Simplex Integration

By Domain Subdivision

Inhaltsverzeichnis

Teil I.
Theory

1. Problem

Total Energy in the System:

$$\Pi = \int_\Omega g(\phi)\psi(\mathbf{u})\,d\Omega + \frac{G_c}{2l}\int_\Omega \phi^2 + l^2\nabla\phi\cdot\nabla\phi\,d\Omega \to \min \qquad [1.1]$$

Degradation Function

$$g(\phi) = \left(1 - \phi^2\right)^2 + k \qquad [1.2]$$

mit

   k ...being a small but finite scalar such as 10 6
   G_c ...critical energy release rate, material parameter
   l ...*width* of phase field
   $\psi(\mathbf{u})$...strain energy density function
   $\mathbf{u}$ ...displacement function
   ϕ ...phase field parameter, ansatz function discussed below
   $\nabla\phi$ ...gradient of phase field parameter

$$\delta_{\mathbf{u}}\Pi = \int_\Omega g(\phi)\sigma(\mathbf{u})\frac{\partial\varepsilon}{\partial\mathbf{u}}\,\delta\mathbf{u} = 0 \qquad [1.3]$$

$$\delta_\phi\Pi = \int_\Omega 2(\phi - 1)\,\delta\phi\,\psi(\mathbf{u})\,d\Omega + \frac{G_c}{l}\int_\Omega \phi\,\delta\phi + l^2\nabla\phi\cdot\nabla\,\delta\phi\,d\Omega = 0 \qquad [1.4]$$

1.1. Ansatz functions

$$\mathbf{u} = \sum_i N_i\mathbf{u}_i + \sum_i N_i F\mathbf{a}_i \qquad [1.5]$$

mit

   N_i ...are quadratic lagrange (standard) shape functions for tetrahedrons
   $\mathbf{U}_i = \mathbf{u}_i, \mathbf{a}_i$ ...are nodal degrees of freedom for displacement function
   F ...is an enrichment function (sigmoid like, depends on ϕ, later)

$$f_{base} = \sum_i N_i\phi_i \qquad [1.6]$$

$$\varsigma = \frac{f_{base}}{\sqrt[4]{f_{base}^2 + k_{res}}} \qquad [1.7]$$

mit

$k_{reg} \ldots$ small but finite parameter

$$\phi = \exp(-\varsigma) \tag{1.8}$$

$$\phi = \exp(-\frac{\varsigma}{l}) \tag{1.9}$$

we need to be able to integrate the residual vectors and the stiffness matrices efficiently and accurately

$$\delta_{u_i}\Pi = \int_\Omega g(\phi)\sigma(u)\frac{\partial\varepsilon}{\partial u} \cdot \frac{\partial u}{\partial U}\, d\Omega \cdot \delta U_i \tag{1.10}$$

$$\delta_{\phi_i}\Pi = \int_\Omega 2(\phi-1)\frac{\partial\phi}{\partial\phi_i}\psi(u)\, d\Omega\, \delta\phi_i + \frac{G_c}{l}\int_\Omega \phi\frac{\partial\phi}{\partial\phi_i} + l^2\nabla\phi \cdot \frac{\partial\nabla\phi}{\partial\phi_i}\, d\Omega\, \delta\phi_i = 0 \tag{1.11}$$

$$\Delta_{u_i}\, \delta_{u_i}\Pi = U_i \cdot \int_\Omega g(\phi)\frac{\partial\varepsilon}{\partial U_i} \cdot \mathbb{C} \cdot \frac{\partial\varepsilon}{\partial U_i}\, d\Omega \cdot \delta U_i \tag{1.12}$$

$$\Delta_{\phi_j}\, \delta_{\phi_i}\Pi = \phi_j \int_\Omega 2\left(\frac{\partial\phi}{\partial\phi_i}\right)^2 \psi(u)\, d\Omega\, \delta\phi_i + \phi_j \int_\Omega 2(\phi-1)\frac{\partial^2\phi}{\partial\phi_i}\phi(u)\, d\Omega\, \delta\phi_i \tag{1.13}$$

$$+ \phi_j\frac{G_c}{l}\int_\Omega \frac{\partial\phi}{\partial\phi_i} + \frac{\partial^2\phi}{\partial\phi_i} + l^2\frac{\partial\nabla\phi}{\partial\phi_j} \cdot \frac{\partial\nabla\phi}{\partial\phi_i} + l^2\nabla\phi \cdot \frac{\partial^2\nabla\phi}{\partial\phi_i^2}\, d\Omega\, \delta\phi_i$$

$$\tag{1.14}$$

2. Element Description