Alternatively, we can implement the hash table with a balanced binary search tree. This gives us an `O(log N)` lookup time. The advantage of this is potentially using less space, since we no longer allocate a large array. We can also iterate through the keys in order, which can be useful sometimes.

▸ ArrayList & Resizable Arrays

In some languages, arrays (often called lists in this case) are automatically resizable. The array or list will grow as you append items. In other languages, like Java, arrays are fixed length. The size is defined when you create the array.

When you need an array-like data structure that offers dynamic resizing, you would usually use an ArrayList. An ArrayList is an array that resizes itself as needed while still providing `O(1)` access. A typical implementation is that when the array is full, the array doubles in size. Each doubling takes `O(n)` time, but happens so rarely that its amortized insertion time is still `O(1)`.

```
ArrayList<String> merge(String[] words, String[] more) {
  ArrayList<String> sentence = new ArrayList<String>();
  for (String w : words) sentence.add(w);
  for (String w : more) sentence.add(w);
  return sentence;
}
```

This is an essential data structure for interviews. Be sure you are comfortable with dynamically resizable arrays/lists in whatever language you will be working with. Note that the name of the data structure as well as the "resizing factor" (which is 2 in Java) can vary.

Why is the amortized insertion runtime O(1)?

Suppose you have an array of size N. We can work backwards to compute how many elements we copied at each capacity increase. Observe that when we increase the array to K elements, the array was previously half that size. Therefore, we needed to copy $K/2$ elements.

```
final capacity increase   : n/2 elements to copy
previous capacity increase: n/4 elements to copy
previous capacity increase: n/8 elements to copy
previous capacity increase: n/16 elements to copy
...
second capacity increase  : 2 elements to copy
first capacity increase   : 1 element to copy
```

Therefore, the total number of copies to insert N elements is roughly $N/2 + N/4 + N/8 + \ldots + 2 + 1$, which is just less than N.

> If the sum of this series isn't obvious to you, imagine this: Suppose you have a kilometer-long walk to the store. You walk 0.5 kilometers, and then 0.25 kilometers, and then 0.125 kilometers, and so on. You will never exceed one kilometer (although you'll get very close to it).

Therefore, inserting N elements takes `O(N)` work total. Each insertion is `O(1)` on average, even though some insertions take `O(N)` time in the worst case.

▸ StringBuilder

Imagine you were concatenating a list of strings, as shown below. What would the running time of this code be? For simplicity, assume that the strings are all the same length (call this x) and that there are n strings.

```
String joinWords(String[] words) {
   String sentence = "";
   for (String w : words) {
      sentence = sentence + w;
   }
   return sentence;
}
```

On each concatenation, a new copy of the string is created, and the two strings are copied over, character by character. The first iteration requires us to copy `x` characters. The second iteration requires copying `2x` characters. The third iteration requires `3x`, and so on. The total time therefore is `O(x + 2x + ... + nx)`. This reduces to $O(xn^2)$.

> Why is it $O(xn^2)$? Because `1 + 2 + ... + n` equals `n(n+1)/2`, or $O(n^2)$.

`StringBuilder` can help you avoid this problem. `StringBuilder` simply creates a resizable array of all the strings, copying them back to a string only when necessary.

```
String joinWords(String[] words) {
   StringBuilder sentence = new StringBuilder();
   for (String w : words) {
      sentence.append(w);
   }
   return sentence.toString();
}
```

A good exercise to practice strings, arrays, and general data structures is to implement your own version of `StringBuilder`, `HashTable` and `ArrayList`.

Additional Reading: Hash Table Collision Resolution (pg 636), Rabin-Karp Substring Search (pg 636).

Interview Questions

1.1 **Is Unique:** Implement an algorithm to determine if a string has all unique characters. What if you cannot use additional data structures?

Hints: #44, #117, #132

pg 192

1.2 **Check Permutation:** Given two strings, write a method to decide if one is a permutation of the other.

Hints: #1, #84, #122, #131

pg 193

1.3 **URLify:** Write a method to replace all spaces in a string with '%20'. You may assume that the string has sufficient space at the end to hold the additional characters, and that you are given the "true" length of the string. (Note: If implementing in Java, please use a character array so that you can perform this operation in place.)

EXAMPLE

Input: `"Mr John Smith    ", 13`

Output: `"Mr%20John%20Smith"`

Hints: #53, #118

pg 194

1.4 **Palindrome Permutation:** Given a string, write a function to check if it is a permutation of a palindrome. A palindrome is a word or phrase that is the same forwards and backwards. A permutation is a rearrangement of letters. The palindrome does not need to be limited to just dictionary words.

EXAMPLE

Input: `Tact Coa`

Output: `True (permutations: "taco cat", "atco cta", etc.)`

Hints: #106, #121, #134, #136

pg 195

1.5 **One Away:** There are three types of edits that can be performed on strings: insert a character, remove a character, or replace a character. Given two strings, write a function to check if they are one edit (or zero edits) away.

EXAMPLE

```
pale,  ple  -> true
pales, pale -> true
pale,  bale -> true
pale,  bake -> false
```

Hints: #23, #97, #130

pg 199

1.6 **String Compression:** Implement a method to perform basic string compression using the counts of repeated characters. For example, the string `aabcccccaaa` would become `a2b1c5a3`. If the "compressed" string would not become smaller than the original string, your method should return the original string. You can assume the string has only uppercase and lowercase letters (a - z).

Hints: #92, #110

pg 201

1.7 **Rotate Matrix:** Given an image represented by an `NxN` matrix, where each pixel in the image is 4 bytes, write a method to rotate the image by 90 degrees. Can you do this in place?

Hints: #51, #100

pg 203

1.8 **Zero Matrix:** Write an algorithm such that if an element in an `MxN` matrix is 0, its entire row and column are set to 0.

Hints: #17, #74, #102

pg 204

1.9 **String Rotation:** Assume you have a method `isSubstring` which checks if one word is a substring of another. Given two strings, `s1` and `s2`, write code to check if `s2` is a rotation of `s1` using only one call to `isSubstring` (e.g., "`waterbottle`" is a rotation of "`erbottlewat`").

Hints: #34, #88, #104

pg 206

Additional Questions: Object-Oriented Design (#7.12), Recursion (#8.3), Sorting and Searching (#10.9), C++ (#12.11), Moderate Problems (#16.8, #16.17, #16.22), Hard Problems (#17.4, #17.7, #17.13, #17.22, #17.26).

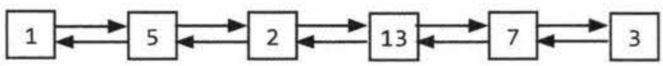
Hints start on page 653.

2

Linked Lists

A linked list is a data structure that represents a sequence of nodes. In a singly linked list, each node points to the next node in the linked list. A doubly linked list gives each node pointers to both the next node and the previous node.

The following diagram depicts a doubly linked list:



Unlike an array, a linked list does not provide constant time access to a particular "index" within the list. This means that if you'd like to find the Kth element in the list, you will need to iterate through K elements.

The benefit of a linked list is that you can add and remove items from the beginning of the list in constant time. For specific applications, this can be useful.

▸ Creating a Linked List

The code below implements a very basic singly linked list.

```
class Node {
   Node next = null;
   int data;

   public Node(int d) {
      data = d;
   }

   void appendToTail(int d) {
      Node end = new Node(d);
      Node n = this;
      while (n.next != null) {
         n = n.next;
      }
      n.next = end;
   }
}
```

In this implementation, we don't have a `LinkedList` data structure. We access the linked list through a reference to the head Node of the linked list. When you implement the linked list this way, you need to be a bit careful. What if multiple objects need a reference to the linked list, and then the head of the linked list changes? Some objects might still be pointing to the old head.

We could, if we chose, implement a `LinkedList` class that wraps the `Node` class. This would essentially just have a single member variable: the head `Node`. This would largely resolve the earlier issue.

Remember that when you're discussing a linked list in an interview, you must understand whether it is a singly linked list or a doubly linked list.

▸ Deleting a Node from a Singly Linked List

Deleting a node from a linked list is fairly straightforward. Given a node `n`, we find the previous node `prev` and set `prev.next` equal to `n.next`. If the list is doubly linked, we must also update `n.next` to set `n.next.prev` equal to `n.prev`. The important things to remember are (1) to check for the null pointer and (2) to update the head or tail pointer as necessary.

Additionally, if you implement this code in C, C++ or another language that requires the developer to do memory management, you should consider if the removed node should be deallocated.

```
Node deleteNode(Node head, int d) {
   Node n = head;

   if (n.data == d) {
      return head.next; /* moved head */
   }

   while (n.next != null) {
      if (n.next.data == d) {
         n.next = n.next.next;
         return head; /* head didn't change */
      }
      n = n.next;
   }
   return head;
}
```

▸ The "Runner" Technique

The "runner" (or second pointer) technique is used in many linked list problems. The runner technique means that you iterate through the linked list with two pointers simultaneously, with one ahead of the other. The "fast" node might be ahead by a fixed amount, or it might be hopping multiple nodes for each one node that the "slow" node iterates through.

For example, suppose you had a linked list a_1->a_2->...->a_n->b_1->b_2->...->b_n and you wanted to rearrange it into a_1->b_1->a_2->b_2->...->a_n->b_n. You do not know the length of the linked list (but you do know that the length is an even number).

You could have one pointer `p1` (the fast pointer) move every two elements for every one move that `p2` makes. When `p1` hits the end of the linked list, `p2` will be at the midpoint. Then, move `p1` back to the front and begin "weaving" the elements. On each iteration, `p2` selects an element and inserts it after `p1`.

▸ Recursive Problems

A number of linked list problems rely on recursion. If you're having trouble solving a linked list problem, you should explore if a recursive approach will work. We won't go into depth on recursion here, since a later chapter is devoted to it.

However, you should remember that recursive algorithms take at least O(n) space, where n is the depth of the recursive call. All recursive algorithms *can* be implemented iteratively, although they may be much more complex.

Interview Questions

2.1 **Remove Dups:** Write code to remove duplicates from an unsorted linked list.

FOLLOW UP

How would you solve this problem if a temporary buffer is not allowed?

Hints: #9, #40

pg 208

2.2 **Return Kth to Last:** Implement an algorithm to find the kth to last element of a singly linked list.

Hints: #8, #25, #41, #67, #126

pg 209

2.3 **Delete Middle Node:** Implement an algorithm to delete a node in the middle (i.e., any node but the first and last node, not necessarily the exact middle) of a singly linked list, given only access to that node.

EXAMPLE

Input: the node `c` from the linked list `a->b->c->d->e->f`

Result: nothing is returned, but the new linked list looks like `a->b->d->e->f`

Hints: #72

pg 211

2.4 **Partition:** Write code to partition a linked list around a value x, such that all nodes less than x come before all nodes greater than or equal to x. If x is contained within the list, the values of x only need to be after the elements less than x (see below). The partition element x can appear anywhere in the "right partition"; it does not need to appear between the left and right partitions.

EXAMPLE

Input: `3 -> 5 -> 8 -> 5 -> 10 -> 2 -> 1` [partition = 5]

Output: `3 -> 1 -> 2 -> 10 -> 5 -> 5 -> 8`

Hints: #3, #24

pg 212

2.5 **Sum Lists:** You have two numbers represented by a linked list, where each node contains a single digit. The digits are stored in *reverse* order, such that the 1's digit is at the head of the list. Write a function that adds the two numbers and returns the sum as a linked list.

EXAMPLE

Input: `(7-> 1 -> 6) + (5 -> 9 -> 2)`. That is, `617 + 295`.

Output: `2 -> 1 -> 9`. That is, `912`.

FOLLOW UP

Suppose the digits are stored in forward order. Repeat the above problem.

EXAMPLE

Input: `(6 -> 1 -> 7) + (2 -> 9 -> 5)`. That is, `617 + 295`.

Output: `9 -> 1 -> 2`. That is, `912`.

Hints: #7, #30, #71, #95, #109

pg 214

2.6 **Palindrome:** Implement a function to check if a linked list is a palindrome.

Hints: #5, #13, #29, #61, #101

pg 216

2.7 **Intersection**: Given two (singly) linked lists, determine if the two lists intersect. Return the intersecting node. Note that the intersection is defined based on reference, not value. That is, if the kth node of the first linked list is the exact same node (by reference) as the `j`th node of the second linked list, then they are intersecting.

Hints: #20, #45, #55, #65, #76, #93, #111, #120, #129

pg 221

2.8 **Loop Detection:** Given a circular linked list, implement an algorithm that returns the node at the beginning of the loop.

DEFINITION

Circular linked list: A (corrupt) linked list in which a node's next pointer points to an earlier node, so as to make a loop in the linked list.

EXAMPLE

Input: `A -> B -> C -> D -> E -> C` [the same C as earlier]

Output: `C`

Hints: #50, #69, #83, #90

pg 223

Additional Questions: Trees and Graphs (#4.3), Object-Oriented Design (#7.12), System Design and Scalability (#9.5), Moderate Problems (#16.25), Hard Problems (#17.12).

Hints start on page 653.

3

Stacks and Queues

Questions on stacks and queues will be much easier to handle if you are comfortable with the ins and outs of the data structure. The problems can be quite tricky, though. While some problems may be slight modifications on the original data structure, others have much more complex challenges.

▸ Implementing a Stack

The stack data structure is precisely what it sounds like: a stack of data. In certain types of problems, it can be favorable to store data in a stack rather than in an array.

A stack uses LIFO (last-in first-out) ordering. That is, as in a stack of dinner plates, the most recent item added to the stack is the first item to be removed.

It uses the following operations:

- `pop()`: Remove the top item from the stack.
- `push(item)`: Add an item to the top of the stack.
- `peek()`: Return the top of the stack.
- `isEmpty()`: Return true if and only if the stack is empty.

Unlike an array, a stack does not offer constant-time access to the `i`th item. However, it does allow constant-time adds and removes, as it doesn't require shifting elements around.

We have provided simple sample code to implement a stack. Note that a stack can also be implemented using a linked list, if items were added and removed from the same side.

```
public class MyStack<T> {
   private static class StackNode<T> {
      private T data;
      private StackNode<T> next;

      public StackNode(T data) {
         this.data = data;
      }
   }

   private StackNode<T> top;

   public T pop() {
      if (top == null) throw new EmptyStackException();
      T item = top.data;
```

```
      top = top.next;
      return item;
   }

   public void push(T item) {
      StackNode<T> t = new StackNode<T>(item);
      t.next = top;
      top = t;
   }

   public T peek() {
      if (top == null) throw new EmptyStackException();
      return top.data;
   }

   public boolean isEmpty() {
      return top == null;
   }
}
```

One case where stacks are often useful is in certain recursive algorithms. Sometimes you need to push temporary data onto a stack as you recurse, but then remove them as you backtrack (for example, because the recursive check failed). A stack offers an intuitive way to do this.

A stack can also be used to implement a recursive algorithm iteratively. (This is a good exercise! Take a simple recursive algorithm and implement it iteratively.)

Implementing a Queue

A queue implements FIFO (first-in first-out) ordering. As in a line or queue at a ticket stand, items are removed from the data structure in the same order that they are added.

It uses the operations:

- `add(item)`: Add an item to the end of the list.
- `remove()`: Remove the first item in the list.
- `peek()`: Return the top of the queue.
- `isEmpty()`: Return true if and only if the queue is empty.

A queue can also be implemented with a linked list. In fact, they are essentially the same thing, as long as items are added and removed from opposite sides.

```
public class MyQueue<T> {
   private static class QueueNode<T> {
      private T data;
      private QueueNode<T> next;

      public QueueNode(T data) {
         this.data = data;
      }
   }

   private QueueNode<T> first;
   private QueueNode<T> last;

   public void add(T item) {
```

```
        QueueNode<T> t = new QueueNode<T>(item);
        if (last != null) {
            last.next = t;
        }
        last = t;
        if (first == null) {
            first = last;
        }
    }

    public T remove() {
        if (first == null) throw new NoSuchElementException();
        T data = first.data;
        first = first.next;
        if (first == null) {
            last = null;
        }
        return data;
    }

    public T peek() {
        if (first == null) throw new NoSuchElementException();
        return first.data;
    }

    public boolean isEmpty() {
        return first == null;
    }
}
```

It is especially easy to mess up the updating of the first and last nodes in a queue. Be sure to double check this.

One place where queues are often used is in breadth-first search or in implementing a cache.

In breadth-first search, for example, we used a queue to store a list of the nodes that we need to process. Each time we process a node, we add its adjacent nodes to the back of the queue. This allows us to process nodes in the order in which they are viewed.

Interview Questions

3.1 **Three in One:** Describe how you could use a single array to implement three stacks.

Hints: #2, #12, #38, #58

pg 227

3.2 **Stack Min:** How would you design a stack which, in addition to `push` and pop, has a function `min` which returns the minimum element? `Push`, pop and `min` should all operate in `O(1)` time.

Hints: #27, #59, #78

pg 232

3.3 **Stack of Plates:** Imagine a (literal) stack of plates. If the stack gets too high, it might topple. Therefore, in real life, we would likely start a new stack when the previous stack exceeds some threshold. Implement a data structure `SetOfStacks` that mimics this. `SetOfStacks` should be composed of several stacks and should create a new stack once the previous one exceeds capacity. `SetOfStacks.push()` and `SetOfStacks.pop()` should behave identically to a single stack (that is, `pop()` should return the same values as it would if there were just a single stack).

FOLLOW UP

Implement a function `popAt(int index)` which performs a pop operation on a specific sub-stack.

Hints: #64, #81

pg 233

3.4 **Queue via Stacks:** Implement a `MyQueue` class which implements a queue using two stacks.

Hints: #98, #114

pg 236

3.5 **Sort Stack:** Write a program to sort a stack such that the smallest items are on the top. You can use an additional temporary stack, but you may not copy the elements into any other data structure (such as an array). The stack supports the following operations: `push`, `pop`, `peek`, and `isEmpty`.

Hints: #15, #32, #43

pg 237

3.6 **Animal Shelter:** An animal shelter, which holds only dogs and cats, operates on a strictly "first in, first out" basis. People must adopt either the "oldest" (based on arrival time) of all animals at the shelter, or they can select whether they would prefer a dog or a cat (and will receive the oldest animal of that type). They cannot select which specific animal they would like. Create the data structures to maintain this system and implement operations such as `enqueue`, `dequeueAny`, `dequeueDog`, and `dequeueCat`. You may use the built-in `LinkedList` data structure.

Hints: #22, #56, #63

pg 239

Additional Questions: Linked Lists (#2.6), Moderate Problems (#16.26), Hard Problems (#17.9).

Hints start on page 653.

4

Trees and Graphs

Many interviewees find tree and graph problems to be some of the trickiest. Searching a tree is more complicated than searching in a linearly organized data structure such as an array or linked list. Additionally, the worst case and average case time may vary wildly, and we must evaluate both aspects of any algorithm. Fluency in implementing a tree or graph from scratch will prove essential.

Because most people are more familiar with trees than graphs (and they're a bit simpler), we'll discuss trees first. This is a bit out of order though, as a tree is actually a type of graph.

> Note: Some of the terms in this chapter can vary slightly across different textbooks and other sources. If you're used to a different definition, that's fine. Make sure to clear up any ambiguity with your interviewer.

▸ Types of Trees

A nice way to understand a tree is with a recursive explanation. A tree is a data structure composed of nodes.

- Each tree has a root node. (Actually, this isn't strictly necessary in graph theory, but it's usually how we use trees in programming, and especially programming interviews.)
- The root node has zero or more child nodes.
- Each child node has zero or more child nodes, and so on.

The tree cannot contain cycles. The nodes may or may not be in a particular order, they could have any data type as values, and they may or may not have links back to their parent nodes.

A very simple class definition for Node is:

```
class Node {
   public String name;
   public Node[] children;
}
```

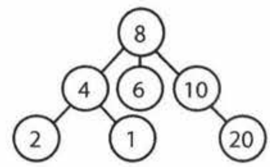
You might also have a `Tree` class to wrap this node. For the purposes of interview questions, we typically do not use a `Tree` class. You can if you feel it makes your code simpler or better, but it rarely does.

```
class Tree {
   public Node root;
}
```

Tree and graph questions are rife with ambiguous details and incorrect assumptions. Be sure to watch out for the following issues and seek clarification when necessary.

Trees vs. Binary Trees

A binary tree is a tree in which each node has up to two children. Not all trees are binary trees. For example, this tree is not a binary tree. You could call it a ternary tree.



There are occasions when you might have a tree that is not a binary tree. For example, suppose you were using a tree to represent a bunch of phone numbers. In this case, you might use a 10-ary tree, with each node having up to 10 children (one for each digit).

A node is called a "leaf" node if it has no children.

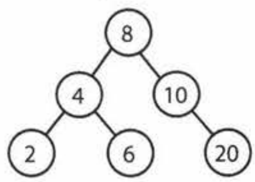
Binary Tree vs. Binary Search Tree

A binary search tree is a binary tree in which every node fits a specific ordering property: `all left descendents <= n < all right descendents`. This must be true for each node `n`.

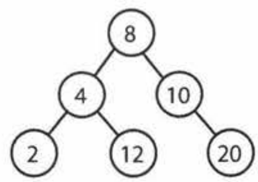
> The definition of a binary search tree can vary slightly with respect to equality. Under some definitions, the tree cannot have duplicate values. In others, the duplicate values will be on the right or can be on either side. All are valid definitions, but you should clarify this with your interviewer.

Note that this inequality must be true for all of a node's descendents, not just its immediate children. The following tree on the left below is a binary search tree. The tree on the right is not, since 12 is to the left of 8.

A binary search tree.



Not a binary search tree.



When given a tree question, many candidates assume the interviewer means a binary *search* tree. Be sure to ask. A binary search tree imposes the condition that, for each node, its left descendents are less than or equal to the current node, which is less than the right descendents.

Balanced vs. Unbalanced

While many trees are balanced, not all are. Ask your interviewer for clarification here. Note that balancing a tree does not mean the left and right subtrees are exactly the same size (like you see under "perfect binary trees" in the following diagram).

One way to think about it is that a "balanced" tree really means something more like "not terribly imbalanced." It's balanced enough to ensure `O(log n)` times for `insert` and `find`, but it's not necessarily as balanced as it could be.

Two common types of balanced trees are red-black trees (pg 639) and AVL trees (pg 637). These are discussed in more detail in the Advanced Topics section.

Complete Binary Trees

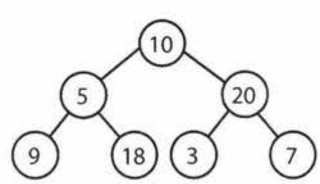
A complete binary tree is a binary tree in which every level of the tree is fully filled, except for perhaps the last level. To the extent that the last level is filled, it is filled left to right.

Full Binary Trees

A full binary tree is a binary tree in which every node has either zero or two children. That is, no nodes have only one child.

Perfect Binary Trees

A perfect binary tree is one that is both full and complete. All leaf nodes will be at the same level, and this level has the maximum number of nodes.



Note that perfect trees are rare in interviews and in real life, as a perfect tree must have exactly $2^k - 1$ nodes (where k is the number of levels). In an interview, do not assume a binary tree is perfect.

▸ Binary Tree Traversal

Prior to your interview, you should be comfortable implementing in-order, post-order, and pre-order traversal. The most common of these is in-order traversal.

In-Order Traversal

In-order traversal means to "visit" (often, print) the left branch, then the current node, and finally, the right branch.

```
void inOrderTraversal(TreeNode node) {
  if (node != null) {
    inOrderTraversal(node.left);
    visit(node);
    inOrderTraversal(node.right);
  }
}
```

When performed on a binary search tree, it visits the nodes in ascending order (hence the name "in-order").

Pre-Order Traversal

Pre-order traversal visits the current node before its child nodes (hence the name "pre-order").

```
void preOrderTraversal(TreeNode node) {
  if (node != null) {
    visit(node);
    preOrderTraversal(node.left);
    preOrderTraversal(node.right);
  }
}
```

In a pre-order traversal, the root is always the first node visited.

Post-Order Traversal

Post-order traversal visits the current node after its child nodes (hence the name "post-order").

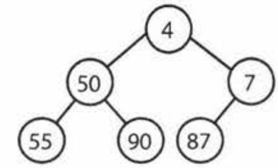
```
void postOrderTraversal(TreeNode node) {
  if (node != null) {
    postOrderTraversal(node.left);
    postOrderTraversal(node.right);
    visit(node);
  }
}
```

In a post-order traversal, the root is always the last node visited.

▸ Binary Heaps (Min-Heaps and Max-Heaps)

We'll just discuss min-heaps here. Max-heaps are essentially equivalent, but the elements are in descending order rather than ascending order.

A min-heap is a *complete* binary tree (that is, totally filled other than the rightmost elements on the last level) where each node is smaller than its children. The root, therefore, is the minimum element in the tree.

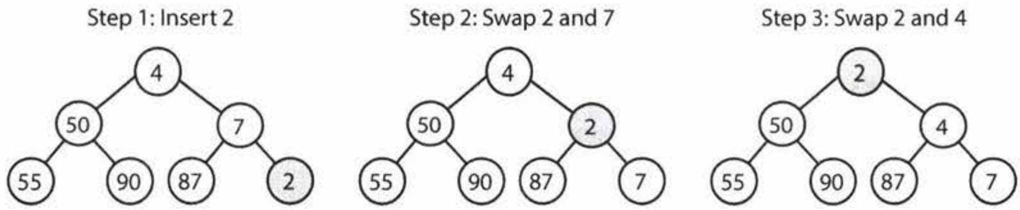


We have two key operations on a min-heap: `insert` and `extract_min`.

Insert

When we insert into a min-heap, we always start by inserting the element at the bottom. We insert at the rightmost spot so as to maintain the complete tree property.

Then, we "fix" the tree by swapping the new element with its parent, until we find an appropriate spot for the element. We essentially bubble up the minimum element.



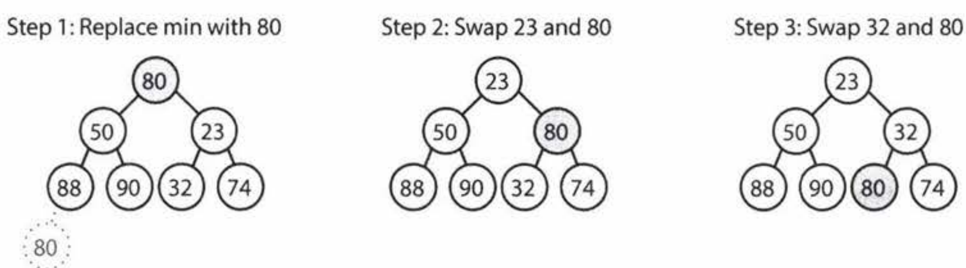
This takes `O(log n)` time, where n is the number of nodes in the heap.

Extract Minimum Element

Finding the minimum element of a min-heap is easy: it's always at the top. The trickier part is how to remove it. (In fact, this isn't that tricky.)

First, we remove the minimum element and swap it with the last element in the heap (the bottommost, rightmost element). Then, we bubble down this element, swapping it with one of its children until the min-heap property is restored.

Do we swap it with the left child or the right child? That depends on their values. There's no inherent ordering between the left and right element, but you'll need to take the smaller one in order to maintain the min-heap ordering.



This algorithm will also take `O(log n)` time.

▸ Tries (Prefix Trees)

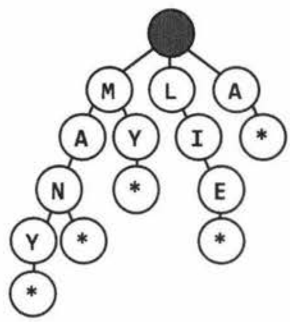
A trie (sometimes called a prefix tree) is a funny data structure. It comes up a lot in interview questions, but algorithm textbooks don't spend much time on this data structure.

A trie is a variant of an n-ary tree in which characters are stored at each node. Each path down the tree may represent a word.

The * nodes (sometimes called "null nodes") are often used to indicate complete words. For example, the fact that there is a * node under MANY indicates that MANY is a complete word. The existence of the MA path indicates there are words that start with MA.

The actual implementation of these * nodes might be a special type of child (such as a `TerminatingTrieNode`, which inherits from `TrieNode`). Or, we could use just a boolean flag `terminates` within the "parent" node.

A node in a trie could have anywhere from 1 through `ALPHABET_SIZE + 1` children (or, 0 through `ALPHABET_SIZE` if a boolean flag is used instead of a * node).



Very commonly, a trie is used to store the entire (English) language for quick prefix lookups. While a hash table can quickly look up whether a string is a valid word, it cannot tell us if a string is a prefix of any valid words. A trie can do this very quickly.

> How quickly? A trie can check if a string is a valid prefix in `O(K)` time, where K is the length of the string. This is actually the same runtime as a hash table will take. Although we often refer to hash table lookups as being `O(1)` time, this isn't entirely true. A hash table must read through all the characters in the input, which takes `O(K)` time in the case of a word lookup.

Many problems involving lists of valid words leverage a trie as an optimization. In situations when we search through the tree on related prefixes repeatedly (e.g., looking up M, then MA, then MAN, then MANY), we might pass around a reference to the current node in the tree. This will allow us to just check if Y is a child of MAN, rather than starting from the root each time.

▸ Graphs

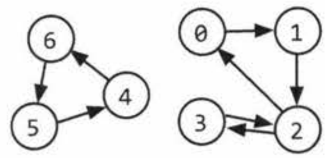
A tree is actually a type of graph, but not all graphs are trees. Simply put, a tree is a connected graph without cycles.

A graph is simply a collection of nodes with edges between (some of) them.

- Graphs can be either directed (like the following graph) or undirected. While directed edges are like a one-way street, undirected edges are like a two-way street.

- The graph might consist of multiple isolated subgraphs. If there is a path between every pair of vertices, it is called a "connected graph."
- The graph can also have cycles (or not). An "acyclic graph" is one without cycles.

Visually, you could draw a graph like this:



In terms of programming, there are two common ways to represent a graph.

Adjacency List

This is the most common way to represent a graph. Every vertex (or node) stores a list of adjacent vertices. In an undirected graph, an edge like `(a, b)` would be stored twice: once in a's adjacent vertices and once in b's adjacent vertices.

A simple class definition for a graph node could look essentially the same as a tree node.

```
class Graph {
  public Node[] nodes;
}

class Node {
   public String name;
   public Node[] children;
}
```

The Graph class is used because, unlike in a tree, you can't necessarily reach all the nodes from a single node.

You don't necessarily need any additional classes to represent a graph. An array (or a hash table) of lists (arrays, arraylists, linked lists, etc.) can store the adjacency list. The graph above could be represented as:

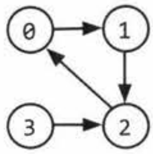
```
0: 1
1: 2
2: 0, 3
3: 2
4: 6
5: 4
6: 5
```

This is a bit more compact, but it isn't quite as clean. We tend to use node classes unless there's a compelling reason not to.

Adjacency Matrices

An adjacency matrix is an NxN boolean matrix (where N is the number of nodes), where a `true` value at `matrix[i][j]` indicates an edge from node `i` to node `j`. (You can also use an integer matrix with 0s and 1s.)

In an undirected graph, an adjacency matrix will be symmetric. In a directed graph, it will not (necessarily) be.



	0	1	2	3
0	0	1	0	0
1	0	0	1	0
2	1	0	0	0
3	0	0	1	0

The same graph algorithms that are used on adjacency lists (breadth-first search, etc.) can be performed with adjacency matrices, but they may be somewhat less efficient. In the adjacency list representation, you can easily iterate through the neighbors of a node. In the adjacency matrix representation, you will need to iterate through all the nodes to identify a node's neighbors.

▸ Graph Search

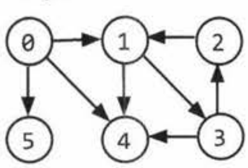
The two most common ways to search a graph are depth-first search and breadth-first search.

In depth-first search (DFS), we start at the root (or another arbitrarily selected node) and explore each branch completely before moving on to the next branch. That is, we go deep first (hence the name *depth-first search*) before we go wide.

In breadth-first search (BFS), we start at the root (or another arbitrarily selected node) and explore each neighbor before going on to any of their children. That is, we go wide (hence *breadth*-first search) before we go deep.

See the below depiction of a graph and its depth-first and breadth-first search (assuming neighbors are iterated in numerical order).

Graph



Depth-First Search

```
Node 0
 Node 1
  Node 3
   Node 2
   Node 4
Node 5
```

Breadth-First Search

```
Node 0
Node 1
Node 4
Node 5
Node 3
Node 2
```

Breadth-first search and depth-first search tend to be used in different scenarios. DFS is often preferred if we want to visit every node in the graph. Both will work just fine, but depth-first search is a bit simpler.

However, if we want to find the shortest path (or just any path) between two nodes, BFS is generally better. Consider representing all the friendships in the entire world in a graph and trying to find a path of friendships between `Ash` and `Vanessa`.

In depth-first search, we could take a path like `Ash -> Brian -> Carleton -> Davis -> Eric -> Farah -> Gayle -> Harry -> Isabella -> John -> Kari`... and then find ourselves very far away. We could go through most of the world without realizing that, in fact, `Vanessa` is `Ash`'s friend. We will still eventually find the path, but it may take a long time. It also won't find us the shortest path.

In breadth-first search, we would stay close to `Ash` for as long as possible. We might iterate through many of `Ash`'s friends, but we wouldn't go to his more distant connections until absolutely necessary. If `Vanessa` is `Ash`'s friend, or his friend-of-a-friend, we'll find this out relatively quickly.

Depth-First Search (DFS)

In DFS, we visit a node a and then iterate through each of a's neighbors. When visiting a node b that is a neighbor of a, we visit all of b's neighbors before going on to a's other neighbors. That is, a exhaustively searches b's branch before any of its other neighbors.

Note that pre-order and other forms of tree traversal are a form of DFS. The key difference is that when implementing this algorithm for a graph, we must check if the node has been visited. If we don't, we risk getting stuck in an infinite loop.

The pseudocode below implements DFS.

```
void search(Node root) {
   if (root == null) return;
   visit(root);
   root.visited = true;
   for each (Node n in root.adjacent) {
      if (n.visited == false) {
         search(n);
      }
   }
}
```

Breadth-First Search (BFS)

BFS is a bit less intuitive, and many interviewees struggle with the implementation unless they are already familiar with it. The main tripping point is the (false) assumption that BFS is recursive. It's not. Instead, it uses a queue.

In BFS, node a visits each of a's neighbors before visiting any of *their* neighbors. You can think of this as searching level by level out from a. An iterative solution involving a queue usually works best.

```
void search(Node root) {
   Queue queue = new Queue();
   root.marked = true;
   queue.enqueue(root); // Add to the end of queue

   while (!queue.isEmpty()) {
      Node r = queue.dequeue(); // Remove from the front of the queue
      visit(r);
      foreach (Node n in r.adjacent) {
         if (n.marked == false) {
            n.marked = true;
            queue.enqueue(n);
         }
      }
   }
}
```

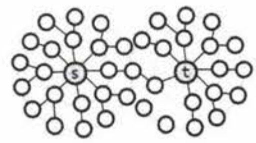
If you are asked to implement BFS, the key thing to remember is the use of the queue. The rest of the algorithm flows from this fact.

Bidirectional Search

Bidirectional search is used to find the shortest path between a source and destination node. It operates by essentially running two simultaneous breadth-first searches, one from each node. When their searches collide, we have found a path.

Breadth-First Search
Single search from s to t that collides after four levels.

Bidirectional Search
Two searches (one from s and one from t) that collide after four levels total (two levels each).



To see why this is faster, consider a graph where every node has at most k adjacent nodes and the shortest path from node s to node t has length d.

- In traditional breadth-first search, we would search up to k nodes in the first "level" of the search. In the second level, we would search up to k nodes for each of those first k nodes, so k^2 nodes total (thus far). We would do this d times, so that's $O(k^d)$ nodes.
- In bidirectional search, we have two searches that collide after approximately $d/2$ levels (the midpoint of the path). The search from s visits approximately $k^{d/2}$, as does the search from t. That's approximately $2 k^{d/2}$, or $O(k^{d/2})$, nodes total.

This might seem like a minor difference, but it's not. It's huge. Recall that $(k^{d/2}) * (k^{d/2}) = k^d$. The bidirectional search is actually faster by a factor of $k^{d/2}$.

Put another way: if our system could only support searching "friend of friend" paths in breadth-first search, it could now likely support "friend of friend of friend of friend" paths. We can support paths that are twice as long.

Additional Reading: Topological Sort (pg 632), Dijkstra's Algorithm (pg 633), AVL Trees (pg 637), Red-Black Trees (pg 639).

Interview Questions

4.1 **Route Between Nodes:** Given a directed graph, design an algorithm to find out whether there is a route between two nodes.

Hints: #127

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4.2 **Minimal Tree:** Given a sorted (increasing order) array with unique integer elements, write an algorithm to create a binary search tree with minimal height.

Hints: #19, #73, #116

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4.3 **List of Depths:** Given a binary tree, design an algorithm which creates a linked list of all the nodes at each depth (e.g., if you have a tree with depth D, you'll have D linked lists).

Hints: #107, #123, #135

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4.4 **Check Balanced:** Implement a function to check if a binary tree is balanced. For the purposes of this question, a balanced tree is defined to be a tree such that the heights of the two subtrees of any node never differ by more than one.

Hints: #21, #33, #49, #105, #124

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4.5 **Validate BST:** Implement a function to check if a binary tree is a binary search tree.

Hints: #35, #57, #86, #113, #128

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4.6 **Successor:** Write an algorithm to find the "next" node (i.e., in-order successor) of a given node in a binary search tree. You may assume that each node has a link to its parent.

Hints: #79, #91

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4.7 **Build Order:** You are given a list of projects and a list of dependencies (which is a list of pairs of projects, where the second project is dependent on the first project). All of a project's dependencies must be built before the project is. Find a build order that will allow the projects to be built. If there is no valid build order, return an error.

EXAMPLE

Input:

```
projects: a, b, c, d, e, f
dependencies: (a, d), (f, b), (b, d), (f, a), (d, c)
```

Output: `f, e, a, b, d, c`

Hints: #26, #47, #60, #85, #125, #133

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4.8 **First Common Ancestor:** Design an algorithm and write code to find the first common ancestor of two nodes in a binary tree. Avoid storing additional nodes in a data structure. NOTE: This is not necessarily a binary search tree.

Hints: #10, #16, #28, #36, #46, #70, #80, #96

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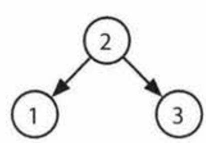
4.9 **BST Sequences:** A binary search tree was created by traversing through an array from left to right and inserting each element. Given a binary search tree with distinct elements, print all possible arrays that could have led to this tree.

EXAMPLE

Input:



Output: `{2, 1, 3}, {2, 3, 1}`

Hints: #39, #48, #66, #82

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4.10 **Check Subtree:** T1 and T2 are two very large binary trees, with T1 much bigger than T2. Create an algorithm to determine if T2 is a subtree of T1.

A tree T2 is a subtree of T1 if there exists a node n in T1 such that the subtree of n is identical to T2. That is, if you cut off the tree at node n, the two trees would be identical.

Hints: #4, #11, #18, #31, #37

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4.11 **Random Node:** You are implementing a binary tree class from scratch which, in addition to `insert`, `find`, and `delete`, has a method `getRandomNode()` which returns a random node from the tree. All nodes should be equally likely to be chosen. Design and implement an algorithm for `getRandomNode`, and explain how you would implement the rest of the methods.

Hints: #42, #54, #62, #75, #89, #99, #112, #119

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4.12 **Paths with Sum:** You are given a binary tree in which each node contains an integer value (which might be positive or negative). Design an algorithm to count the number of paths that sum to a given value. The path does not need to start or end at the root or a leaf, but it must go downwards (traveling only from parent nodes to child nodes).

Hints: #6, #14, #52, #68, #77, #87, #94, #103, #108, #115

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Additional Questions: Recursion (#8.10), System Design and Scalability (#9.2, #9.3), Sorting and Searching (#10.10), Hard Problems (#17.7, #17.12, #17.13, #17.14, #17.17, #17.20, #17.22, #17.25).

Hints start on page 653.

5

Bit Manipulation

Bit manipulation is used in a variety of problems. Sometimes, the question explicitly calls for bit manipulation. Other times, it's simply a useful technique to optimize your code. You should be comfortable doing bit manipulation by hand, as well as with code. Be careful; it's easy to make little mistakes.

▸ Bit Manipulation By Hand

If you're rusty on bit manipulation, try the following exercises by hand. The items in the third column can be solved manually or with "tricks" (described below). For simplicity, assume that these are four-bit numbers.

If you get confused, work them through as a base 10 number. You can then apply the same process to a binary number. Remember that ^ indicates an XOR, and ~ is a NOT (negation).

`0110 + 0010`	`0011 * 0101`	`0110 + 0110`
`0011 + 0010`	`0011 * 0011`	`0100 * 0011`
`0110 - 0011`	`1101 >> 2`	`1101 ^ (~1101)`
`1000 - 0110`	`1101 ^ 0101`	`1011 & (~0 << 2)`

Solutions: line 1 (1000, 1111, 1100); line 2 (0101, 1001, 1100); line 3 (0011, 0011, 1111); line 4 (0010, 1000, 1000).

The tricks in Column 3 are as follows:

1. `0110 + 0110` is equivalent to `0110 * 2`, which is equivalent to shifting `0110` left by 1.
2. `0100` equals 4, and multiplying by 4 is just left shifting by 2. So we shift `0011` left by 2 to get `1100`.
3. Think about this operation bit by bit. If you XOR a bit with its own negated value, you will always get 1. Therefore, the solution to `a^(~a)` will be a sequence of 1s.
4. `~0` is a sequence of 1s, so `~0 << 2` is 1s followed by two 0s. ANDing that with another value will clear the last two bits of the value.

If you didn't see these tricks immediately, think about them logically.

▸ Bit Facts and Tricks

The following expressions are useful in bit manipulation. Don't just memorize them, though; think deeply about why each of these is true. We use "1s" and "0s" to indicate a sequence of 1s or 0s, respectively.

```
x ^ 0s = x       x & 0s = 0       x | 0s = x
x ^ 1s = ~x      x & 1s = x       x | 1s = 1s
x ^ x = 0        x & x = x        x | x = x
```

To understand these expressions, recall that these operations occur bit-by-bit, with what's happening on one bit never impacting the other bits. This means that if one of the above statements is true for a single bit, then it's true for a sequence of bits.

▸ Two's Complement and Negative Numbers

Computers typically store integers in two's complement representation. A positive number is represented as itself while a negative number is represented as the two's complement of its absolute value (with a 1 in its sign bit to indicate that a negative value). The two's complement of an N-bit number (where N is the number of bits used for the number, *excluding* the sign bit) is the complement of the number with respect to 2^N.

Let's look at the 4-bit integer -3 as an example. If it's a 4-bit number, we have one bit for the sign and three bits for the value. We want the complement with respect to 2^3, which is 8. The complement of 3 (the absolute value of -3) with respect to 8 is 5. 5 in binary is `101`. Therefore, -3 in binary as a 4-bit number is 1101, with the first bit being the sign bit.

In other words, the binary representation of -K (negative K) as a N-bit number is $\texttt{concat(1, }2^{N-1}\texttt{ - K)}$.

Another way to look at this is that we invert the bits in the positive representation and then add 1. 3 is `011` in binary. Flip the bits to get `100`, add 1 to get `101`, then prepend the sign bit (1) to get `1101`.

In a four-bit integer, this would look like the following.

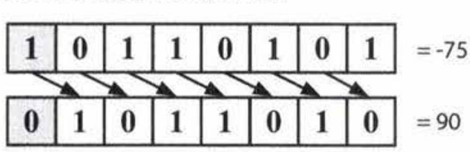
Positive Values		Negative Values	
7	0 111	-1	1 111
6	0 110	-2	1 110
5	0 101	-3	1 101
4	0 100	-4	1 100
3	0 011	-5	1 011
2	0 010	-6	1 010
1	0 001	-7	1 001
0	0 000		

Observe that the absolute values of the integers on the left and right always sum to 2^3, and that the binary values on the left and right sides are identical, other than the sign bit. Why is that?

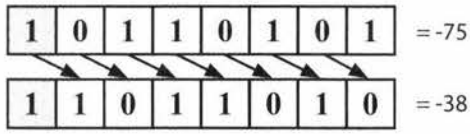
▸ Arithmetic vs. Logical Right Shift

There are two types of right shift operators. The arithmetic right shift essentially divides by two. The logical right shift does what we would visually see as shifting the bits. This is best seen on a negative number.

In a logical right shift, we shift the bits and put a `0` in the most significant bit. It is indicated with a >>> operator. On an 8-bit integer (where the sign bit is the most significant bit), this would look like the image below. The sign bit is indicated with a gray background.



In an arithmetic right shift, we shift values to the right but fill in the new bits with the value of the sign bit. This has the effect of (roughly) dividing by two. It is indicated by a >> operator.



What do you think these functions would do on parameters `x = -93242` and `count = 40`?

```
int repeatedArithmeticShift(int x, int count) {
  for (int i = 0; i < count; i++) {
    x >>= 1; // Arithmetic shift by 1
  }
  return x;
}

int repeatedLogicalShift(int x, int count) {
  for (int i = 0; i < count; i++) {
    x >>>= 1; // Logical shift by 1
  }
  return x;
}
```

With the logical shift, we would get `0` because we are shifting a zero into the most significant bit repeatedly.

With the arithmetic shift, we would get `-1` because we are shifting a one into the most significant bit repeatedly. A sequence of all `1`s in a (signed) integer represents `-1`.

▸ Common Bit Tasks: Getting and Setting

The following operations are very important to know, but do not simply memorize them. Memorizing leads to mistakes that are impossible to recover from. Rather, understand *how* to implement these methods, so that you can implement these, and other, bit problems.

Get Bit

This method shifts `1` over by `i` bits, creating a value that looks like `00010000`. By performing an AND with `num`, we clear all bits other than the bit at bit `i`. Finally, we compare that to `0`. If that new value is not zero, then bit `i` must have a `1`. Otherwise, bit `i` is a `0`.

```
boolean getBit(int num, int i) {
  return ((num & (1 << i)) != 0);
}
```

Set Bit

`SetBit` shifts `1` over by `i` bits, creating a value like `00010000`. By performing an OR with `num`, only the value at bit `i` will change. All other bits of the mask are zero and will not affect `num`.

```
int setBit(int num, int i) {
  return num | (1 << i);
}
```

Clear Bit

This method operates in almost the reverse of `setBit`. First, we create a number like `11101111` by creating the reverse of it (`00010000`) and negating it. Then, we perform an AND with `num`. This will clear the `i`th bit and leave the remainder unchanged.

```
int clearBit(int num, int i) {
  int mask = ~(1 << i);
  return num & mask;
}
```

To clear all bits from the most significant bit through `i` (inclusive), we create a mask with a `1` at the `i`th bit (`1 << i`). Then, we subtract `1` from it, giving us a sequence of `0`s followed by `i` `1`s. We then AND our number with this mask to leave just the last `i` bits.

```
int clearBitsMSBthroughI(int num, int i) {
  int mask = (1 << i) - 1;
  return num & mask;
}
```

To clear all bits from `i` through `0` (inclusive), we take a sequence of all `1`s (which is `-1`) and shift it left by `i + 1` bits. This gives us a sequence of 1s (in the most significant bits) followed by `i` `0` bits.

```
int clearBitsIthrough0(int num, int i) {
  int mask = (-1 << (i + 1));
  return num & mask;
}
```

Update Bit

To set the `i`th bit to a value v, we first clear the bit at position `i` by using a mask that looks like `11101111`. Then, we shift the intended value, v, left by `i` bits. This will create a number with bit `i` equal to v and all other bits equal to `0`. Finally, we OR these two numbers, updating the `i`th bit if v is `1` and leaving it as `0` otherwise.

```
int updateBit(int num, int i, boolean bitIs1) {
  int value = bitIs1 ? 1 : 0;
  int mask = ~(1 << i);
  return (num & mask) | (value << i);
}
```

Interview Questions

5.1 **Insertion:** You are given two 32-bit numbers, `N` and `M`, and two bit positions, `i` and `j`. Write a method to insert `M` into `N` such that `M` starts at bit `j` and ends at bit `i`. You can assume that the bits `j` through `i` have enough space to fit all of `M`. That is, if `M = 10011`, you can assume that there are at least 5 bits between `j` and `i`. You would not, for example, have `j` = 3 and `i` = 2, because `M` could not fully fit between bit 3 and bit 2.

EXAMPLE

Input: `N = 10000000000, M = 10011, i = 2, j = 6`

Output: `N = 10001001100`

Hints: #137, #169, #215

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5.2 **Binary to String:** Given a real number between 0 and 1 (e.g., 0.72) that is passed in as a double, print the binary representation. If the number cannot be represented accurately in binary with at most 32 characters, print "ERROR."

Hints: #143, #167, #173, #269, #297

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5.3 **Flip Bit to Win:** You have an integer and you can flip exactly one bit from a `0` to a `1`. Write code to find the length of the longest sequence of 1s you could create.

EXAMPLE

Input: `1775 (or: 11011101111)`

Output: `8`

Hints: #159, #226, #314, #352

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5.4 **Next Number:** Given a positive integer, print the next smallest and the next largest number that have the same number of 1 bits in their binary representation.

Hints: #147, #175, #242, #312, #339, #358, #375, #390

pg 280

5.5 **Debugger:** Explain what the following code does: `((n & (n-1)) == 0)`.

Hints: #151, #202, #261, #302, #346, #372, #383, #398

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5.6 **Conversion:** Write a function to determine the number of bits you would need to flip to convert integer A to integer B.

EXAMPLE

Input: `29 (or: 11101), 15 (or: 01111)`

Output: `2`

Hints: #336, #369

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5.7 **Pairwise Swap:** Write a program to swap odd and even bits in an integer with as few instructions as possible (e.g., bit `0` and bit `1` are swapped, bit `2` and bit `3` are swapped, and so on).

Hints: #145, #248, #328, #355

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5.8 **Draw Line:** A monochrome screen is stored as a single array of bytes, allowing eight consecutive pixels to be stored in one byte. The screen has width `w`, where `w` is divisible by 8 (that is, no byte will be split across rows). The height of the screen, of course, can be derived from the length of the array and the width. Implement a function that draws a horizontal line from `(x1, y)` to `(x2, y)`.

The method signature should look something like:

```
drawLine(byte[] screen, int width, int x1, int x2, int y)
```

Hints: #366, #381, #384, #391

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Additional Questions: Arrays and Strings (#1.1, #1.4, #1.8), Math and Logic Puzzles (#6.10), Recursion (#8.4, #8.14), Sorting and Searching (#10.7, #10.8), C++ (#12.10), Moderate Problems (#16.1, #16.7), Hard Problems (#17.1).

Hints start on page 662.

6

Math and Logic Puzzles

So-called "puzzles" (or brain teasers) are some of the most hotly debated questions, and many companies have policies banning them. Unfortunately, even when these questions are banned, you still may find yourself being asked one of them. Why? Because no one can agree on a definition of what a brainteaser is.

The good news is that if you are asked a puzzle or brainteaser, it's likely to be a reasonably fair one. It probably won't rely on a trick of wording, and it can almost always be logically deduced. Many have their foundations in mathematics or computer science, and almost all have solutions that can be logically deduced.

We'll go through some common approaches for tackling these questions, as well as some of the essential knowledge.

▸ Prime Numbers

As you probably know, every positive integer can be decomposed into a product of primes. For example:

$$84 = 2^2 * 3^1 * 5^0 * 7^1 * 11^0 * 13^0 * 17^0 * \ldots$$

Note that many of these primes have an exponent of zero.

Divisibility

The prime number law stated above means that, in order for a number x to divide a number y (written x\y, or `mod(y, x) = 0`), all primes in x's prime factorization must be in y's prime factorization. Or, more specifically:

Let $x = 2^{j0} * 3^{j1} * 5^{j2} * 7^{j3} * 11^{j4} * \ldots$

Let $y = 2^{k0} * 3^{k1} * 5^{k2} * 7^{k3} * 11^{k4} * \ldots$

If x\y, then for all `i, ji <= ki`.

In fact, the greatest common divisor of x and y will be:

$$gcd(x, y) = 2^{min(j0, k0)} * 3^{min(j1, k1)} * 5^{min(j2, k2)} * \ldots$$

The least common multiple of x and y will be:

$$lcm(x, y) = 2^{max(j0, k0)} * 3^{max(j1, k1)} * 5^{max(j2, k2)} * \ldots$$

As a fun exercise, stop for a moment and think what would happen if you did `gcd * lcm`:

$$\begin{aligned} gcd * lcm &= 2^{min(j0, k0)} * 2^{max(j0, k0)} * 3^{min(j1, k1)} * 3^{max(j1, k1)} * \ldots \\ &= 2^{min(j0, k0) + max(j0, k0)} * 3^{min(j1, k1) + max(j1, k1)} * \ldots \\ &= 2^{j0 + k0} * 3^{j1 + k1} * \ldots \\ &= 2^{j0} * 2^{k0} * 3^{j1} * 3^{k1} * \ldots \end{aligned}$$

```
              = xy
```

Checking for Primality

This question is so common that we feel the need to specifically cover it. The naive way is to simply iterate from 2 through `n-1`, checking for divisibility on each iteration.

```java
boolean primeNaive(int n) {
  if (n < 2) {
    return false;
  }
  for (int i = 2; i < n; i++) {
    if (n % i == 0) {
      return false;
    }
  }
  return true;
}
```

A small but important improvement is to iterate only up through the square root of n.

```java
boolean primeSlightlyBetter(int n) {
  if (n < 2) {
    return false;
  }
  int sqrt = (int) Math.sqrt(n);
  for (int i = 2; i <= sqrt; i++) {
    if (n % i == 0) return false;
  }
  return true;
}
```

The $\sqrt{n}$ is sufficient because, for every number a which divides n evenly, there is a complement b, where `a * b = n`. If `a >` $\sqrt{n}$, then `b <` $\sqrt{n}$ (since $(\sqrt{n})^2 = n$). We therefore don't need a to check n's primality, since we would have already checked with b.

Of course, in reality, all we *really* need to do is to check if n is divisible by a prime number. This is where the Sieve of Eratosthenes comes in.

Generating a List of Primes: The Sieve of Eratosthenes

The Sieve of Eratosthenes is a highly efficient way to generate a list of primes. It works by recognizing that all non-prime numbers are divisible by a prime number.

We start with a list of all the numbers up through some value `max`. First, we cross off all numbers divisible by 2. Then, we look for the next prime (the next non-crossed off number) and cross off all numbers divisible by it. By crossing off all numbers divisible by 2, 3, 5, 7, 11, and so on, we wind up with a list of prime numbers from 2 through `max`.

The code below implements the Sieve of Eratosthenes.

```java
boolean[] sieveOfEratosthenes(int max) {
  boolean[] flags = new boolean[max + 1];
  int count = 0;

  init(flags); // Set all flags to true other than 0 and 1
  int prime = 2;

  while (prime <= Math.sqrt(max)) {
```

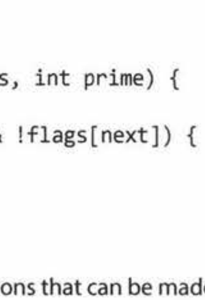
```
      /* Cross off remaining multiples of prime */
      crossOff(flags, prime);

      /* Find next value which is true */
      prime = getNextPrime(flags, prime);
    }

    return flags;
}

void crossOff(boolean[] flags, int prime) {
    /* Cross off remaining multiples of prime. We can start with (prime*prime),
     * because if we have a k * prime, where k < prime, this value would have
     * already been crossed off in a prior iteration. */
    for (int i = prime * prime; i < flags.length; i += prime) {
      flags[i] = false;
    }
}

int getNextPrime(boolean[] flags, int prime) {
    int next = prime + 1;
    while (next < flags.length && !flags[next]) {
      next++;
    }
    return next;
}
```

Of course, there are a number of optimizations that can be made to this. One simple one is to only use odd numbers in the array, which would allow us to reduce our space usage by half.

▸ Probability

Probability can be a complex topic, but it's based in a few basic laws that can be logically derived.

Let's look at a Venn diagram to visualize two events A and B. The areas of the two circles represent their relative probability, and the overlapping area is the event {`A and B`}.



Probability of A and B

Imagine you were throwing a dart at this Venn diagram. What is the probability that you would land in the intersection between A and B? If you knew the odds of landing in A, and you also knew the percent of A that's also in B (that is, the odds of being in B given that you were in A), then you could express the probability as:

```
P(A and B) = P(B given A) P(A)
```

For example, imagine we were picking a number between 1 and 10 (inclusive). What's the probability of picking an even number *and* a number between 1 and 5? The odds of picking a number between 1 and 5 is 50%, and the odds of a number between 1 and 5 being even is 40%. So, the odds of doing both are:

```
P(x is even and x <= 5)
```

```
    = P(x is even given x <= 5) P(x <= 5)
    = (2/5) * (1/2)
    = 1/5
```

Observe that since `P(A and B) = P(B given A) P(A) = P(A given B) P(B)`, you can express the probability of A given B in terms of the reverse:

```
P(A given B) = P(B given A) P(A) / P(B)
```

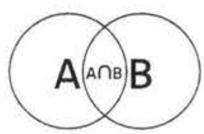
The above equation is called Bayes' Theorem.

Probability of A or B

Now, imagine you wanted to know what the probability of landing in A or B is. If you knew the odds of landing in each individually, and you also knew the odds of landing in their intersection, then you could express the probability as:

```
P(A or B) = P(A) + P(B) - P(A and B)
```

Logically, this makes sense. If we simply added their sizes, we would have double-counted their intersection. We need to subtract this out. We can again visualize this through a Venn diagram:



For example, imagine we were picking a number between 1 and 10 (inclusive). What's the probability of picking an even number *or* a number between 1 and 5? We have a 50% probability of picking an even number and a 50% probability of picking a number between 1 and 5. The odds of doing both are 20%. So the odds are:

```
P(x is even or x <=5)
    = P(x is even) + P(x <= 5) - P(x is even and x <= 5)
    = 1/2 + 1/2 - 1/5
    = 4/5
```

From here, getting the special case rules for independent events and for mutually exclusive events is easy.

Independence

If A and B are independent (that is, one happening tells you nothing about the other happening), then `P(A and B) = P(A) P(B)`. This rule simply comes from recognizing that `P(B given A) = P(B)`, since A indicates nothing about B.

Mutual Exclusivity

If A and B are mutually exclusive (that is, if one happens, then the other cannot happen), then `P(A or B) = P(A) + P(B)`. This is because `P(A and B) = 0`, so this term is removed from the earlier `P(A or B)` equation.

Many people, strangely, mix up the concepts of independence and mutual exclusivity. They are *entirely* different. In fact, two events cannot be both independent and mutually exclusive (provided both have probabilities greater than 0). Why? Because mutual exclusivity means that if one happens then the other cannot. Independence, however, says that one event happening means absolutely *nothing* about the other event. Thus, as long as two events have non-zero probabilities, they will never be both mutually exclusive and independent.

If one or both events have a probability of zero (that is, it is impossible), then the events are both independent and mutually exclusive. This is provable through a simple application of the definitions (that is, the formulas) of independence and mutual exclusivity.

▸ Start Talking

Don't panic when you get a brainteaser. Like algorithm questions, interviewers want to see how you tackle a problem; they don't expect you to immediately know the answer. Start talking, and show the interviewer how you approach a problem.

▸ Develop Rules and Patterns

In many cases, you will find it useful to write down "rules" or patterns that you discover while solving the problem. And yes, you really should write these down—it will help you remember them as you solve the problem. Let's demonstrate this approach with an example.

You have two ropes, and each takes exactly one hour to burn. How would you use them to time exactly 15 minutes? Note that the ropes are of uneven densities, so half the rope length-wise does not necessarily take half an hour to burn.

> *Tip: Stop here and spend some time trying to solve this problem on your own. If you absolutely must, read through this section for hints—but do so slowly. Every paragraph will get you a bit closer to the solution.*

From the statement of the problem, we immediately know that we can time one hour. We can also time two hours, by lighting one rope, waiting until it is burnt, and then lighting the second. We can generalize this into a rule.

Rule 1: Given a rope that takes x minutes to burn and another that takes y minutes, we can time x+y minutes.

What else can we do with the rope? We can probably assume that lighting a rope in the middle (or anywhere other than the ends) won't do us much good. The flames would expand in both directions, and we have no idea how long it would take to burn.

However, we can light a rope at both ends. The two flames would meet after 30 minutes.

Rule 2: Given a rope that takes x minutes to burn, we can time $x/2$ minutes.

We now know that we can time 30 minutes using a single rope. This also means that we can remove 30 minutes of burning time from the second rope, by lighting rope 1 on both ends and rope 2 on just one end.

Rule 3: If rope 1 takes x minutes to burn and rope 2 takes y minutes, we can turn rope 2 into a rope that takes (y-x) minutes or ($y-x/2$) minutes.

Now, let's piece all of these together. We can turn rope 2 into a rope with 30 minutes of burn time. If we then light rope 2 on the other end (see rule 2), rope 2 will be done after 15 minutes.

From start to end, our approach is as follows:

1. Light rope 1 at both ends and rope 2 at one end.
2. When the two flames on Rope 1 meet, 30 minutes will have passed. Rope 2 has 30 minutes left of burn-time.

3. At that point, light Rope 2 at the other end.
4. In exactly fifteen minutes, Rope 2 will be completely burnt.

Note how solving this problem is made easier by listing out what you've learned and what "rules" you've discovered.

▸ Worst Case Shifting

Many brainteasers are worst-case minimization problems, worded either in terms of *minimizing* an action or in doing something at most a specific number of times. A useful technique is to try to "balance" the worst case. That is, if an early decision results in a skewing of the worst case, we can sometimes change the decision to balance out the worst case. This will be clearest when explained with an example.

The "nine balls" question is a classic interview question. You have nine balls. Eight are of the same weight, and one is heavier. You are given a balance which tells you only whether the left side or the right side is heavier. Find the heavy ball in just two uses of the scale.

A first approach is to divide the balls in sets of four, with the ninth ball sitting off to the side. The heavy ball is in the heavier set. If they are the same weight, then we know that the ninth ball is the heavy one. Replicating this approach for the remaining sets would result in a worst case of three weighings—one too many!

This is an imbalance in the worst case: the ninth ball takes just one weighing to discover if it's heavy, whereas others take three. If we *penalize* the ninth ball by putting more balls off to the side, we can lighten the load on the others. This is an example of "worst case balancing."

If we divide the balls into sets of three items each, we will know after just one weighing which set has the heavy one. We can even formalize this into a *rule*: given N balls, where N is divisible by 3, one use of the scale will point us to a set of $^{N}/_{3}$ balls with the heavy ball.

For the final set of three balls, we simply repeat this: put one ball off to the side and weigh two. Pick the heavier of the two. Or, if the balls are the same weight, pick the third one.

▸ Algorithm Approaches

If you're stuck, consider applying one of the approaches for solving algorithm questions (starting on page 67). Brainteasers are often nothing more than algorithm questions with the technical aspects removed. Base Case and Build and Do It Yourself (DIY) can be especially useful.

Additional Reading: Useful Math (pg 629).

Interview Questions

6.1 **The Heavy Pill:** You have 20 bottles of pills. 19 bottles have 1.0 gram pills, but one has pills of weight 1.1 grams. Given a scale that provides an exact measurement, how would you find the heavy bottle? You can only use the scale once.

Hints: #186, #252, #319, #387

pg 289

6.2 **Basketball:** You have a basketball hoop and someone says that you can play one of two games.

Game 1: You get one shot to make the hoop.

Game 2: You get three shots and you have to make two of three shots.

If p is the probability of making a particular shot, for which values of p should you pick one game or the other?

Hints: #181, #239, #284, #323

pg 290

6.3 **Dominos:** There is an 8x8 chessboard in which two diagonally opposite corners have been cut off. You are given 31 dominos, and a single domino can cover exactly two squares. Can you use the 31 dominos to cover the entire board? Prove your answer (by providing an example or showing why it's impossible).

Hints: #367, #397

pg 291

6.4 **Ants on a Triangle:** There are three ants on different vertices of a triangle. What is the probability of collision (between any two or all of them) if they start walking on the sides of the triangle? Assume that each ant randomly picks a direction, with either direction being equally likely to be chosen, and that they walk at the same speed.

Similarly, find the probability of collision with n ants on an n-vertex polygon.

Hints: #157, #195, #296

pg 291

6.5 **Jugs of Water:** You have a five-quart jug, a three-quart jug, and an unlimited supply of water (but no measuring cups). How would you come up with exactly four quarts of water? Note that the jugs are oddly shaped, such that filling up exactly "half" of the jug would be impossible.

Hints: #149, #379, #400

pg 292

6.6 **Blue-Eyed Island:** A bunch of people are living on an island, when a visitor comes with a strange order: all blue-eyed people must leave the island as soon as possible. There will be a flight out at 8:00 pm every evening. Each person can see everyone else's eye color, but they do not know their own (nor is anyone allowed to tell them). Additionally, they do not know how many people have blue eyes, although they do know that at least one person does. How many days will it take the blue-eyed people to leave?

Hints: #218, #282, #341, #370

pg 293

6.7 **The Apocalypse**: In the new post-apocalyptic world, the world queen is desperately concerned about the birth rate. Therefore, she decrees that all families should ensure that they have one girl or else they face massive fines. If all families abide by this policy—that is, they have continue to have children until they have one girl, at which point they immediately stop—what will the gender ratio of the new generation be? (Assume that the odds of someone having a boy or a girl on any given pregnancy is equal.) Solve this out logically and then write a computer simulation of it.

Hints: #154, #160, #171, #188, #201

pg 293

6.8 **The Egg Drop Problem:** There is a building of 100 floors. If an egg drops from the Nth floor or above, it will break. If it's dropped from any floor below, it will not break. You're given two eggs. Find N, while minimizing the number of drops for the worst case.

Hints: #156, #233, #294, #333, #357, #374, #395

pg 296

6.9 **100 Lockers:** There are 100 closed lockers in a hallway. A man begins by opening all 100 lockers. Next, he closes every second locker. Then, on his third pass, he toggles every third locker (closes it if it is open or opens it if it is closed). This process continues for 100 passes, such that on each pass `i`, the man toggles every `i`th locker. After his 100th pass in the hallway, in which he toggles only locker #100, how many lockers are open?

Hints: #139, #172, #264, #306

pg 297

6.10 **Poison**: You have 1000 bottles of soda, and exactly one is poisoned. You have 10 test strips which can be used to detect poison. A single drop of poison will turn the test strip positive permanently. You can put any number of drops on a test strip at once and you can reuse a test strip as many times as you'd like (as long as the results are negative). However, you can only run tests once per day and it takes seven days to return a result. How would you figure out the poisoned bottle in as few days as possible?

FOLLOW UP

Write code to simulate your approach.

Hints: #146, #163, #183, #191, #205, #221, #230, #241, #249

pg 298

Additional Problems: Moderate Problems (#16.5), Hard Problems (#17.19)

Hints start on page 662.

7

Object-Oriented Design

Object-oriented design questions require a candidate to sketch out the classes and methods to implement technical problems or real-life objects. These problems give—or at least are believed to give—an interviewer insight into your coding style.

These questions are not so much about regurgitating design patterns as they are about demonstrating that you understand how to create elegant, maintainable object-oriented code. Poor performance on this type of question may raise serious red flags.

▸ How to Approach

Regardless of whether the object is a physical item or a technical task, object-oriented design questions can be tackled in similar ways. The following approach will work well for many problems.

Step 1: Handle Ambiguity

Object-oriented design (OOD) questions are often intentionally vague in order to test whether you'll make assumptions or if you'll ask clarifying questions. After all, a developer who just codes something without understanding what she is expected to create wastes the company's time and money, and may create much more serious issues.

When being asked an object-oriented design question, you should inquire *who* is going to use it and *how* they are going to use it. Depending on the question, you may even want to go through the "six Ws": who, what, where, when, how, why.

For example, suppose you were asked to describe the object-oriented design for a coffee maker. This seems straightforward enough, right? Not quite.

Your coffee maker might be an industrial machine designed to be used in a massive restaurant servicing hundreds of customers per hour and making ten different kinds of coffee products. Or it might be a very simple machine, designed to be used by the elderly for just simple black coffee. These use cases will significantly impact your design.

Step 2: Define the Core Objects

Now that we understand what we're designing, we should consider what the "core objects" in a system are. For example, suppose we are asked to do the object-oriented design for a restaurant. Our core objects might be things like `Table`, `Guest`, `Party`, `Order`, `Meal`, `Employee`, `Server`, and `Host`.

Step 3: Analyze Relationships

Having more or less decided on our core objects, we now want to analyze the relationships between the objects. Which objects are members of which other objects? Do any objects inherit from any others? Are relationships many-to-many or one-to-many?

For example, in the restaurant question, we may come up with the following design:

- `Party` should have an array of `Guests`.
- `Server` and `Host` inherit from `Employee`.
- Each `Table` has one `Party`, but each `Party` may have multiple `Tables`.
- There is one `Host` for the `Restaurant`.

Be very careful here—you can often make incorrect assumptions. For example, a single `Table` may have multiple `Parties` (as is common in the trendy "communal tables" at some restaurants). You should talk to your interviewer about how general purpose your design should be.

Step 4: Investigate Actions

At this point, you should have the basic outline of your object-oriented design. What remains is to consider the key actions that the objects will take and how they relate to each other. You may find that you have forgotten some objects, and you will need to update your design.

For example, a `Party` walks into the `Restaurant`, and a `Guest` requests a `Table` from the `Host`. The `Host` looks up the `Reservation` and, if it exists, assigns the `Party` to a `Table`. Otherwise, the `Party` is added to the end of the list. When a `Party` leaves, the `Table` is freed and assigned to a new `Party` in the list.

▸ Design Patterns

Because interviewers are trying to test your capabilities and not your knowledge, design patterns are mostly beyond the scope of an interview. However, the Singleton and Factory Method design patterns are widely used in interviews, so we will cover them here.

There are far more design patterns than this book could possibly discuss. A great way to improve your software engineering skills is to pick up a book that focuses on this area specifically.

Be careful you don't fall into a trap of constantly trying to find the "right" design pattern for a particular problem. You should create the design that works for that problem. In some cases it might be an established pattern, but in many other cases it is not.

Singleton Class

The Singleton pattern ensures that a class has only one instance and ensures access to the instance through the application. It can be useful in cases where you have a "global" object with exactly one instance. For example, we may want to implement `Restaurant` such that it has exactly one instance of `Restaurant`.

```
public class Restaurant {
  private static Restaurant _instance = null;
  protected Restaurant() { ... }
  public static Restaurant getInstance() {
    if (_instance == null) {
      _instance = new Restaurant();
    }
```

```
        return _instance;
    }
}
```

It should be noted that many people dislike the Singleton design pattern, even calling it an "anti-pattern." One reason for this is that it can interfere with unit testing.

Factory Method

The Factory Method offers an interface for creating an instance of a class, with its subclasses deciding which class to instantiate. You might want to implement this with the creator class being abstract and not providing an implementation for the Factory method. Or, you could have the Creator class be a concrete class that provides an implementation for the Factory method. In this case, the Factory method would take a parameter representing which class to instantiate.

```
public class CardGame {
  public static CardGame createCardGame(GameType type) {
    if (type == GameType.Poker) {
      return new PokerGame();
    } else if (type == GameType.BlackJack) {
      return new BlackJackGame();
    }
    return null;
  }
}
```

Interview Questions

7.1 **Deck of Cards:** Design the data structures for a generic deck of cards. Explain how you would subclass the data structures to implement blackjack.

Hints: #153, #275

pg 305

7.2 **Call Center:** Imagine you have a call center with three levels of employees: respondent, manager, and director. An incoming telephone call must be first allocated to a respondent who is free. If the respondent can't handle the call, he or she must escalate the call to a manager. If the manager is not free or not able to handle it, then the call should be escalated to a director. Design the classes and data structures for this problem. Implement a method `dispatchCall()` which assigns a call to the first available employee.

Hints: #363

pg 307

7.3 **Jukebox:** Design a musical jukebox using object-oriented principles.

Hints: #198

pg 310

7.4 **Parking Lot:** Design a parking lot using object-oriented principles.

Hints: #258

pg 312

7.5 **Online Book Reader:** Design the data structures for an online book reader system.

Hints: #344

pg 318

7.6 **Jigsaw:** Implement an NxN jigsaw puzzle. Design the data structures and explain an algorithm to solve the puzzle. You can assume that you have a `fitsWith` method which, when passed two puzzle edges, returns true if the two edges belong together.

Hints: #192, #238, #283

pg 318

7.7 **Chat Server:** Explain how you would design a chat server. In particular, provide details about the various backend components, classes, and methods. What would be the hardest problems to solve?

Hints: #213, #245, #271

pg 326

7.8 **Othello:** Othello is played as follows: Each Othello piece is white on one side and black on the other. When a piece is surrounded by its opponents on both the left and right sides, or both the top and bottom, it is said to be captured and its color is flipped. On your turn, you must capture at least one of your opponent's pieces. The game ends when either user has no more valid moves. The win is assigned to the person with the most pieces. Implement the object-oriented design for Othello.

Hints: #179, #228

pg 326

7.9 **Circular Array:** Implement a `CircularArray` class that supports an array-like data structure which can be efficiently rotated. If possible, the class should use a generic type (also called a template), and should support iteration via the standard `for (Obj o : circularArray)` notation.

Hints: #389

pg 329

7.10 **Minesweeper:** Design and implement a text-based Minesweeper game. Minesweeper is the classic single-player computer game where an NxN grid has B mines (or bombs) hidden across the grid. The remaining cells are either blank or have a number behind them. The numbers reflect the number of bombs in the surrounding eight cells. The user then uncovers a cell. If it is a bomb, the player loses. If it is a number, the number is exposed. If it is a blank cell, this cell and all adjacent blank cells (up to and including the surrounding numeric cells) are exposed. The player wins when all non-bomb cells are exposed. The player can also flag certain places as potential bombs. This doesn't affect game play, other than to block the user from accidentally clicking a cell that is thought to have a bomb. (Tip for the reader: if you're not familiar with this game, please play a few rounds online first.)

This is a fully exposed board with 3 bombs. This is not shown to the user.

	1	1	1			
	1	*	1			
	2	2	2			
	1	*	1			
	1	1	1			
			1	1	1	
			1	*	1	

The player initially sees a board with nothing exposed.

?	?	?	?	?	?	?
?	?	?	?	?	?	?
?	?	?	?	?	?	?
?	?	?	?	?	?	?
?	?	?	?	?	?	?
?	?	?	?	?	?	?
?	?	?	?	?	?	?

Clicking on cell (row = 1, col = 0) would expose this:

	1	?	?	?	?	?
	1	?	?	?	?	?
	2	?	?	?	?	?
	1	?	?	?	?	?
	1	1	1	?	?	?
			1	?	?	?
			1	?	?	?

The user wins when everything other than bombs has been exposed.

	1	1	1			
	1	?	1			
	2	2	2			
	1	?	1			
	1	1	1			
			1	1	1	
			1	?	1	

Hints: #351, #361, #377, #386, #399

pg 332

7.11 **File System:** Explain the data structures and algorithms that you would use to design an in-memory file system. Illustrate with an example in code where possible.

Hints: #141, #216

pg 337

7.12 **Hash Table:** Design and implement a hash table which uses chaining (linked lists) to handle collisions.

Hints: #287, #307

pg 339

Additional Questions: Threads and Locks (#16.3)

Hints start on page 662.

8

Recursion and Dynamic Programming

While there are a large number of recursive problems, many follow similar patterns. A good hint that a problem is recursive is that it can be built off of subproblems.

When you hear a problem beginning with the following statements, it's often (though not always) a good candidate for recursion: "Design an algorithm to compute the nth ...", "Write code to list the first n...", "Implement a method to compute all...", and so on.

> Tip: In my experience coaching candidates, people typically have about 50% accuracy in their "this sounds like a recursive problem" instinct. Use that instinct, since that 50% is valuable. But don't be afraid to look at the problem in a different way, even if you initially thought it seemed recursive. There's also a 50% chance that you were wrong.

Practice makes perfect! The more problems you do, the easier it will be to recognize recursive problems.

▸ How to Approach

Recursive solutions, by definition, are built off of solutions to subproblems. Many times, this will mean simply to compute `f(n)` by adding something, removing something, or otherwise changing the solution for `f(n-1)`. In other cases, you might solve the problem for the first half of the data set, then the second half, and then merge those results.

There are many ways you might divide a problem into subproblems. Three of the most common approaches to develop an algorithm are bottom-up, top-down, and half-and-half.

Bottom-Up Approach

The bottom-up approach is often the most intuitive. We start with knowing how to solve the problem for a simple case, like a list with only one element. Then we figure out how to solve the problem for two elements, then for three elements, and so on. The key here is to think about how you can *build* the solution for one case off of the previous case (or multiple previous cases).

Top-Down Approach

The top-down approach can be more complex since it's less concrete. But sometimes, it's the best way to think about the problem.

In these problems, we think about how we can divide the problem for case N into subproblems.

Be careful of overlap between the cases.

Half-and-Half Approach

In addition to top-down and bottom-up approaches, it's often effective to divide the data set in half.

For example, binary search works with a "half-and-half" approach. When we look for an element in a sorted array, we first figure out which half of the array contains the value. Then we recurse and search for it in that half.

Merge sort is also a "half-and-half" approach. We sort each half of the array and then merge together the sorted halves.

▸ Recursive vs. Iterative Solutions

Recursive algorithms can be very space inefficient. Each recursive call adds a new layer to the stack, which means that if your algorithm recurses to a depth of n, it uses at least O(n) memory.

For this reason, it's often better to implement a recursive algorithm iteratively. *All* recursive algorithms can be implemented iteratively, although sometimes the code to do so is much more complex. Before diving into recursive code, ask yourself how hard it would be to implement it iteratively, and discuss the tradeoffs with your interviewer.

▸ Dynamic Programming & Memoization

Although people make a big deal about how scary dynamic programming problems are, there's really no need to be afraid of them. In fact, once you get the hang of them, these can actually be very easy problems.

Dynamic programming is mostly just a matter of taking a recursive algorithm and finding the overlapping subproblems (that is, the repeated calls). You then cache those results for future recursive calls.

Alternatively, you can study the pattern of the recursive calls and implement something iterative. You still "cache" previous work.

> A note on terminology: Some people call top-down dynamic programming "memoization" and only use "dynamic programming" to refer to bottom-up work. We do not make such a distinction here. We call both dynamic programming.

One of the simplest examples of dynamic programming is computing the nth Fibonacci number. A good way to approach such a problem is often to implement it as a normal recursive solution, and then add the caching part.

Fibonacci Numbers

Let's walk through an approach to compute the nth Fibonacci number.

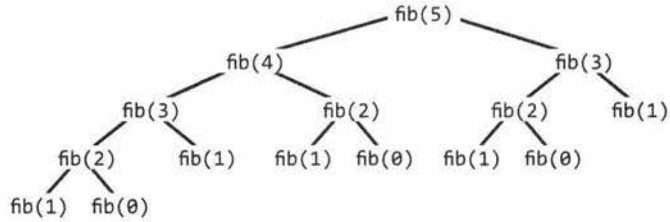
Recursive

We will start with a recursive implementation. Sounds simple, right?

```
int fibonacci(int i) {
  if (i == 0) return 0;
  if (i == 1) return 1;
  return fibonacci(i - 1) + fibonacci(i - 2);
}
```

What is the runtime of this function? Think for a second before you answer.

If you said $O(n)$ or $O(n^2)$ (as many people do), think again. Study the code path that the code takes. Drawing the code paths as a tree (that is, the recursion tree) is useful on this and many recursive problems.



Observe that the leaves on the tree are all `fib(1)` and `fib(0)`. Those signify the base cases.

The total number of nodes in the tree will represent the runtime, since each call only does $O(1)$ work outside of its recursive calls. Therefore, the number of calls is the runtime.

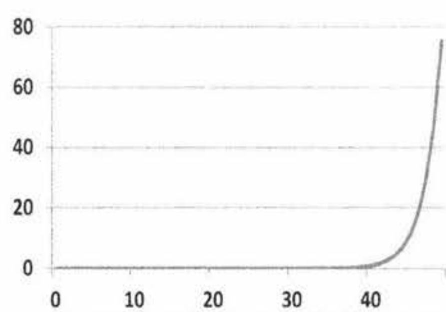
> Tip: Remember this for future problems. Drawing the recursive calls as a tree is a great way to figure out the runtime of a recursive algorithm.

How many nodes are in the tree? Until we get down to the base cases (leaves), each node has two children. Each node branches out twice.

The root node has two children. Each of those children has two children (so four children total in the "grand-children" level). Each of those grandchildren has two children, and so on. If we do this n times, we'll have roughly $O(2^n)$ nodes. This gives us a runtime of roughly $O(2^n)$.

> Actually, it's slightly better than $O(2^n)$. If you look at the subtree, you might notice that (excluding the leaf nodes and those immediately above it) the right subtree of any node is always smaller than the left subtree. If they were the same size, we'd have an $O(2^n)$ runtime. But since the right and left subtrees are not the same size, the true runtime is closer to $O(1.6^n)$. Saying $O(2^n)$ is still technically correct though as it describes an upper bound on the runtime (see "Big O, Big Theta, and Big Omega" on page 39). Either way, we still have an exponential runtime.

Indeed, if we implemented this on a computer, we'd see the number of seconds increase exponentially.



Seconds to Generate Nth Fibonacci

We should look for a way to optimize this.

Top-Down Dynamic Programming (or Memoization)

Study the recursion tree. Where do you see identical nodes?

There are lots of identical nodes. For example, `fib(3)` appears twice and fib(2) appears three times. Why should we recompute these from scratch each time?

In fact, when we call `fib(n)`, we shouldn't have to do much more than `O(n)` calls, since there's only `O(n)` possible values we can throw at `fib`. Each time we compute `fib(i)`, we should just cache this result and use it later.

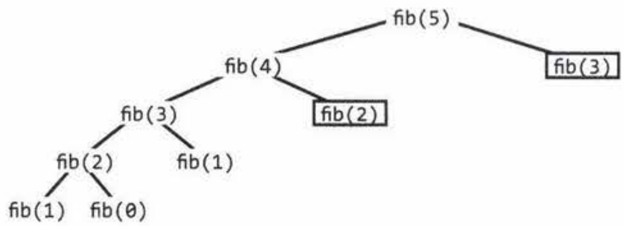
This is exactly what memoization is.

With just a small modification, we can tweak this function to run in `O(n)` time. We simply cache the results of `fibonacci(i)` between calls.

```
int fibonacci(int n) {
  return fibonacci(n, new int[n + 1]);
}

int fibonacci(int i, int[] memo) {
  if (i == 0 || i == 1) return i;

  if (memo[i] == 0) {
    memo[i] = fibonacci(i - 1, memo) + fibonacci(i - 2, memo);
  }
  return memo[i];
}
```

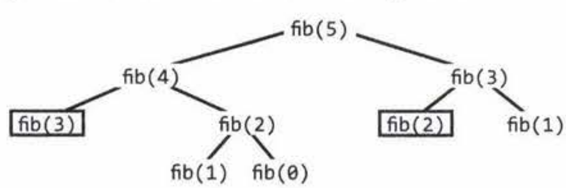
While the first recursive function may take over a minute to generate the 50th Fibonacci number on a typical computer, the dynamic programming method can generate the 10,000th Fibonacci number in just fractions of a millisecond. (Of course, with this exact code, the `int` would have overflowed very early on.)

Now, if we draw the recursion tree, it looks something like this (the black boxes represent cached calls that returned immediately):



How many nodes are in this tree now? We might notice that the tree now just shoots straight down, to a depth of roughly n. Each node of those nodes has one other child, resulting in roughly 2n children in the tree. This gives us a runtime of `O(n)`.

Often it can be useful to picture the recursion tree as something like this:



This is *not* actually how the recursion occurred. However, by expanding the further up nodes rather than the lower nodes, you have a tree that grows wide before it grows deep. (It's like doing this breadth-first rather than depth-first.) Sometimes this makes it easier to compute the number of nodes in the tree. All you're really doing is changing which nodes you expand and which ones return cached values. Try this if you're stuck on computing the runtime of a dynamic programming problem.

Bottom-Up Dynamic Programming

We can also take this approach and implement it with bottom-up dynamic programming. Think about doing the same things as the recursive memoized approach, but in reverse.

First, we compute `fib(1)` and `fib(0)`, which are already known from the base cases. Then we use those to compute `fib(2)`. Then we use the prior answers to compute `fib(3)`, then `fib(4)`, and so on.

```
int fibonacci(int n) {
   if (n == 0) return 0;
   else if (n == 1) return 1;

   int[] memo = new int[n];
   memo[0] = 0;
   memo[1] = 1;
   for (int i = 2; i < n; i++) {
      memo[i] = memo[i - 1] + memo[i - 2];
   }
   return memo[n - 1] + memo[n - 2];
}
```

If you really think about how this works, you only use `memo[i]` for `memo[i+1]` and `memo[i+2]`. You don't need it after that. Therefore, we can get rid of the memo table and just store a few variables.

```
int fibonacci(int n) {
   if (n == 0) return 0;
   int a = 0;
   int b = 1;
   for (int i = 2; i < n; i++) {
      int c = a + b;
      a = b;
      b = c;
   }
   return a + b;
}
```

This is basically storing the results from the last two Fibonacci values into a and b. At each iteration, we compute the next value (`c = a + b`) and then move (`b, c = a + b`) into (`a, b`).

This explanation might seem like overkill for such a simple problem, but truly understanding this process will make more difficult problems much easier. Going through the problems in this chapter, many of which use dynamic programming, will help solidify your understanding.

Additional Reading: Proof by Induction (pg 631).

Interview Questions

8.1 **Triple Step:** A child is running up a staircase with n steps and can hop either 1 step, 2 steps, or 3 steps at a time. Implement a method to count how many possible ways the child can run up the stairs.

Hints: #152, #178, #217, #237, #262, #359

pg 342

8.2 **Robot in a Grid:** Imagine a robot sitting on the upper left corner of grid with `r` rows and `c` columns. The robot can only move in two directions, right and down, but certain cells are "off limits" such that the robot cannot step on them. Design an algorithm to find a path for the robot from the top left to the bottom right.

Hints: #331, #360, #388

pg 344

8.3 **Magic Index:** A magic index in an array `A[0...n-1]` is defined to be an index such that `A[i] = i`. Given a sorted array of distinct integers, write a method to find a magic index, if one exists, in array A.

FOLLOW UP

What if the values are not distinct?

Hints: #170, #204, #240, #286, #340

pg 346

8.4 **Power Set:** Write a method to return all subsets of a set.

Hints: #273, #290, #338, #354, #373

pg 348

8.5 **Recursive Multiply:** Write a recursive function to multiply two positive integers without using the * operator. You can use addition, subtraction, and bit shifting, but you should minimize the number of those operations.

Hints: #166, #203, #227, #234, #246, #280

pg 350

8.6 **Towers of Hanoi:** In the classic problem of the Towers of Hanoi, you have 3 towers and `N` disks of different sizes which can slide onto any tower. The puzzle starts with disks sorted in ascending order of size from top to bottom (i.e., each disk sits on top of an even larger one). You have the following constraints:

(1) Only one disk can be moved at a time.

(2) A disk is slid off the top of one tower onto another tower.

(3) A disk cannot be placed on top of a smaller disk.

Write a program to move the disks from the first tower to the last using stacks.

Hints: #144, #224, #250, #272, #318

pg 353

8.7 **Permutations without Dups:** Write a method to compute all permutations of a string of unique characters.

Hints: #150, #185, #200, #267, #278, #309, #335, #356

pg 355

8.8 **Permutations with Dups:** Write a method to compute all permutations of a string whose characters are not necessarily unique. The list of permutations should not have duplicates.

Hints: #161, #190, #222, #255

pg 357

8.9 **Parens:** Implement an algorithm to print all valid (e.g., properly opened and closed) combinations of n pairs of parentheses.

EXAMPLE

Input: 3

Output: `((()))`, `(()())`, `(())()`, `()(())`, `()()()`

Hints: #138, #174, #187, #209, #243, #265, #295

pg 359

8.10 **Paint Fill:** Implement the "paint fill" function that one might see on many image editing programs. That is, given a screen (represented by a two-dimensional array of colors), a point, and a new color, fill in the surrounding area until the color changes from the original color.

Hints: #364, #382

pg 361

8.11 **Coins:** Given an infinite number of quarters (25 cents), dimes (10 cents), nickels (5 cents), and pennies (1 cent), write code to calculate the number of ways of representing n cents.

Hints: #300, #324, #343, #380, #394

pg 362

8.12 **Eight Queens:** Write an algorithm to print all ways of arranging eight queens on an 8x8 chess board so that none of them share the same row, column, or diagonal. In this case, "diagonal" means all diagonals, not just the two that bisect the board.

Hints: #308, #350, #371

pg 364

8.13 **Stack of Boxes:** You have a stack of n boxes, with widths w_i, heights h_i, and depths d_i. The boxes cannot be rotated and can only be stacked on top of one another if each box in the stack is strictly larger than the box above it in width, height, and depth. Implement a method to compute the height of the tallest possible stack. The height of a stack is the sum of the heights of each box.

Hints: #155, #194, #214, #260, #322, #368, #378

pg 366

8.14 **Boolean Evaluation:** Given a boolean expression consisting of the symbols `0` (false), `1` (true), & (AND), | (OR), and ^ (XOR), and a desired boolean result value `result`, implement a function to count the number of ways of parenthesizing the expression such that it evaluates to `result`.

EXAMPLE

```
countEval("1^0|0|1", false) -> 2
countEval("0&0&0&1^1|0", true) -> 10
```

Hints: #148, #168, #197, #305, #327

pg 368

Additional Questions: Linked Lists (#2.2, #2.5, #2.6), Stacks and Queues (#3.3), Trees and Graphs (#4.2, #4.3, #4.4, #4.5, #4.8, #4.10, #4.11, #4.12), Math and Logic Puzzles (#6.6), Sorting and Searching (#10.5, #10.9, #10.10), C++ (#12.8), Moderate Problems (#16.11), Hard Problems (#17.4, #17.6, #17.8, #17.12, #17.13, #17.15, #17.16, #17.24, #17.25).

Hints start on page 662.

9

System Design and Scalability

Despite how intimidating they seem, scalability questions can be among the easiest questions. There are no "gotchas," no tricks, and no fancy algorithms—at least not usually. What trips up many people is that they believe there's something "magic" to these problems—some hidden bit of knowledge.

It's not like that. These questions are simply designed to see how you would perform in the real world. If you were asked by your manager to design some system, what would you do?

That's why you should approach it just like this. Tackle the problem by doing it just like you would at work. Ask questions. Engage the interviewer. Discuss the tradeoffs.

We will touch on some key concepts in this chapter, but recognize it's not really about memorizing these concepts. Yes, understanding some big components of system design can be useful, but it's much more about the process you take. There are good solutions and bad solutions. There is no perfect solution.

▸ Handling the Questions

- **Communicate**: A key goal of system design questions is to evaluate your ability to communicate. Stay engaged with the interviewer. Ask them questions. Be open about the issues of your system.
- **Go broad first**: Don't dive straight into the algorithm part or get excessively focused on one part.
- **Use the whiteboard**: Using a whiteboard helps your interviewer follow your proposed design. Get up to the whiteboard in the very beginning and use it to draw a picture of what you're proposing.
- **Acknowledge interviewer concerns**: Your interviewer will likely jump in with concerns. Don't brush them off; validate them. Acknowledge the issues your interviewer points out and make changes accordingly.
- **Be careful about assumptions**: An incorrect assumption can dramatically change the problem. For example, if your system produces analytics / statistics for a dataset, it matters whether those analytics must be totally up to date.
- **State your assumptions explicitly**: When you do make assumptions, state them. This allows your interviewer to correct you if you're mistaken, and shows that you at least know what assumptions you're making.
- **Estimate when necessary**: In many cases, you might not have the data you need. For example, if you're designing a web crawler, you might need to estimate how much space it will take to store all the URLs. You can estimate this with other data you know.
- **Drive**: As the candidate, you should stay in the driver's seat. This doesn't mean you don't talk to your interviewer; in fact, you *must* talk to your interviewer. However, you should be driving through the ques-

tion. Ask questions. Be open about tradeoffs. Continue to go deeper. Continue to make improvements.

These questions are largely about the process rather than the ultimate design.

▸ Design: Step-By-Step

If your manager asked you to design a system such as TinyURL, you probably wouldn't just say, "Okay", then lock yourself in your office to design it by yourself. You would probably have a lot more questions before you do it. This is the way you should handle it in an interview.

Step 1: Scope the Problem

You can't design a system if you don't know what you're designing. Scoping the problem is important because you want to ensure that you're building what the interviewer wants and because this might be something that interviewer is specifically evaluating.

If you're asked something such as "Design TinyURL", you'll want to understand what exactly you need to implement. Will people be able to specify their own short URLs? Or will it all be auto-generated? Will you need to keep track of any stats on the clicks? Should the URLs stay alive forever, or do they have a timeout?

These are questions that must be answered before going further.

Make a list here as well of the major features or use cases. For example, for TinyURL, it might be:

- Shortening a URL to a TinyURL.
- Analytics for a URL.
- Retrieving the URL associated with a TinyURL.
- User accounts and link management.

Step 2: Make Reasonable Assumptions

It's okay to make some assumptions (when necessary), but they should be reasonable. For example, it would not be reasonable to assume that your system only needs to process 100 users per day, or to assume that you have infinite memory available.

However, it might be reasonable to design for a max of one million new URLs per day. Making this assumption can help you calculate how much data your system might need to store.

Some assumptions might take some "product sense" (which is not a bad thing). For example, is it okay for the data to be stale by a max of ten minutes? That all depends. If it takes 10 minutes for a just-entered URL to work, that's a deal-breaking issue. People usually want these URLs to be active immediately. However, if the statistics are ten minutes out of date, that might be okay. Talk to your interviewer about these sorts of assumptions.

Step 3: Draw the Major Components

Get up out of that chair and go to the whiteboard. Draw a diagram of the major components. You might have something like a frontend server (or set of servers) that pull data from the backend's data store. You might have another set of servers that crawl the internet for some data, and another set that process analytics. Draw a picture of what this system might look like.

Walk through your system from end-to-end to provide a flow. A user enters a new URL. Then what?

It may help here to ignore major scalability challenges and just pretend that the simple, obvious approaches will be okay. You'll handle the big issues in Step 4.

Step 4: Identify the Key Issues

Once you have a basic design in mind, focus on the key issues. What will be the bottlenecks or major challenges in the system?

For example, if you were designing TinyURL, one situation you might consider is that while some URLs will be infrequently accessed, others can suddenly peak. This might happen if a URL is posted on Reddit or another popular forum. You don't necessarily want to constantly hit the database.

Your interviewer might provide some guidance here. If so, take this guidance and use it.

Step 5: Redesign for the Key Issues

Once you have identified the key issues, it's time to adjust your design for it. You might find that it involves a major redesign or just some minor tweaking (like using a cache).

Stay up at the whiteboard here and update your diagram as your design changes.

Be open about any limitations in your design. Your interviewer will likely be aware of them, so it's important to communicate that you're aware of them, too.

▸ Algorithms that Scale: Step-By-Step

In some cases, you're not being asked to design an entire system. You're just being asked to design a single feature or algorithm, but you have to do it in a scalable way. Or, there might be one algorithm part that is the "real" focus of a broader design question.

In these cases, try the following approach.

Step 1: Ask Questions

As in the earlier approach, ask questions to make sure you really understand the question. There might be details the interviewer left out (intentionally or unintentionally). You can't solve a problem if you don't understand exactly what the problem is.

Step 2: Make Believe

Pretend that the data can all fit on one machine and there are no memory limitations. How would you solve the problem? The answer to this question will provide the general outline for your solution.

Step 3: Get Real

Now go back to the original problem. How much data can you fit on one machine, and what problems will occur when you split up the data? Common problems include figuring out how to logically divide the data up, and how one machine would identify where to look up a different piece of data.

Step 4: Solve Problems

Finally, think about how to solve the issues you identified in Step 2. Remember that the solution for each issue might be to actually remove the issue entirely, or it might be to simply mitigate the issue. Usually, you can continue using (with modifications) the approach you outlined in Step 1, but occasionally you will need to fundamentally alter the approach.

Note that an iterative approach is typically useful. That is, once you have solved the problems from Step 3, new problems may have emerged, and you must tackle those as well.

Your goal is not to re-architect a complex system that companies have spent millions of dollars building, but rather to demonstrate that you can analyze and solve problems. Poking holes in your own solution is a fantastic way to demonstrate this.

▸ Key Concepts

While system design questions aren't really tests of what you know, certain concepts can make things a lot easier. We will give a brief overview here. All of these are deep, complex topics, so we encourage you to use online resources for more research.

Horizontal vs. Vertical Scaling

A system can be scaled one of two ways.

- Vertical scaling means increasing the resources of a specific node. For example, you might add additional memory to a server to improve its ability to handle load changes.
- Horizontal scaling means increasing the number of nodes. For example, you might add additional servers, thus decreasing the load on any one server.

Vertical scaling is generally easier than horizontal scaling, but it's limited. You can only add so much memory or disk space.

Load Balancer

Typically, some frontend parts of a scalable website will be thrown behind a load balancer. This allows a system to distribute the load evenly so that one server doesn't crash and take down the whole system. To do so, of course, you have to build out a network of cloned servers that all have essentially the same code and access to the same data.

Database Denormalization and NoSQL

Joins in a relational database such as SQL can get very slow as the system grows bigger. For this reason, you would generally avoid them.

Denormalization is one part of this. Denormalization means adding redundant information into a database to speed up reads. For example, imagine a database describing projects and tasks (where a project can have multiple tasks). You might need to get the project name and the task information. Rather than doing a join across these tables, you can store the project name within the task table (in addition to the project table).

Or, you can go with a NoSQL database. A NoSQL database does not support joins and might structure data in a different way. It is designed to scale better.

Database Partitioning (Sharding)

Sharding means splitting the data across multiple machines while ensuring you have a way of figuring out which data is on which machine.

A few common ways of partitioning include:

- **Vertical Partitioning:** This is basically partitioning by feature. For example, if you were building a social network, you might have one partition for tables relating to profiles, another one for messages, and so on. One drawback of this is that if one of these tables gets very large, you might need to repartition that database (possibly using a different partitioning scheme).
- **Key-Based (or Hash-Based) Partitioning:** This uses some part of the data (for example an ID) to partition it. A very simple way to do this is to allocate N servers and put the data on `mod(key, n)`. One issue with this is that the number of servers you have is effectively fixed. Adding additional servers means reallocating all the data—a very expensive task.
- **Directory-Based Partitioning:** In this scheme, you maintain a lookup table for where the data can be found. This makes it relatively easy to add additional servers, but it comes with two major drawbacks. First, the lookup table can be a single point of failure. Second, constantly accessing this table impacts performance.

Many architectures actually end up using multiple partitioning schemes.

Caching

An in-memory cache can deliver very rapid results. It is a simple key-value pairing and typically sits between your application layer and your data store.

When an application requests a piece of information, it first tries the cache. If the cache does not contain the key, it will then look up the data in the data store. (At this point, the data might—or might not—be stored in the data store.)

When you cache, you might cache a query and its results directly. Or, alternatively, you can cache the specific object (for example, a rendered version of a part of the website, or a list of the most recent blog posts).

Asynchronous Processing & Queues

Slow operations should ideally be done asynchronously. Otherwise, a user might get stuck waiting and waiting for a process to complete.

In some cases, we can do this in advance (i.e., we can pre-process). For example, we might have a queue of jobs to be done that update some part of the website. If we were running a forum, one of these jobs might be to re-render a page that lists the most popular posts and the number of comments. That list might end up being slightly out of date, but that's perhaps okay. It's better than a user stuck waiting on the website to load simply because someone added a new comment and invalidated the cached version of this page.

In other cases, we might tell the user to wait and notify them when the process is done. You've probably seen this on websites before. Perhaps you enabled some new part of a website and it says it needs a few minutes to import your data, but you'll get a notification when it's done.

Networking Metrics

Some of the most important metrics around networking include:

- **Bandwidth:** This is the maximum amount of data that can be transferred in a unit of time. It is typically expressed in bits per second (or some similar ways, such as gigabytes per second).
- **Throughput:** Whereas bandwidth is the maximum data that can be transferred in a unit of time, throughput is the actual amount of data that is transferred.
- **Latency:** This is how long it takes data to go from one end to the other. That is, it is the delay between the sender sending information (even a very small chunk of data) and the receiver receiving it.

Imagine you have a conveyor belt that transfers items across a factory. Latency is the time it takes an item to go from one side to another. Throughput is the number of items that roll off the conveyor belt per second.

- Building a fatter conveyor belt will not change latency. It will, however, change throughput and bandwidth. You can get more items on the belt, thus transferring more in a given unit of time.
- Shortening the belt will decrease latency, since items spend less time in transit. It won't change the throughput or bandwidth. The same number of items will roll off the belt per unit of time.
- Making a faster conveyor belt will change all three. The time it takes an item to travel across the factory decreases. More items will also roll off the conveyor belt per unit of time.
- Bandwidth is the number of items that can be transferred per unit of time, in the best possible conditions. Throughput is the time it really takes, when the machines perhaps aren't operating smoothly.

Latency can be easy to disregard, but it can be very important in particular situations. For example, if you're playing certain online games, latency can be a very big deal. How can you play a typical online sports game (like a two-player football game) if you aren't notified very quickly of your opponent's movement? Additionally, unlike throughput where at least you have the option of speeding things up through data compression, there is often little you can do about latency.

MapReduce

MapReduce is often associated with Google, but it's used much more broadly than that. A MapReduce program is typically used to process large amounts of data.

As its name suggests, a MapReduce program requires you to write a Map step and a Reduce step. The rest is handled by the system.

- `Map` takes in some data and emits a `<key, value>` pair.
- `Reduce` takes a key and a set of associated values and "reduces" them in some way, emitting a new key and value. The results of this might be fed back into the `Reduce` program for more reducing.

MapReduce allows us to do a lot of processing in parallel, which makes processing huge amounts of data more scalable.

For more information, see "MapReduce" on page 642.

▸ Considerations

In addition to the earlier concepts to learn, you should consider the following issues when designing a system.

- **Failures**: Essentially any part of a system can fail. You'll need to plan for many or all of these failures.
- **Availability and Reliability**: Availability is a function of the percentage of time the system is operational. Reliability is a function of the probability that the system is operational for a certain unit of time.
- **Read-heavy vs. Write-heavy**: Whether an application will do a lot of reads or a lot of writes impacts the design. If it's write-heavy, you could consider queuing up the writes (but think about potential failure here!). If it's read-heavy, you might want to cache. Other design decisions could change as well.
- **Security**: Security threats can, of course, be devastating for a system. Think about the types of issues a system might face and design around those.

This is just to get you started with the potential issues for a system. Remember to be open in your interview about the tradeoffs.

▸ There is no "perfect" system.

There is no single design for TinyURL or Google Maps or any other system that works perfectly (although there are a great number that would work terribly). There are always tradeoffs. Two people could have substantially different designs for a system, with both being excellent given different assumptions.

Your goal in these problems is to be able to understand use cases, scope a problem, make reasonable assumptions, create a solid design based on those assumptions, and be open about the weaknesses of your design. Do not expect something perfect.

▸ Example Problem

Given a list of millions of documents, how would you find all documents that contain a list of words? The words can appear in any order, but they must be complete words. That is, "book" does not match "bookkeeper."

Before we start solving the problem, we need to understand whether this is a one time only operation, or if this `findWords` procedure will be called repeatedly. Let's assume that we will be calling `findWords` many times for the same set of documents, and, therefore, we can accept the burden of pre-processing.

Step 1

The first step is to pretend we just have a few dozen documents. How would we implement `findWords` in this case? (Tip: stop here and try to solve this yourself before reading on.)

One way to do this is to pre-process each document and create a hash table index. This hash table would map from a word to a list of the documents that contain that word.

```
"books" -> {doc2, doc3, doc6, doc8}
"many"  -> {doc1, doc3, doc7, doc8, doc9}
```

To search for "`many books`," we would simply do an intersection on the values for "`books`" and "`many`", and return `{doc3, doc8}` as the result.

Step 2

Now go back to the original problem. What problems are introduced with millions of documents? For starters, we probably need to divide up the documents across many machines. Also, depending on a variety of factors, such as the number of possible words and the repetition of words in a document, we may not be able to fit the full hash table on one machine. Let's assume that this is the case.

This division introduces the following key concerns:

1. How will we divide up our hash table? We could divide it up by keyword, such that a given machine contains the full document list for a given word. Or, we could divide by document, such that a machine contains the keyword mapping for only a subset of the documents.
2. Once we decide how to divide up the data, we may need to process a document on one machine and push the results off to other machines. What does this process look like? (Note: if we divide the hash table by document, this step may not be necessary.)
3. We will need a way of knowing which machine holds a piece of data. What does this lookup table look like, and where is it stored?

These are just three concerns. There may be many others.

Step 3

In Step 3, we find solutions to each of these issues. One solution is to divide up the words alphabetically by keyword, such that each machine controls a range of words (e.g., "after" through "apple").

We can implement a simple algorithm in which we iterate through the keywords alphabetically, storing as much data as possible on one machine. When that machine is full, we can move to the next machine.

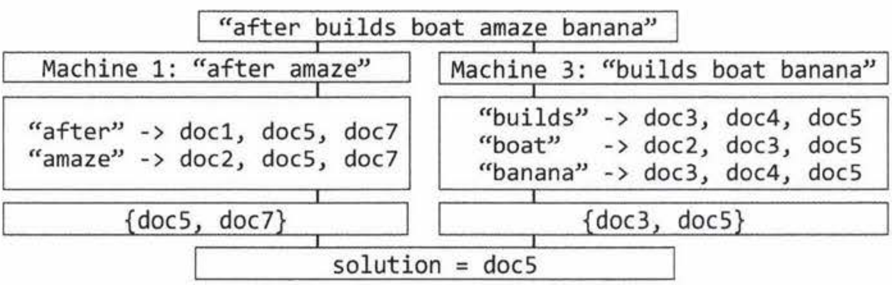
The advantage of this approach is that the lookup table is small and simple (since it must only specify a range of values), and each machine can store a copy of the lookup table. However, the disadvantage is that if new documents or words are added, we may need to perform an expensive shift of keywords.

To find all the documents that match a list of strings, we would first sort the list and then send each machine a lookup request for the strings that the machine owns. For example, if our string is `"after builds boat amaze banana"`, machine 1 would get a lookup request for `{"after", "amaze"}`.

Machine 1 looks up the documents containing "after" and "amaze," and performs an intersection on these document lists. Machine 3 does the same for `{"banana", "boat", "builds"}`, and intersects their lists.

In the final step, the initial machine would do an intersection on the results from Machine 1 and Machine 3.

The following diagram explains this process.



Interview Questions

These questions are designed to mirror a real interview, so they will not always be well defined. Think about what questions you would ask your interviewer and then make reasonable assumptions. You may make different assumptions than us, and that will lead you to a very different design. That's okay!

9.1 **Stock Data:** Imagine you are building some sort of service that will be called by up to 1,000 client applications to get simple end-of-day stock price information (open, close, high, low). You may assume that you already have the data, and you can store it in any format you wish. How would you design the client-facing service that provides the information to client applications? You are responsible for the development, rollout, and ongoing monitoring and maintenance of the feed. Describe the different methods you considered and why you would recommend your approach. Your service can use any technologies you wish, and can distribute the information to the client applications in any mechanism you choose.

Hints: #385, #396

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9.2 **Social Network:** How would you design the data structures for a very large social network like Facebook or LinkedIn? Describe how you would design an algorithm to show the shortest path between two people (e.g., Me -> Bob -> Susan -> Jason -> You).

Hints: #270, #285, #304, #321

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9.3 **Web Crawler:** If you were designing a web crawler, how would you avoid getting into infinite loops?

Hints: #334, #353, #365

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9.4 **Duplicate URLs:** You have 10 billion URLs. How do you detect the duplicate documents? In this case, assume "duplicate" means that the URLs are identical.

Hints: #326, #347

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9.5 **Cache:** Imagine a web server for a simplified search engine. This system has 100 machines to respond to search queries, which may then call out using `processSearch(string query)` to another cluster of machines to actually get the result. The machine which responds to a given query is chosen at random, so you cannot guarantee that the same machine will always respond to the same request. The method `processSearch` is very expensive. Design a caching mechanism for the most recent queries. Be sure to explain how you would update the cache when data changes.

Hints: #259, #274, #293, #311

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9.6 **Sales Rank**: A large eCommerce company wishes to list the best-selling products, overall and by category. For example, one product might be the #1056th best-selling product overall but the #13th best-selling product under "Sports Equipment" and the #24th best-selling product under "Safety." Describe how you would design this system.

Hints: #142, #158, #176, #189, #208, #223, #236, #244

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9.7 **Personal Financial Manager**: Explain how you would design a personal financial manager (like Mint.com). This system would connect to your bank accounts, analyze your spending habits, and make recommendations.

Hints: #162, #180, #199, #212, #247, #276

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9.8 **Pastebin**: Design a system like Pastebin, where a user can enter a piece of text and get a randomly generated URL to access it.

Hints: #165, #184, #206, #232

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Additional Questions: Object-Oriented Design (#7.7)

Hints start on page 662.

10

Sorting and Searching

Understanding the common sorting and searching algorithms is incredibly valuable, as many sorting and searching problems are tweaks of the well-known algorithms. A good approach is therefore to run through the different sorting algorithms and see if one applies particularly well.

For example, suppose you are asked the following question: Given a very large array of `Person` objects, sort the people in increasing order of age.

We're given two interesting bits of knowledge here:

1. It's a large array, so efficiency is very important.
2. We are sorting based on ages, so we know the values are in a small range.

By scanning through the various sorting algorithms, we might notice that bucket sort (or radix sort) would be a perfect candidate for this algorithm. In fact, we can make the buckets small (just 1 year each) and get `O(n)` running time.

▸ Common Sorting Algorithms

Learning (or re-learning) the common sorting algorithms is a great way to boost your performance. Of the five algorithms explained below, Merge Sort, Quick Sort and Bucket Sort are the most commonly used in interviews.

Bubble Sort | Runtime: $O(n^2)$ **average and worst case. Memory:** `O(1)`.

In bubble sort, we start at the beginning of the array and swap the first two elements if the first is greater than the second. Then, we go to the next pair, and so on, continuously making sweeps of the array until it is sorted. In doing so, the smaller items slowly "bubble" up to the beginning of the list.

Selection Sort | Runtime: $O(n^2)$ **average and worst case. Memory:** `O(1)`.

Selection sort is the child's algorithm: simple, but inefficient. Find the smallest element using a linear scan and move it to the front (swapping it with the front element). Then, find the second smallest and move it, again doing a linear scan. Continue doing this until all the elements are in place.

Merge Sort | Runtime: `O(n log(n))` **average and worst case. Memory:** Depends.

Merge sort divides the array in half, sorts each of those halves, and then merges them back together. Each of those halves has the same sorting algorithm applied to it. Eventually, you are merging just two single-element arrays. It is the "merge" part that does all the heavy lifting.

The merge method operates by copying all the elements from the target array segment into a helper array, keeping track of where the start of the left and right halves should be (`helperLeft` and `helperRight`). We then iterate through `helper`, copying the smaller element from each half into the array. At the end, we copy any remaining elements into the target array.

```
void mergesort(int[] array) {
  int[] helper = new int[array.length];
  mergesort(array, helper, 0, array.length - 1);
}

void mergesort(int[] array, int[] helper, int low, int high) {
  if (low < high) {
    int middle = (low + high) / 2;
    mergesort(array, helper, low, middle); // Sort left half
    mergesort(array, helper, middle+1, high); // Sort right half
    merge(array, helper, low, middle, high); // Merge them
  }
}

void merge(int[] array, int[] helper, int low, int middle, int high) {
  /* Copy both halves into a helper array */
  for (int i = low; i <= high; i++) {
    helper[i] = array[i];
  }

  int helperLeft = low;
  int helperRight = middle + 1;
  int current = low;

  /* Iterate through helper array. Compare the left and right half, copying back
   * the smaller element from the two halves into the original array. */
  while (helperLeft <= middle && helperRight <= high) {
    if (helper[helperLeft] <= helper[helperRight]) {
      array[current] = helper[helperLeft];
      helperLeft++;
    } else { // If right element is smaller than left element
      array[current] = helper[helperRight];
      helperRight++;
    }
    current++;
  }

  /* Copy the rest of the left side of the array into the target array */
  int remaining = middle - helperLeft;
  for (int i = 0; i <= remaining; i++) {
    array[current + i] = helper[helperLeft + i];
  }
}
```

You may notice that only the remaining elements from the left half of the helper array are copied into the target array. Why not the right half? The right half doesn't need to be copied because it's *already* there.

Consider, for example, an array like `[1, 4, 5 || 2, 8, 9]` (the "||" indicates the partition point). Prior to merging the two halves, both the helper array and the target array segment will end with `[8, 9]`. Once we copy over four elements (`1, 4, 5`, and 2) into the target array, the `[8, 9]` will still be in place in both arrays. There's no need to copy them over.

The space complexity of merge sort is $O(n)$ due to the auxiliary space used to merge parts of the array.

Quick Sort | Runtime: $O(n \log(n))$ **average,** $O(n^2)$ **worst case. Memory:** $O(\log(n))$.

In quick sort, we pick a random element and partition the array, such that all numbers that are less than the partitioning element come before all elements that are greater than it. The partitioning can be performed efficiently through a series of swaps (see below).

If we repeatedly partition the array (and its sub-arrays) around an element, the array will eventually become sorted. However, as the partitioned element is not guaranteed to be the median (or anywhere near the median), our sorting could be very slow. This is the reason for the $O(n^2)$ worst case runtime.

```
void quickSort(int[] arr, int left, int right) {
  int index = partition(arr, left, right);
  if (left < index - 1) { // Sort left half
    quickSort(arr, left, index - 1);
  }
  if (index < right) { // Sort right half
    quickSort(arr, index, right);
  }
}

int partition(int[] arr, int left, int right) {
  int pivot = arr[(left + right) / 2]; // Pick pivot point
  while (left <= right) {
    // Find element on left that should be on right
    while (arr[left] < pivot) left++;

    // Find element on right that should be on left
    while (arr[right] > pivot) right--;

    // Swap elements, and move left and right indices
    if (left <= right) {
      swap(arr, left, right); // swaps elements
      left++;
      right--;
    }
  }
  return left;
}
```

Radix Sort | Runtime: $O(kn)$ **(see below)**

Radix sort is a sorting algorithm for integers (and some other data types) that takes advantage of the fact that integers have a finite number of bits. In radix sort, we iterate through each digit of the number, grouping numbers by each digit. For example, if we have an array of integers, we might first sort by the first digit, so that the 0s are grouped together. Then, we sort each of these groupings by the next digit. We repeat this process sorting by each subsequent digit, until finally the whole array is sorted.

Unlike comparison sorting algorithms, which cannot perform better than $O(n \log(n))$ in the average case, radix sort has a runtime of $O(kn)$, where n is the number of elements and k is the number of passes of the sorting algorithm.

▸ Searching Algorithms

When we think of searching algorithms, we generally think of binary search. Indeed, this is a very useful algorithm to study.

In binary search, we look for an element x in a sorted array by first comparing x to the midpoint of the array. If x is less than the midpoint, then we search the left half of the array. If x is greater than the midpoint, then we search the right half of the array. We then repeat this process, treating the left and right halves as subarrays. Again, we compare x to the midpoint of this subarray and then search either its left or right side. We repeat this process until we either find x or the subarray has size 0.

Note that although the concept is fairly simple, getting all the details right is far more difficult than you might think. As you study the code below, pay attention to the plus ones and minus ones.

```
int binarySearch(int[] a, int x) {
  int low = 0;
  int high = a.length - 1;
  int mid;

  while (low <= high) {
    mid = (low + high) / 2;
    if (a[mid] < x) {
      low = mid + 1;
    } else if (a[mid] > x) {
      high = mid - 1;
    } else {
      return mid;
    }
  }
  return -1; // Error
}

int binarySearchRecursive(int[] a, int x, int low, int high) {
  if (low > high) return -1; // Error

  int mid = (low + high) / 2;
  if (a[mid] < x) {
    return binarySearchRecursive(a, x, mid + 1, high);
  } else if (a[mid] > x) {
    return binarySearchRecursive(a, x, low, mid - 1);
  } else {
    return mid;
  }
}
```

Potential ways to search a data structure extend beyond binary search, and you would do best not to limit yourself to just this option. You might, for example, search for a node by leveraging a binary tree, or by using a hash table. Think beyond binary search!

Interview Questions

10.1 Sorted Merge: You are given two sorted arrays, A and B, where A has a large enough buffer at the end to hold B. Write a method to merge B into A in sorted order.

Hints: #332

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10.2 **Group Anagrams:** Write a method to sort an array of strings so that all the anagrams are next to each other.

Hints: #177, #182, #263, #342

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10.3 **Search in Rotated Array:** Given a sorted array of `n` integers that has been rotated an unknown number of times, write code to find an element in the array. You may assume that the array was originally sorted in increasing order.

EXAMPLE

Input: find 5 in `{15, 16, 19, 20, 25, 1, 3, 4, 5, 7, 10, 14}`

Output: 8 (the index of 5 in the array)

Hints: #298, #310

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10.4 **Sorted Search, No Size:** You are given an array-like data structure `Listy` which lacks a size method. It does, however, have an `elementAt(i)` method that returns the element at index `i` in `O(1)` time. If `i` is beyond the bounds of the data structure, it returns `-1`. (For this reason, the data structure only supports positive integers.) Given a `Listy` which contains sorted, positive integers, find the index at which an element x occurs. If x occurs multiple times, you may return any index.

Hints: #320, #337, #348

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10.5 **Sparse Search:** Given a sorted array of strings that is interspersed with empty strings, write a method to find the location of a given string.

EXAMPLE

Input: `ball`, `{"at", "", "", "", "ball", "", "", "car", "", "", "dad", "", ""}`

Output: 4

Hints: #256

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10.6 **Sort Big File:** Imagine you have a 20 GB file with one string per line. Explain how you would sort the file.

Hints: #207

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10.7 **Missing Int:** Given an input file with four billion non-negative integers, provide an algorithm to generate an integer that is not contained in the file. Assume you have 1 GB of memory available for this task.

FOLLOW UP

What if you have only 10 MB of memory? Assume that all the values are distinct and we now have no more than one billion non-negative integers.

Hints: #235, #254, #281

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10.8 **Find Duplicates:** You have an array with all the numbers from 1 to N, where N is at most 32,000. The array may have duplicate entries and you do not know what N is. With only 4 kilobytes of memory available, how would you print all duplicate elements in the array?

Hints: #289, #315

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10.9 **Sorted Matrix Search:** Given an `M x N` matrix in which each row and each column is sorted in ascending order, write a method to find an element.

Hints: #193, #211, #229, #251, #266, #279, #288, #291, #303, #317, #330

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10.10 **Rank from Stream:** Imagine you are reading in a stream of integers. Periodically, you wish to be able to look up the rank of a number x (the number of values less than or equal to x). Implement the data structures and algorithms to support these operations. That is, implement the method `track(int x)`, which is called when each number is generated, and the method `getRankOfNumber(int x)`, which returns the number of values less than or equal to x (not including x itself).

EXAMPLE

Stream (in order of appearance): `5, 1, 4, 4, 5, 9, 7, 13, 3`

```
getRankOfNumber(1) = 0
getRankOfNumber(3) = 1
getRankOfNumber(4) = 3
```

Hints: #301, #376, #392

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10.11 **Peaks and Valleys**: In an array of integers, a "peak" is an element which is greater than or equal to the adjacent integers and a "valley" is an element which is less than or equal to the adjacent integers. For example, in the array {5, 8, 6, 2, 3, 4, 6}, {8, 6} are peaks and {5, 2} are valleys. Given an array of integers, sort the array into an alternating sequence of peaks and valleys.

EXAMPLE

Input: {5, 3, 1, 2, 3}

Output: {5, 1, 3, 2, 3}

Hints: #196, #219, #231, #253, #277, #292, #316

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Additional Questions: Arrays and Strings (#1.2), Recursion (#8.3), Moderate (#16.10, #16.16, #16.21, #16.24), Hard (#17.11, #17.26).

Hints start on page 662.

11

Testing

Before you flip past this chapter saying, "but I'm not a tester," stop and think. Testing is an important task for a software engineer, and for this reason, testing questions may come up during your interview. Of course, if you are applying for Testing roles (or Software Engineer in Test), then that's all the more reason why you need to pay attention.

Testing problems usually fall under one of four categories: (1) Test a real world object (like a pen); (2) Test a piece of software; (3) Write test code for a function; (4) Troubleshoot an existing issue. We'll cover approaches for each of these four types.

Remember that all four types require you to not make an assumption that the input or the user will play nice. Expect abuse and plan for it.

▸ What the Interviewer Is Looking For

At their surface, testing questions seem like they're just about coming up with an extensive list of test cases. And to some extent, that's right. You do need to come up with a reasonable list of test cases.

But in addition, interviewers want to test the following:

- *Big Picture Understanding:* Are you a person who understands what the software is really about? Can you prioritize test cases properly? For example, suppose you're asked to test an e-commerce system like Amazon. It's great to make sure that the product images appear in the right place, but it's even more important that payments work reliably, products are added to the shipment queue, and customers are never double charged.
- *Knowing How the Pieces Fit Together:* Do you understand how software works, and how it might fit into a greater ecosystem? Suppose you're asked to test Google Spreadsheets. It's important that you test opening, saving, and editing documents. But, Google Spreadsheets is part of a larger ecosystem. You need to test integration with Gmail, with plug-ins, and with other components.
- *Organization:* Do you approach the problem in a structured manner, or do you just spout off anything that comes to your head? Some candidates, when asked to come up with test cases for a camera, will just state anything and everything that comes to their head. A good candidate will break down the parts into categories like Taking Photos, Image Management, Settings, and so on. This structured approach will also help you to do a more thorough job creating the test cases.
- *Practicality:* Can you actually create reasonable testing plans? For example, if a user reports that the software crashes when they open a specific image, and you just tell them to reinstall the software, that's typically not very practical. Your testing plans need to be feasible and realistic for a company to implement.

Demonstrating these aspects will show that you will be a valuable member of the testing team.

▸ Testing a Real World Object

Some candidates are surprised to be asked questions like how to test a pen. After all, you should be testing software, right? Maybe, but these "real world" questions are still very common. Let's walk through this with an example.

Question: How would you test a paperclip?

Step 1: Who will use it? And why?

You need to discuss with your interviewer who is using the product and for what purpose. The answer may not be what you think. The answer could be "by teachers, to hold papers together," or it could be "by artists, to bend into the shape of animal." Or, it could be both. The answer to this question will shape how you handle the remaining questions.

Step 2: What are the use cases?

It will be useful for you to make a list of the use cases. In this case, the use case might be simply fastening paper together in a non-damaging (to the paper) way.

For other questions, there might be multiple use cases. It might be, for example, that the product needs to be able to send and receive content, or write and erase, and so on.

Step 3: What are the bounds of use?

The bounds of use might mean holding up to thirty sheets of paper in a single usage without permanent damage (e.g., bending), and thirty to fifty sheets with minimal permanent bending.

The bounds also extend to environmental factors as well. For example, should the paperclip work during very warm temperatures (90 - 110 degrees Fahrenheit)? What about extreme cold?

Step 4: What are the stress / failure conditions?

No product is fail-proof, so analyzing failure conditions needs to be part of your testing. A good discussion to have with your interviewer is about when it's acceptable (or even necessary) for the product to fail, and what failure should mean.

For example, if you were testing a laundry machine, you might decide that the machine should be able to handle at least 30 shirts or pants. Loading 30 - 45 pieces of clothing may result in minor failure, such as the clothing being inadequately cleaned. At more than 45 pieces of clothing, extreme failure might be acceptable. However, extreme failure in this case should probably mean the machine never turning on the water. It should certainly *not* mean a flood or a fire.

Step 5: How would you perform the testing?

In some cases, it might also be relevant to discuss the details of performing the testing. For example, if you need to make sure a chair can withstand normal usage for five years, you probably can't actually place it in a home and wait five years. Instead, you'd need to define what "normal" usage is (How many "sits" per year on the seat? What about the armrest?). Then, in addition to doing some manual testing, you would likely want a machine to automate some of the usage.

▸ Testing a Piece of Software

Testing a piece of software is actually very similar to testing a real world object. The major difference is that software testing generally places a greater emphasis on the details of performing testing.

Note that software testing has two core aspects to it:

- *Manual vs. Automated Testing:* In an ideal world, we might love to automate everything, but that's rarely feasible. Some things are simply much better with manual testing because some features are too qualitative for a computer to effectively examine (such as if content represents pornography). Additionally, whereas a computer can generally recognize only issues that it's been told to look for, human observation may reveal new issues that haven't been specifically examined. Both humans and computers form an essential part of the testing process.
- *Black Box Testing vs. White Box Testing:* This distinction refers to the degree of access we have into the software. In black box testing, we're just given the software as-is and need to test it. With white box testing, we have additional programmatic access to test individual functions. We can also automate some black box testing, although it's certainly much harder.

Let's walk through an approach from start to end.

Step 1: Are we doing Black Box Testing or White Box Testing?

Though this question can often be delayed to a later step, I like to get it out of the way early on. Check with your interviewer as to whether you're doing black box testing or white box testing—or both.

Step 2: Who will use it? And why?

Software typically has one or more target users, and the features are designed with this in mind. For example, if you're asked to test software for parental controls on a web browser, your target users include both parents (who are implementing the blocking) and children (who are the recipients of blocking). You may also have "guests" (people who should neither be implementing nor receiving blocking).

Step 3: What are the use cases?

In the software blocking scenario, the use cases of the parents include installing the software, updating controls, removing controls, and of course their own personal internet usage. For the children, the use cases include accessing legal content as well as "illegal" content.

Remember that it's not up to you to just magically decide the use cases. This is a conversation to have with your interviewer.

Step 4: What are the bounds of use?

Now that we have the vague use cases defined, we need to figure out what exactly this means. What does it mean for a website to be blocked? Should just the "illegal" page be blocked, or the entire website? Is the application supposed to "learn" what is bad content, or is it based on a white list or black list? If it's supposed to learn what inappropriate content is, what degree of false positives or false negatives is acceptable?

Step 5: What are the stress conditions / failure conditions?

When the software fails—which it inevitably will—what should the failure look like? Clearly, the software failure shouldn't crash the computer. Instead, it's likely that the software should just permit a blocked site, or ban an allowable site. In the latter case, you might want to discuss the possibility of a selective override with a password from the parents.

Step 6: What are the test cases? How would you perform the testing?

Here is where the distinctions between manual and automated testing, and between black box and white box testing, really come into play.

Steps 3 and 4 should have roughly defined the use cases. In step 6, we further define them and discuss how to perform the testing. What exact situations are you testing? Which of these steps can be automated? Which require human intervention?

Remember that while automation allows you to do some very powerful testing, it also has some significant drawbacks. Manual testing should usually be part of your test procedures.

When you go through this list, don't just rattle off every scenario you can think of. It's disorganized, and you're sure to miss major categories. Instead, approach this in a structured manner. Break down your testing into the main components, and go from there. Not only will you give a more complete list of test cases, but you'll also show that you're a structured, methodical person.

▸ Testing a Function

In many ways, testing a function is the easiest type of testing. The conversation is typically briefer and less vague, as the testing is usually limited to validating input and output.

However, don't overlook the value of some conversation with your interviewer. You should discuss any assumptions with your interviewer, particularly with respect to how to handle specific situations.

Suppose you were asked to write code to test `sort(int[] array)`, which sorts an array of integers. You might proceed as follows.

Step 1: Define the test cases

In general, you should think about the following types of test cases:

- *The normal case:* Does it generate the correct output for typical inputs? Remember to think about potential issues here. For example, because sorting often requires some sort of partitioning, it's reasonable to think that the algorithm might fail on arrays with an odd number of elements, since they can't be evenly partitioned. Your test case should list both examples.
- *The extremes:* What happens when you pass in an empty array? Or a very small (one element) array? What if you pass in a very large one?
- *Nulls and "illegal" input:* It is worthwhile to think about how the code should behave when given illegal input. For example, if you're testing a function to generate the nth Fibonacci number, your test cases should probably include the situation where n is negative.
- *Strange input:* A fourth kind of input sometimes comes up: strange input. What happens when you pass in an already sorted array? Or an array that's sorted in reverse order?

Generating these tests does require knowledge of the function you are writing. If you are unclear as to the constraints, you will need to ask your interviewer about this first.

Step 2: Define the expected result

Often, the expected result is obvious: the right output. However, in some cases, you might want to validate additional aspects. For instance, if the `sort` method returns a new sorted copy of the array, you should probably validate that the original array has not been touched.

Step 3: Write test code

Once you have the test cases and results defined, writing the code to implement the test cases should be fairly straightforward. Your code might look something like:

```
void testAddThreeSorted() {
  MyList list = new MyList();
  list.addThreeSorted(3, 1, 2); // Adds 3 items in sorted order
  assertEquals(list.getElement(0), 1);
  assertEquals(list.getElement(1), 2);
  assertEquals(list.getElement(2), 3);
}
```

▸ Troubleshooting Questions

A final type of question is explaining how you would debug or troubleshoot an existing issue. Many candidates balk at a question like this, giving unrealistic answers like "reinstall the software." You can approach these questions in a structured manner, like anything else.

Let's walk through this problem with an example: You're working on the Google Chrome team when you receive a bug report: Chrome crashes on launch. What would you do?

Reinstalling the browser might solve this user's problem, but it wouldn't help the other users who might be experiencing the same issue. Your goal is to understand what's *really* happening, so that the developers can fix it.

Step 1: Understand the Scenario

The first thing you should do is ask questions to understand as much about the situation as possible.

- How long has the user been experiencing this issue?
- What version of the browser is it? What operating system?
- Does the issue happen consistently, or how often does it happen? When does it happen?
- Is there an error report that launches?

Step 2: Break Down the Problem

Now that you understand the details of the scenario, you want to break down the problem into testable units. In this case, you can imagine the flow of the situation as follows:

1. Go to Windows Start menu.
2. Click on Chrome icon.
3. Browser instance starts.
4. Browser loads settings.
5. Browser issues HTTP request for homepage.

6. Browser gets HTTP response.
7. Browser parses webpage.
8. Browser displays content.

At some point in this process, something fails and it causes the browser to crash. A strong tester would iterate through the elements of this scenario to diagnose the problem.

Step 3: Create Specific, Manageable Tests

Each of the above components should have realistic instructions—things that you can ask the user to do, or things that you can do yourself (such as replicating steps on your own machine). In the real world, you will be dealing with customers, and you can't give them instructions that they can't or won't do.

Interview Questions

11.1 **Mistake:** Find the mistake(s) in the following code:

```
unsigned int i;
for (i = 100; i >= 0; --i)
  printf("%d\n", i);
```

Hints: #257, #299, #362

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11.2 **Random Crashes:** You are given the source to an application which crashes when it is run. After running it ten times in a debugger, you find it never crashes in the same place. The application is single threaded, and uses only the C standard library. What programming errors could be causing this crash? How would you test each one?

Hints: #325

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11.3 **Chess Test:** We have the following method used in a chess game: `boolean canMoveTo(int x, int y)`. This method is part of the `Piece` class and returns whether or not the piece can move to position `(x, y)`. Explain how you would test this method.

Hints: #329, #401

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11.4 **No Test Tools:** How would you load test a webpage without using any test tools?

Hints: #313, #345

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11.5 **Test a Pen:** How would you test a pen?

Hints: #140, #164, #220

pg 420

11.6 **Test an ATM:** How would you test an ATM in a distributed banking system?

Hints: #210, #225, #268, #349, #393

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Hints start on page 662.

12

C and C++

A good interviewer won't demand that you code in a language you don't profess to know. Hopefully, if you're asked to code in C++, it's listed on your resume. If you don't remember all the APIs, don't worry—most interviewers (though not all) don't care that much. We do recommend, however, studying up on basic C++ syntax so that you can approach these questions with ease.

▸ Classes and Inheritance

Though C++ classes have similar characteristics to those of other languages, we'll review some of the syntax below.

The code below demonstrates the implementation of a basic class with inheritance.

```
#include <iostream>
using namespace std;

#define NAME_SIZE 50 // Defines a macro

class Person {
  int id; // all members are private by default
  char name[NAME_SIZE];

 public:
  void aboutMe() {
    cout << “I am a person.”;
  }
};

class Student : public Person {
 public:
  void aboutMe() {
    cout << “I am a student.”;
  }
};

int main() {
  Student * p = new Student();
  p->aboutMe(); // prints “I am a student.”
  delete p; // Important! Make sure to delete allocated memory.
  return 0;
}
```

All data members and methods are private by default in C++. One can modify this by introducing the keyword `public`.

▸ Constructors and Destructors

The constructor of a class is automatically called upon an object's creation. If no constructor is defined, the compiler automatically generates one called the Default Constructor. Alternatively, we can define our own constructor.

If you just need to initialize primitive types, a simple way to do it is this:

```
Person(int a) {
  id = a;
}
```

This works for primitive types, but you might instead want to do this:

```
Person(int a) : id(a) {
  ...
}
```

The data member `id` is assigned before the actual object is created and before the remainder of the constructor code is called. This approach is necessary when the fields are constant or class types.

The destructor cleans up upon object deletion and is automatically called when an object is destroyed. It cannot take an argument as we don't explicitly call a destructor.

```
~Person() {
  delete obj; // free any memory allocated within class
}
```

▸ Virtual Functions

In an earlier example, we defined p to be of type `Student`:

```
Student * p = new Student();
p->aboutMe();
```

What would happen if we defined p to be a `Person*`, like so?

```
Person * p = new Student();
p->aboutMe();
```

In this case, "`I am a person`" would be printed instead. This is because the function `aboutMe` is resolved at compile-time, in a mechanism known as *static binding*.

If we want to ensure that the `Student`'s implementation of `aboutMe` is called, we can define `aboutMe` in the `Person` class to be `virtual`.

```
class Person {
  ...
  virtual void aboutMe() {
    cout << "I am a person.";
  }
};

class Student : public Person {
 public:
  void aboutMe() {
    cout << "I am a student.";
  }
```

```
};
```

Another usage for virtual functions is when we can't (or don't want to) implement a method for the parent class. Imagine, for example, that we want `Student` and `Teacher` to inherit from `Person` so that we can implement a common method such as `addCourse(string s)`. Calling `addCourse` on `Person`, however, wouldn't make much sense since the implementation depends on whether the object is actually a `Student` or `Teacher`.

In this case, we might want `addCourse` to be a virtual function defined within `Person`, with the implementation being left to the subclass.

```
class Person {
  int id; // all members are private by default
  char name[NAME_SIZE];
  public:
  virtual void aboutMe() {
    cout << “I am a person.” << endl;
  }
  virtual bool addCourse(string s) = 0;
};

class Student : public Person {
  public:
  void aboutMe() {
    cout << “I am a student.” << endl;
  }

  bool addCourse(string s) {
    cout << “Added course ” << s << “ to student.” << endl;
    return true;
  }
};

int main() {
  Person * p = new Student();
  p->aboutMe(); // prints “I am a student.”
  p->addCourse(“History”);
  delete p;
}
```

Note that by defining `addCourse` to be a "pure virtual function," `Person` is now an abstract class and we cannot instantiate it.

▸ Virtual Destructor

The virtual function naturally introduces the concept of a "virtual destructor." Suppose we wanted to implement a destructor method for `Person` and `Student`. A naive solution might look like this:

```
class Person {
 public:
  ~Person() {
    cout << “Deleting a person.” << endl;
  }
};

class Student : public Person {
 public:
```

```
  ~Student() {
     cout << "Deleting a student." << endl;
  }
};

int main() {
  Person * p = new Student();
  delete p; // prints "Deleting a person."
}
```

As in the earlier example, since p is a `Person`, the destructor for the `Person` class is called. This is problematic because the memory for `Student` may not be cleaned up.

To fix this, we simply define the destructor for `Person` to be virtual.

```
class Person {
 public:
  virtual ~Person() {
     cout << "Deleting a person." << endl;
  }
};

class Student : public Person {
 public:
  ~Student() {
     cout << "Deleting a student." << endl;
  }
};

int main() {
  Person * p = new Student();
  delete p;
}
```

This will output the following:

```
Deleting a student.
Deleting a person.
```

Default Values

Functions can specify default values, as shown below. Note that all default parameters must be on the right side of the function declaration, as there would be no other way to specify how the parameters line up.

```
int func(int a, int b = 3) {
  x = a;
  y = b;
  return a + b;
}

w = func(4);
z = func(4, 5);
```

Operator Overloading

Operator overloading enables us to apply operators like + to objects that would otherwise not support these operations. For example, if we wanted to merge two `BookShelves` into one, we could overload the + operator as follows.

```
BookShelf BookShelf::operator+(BookShelf &other) { ... }
```

▸ Pointers and References

A pointer holds the address of a variable and can be used to perform any operation that could be directly done on the variable, such as accessing and modifying it.

Two pointers can equal each other, such that changing one's value also changes the other's value (since they, in fact, point to the same address).

```
int * p = new int;
*p = 7;
int * q = p;
*p = 8;
cout << *q; // prints 8
```

Note that the size of a pointer varies depending on the architecture: 32 bits on a 32-bit machine and 64 bits on a 64-bit machine. Pay attention to this difference, as it's common for interviewers to ask exactly how much space a data structure takes up.

References

A reference is another name (an alias) for a pre-existing object and it does not have memory of its own. For example:

```
1 int a = 5;
2 int & b = a;
3 b = 7;
4 cout << a; // prints 7
```

In line 2 above, b is a reference to a; modifying b will also modify a.

You cannot create a reference without specifying where in memory it refers to. However, you can create a free-standing reference as shown below:

```
/* allocates memory to store 12 and makes b a reference to this
 * piece of memory. */
const int & b = 12;
```

Unlike pointers, references cannot be null and cannot be reassigned to another piece of memory.

Pointer Arithmetic

One will often see programmers perform addition on a pointer, such as what you see below:

```
int * p = new int[2];
p[0] = 0;
p[1] = 1;
p++;
cout << *p; // Outputs 1
```

Performing p++ will skip ahead by `sizeof(int)` bytes, such that the code outputs 1. Had p been of different type, it would skip ahead as many bytes as the size of the data structure.

▸ Templates

Templates are a way of reusing code to apply the same class to different data types. For example, we might have a list-like data structure which we would like to use for lists of various types. The code below implements this with the `ShiftedList` class.

```
template <class T>class ShiftedList {
  T* array;
  int offset, size;
 public:
  ShiftedList(int sz) : offset(0), size(sz) {
    array = new T[size];
  }

  ~ShiftedList() {
    delete [] array;
  }

  void shiftBy(int n) {
    offset = (offset + n) % size;
  }

  T getAt(int i) {
    return array[convertIndex(i)];
  }

  void setAt(T item, int i) {
    array[convertIndex(i)] = item;
  }

private:
  int convertIndex(int i) {
    int index = (i - offset) % size;
    while (index < 0) index += size;
    return index;
  }
};
```

Interview Questions

12.1 **Last K Lines:** Write a method to print the last K lines of an input file using C++.

Hints: #449, #459

pg 422

12.2 **Reverse String:** Implement a function `void reverse(char* str)` in C or C++ which reverses a null-terminated string.

Hints: #410, #452

pg 423

12.3 **Hash Table vs. STL Map:** Compare and contrast a hash table and an STL map. How is a hash table implemented? If the number of inputs is small, which data structure options can be used instead of a hash table?

Hints: #423

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12.4 Virtual Functions: How do virtual functions work in C++?

Hints: #463

pg 424

12.5 Shallow vs. Deep Copy: What is the difference between deep copy and shallow copy? Explain how you would use each.

Hints: #445

pg 425

12.6 Volatile: What is the significance of the keyword "volatile" in C?

Hints: #456

pg 426

12.7 Virtual Base Class: Why does a destructor in base class need to be declared virtual?

Hints: #421, #460

pg 427

12.8 Copy Node: Write a method that takes a pointer to a Node structure as a parameter and returns a complete copy of the passed in data structure. The Node data structure contains two pointers to other Nodes.

Hints: #427, #462

pg 427

12.9 Smart Pointer: Write a smart pointer class. A smart pointer is a data type, usually implemented with templates, that simulates a pointer while also providing automatic garbage collection. It automatically counts the number of references to a SmartPointer<T*> object and frees the object of type T when the reference count hits zero.

Hints: #402, #438, #453

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12.10 Malloc: Write an aligned malloc and free function that supports allocating memory such that the memory address returned is divisible by a specific power of two.

EXAMPLE

align_malloc(1000,128) will return a memory address that is a multiple of 128 and that points to memory of size 1000 bytes.

aligned_free() will free memory allocated by align_malloc.

Hints: #413, #432, #440

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12.11 2D Alloc: Write a function in C called my2DAlloc which allocates a two-dimensional array. Minimize the number of calls to malloc and make sure that the memory is accessible by the notation arr[i][j].

Hints: #406, #418, #426

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Additional Questions: Linked Lists (#2.6), Testing (#11.1), Java (#13.4), Threads and Locks (#15.3).

Hints start on page 676.

13

Java

While Java-related questions are found throughout this book, this chapter deals with questions about the language and syntax. Such questions are more unusual at bigger companies, which believe more in testing a candidate's aptitude than a candidate's knowledge (and which have the time and resources to train a candidate in a particular language). However, at other companies, these pesky questions can be quite common.

▸ How to Approach

As these questions focus so much on knowledge, it may seem silly to talk about an approach to these problems. After all, isn't it just about knowing the right answer?

Yes and no. Of course, the best thing you can do to master these questions is to learn Java inside and out. But, if you do get stumped, you can try to tackle it with the following approach:

1. Create an example of the scenario, and ask yourself how things should play out.
2. Ask yourself how other languages would handle this scenario.
3. Consider how you would design this situation if you were the language designer. What would the implications of each choice be?

Your interviewer may be equally—or more—impressed if you can derive the answer than if you automatically knew it. Don't try to bluff though. Tell the interviewer, "I'm not sure I can recall the answer, but let me see if I can figure it out. Suppose we have this code..."

▸ Overloading vs. Overriding

Overloading is a term used to describe when two methods have the same name but differ in the type or number of arguments.

```
public double computeArea(Circle c) { ... }
public double computeArea(Square s) { ... }
```

Overriding, however, occurs when a method shares the same name and function signature as another method in its super class.

```
public abstract class Shape {
  public void printMe() {
    System.out.println("I am a shape.");
  }
  public abstract double computeArea();
}
```

```java

public class Circle extends Shape {
  private double rad = 5;
  public void printMe() {
    System.out.println(“I am a circle.”);
  }

  public double computeArea() {
    return rad * rad * 3.15;
  }
}

public class Ambiguous extends Shape {
  private double area = 10;
  public double computeArea() {
    return area;
  }
}

public class IntroductionOverriding {
  public static void main(String[] args) {
    Shape[] shapes = new Shape[2];
    Circle circle = new Circle();
    Ambiguous ambiguous = new Ambiguous();

    shapes[0] = circle;
    shapes[1] = ambiguous;

    for (Shape s : shapes) {
      s.printMe();
      System.out.println(s.computeArea());
    }
  }
}
```

The above code will print:

```
I am a circle.
78.75
I am a shape.
10.0
```

Observe that `Circle` overrode `printMe()`, whereas `Ambiguous` just left this method as-is.

▸ Collection Framework

Java's collection framework is incredibly useful, and you will see it used throughout this book. Here are some of the most useful items:

`ArrayList`: An `ArrayList` is a dynamically resizing array, which grows as you insert elements.

```java
ArrayList<String> myArr = new ArrayList<String>();
myArr.add(“one”);
myArr.add(“two”);
System.out.println(myArr.get(0)); /* prints <one> */
```

`Vector`: A `vector` is very similar to an `ArrayList`, except that it is synchronized. Its syntax is almost identical as well.

```
Vector<String> myVect = new Vector<String>();
myVect.add("one");
myVect.add("two");
System.out.println(myVect.get(0));
```

`LinkedList`: `LinkedList` is, of course, Java's built-in `LinkedList` class. Though it rarely comes up in an interview, it's useful to study because it demonstrates some of the syntax for an iterator.

```
LinkedList<String> myLinkedList = new LinkedList<String>();
myLinkedList.add("two");
myLinkedList.addFirst("one");
Iterator<String> iter = myLinkedList.iterator();
while (iter.hasNext()) {
   System.out.println(iter.next());
}
```

`HashMap`: The `HashMap` collection is widely used, both in interviews and in the real world. We've provided a snippet of the syntax below.

```
HashMap<String, String> map = new HashMap<String, String>();
map.put("one", "uno");
map.put("two", "dos");
System.out.println(map.get("one"));
```

Before your interview, make sure you're very comfortable with the above syntax. You'll need it.

Interview Questions

Please note that because virtually all the solutions in this book are implemented with Java, we have selected only a small number of questions for this chapter. Moreover, most of these questions deal with the "trivia" of the languages, since the rest of the book is filled with Java programming questions.

13.1 Private Constructor: In terms of inheritance, what is the effect of keeping a constructor private?

Hints: #404

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13.2 Return from Finally: In Java, does the `finally` block get executed if we insert a `return` statement inside the try block of a `try-catch-finally`?

Hints: #409

pg 433

13.3 Final, etc.: What is the difference between `final`, `finally`, and `finalize`?

Hints: #412

pg 433

13.4 Generics vs. Templates: Explain the difference between templates in C++ and generics in Java.

Hints: #416, #425

pg 435

13.5 TreeMap, HashMap, LinkedHashMap: Explain the differences between `TreeMap`, `HashMap`, and `LinkedHashMap`. Provide an example of when each one would be best.

Hints: #420, #424, #430, #454

pg 436

13.6 Object Reflection: Explain what object reflection is in Java and why it is useful.

Hints: #435

pg 437

13.7 Lambda Expressions: There is a class Country that has methods getContinent() and getPopulation(). Write a function int getPopulation(List<Country> countries, String continent) that computes the total population of a given continent, given a list of all countries and the name of a continent.

Hints: #448, #461, #464

pg 438

13.8 Lambda Random: Using Lambda expressions, write a function List<Integer> getRandomSubset(List<Integer> list) that returns a random subset of arbitrary size. All subsets (including the empty set) should be equally likely to be chosen.

Hints: #443, #450, #457

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Additional Questions: Arrays and Strings (#1.3), Object-Oriented Design (#7.12), Threads and Locks (#15.3)

Hints start on page 676.

14

Databases

If you profess knowledge of databases, you might be asked some questions on it. We'll review some of the key concepts and offer an overview of how to approach these problems. As you read these queries, don't be surprised by minor variations in syntax. There are a variety of flavors of SQL, and you might have worked with a slightly different one. The examples in this book have been tested against Microsoft SQL Server.

▸ SQL Syntax and Variations

Implicit and explicit joins are shown below. These two statements are equivalent, and it's a matter of personal preference which one you choose. For consistency, we will stick to the explicit join.

Explicit Join

```
SELECT CourseName, TeacherName
FROM Courses INNER JOIN Teachers
ON Courses.TeacherID = Teachers.TeacherID
```

Implicit Join

```
SELECT CourseName, TeacherName
FROM   Courses, Teachers
WHERE  Courses.TeacherID =
       Teachers.TeacherID
```

▸ Denormalized vs. Normalized Databases

Normalized databases are designed to minimize redundancy, while denormalized databases are designed to optimize read time.

In a traditional normalized database with data like `Courses` and `Teachers`, `Courses` might contain a column called `TeacherID`, which is a foreign key to `Teacher`. One benefit of this is that information about the teacher (name, address, etc.) is only stored once in the database. The drawback is that many common queries will require expensive joins.

Instead, we can denormalize the database by storing redundant data. For example, if we knew that we would have to repeat this query often, we might store the teacher's name in the `Courses` table. Denormalization is commonly used to create highly scalable systems.

▸ SQL Statements

Let's walk through a review of basic SQL syntax, using as an example the database that was mentioned earlier. This database has the following simple structure (* indicates a primary key):

```
Courses: CourseID*, CourseName, TeacherID
Teachers: TeacherID*, TeacherName
Students: StudentID*, StudentName
```

```
StudentCourses: CourseID*, StudentID*
```

Using the above table, implement the following queries.

Query 1: Student Enrollment

Implement a query to get a list of all students and how many courses each student is enrolled in.

At first, we might try something like this:

```
/* Incorrect Code */
SELECT Students.StudentName, count(*)
FROM Students INNER JOIN StudentCourses
ON Students.StudentID = StudentCourses.StudentID
GROUP BY Students.StudentID
```

This has three problems:

1. We have excluded students who are not enrolled in any courses, since `StudentCourses` only includes enrolled students. We need to change this to a `LEFT JOIN`.
2. Even if we changed it to a `LEFT JOIN`, the query is still not quite right. Doing `count(*)` would return how many items there are in a given group of `StudentID`s. Students enrolled in zero courses would still have one item in their group. We need to change this to count the number of `CourseID`s in each group: `count(StudentCourses.CourseID)`.
3. We've grouped by `Students.StudentID`, but there are still multiple `StudentName`s in each group. How will the database know which `StudentName` to return? Sure, they may all have the same value, but the database doesn't understand that. We need to apply an *aggregate* function to this, such as `first(Students.StudentName)`.

Fixing these issues gets us to this query:

```
1  /* Solution 1: Wrap with another query */
2  SELECT StudentName, Students.StudentID, Cnt
3  FROM (
4    SELECT  Students.StudentID, count(StudentCourses.CourseID) as [Cnt]
5    FROM Students LEFT JOIN StudentCourses
6    ON Students.StudentID = StudentCourses.StudentID
7    GROUP BY Students.StudentID
8  ) T INNER JOIN Students on T.studentID = Students.StudentID
```

Looking at this code, one might ask why we don't just select the student name on line 3 to avoid having to wrap lines 3 through 6 with another query. This (incorrect) solution is shown below.

```
/* Incorrect Code */
SELECT StudentName, Students.StudentID, count(StudentCourses.CourseID) as [Cnt]
FROM Students LEFT JOIN StudentCourses
ON Students.StudentID = StudentCourses.StudentID
GROUP BY Students.StudentID
```

The answer is that we *can't* do that - at least not exactly as shown. We can only select values that are in an aggregate function or in the GROUP BY clause.

Alternatively, we could resolve the above issues with either of the following statements:

```
/* Solution 2: Add StudentName to GROUP BY clause. */
SELECT StudentName, Students.StudentID, count(StudentCourses.CourseID) as [Cnt]
FROM Students LEFT JOIN StudentCourses
ON Students.StudentID = StudentCourses.StudentID
GROUP BY Students.StudentID, Students.StudentName
```

OR

```
/* Solution 3: Wrap with aggregate function. */
SELECT  max(StudentName) as [StudentName], Students.StudentID,
        count(StudentCourses.CourseID) as [Count]
FROM Students LEFT JOIN StudentCourses
ON Students.StudentID = StudentCourses.StudentID
GROUP BY Students.StudentID
```

Query 2: Teacher Class Size

Implement a query to get a list of all teachers and how many students they each teach. If a teacher teaches the same student in two courses, you should double count the student. Sort the list in descending order of the number of students a teacher teaches.

We can construct this query step by step. First, let's get a list of `TeacherID`s and how many students are associated with each `TeacherID`. This is very similar to the earlier query.

```
SELECT TeacherID, count(StudentCourses.CourseID) AS [Number]
FROM Courses INNER JOIN StudentCourses
ON Courses.CourseID = StudentCourses.CourseID
GROUP BY Courses.TeacherID
```

Note that this `INNER JOIN` will not select teachers who aren't teaching classes. We'll handle that in the below query when we join it with the list of all teachers.

```
SELECT TeacherName, isnull(StudentSize.Number, 0)
FROM Teachers LEFT JOIN
     (SELECT TeacherID, count(StudentCourses.CourseID) AS [Number]
      FROM Courses INNER JOIN StudentCourses
      ON Courses.CourseID = StudentCourses.CourseID
      GROUP BY Courses.TeacherID) StudentSize
ON Teachers.TeacherID = StudentSize.TeacherID
ORDER BY StudentSize.Number DESC
```

Note how we handled the `NULL` values in the `SELECT` statement to convert the `NULL` values to zeros.

▸ Small Database Design

Additionally, you might be asked to design your own database. We'll walk you through an approach for this. You might notice the similarities between this approach and the approach for object-oriented design.

Step 1: Handle Ambiguity

Database questions often have some ambiguity, intentionally or unintentionally. Before you proceed with your design, you must understand exactly what you need to design.

Imagine you are asked to design a system to represent an apartment rental agency. You will need to know whether this agency has multiple locations or just one. You should also discuss with your interviewer how general you should be. For example, it would be extremely rare for a person to rent two apartments in the same building. But does that mean you shouldn't be able to handle that? Maybe, maybe not. Some very rare conditions might be best handled through a work around (like duplicating the person's contact information in the database).

Step 2: Define the Core Objects

Next, we should look at the core objects of our system. Each of these core objects typically translates into a table. In this case, our core objects might be `Property`, `Building`, `Apartment`, `Tenant` and `Manager`.

Step 3: Analyze Relationships

Outlining the core objects should give us a good sense of what the tables should be. How do these tables relate to each other? Are they many-to-many? One-to-many?

If `Buildings` has a one-to-many relationship with `Apartments` (one `Building` has many `Apartments`), then we might represent this as follows:

Apartments	
ApartmentID	int
ApartmentAddress	varchar(100)
BuildingID	int

Buildings	
BuildingID	int
BuildingName	varchar(100)
BuildingAddress	varchar(500)

Note that the `Apartments` table links back to `Buildings` with a `BuildingID` column.

If we want to allow for the possibility that one person rents more than one apartment, we might want to implement a many-to-many relationship as follows:

TenantApartments	
TenantID	int
ApartmentID	int

Apartments	
ApartmentID	int
ApartmentAddress	varchar(500)
BuildingID	int

Tenants	
TenantID	int
TenantName	varchar(100)
TenantAddress	varchar(500)

The `TenantApartments` table stores a relationship between `Tenants` and `Apartments`.

Step 4: Investigate Actions

Finally, we fill in the details. Walk through the common actions that will be taken and understand how to store and retrieve the relevant data. We'll need to handle lease terms, moving out, rent payments, etc. Each of these actions requires new tables and columns.

▸ Large Database Design

When designing a large, scalable database, joins (which are required in the above examples) are generally very slow. Thus, you must *denormalize* your data. Think carefully about how data will be used—you'll probably need to duplicate the data in multiple tables.

Interview Questions

Questions 1 through 3 refer to the database schema at the end of the chapter. Each apartment can have multiple tenants, and each tenant can have multiple apartments. Each apartment belongs to one building, and each building belongs to one complex.

14.1 Multiple Apartments: Write a SQL query to get a list of tenants who are renting more than one apartment.

Hints: #408

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14.2 **Open Requests:** Write a SQL query to get a list of all buildings and the number of open requests (`Requests` in which `status` equals `'Open'`).

Hints: #411

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14.3 **Close All Requests:** Building #11 is undergoing a major renovation. Implement a query to close all requests from apartments in this building.

Hints: #431

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14.4 **Joins:** What are the different types of joins? Please explain how they differ and why certain types are better in certain situations.

Hints: #451

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14.5 **Denormalization:** What is denormalization? Explain the pros and cons.

Hints: #444, #455

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14.6 **Entity-Relationship Diagram:** Draw an entity-relationship diagram for a database with companies, people, and professionals (people who work for companies).

Hints: #436

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14.7 **Design Grade Database:** Imagine a simple database storing information for students' grades. Design what this database might look like and provide a SQL query to return a list of the honor roll students (top 10%), sorted by their grade point average.

Hints: #428, #442

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Additional Questions: Object-Oriented Design (#7.7), System Design and Scalability (#9.6)

Hints start on page 676.

Apartments	
AptID	int
UnitNumber	varchar(10)
BuildingID	int

Buildings	
BuildingID	int
ComplexID	int
BuildingName	varchar(100)
Address	varchar(500)

Requests	
RequestID	int
Status	varchar(100)
AptID	int
Description	varchar(500)

Complexes	
ComplexID	int
ComplexName	varchar(100)

AptTenants	
TenantID	int
AptID	int

Tenants	
TenantID	int
TenantName	varchar(100)

15

Threads and Locks

In a Microsoft, Google or Amazon interview, it's not terribly common to be asked to implement an algorithm with threads (unless you're working in a team for which this is a particularly important skill). It is, however, relatively common for interviewers at any company to assess your general understanding of threads, particularly your understanding of deadlocks.

This chapter will provide an introduction to this topic.

▸ Threads in Java

Every thread in Java is created and controlled by a unique object of the `java.lang.Thread` class. When a standalone application is run, a user thread is automatically created to execute the `main()` method. This thread is called the main thread.

In Java, we can implement threads in one of two ways:

- By implementing the `java.lang.Runnable` interface
- By extending the `java.lang.Thread` class

We will cover both of these below.

Implementing the Runnable Interface

The `Runnable` interface has the following very simple structure.

```
public interface Runnable {
  void run();
}
```

To create and use a thread using this interface, we do the following:

1. Create a class which implements the `Runnable` interface. An object of this class is a `Runnable` object.
2. Create an object of type `Thread` by passing a `Runnable` object as argument to the `Thread` constructor. The `Thread` object now has a `Runnable` object that implements the `run()` method.
3. The `start()` method is invoked on the `Thread` object created in the previous step.

For example:

```
public class RunnableThreadExample implements Runnable {
  public int count = 0;

  public void run() {
    System.out.println("RunnableThread starting.");
```

```
6          try {
7             while (count < 5) {
8                Thread.sleep(500);
9                count++;
10            }
11         } catch (InterruptedException exc) {
12            System.out.println(“RunnableThread interrupted.”);
13         }
14         System.out.println(“RunnableThread terminating.”);
15      }
16   }
17
18   public static void main(String[] args) {
19      RunnableThreadExample instance = new RunnableThreadExample();
20      Thread thread = new Thread(instance);
21      thread.start();
22
23      /* waits until above thread counts to 5 (slowly) */
24      while (instance.count != 5) {
25         try {
26            Thread.sleep(250);
27         } catch (InterruptedException exc) {
28            exc.printStackTrace();
29         }
30      }
31   }
```

In the above code, observe that all we really needed to do is have our class implement the `run()` method (line 4). Another method can then pass an instance of the class to `new Thread(obj)` (lines 19 - 20) and call `start()` on the thread (line 21).

Extending the Thread Class

Alternatively, we can create a thread by extending the `Thread` class. This will almost always mean that we override the `run()` method, and the subclass may also call the thread constructor explicitly in its constructor.

The below code provides an example of this.

```
public class ThreadExample extends Thread {
   int count = 0;

   public void run() {
      System.out.println(“Thread starting.”);
      try {
         while (count < 5) {
            Thread.sleep(500);
            System.out.println(“In Thread, count is ” + count);
            count++;
         }
      } catch (InterruptedException exc) {
         System.out.println(“Thread interrupted.”);
      }
      System.out.println(“Thread terminating.”);
   }
}

```

```
public class ExampleB {
  public static void main(String args[]) {
    ThreadExample instance = new ThreadExample();
    instance.start();

    while (instance.count != 5) {
      try {
        Thread.sleep(250);
      } catch (InterruptedException exc) {
        exc.printStackTrace();
      }
    }
  }
}
```

This code is very similar to the first approach. The difference is that since we are extending the `Thread` class, rather than just implementing an interface, we can call `start()` on the instance of the class itself.

Extending the Thread Class vs. Implementing the Runnable Interface

When creating threads, there are two reasons why implementing the `Runnable` interface may be preferable to extending the `Thread` class:

- Java does not support multiple inheritance. Therefore, extending the `Thread` class means that the subclass cannot extend any other class. A class implementing the `Runnable` interface will be able to extend another class.
- A class might only be interested in being runnable, and therefore, inheriting the full overhead of the `Thread` class would be excessive.

Synchronization and Locks

Threads within a given process share the same memory space, which is both a positive and a negative. It enables threads to share data, which can be valuable. However, it also creates the opportunity for issues when two threads modify a resource at the same time. Java provides synchronization in order to control access to shared resources.

The keyword `synchronized` and the `lock` form the basis for implementing synchronized execution of code.

Synchronized Methods

Most commonly, we restrict access to shared resources through the use of the `synchronized` keyword. It can be applied to methods and code blocks, and restricts multiple threads from executing the code simultaneously *on the same object.*

To clarify the last point, consider the following code:

```
public class MyClass extends Thread  {
  private String name;
  private MyObject myObj;

  public MyClass(MyObject obj, String n) {
    name = n;
    myObj = obj;
  }
```

```

  public void run() {
    myObj.foo(name);
  }
}

public class MyObject {
  public synchronized void foo(String name) {
    try {
      System.out.println("Thread " + name + ".foo(): starting");
      Thread.sleep(3000);
      System.out.println("Thread " + name + ".foo(): ending");
    } catch (InterruptedException exc) {
      System.out.println("Thread " + name + ": interrupted.");
    }
  }
}
```

Can two instances of `MyClass` call `foo` at the same time? It depends. If they have the same instance of `MyObject`, then no. But, if they hold different references, then the answer is yes.

```
/* Difference references - both threads can call MyObject.foo() */
MyObject obj1 = new MyObject();
MyObject obj2 = new MyObject();
MyClass thread1 = new MyClass(obj1, "1");
MyClass thread2 = new MyClass(obj2, "2");
thread1.start();
thread2.start()

/* Same reference to obj. Only one will be allowed to call foo,
 * and the other will be forced to wait. */
MyObject obj = new MyObject();
MyClass thread1 = new MyClass(obj, "1");
MyClass thread2 = new MyClass(obj, "2");
thread1.start()
thread2.start()
```

Static methods synchronize on the *class lock*. The two threads above could not simultaneously execute synchronized static methods on the same class, even if one is calling `foo` and the other is calling `bar`.

```
public class MyClass extends Thread  {
  ...
  public void run() {
    if (name.equals("1")) MyObject.foo(name);
    else if (name.equals("2")) MyObject.bar(name);
  }
}

public class MyObject {
  public static synchronized void foo(String name) { /* same as before */ }
  public static synchronized void bar(String name) { /* same as foo */ }
}
```

If you run this code, you will see the following printed:

```
Thread 1.foo(): starting
Thread 1.foo(): ending
Thread 2.bar(): starting
Thread 2.bar(): ending
```

Synchronized Blocks

Similarly, a block of code can be synchronized. This operates very similarly to synchronizing a method.

```
public class MyClass extends Thread  {
  ...
  public void run() {
    myObj.foo(name);
  }
}
public class MyObject {
  public void foo(String name) {
    synchronized(this) {
      ...
    }
  }
}
```

Like synchronizing a method, only one thread per instance of `MyObject` can execute the code within the `synchronized` block. That means that, if `thread1` and `thread2` have the same instance of `MyObject`, only one will be allowed to execute the code block at a time.

Locks

For more granular control, we can utilize a lock. A lock (or monitor) is used to synchronize access to a shared resource by associating the resource with the lock. A thread gets access to a shared resource by first acquiring the lock associated with the resource. At any given time, at most one thread can hold the lock and, therefore, only one thread can access the shared resource.

A common use case for locks is when a resource is accessed from multiple places, but should be only accessed by one thread *at a time*. This case is demonstrated in the code below.

```
public class LockedATM {
  private Lock lock;
  private int balance = 100;

  public LockedATM() {
    lock = new ReentrantLock();
  }

  public int withdraw(int value) {
    lock.lock();
    int temp = balance;
    try {
      Thread.sleep(100);
      temp = temp - value;
      Thread.sleep(100);
      balance = temp;
    } catch (InterruptedException e) {      }
    lock.unlock();
    return temp;
  }

  public int deposit(int value) {
    lock.lock();
    int temp = balance;
    try {
      Thread.sleep(100);
```

```
            temp = temp + value;
            Thread.sleep(300);
            balance = temp;
        } catch (InterruptedException e) {      }
        lock.unlock();
        return temp;
    }
}
```

Of course, we've added code to intentionally slow down the execution of `withdraw` and `deposit`, as it helps to illustrate the potential problems that can occur. You may not write code exactly like this, but the situation it mirrors is very, very real. Using a lock will help protect a shared resource from being modified in unexpected ways.

▸ Deadlocks and Deadlock Prevention

A deadlock is a situation where a thread is waiting for an object lock that another thread holds, and this second thread is waiting for an object lock that the first thread holds (or an equivalent situation with several threads). Since each thread is waiting for the other thread to relinquish a lock, they both remain waiting forever. The threads are said to be deadlocked.

In order for a deadlock to occur, you must have all four of the following conditions met:

1. *Mutual Exclusion:* Only one process can access a resource at a given time. (Or, more accurately, there is limited access to a resource. A deadlock could also occur if a resource has limited quantity.)
2. *Hold and Wait:* Processes already holding a resource can request additional resources, without relinquishing their current resources.
3. *No Preemption:* One process cannot forcibly remove another process' resource.
4. *Circular Wait:* Two or more processes form a circular chain where each process is waiting on another resource in the chain.

Deadlock prevention entails removing any of the above conditions, but it gets tricky because many of these conditions are difficult to satisfy. For instance, removing #1 is difficult because many resources can only be used by one process at a time (e.g., printers). Most deadlock prevention algorithms focus on avoiding condition #4: circular wait.

Interview Questions

15.1 **Thread vs. Process:** What's the difference between a thread and a process?

Hints: #405

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15.2 **Context Switch:** How would you measure the time spent in a context switch?

Hints: #403, #407, #415, #441

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15.3 **Dining Philosophers:** In the famous dining philosophers problem, a bunch of philosophers are sitting around a circular table with one chopstick between each of them. A philosopher needs both chopsticks to eat, and always picks up the left chopstick before the right one. A deadlock could potentially occur if all the philosophers reached for the left chopstick at the same time. Using threads and locks, implement a simulation of the dining philosophers problem that prevents deadlocks.

Hints: #419, #437

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15.4 **Deadlock-Free Class:** Design a class which provides a lock only if there are no possible deadlocks.

Hints: #422, #434

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15.5 **Call In Order:** Suppose we have the following code:

```
public class Foo {
   public Foo() { ... }
   public void first() { ... }
   public void second() { ... }
   public void third() { ... }
}
```

The same instance of `Foo` will be passed to three different threads. `ThreadA` will call `first`, `threadB` will call `second`, and `threadC` will call `third`. Design a mechanism to ensure that `first` is called before `second` and `second` is called before `third`.

Hints: #417, #433, #446

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15.6 **Synchronized Methods:** You are given a class with synchronized method A and a normal method B. If you have two threads in one instance of a program, can they both execute A at the same time? Can they execute A and B at the same time?

Hints: #429

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15.7 **FizzBuzz**: In the classic problem FizzBuzz, you are told to print the numbers from 1 to n. However, when the number is divisible by 3, print "Fizz". When it is divisible by 5, print "Buzz". When it is divisible by 3 and 5, print "FizzBuzz". In this problem, you are asked to do this in a multithreaded way. Implement a multithreaded version of FizzBuzz with four threads. One thread checks for divisibility of 3 and prints "Fizz". Another thread is responsible for divisibility of 5 and prints "Buzz". A third thread is responsible for divisibility of 3 and 5 and prints "FizzBuzz". A fourth thread does the numbers.

Hints: #414, #439, #447, #458

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Hints start on page 676.

16

Moderate

16.1 **Number Swapper:** Write a function to swap a number in place (that is, without temporary variables).

Hints: #492, #716, #737

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16.2 **Word Frequencies:** Design a method to find the frequency of occurrences of any given word in a book. What if we were running this algorithm multiple times?

Hints: #489, #536

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16.3 **Intersection:** Given two straight line segments (represented as a start point and an end point), compute the point of intersection, if any.

Hints: #465, #472, #497, #517, #527

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16.4 **Tic Tac Win:** Design an algorithm to figure out if someone has won a game of tic-tac-toe.

Hints: #710, #732

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16.5 **Factorial Zeros:** Write an algorithm which computes the number of trailing zeros in n factorial.

Hints: #585, #711, #729, #733, #745

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16.6 **Smallest Difference:** Given two arrays of integers, compute the pair of values (one value in each array) with the smallest (non-negative) difference. Return the difference.

EXAMPLE

Input: {1, 3, 15, 11, 2}, {23, 127, 235, 19, 8}

Output: 3. That is, the pair (11, 8).

Hints: #632, #670, #679

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16.7 **Number Max:** Write a method that finds the maximum of two numbers. You should not use if-else or any other comparison operator.

Hints: #473, #513, #707, #728

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16.8 English Int: Given any integer, print an English phrase that describes the integer (e.g., "One Thousand, Two Hundred Thirty Four").

Hints: #502, #588, #688

pg 477

16.9 Operations: Write methods to implement the multiply, subtract, and divide operations for integers. The results of all of these are integers. Use only the add operator.

Hints: #572, #600, #613, #648

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16.10 Living People: Given a list of people with their birth and death years, implement a method to compute the year with the most number of people alive. You may assume that all people were born between 1900 and 2000 (inclusive). If a person was alive during any portion of that year, they should be included in that year's count. For example, Person (birth = 1908, death = 1909) is included in the counts for both 1908 and 1909.

Hints: #476, #490, #507, #514, #523, #532, #541, #549, #576

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16.11 Diving Board: You are building a diving board by placing a bunch of planks of wood end-to-end. There are two types of planks, one of length `shorter` and one of length `longer`. You must use exactly K planks of wood. Write a method to generate all possible lengths for the diving board.

Hints: #690, #700, #715, #722, #740, #747

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16.12 XML Encoding: Since XML is very verbose, you are given a way of encoding it where each tag gets mapped to a pre-defined integer value. The language/grammar is as follows:

```
Element   --> Tag Attributes END Children END
Attribute --> Tag Value
END       --> 0
Tag       --> some predefined mapping to int
Value     --> string value
```

For example, the following XML might be converted into the compressed string below (assuming a mapping of `family -> 1, person ->2, firstName -> 3, lastName -> 4, state -> 5`).

```
<family lastName="McDowell" state="CA">
  <person firstName="Gayle">Some Message</person>
</family>
```

Becomes:

```
1 4 McDowell 5 CA 0 2 3 Gayle 0 Some Message 0 0
```

Write code to print the encoded version of an XML element (passed in `Element` and `Attribute` objects).

Hints: #466

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16.13 Bisect Squares: Given two squares on a two-dimensional plane, find a line that would cut these two squares in half. Assume that the top and the bottom sides of the square run parallel to the x-axis.

Hints: #468, #479, #528, #560

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16.14 Best Line: Given a two-dimensional graph with points on it, find a line which passes the most number of points.

Hints: #491, #520, #529, #563

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16.15 Master Mind: The Game of Master Mind is played as follows:

The computer has four slots, and each slot will contain a ball that is red (R), yellow (Y), green (G) or blue (B). For example, the computer might have RGGB (Slot #1 is red, Slots #2 and #3 are green, Slot #4 is blue).

You, the user, are trying to guess the solution. You might, for example, guess YRGB.

When you guess the correct color for the correct slot, you get a "hit." If you guess a color that exists but is in the wrong slot, you get a "pseudo-hit." Note that a slot that is a hit can never count as a pseudo-hit.

For example, if the actual solution is RGBY and you guess GGRR, you have one hit and one pseudo-hit.

Write a method that, given a guess and a solution, returns the number of hits and pseudo-hits.

Hints: #639, #730

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16.16 Sub Sort: Given an array of integers, write a method to find indices `m` and `n` such that if you sorted elements `m` through `n`, the entire array would be sorted. Minimize `n - m` (that is, find the smallest such sequence).

EXAMPLE

Input: `1, 2, 4, 7, 10, 11, 7, 12, 6, 7, 16, 18, 19`

Output: `(3, 9)`

Hints: #482, #553, #667, #708, #735, #746

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16.17 Contiguous Sequence: You are given an array of integers (both positive and negative). Find the contiguous sequence with the largest sum. Return the sum.

EXAMPLE

Input: `2, -8, 3, -2, 4, -10`

Output: `5 (i.e., {3, -2, 4})`

Hints: #531, #551, #567, #594, #614

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16.18 Pattern Matching: You are given two strings, `pattern` and `value`. The `pattern` string consists of just the letters a and b, describing a pattern within a string. For example, the string `catcatgocatgo` matches the pattern `aabab` (where `cat` is a and go is b). It also matches patterns like a, ab, and b. Write a method to determine if `value` matches `pattern`.

Hints: #631, #643, #653, #663, #685, #718, #727

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16.19 Pond Sizes: You have an integer matrix representing a plot of land, where the value at that location represents the height above sea level. A value of zero indicates water. A pond is a region of water connected vertically, horizontally, or diagonally. The size of the pond is the total number of connected water cells. Write a method to compute the sizes of all ponds in the matrix.

EXAMPLE

Input:

```
0 2 1 0
0 1 0 1
1 1 0 1
0 1 0 1
```

Output: 2, 4, 1 (in any order)

Hints: #674, #687, #706, #723

pg 503

16.20 T9: On old cell phones, users typed on a numeric keypad and the phone would provide a list of words that matched these numbers. Each digit mapped to a set of 0 - 4 letters. Implement an algorithm to return a list of matching words, given a sequence of digits. You are provided a list of valid words (provided in whatever data structure you'd like). The mapping is shown in the diagram below:

1	**2** abc	**3** def
4 ghi	**5** jkl	**6** mno
7 pqrs	**8** tuv	**9** wxyz
	0	

EXAMPLE

Input: `8733`

Output: `tree, used`

Hints: #471, #487, #654, #703, #726, #744

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16.21 Sum Swap: Given two arrays of integers, find a pair of values (one value from each array) that you can swap to give the two arrays the same sum.

EXAMPLE

Input: {4, 1, 2, 1, 1, 2} and {3, 6, 3, 3}

Output: {1, 3}

Hints: #545, #557, #564, #571, #583, #592, #602, #606, #635

pg 509

16.22 Langton's Ant: An ant is sitting on an infinite grid of white and black squares. It initially faces right. At each step, it does the following:

(1) At a white square, flip the color of the square, turn 90 degrees right (clockwise), and move forward one unit.

(2) At a black square, flip the color of the square, turn 90 degrees left (counter-clockwise), and move forward one unit.

Write a program to simulate the first K moves that the ant makes and print the final board as a grid. Note that you are not provided with the data structure to represent the grid. This is something you must design yourself. The only input to your method is K. You should print the final grid and return nothing. The method signature might be something like `void printKMoves(int K)`.

Hints: #474, #481, #533, #540, #559, #570, #599, #616, #627

pg 512

16.23 Rand7 from Rand5: Implement a method `rand7()` given `rand5()`. That is, given a method that generates a random number between 0 and 4 (inclusive), write a method that generates a random number between 0 and 6 (inclusive).

Hints: #505, #574, #637, #668, #697, #720

pg 518

16.24 Pairs with Sum: Design an algorithm to find all pairs of integers within an array which sum to a specified value.

Hints: #548, #597, #644, #673

pg 520

16.25 LRU Cache: Design and build a "least recently used" cache, which evicts the least recently used item. The cache should map from keys to values (allowing you to insert and retrieve a value associated with a particular key) and be initialized with a max size. When it is full, it should evict the least recently used item.

Hints: #524, #630, #694

pg 521

16.26 Calculator: Given an arithmetic equation consisting of positive integers, +, -, * and / (no parentheses), compute the result.

EXAMPLE

Input: `2*3+5/6*3+15`

Output: `23.5`

Hints: #521, #624, #665, #698

pg 524

17

Hard

17.1 **Add Without Plus:** Write a function that adds two numbers. You should not use + or any arithmetic operators.

Hints: #467, #544, #601, #628, #642, #664, #692, #712, #724

pg 530

17.2 **Shuffle:** Write a method to shuffle a deck of cards. It must be a perfect shuffle—in other words, each of the 52! permutations of the deck has to be equally likely. Assume that you are given a random number generator which is perfect.

Hints: #483, #579, #634

pg 531

17.3 **Random Set:** Write a method to randomly generate a set of `m` integers from an array of size n. Each element must have equal probability of being chosen.

Hints: #494, #596

pg 532

17.4 **Missing Number:** An array A contains all the integers from 0 to n, except for one number which is missing. In this problem, we cannot access an entire integer in A with a single operation. The elements of A are represented in binary, and the only operation we can use to access them is "fetch the `j`th bit of `A[i]`," which takes constant time. Write code to find the missing integer. Can you do it in `O(n)` time?

Hints: #610, #659, #683

pg 533

17.5 **Letters and Numbers:** Given an array filled with letters and numbers, find the longest subarray with an equal number of letters and numbers.

Hints: #485, #515, #619, #671, #713

pg 536

17.6 **Count of 2s:** Write a method to count the number of 2s that appear in all the numbers between 0 and n (inclusive).

EXAMPLE

Input: 25

Output: 9 (2, 12, 20, 21, 22, 23, 24 and 25. Note that 22 counts for two 2s.)

Hints: #573, #612, #641

pg 538

17.7 **Baby Names:** Each year, the government releases a list of the 10000 most common baby names and their frequencies (the number of babies with that name). The only problem with this is that some names have multiple spellings. For example, "John" and "Jon" are essentially the same name but would be listed separately in the list. Given two lists, one of names/frequencies and the other of pairs of equivalent names, write an algorithm to print a new list of the true frequency of each name. Note that if John and Jon are synonyms, and Jon and Johnny are synonyms, then John and Johnny are synonyms. (It is both transitive and symmetric.) In the final list, any name can be used as the "real" name.

EXAMPLE

Input:

Names: John (15), Jon (12), Chris (13), Kris (4), Christopher (19)

Synonyms: (Jon, John), (John, Johnny), (Chris, Kris), (Chris, Christopher)

Output: John (27), Kris (36)

Hints: #478, #493, #512, #537, #586, #605, #655, #675, #704

pg 541

17.8 **Circus Tower:** A circus is designing a tower routine consisting of people standing atop one another's shoulders. For practical and aesthetic reasons, each person must be both shorter and lighter than the person below him or her. Given the heights and weights of each person in the circus, write a method to compute the largest possible number of people in such a tower.

EXAMPLE

Input (ht, wt): `(65, 100) (70, 150) (56, 90) (75, 190) (60, 95) (68, 110)`

Output: The longest tower is length 6 and includes from top to bottom:

`(56, 90) (60,95) (65,100) (68,110) (70,150) (75,190)`

Hints: #638, #657, #666, #682, #699

pg 546

17.9 **Kth Multiple:** Design an algorithm to find the kth number such that the only prime factors are 3, 5, and 7. Note that 3, 5, and 7 do not have to be factors, but it should not have any other prime factors. For example, the first several multiples would be (in order) 1, 3, 5, 7, 9, 15, 21.

Hints: #488, #508, #550, #591, #622, #660, #686

pg 549

17.10 **Majority Element:** A majority element is an element that makes up more than half of the items in an array. Given a positive integers array, find the majority element. If there is no majority element, return -1. Do this in `O(N)` time and `O(1)` space.

EXAMPLE

Input: `1 2 5 9 5 9 5 5 5`

Output: 5

Hints: #522, #566, #604, #620, #650

pg 554

17.11 **Word Distance:** You have a large text file containing words. Given any two words, find the shortest distance (in terms of number of words) between them in the file. If the operation will be repeated many times for the same file (but different pairs of words), can you optimize your solution?

Hints: #486, #501, #538, #558, #633

pg 557

17.12 BiNode: Consider a simple data structure called `BiNode`, which has pointers to two other nodes.

```
public class BiNode {
   public BiNode node1, node2;
   public int data;
}
```

The data structure `BiNode` could be used to represent both a binary tree (where `node1` is the left node and `node2` is the right node) or a doubly linked list (where `node1` is the previous node and `node2` is the next node). Implement a method to convert a binary search tree (implemented with `BiNode`) into a doubly linked list. The values should be kept in order and the operation should be performed in place (that is, on the original data structure).

Hints: #509, #608, #646, #680, #701, #719

pg 560

17.13 Re-Space: Oh, no! You have accidentally removed all spaces, punctuation, and capitalization in a lengthy document. A sentence like "`I reset the computer. It still didn't boot!`" became "`iresetthecomputeritstilldidntboot`". You'll deal with the punctuation and capitalization later; right now you need to re-insert the spaces. Most of the words are in a dictionary but a few are not. Given a dictionary (a list of strings) and the document (a string), design an algorithm to unconcatenate the document in a way that minimizes the number of unrecognized characters.

EXAMPLE:

Input: `jesslookedjustliketimherbrother`

Output: `jess looked just like tim her brother` (7 unrecognized characters)

Hints: #496, #623, #656, #677, #739, #749

pg 563

17.14 Smallest K: Design an algorithm to find the smallest K numbers in an array.

Hints: #470, #530, #552, #593, #625, #647, #661, #678

pg 567

17.15 Longest Word: Given a list of words, write a program to find the longest word made of other words in the list.

EXAMPLE

Input: `cat, banana, dog, nana, walk, walker, dogwalker`

Output: `dogwalker`

Hints: #475, #499, #543, #589

pg 572

17.16 The Masseuse: A popular masseuse receives a sequence of back-to-back appointment requests and is debating which ones to accept. She needs a 15-minute break between appointments and therefore she cannot accept any adjacent requests. Given a sequence of back-to-back appointment requests (all multiples of 15 minutes, none overlap, and none can be moved), find the optimal (highest total booked minutes) set the masseuse can honor. Return the number of minutes.

EXAMPLE

Input: `{30, 15, 60, 75, 45, 15, 15, 45}`

Output: `180 minutes ({30, 60, 45, 45}).`

Hints: #495, #504, #516, #526, #542, #554, #562, #568, #578, #587, #607

pg 574

17.17 Multi Search: Given a string b and an array of smaller strings T, design a method to search b for each small string in T.

Hints: #480, #582, #617, #743

pg 578

17.18 Shortest Supersequence: You are given two arrays, one shorter (with all distinct elements) and one longer. Find the shortest subarray in the longer array that contains all the elements in the shorter array. The items can appear in any order.

EXAMPLE

Input: `{1, 5, 9}` | `{7, 5, 9, 0, 2, 1, 3, 5, 7, 9, 1, 1, 5, 8, 8, 9, 7}`

Output: `[7, 10]` (the underlined portion above)

Hints: #645, #652, #669, #681, #691, #725, #731, #741

pg 584

17.19 Missing Two: You are given an array with all the numbers from 1 to N appearing exactly once, except for one number that is missing. How can you find the missing number in `O(N)` time and `O(1)` space? What if there were two numbers missing?

Hints: #503, #590, #609, #626, #649, #672, #689, #696, #702, #717

pg 591

17.20 Continuous Median: Numbers are randomly generated and passed to a method. Write a program to find and maintain the median value as new values are generated.

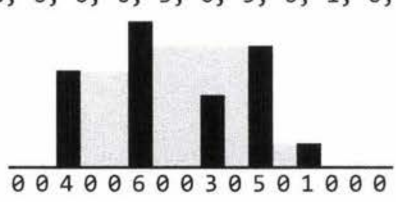
Hints: #519, #546, #575, #709

pg 595

17.21 Volume of Histogram: Imagine a histogram (bar graph). Design an algorithm to compute the volume of water it could hold if someone poured water across the top. You can assume that each histogram bar has width 1.

EXAMPLE (Black bars are the histogram. Gray is water.)

Input: `{0, 0, 4, 0, 0, 6, 0, 0, 3, 0, 5, 0, 1, 0, 0, 0}`



Output: 26

Hints: #629, #640, #651, #658, #662, #676, #693, #734, #742

pg 596

17.22 Word Transformer: Given two words of equal length that are in a dictionary, write a method to transform one word into another word by changing only one letter at a time. The new word you get in each step must be in the dictionary.

EXAMPLE

Input: DAMP, LIKE

Output: DAMP -> LAMP -> LIMP -> LIME -> LIKE

Hints: #506, #535, #556, #580, #598, #618, #738

pg 602

17.23 Max Black Square: Imagine you have a square matrix, where each cell (pixel) is either black or white Design an algorithm to find the maximum subsquare such that all four borders are filled with black pixels.

Hints: #684, #695, #705, #714, #721, #736

pg 608

17.24 Max Submatrix: Given an NxN matrix of positive and negative integers, write code to find the submatrix with the largest possible sum.

Hints: #469, #511, #525, #539, #565, #581, #595, #615, #621

pg 611

17.25 Word Rectangle: Given a list of millions of words, design an algorithm to create the largest possible rectangle of letters such that every row forms a word (reading left to right) and every column forms a word (reading top to bottom). The words need not be chosen consecutively from the list, but all rows must be the same length and all columns must be the same height.

Hints: #477, #500, #748

pg 615

17.26 Sparse Similarity: The similarity of two documents (each with distinct words) is defined to be the size of the intersection divided by the size of the union. For example, if the documents consist of integers, the similarity of `{1, 5, 3}` and `{1, 7, 2, 3}` is `0.4`, because the intersection has size 2 and the union has size 5.

We have a long list of documents (with distinct values and each with an associated ID) where the similarity is believed to be "sparse." That is, any two arbitrarily selected documents are very likely to have similarity 0. Design an algorithm that returns a list of pairs of document IDs and the associated similarity.

Print only the pairs with similarity greater than 0. Empty documents should not be printed at all. For simplicity, you may assume each document is represented as an array of distinct integers.

EXAMPLE

Input:

```
13: {14, 15, 100, 9, 3}
16: {32, 1, 9, 3, 5}
19: {15, 29, 2, 6, 8, 7}
24: {7, 10}
```

Output:

```
ID1, ID2 : SIMILARITY
13, 19   : 0.1
13, 16   : 0.25
19, 24   : 0.14285714285714285
```

Hints: #484, #498, #510, #518, #534, #547, #555, #561, #569, #577, #584, #603, #611, #636

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