Simplex Integration

By Domain Subdivision

Inhaltsverzeichnis

Teil I.
Theory

1. Problem

Total Energy in the System:

$$\Pi = \int_\Omega g(\phi)\psi(\mathbf{u})\,d\Omega + \frac{G_c}{2l}\int_\Omega \phi^2 + l^2\nabla\phi\cdot\nabla\phi\,d\Omega \rightarrow \min \qquad [1.1]$$

Degradation Function

$$g(\phi) = \left(1-\phi^2\right)^2 + k \qquad [1.2]$$

mit

k ... being a small but finite scalar such as 10 6
G_c ... critical energy release rate, material parameter
l ... *width* of phase field
$\psi(u)$ ... strain energy density function
u ... displacement function
ϕ ... phase field parameter, ansatz function discussed below
$\nabla\phi$ ... gradient of phase field parameter

$$\delta_u\Pi = \int_\Omega g(\phi)\sigma(\mathbf{u})\frac{\partial\varepsilon}{\partial\mathbf{u}}\,\delta\mathbf{u} = 0 \qquad [1.3]$$

$$\delta_\phi\Pi = \int_\Omega 2(\phi-1)\,\delta\phi\psi(\mathbf{u})\,d\Omega + \frac{G_c}{l}\int_\Omega \phi\,\delta\phi + l^2\nabla\phi\cdot\nabla\,\delta\phi\,d\Omega = 0 \qquad [1.4]$$

1.1. Ansatz functions

$$\mathbf{u} = \sum_i N_i\mathbf{u}_i + \sum_i N_i F\mathbf{a}_i \qquad [1.5]$$

mit

N_i ... are quadratic lagrange (standard) shape functions for tetrahedrons
$\mathbf{U}_i = \mathbf{u}_i, \mathbf{a}_i$ ... are nodal degrees of freedom for displacement function
F ... is an enrichment function (sigmoid like, depends on ϕ, later)

$$f_{base} = \sum_i N_i\phi_i \qquad [1.6]$$

$$\varsigma = \frac{f_{base}}{\sqrt[4]{f_{base}^2 + k_{res}}} \qquad [1.7]$$

mit

k_{reg} . . . small but finite parameter

$$\phi = \exp(-\varsigma) \quad [1.8]$$
$$\phi = \exp(-\frac{\varsigma}{l}) \quad [1.9]$$

we need to be able to integrate the residual vectors and the stiffness matrices efficiently and accurately

$$\delta_{\mathbf{U}_i}\Pi = \int_\Omega g(\phi)\boldsymbol{\sigma}(\mathbf{u})\frac{\partial \varepsilon}{\partial \mathbf{u}} \cdot \frac{\partial \mathbf{u}}{\partial \mathbf{U}}\, d\Omega \cdot \delta \mathbf{U}_i \quad [1.10]$$

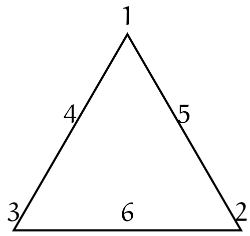
$$\delta_{\phi_i}\Pi = \int_\Omega 2(\phi-1)\frac{\partial\phi}{\partial\phi_i}\psi(\mathbf{u})\, d\Omega\, \delta\phi_i + \frac{G_c}{l}\int_\Omega \phi\frac{\partial\phi}{\partial\phi_i} + l^2\nabla\phi\cdot\frac{\partial\nabla\phi}{\partial\phi_i}\, d\Omega\, \delta\phi_i = 0 \quad [1.11]$$

$$\Delta_{U_i}\,\delta_{U_i}\Pi = U_i \cdot \int_\Omega g(\phi)\frac{\partial\varepsilon}{\partial U_i}\cdot\mathbb{C}\cdot\frac{\partial\varepsilon}{\partial U_i}\, d\Omega\cdot\delta U_i \quad [1.12]$$

$$\Delta_{\phi_j}\,\delta_{\phi_i}\Pi = \phi_j\int_\Omega 2\left(\frac{\partial\phi}{\partial\phi_i}\right)^2\psi(u)\, d\Omega\,\delta\phi_i + \phi_j\int_\Omega 2(\phi-1)\frac{\partial^2\phi}{\partial\phi_i}\phi(u)\, d\Omega\,\delta\phi_i \quad [1.13]$$
$$+\phi_j\frac{G_c}{l}\int_\Omega \frac{\partial\phi}{\partial\phi_i} + \frac{\partial^2\phi}{\partial\phi_i} + l^2\frac{\partial\nabla\phi}{\partial\phi_j}\cdot\frac{\partial\nabla\phi}{\partial\phi_i} + l^2\nabla\phi\cdot\frac{\partial^2\nabla\phi}{\partial\phi_i^2}\, d\Omega\,\delta\phi_i \quad [1.14]$$

2. Element Description

Shape Functions in barycentric coordinates



$$N_1(\xi_1,\xi_2,\xi_3) = \xi_1 \quad [2.1]$$
$$N_2(\xi_1,\xi_2,\xi_3) = \xi_2 \quad [2.2]$$
$$N_3(\xi_1,\xi_2,\xi_3) = \xi_3 \quad [2.3]$$
$$N_4(\xi_1,\xi_2,\xi_3) = 4\xi_1\xi_3 \quad [2.4]$$
$$N_5(\xi_1,\xi_2,\xi_3) = 4\xi_1\xi_2 \quad [2.5]$$
$$N_6(\xi_1,\xi_2,\xi_3) = 4\xi_2\xi_3 \quad [2.6]$$

Barycentric Interpolation Formula

$$P(\xi_1,\xi_2,\xi_3) = p_1\xi_1 + p_2\xi_2 + p_3\xi_3 \quad [2.7]$$

$\xi - \eta$-Transformation

$$\xi := \xi_1 \quad [2.8]$$
$$\eta := \xi_2 \quad [2.9]$$
$$\xi_3 := 1 - \xi - \eta \quad [2.10]$$