

Math 300 Cheat Sheet

Chenkai Luo

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1 Propositional Logic

1.1 Logical Equivalencies

De Morgan's Laws

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

Other Equivalencies

$$P \rightarrow Q \equiv \neg Q \rightarrow \neg P \text{ (contrapositive)}$$

$$P \wedge P \equiv P \vee P \equiv P$$

$$\neg \neg P \equiv P$$

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

$$P \rightarrow Q \equiv \neg P \vee Q$$

$$\neg(P \rightarrow Q) \equiv P \wedge \neg Q$$

1.2 Quantifiers

De Morgan's Laws for Quantifiers

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

2 Set Theory

2.1 Definitions

Injective means one to one

Surjective means onto

Laws

Identity Laws	$A \cup \emptyset = A$	$A \cap U = A$
Domination Laws	$A \cup U = U$	$A \cap \emptyset = \emptyset$
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative Laws	$A \cup (B \cap C) = (A \cup B) \cap C$	$A \cap (B \cap C) = (A \cap B) \cap C$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Basic Properties	$(A^c)^c = A$	$A - B = A \cap (B^c)$
Empty/Universal Set Properties	$A - \emptyset = A$	$A - U = \emptyset$
	$\emptyset^c = U$	$U^c = \emptyset$
De Morgan's Laws	$(A \cap B)^c = A^c \cup B^c$	$(A \cup B)^c = A^c \cap B^c$
Subsets/Complements	$A \subseteq B$ if and only if $B^c \subseteq A^c$	
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Complement Laws	$A \cup A^c = U$	$A \cap A^c = \emptyset$

Notation

Name	Meaning	Symbol	LaTeX
Empty set	The set containing zero elements	\emptyset OR $\{\}$	<code>\emptyset</code>
In	a is an element of b	$a \in b$	<code>\in</code>
Not in	a is not an element of b	$a \notin b$	
Subset	All elements of a are in b	$a \subseteq b$	<code>\subseteq</code>
Proper subset	A is a subset of b but not equal to b	$a \subset b$	<code>\subset</code>

Universal set	Set of all possible elements	U	
Union	Elements in either A or B or both	$A \cup B$	
Intersection	Elements in both A and B	$A \cap B$	
Set difference	Elements in A that are not in B	$A - B$	
Complement (sets)	Set difference $U - A$	\bar{A} or A^c	
Power set	Set of all possible subsets of A	$P(A)$	
Cardinality	Number of distinct elements in A	$ A $ or $\text{card}(A)$	

3 Axioms of \mathbb{R} (the real numbers)

1	Closure	$a + b \in \mathbb{R}, ab \in \mathbb{R}$
2	Commutativity	$a + b = b + a, ab = ba$
3	Associativity	$(a + b) + c = a + (b + c), (ab) * c = a * (bc)$
4	Distributivity	$(a + b) * c = ac + bc, a*(b + c) = ab + ac$
5	Existence of identities	There are two different numbers, 0 and 1, such that for all real numbers $a \in \mathbb{R}$ such that $a + 0 = a$ and $a * 1 = a$
6	Additive inverse	There exists a number $-a \in \mathbb{R}$ such that $a + (-a) = 0$
7	Multiplicative inverse	For all $a \in \mathbb{R}$ with $a \neq 0$, there exists a number $a^{-1} \in \mathbb{R}$ such that $a * a^{-1} = 1$

Also

- \mathbb{N} (natural numbers - 1 and up) satisfies 1 - 4
- \mathbb{Z} (integers) satisfies 1 - 6
- \mathbb{Q} (rational numbers) satisfies 1 - 7

$a + (-b)$ can be written as $a - b$

Equality axioms for \mathbb{R} (also holds for sets)

8	Reflexivity	$a = a$
9	Symmetry	If $a = b$, then $b = a$
10	Transitivity	If $a = b$ and $b = c$, then $c = a$
11	Substitution	If $a = b$, then b may be substituted for a without changing the truth value

Order axioms

12	Positive Closure	If $a, b \in \mathbb{R}$, and $a, b > 0$, then $a + b$ and $a * b$ stays positive
13	Trichotomy	If $a \in \mathbb{R}$, then exactly one of the following is true: a is positive, a is negative, a equals 0
14	Well-Ordering Property	A set of positive integers has a minimum element.