Simplex Integration

By Domain Subdivision

Inhaltsverzeichnis

Teil I.
Theory

1. Problem

Total Energy in the System:

$$\Pi = \int_\Omega g(\phi)\psi(\mathbf{u})\,d\Omega + \frac{G_c}{2l}\int_\Omega \phi^2 + l^2\nabla\phi\cdot\nabla\phi\,d\Omega \rightarrow \min \qquad [1.1]$$

Degradation Function

$$g(\phi) = \left(1-\phi^2\right)^2 + k \qquad [1.2]$$

mit

k . . . being a small but finite scalar such as 10 6
G_c . . . critical energy release rate, material parameter
l . . . *width* of phase field
$\psi(u)$ . . . strain energy density function
u . . . displacement function
ϕ . . . phase field parameter, ansatz function discussed below
$\nabla\phi$ . . . gradient of phase field parameter

$$\delta_u\Pi = \int_\Omega g(\phi)\sigma(\mathbf{u})\frac{\partial\varepsilon}{\partial\mathbf{u}}\,\delta\mathbf{u} = 0 \qquad [1.3]$$

$$\delta_\phi\Pi = \int_\Omega 2(\phi-1)\,\delta\phi\psi(\mathbf{u})\,d\Omega + \frac{G_c}{l}\int_\Omega \phi\,\delta\phi + l^2\nabla\phi\cdot\nabla\,\delta\phi\,d\Omega = 0 \qquad [1.4]$$

1.1. Ansatz functions

$$\mathbf{u} = \sum_i N_i\mathbf{u}_i + \sum_i N_i F\mathbf{a}_i \qquad [1.5]$$

mit

N_i . . . are quadratic lagrange (standard) shape functions for tetrahedrons
$\mathbf{U}_i = \mathbf{u}_i, \mathbf{a}_i$ . . . are nodal degrees of freedom for displacement function
F . . . is an enrichment function (sigmoid like, depends on ϕ, later)

$$f_{\text{base}} = \sum_i N_i\phi_i \qquad [1.6]$$

$$\varsigma = \frac{f_{\text{base}}}{\sqrt[4]{f_{\text{base}}^2 + k_{\text{res}}}} \qquad [1.7]$$

mit

k_{reg} . . . small but finite parameter

$$\phi = \exp(-\varsigma) \quad [1.8]$$

$$\phi = \exp(-\frac{\varsigma}{l}) \quad [1.9]$$

we need to be able to integrate the residual vectors and the stiffness matrices efficiently and accurately

$$\delta_{\mathbf{U}_i}\Pi = \int_\Omega g(\phi)\boldsymbol{\sigma}(\mathbf{u})\frac{\partial\varepsilon}{\partial\mathbf{u}} \cdot \frac{\partial\mathbf{u}}{\partial\mathbf{U}}\,d\Omega \cdot \delta\mathbf{U}_i \quad [1.10]$$

$$\delta_{\phi_i}\Pi = \int_\Omega 2(\phi-1)\frac{\partial\phi}{\partial\phi_i}\psi(\mathbf{u})\,d\Omega\,\delta\phi_i + \frac{G_c}{l}\int_\Omega \phi\frac{\partial\phi}{\partial\phi_i} + l^2\nabla\phi\cdot\frac{\partial\nabla\phi}{\partial\phi_i}\,d\Omega\,\delta\phi_i = 0 \quad [1.11]$$

$$\Delta_{U_i}\,\delta_{U_i}\Pi = U_i \cdot \int_\Omega g(\phi)\frac{\partial\varepsilon}{\partial U_i}\cdot\mathbb{C}\cdot\frac{\partial\varepsilon}{\partial U_i}\,d\Omega\cdot\delta U_i \quad [1.12]$$

$$\Delta_{\phi_j}\,\delta_{\phi_i}\Pi = \phi_j\int_\Omega 2\left(\frac{\partial\phi}{\partial\phi_i}\right)^2\psi(u)\,d\Omega\,\delta\phi_i + \phi_j\int_\Omega 2(\phi-1)\frac{\partial^2\phi}{\partial\phi_i}\phi(u)\,d\Omega\,\delta\phi_i \quad [1.13]$$

$$+\,\phi_j\frac{G_c}{l}\int_\Omega \frac{\partial\phi}{\partial\phi_i} + \frac{\partial^2\phi}{\partial\phi_i} + l^2\frac{\partial\nabla\phi}{\partial\phi_j}\cdot\frac{\partial\nabla\phi}{\partial\phi_i} + l^2\nabla\phi\cdot\frac{\partial^2\nabla\phi}{\partial\phi_i^2}\,d\Omega\,\delta\phi_i \quad [1.14]$$