Math 300 Cheat Sheet

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1 Propositional Logic

1.1 Logical Equivalencies

De Morgan's Laws

$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$

$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

Other Equivalencies

$P \to Q \equiv \neg Q \to \neg P$ (contrapositive)

$P \wedge P \equiv P \vee P \equiv P$

$\neg\neg P \equiv P$

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

$P \to Q \equiv \neg P \vee Q$

$\neg(P \to Q) \equiv P \wedge \neg Q$

1.2 Quantifiers

De Morgan's Laws for Quantifiers

$\neg \forall x P(x) \equiv \exists x \neg P(x)$

$\neg \exists x P(x) \equiv \forall x \neg P(x)$

2 Set Theory

2.1 Definitions

Injective means one to one
Surjective means onto

Laws

Identity Laws	$A \cup \emptyset = A$	$A \cap U = A$
Domination Laws	$A \cup U = U$	$A \cap \emptyset = \emptyset$
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative Laws	$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cap (B \cap C) = (A \cap B) \cap C$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Basic Properties	$(A^{\mathsf{c}})^{\mathsf{c}} = A$	$A - B = A \cap (B^{\mathsf{c}})$
Empty/Universal Set Properties	$A - \emptyset = A$	$A - U = \emptyset$
	$\emptyset^{\mathsf{c}} = U$	$U^{\mathsf{c}} = \emptyset$
De Morgan's Laws	$(A \cap B)^{\mathsf{c}} = A^{\mathsf{c}} \cup B^{\mathsf{c}}$	$(A \cup B)^{\mathsf{c}} = A^{\mathsf{c}} \cap B^{\mathsf{c}}$
Subsets/Complements	$A \subseteq B$ if and only if $B^{\mathsf{c}} \subseteq A^{\mathsf{c}}$	
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Complement Laws	$A \cup A^{\mathsf{c}} = U$	$A \cap A^{\mathsf{c}} = \emptyset$

Notation

Name	**Meaning**	**Symbol**	**LaTeX**
Empty set	The set containing zero elements	$\emptyset$ or $\{\}$	\emptyset
In	a is an element of B	$a \in B$	\in
Not in	a is not an element of B	$a \notin B$	\notin
Subset	All elements of A are in B	$A \subseteq B$	\subseteq
Proper subset	A is a subset of B but not equal to B	$A \subset B$	\subset
Universal set	Set of all possible elements	U	
Union	Elements in either A or B or both	$A \cup B$	\cup
Intersection	Elements in both A and B	$A \cap B$	\cap
Set difference	Elements in A that are not in B	$A \setminus B$	\setminus
Complement (sets)	Set difference $U \setminus A$	$\bar{A}$ or A^{c}	\bar{A} or A^ c
Power set	Set of all possible subsets of A	$P(A)$	
Cardinality	Number of distinct elements in A	$\|A\|$ or card(A)	

3 Axioms of $\mathbb{R}$ (the real numbers)

1	Closure	$a + b \in \mathbb{R}$, $ab \in \mathbb{R}$
2	Commutativity	$a + b = b + a$, $ab = ba$
3	Associativity	$(a + b) + c = a + (b + c)$, $(ab) \cdot c = a \cdot (bc)$
4	Distributivity	$(a + b) \cdot c = ac + bc$, $a \cdot (b + c) = ab + ac$
5	Existence of identities	There are two different numbers, 0 and 1, such that for all real numbers $a \in \mathbb{R}$, $a + 0 = a$ and $a \cdot 1 = a$
6	Additive inverse	There exists a number $-a \in \mathbb{R}$ such that $a + (-a) = 0$
7	Multiplicative inverse	For all $a \in \mathbb{R}$ with a $\neq 0$, there exists a number $a^{-1} \in \mathbb{R}$ such that $a \cdot a^{-1} = 1$

Also

- $\mathbb{N}$ (natural numbers - 1 and up) satisfies 1 - 4
- $\mathbb{Z}$ (integers) satisfies 1 - 6
- $\mathbb{Q}$ (rational numbers) satisfies 1 - 7

$a + (-b)$ can be written as $a - b$

Equality axioms for $\mathbb{R}$ (also holds for sets)

8	Reflexivity	$a = a$
9	Symmetry	If $a = b$, then $b = a$
10	Transitivity	If $a = b$ and $b = c$, then $c = a$
11	Substitution	If $a = b$, then b may be substituted for a without changing the truth value

Order axioms

12	Positive Closure	If a, $b \in \mathbb{R}$, and a, $b > 0$, then $a + b$ and $a \cdot b$ are positive
13	Trichotomy	If $a \in \mathbb{R}$, then exactly one of the following is true: a is positive, a is negative, $a = 0$
14	Well-Ordering Property	A set of positive integers has a minimum element.