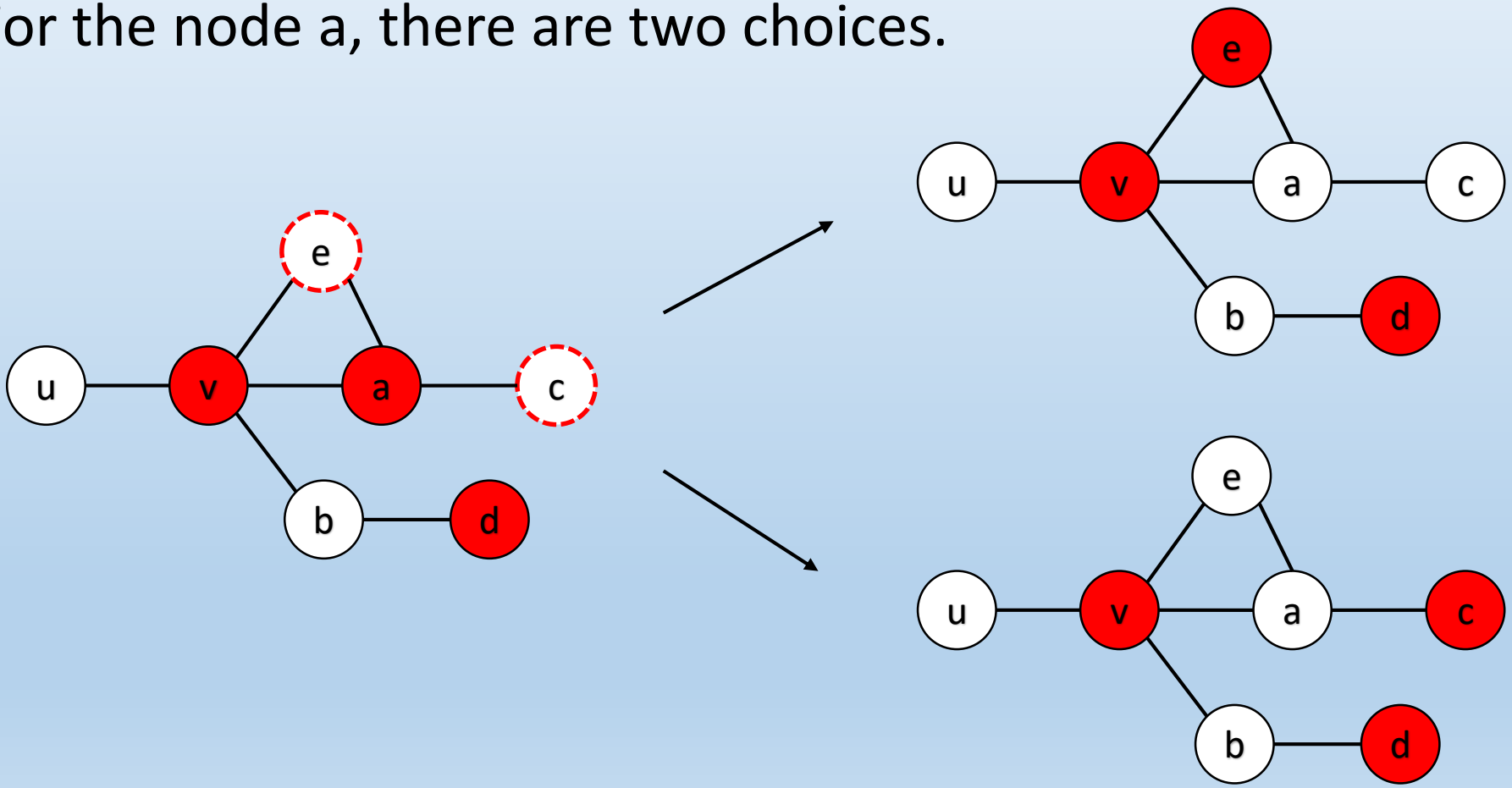
However, we usually need to do this recursively.




Recursively move both a and b out.

# Move Out u from the MIS

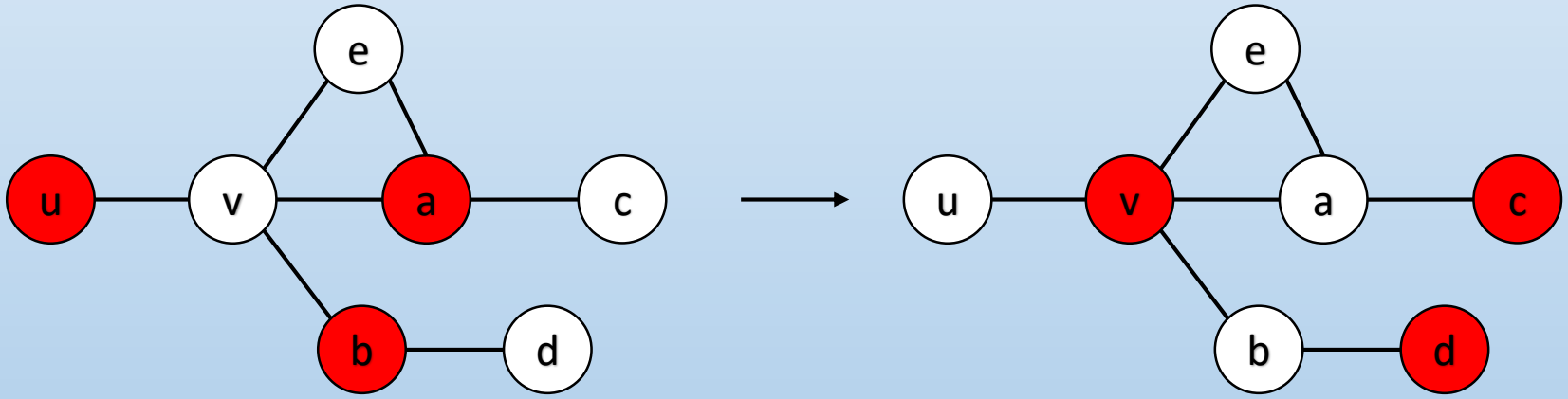
For the node a, there are two choices.



# Valid Swap

Move out: {u, a, b}

Move in: {v, c, d}

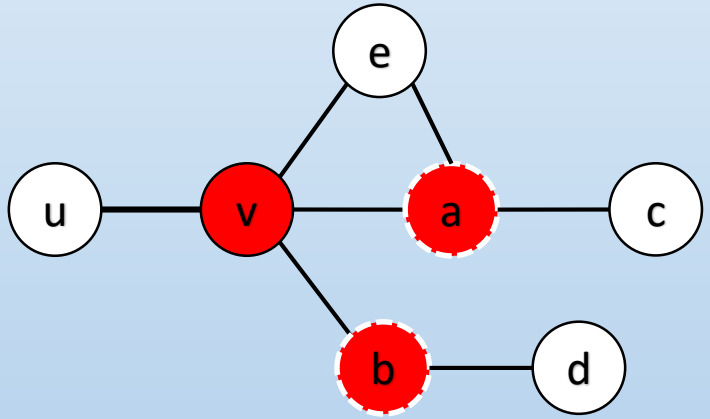


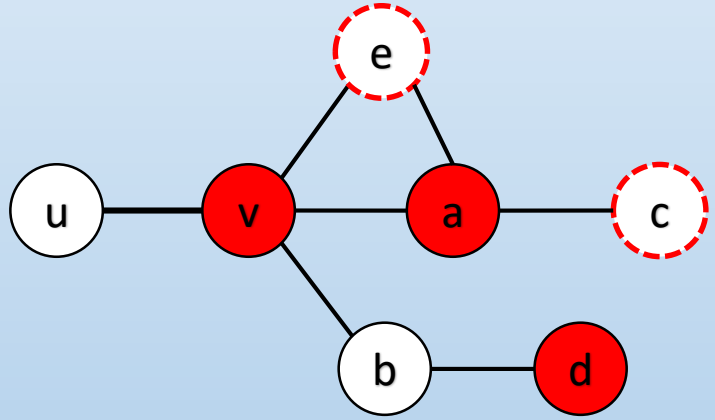

Happy ending!

# Brute-Force Search

Move in a white node.

-> Move out all its red neighbors.

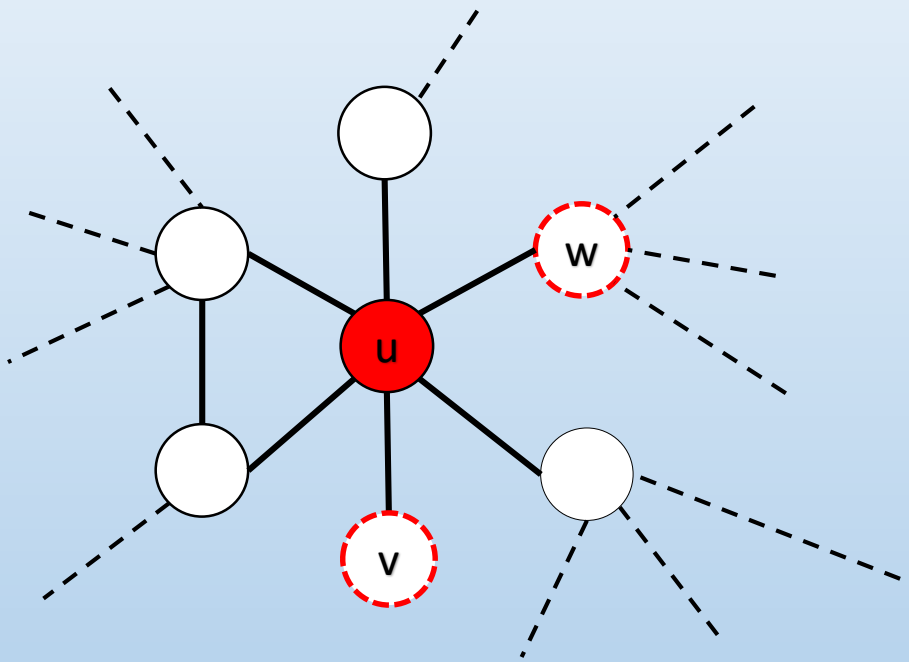


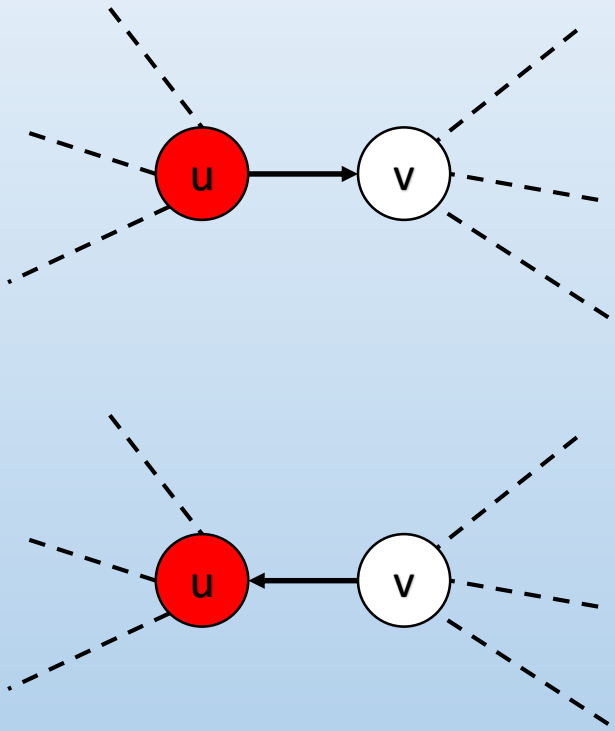

Move out a red node.

-> Move in one of its white neighbors.

# Two Ways of Pruning

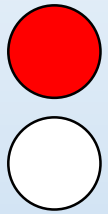



To move out a red node,
search at most two given neighbors.




Only search in one direction.

# Dependency Graph



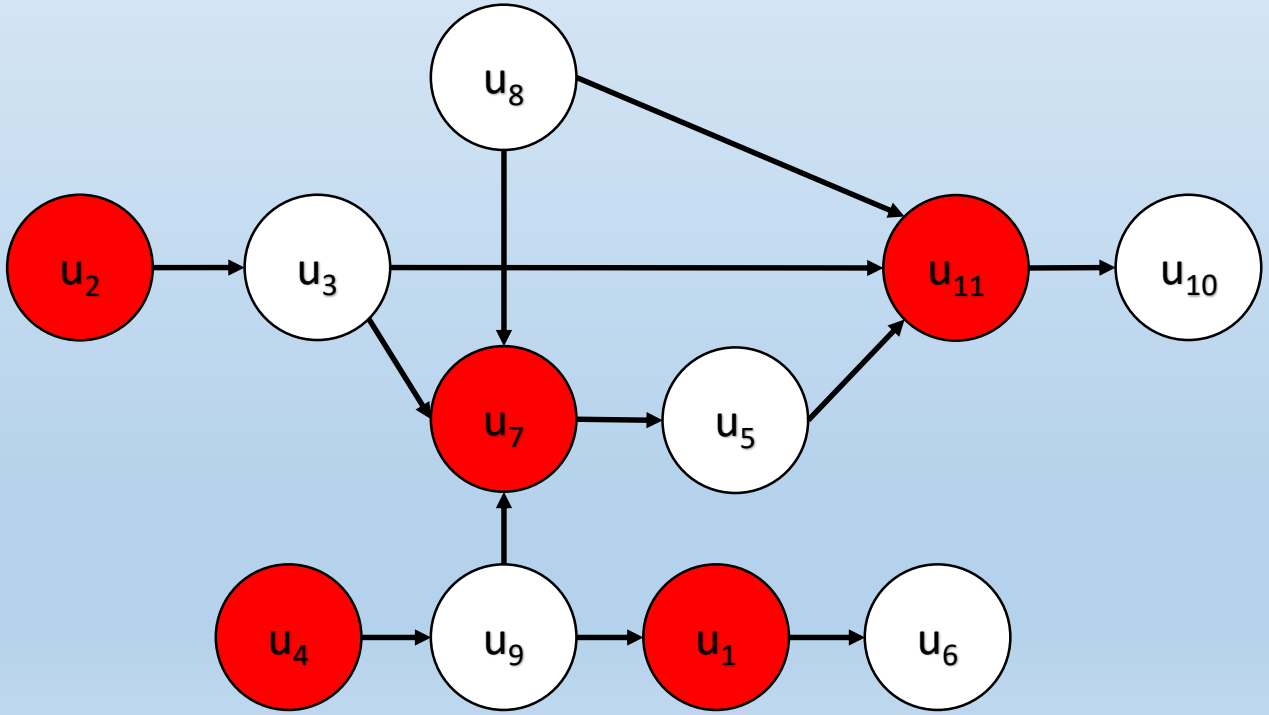
We call the nodes in the independent set **reducing nodes**.

Others are called **dependent nodes**.

- Remain all edges between reducing and dependent nodes.
- Each edge gets directed.
- Each reducing node has at most two out-neighbors.
- Each dependent node has at most one in-neighbor.

# Dependency Graph

- The direction implies that whether a node can be moved in/out depends on its out-neighbor(s).



# Dependency Graph

Efficiency

- Searching in such a dependency graph is especially efficient.
- In our paper, we prove that the average time complexity is merely O(d).

Effectiveness

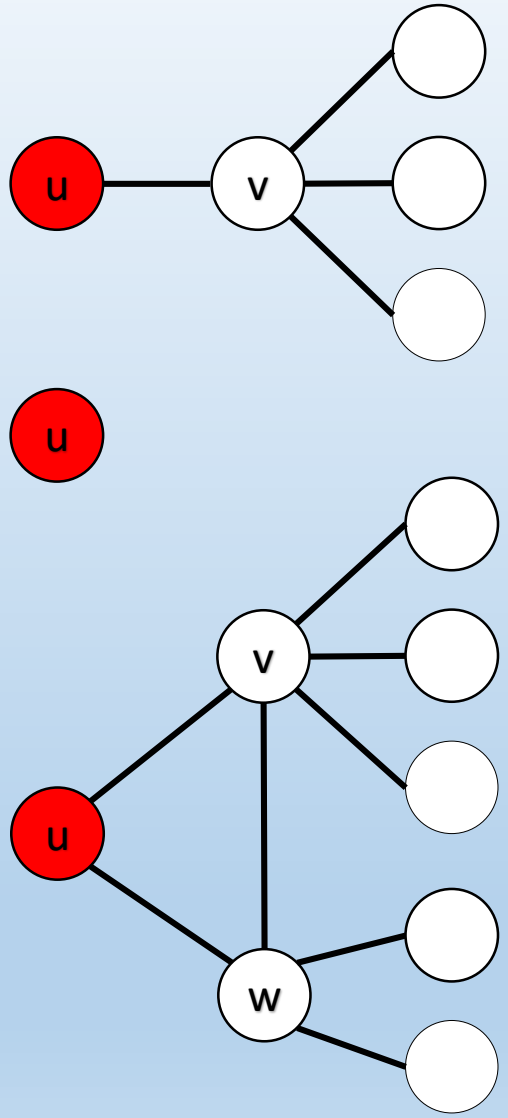
- However, dramatically reducing searching space may damage the accuracy.
- It is vital to construct a good dependency graph.

# DG: Construction

Let us go back to the reducing-peeling framework.

These simple reduction rules are only applied on a node whose degree is 0, 1, or 2.

The degree on original graph may be much larger because other neighbors have already been deleted.

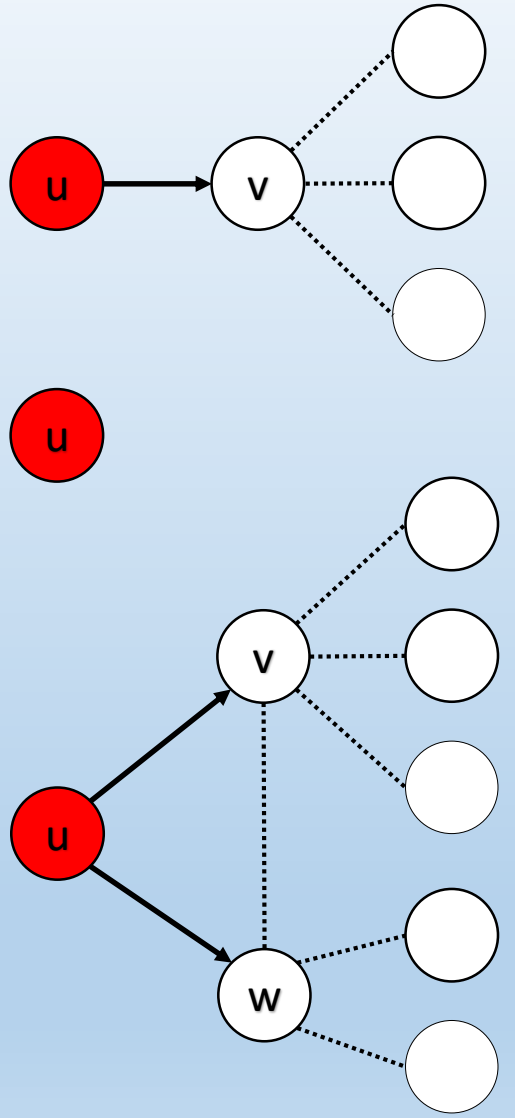


# DG: Construction

The reducing-peeling framework does not merely output a Near-MIS, but also implies a topological order of the graph.

Based on this point, we can construct a good dependency graph.

- For each reducing node, its out-neighbors are decided when a reduction rule works.
- Then, the others become its in-neighbors.



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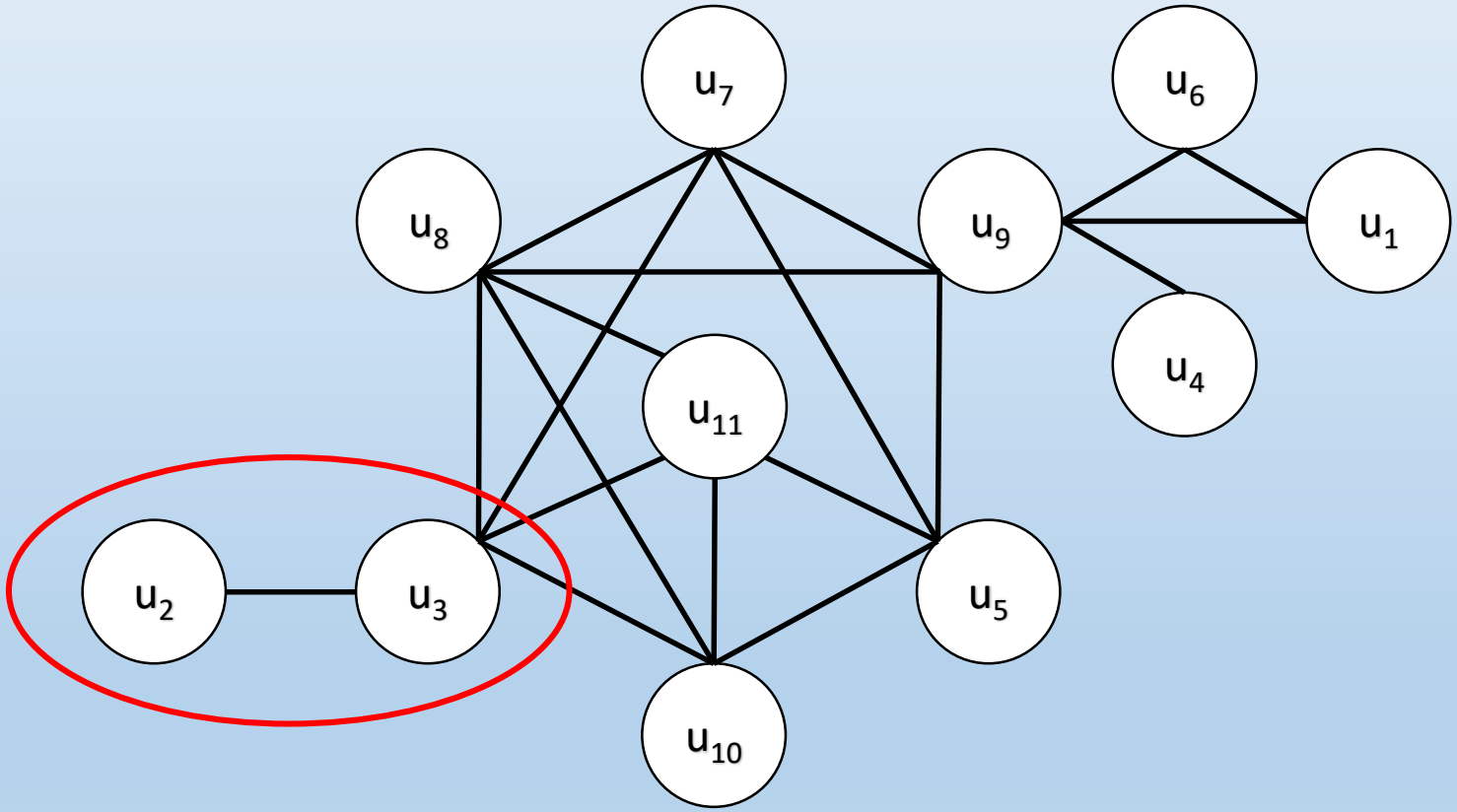

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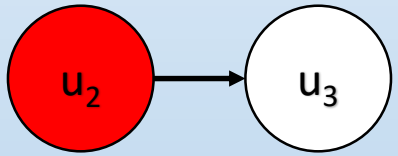

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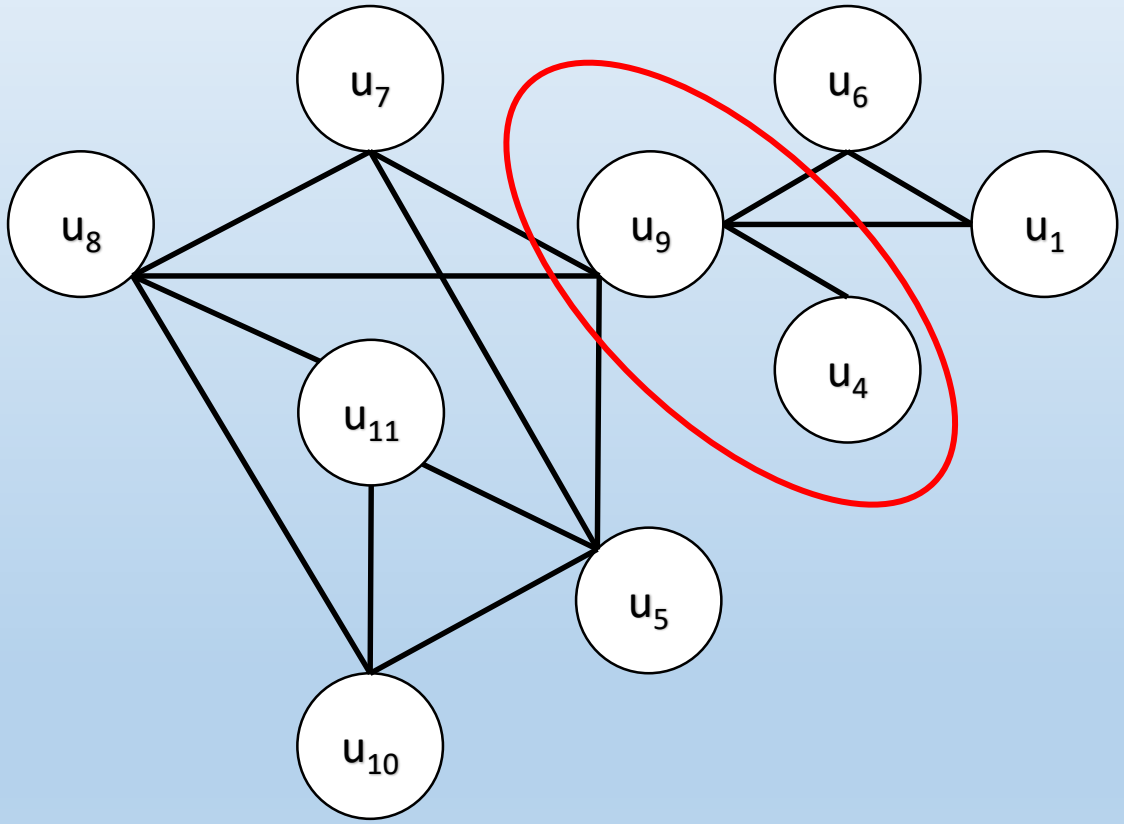

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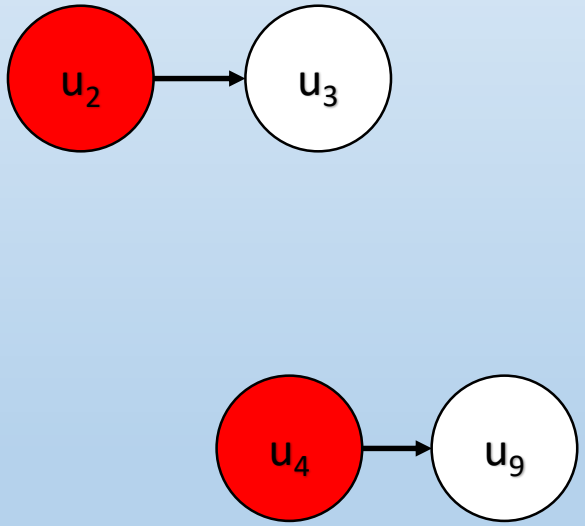

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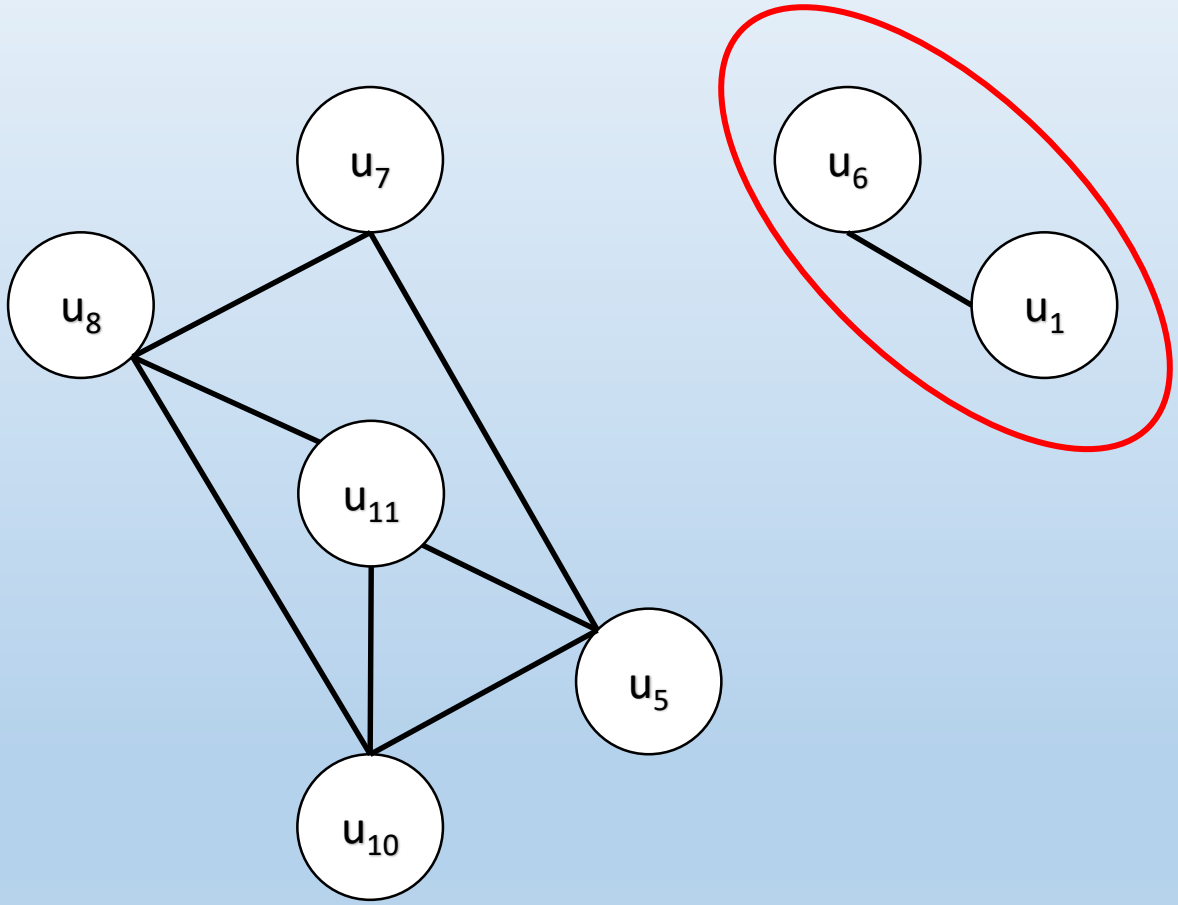

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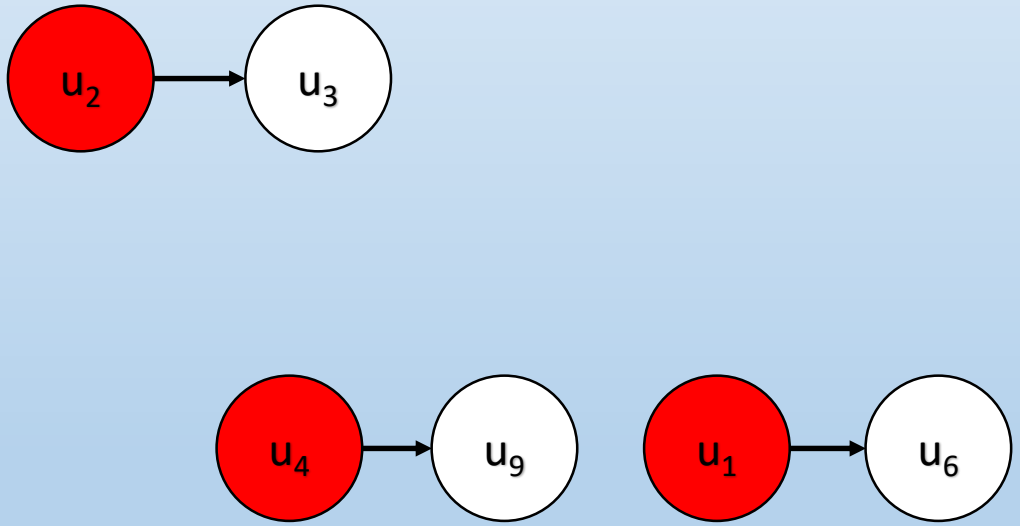

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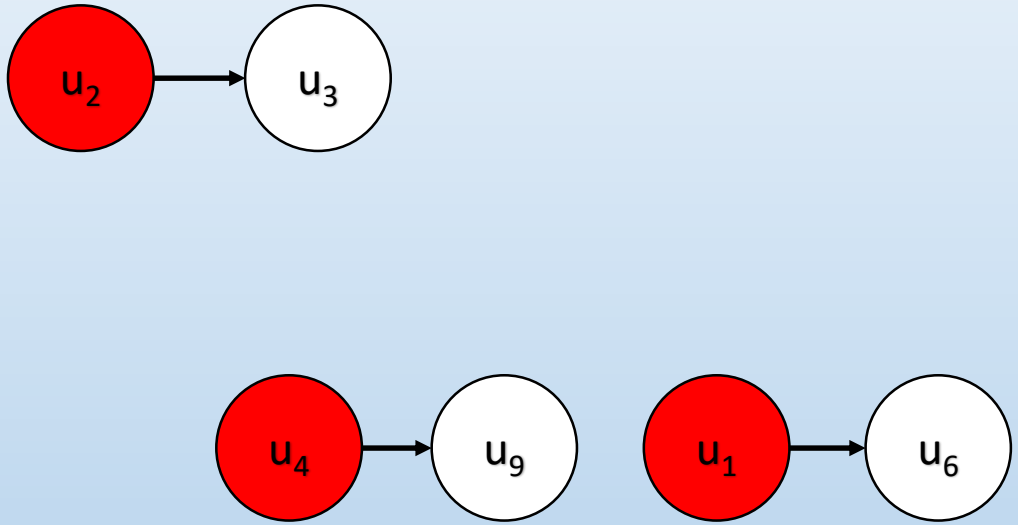


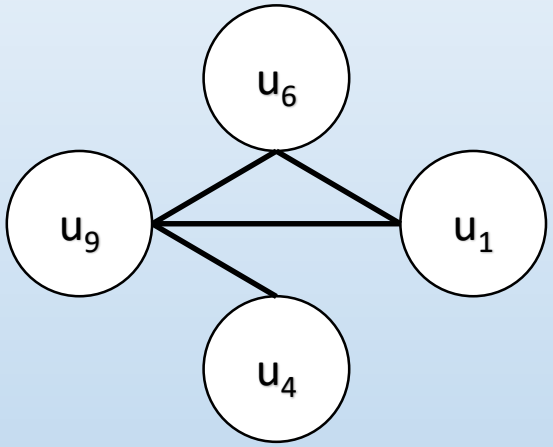

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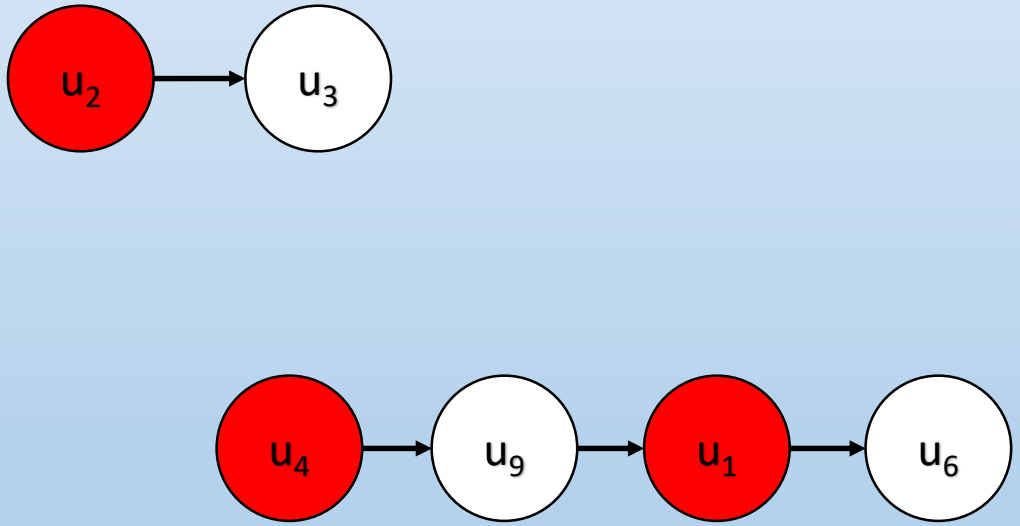

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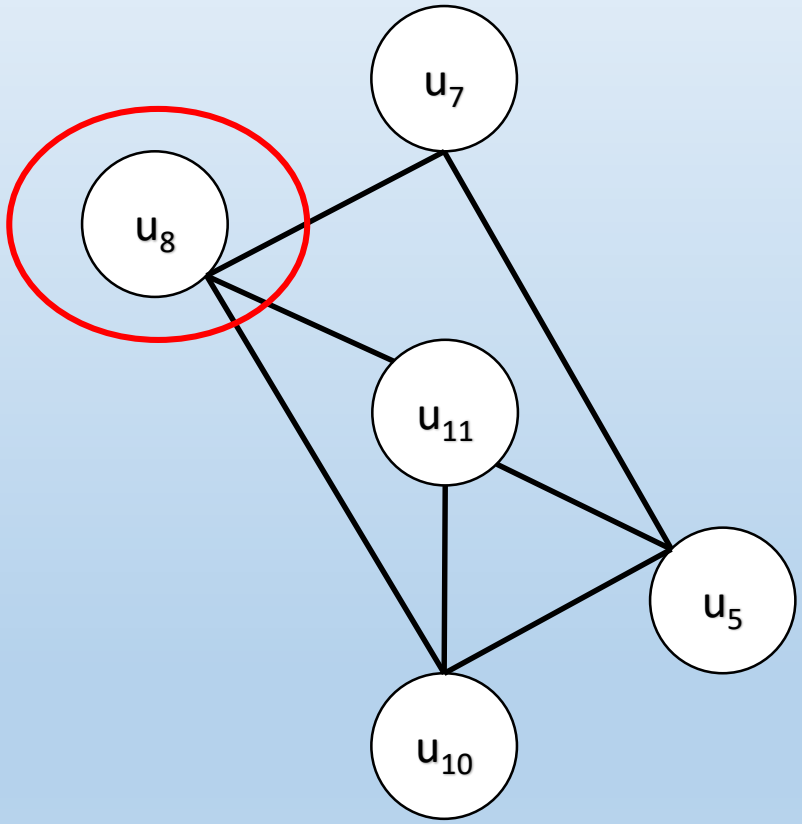

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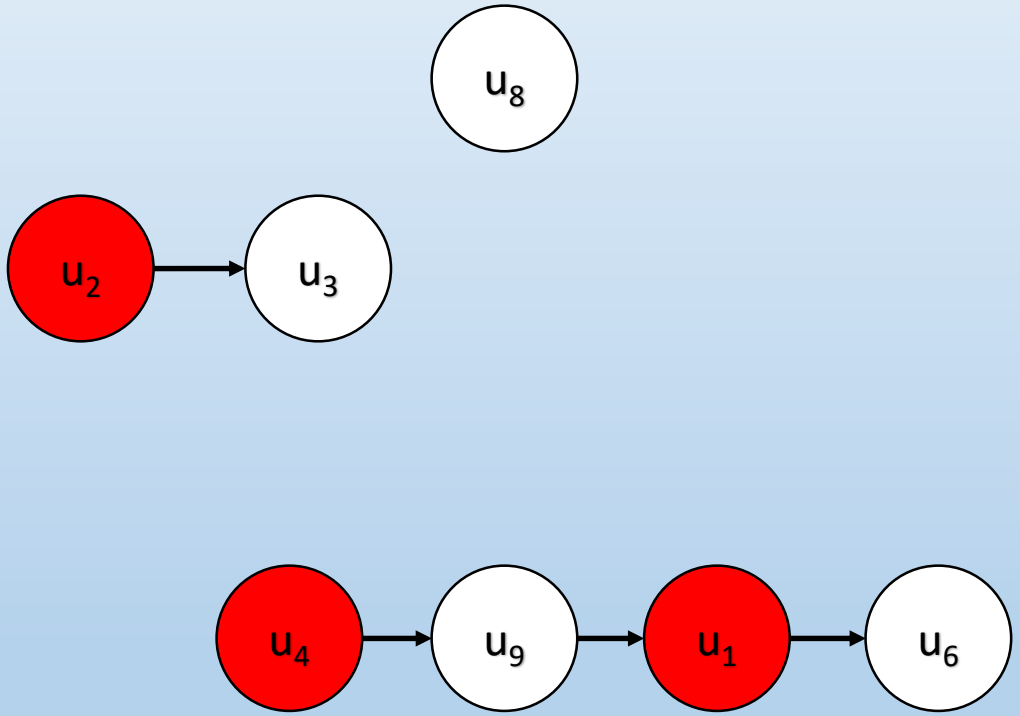

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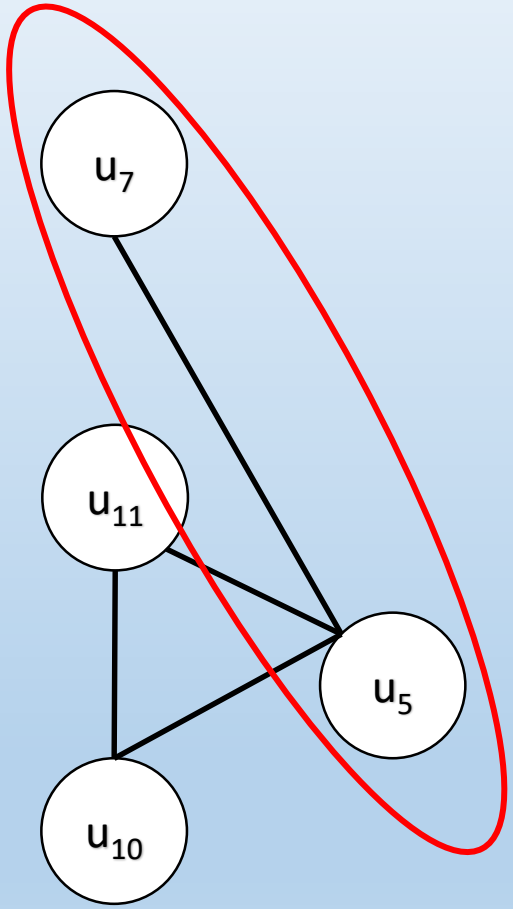

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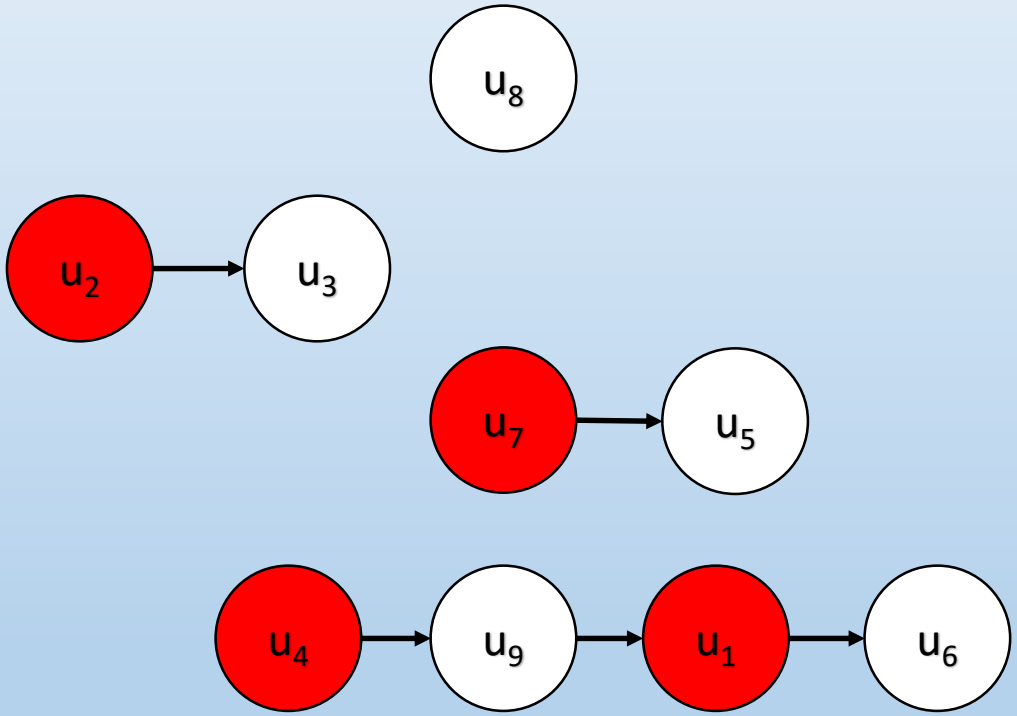

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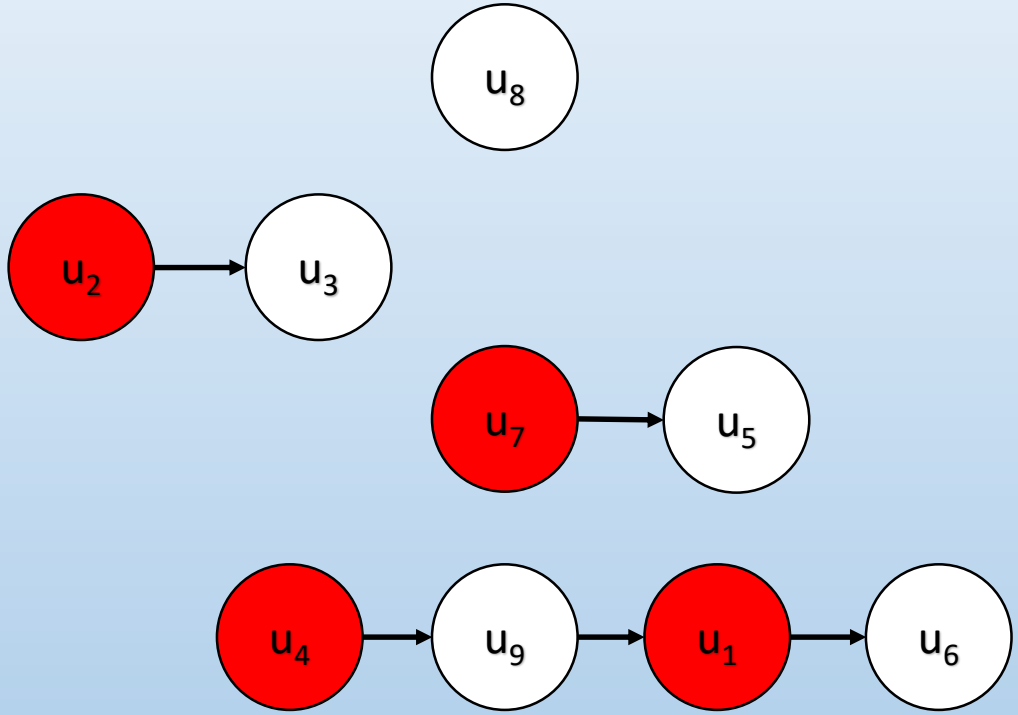


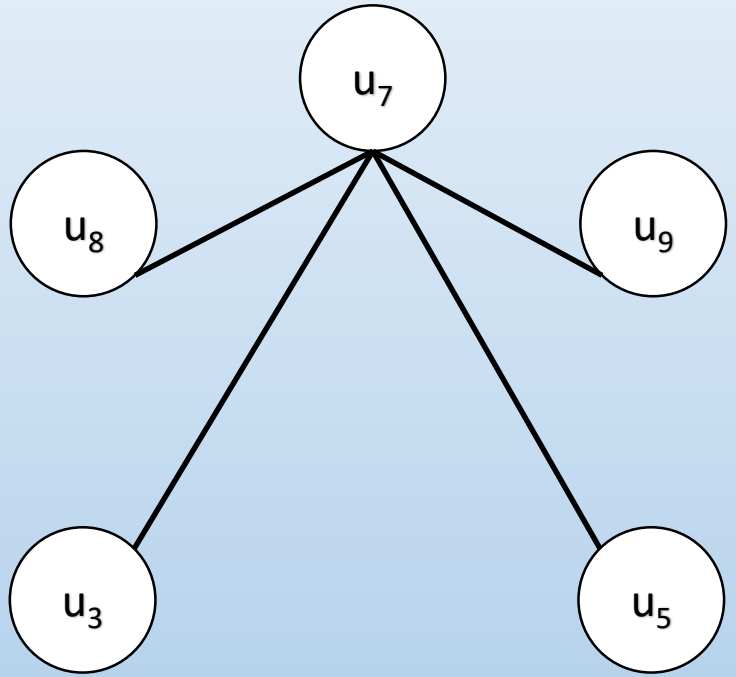

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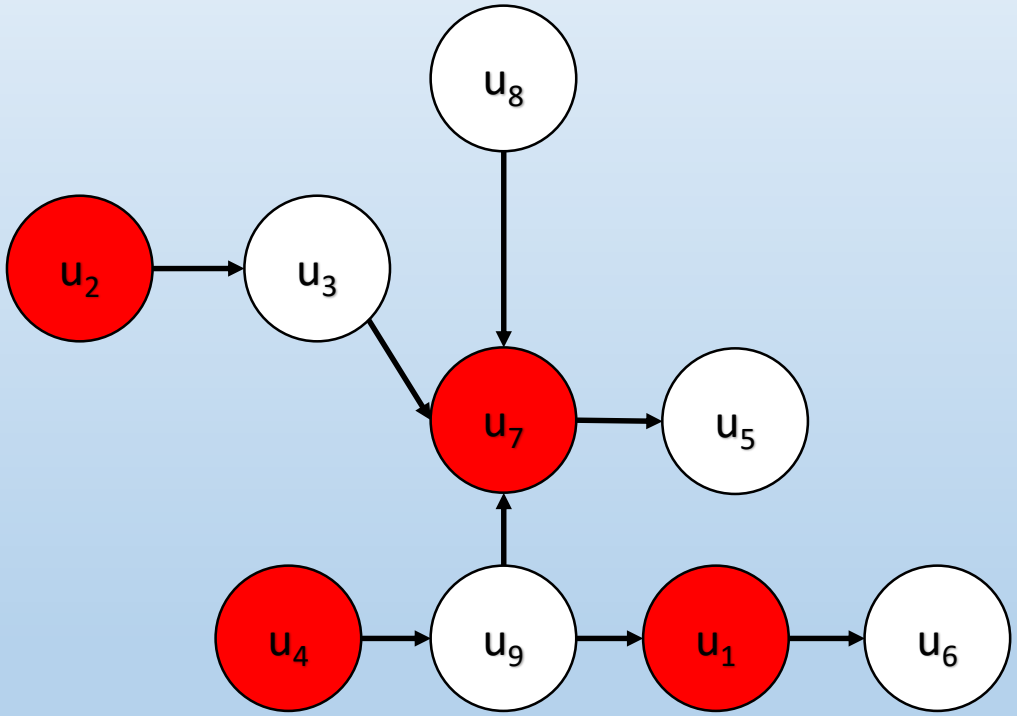

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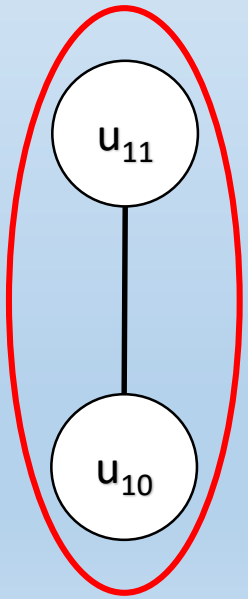

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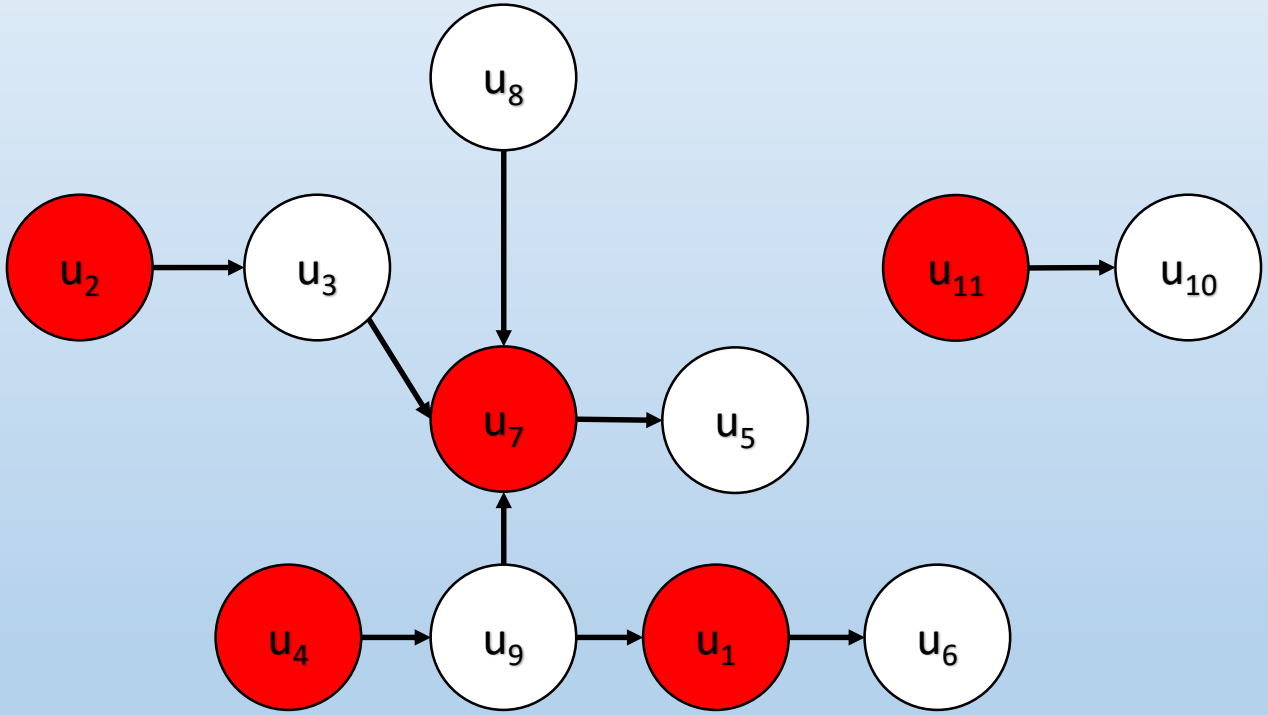

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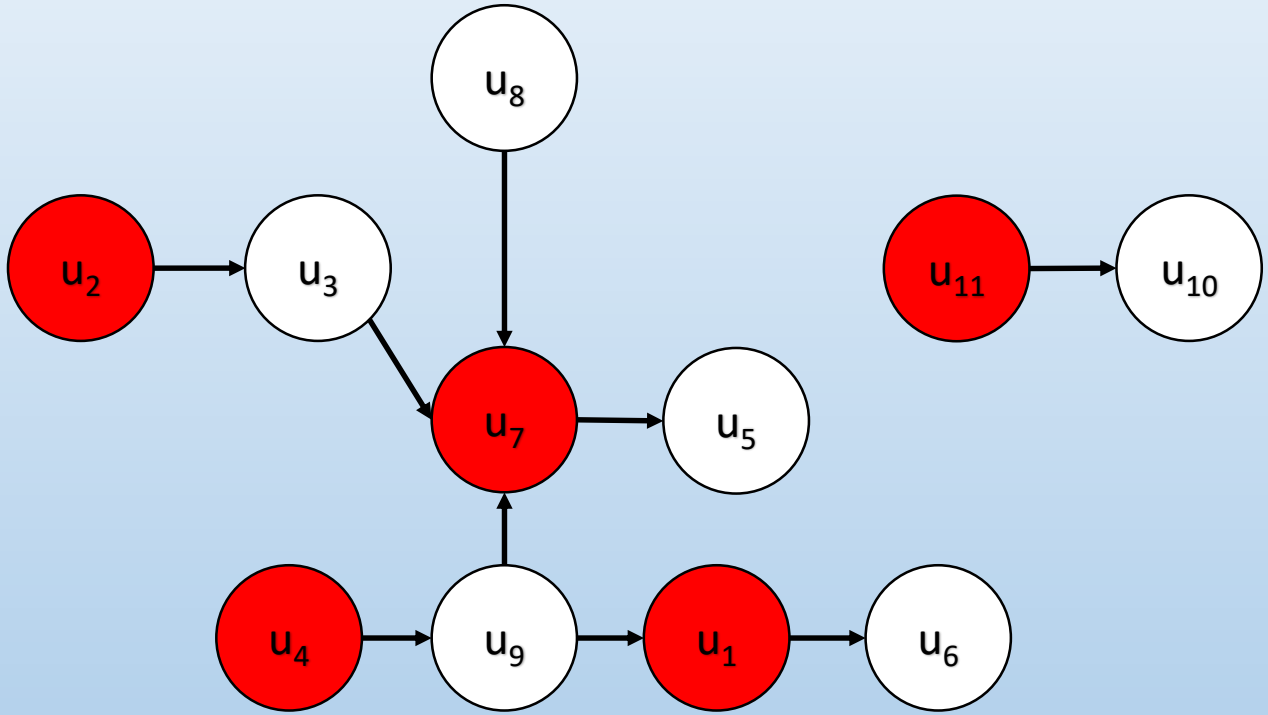


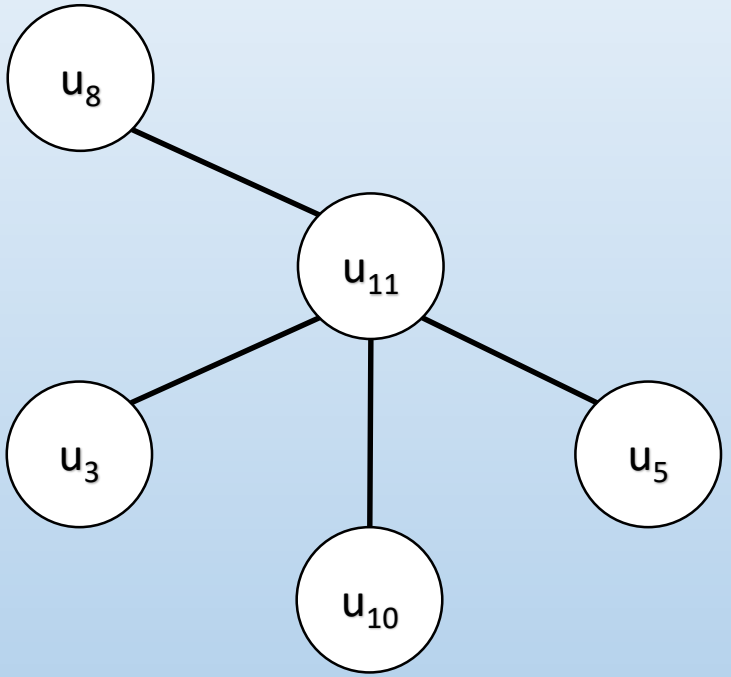

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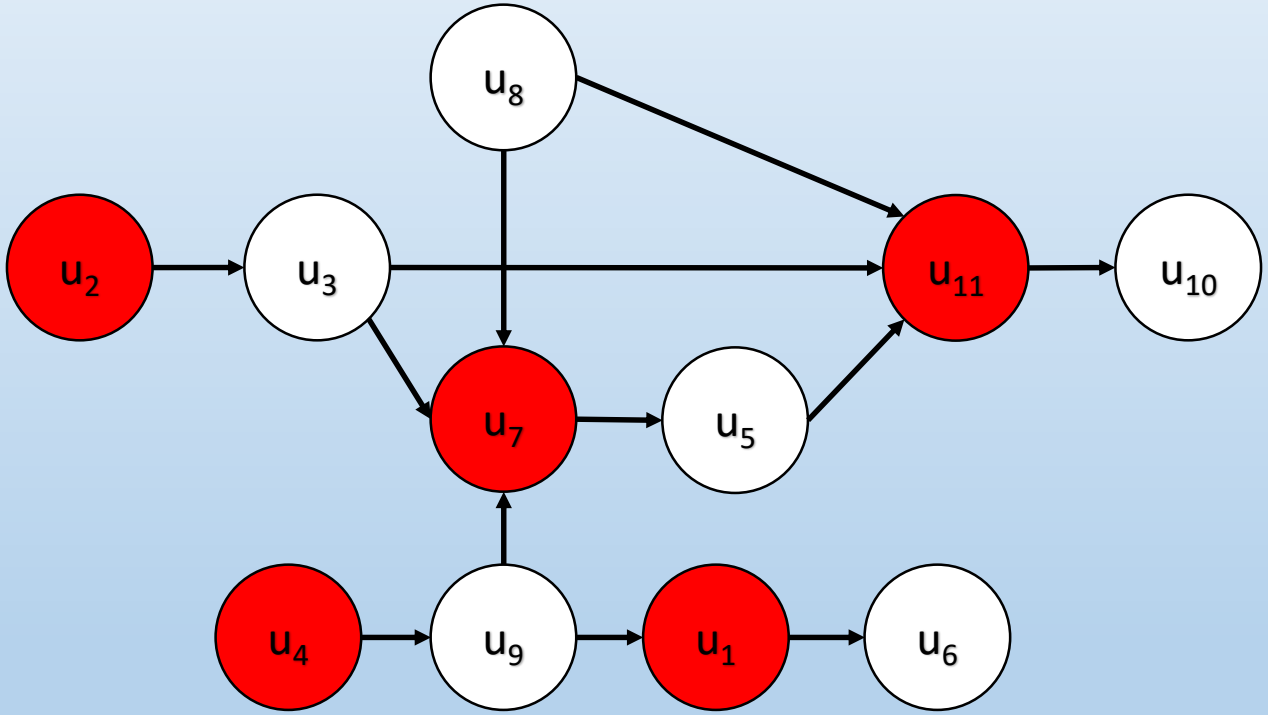


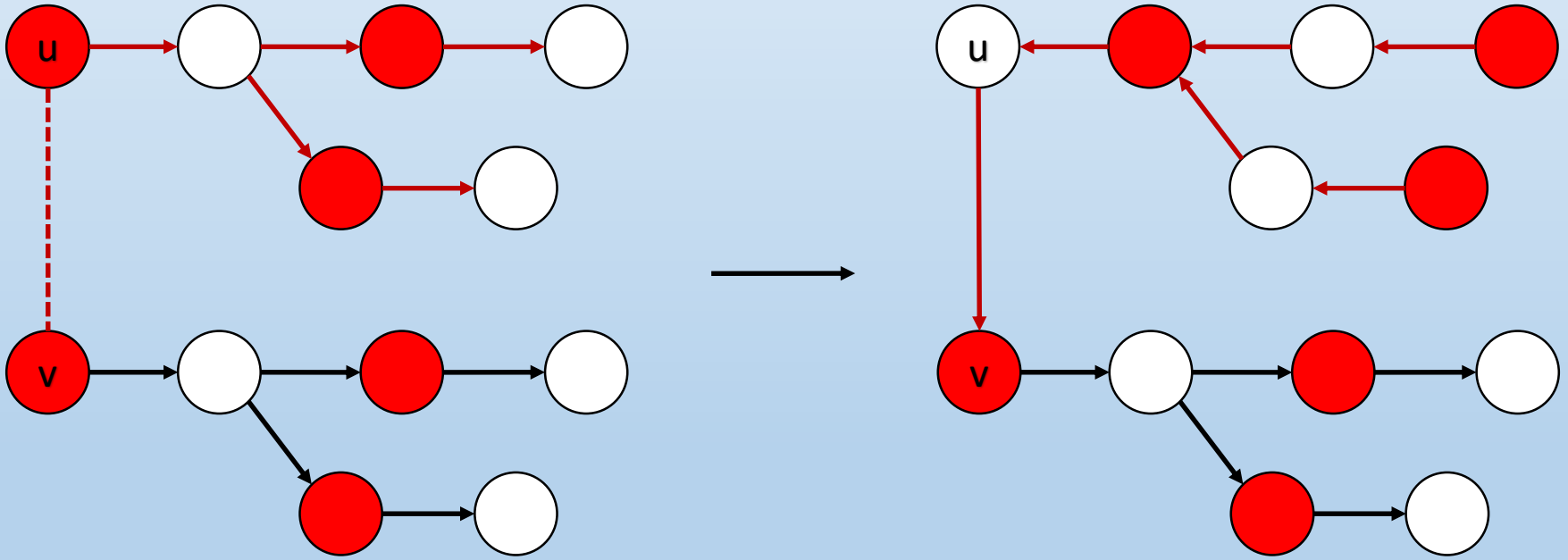
# DG: Dealing with Updates

We mainly focus on the following three operations.

- Delete a node u
  - u is a reducing node.
  - Try to move out u.
- Add an edge (u, v)
  - Both u and v are reducing nodes.
  - Try to move out u or v. Simply abandon either if it fails.
- Delete an edge (u, v)
  - Try to move in the dependent node.

# DG: Maintenance

add an edge (u, v)



# DGOracle

Actually, we do not use the given Near-$MIS_0$. (How tricky!!!)

Because the reducing-peeling framework can generate an initial solution Near-$MIS_{RP}$. (Its original job, is not it?)

Can we make use of Near-$MIS_0$ if it is better than NEAR-$MIS_{RP}$?

Specifically, what if Near-$MIS_0$ is exactly a MIS. ($Gap_0 = 0$)

# DGOracle


Here, the inexact reduction is guided by Near-MIS$_0$.

If Near-MIS$_0$ contains node u, we believe that u is a good node even if it has the largest degree.

When the Near-MIS$_0$ is optimal(MIS), this step even becomes exact.

Therefore, we call the dependency graph constructed in this method DGOracle.

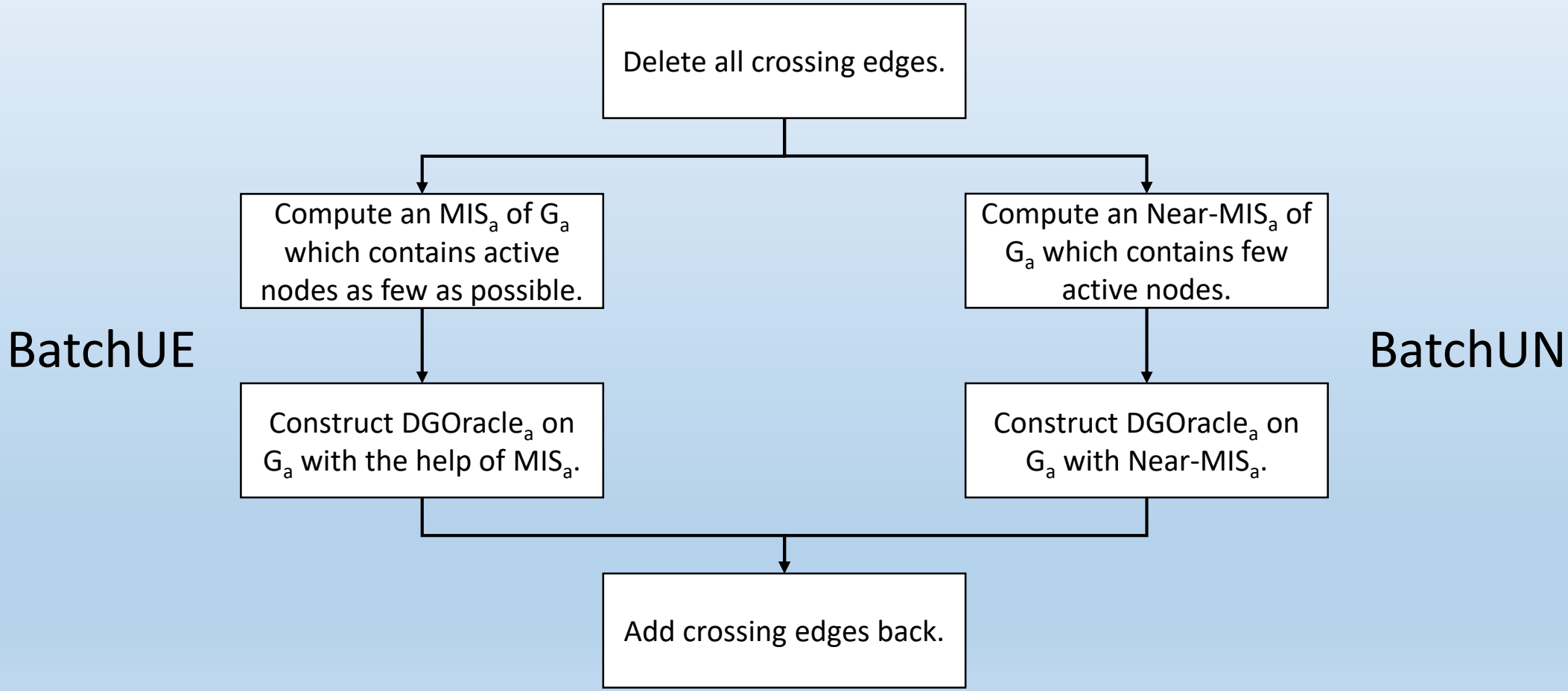
# Batch Update

How to compute Near-$MIS_k$ based on Near-$MIS_0$ directly?

In other words, we want to solve k updates in a batch.

This problem is very reasonable when many updates are concentrated in a small region.

# Batch Update



# Experiment

# Experimental Setting

We use 18 real-world networks and randomly generate 10,000 updates on each graph.

- LSTwo+LazySearch[8]
- DGOne-DIS(without d2 reduction)
- DGTwo-DIS
- DGOracle-DIS
- BatchUE/BatchUN

| ID | Graph | \|V(G)\| | \|E(G)\| | d |
|---|---|---|---|---|
| G01 | Wiki-Vote | 7,115 | 100,762 | 28.32 |
| G02 | CA-HepPh | 12,006 | 118,489 | 19.74 |
| G03 | AstroPh | 18,771 | 198,050 | 21.1 |
| G04 | Brightkite | 58,228 | 214,078 | 7.35 |
| G05 | Epinions | 75,879 | 405,740 | 10.69 |
| G06 | Slashdot | 82,168 | 504,230 | 12.27 |
| G07 | com-dblp | 317,080 | 1,049,866 | 6.62 |
| G08 | com-amazon | 334,863 | 925,872 | 5.53 |
| G09 | BerkStan | 685,230 | 6,649,470 | 19.41 |
| G10 | web-Google | 875,713 | 4,322,051 | 9.87 |
| G11 | soc-pokec | 1,632,803 | 22,301,964 | 27.32 |
| G12 | as-skitter | 1,696,415 | 11,095,298 | 13.08 |
| G13 | wiki-tomcats | 1,791,489 | 25,444,207 | 28.41 |
| G14 | Wiki-Talk | 2,394,385 | 4,659,565 | 3.89 |
| G15 | com-orkut | 3,072,441 | 117,185,083 | 76.28 |
| G16 | cit-Patents | 3,774,768 | 16,518,947 | 8.75 |
| G17 | com-lj | 3,997,962 | 34,681,189 | 17.35 |
| G18 | LiveJournal | 4,846,609 | 42,851,237 | 17.68 |

# Gap(Accuracy)

TABLE II
THE GAPS CAUSED BY 1000 UPDATES AND THE MEMORY USAGE

| Graphs | *LSTwo+LazySearch*$^+$ | | *DGOneDIS* | | *DGTwoDIS* | | *DGOracleDIS* | | *BatchUE* | | *BatchUN* | | Memory Usage (MB) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *AGap* | *RGap* | *AGap* | *RGap* | *AGap* | *RGap* | *AGap* | *RGap* | *AGap* | *RGap* | *AGap* | *RGap* | *DGOneDIS* | *DGTwoDIS* | *BatchUE* |
| Wiki-Vote | 23 | 23 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 2.48 | 2.51 | 2.52 |
| CA-HepPh | 62 | 62 | 10 | 9 | 6 | 6 | 6 | 6 | 5 | 2 | 4 | -5 | 2.99 | 3.04 | 3.76 |
| AstroPh | 35 | 35 | 12 | 11 | 5 | 5 | 3 | 3 | 7 | 7 | 5 | 5 | 4.98 | 5.05 | 5.14 |
| Brightkite | 17 | 17 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 6.29 | 6.51 | 6.76 |
| Epinions | 19 | 19 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 11.09 | 11.39 | 11.48 |
| Slashdot | 26 | 26 | 3 | 1 | 2 | 0 | 0 | 0 | 1 | 1 | 2 | 1 | 13.49 | 13.81 | 14.04 |
| com-dblp | 72 | 72 | 22 | 10 | 6 | 5 | 5 | 5 | 5 | 4 | 6 | 4 | 31.59 | 32.81 | 32.93 |
| com-amazon | 89 | 89 | 150 | -2 | 21 | -2 | 0 | 0 | 3 | -5 | 4 | -5 | 29.18 | 30.45 | 30.86 |
| BerkStan | 838 | 838 | 1074 | 5 | 761 | 4 | 18 | 18 | 15 | 0 | 12 | 3 | 168.53 | 171.15 | 172.28 |
| web-Google | 306 | 306 | 314 | 18 | 173 | 7 | 10 | 10 | 11 | 5 | 16 | 5 | 119.8 | 123.14 | 124.21 |
| as-skitter | 371 | 371 | 393 | -2 | 295 | 0 | 0 | 0 | 7 | 0 | 14 | -1 | 294.39 | 300.86 | 302.38 |
| Wiki-Talk | 74 | 74 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 163.74 | 172.87 | 173.52 |
| LiveJournal | 414 | 414 | 1458 | 4 | 388 | -2 | 2 | 2 | 10 | -2 | 18 | -2 | 1096.34 | 1114.83 | 1115.12 |

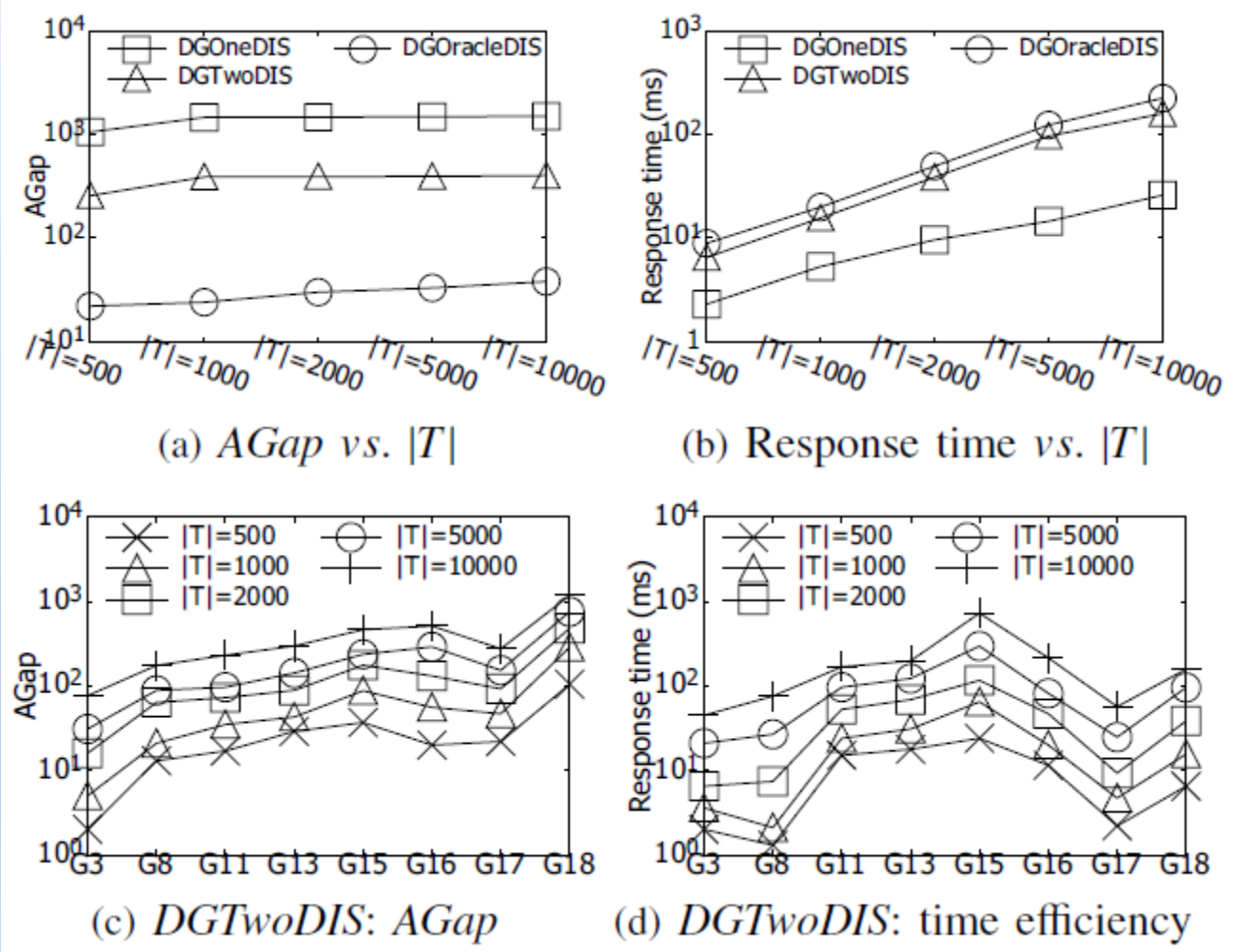
# Efficiency Test

TABLE III

VISITED VERTICES ON AVERAGE AND RESPONSE TIME CONSUMED BY 1000 UPDATES

| Graphs | *LSTwo+LazySearch*$^+$ | *DGOneDIS* | | *DGTwoDIS* | | *DGOracleDIS* | | *BatchUE* | | *BatchUN* | | *DGOneDY* | | *DGTwoDY* | | *DGOracleDY* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *ms* | # | *ms* | # | *ms* | # | *ms* | # | *ms* | # | *ms* | # | *ms* | # | *ms* | # | *ms* |
| Wiki-Vote | 19.3 | 1.1 | 1.3 | 5.6 | 7.2 | 3.6 | 4.5 | 1.3 | 1.5 | 0.28 | 0.3 | 1.3 | 1.5 | 5.5 | 7.1 | 3.9 | 4.6 |
| CA-HepPh | 36.6 | 2.7 | 3.2 | 3.7 | 4.6 | 4.4 | 5.4 | 1.7 | 1.9 | 0.78 | 0.9 | 2.6 | 3.3 | 4.2 | 4.8 | 4.9 | 5.7 |
| AstroPh | 63.7 | 1.4 | 1.5 | 2.8 | 3.3 | 5.8 | 6.9 | 0.76 | 0.9 | 0.31 | 0.4 | 1.3 | 1.5 | 3.1 | 3.7 | 6.7 | 7.1 |
| Brightkite | 104.6 | 3.6 | 4.1 | 3.9 | 4.5 | 7.3 | 8.3 | 1.8 | 2.2 | 0.91 | 1.1 | 4.0 | 4.6 | 3.8 | 4.5 | 6.9 | 7.7 |
| Epinions | 72.1 | 0.92 | 1.2 | 1.4 | 1.7 | 5.2 | 6.6 | 0.71 | 0.85 | 0.28 | 0.33 | 1.1 | 1.4 | 1.2 | 1.8 | 5.1 | 6.3 |
| Slashdot | 166.8 | 1.8 | 2.3 | 2.3 | 2.8 | 7.5 | 9.3 | 0.79 | 0.9 | 0.17 | 0.2 | 1.9 | 2.1 | 2.2 | 2.7 | 7.4 | 8.4 |
| com-dblp | 88.7 | 1.1 | 1.4 | 1.7 | 1.9 | 3.2 | 4.2 | 0.56 | 0.66 | 0.29 | 0.41 | 1.3 | 1.6 | 2.1 | 2.5 | 3.1 | 4.2 |
| com-amazon | 1209.3 | 2.1 | 2.6 | 2.2 | 2.7 | 4.3 | 5.1 | 1.2 | 1.4 | 0.41 | 0.52 | 2.2 | 2.8 | 2.3 | 2.9 | 4.3 | 5.6 |
| BerkStan | 2723.4 | 6.4 | 7.7 | 7.5 | 8.6 | 8.9 | 10.5 | 2.8 | 3.3 | 1.3 | 1.5 | 5.8 | 7.2 | 6.1 | 7.8 | 8.2 | 9.6 |
| web-Google | 4045.8 | 4.7 | 5.4 | 5.6 | 6.3 | 7.7 | 9.3 | 2.2 | 2.6 | 0.64 | 0.72 | 4.3 | 5.5 | 5.2 | 6.5 | 8.6 | 9.7 |
| as-skitter | 3659.3 | 7.2 | 8.6 | 8.2 | 9.8 | 9.6 | 11.5 | 2.8 | 3.4 | 0.26 | 0.29 | 7.1 | 8.2 | 7.9 | 9.1 | 8.9 | 10.8 |
| Wiki-Talk | 2325.5 | 0.9 | 1.1 | 1.4 | 1.6 | 4.7 | 5.8 | 0.55 | 0.62 | 0.1 | 0.11 | 1.2 | 1.5 | 1.4 | 1.8 | 5.6 | 6.4 |
| LiveJournal | 7521.4 | 4.2 | 5.3 | 12.9 | 15.2 | 10.1 | 12.4 | 1.1 | 1.6 | 0.29 | 0.39 | 4.6 | 5.5 | 12.9 | 14.8 | 9.5 | 11.2 |

# Scalability




(a) *AGap vs.* $|T|$ (b) Response time *vs.* $|T|$

(c) *DGTwoDIS*: *AGap* (d) *DGTwoDIS*: time efficiency

# Conclusion

In our paper, we propose a new method to maintain a Near-MIS in dynamic graphs:

Construct a dependency graph to guide the searching procedure based on the traditional reducing-peeling framework.

- Maintain the high quality of a Near-MIS during updates.
- Reduce the time complexity of each update to O(d) in average case.

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