

1. (1) 由于 $1 = \int_{-\infty}^{+\infty} f(x)dx = \int_0^3 cx^2 dx = 9c$, 所以, $c=1/9$.

$$(2) P\{1 < X < 2\} = \int_1^2 f(x)dx = \int_1^2 x^2 / 9 dx = 7/27$$

$$P\{X \leq 1\} = \int_{-\infty}^1 f(x)dx = \int_0^1 x^2 / 9 dx = 1/27$$

$$P\{X=2\}=0.$$

2. (1) X 的密度函数为: $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & \text{其它} \end{cases}$

$$(2) P\{-1 < X < 2\} = P\{0 \leq X < 2\} = 1 - e^{-4} \approx 0.9817$$

$$p\{1 < x < 3\} = e^{-2} - e^{-6} \approx 0.1329$$

$$p\{x \leq 5\} = 1 - e^{-10} \approx 0.99995$$

$$p\{x < 4\} = e^{-8} \approx 0.0003355$$

3. 解 由于 $P\{X < 0.1\} = \int_{-\infty}^{0.1} f(x)dx = \int_0^{0.1} 2x dx = 0.01$

所以, $V_n \sim B(n, 0.01)$, 故, V_n 的分布律为:

$$P\{V_n = k\} = C_n^k \times 0.01^k \times 0.99^{n-k}, \quad k=0, 1, 2, \dots, n$$

4. (1) 由于, $F(+\infty)=1$, 所以 $A=1$.

又由 $F(0+)=F(0)$ 得 $B=-1$.

$$(2) P\{-\sqrt{2} < X < \sqrt{2}\} = F(\sqrt{2}) - F(-\sqrt{2}) = 1 - e^{-1}$$

(3) 由于 $F(x)$ 是连续的, 所以 X 是连续型随机变量.

X 的密度函数为:

$$f(x) = F'(x) = \begin{cases} xe^{-x^2/2}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

5. 解 对任意 $0 < x < 2$, 有

$$FY(x) = P\{Y \leq x\} = P\{X \leq x/2\} = \int_{-\infty}^{x/2} f(t)dt = \int_0^{x/2} 2t dt = \frac{x^2}{4}$$

所以, $f_Y(x) = F'_Y(x) = x/2$. 即 Y 的密度函数为:

$$f_Y(x) = \begin{cases} \frac{x}{2}, & 0 < x < 2 \\ 0, & \text{其它} \end{cases}$$

对任意 $0 < x < 1$, 有

$$FZ(x) = P\{Z \leq x\} = P\{X \geq 1 - x\} = \int_{1-x}^1 2t dt = 1 - (1 - x)^2$$

所以, $f_Z(x) = F'_Z(x) = 2(1 - x)$. 即 Z 的密度为:

$$f_Z(x) = \begin{cases} 2(1 - x), & 0 < x < 1 \\ 0, & \text{其它} \end{cases}$$

对任意 $0 < x < 1$, 有

$$FU(x) = P\{U \leq x\} = P\{-x/2 \leq X \leq x/2\} = \int_{-x/2}^{x/2} 2t dt = x$$

所以, $f_U(x) = F'_U(x) = 1$. 即 U 的密度函数为:

$$f_U(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{其它} \end{cases}$$

6. 解 由于 $X < 2$ 时 $Y = X$, $X \geq 2$ 时 $Y = 2$. 所以

$$F_Y(y) = \begin{cases} 0, & y < 0 \\ 1 - e^{-5y}, & 0 \leq y < 2 \\ 1, & y \geq 2 \end{cases}$$