

# 2018-2019 期中模拟练习-陈越

浙江大学2018-19春夏《高级数据结构与算法分析》期中模拟练习-陈越



判断 10

A. 单选题 8

程序填空题 2

1-1 When measuring the relevancy of the answer set, if the precision is low but the recall is high, it means that most of the relevant documents are missing, but most of the retrieved documents are relevant. (4分)

☐ T ☒ F

1-2 In amortized analysis, a good potential function should always assume its minimum at the start of the sequence. (3分)

☒ T ☐ F

1-3 Finding the maximum key from a splay tree will result in a tree with its root having no right subtree. (4分)

☒ T ☐ F

1-4 In backtracking, if different solution spaces have different sizes, start testing from the partial solution with the largest space size would have a better chance to reduce the time cost. (3分)

☒ T ☐ F

1-5 A perfectly balanced tree forms if keys 1 to  $2^k - 1$  are inserted in order into an initially empty leftist heap. (4分)

☒ T ☐ F

1-6 In a red-black tree, the number of rotations in the DELETE operation is  $O(1)$ . (3分)

☒ T ☒ F

1-7 The time bound of the FIND operation in a B+ tree containing  $N$  numbers is  $O(\log N)$ , no matter what the degree of the tree is. (3分)

☒ T ☐ F



1-8 Making  $N$  insertions into an initially empty binomial queue takes  $\Theta(N \log N)$  time in the worst case. (3分)

☐ T ☒ F

1-9 In an AVL tree, it is possible to have this situation that the balance factors of a node and both of its children are all +1. (4分)

☒ T ☒ F

1-10 For the recurrence equation  $T(N) = aT(N/b) + f(N)$ , if  $af(N/b) = f(N)$ , then  $T(N) = \Theta(N \log_b N)$ . (4分)

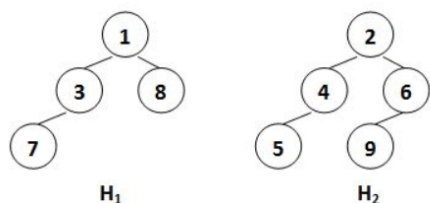
☐ T ☒ F

2-1 Delete a node  $v$  from an AVL tree  $T_1$ , we can obtain another AVL tree  $T_2$ . Then insert  $v$  into  $T_2$ , we can obtain another AVL tree  $T_3$ . Which one(s) of the following statements about  $T_1$  and  $T_3$  is(are) true? (5分)

- I. If  $v$  is a leaf node in  $T_1$ , then  $T_1$  and  $T_3$  might be different.
- II. If  $v$  is not a leaf node in  $T_1$ , then  $T_1$  and  $T_3$  must be different.
- III. If  $v$  is not a leaf node in  $T_1$ , then  $T_1$  and  $T_3$  must be the same.

- ☒ A. I only  
☐ B. II only  
☐ C. I and II only  
☐ D. I and III only

2-2 Merge the two leftist heaps in the following figure. Which one of the following statements is FALSE? (5分)



- ☐ A. the null path length of 6 is the same as that of 2  
☐ B. 1 is the root with 3 being its right child  
☐ C. Along the left most path from top down, we have 1, 2, 4, and 5  
☐ D. 6 is the left child of 2

2-3 A queue can be implemented by using two stacks  $S_A$  and  $S_B$  as follows:

- To enqueue  $x$ , we push  $x$  onto  $S_A$ .
- To dequeue from the queue, we pop and return the top item from  $S_B$ . However, if  $S_B$  is empty, we first fill it (and empty  $S_A$ ) by popping the top item from  $S_A$ , pushing this item onto  $S_B$ , and repeat until  $S_A$  is empty.

Assuming that push and pop operations take  $O(1)$  worst-case time, please select a potential function  $\phi$  which can help us prove that enqueue and dequeue operations take  $O(1)$  amortized time (when starting from an empty queue). (5分)

- ☐ A.  $\phi = |S_B|$   
☒ B.  $\phi = 2|S_B|$   
☐ C.  $\phi = |S_A|$   
☒ D.  $\phi = 2|S_A|$

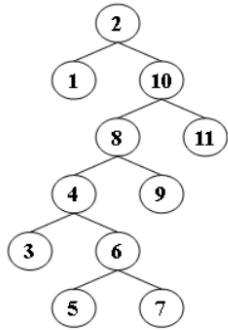
2-4 3-way-mergesort : Suppose instead of dividing in two halves at each step of the mergesort, we divide into three one thirds, sort each part, and finally combine all of them using a three-way-merge. What is the overall time complexity of this algorithm ? (5分)

- ☐ A.  $O(n(\log^2 n))$   
☐ B.  $O(n^2 \log n)$   
☒ C.  $O(n \log n)$   
☐ D.  $O(n)$

2-5 Insert  $\{ 3, 1, 4, 5, 0, 9, 2, 6, 8, 7 \}$  into an initially empty 2-3 tree (with splitting). Which one of the following statements is FALSE? (5分)

- ☐ A. 6 and 7 are in the same node  
☐ B. the parent of the node containing 5 has 3 children  
☒ C. the first key stored in the root is 6  
☐ D. there are 5 leaf nodes

2-6 For the result of accessing 5 in the splay tree in the following figure, besides saying that 5 must be the root, which one of the following statements is also TRUE? (5分)



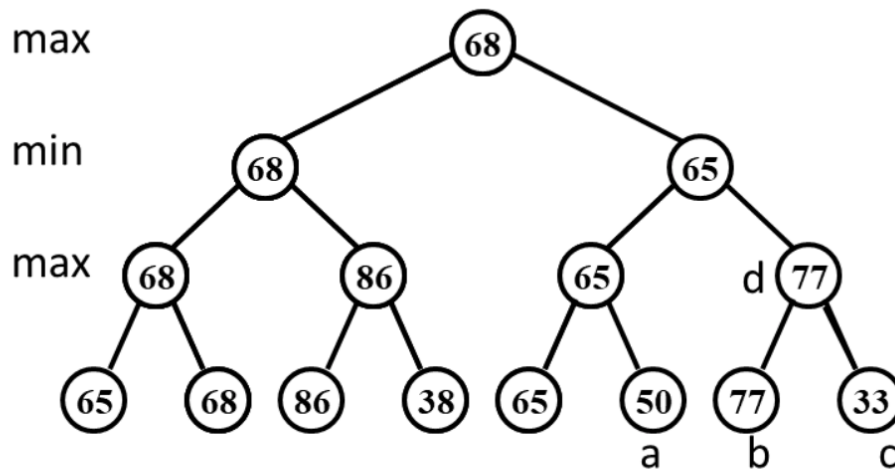
- ☐ A. 2 and 10 are siblings
- ☐ B. 4 and 10 are siblings
- ☒ C. 6 and 10 are siblings
- ☐ D. 6 is a leaf node

2-7 There are 14000 documents in the database. The statistic data for one query are shown in the following table. The recall is: \_\_ (5分)

	Relevant	Irrelevant
Retrieved	2000	6000
Not Retrieved	4000	2000

- ☐ A. 50%
- ☒ B. 33%
- ☐ C. 25%
- ☐ D. 14%

2-8 Given the following game tree, which node in the right subtree is the first node to be pruned with  $\alpha$ - $\beta$  pruning algorithm? (5分)



- ☒ A. a
- ☐ B. b
- ☒ C. c
- ☐ D. d

## 2017-2018 期中模拟

判断 11

A. 单选题 6

程序填空题 2

1-1 To solve a problem by dynamic programming instead of recursions, the key approach is to store the results of computations for the subproblems so that we only have to compute each different subproblem once. Those solutions can be stored in an array or a hash table. (3分)

☒ T ☐ F

1-2 In the 4-queens problem,  $(x_1, x_2, x_3, x_4)$  correspond to the 4 queens' column indices. During backtracking,  $(1, 4, 2, ?)$  will be checked before  $(1, 3, 4, ?)$ , and none of them has any solution in their branches. (4分)

☐ T ☒ F

1-3 For the recurrence equation  $T(N) = aT(N/b) + f(N)$ , if  $af(N/b) = Kf(N)$  for some constant  $K > 1$ , then  $T(N) = \Theta(f(N))$ . (5分)

☐ T ☒ F

1-4 In a B+ tree, leaves and nonleaf nodes have some key values in common. (3分)

☒ T ☐ F

1-5 Word stemming is to eliminate the commonly used words from the original documents. (3分)

☐ T ☒ F

1-6 For one operation, if its worst-case time bound is  $\Theta(\log N)$ , then its amortized time bound must be  $O(\log N)$ . (3分)

☒ T ☐ F

1-7 In a red-black tree, an internal red node cannot be a node of degree 1. (4分)

☒ T ☐ F

1-8 Insert { 1, 2, 5, 3, 8, 4, -1, 10, 128, 34, 15, 63, 18, -24, 186 } into an initially empty binomial queue, the resulting roots are 186, -24, 15 and -1. (5分)

☒ T ☐ F

1-9 All of the Zig, Zig-zig, and Zig-zag rotations not only move the accessed node to the root, but also roughly half the depth of most nodes on the path. (3分)

☒ T ☐ F

1-10 While accessing a term, hashing is faster than search trees. (3分)

☐ T ☒ F

1-11 With the same operations, the resulting skew heap is always more balanced than the leftist heap. (3分)

☐ T ☒ F

1-1 For any node in an AVL tree, the height of the left subtree must be greater than that of the right subtree. (3分)

☐ T ☒ F

1-2 Word stemming is to eliminate the commonly used words from the original documents. (3分)

☐ T ☒ F

1-3 Given that problem A is NP-complete. If problem B is in NP and can be polynomially reduced to problem A, then problem B is NP-complete. (3分)

☒ T ☐ F

1-4 If a problem can be solved by dynamic programming, it must be solved in polynomial time. (3分)

☐ T ☒ F

1-5 For one operation, if its amortized time bound is  $O(\log N)$ , then its worst-case time bound must be  $O(\log N)$ . (3分)

☐ T ☒ F

1-6 In a red-black tree, the number of internal nodes in the subtree rooted at  $x$  is no more than  $2^{bh(x)} - 1$  where  $bh(x)$  is the black-height of  $x$ . (3分)

☒ T ☐ F

1-7 For one operation, if its worst-case time bound is  $\Theta(\log N)$ , then its amortized time bound must be  $O(\log N)$ . (3分)

☒ T ☐ F

1-8 When measuring the relevancy of the answer set, if the precision is high but the recall is low, it means that most of the relevant documents are retrieved, but too many irrelevant documents are returned as well. (3分)

☐ T ☒ F

1-9 For any node in an AVL tree, the left and right subtrees must have the same height. (3分)

☐ T ☒ F

1-10 The right path of a skew heap can be arbitrarily long. (3分)

☒ T ☐ F

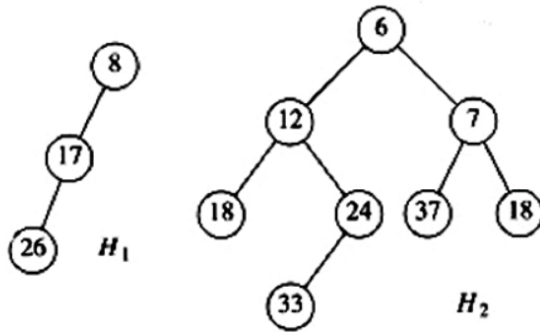
1-11 All the languages can be decided by a non-deterministic machine. (3分)

☐ T ☒ F

1-12 In a B+ tree, leaves and nonleaf nodes have some key values in common. (3分)

☒ T ☐ F

2-1 Merge the two leftist heaps in the following figure. Which one of the following statements is FALSE? (4分)



- ☐ A. 6 is the root with 7 being its right child
- ☒ B. 37 is the left child of 7
- ☐ C. the depths of 24 and 8 are the same
- ☐ D. the null path length of 7 is the same as that of 12

2-2 A queue can be implemented by using two stacks  $S_A$  and  $S_B$  as follows:

- To enqueue  $x$ , we push  $x$  onto  $S_A$ .
- To dequeue from the queue, we pop and return the top item from  $S_B$ . However, if  $S_B$  is empty, we first fill it (and empty  $S_A$ ) by popping the top item from  $S_A$ , pushing this item onto  $S_B$ , and repeat until  $S_A$  is empty.

Assuming that push and pop operations take  $O(1)$  worst-case time, please select a potential function  $\phi$  which can help us prove that enqueue and dequeue operations take  $O(1)$  amortized time (when starting from an empty queue). (6分)

- ☒ A.  $\phi = 2|S_A|$
- ☐ B.  $\phi = |S_A|$
- ☐ C.  $\phi = 2|S_B|$
- ☐ D.  $\phi = |S_B|$

2-3 When solving a problem with input size  $N$  by divide and conquer, if at each stage the problem is divided into 4 sub-problems of equal size  $N/5$ , and the conquer step takes  $O(\log N)$  to form the solution from the sub-solutions, then the overall time complexity is: (5分)

- ☐ A.  $O(N)$
- ☒ B.  $O(N^{\log 4 / \log 5})$
- ☐ C.  $O(\log^2 N)$
- ☐ D.  $O(\log N)$

2-4 Which one of the following statements is TRUE? (4分)

- ☐ A. The relationship of skew heaps to leftist heaps is analogous to the relation between splay trees and AVL trees
- ☐ B. For leftist heaps and skew heaps, the worst-case running time of a single insertion are both  $O(N)$
- ☐ C. With the same operations, the resulting skew heap is always more balanced than the leftist heap
- ☐ D. None of the above is true

2-5 Insert  $\{2, 9, 6, 7, 0, 3, 8, 10\}$  into an initially empty 2-3 tree (with splitting), and then delete 7. Which one of the following statements is FALSE about the resulting tree? (5分)

- ☒ A. there are 4 leaf nodes
- ☐ B. the parent of the node containing 2 has 3 children
- ☐ C. the first key stored in the root is 3
- ☐ D. 9 and 8 are in the same node

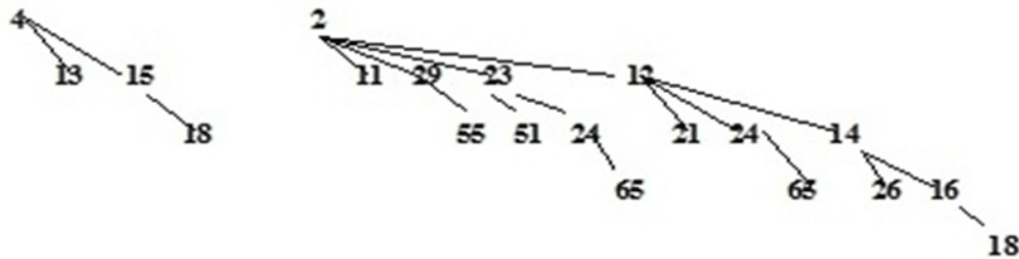
2-6 Rod-cutting Problem: Given a rod of total length  $N$  inches and a table of selling prices  $P_L$  for lengths  $L = 1, 2, \dots, M$ . You are asked to find the maximum revenue  $R_N$  obtainable by cutting up the rod and selling the pieces. For example, based on the following table of prices, if we are to sell an 8-inch rod, the optimal solution is to cut it into two pieces of lengths 2 and 6, which produces revenue  $R_8 = P_2 + P_6 = 5 + 17 = 22$ . And if we are to sell a 3-inch rod, the best way is not to cut it at all.

Length $L$	1	2	3	4	5	6	7	8	9	10
Price $P_L$	1	5	8	9	10	17	17	20	23	28

Which one of the following statements is FALSE? (5分)

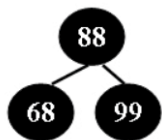
- ☐ A. This problem can be solved by dynamic programming
- ☐ B. The time complexity of this algorithm is  $O(N^2)$
- ☐ C. If  $N \leq M$ , we have  $R_N = \max\{P_N, \max_{1 \leq i < N} \{R_i + R_{N-i}\}\}$
- ☒ D. If  $N > M$ , we have  $R_N = \max_{1 \leq i < N} \{R_i + R_{N-M}\}$

2-7 Delete the minimum number from the given binomial queue in the following figure. Which one of the following statements is FALSE? (5分)



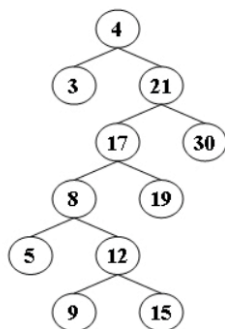
- ☐ A. there are three binomial trees after deletion, which are  $B_0$ ,  $B_1$  and  $B_4$
- ☐ B. 11 is the root of a binomial tree
- ☒ C. 29 and 23 are both children of 4
- ☐ D. 23 is not the root of any resulting binomial tree

2-8 Starting from the red-black tree given in the figure, after successively inserting the keys  $\{80, 70, 35\}$ , which one of the following statements is FALSE? (4分)



- ☐ A. 35 is the deepest red node
- ☐ B. there are two red nodes
- ☐ C. 70 is the parent of 80
- ☒ D. 70 and 99 are siblings, and they are both black

2-9 For the result of accessing 9 in the splay tree in the following figure, besides saying that 9 must be the root, which one of the following statements is also TRUE? (4分)



- ☐ A. 12 is a leaf node
- ☐ B. 4 and 21 are siblings
- ☐ C. 8 and 21 are siblings
- ☒ D. 12 and 21 are siblings

2-10 We can perform BuildHeap for leftist heaps by considering each element as a one-node leftist heap, placing all these heaps on a queue, and performing the following step: Until only one heap is on the queue, dequeue two heaps, merge them, and enqueue the result. Which one of the following statements is FALSE? (5分)

- ☐ A. in the  $k$ -th run,  $\lceil N/2^k \rceil$  leftist heaps are formed, each contains  $2^k$  nodes
- ☐ B. the worst case is when  $N = 2^K$  for some integer  $K$
- ☐ C. the time complexity  $T(N) = O(\frac{N}{2} \log 2^0 + \frac{N}{2^2} \log 2^1 + \frac{N}{2^3} \log 2^2 + \cdots + \frac{N}{2^K} \log 2^{K-1})$  for some integer  $K$  so that  $N = 2^K$
- ☒ D. the worst case time complexity of this algorithm is  $\Theta(N \log N)$

2-11 If the depth of an AVL tree is 6 (the depth of an empty tree is defined to be  $-1$ ), then the minimum possible number of nodes in this tree is: (5分)

- ☐ A. 12
- ☐ B. 20
- ☒ C. 33
- ☐ D. 64