

CSU44054/CS7GV4:

~~Augmented~~ Reality

Gareth W. Young

*extended*



# The Geometry of Virtual Worlds

- Most of the matrix transforms appear in standard computer graphics texts.

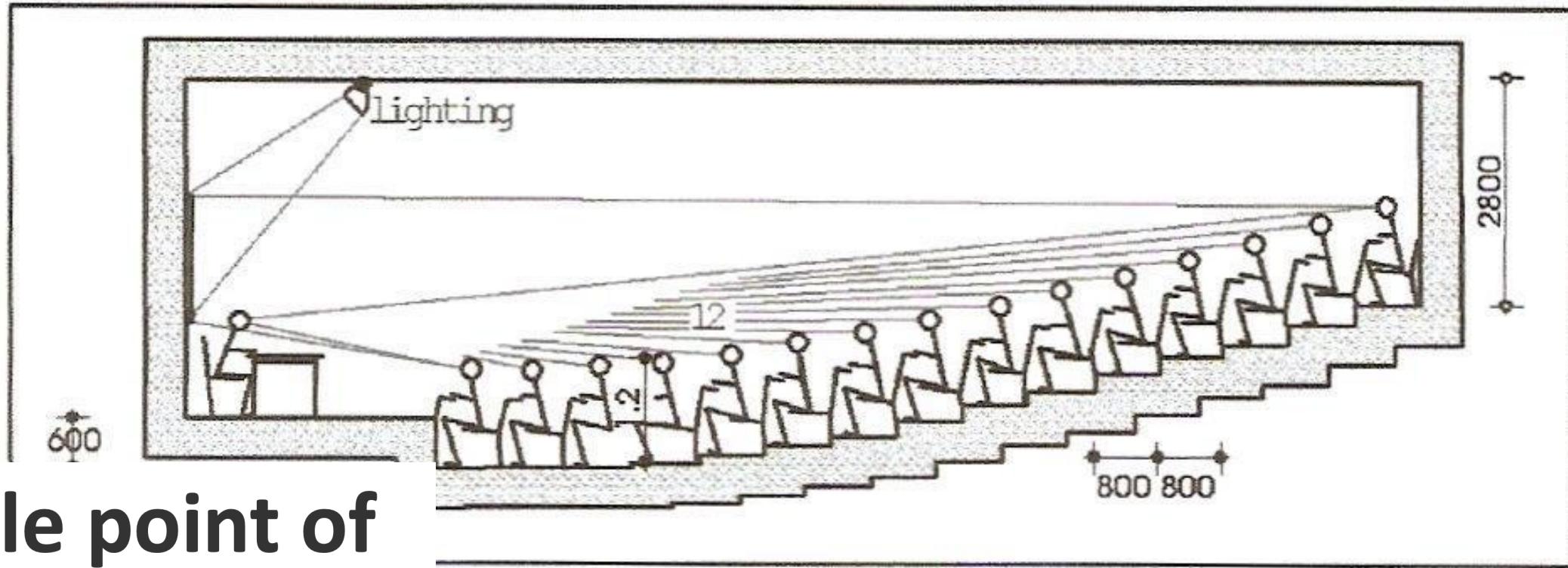


# What do we see?

- 3D world
- Single point of observation

# What do we see?

## 3D World



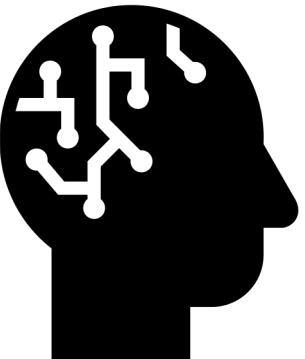
## Single point of observation

# What do we see?

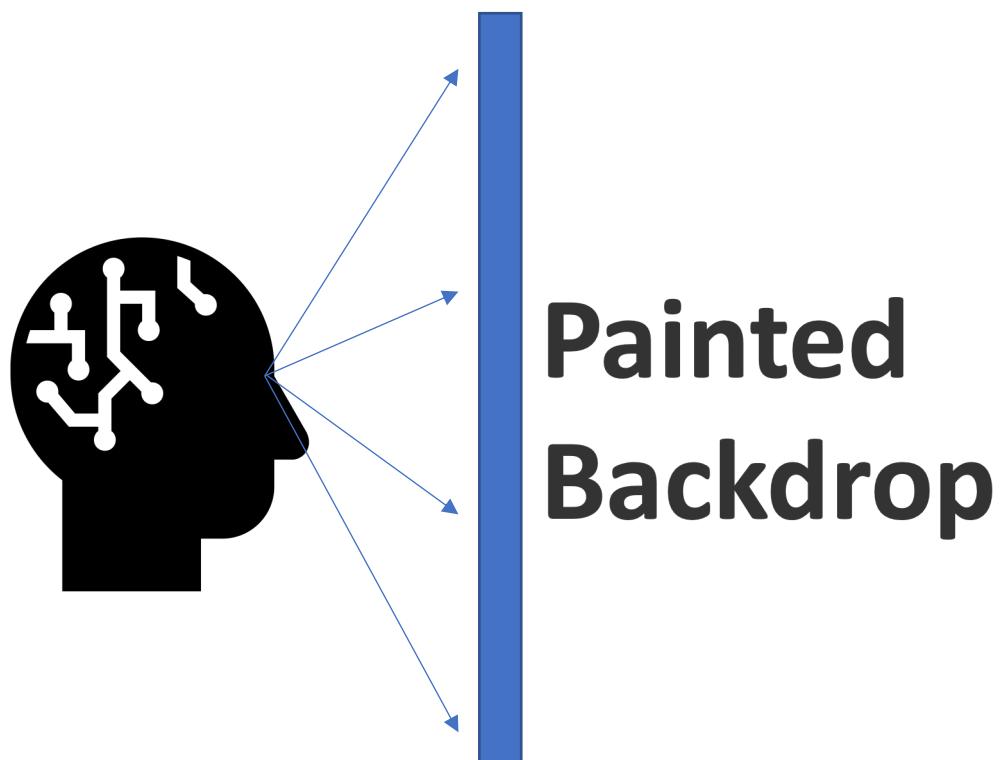
2D World



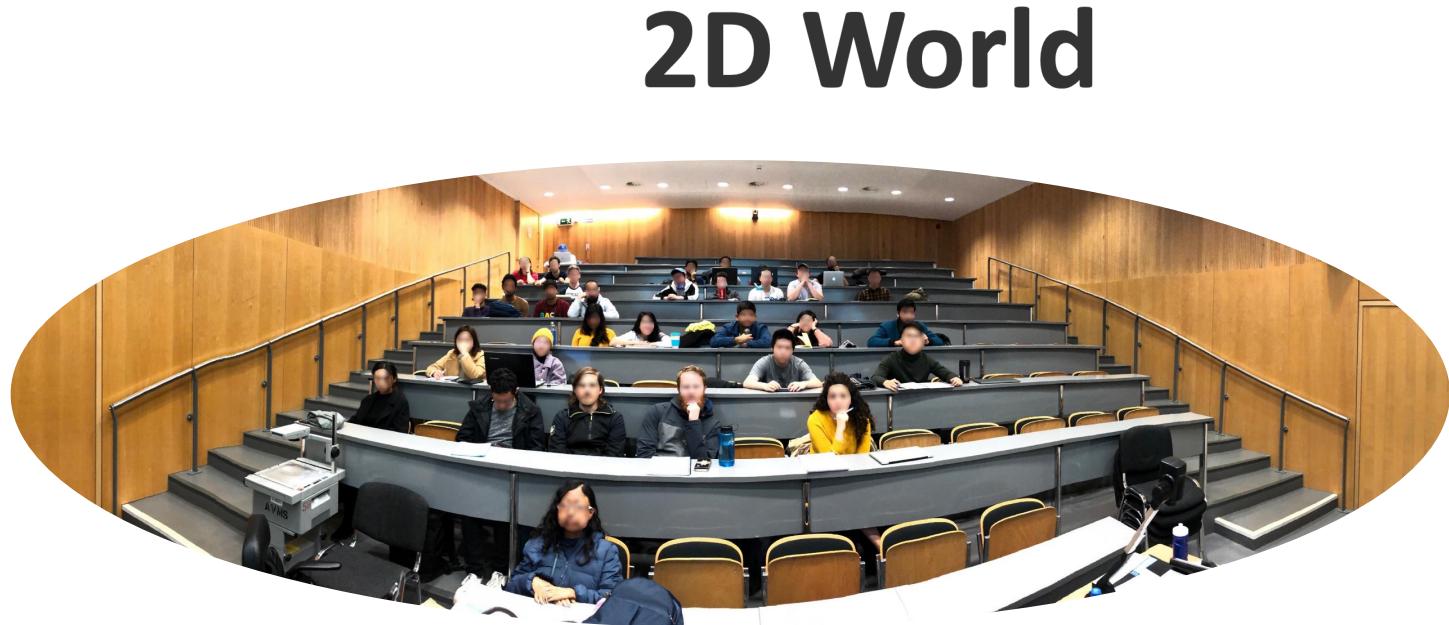
Single point of  
observation



# What do we see?



**Single point of  
observation**

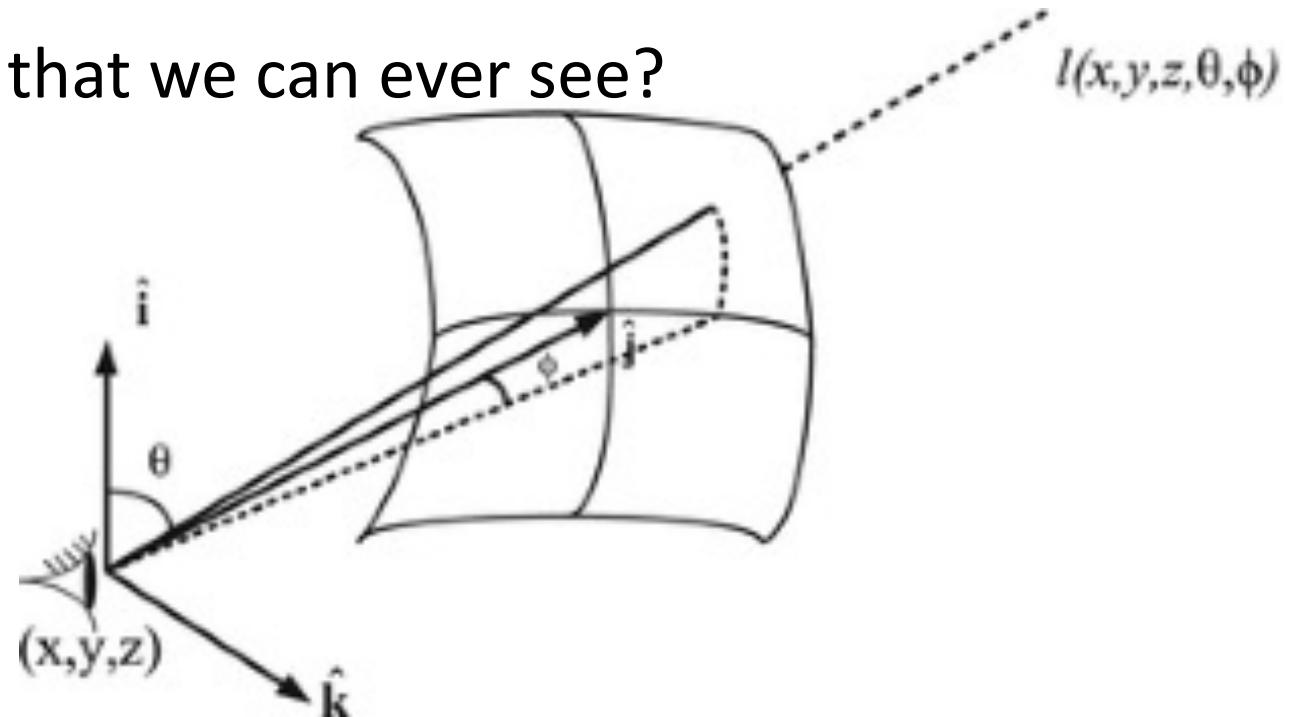




What do I see?

# What do we see?

Q: What is the set of all things that we can ever see?



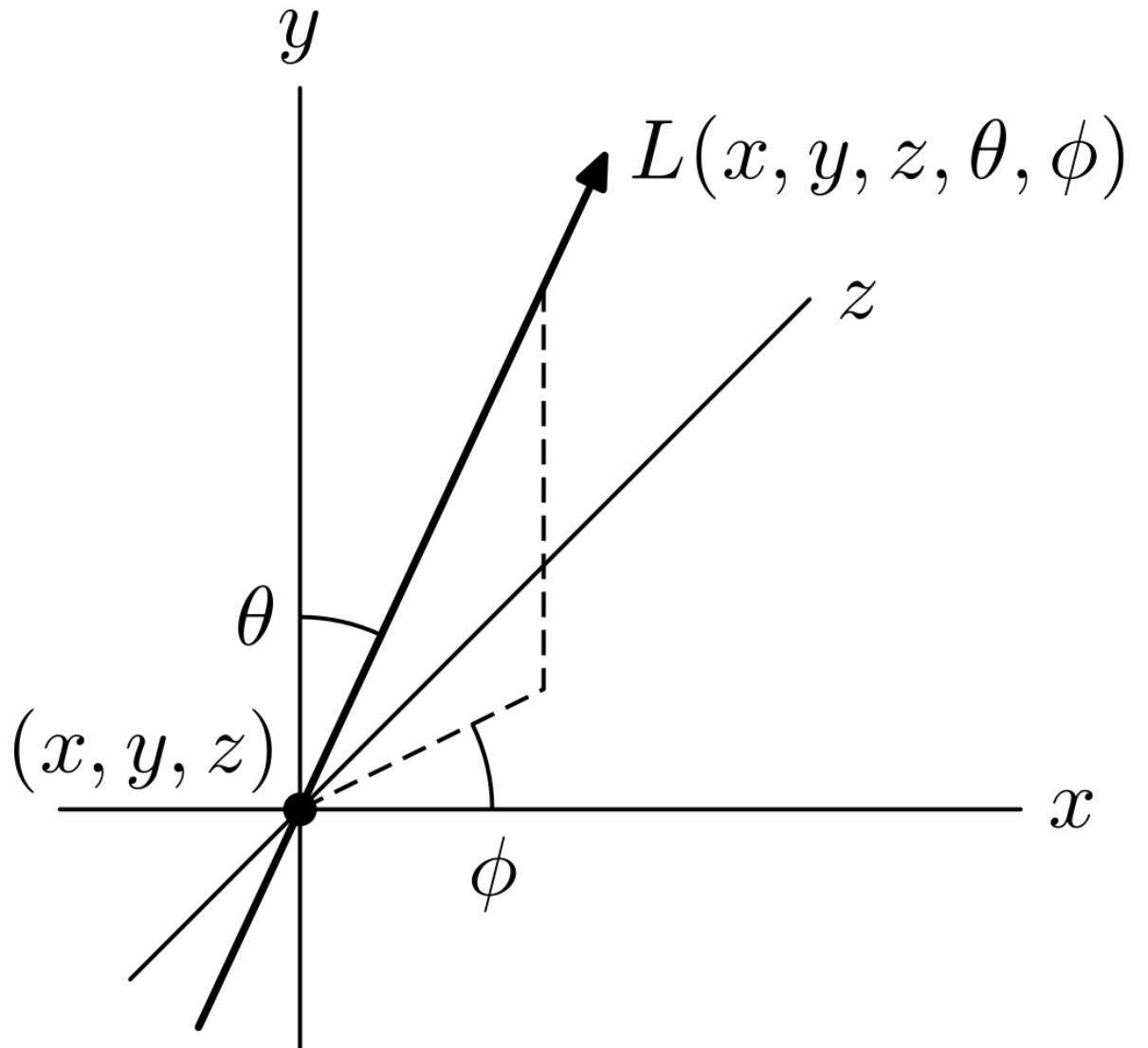
A: The Plenoptic Function

# The Plenoptic Function

## Definition

- The plenoptic function describes the intensity of each light ray in the world as a function of visual angle, wavelength, time, and viewing position.

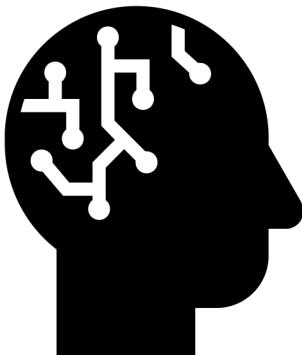
[https://link.springer.com/referenceworkentry/10.1007/978-0-387-31439-6\\_7](https://link.springer.com/referenceworkentry/10.1007/978-0-387-31439-6_7)



Intensity of light:  $P(\theta, \phi)$



- Seen from a single viewpoint
- At a single time
- Averaged over the wavelengths of the visible spectrum

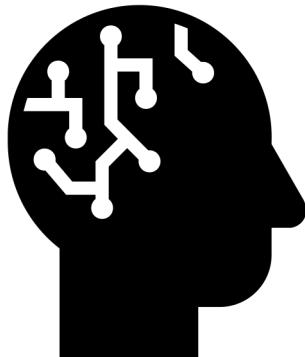


Note:  $P(x, y)$ , but spherical coordinate are nicer

# Color Snapshot: $P(\theta, \Phi, \lambda)$



- Seen from a single viewpoint
- At a single time
- As a function of wavelength

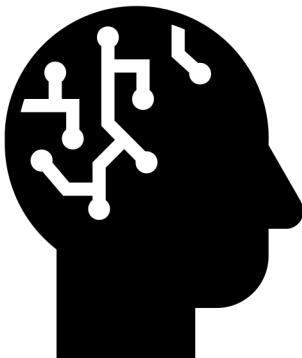


A Video:

$$P(\theta, \Phi, \lambda, t)$$



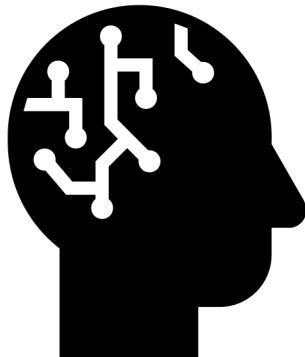
- Seen from a single viewpoint
- Over time
- As a function of wavelength



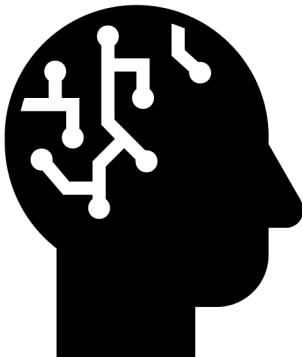
# A Free Viewpoint Video: $P(\theta, \Phi, \lambda, t, V_x, V_y, V_z)$



- Seen from ANY viewpoint
- Over time
- As a function of wavelength



What is it good for?:  $P(\theta, \Phi, \lambda, t, V_x, V_y, V_z)$



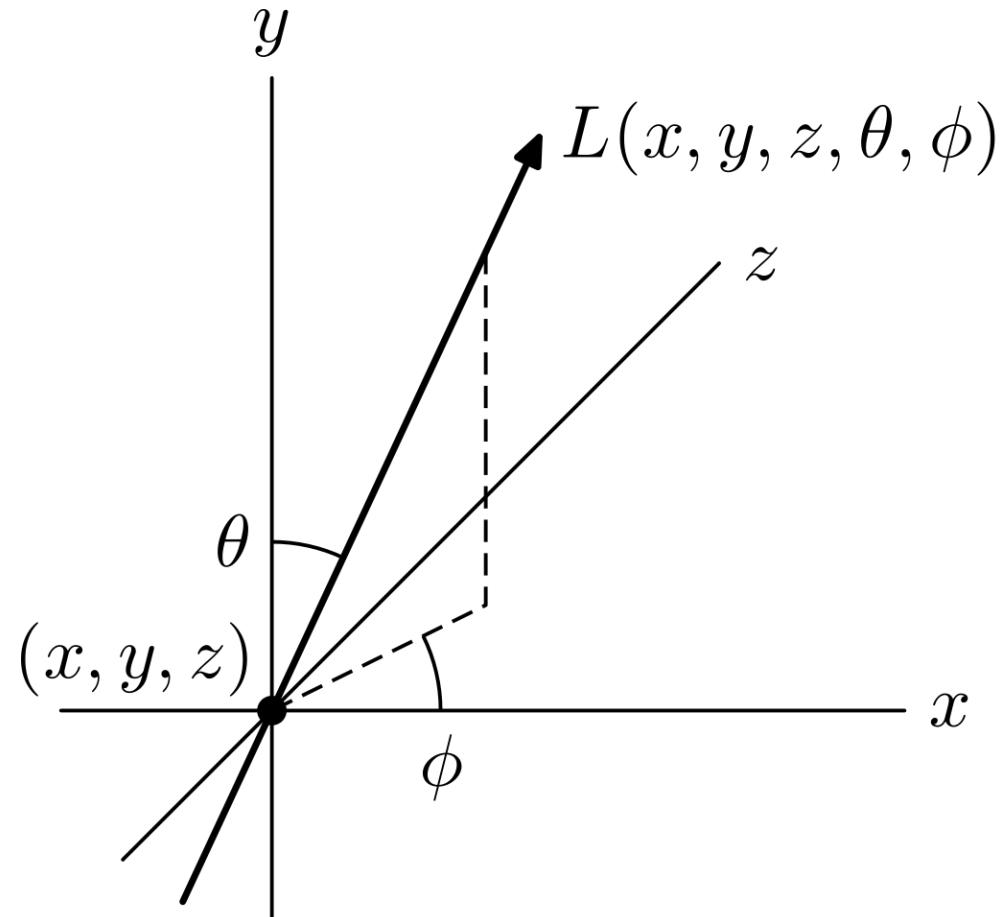
- It can reconstruct every possible view, at every moment, from every position, at every wavelength
- It completely captures our visual reality.



# The 5D Plenoptic Function: $P(X,Y,Z,\theta,\Phi)$

- Ignore time and wavelength
- Focus just on spatial structure of light

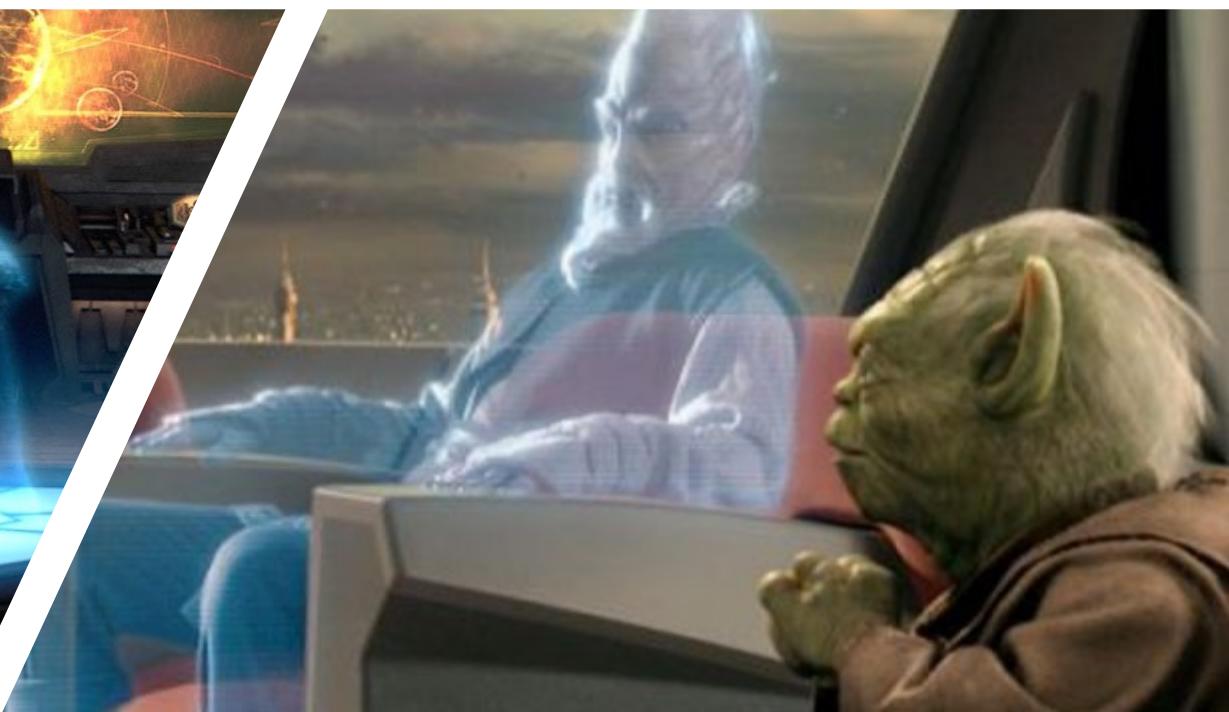
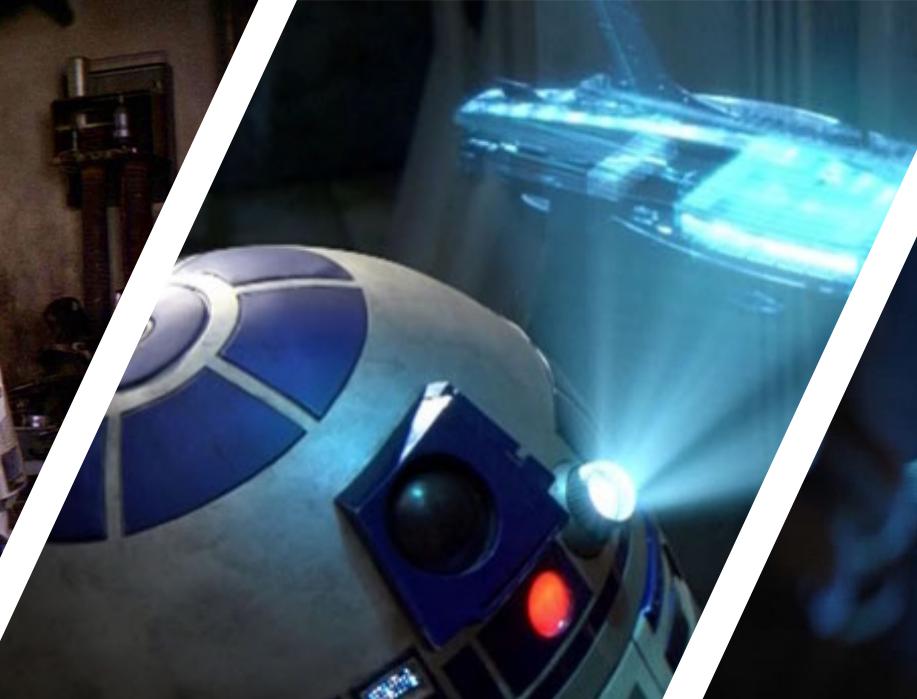
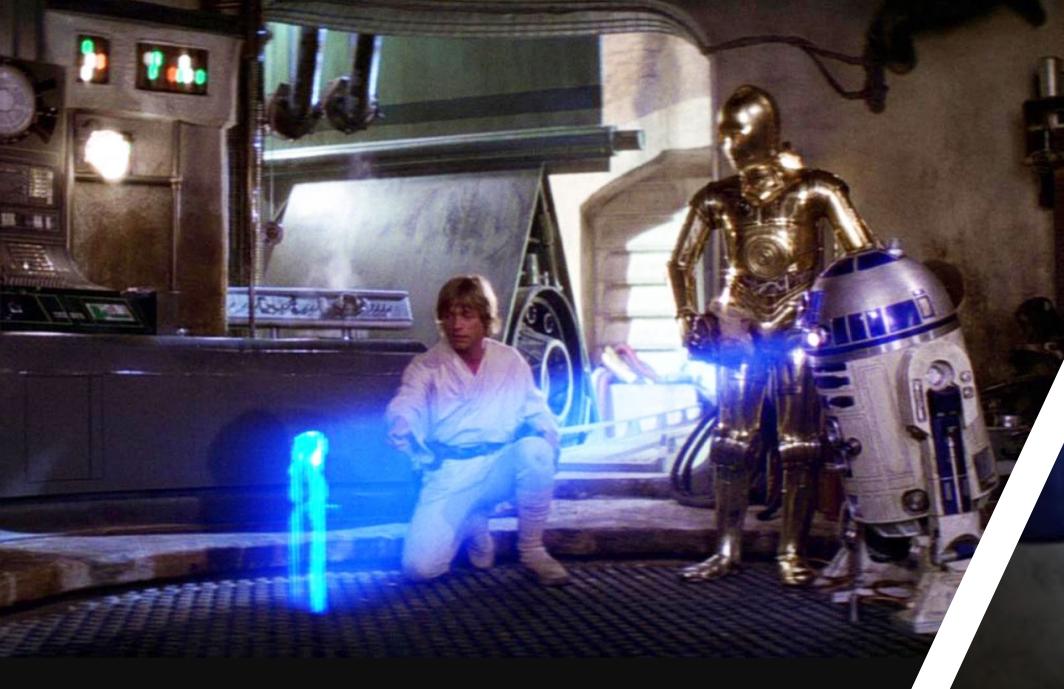
# Why is this so important?

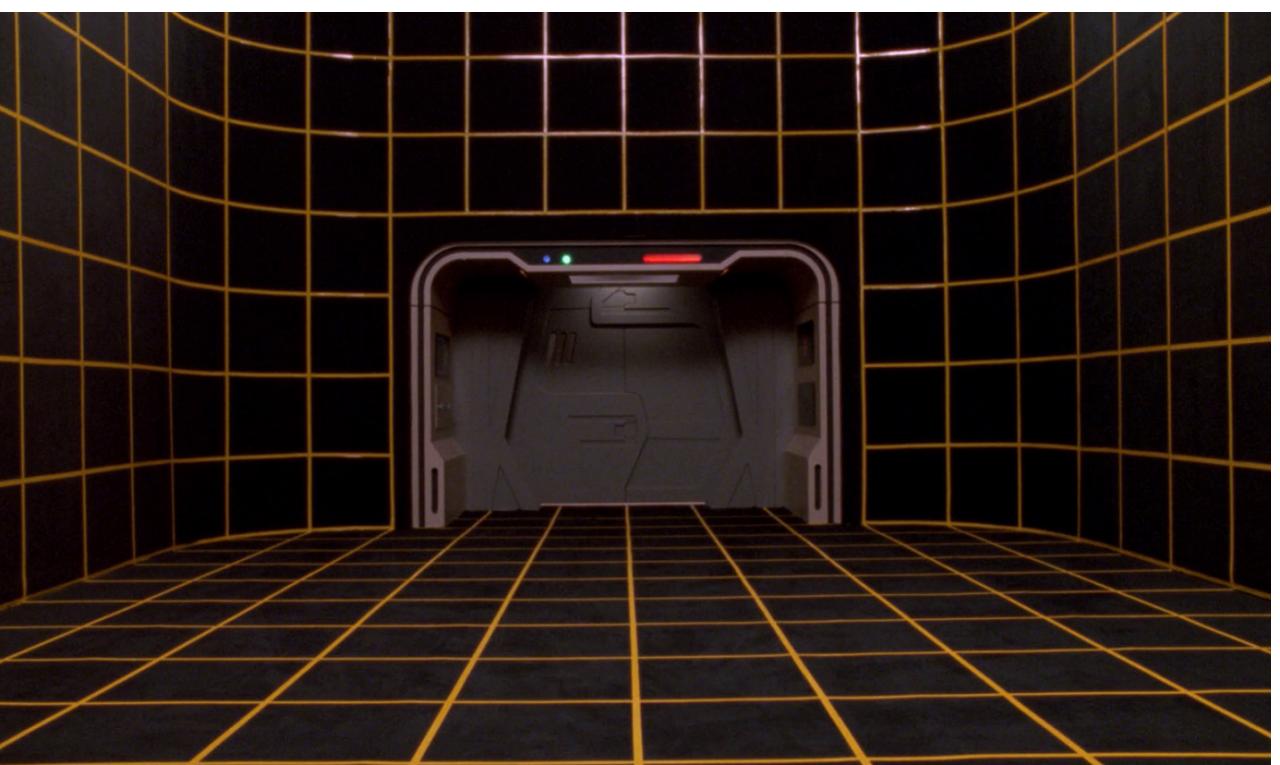
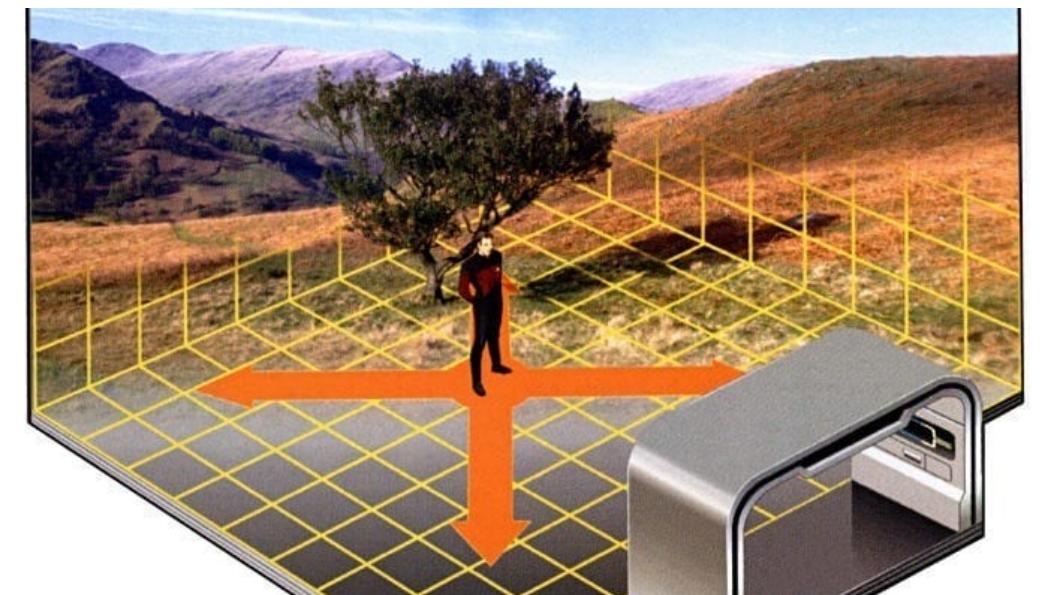


- It contains every photograph, every movie, everything that anyone has ever seen!

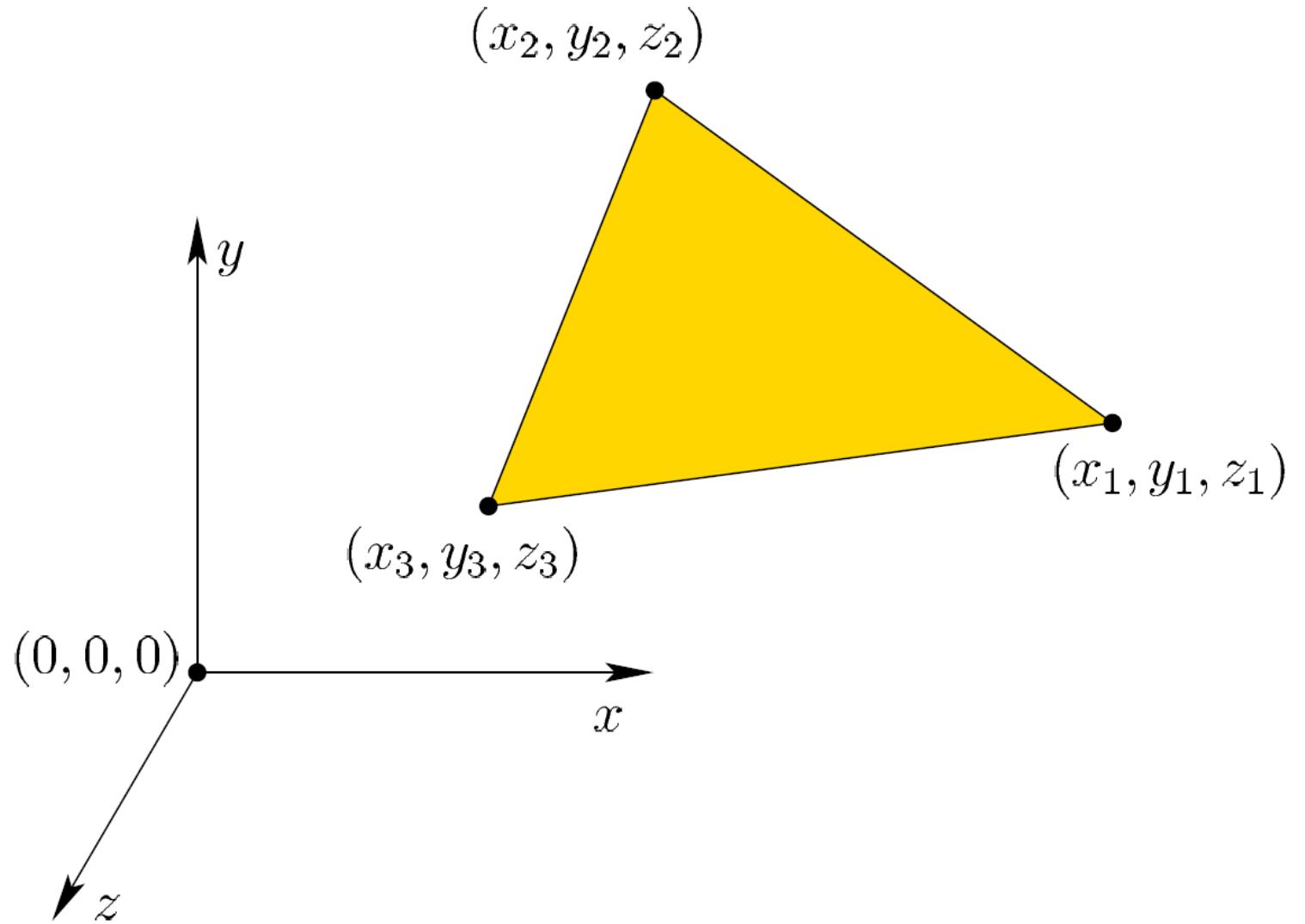
A scene from Star Wars: Episode V - The Empire Strikes Back. Luke Skywalker (Mark Hamill) and Obi-Wan Kenobi (Alec Guinness) are seated in the cockpit of a Millenium Falcon. They are looking towards the front of the ship. A bright blue lightsaber blade is positioned vertically between them, its hilt pointing downwards. The cockpit has various controls and a circular window showing the dark space outside.

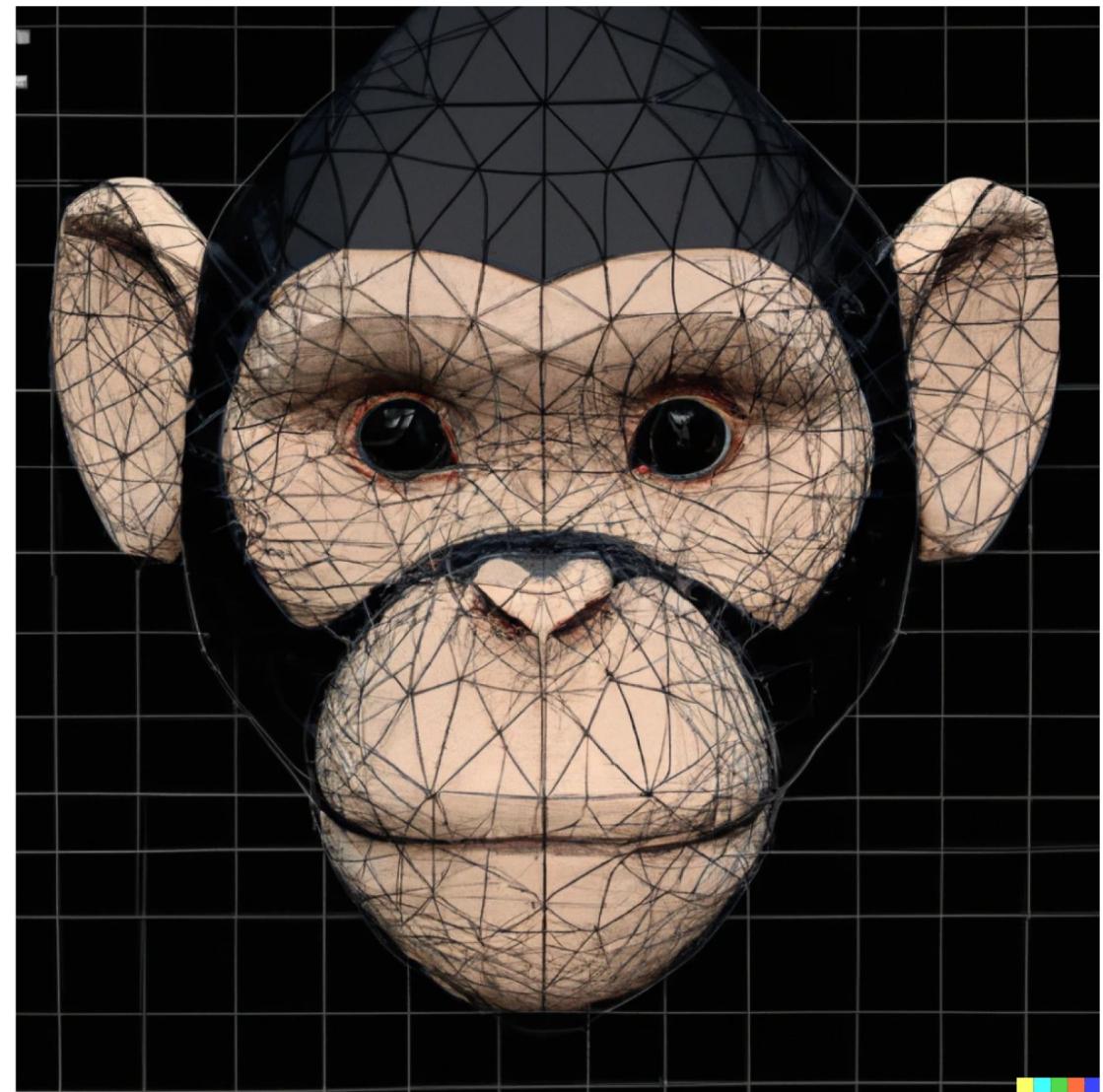
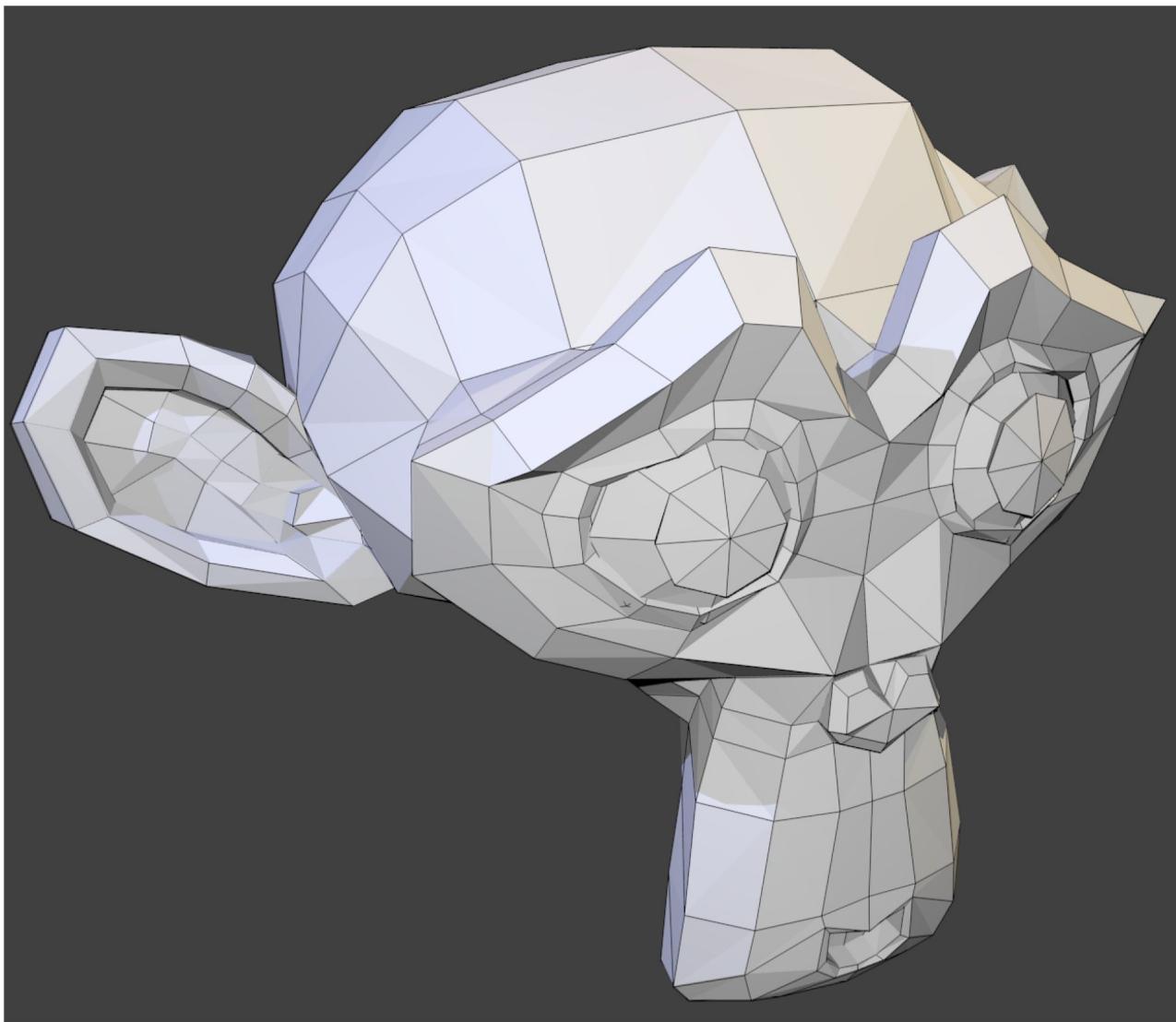
Because...

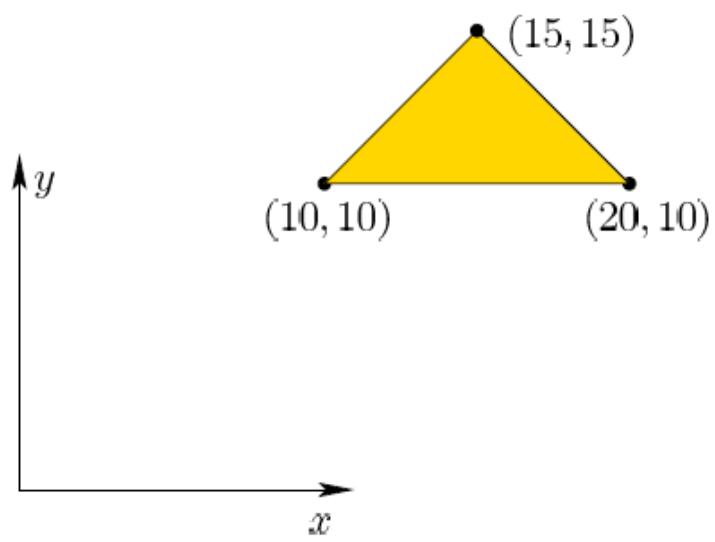




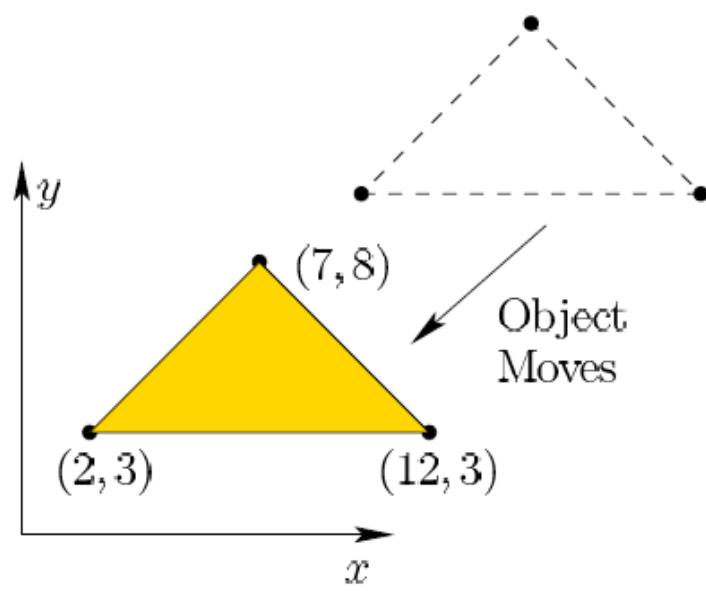
# The Geometry of XR Worlds



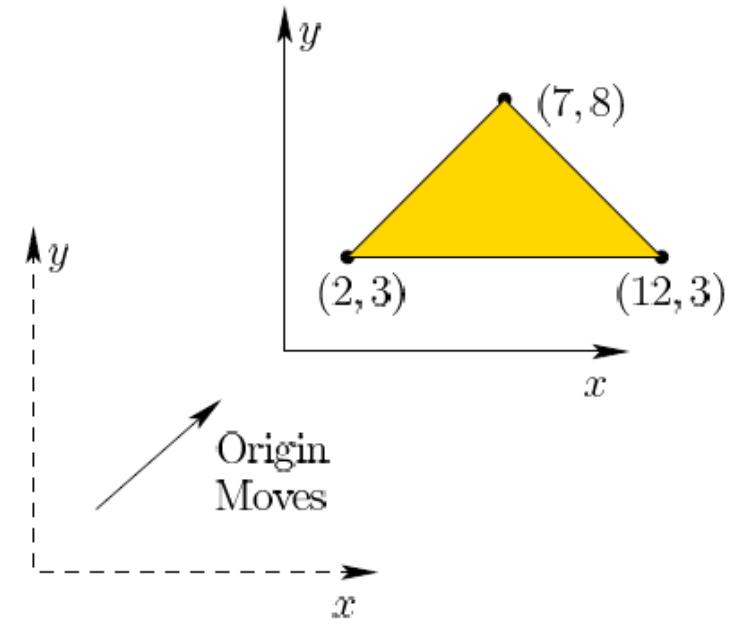




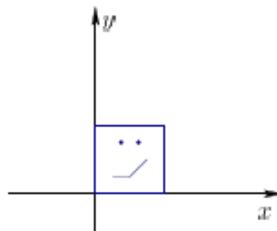
(a) Original object



(b) Object moves

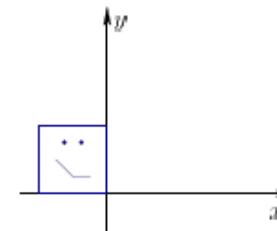


(c) Origin moves



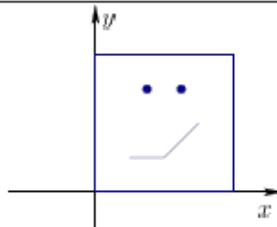
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Identity



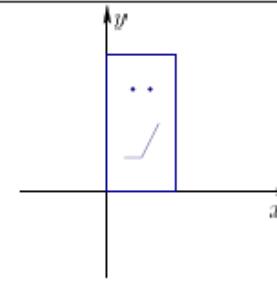
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Mirror



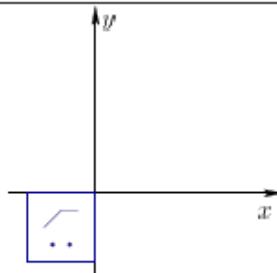
$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Scale



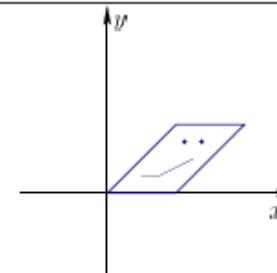
$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Stretch



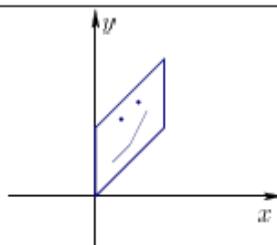
$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Rotate 180



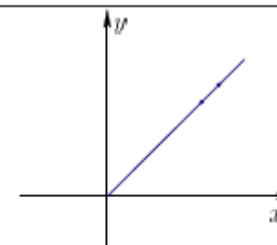
$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

*x*-shear



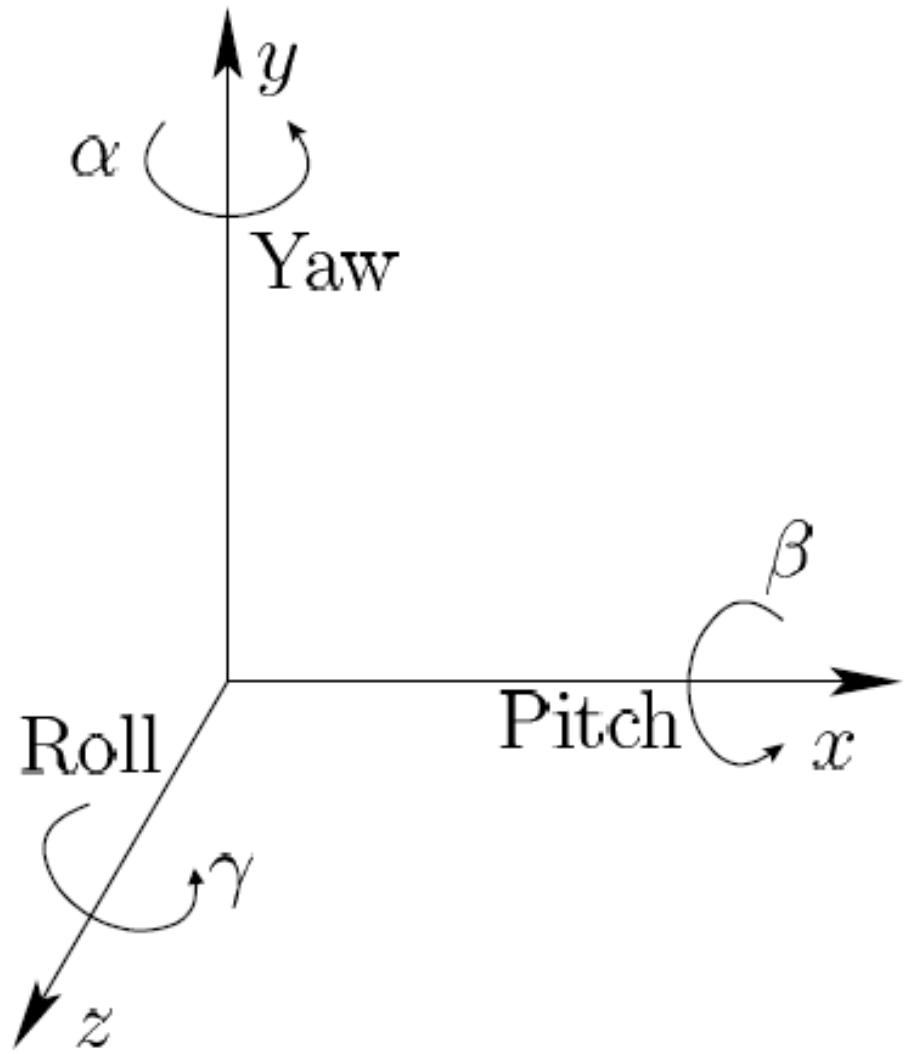
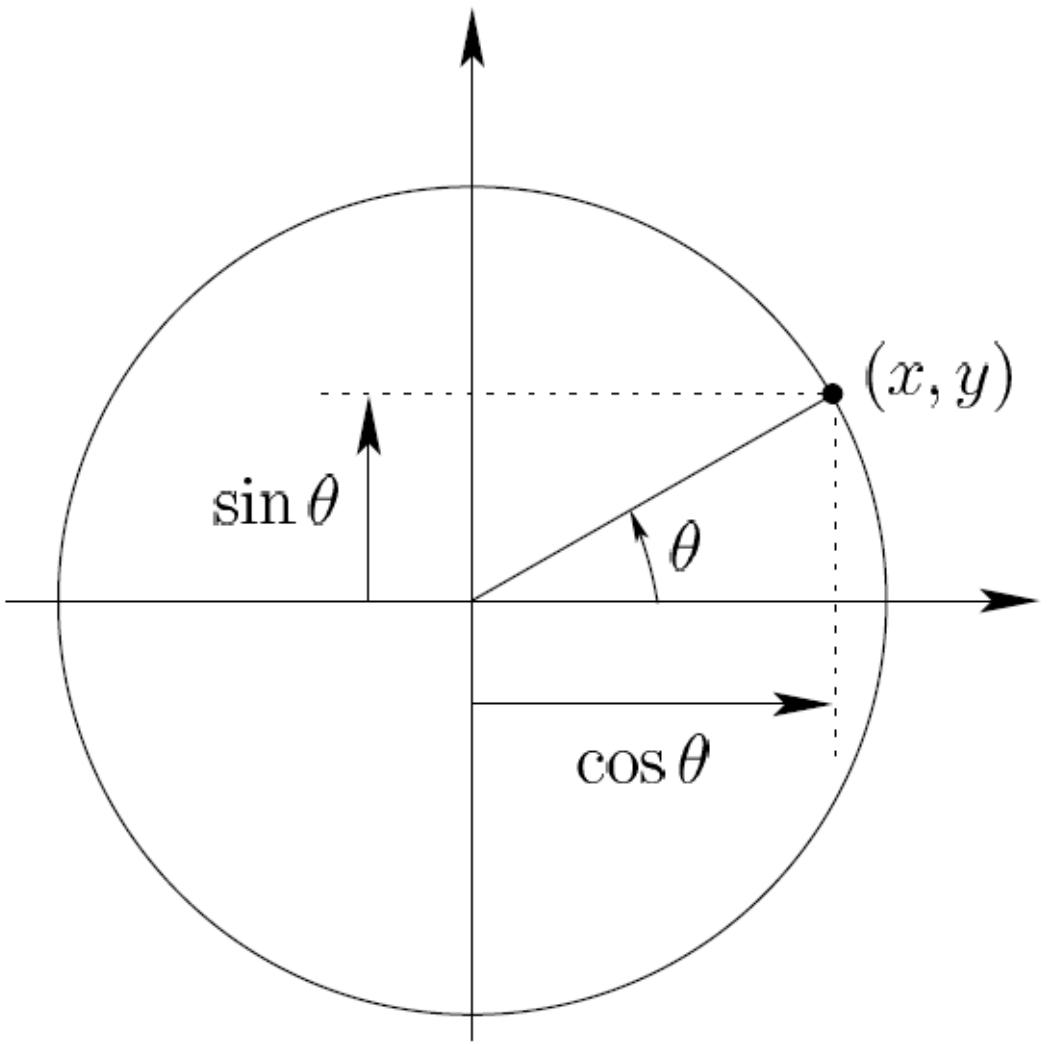
$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

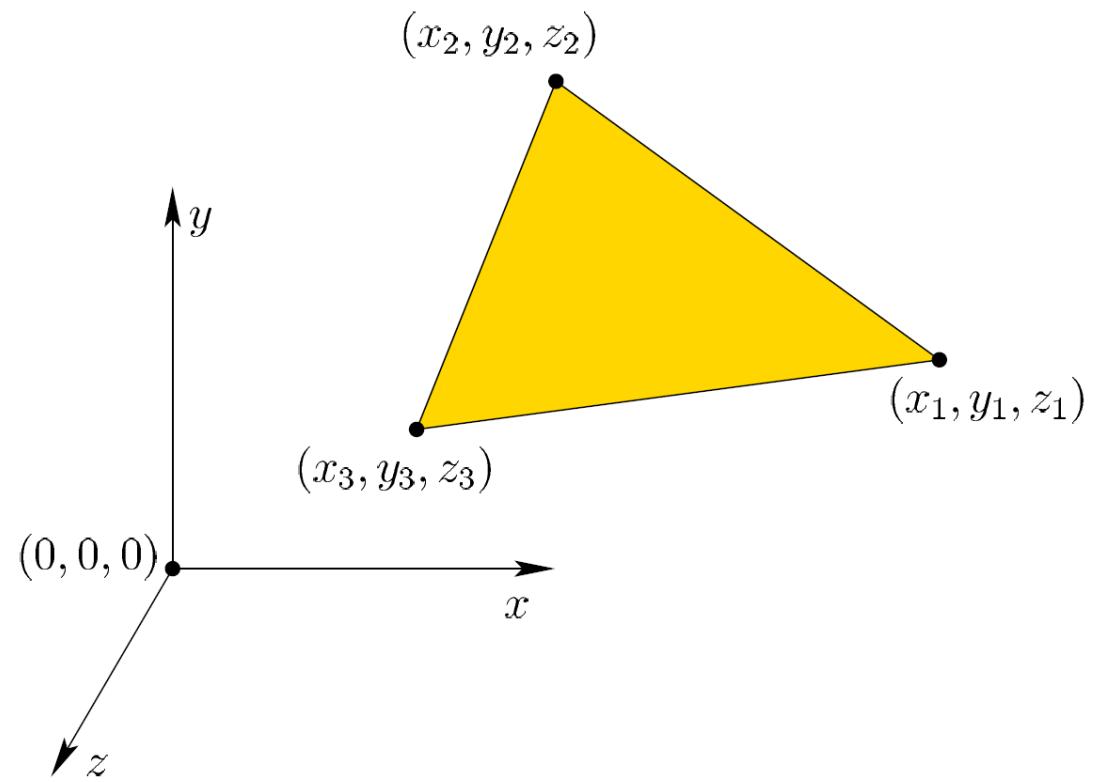
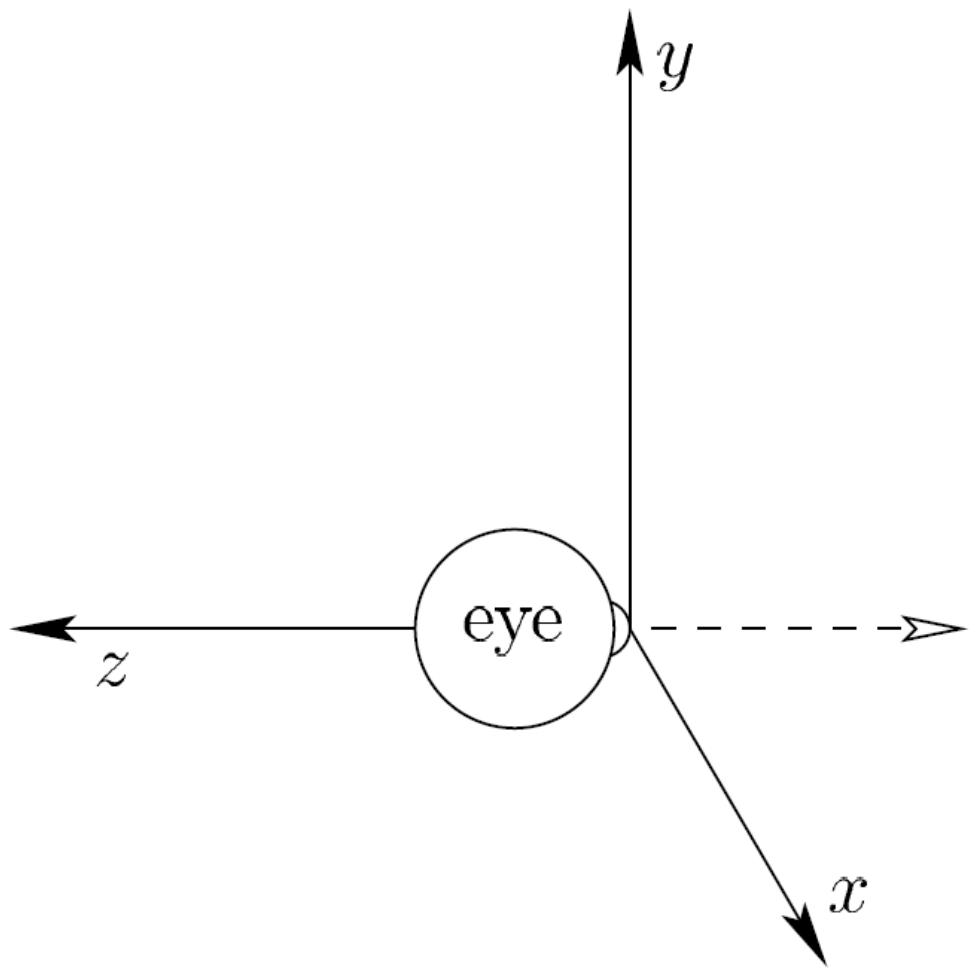
*y*-shear

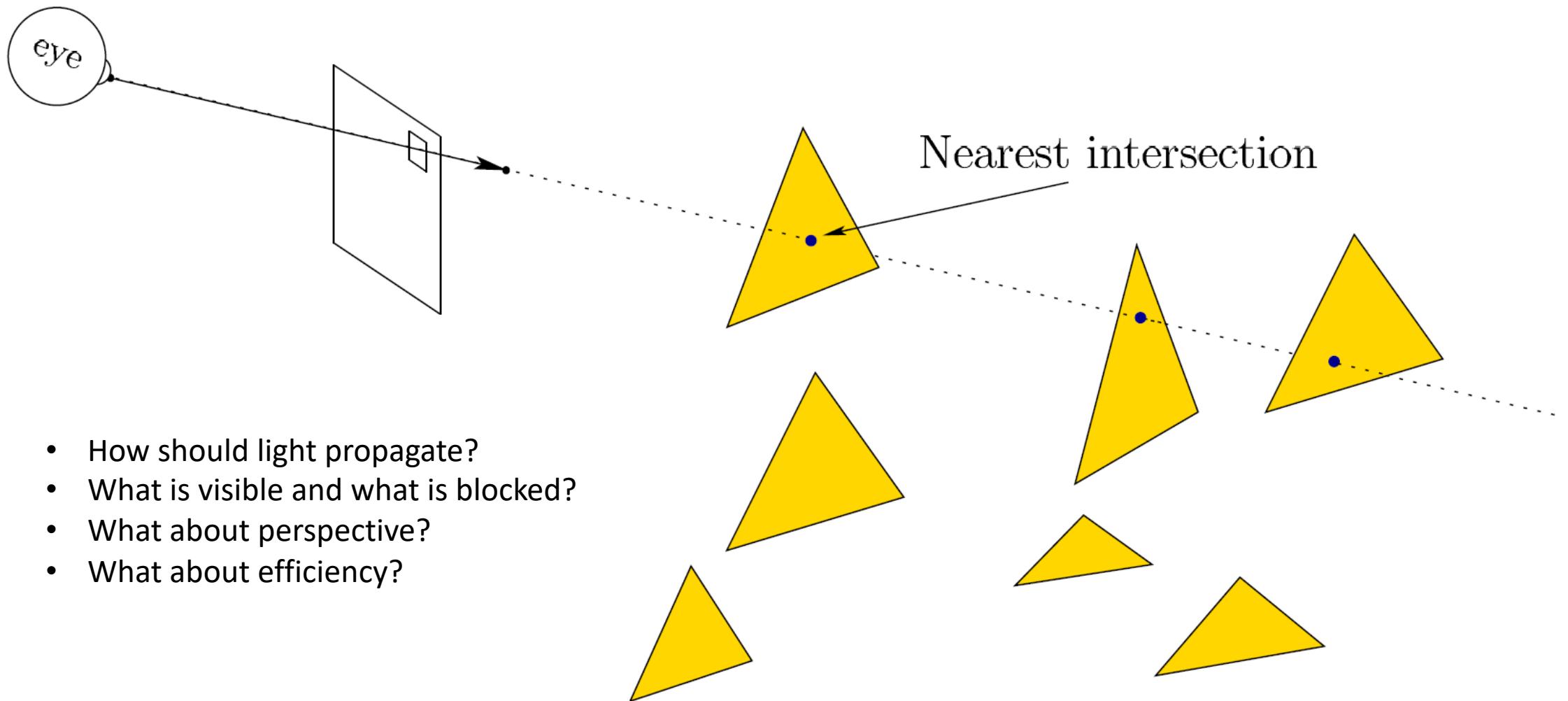


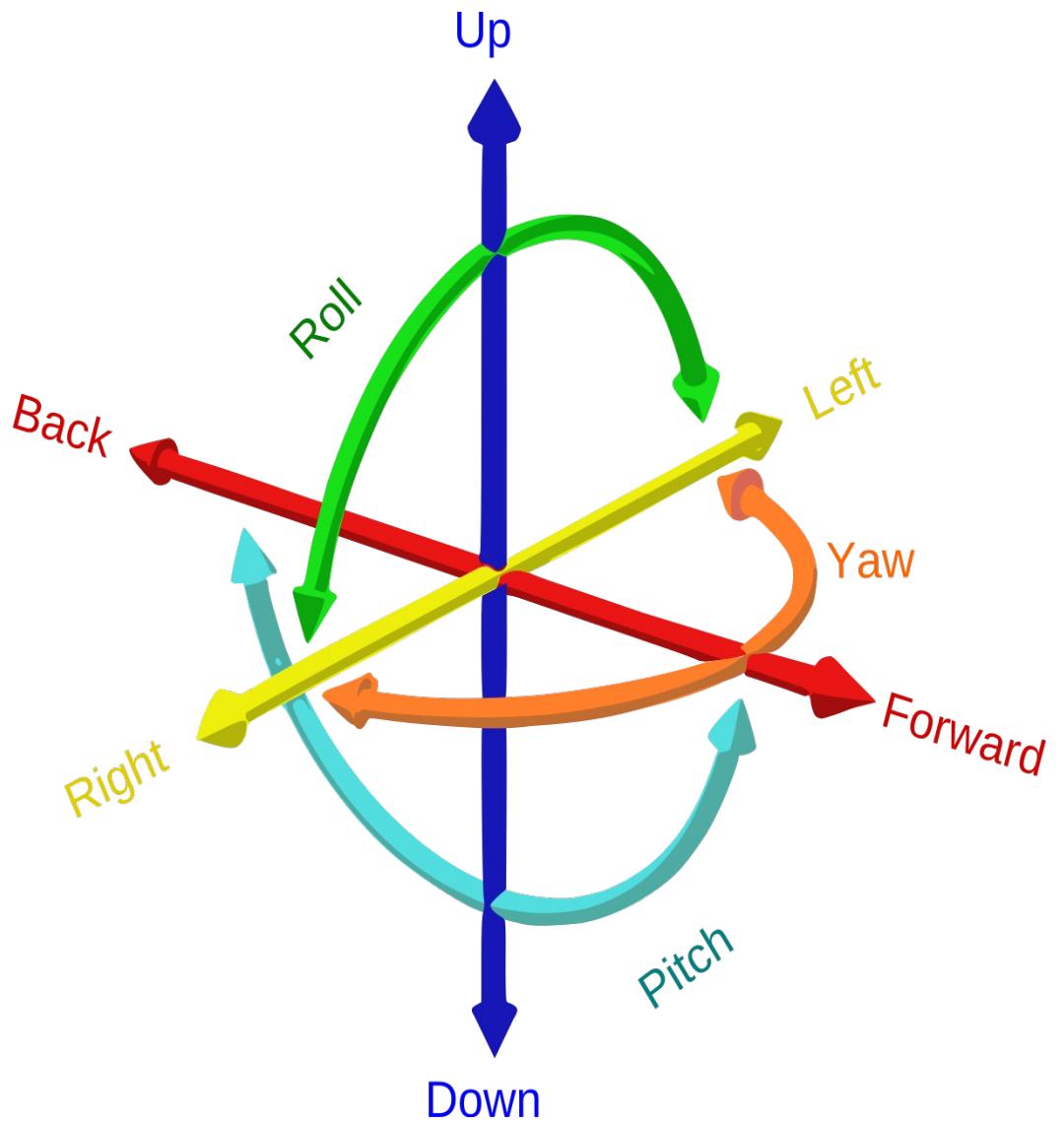
$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Singular









# Further Reading

- Hartley, R., & Zisserman, A. (2003). Multiple view geometry in computer vision. Cambridge university press.
- Adelson EH, Bergen JR (1991) The plenoptic function and the elements of early vision. In: Landy M, Movshon JA (eds) Computational models of visual processing. MIT, Cambridge, MA
- Examples:
  - <http://www.robots.ox.ac.uk/~vgg/hzbook/>
  - <http://www.robots.ox.ac.uk/~vgg/hzbook/hzbook2/HZepipolar.pdf>
  - <http://www.robots.ox.ac.uk/~vgg/hzbook/code/>