

**浙江大学 2010 – 2011 学年冬季学期**  
**《高级数据结构与算法分析》课程期末考试试卷**

课程号：\_\_\_\_\_，开课学院：软件学院、计算机学院、竺可桢学院

考试试卷： A 卷、B 卷（请在选定项上打 ）

考试形式： 闭、开卷（请在选定项上打 ），允许带\_\_\_无\_\_\_入场

考试日期： 2011 年 1 月 18 日，考试时间： 120 分钟

**诚信考试，沉着应考，杜绝违纪。**

考生姓名：\_\_\_\_\_学号：\_\_\_\_\_所属院系：\_\_\_\_\_

题序	一	二	三	四	总分
得分					
评卷人					

**Answer Sheet**

Part I				
1 .	2 .	3 .	4 .	5 .
6 .	7 .	8 .	9 .	10 .
Part II				
<div>1.</div> <div>① _____</div> <div>② _____</div> <div>③ _____</div>		<div>2.</div> <div>① _____</div> <div>② _____</div>		
Part III				
<div>1.</div>				

2.

3.

4.

5.

Part IV

**NOTE: Please write your answers on the answer sheet.**

**注意：请将答案填写在答题纸上。**

**I. Please fill in the blanks ( the answer for each blank is unique ). ( 2 points each )**

1. Given A as the root of a subtree in an AVL-tree. Suppose that before insertion, the balance factors of left child and right child of A are -1 and 0 respectively. If inserting a new node X to that subtree will make the tree unbalanced, what kind of rotation should be taken to keep the subtree balanced?  
a. RR rotation    b. LL rotation    c. LR rotation    d. RL rotation
2. Given a 12-node splay tree with keys 1, 2, ..., 11, and 12. What will the splay tree look like after Find(1), Find(2), ... Find(11), and Find(12)?  
a. a complete tree of 12 nodes    b. a tree with height 11  
c. a tree with height 6    d. depends on the initial tree structure
3. Which one of the following is **NOT** true about B+ trees and AVL trees: they are both --  
a. balanced binary trees    b. fit for sequential searches  
c. fit for random searches    d. fit for range searches
4. For a skew heap, which statement is **NOT** correct?  
a. Skew heap may or may not be a leftist heap  
b. The right path of a skew heap can be arbitrarily long  
c. Comparing to leftist heaps, skew heaps are always more efficient in running time for every merge  
d. Comparing to leftist heaps, skew heaps are always more efficient in space
5. To delete the minimum key from a binomial queue H, there are 4 steps to go:  
Step 1: FindMin in  $B_k$ ;  
Step 2: Remove  $B_k$  from H and get  $H'$ ;  
Step 3: Remove root from  $B_k$  and get  $H''$ ;  
Step 4: Merge ( $H'$ ,  $H''$ ).  
Which step(s) will **have to** take  $O(\log N)$  time?  
a. only 1    b. 1 and 2    c. 3 and 4    d. only 4
6. Given 3 items and a knapsack with capacity 25. The items have weights 20, 15 and 10, and profits 23, 18 and 11, respectively. The percentages of the items that should be packed to obtain maximum profit is \_\_\_\_\_.  
a. 0, 100%, 100%    b. 100%, 33.33%, 0  
c. 100%, 0, 50%    d. 50%, 100%, 0
7. Which one of the following structures has the property that any M consecutive insertions starting from empty one take at most  $O(M \log N)$  time and a single insertion takes at most  $O(\log N)$  time?  
a. Splay tree    b. Skew heap    c. Leftist heap    d. Binomial Queue

8. To solve a problem with input size  $N$  by divide and conquer algorithm, among the following methods, \_\_\_\_ is the worst.
- divide into 3 sub-problems of equal complexity  $N/2$  and conquer in  $O(N)$
  - divide into 3 sub-problems of equal complexity  $N/3$  and conquer in  $O(N\log N)$
  - divide into 2 sub-problems of equal complexity  $N/3$  and conquer in  $O(N)$
  - divide into 2 sub-problems of equal complexity  $N/3$  and conquer in  $O(N\log N)$
9. The problem of "N queens" is to place  $N$  queens on an  $N \times N$  chessboard such that no two queens attack. If the problem is to be solved by backtracking method, we need to check \_\_\_\_\_ edges of the game tree with  $N=3$  to see that there is no solution.
- 9
  - 11
  - 13
  - 15
10. Which of the following statements is **NOT** correct?
- Undirected Hamilton Circuit and satisfiability problem are NP-complete.
  - Although a nondeterministic Turing Machine can always choose the correct step to solve problems, we cannot use it to solve undecidable problems.
  - The problem of determining whether a graph does not have a Hamiltonian cycle is in NP-class.
  - If we can solve any NP-complete problem in polynomial time, then we will be able to solve, in polynomial time, all the problems in NP.

## II. Given the function descriptions of the following two (pseudo-code) programs, please fill in the blank lines. (15 points)

1. The function is to merge two leftist heaps  $H1$  and  $H2$ . (9 points)

```

PriorityQueue Merge( PriorityQueue H1, PriorityQueue H2 )
{
    if (H1==NULL) return H2;
    if (H2==NULL) return H1;
    if ( _____ )
        swap(H1, H2); //swap H1 and H2
    if ( H1->Left == NULL )
        _____;
    else {
        H1->Right = Merge( H1->Right, H2 );
        if ( H1->Left->Npl < H1->Right->Npl )
            SwapChildren( H1 ); //swap the left child and right child of H1
        _____;
    }
    return H1;
}

```

2. The function is to insert  $X$  into a binomial queue  $H$ . (6 points)

```

BinQueue Insert( ElementType X, BinQueue H )
{
    BinTree Carry;
    int i;

    H->CurrentSize ++;
    Carry = malloc( sizeof( struct BinNode ) );
    Carry->Element = X;
    Carry->LeftChild = Carry->NextSibling = NULL;
}

```

```

i = 0;
while ( H->TheTrees[ i ] ) {
    Carry = CombineTrees( Carry, _____ ); //combine two equal-sized trees
    H->TheTrees[ i ++ ] = NULL;
}
_____ ;
return H;
}

```

### III. Please write or draw your answers for the following problems on the answer sheet. (50 points)

1. What is the height of the highest AVL-tree of 12 nodes? (3 points)

Please draw the highest AVL-tree with keys 1, 2, ..., 12, and at the mean time, keep the tree as a **leftist-tree** as well. (5 points)

2. Given the keywords and searching frequencies, in order to minimize the expected total access time, there is a **greedy** way to construct the binary search tree: first arrange all the keywords in descendent order of frequencies; and then insert the keywords in that order to an initially empty AVL-tree. Given the following keywords and their frequencies:

keywords	break	case	char	do	return	switch	void
frequencies	22	18	20	5	25	2	8

Please:

(1) show the AVL-tree obtained by the above greedy method; and (5 points)

(2) give the total search cost of the constructed search tree. (4 points)

(3) Compare this AVL-tree with the optimal binary search tree and tell the difference. (5 points)

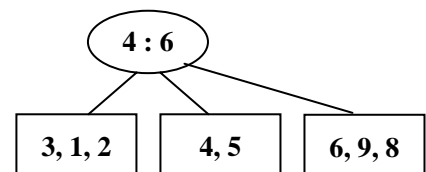
3. Given a 2-3 tree as shown in the figure.

Please:

(1) show the result of inserting 7 with splitting strategy; (2 points)

(2) show the result of deleting 4 and then 5 from the tree obtained in (1); (2 points) and

(3) show the result of deleting 1, 2, 8 and then 7 from the tree obtained in (2). (2 points)



4. (1) Please briefly describe the process of building an inverted file index. (4 points)

(2) Given 1 billion files of 10KB each, and with 500 million distinct terms. Suppose that the posting list of each term is a linked list with the structure:

DocID (4 bytes)	TermPosition (2 bytes)	NextPointer (4 bytes)
--------------------	---------------------------	--------------------------

How many bytes are required to store the index? How about the files? (2 points)

(3) If you only have *some* personal computers with 1GB memory and 10GB hard disks, how would you handle this indexing problem? (4 points)

5. Given 8 characters and their frequencies as the following:

character	a	b	c	d	e	f	g	h
frequency	13	7	9	15	26	14	11	5

Please:

- (1) draw the corresponding Huffman tree with the weight on the left child being no more than that on the right child for every node; (5 points)
- (2) give the Huffman code of each character following the "left-0 right-1" rule; (4 points) and
- (3) decode "100010111100111110". (3 points)

#### IV. Partial Sum

You are given a sequence of positive integers  $a_1, a_2, \dots, a_N$ , and a positive integer  $B$ . Your goal is to determine if some subsequence of  $a_1, a_2, \dots, a_N$  sums to exactly  $B$ . In other words, determine if there is a subset  $S$  of (not necessarily consecutive) indices so that  $\sum_{i \in S} a_i = B$ . Please give an algorithm description for solving this problem with **no more than  $O(NB)$  complexity**. (15 points)

*Hint: Consider dynamic programming with subsequence of  $a_1, \dots, a_i$  summing up to exactly  $b$  for every  $i \in \{1, \dots, N\}$  and  $b \in \{1, \dots, B\}$ .*