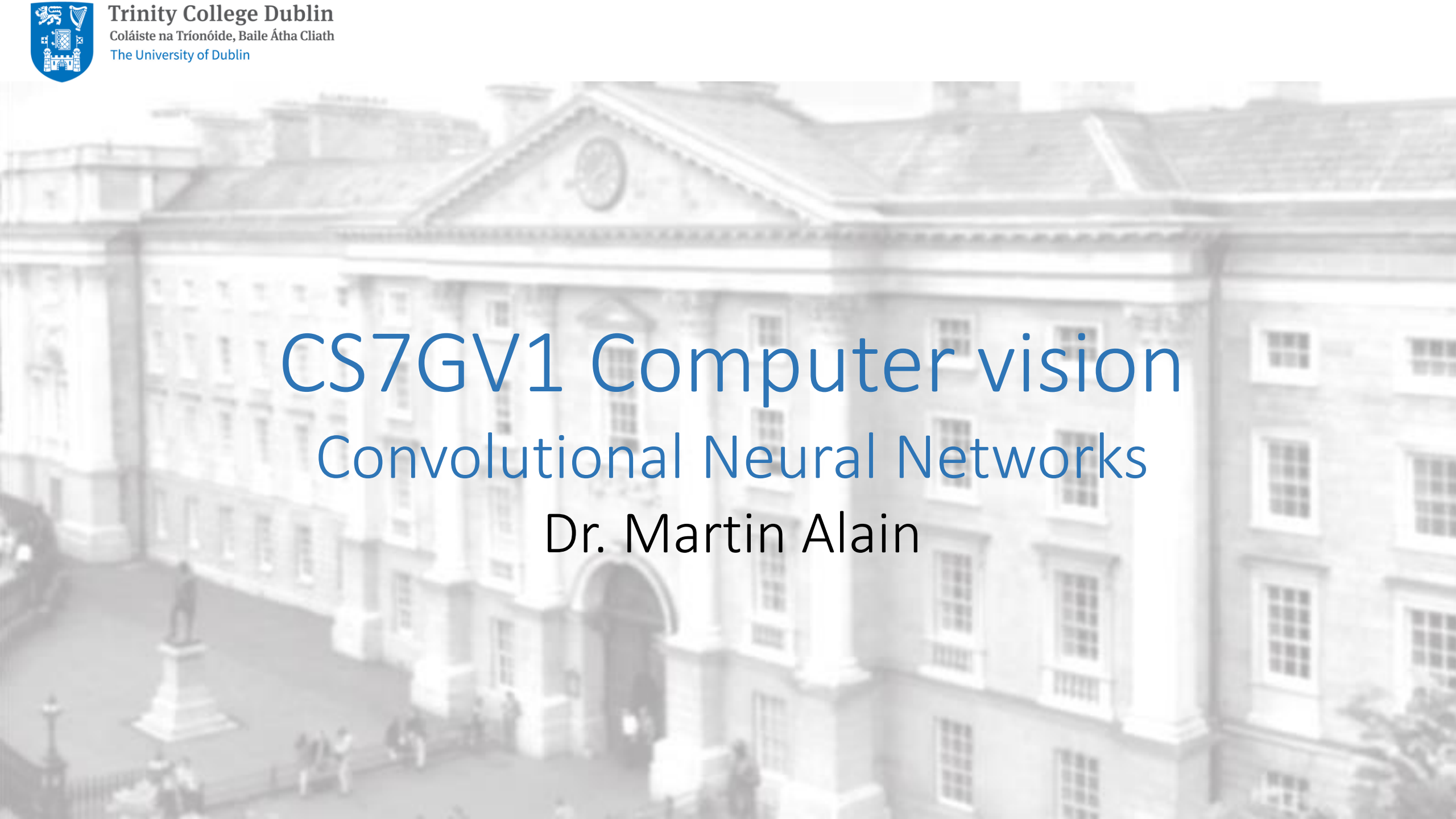
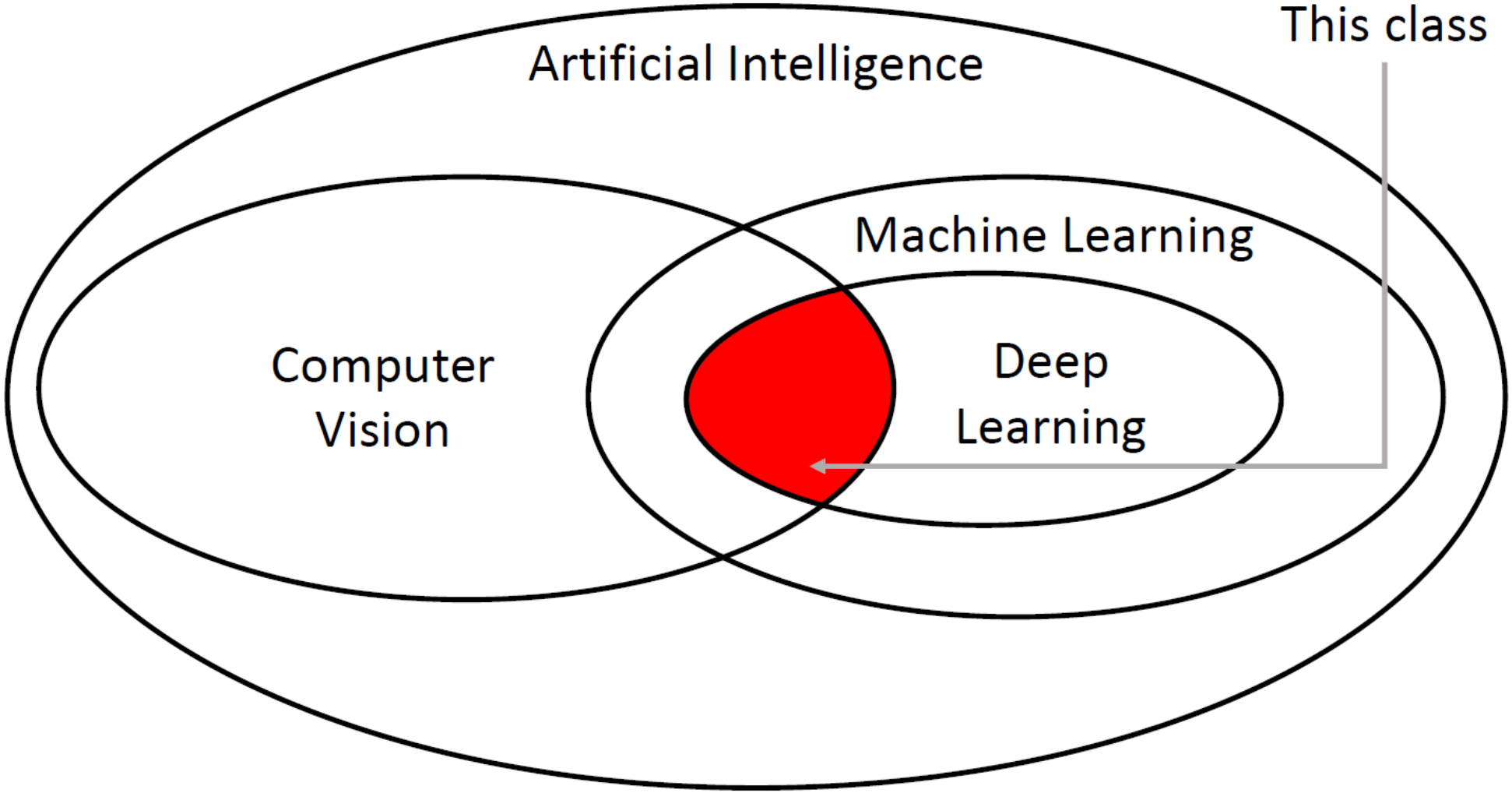


CS7GV1 Computer vision

Convolutional Neural Networks

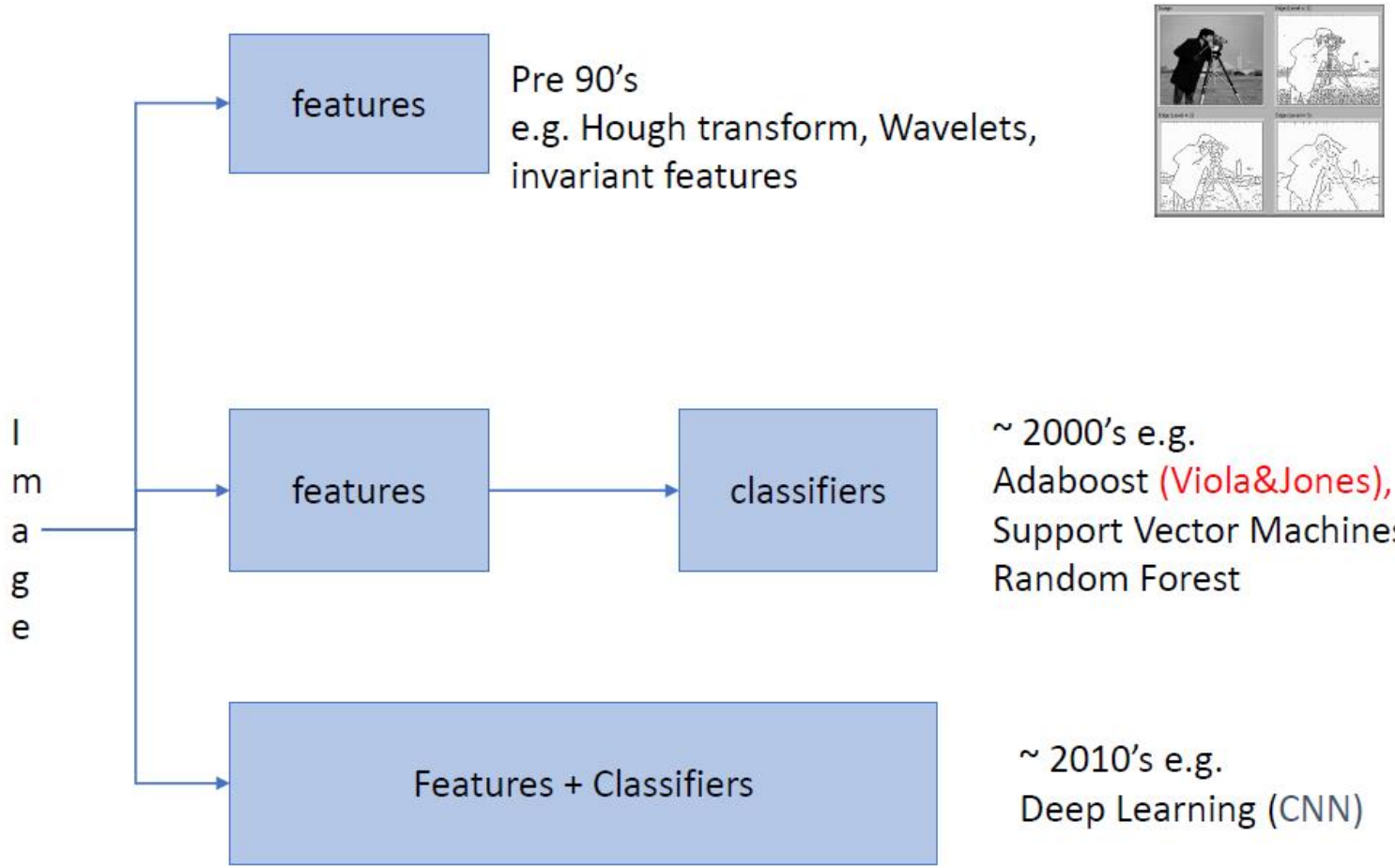
Dr. Martin Alain

Introduction



Source: Justin Johnson

Computer vision and machine learning



Machine learning

Background

A Recipe for Machine Learning

1. Given training data:

$$\{\boldsymbol{x}_i, \boldsymbol{y}_i\}_{i=1}^{N}$$

2. Choose each of these:
   - Decision function

$$\hat{\boldsymbol{y}} = f_{\boldsymbol{\theta}}(\boldsymbol{x}_i)$$

   - Loss function

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}_i) \in \mathbb{R}$$

3. Define goal:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$$

4. Train with SGD:
(take small steps opposite the gradient)

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \eta_t \nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$$

@RDahyot

Source:
https://www.cs.cmu.edu/~mgormley/courses/10601-s17/slides/lecture20-backprop.pdf

Neuron

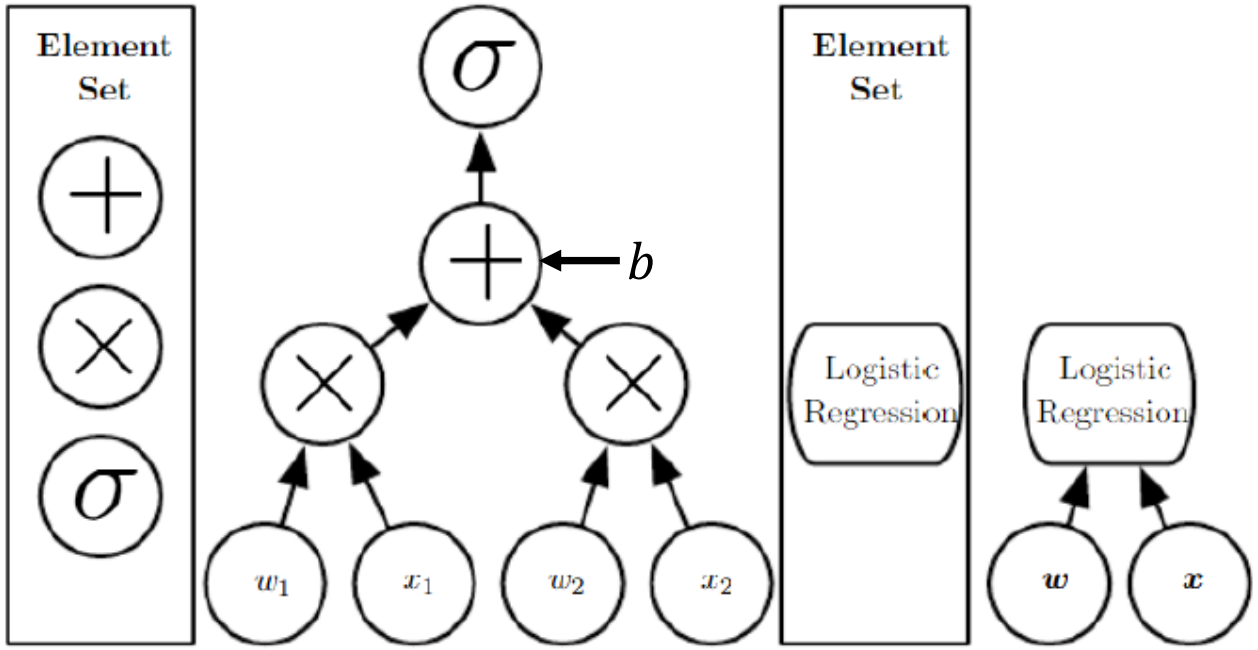


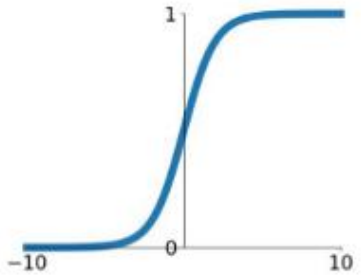
Figure 1.3: Illustration of computational graphs mapping an input to an output where each node performs an operation. Depth is the length of the longest path from input to output but depends on the definition of what constitutes a possible computational step. The computation depicted in these graphs is the output of a logistic regression model, $\sigma(\boldsymbol{w}^T \boldsymbol{x})$, where σ is the logistic sigmoid function. If we use addition, multiplication and logistic sigmoids as the elements of our computer language, then this model has depth three. If we view logistic regression as an element itself, then this model has depth one.

σ is an activation function chosen here as the logit function but other can be chosen as part of a network

Activation function

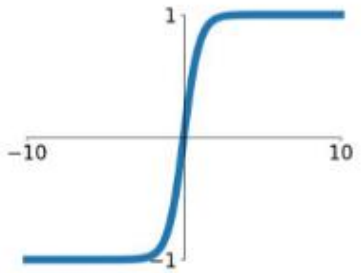
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



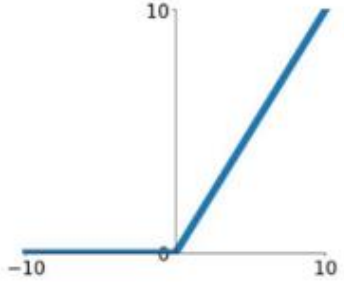
tanh

$$\tanh(x)$$



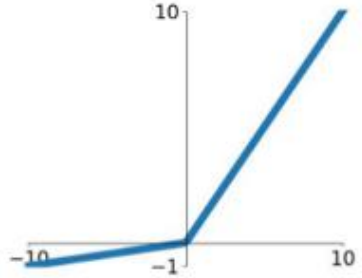
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

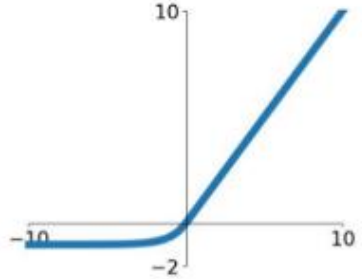


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

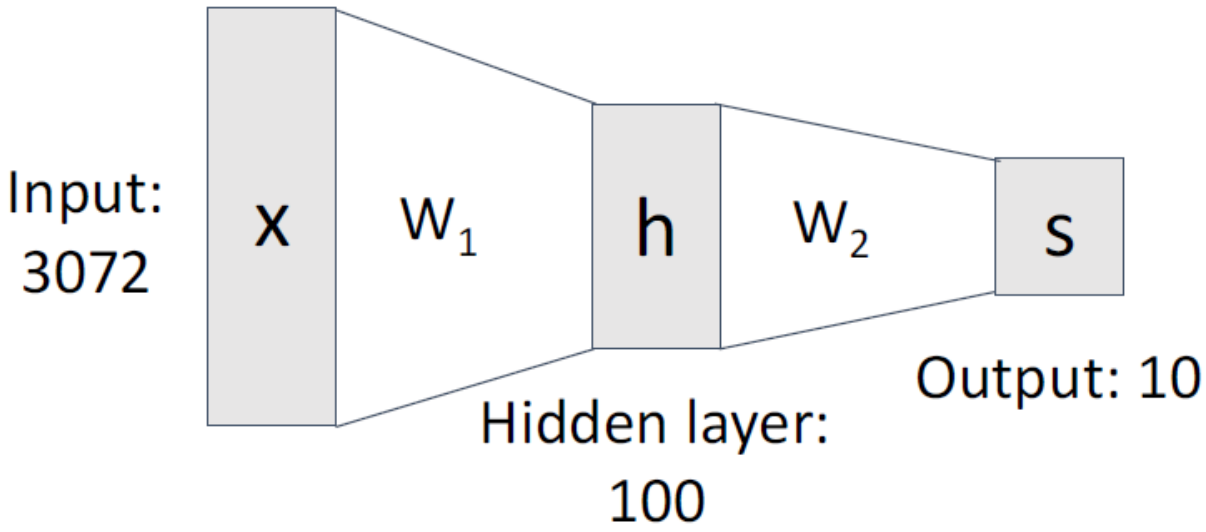
ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Neural Networks

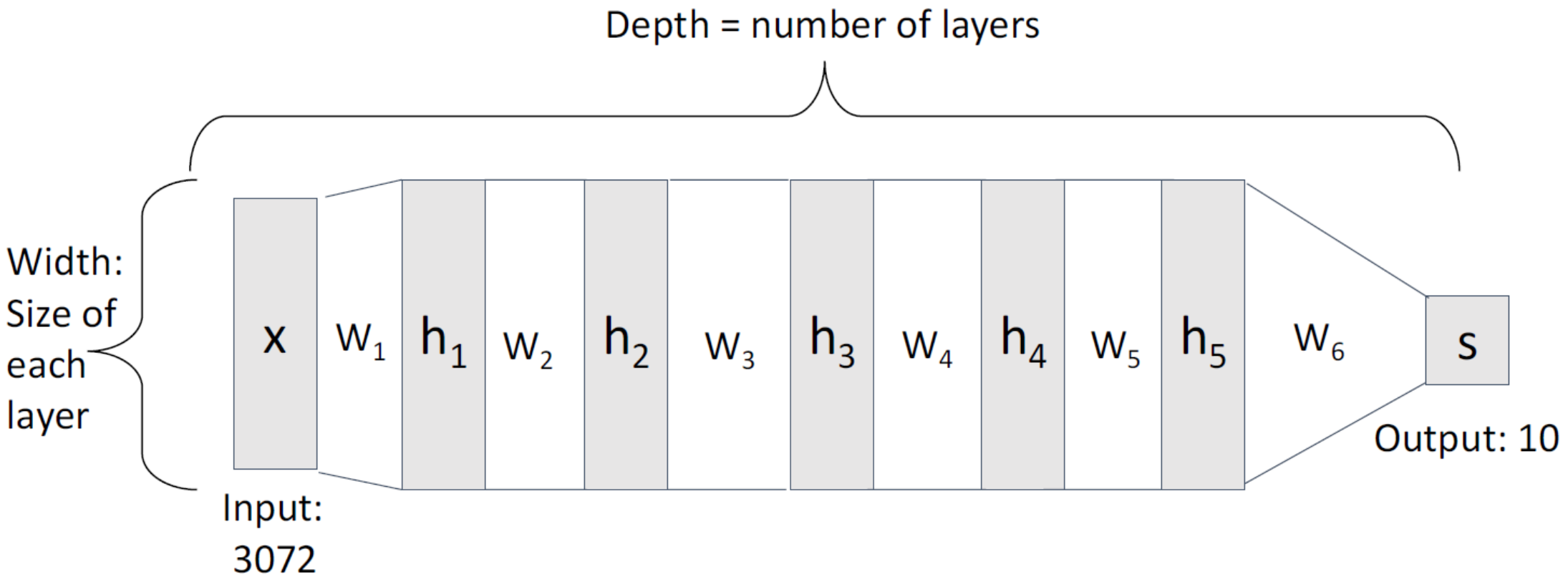
- Fully connected neural network also called Multi-layer perceptron (MLP)



$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

Source: Justin Johnson

Deep Neural Networks



Source: Justin Johnson

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$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \eta_t \nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$$

Source: https://www.cs.cmu.edu/~mgormley/courses/10601-s17/slides/lecture20-backprop.pdf

Backpropragation

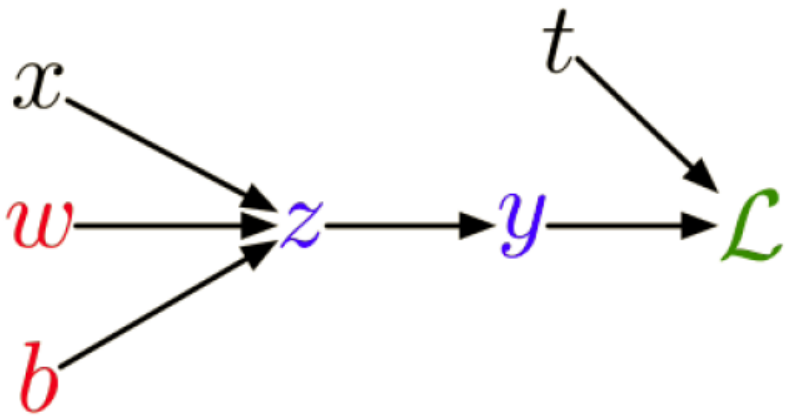
- We've already been using the univariate Chain Rule.
- Recall: if $f(x)$ and $x(t)$ are univariate functions, then

$$\frac{\mathrm{d}}{\mathrm{d}t} f(x(t)) = \frac{\mathrm{d}f}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t}.$$

Source: http://www.cs.toronto.edu/~rgrosse/courses/csc321_2017/slides/lec6.pdf

Backpropragation

Recall: Univariate logistic least squares model



$$z = wx + b$$
$$y = \sigma(z)$$
$$\mathcal{L} = \frac{1}{2}(y - t)^2$$

Let's compute the loss derivatives.

Source: http://www.cs.toronto.edu/~rgrosse/courses/csc321_2017/slides/lec6.pdf

Backpropragation

How you would have done it in calculus class

$$\mathcal{L} = \frac{1}{2}(\sigma(wx+b)-t)^2$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w} &= \frac{\partial}{\partial w}\left[\frac{1}{2}(\sigma(wx+b)-t)^2\right] \\
&= \frac{1}{2}\frac{\partial}{\partial w}(\sigma(wx+b)-t)^2 \\
&= (\sigma(wx+b)-t)\frac{\partial}{\partial w}(\sigma(wx+b)-t) \\
&= (\sigma(wx+b)-t)\sigma'(wx+b)\frac{\partial}{\partial w}(wx+b) \\
&= (\sigma(wx+b)-t)\sigma'(wx+b)x
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial b} &= \frac{\partial}{\partial b}\left[\frac{1}{2}(\sigma(wx+b)-t)^2\right] \\
&= \frac{1}{2}\frac{\partial}{\partial b}(\sigma(wx+b)-t)^2 \\
&= (\sigma(wx+b)-t)\frac{\partial}{\partial b}(\sigma(wx+b)-t) \\
&= (\sigma(wx+b)-t)\sigma'(wx+b)\frac{\partial}{\partial b}(wx+b) \\
&= (\sigma(wx+b)-t)\sigma'(wx+b)
\end{aligned}$$

What are the disadvantages of this approach?

Source: http://www.cs.toronto.edu/~rgrosse/courses/csc321_2017/slides/lec6.pdf

Backpropragation

A more structured way to do it

Computing the loss:

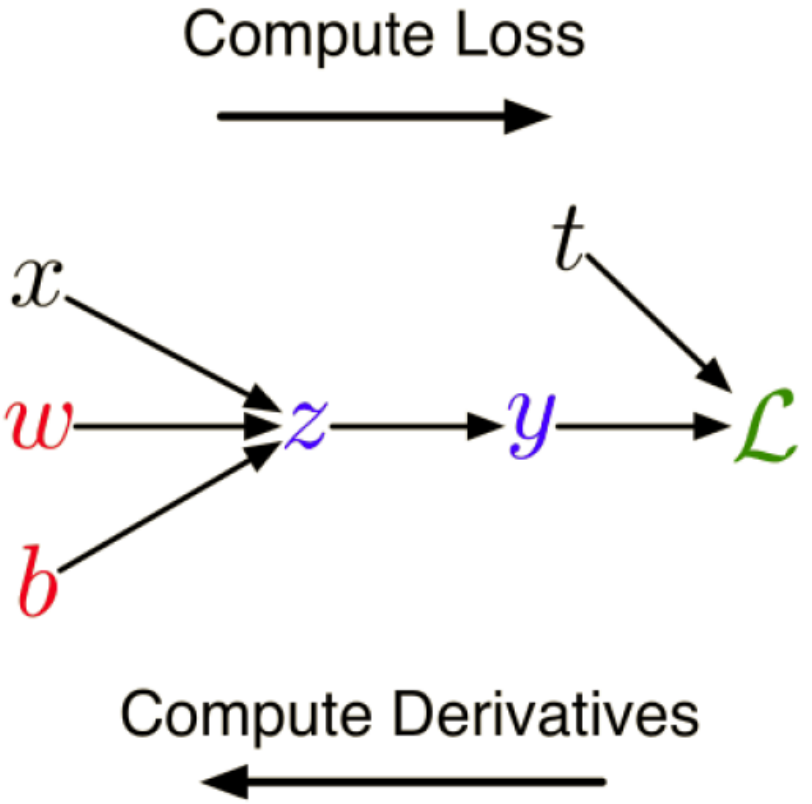
$$z = wx + b$$
$$y = \sigma(z)$$
$$\mathcal{L} = \frac{1}{2}(y - t)^2$$

(Forward pass)

Computing the derivatives:

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}y} = y - t$$
$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}z} = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}y} \cdot \sigma'(z)$$
$$\frac{\partial\mathcal{L}}{\partial w} = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}z} \cdot x$$
$$\frac{\partial\mathcal{L}}{\partial b} = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}z}$$

(backpropagation)



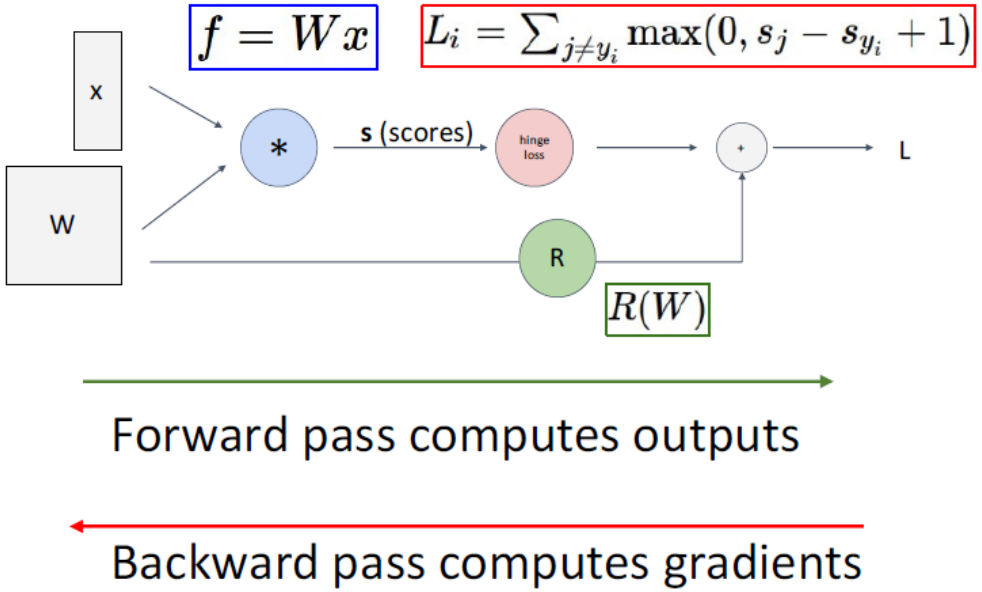
Remember, the goal isn't to obtain closed-form solutions, but to be able to write a program that efficiently computes the derivatives.

Source: http://www.cs.toronto.edu/~rgrosse/courses/csc321_2017/slides/lec6.pdf

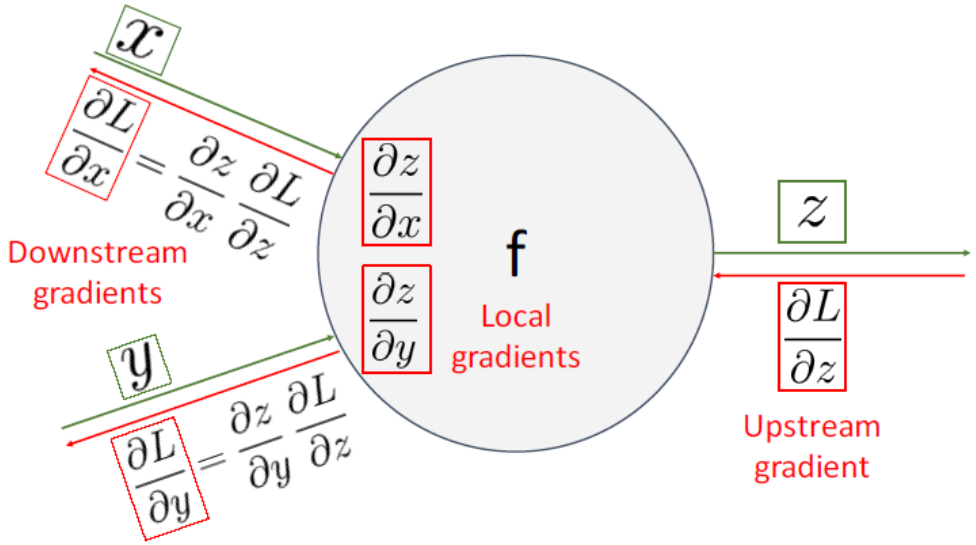
Backpropagation: summary

Last Time: Backpropagation

Represent complex expressions as **computational graphs**

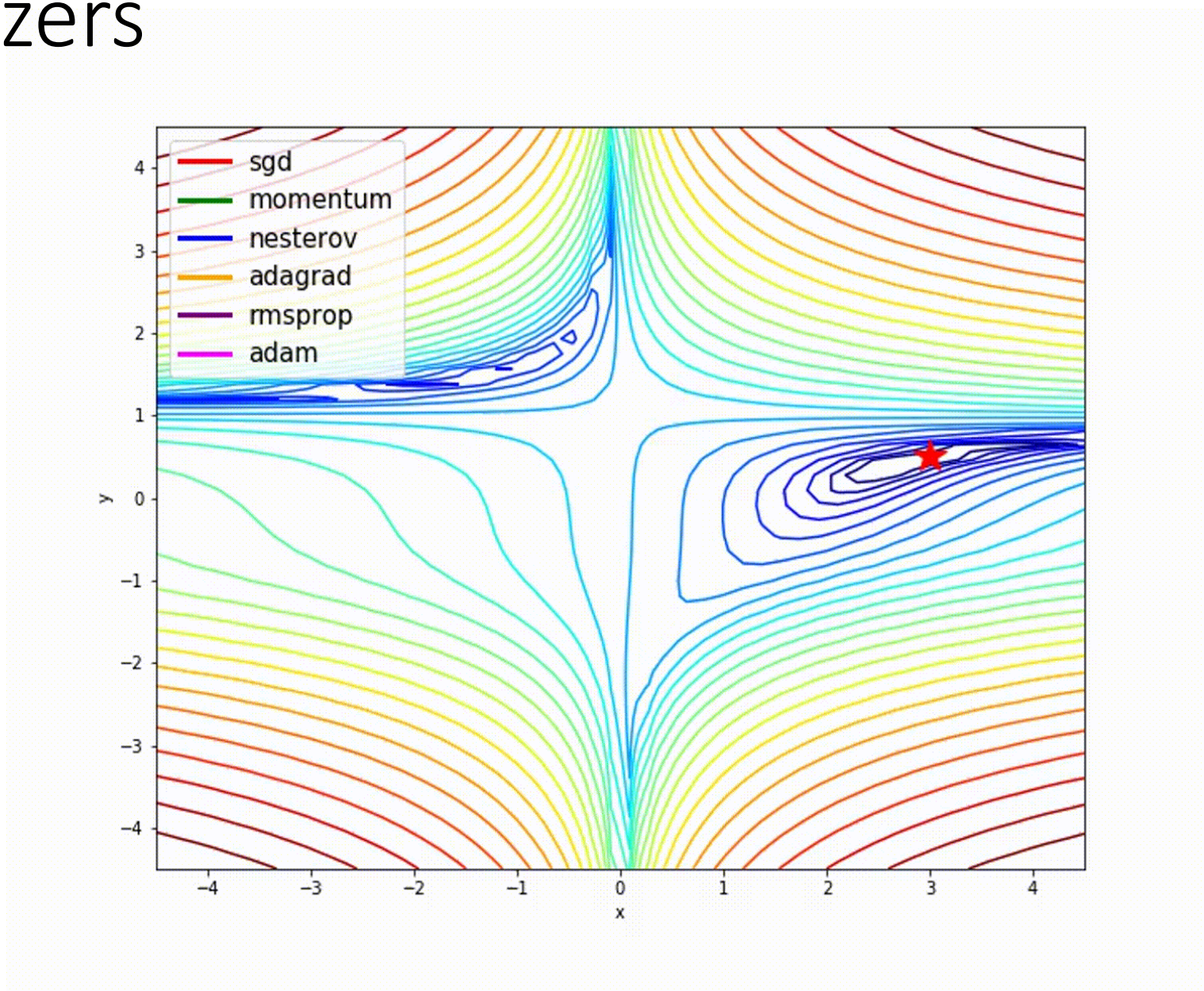


During the backward pass, each node in the graph receives **upstream gradients** and multiplies them by **local gradients** to compute **downstream gradients**

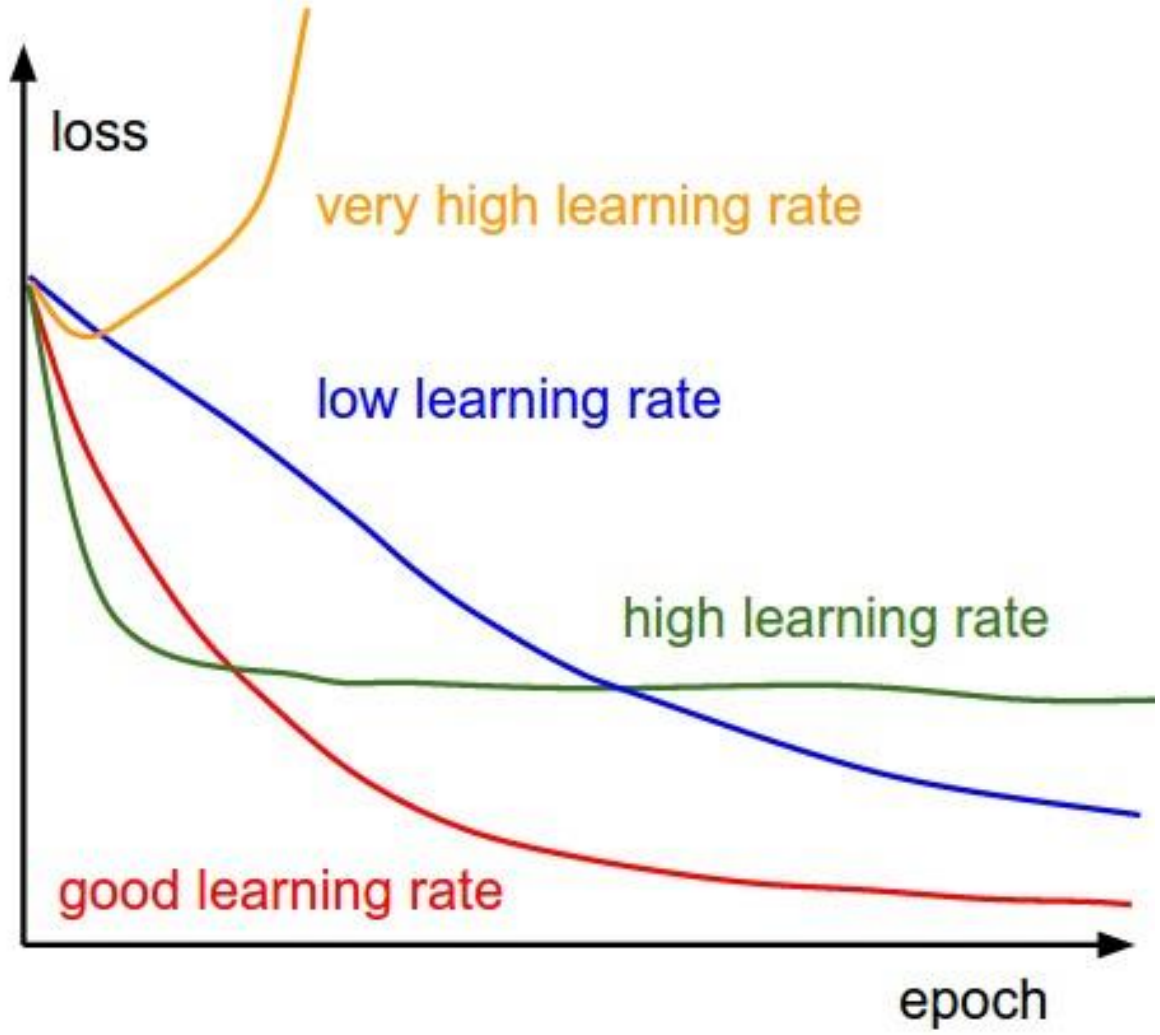


Source: Justin Johnson

Optimizers

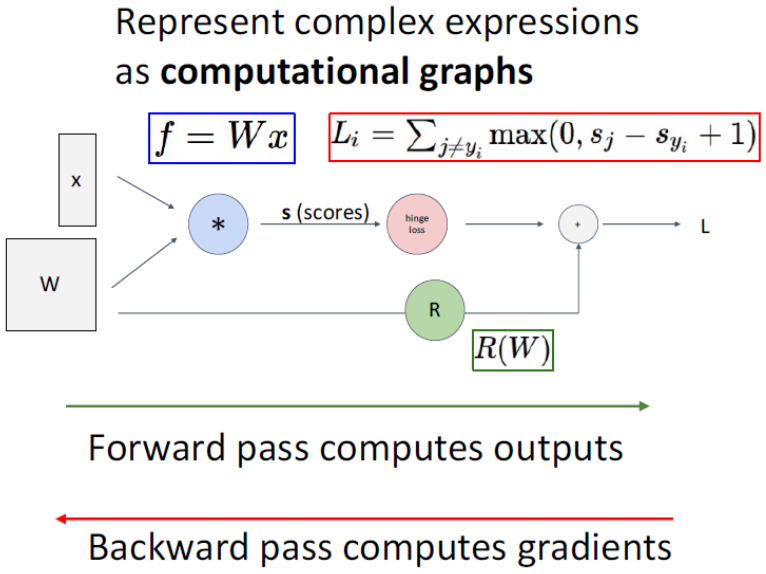

sgd
momentum
nesterov
adagrad
rmsprop
adam
4
3
2
1
0
−1
−2
−3
−4
y
−4
−3
−2
−1
0
1
2
3
4
x

Learning rate



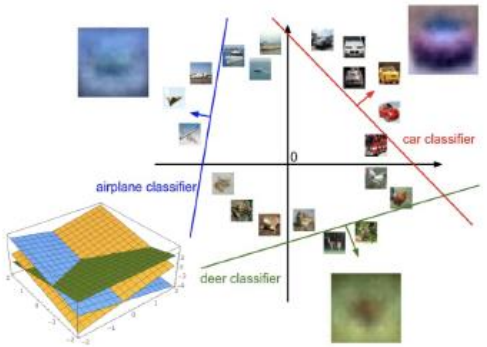
Epochs and (mini-)batches

- Epoch: Forward + backward passes for entire training dataset
- Batch: Subset of training dataset used for forward + backward passes
- Batch size: number of elements in a batch
- Iterations: number of batches needed to complete 1 epoch

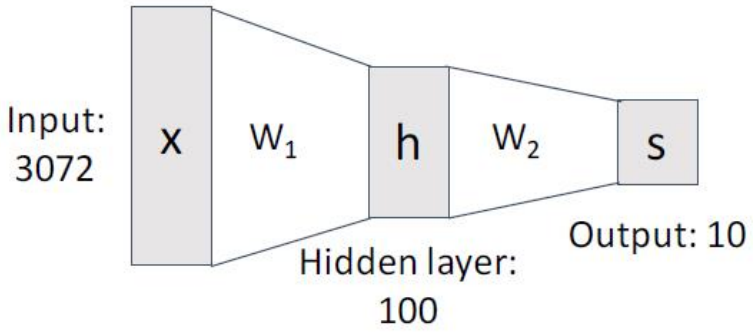


Why Convolutional Networks?

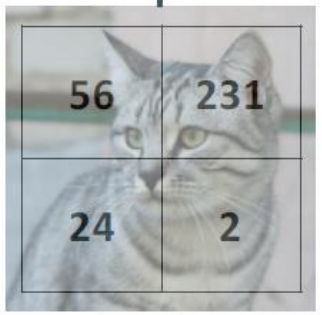
f(x,W) = Wx



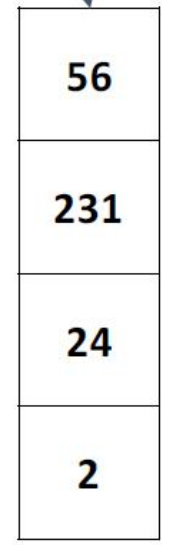
$$f = W_2 \max(0, W_1 x)$$



Stretch pixels into column



Input image
(2, 2)



(4,)

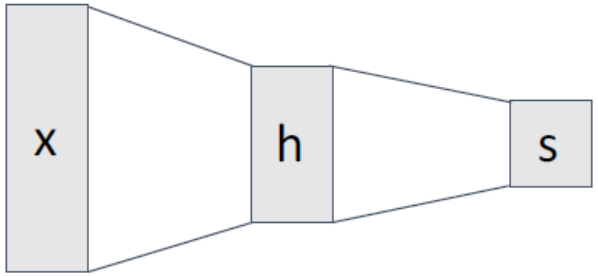
Problem: So far our classifiers don't respect the spatial structure of images!

Solution: Define new computational nodes that operate on images!

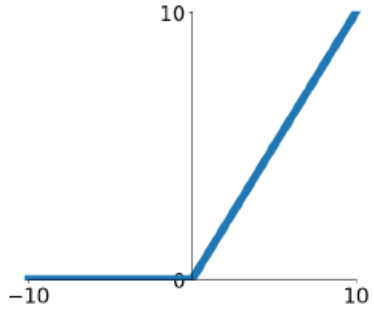
Source: Justin Johnson

Components of a Convolution Network

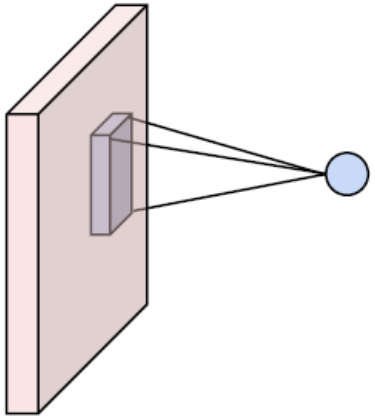
Fully-Connected Layers



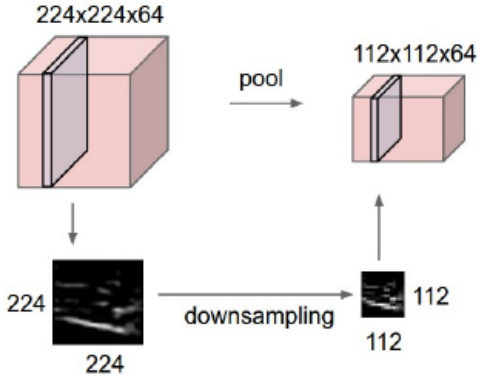
Activation Function



Convolution Layers



Pooling Layers

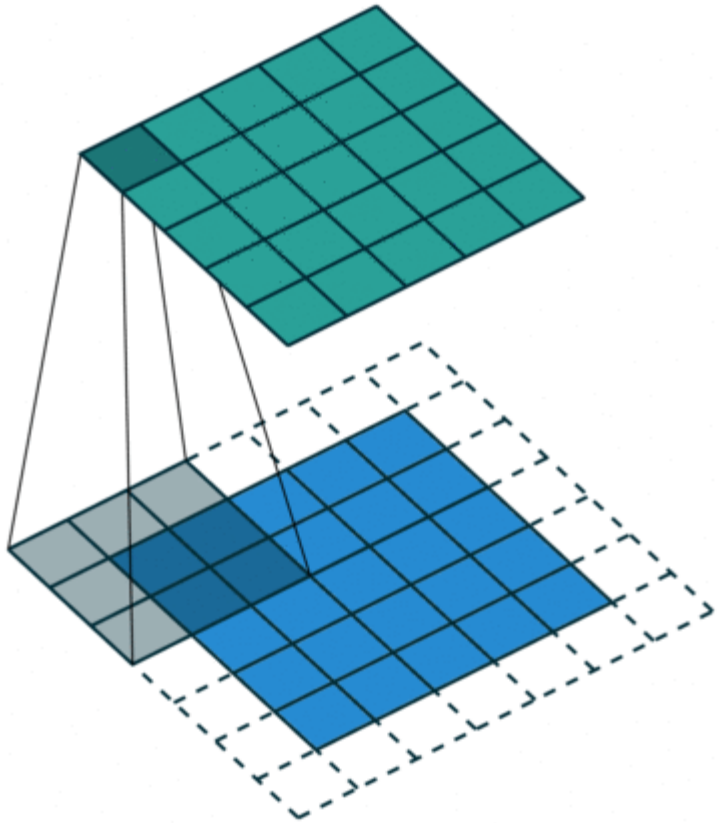


Normalization

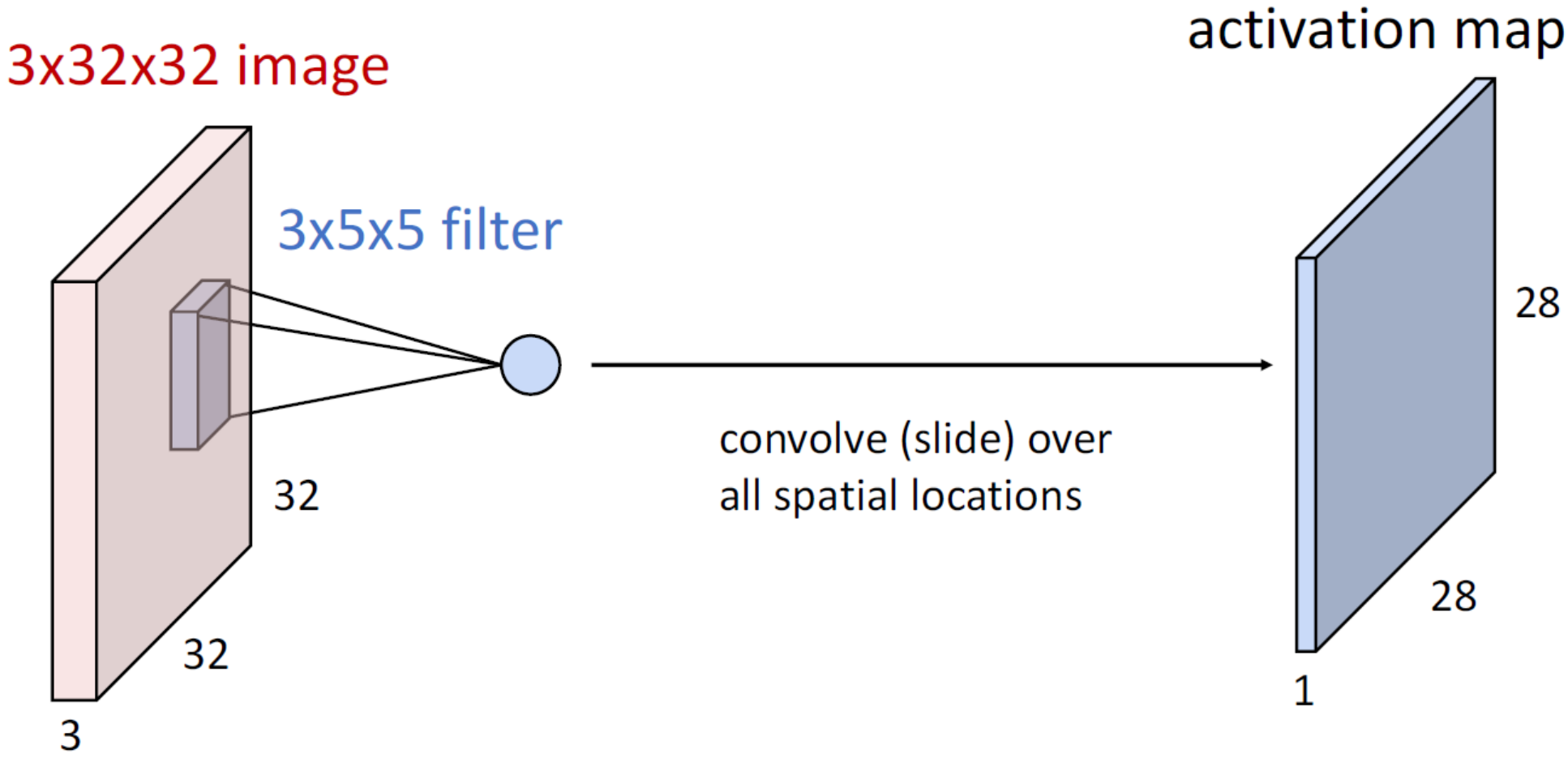
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Source: Justin Johnson

Convolution layers

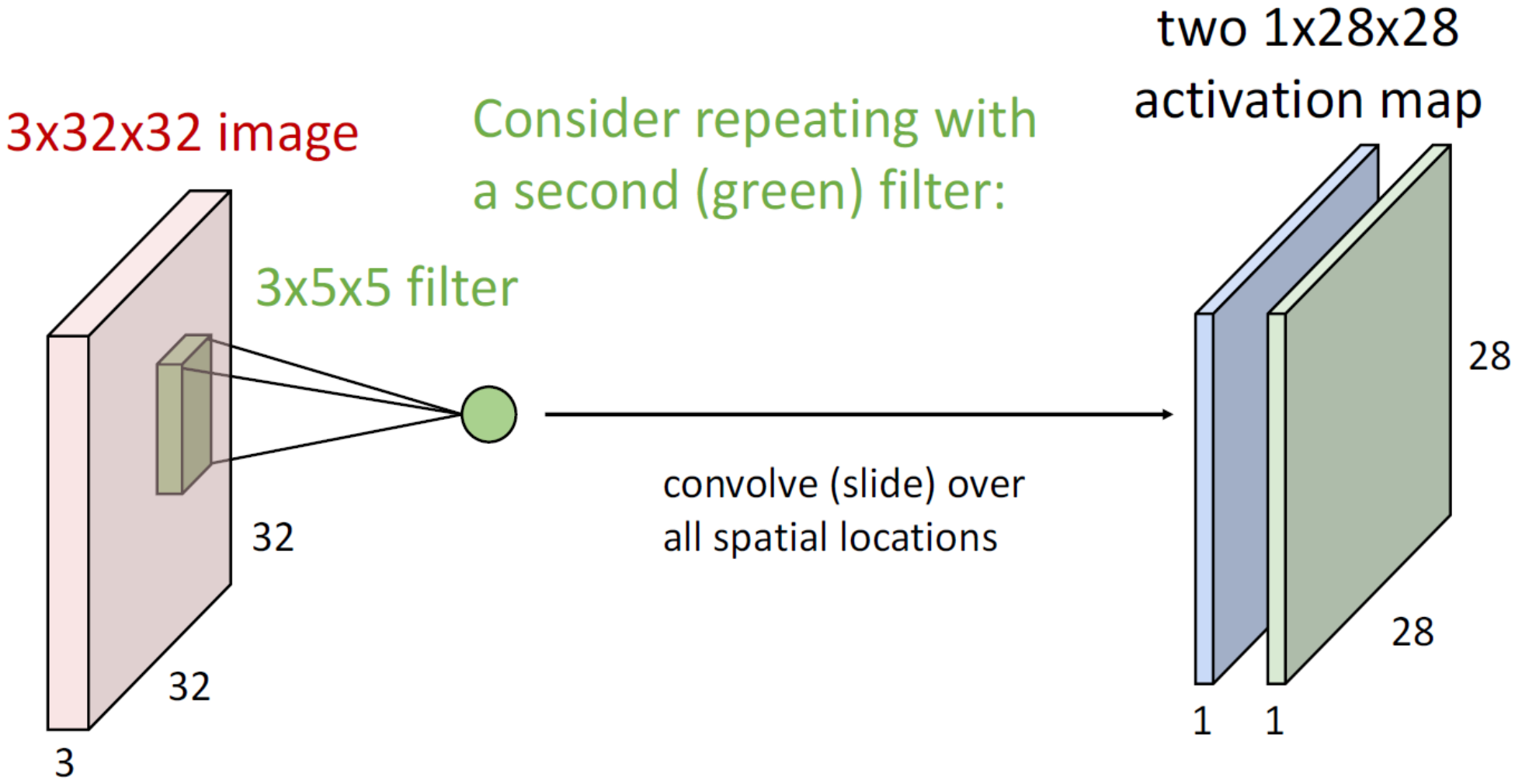


Convolution layers



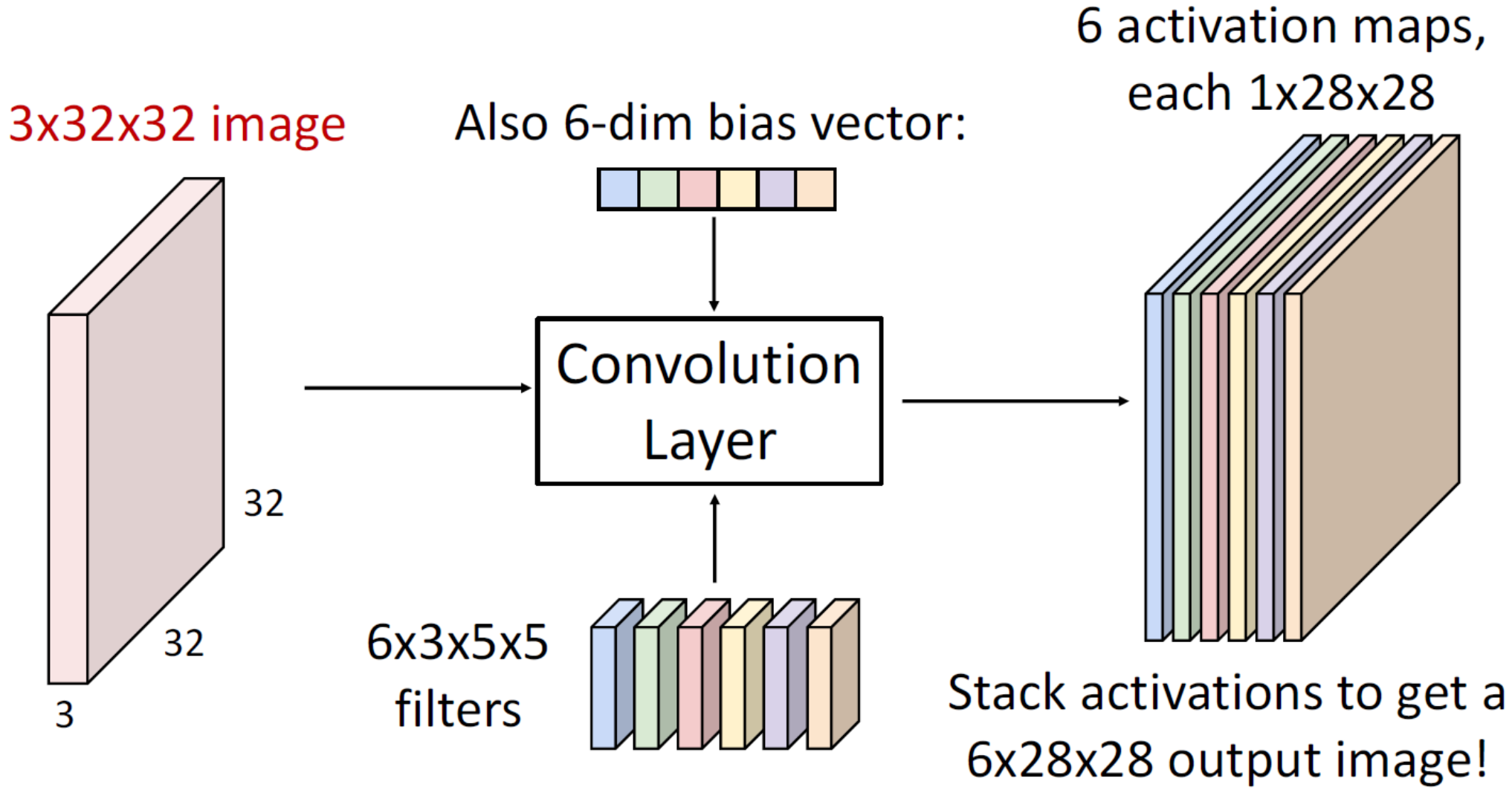
Source: Justin Johnson

Convolution layers



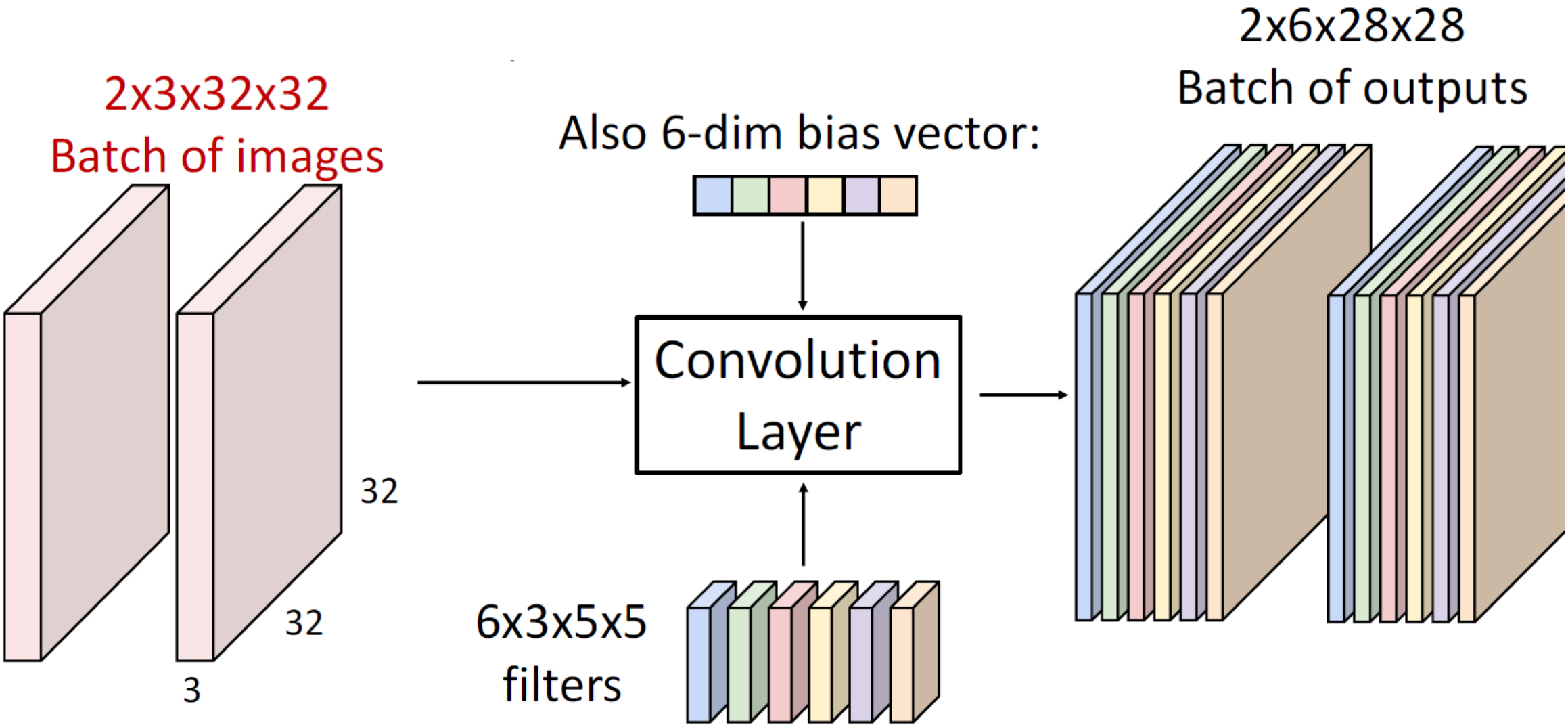
Source: Justin Johnson

Convolution layers



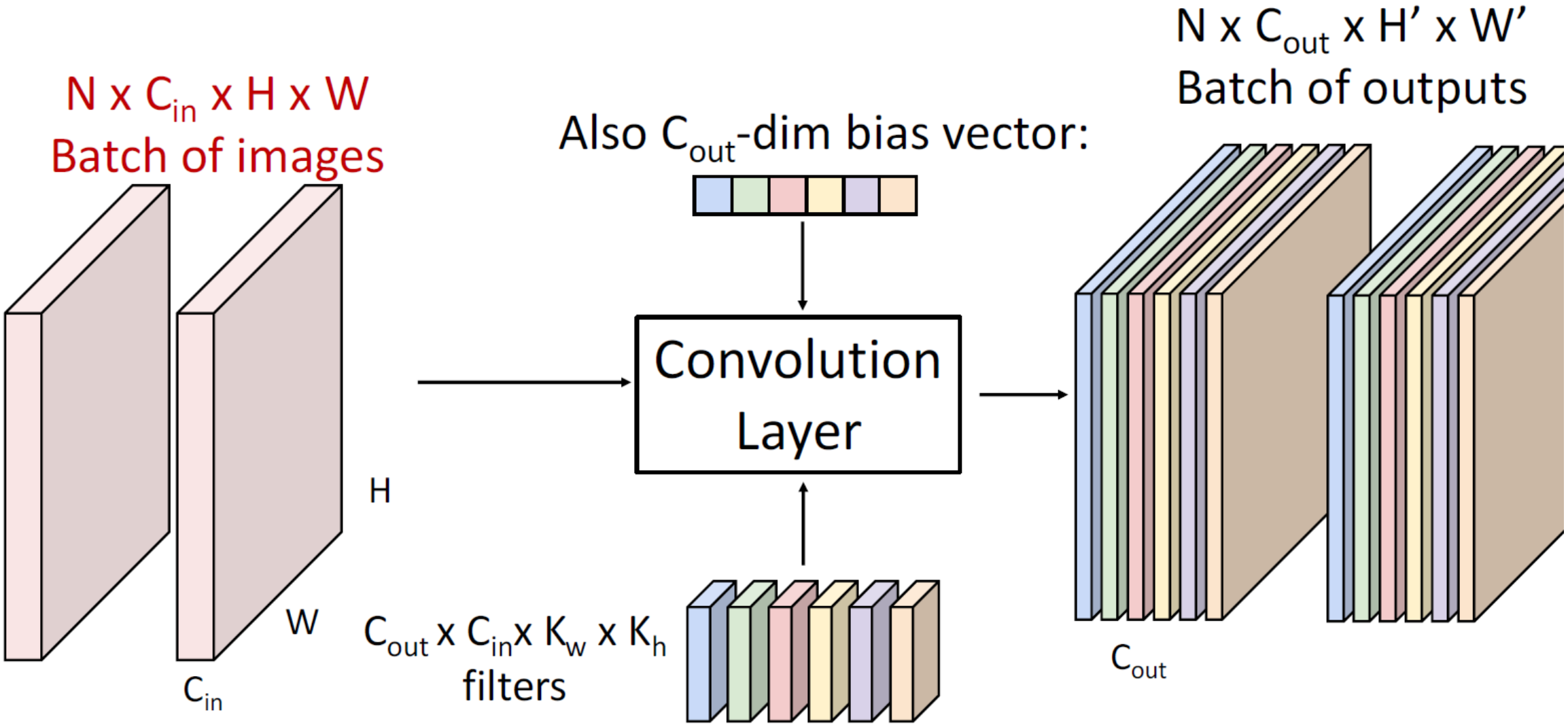
Source: Justin Johnson

Convolution layers



Source: Justin Johnson

Convolution layers



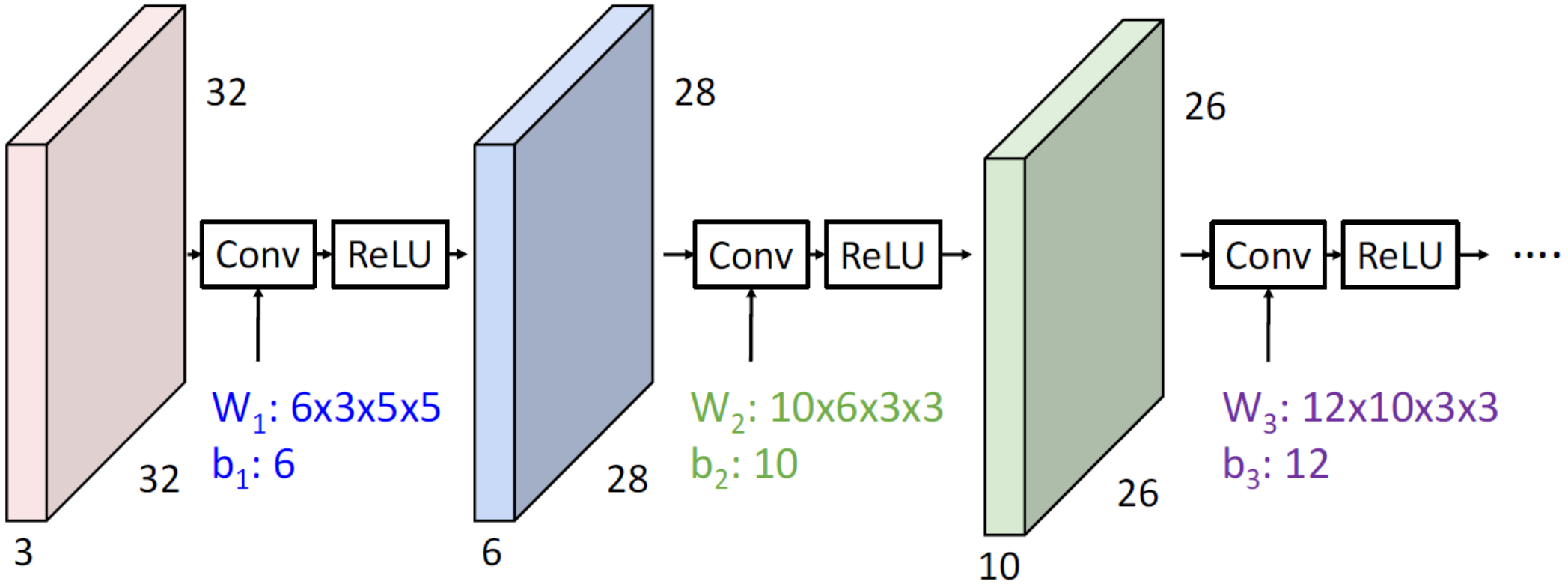
Source: Justin Johnson

Stacking convolutions

Q: What happens if we stack two convolution layers?

A: We get another convolution!

(Recall $y=W_2W_1x$ is a linear classifier)



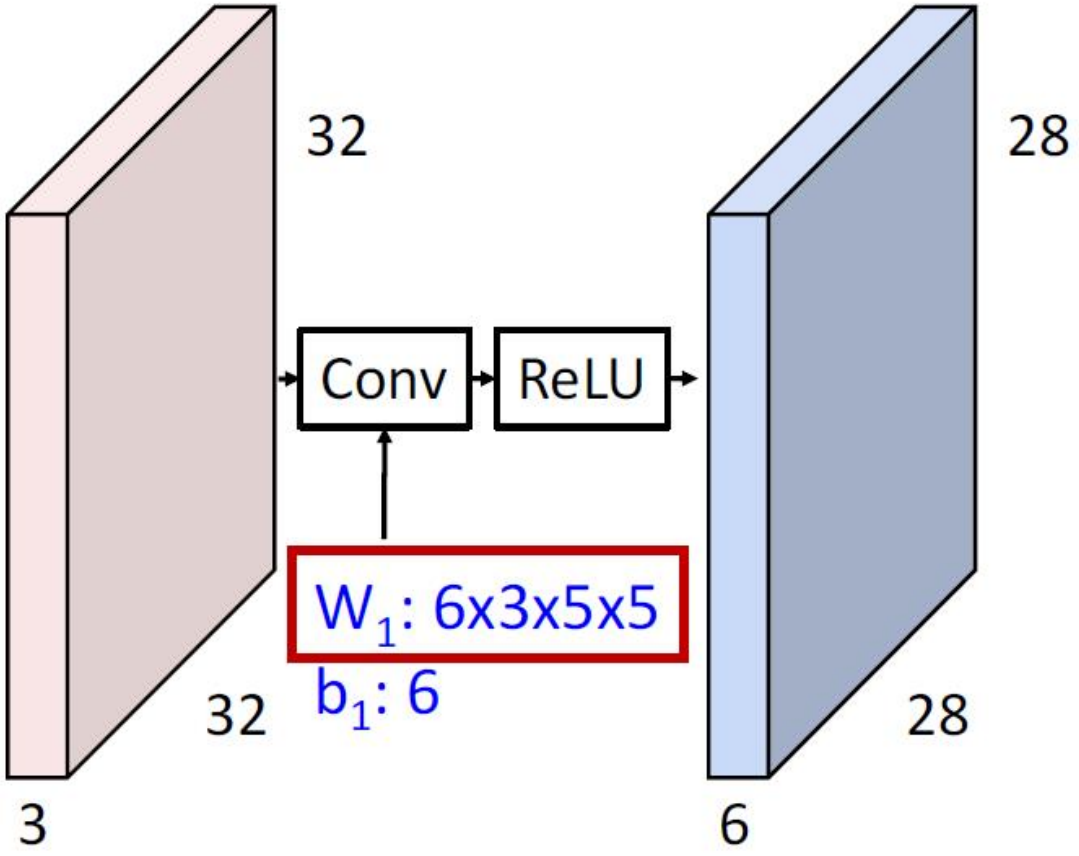
Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

Second hidden layer:
N x 10 x 26 x 26

Source: Justin Johnson

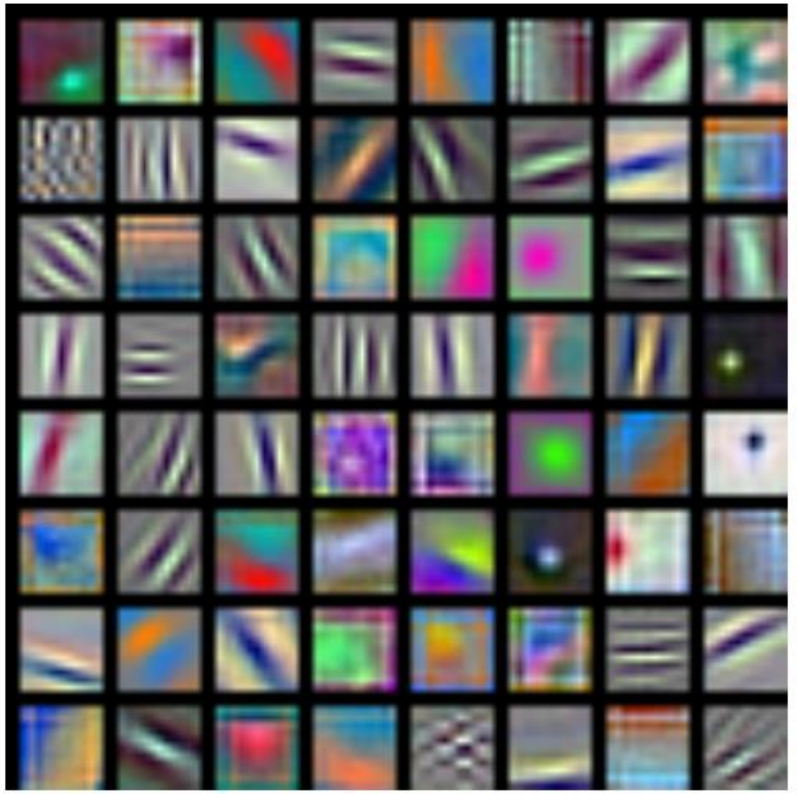
A closer look at convolutional filters



Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

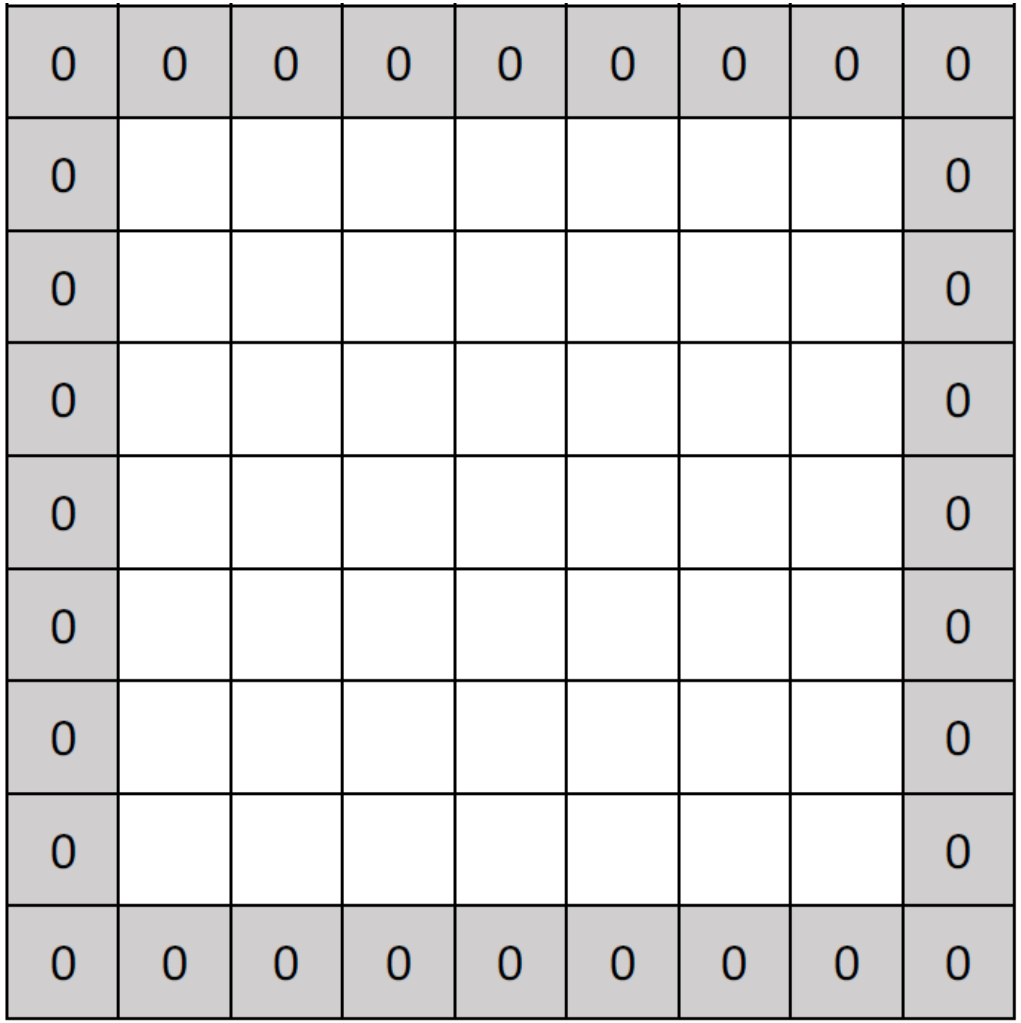
First-layer conv filters: local image templates
(Often learns oriented edges, opposing colors)



AlexNet: 64 filters, each 3x11x11

Source: Justin Johnson

Padding



Input: 7x7
Filter: 3x3
Output: 5x5

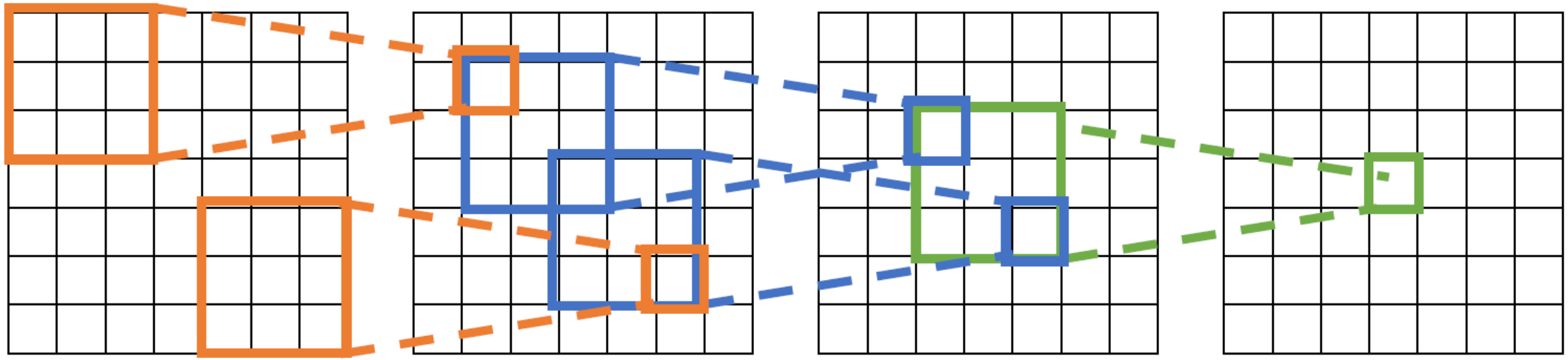
In general:
Input: W
Filter: K
Padding: P
Output: W – K + 1 + 2P

Very common:
Set P = (K – 1) / 2 to make output have same size as input!

Source: Justin Johnson

Receptive fields

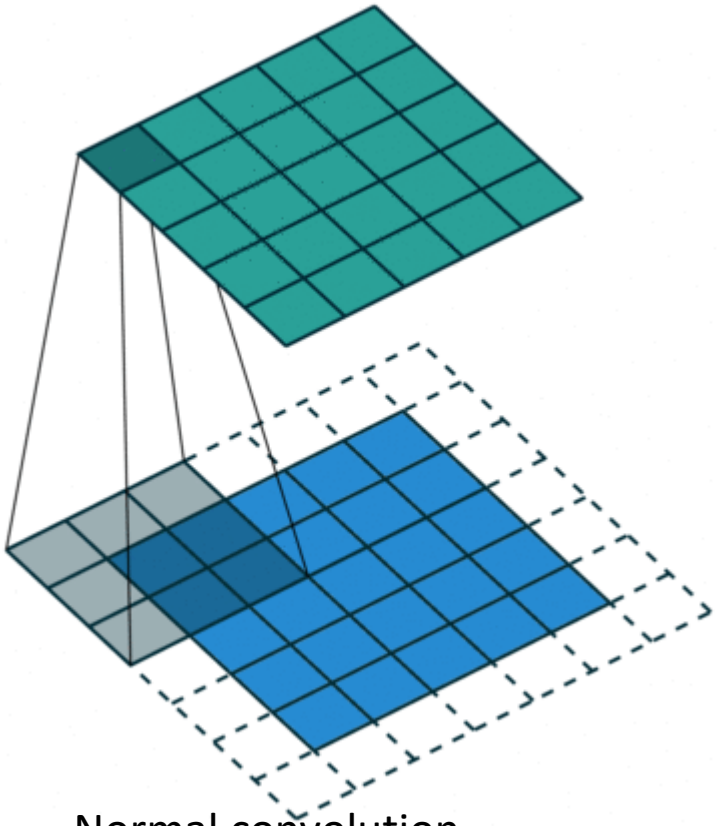
Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



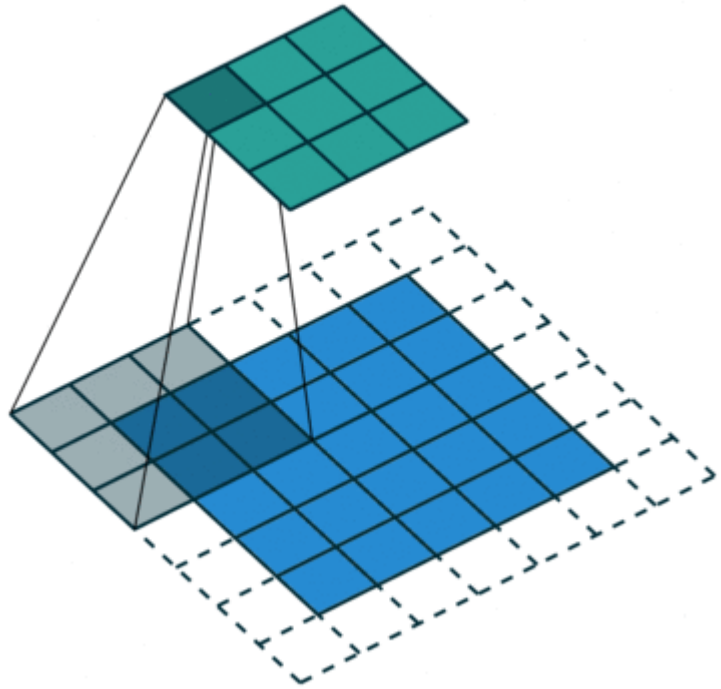
Be careful – ”receptive field in the input” vs “receptive field in the previous layer”
Hopefully clear from context!

Source: Justin Johnson

Strided convolution

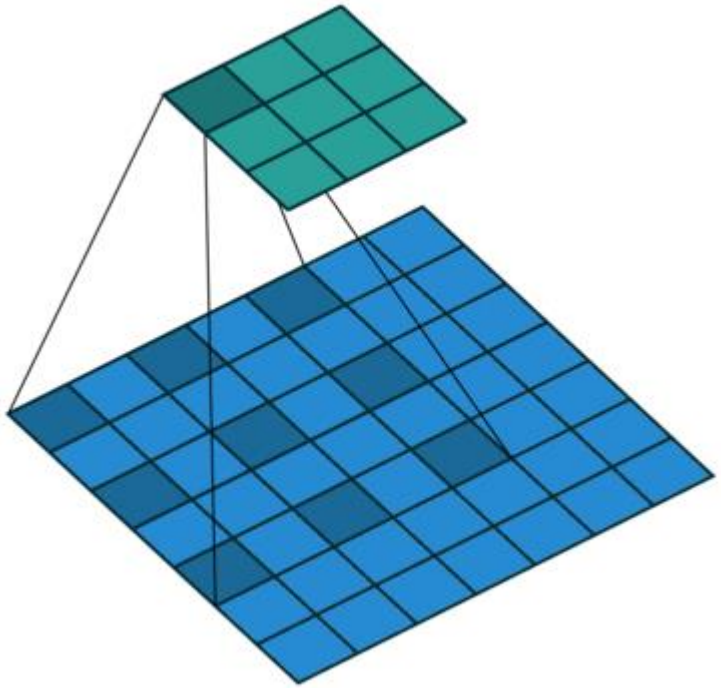


Normal convolution

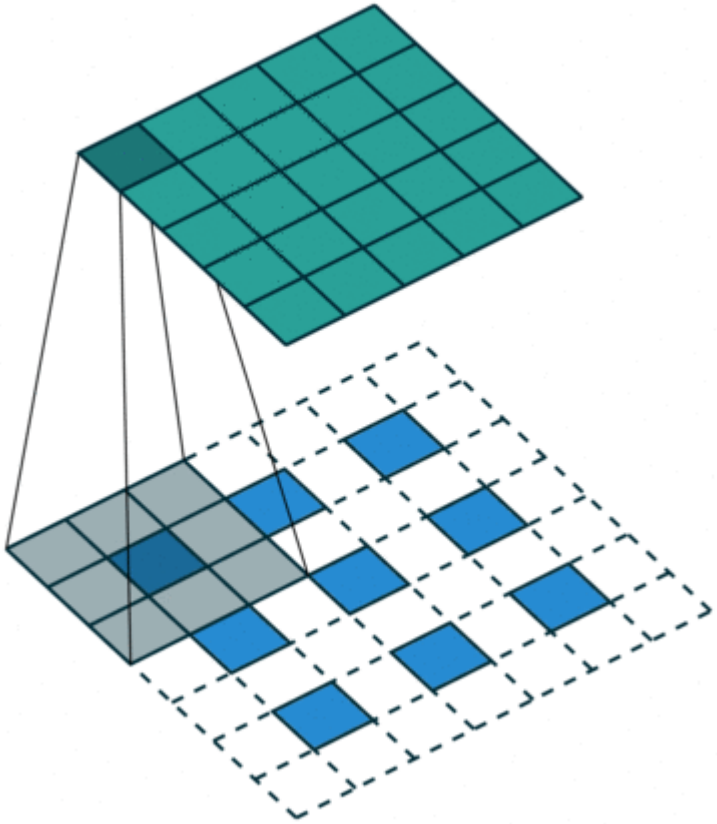


Strided convolution

Dilated convolution



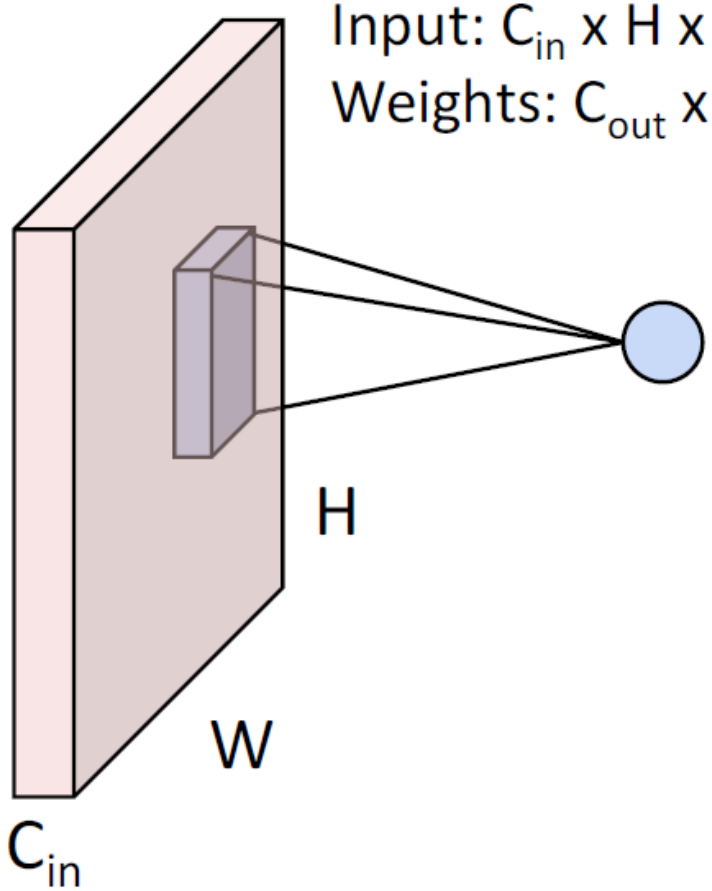
Transposed convolution



Other types of convolution

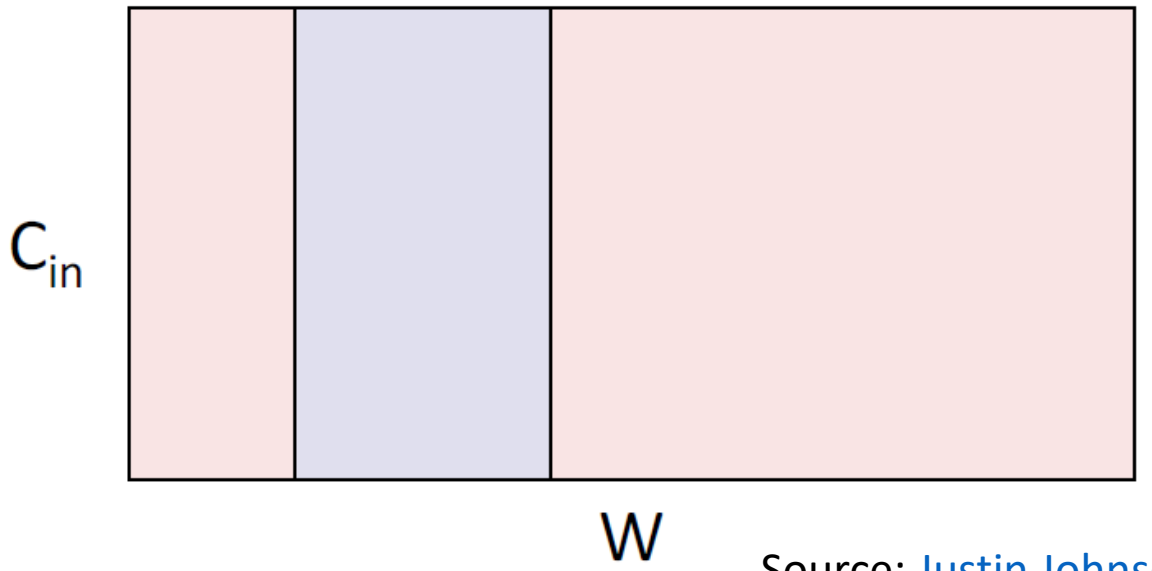
So far: 2D Convolution

Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



1D Convolution

Input: C_{in} x W
Weights: C_{out} x C_{in} x K

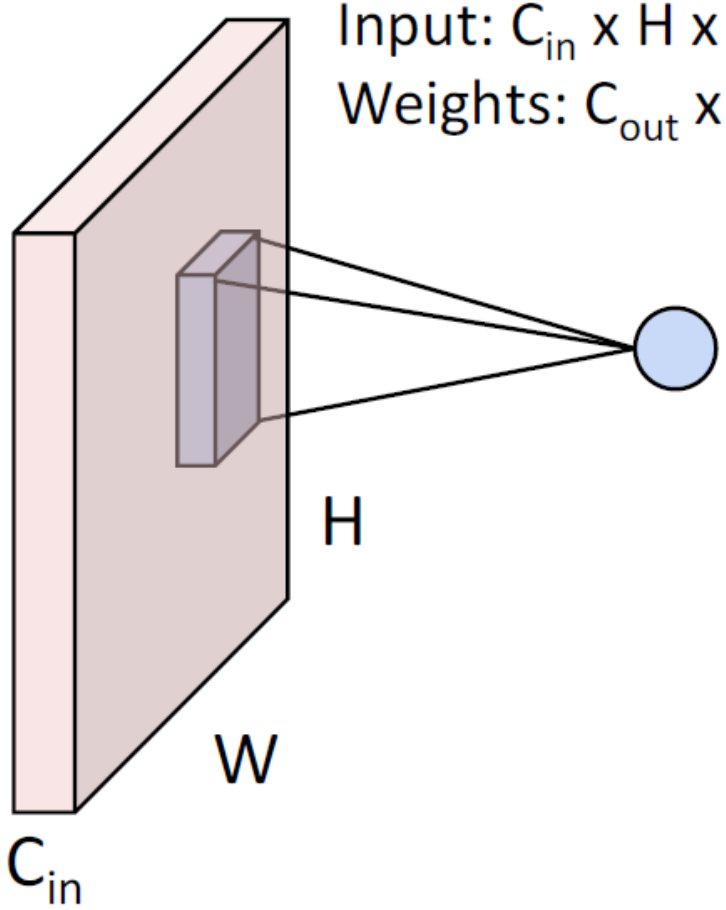


Source: Justin Johnson

Other types of convolution

So far: 2D Convolution

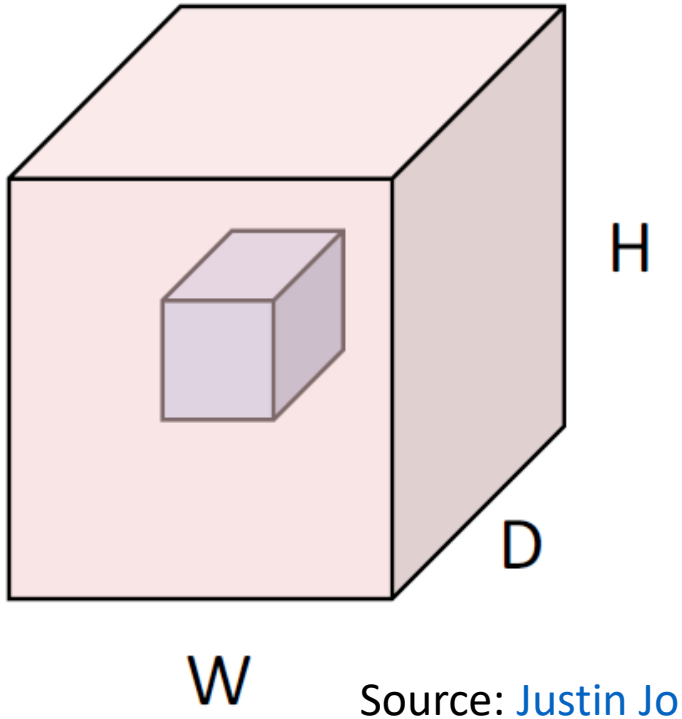
Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



3D Convolution

Input: C_{in} x H x W x D
Weights: C_{out} x C_{in} x K x K x K

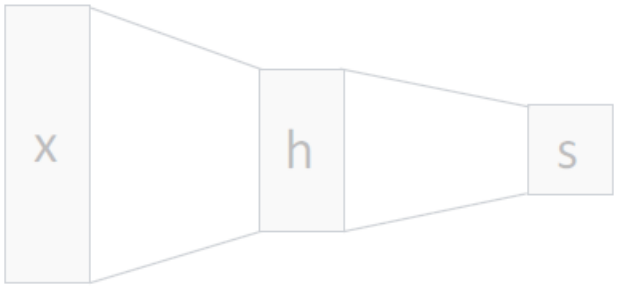
C_{in}-dim vector
at each point
in the volume



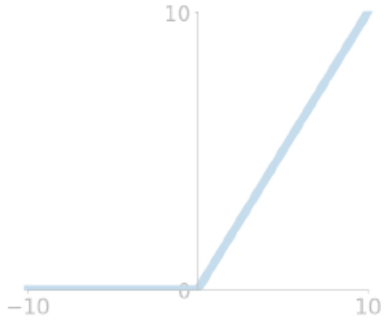
Source: Justin Johnson

Components of a Convolution Network

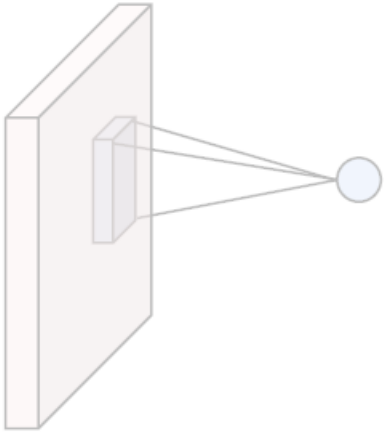
Fully-Connected Layers



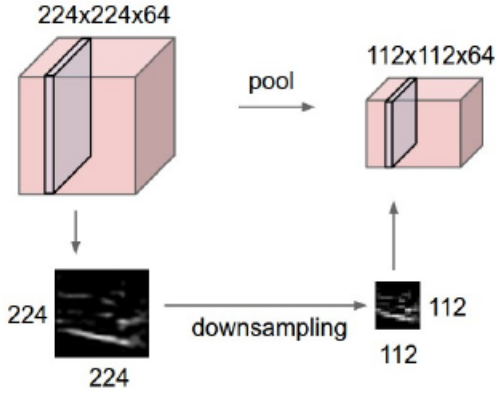
Activation Function



Convolution Layers



Pooling Layers

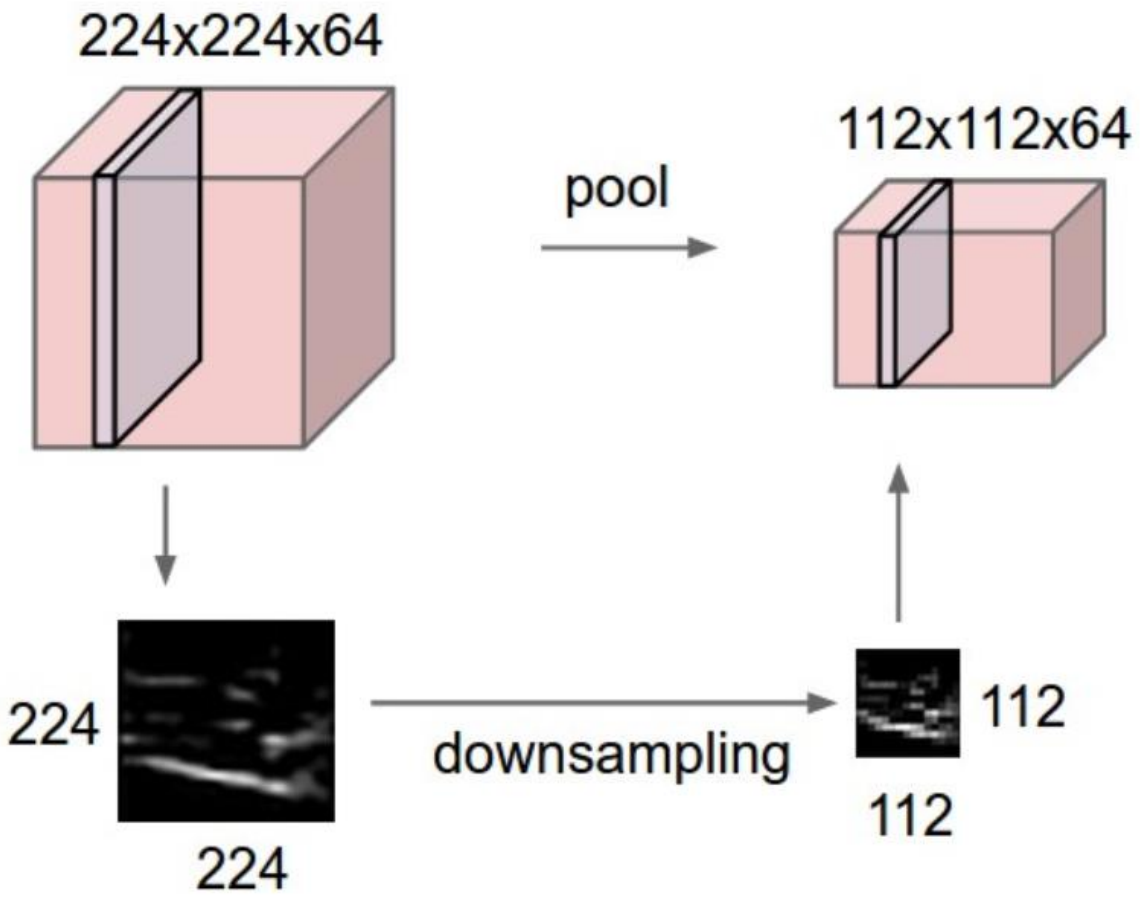


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Source: Justin Johnson

Pooling layers

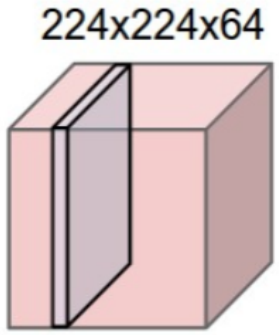


Hyperparameters:
Kernel Size
Stride
Pooling function

Source: Justin Johnson

Max pooling

Geoffrey Hinton: “The pooling operation used in convolutional neural networks is a big mistake and the fact that it works so well is a disaster.”



Single depth slice

x

1	1	2	4
5	**6**	7	**8**
3	2	1	0
1	2	3	**4**

y

Max pooling with 2x2 kernel size and stride 2

6	8
3	4

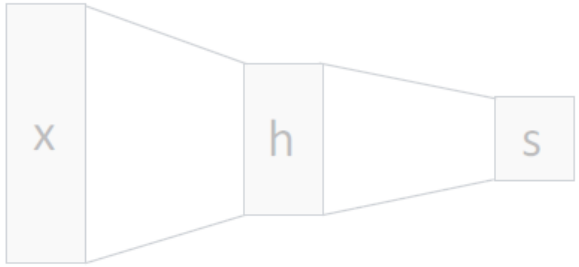
Introduces **invariance** to small spatial shifts

No learnable parameters!

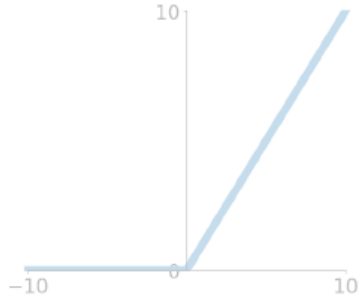
Source: Justin Johnson

Components of a Convolution Network

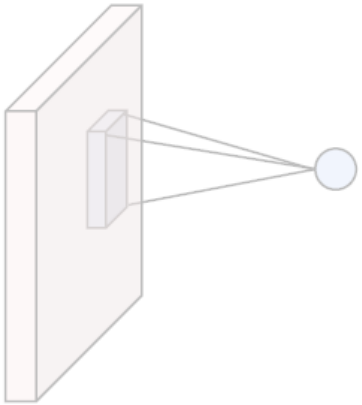
Fully-Connected Layers



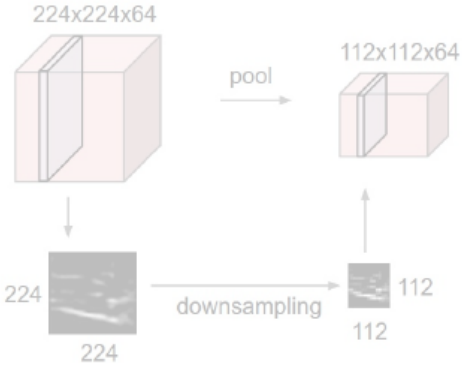
Activation Function



Convolution Layers



Pooling Layers



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Source: Justin Johnson

Batch normalization

Idea: “Normalize” the outputs of a layer so they have zero mean and unit variance

Why? Helps reduce “internal covariate shift”, improves optimization

We can normalize a batch of activations like this:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

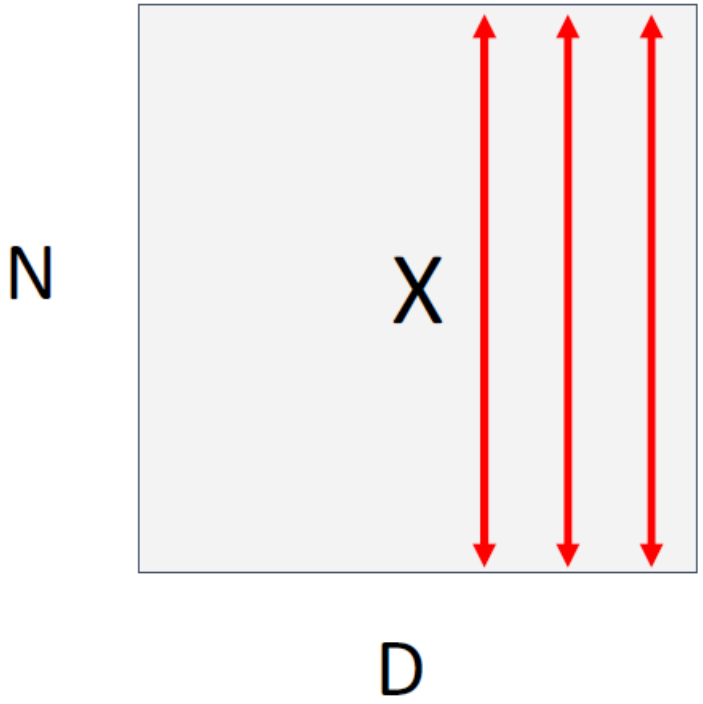
This is a **differentiable function**, so we can use it as an operator in our networks and backprop through it!

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Source: Justin Johnson

Batch normalization

Input: $x : N \times D$



$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{i,j} - \mu_j)^2$$

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

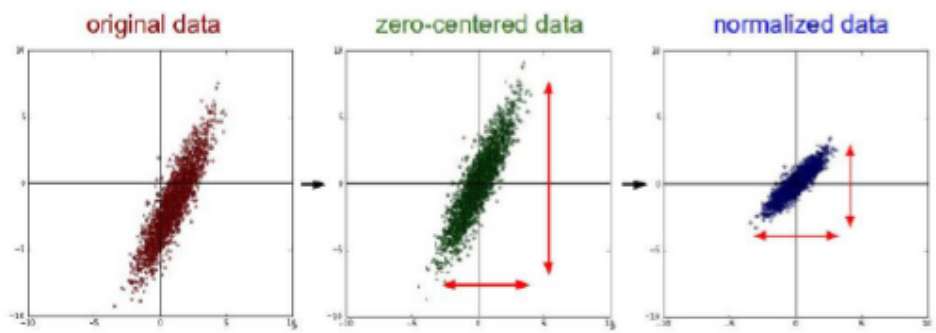
Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Source: Justin Johnson

Whitening http://cs231n.stanford.edu/slides/2018/cs231n_2018_lecture07.pdf

Decorrelated Batch Normalization

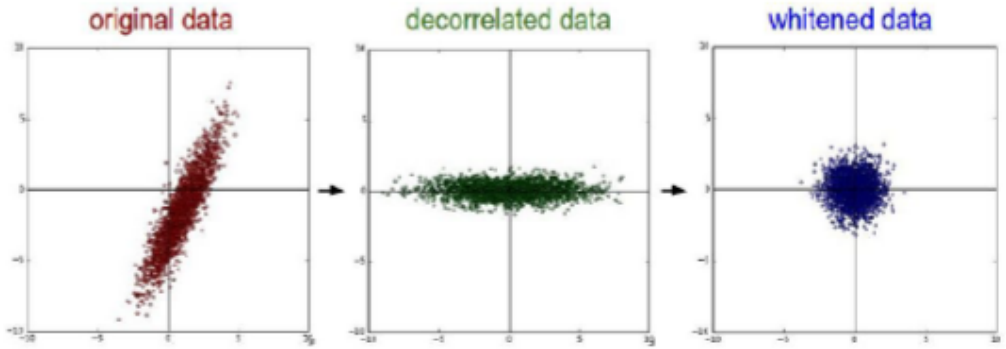
Batch Normalization



$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

BatchNorm normalizes the data, but cannot correct for correlations among the input features

Decorrelated Batch Normalization



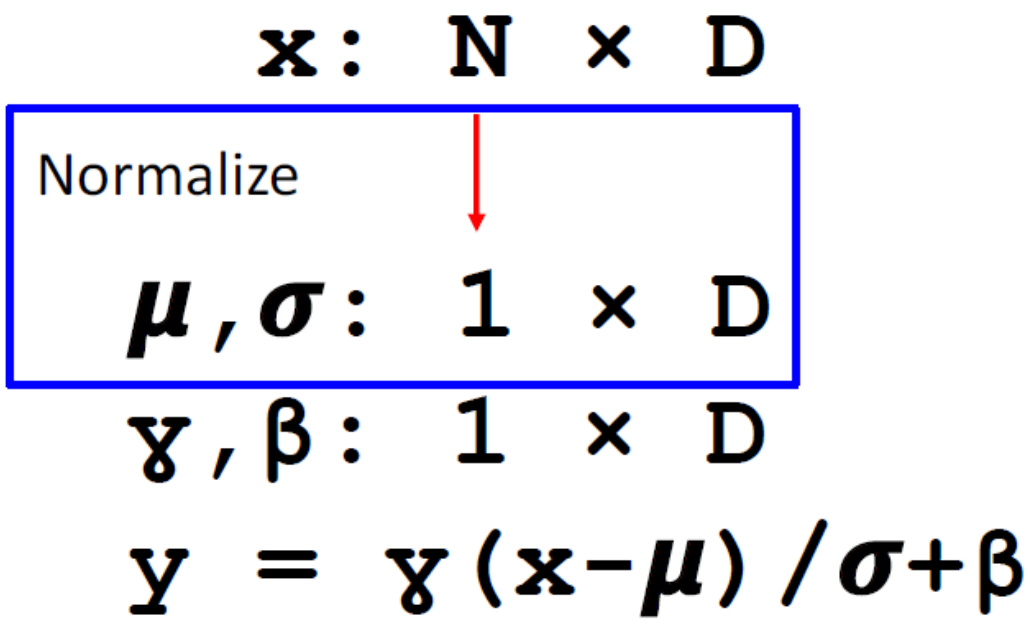
$$\hat{x}_i = \Sigma^{-\frac{1}{2}}(x_i - \mu)$$

DBN **whitens** the data using the full covariance matrix of the minibatch; this corrects for correlations

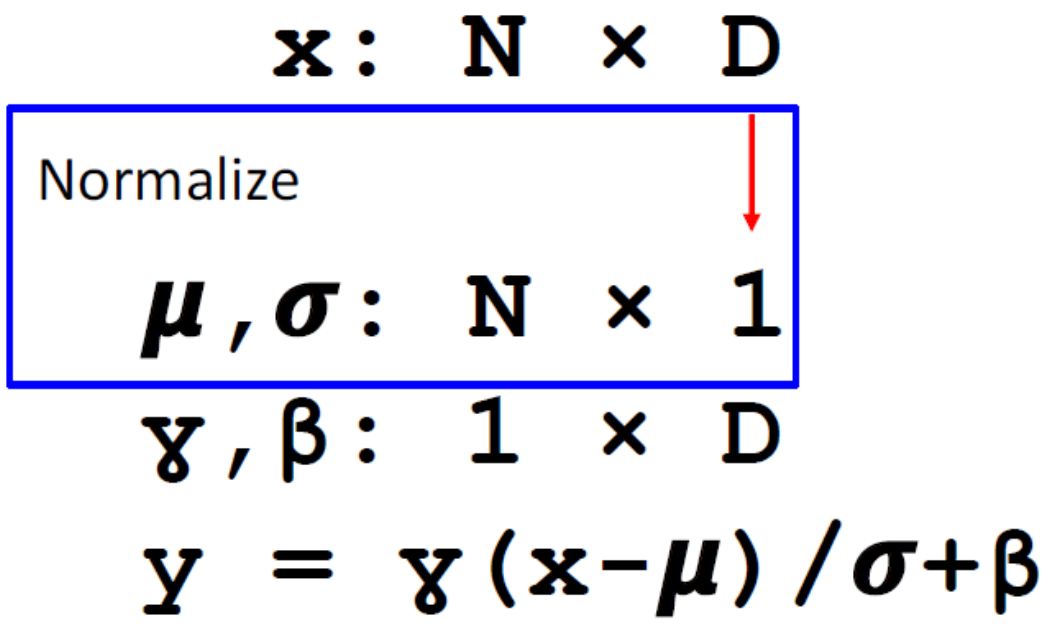
Huang et al, "Decorrelated Batch Normalization", arXiv 2018 (Appeared 4/23/2018)

Layer normalization

Batch Normalization for fully-connected networks



Layer Normalization for fully-connected networks
Same behavior at train and test!
Used in RNNs, Transformers

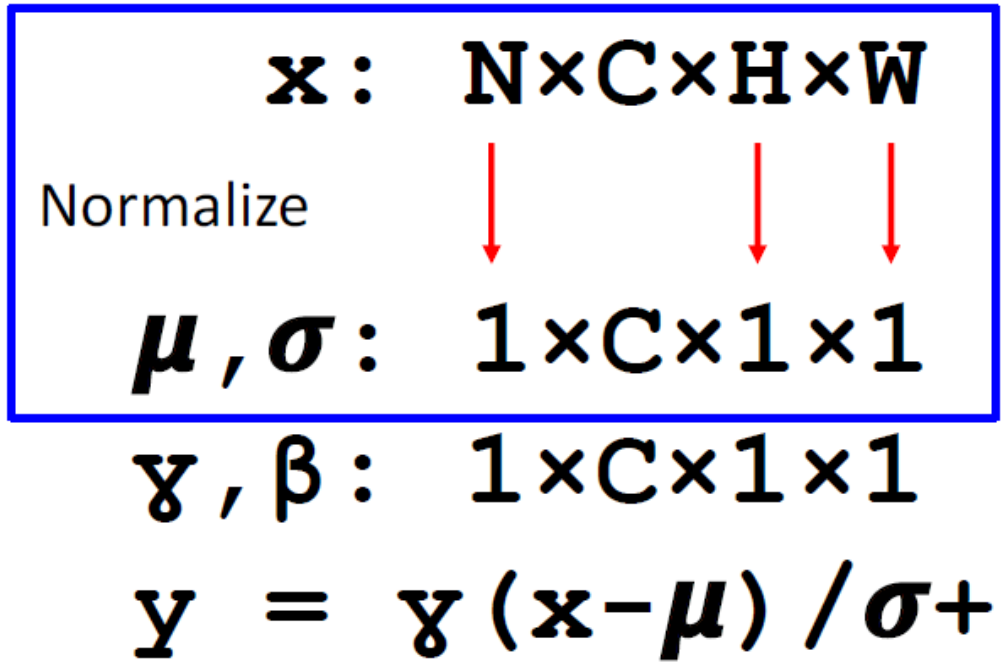


Ba, Kiros, and Hinton, "Layer Normalization", arXiv 2016

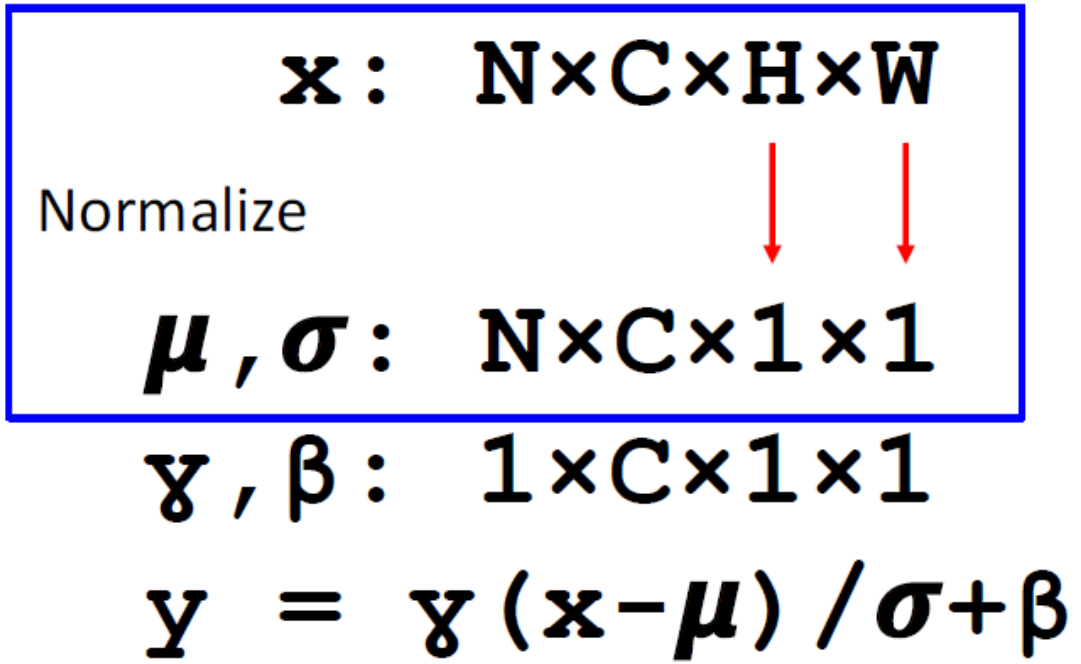
Source: Justin Johnson

Instance normalization

Batch Normalization for convolutional networks



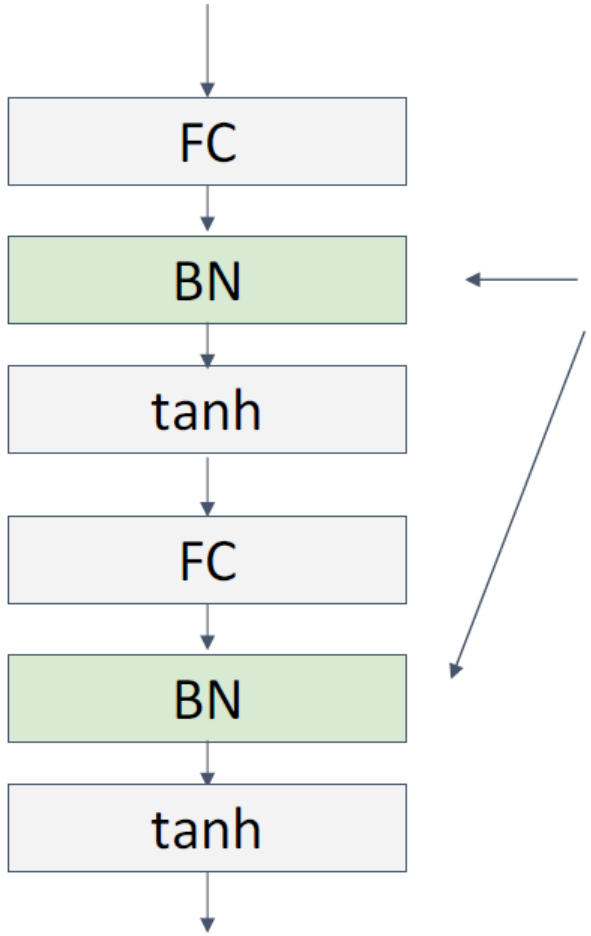
Instance Normalization for convolutional networks
Same behavior at train / test!



Ulyanov et al, Improved Texture Networks: Maximizing Quality and Diversity in Feed-forward Stylization and Texture Synthesis, CVPR 2017

Source: Justin Johnson

Batch normalization



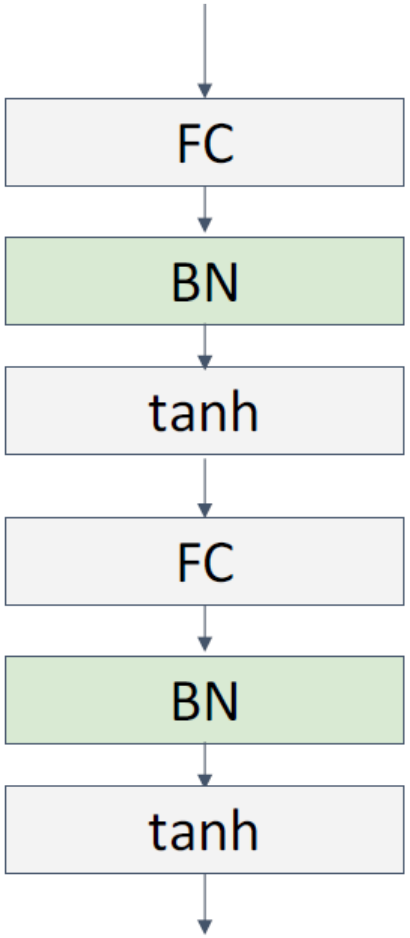
Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Source: Justin Johnson

Batch normalization



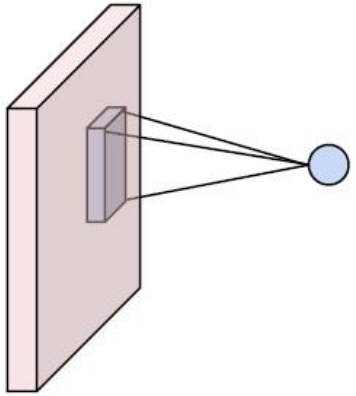
- Makes deep networks **much** easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!
- Not well-understood theoretically (yet)
- Behaves differently during training and testing: this is a very common source of bugs!

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

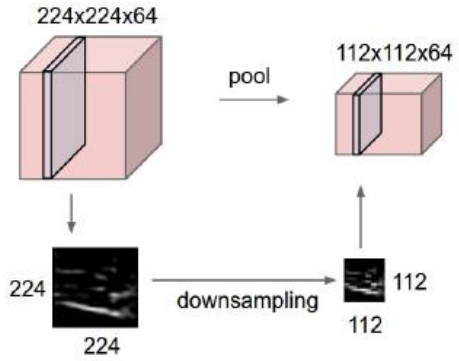
Source: Justin Johnson

Summary:
Components of a Convolution Network

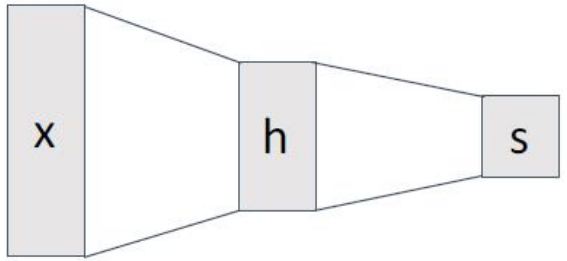
Convolution Layers



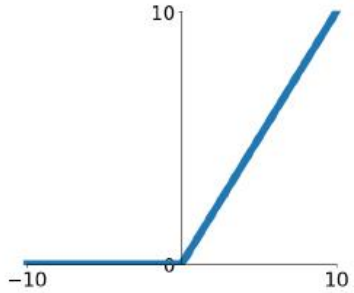
Pooling Layers



Fully-Connected Layers



Activation Function

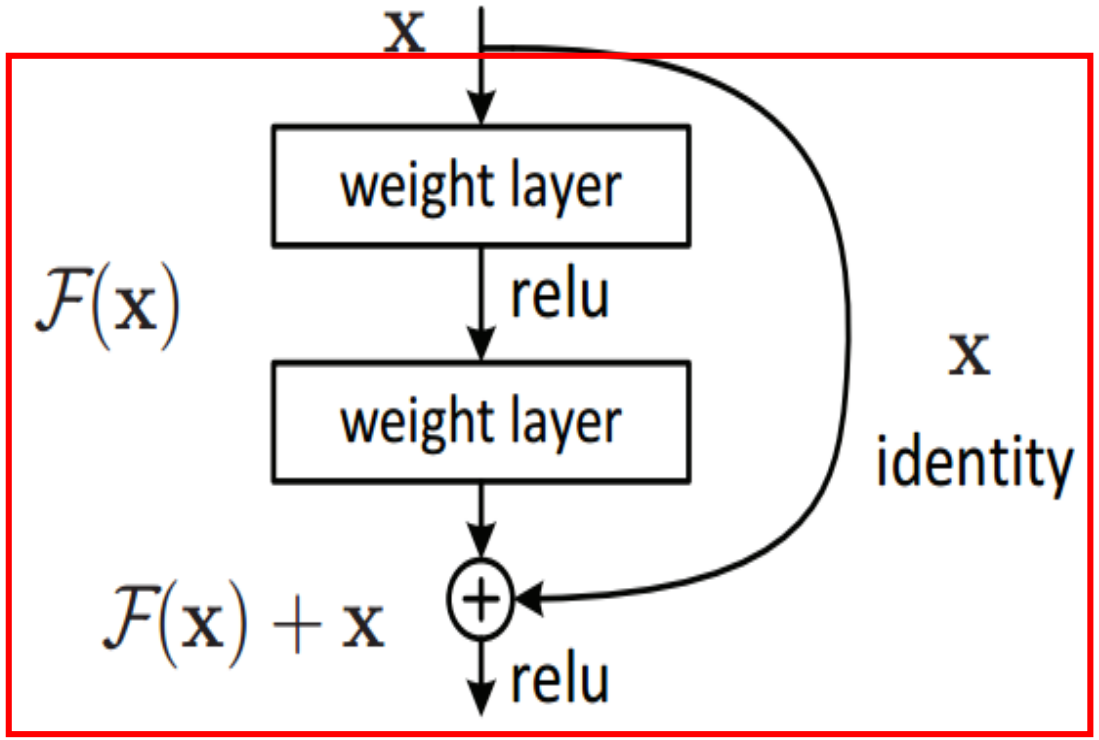


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

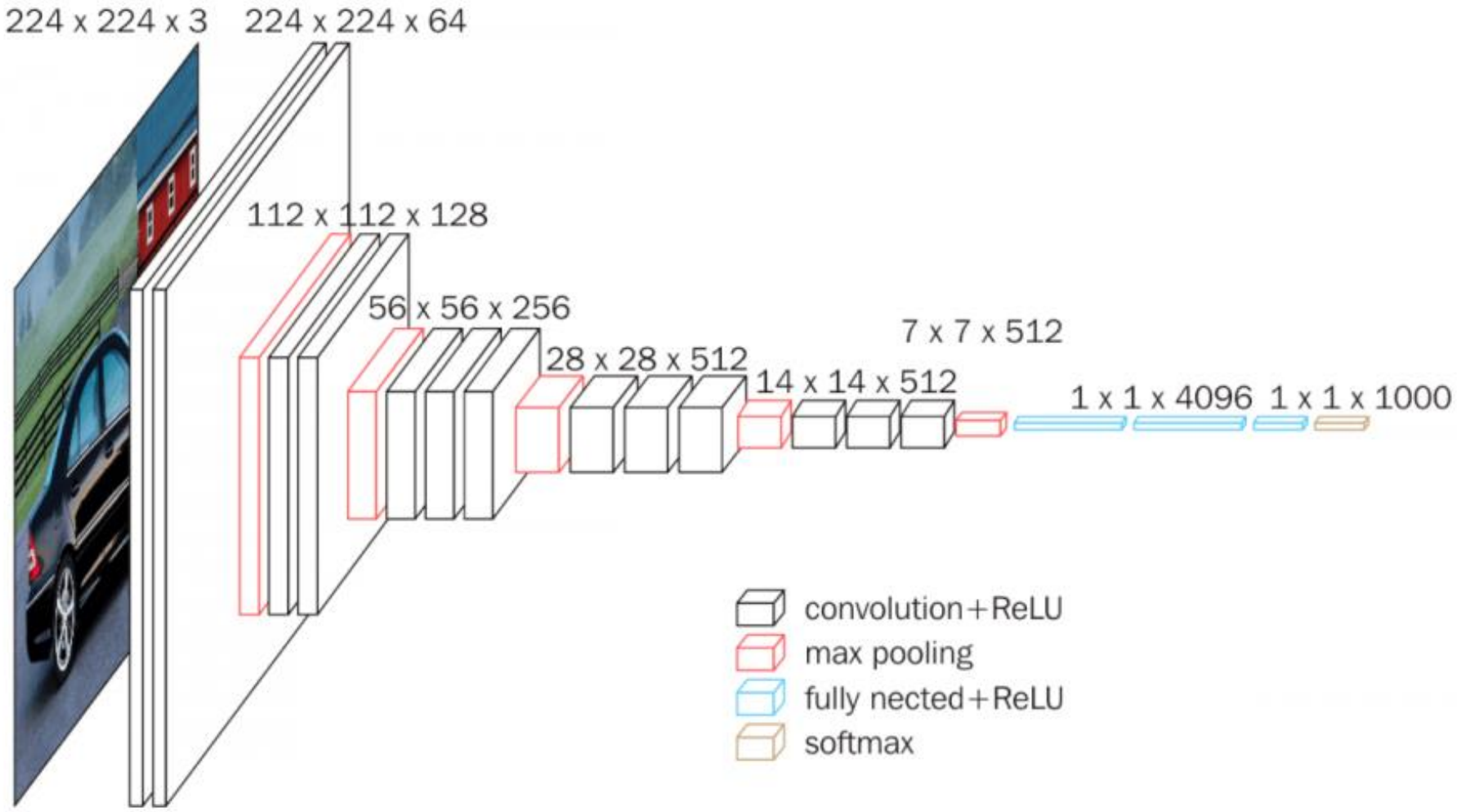
Source: Justin Johnson

Residual connections

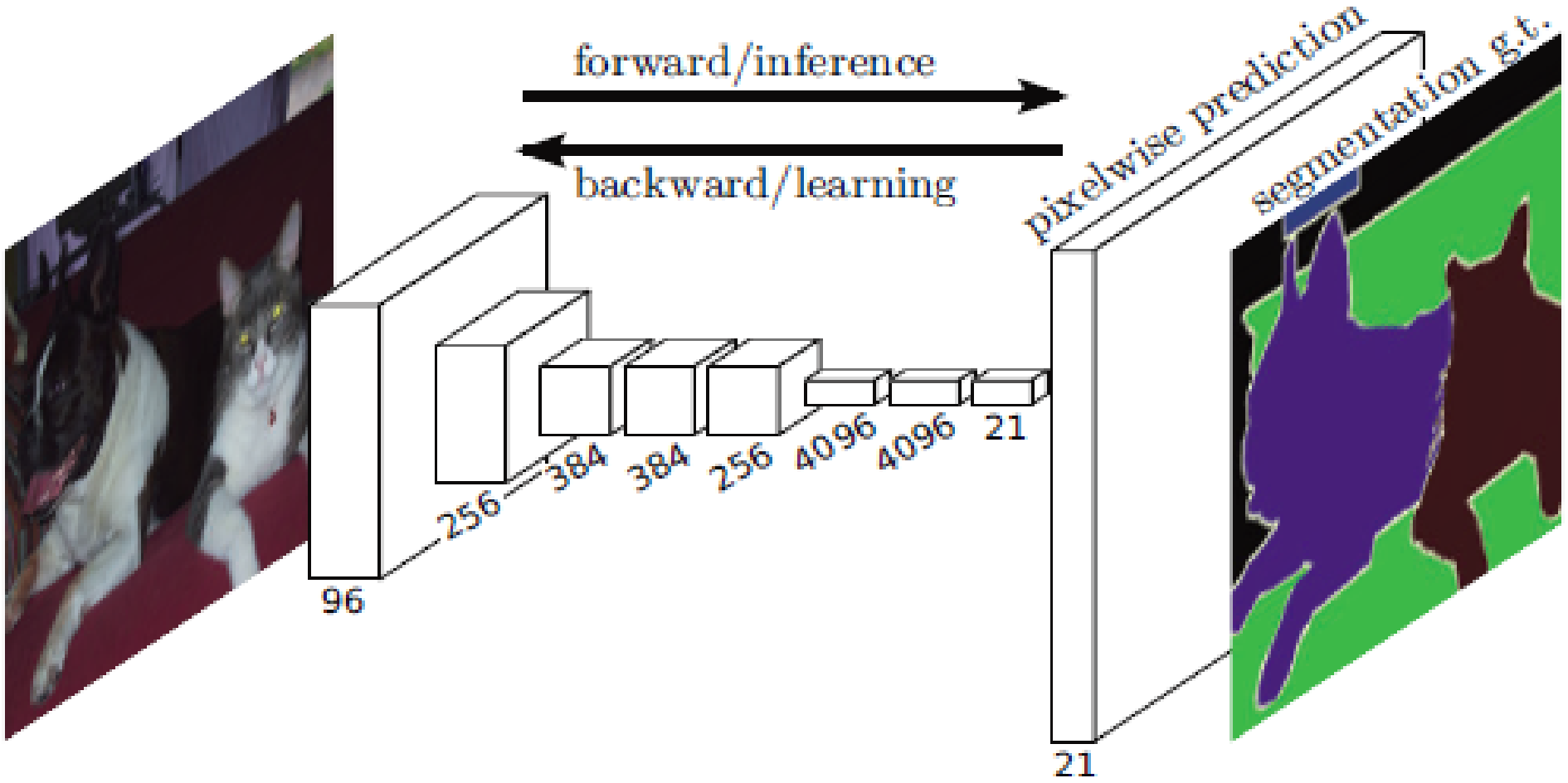


Residual block

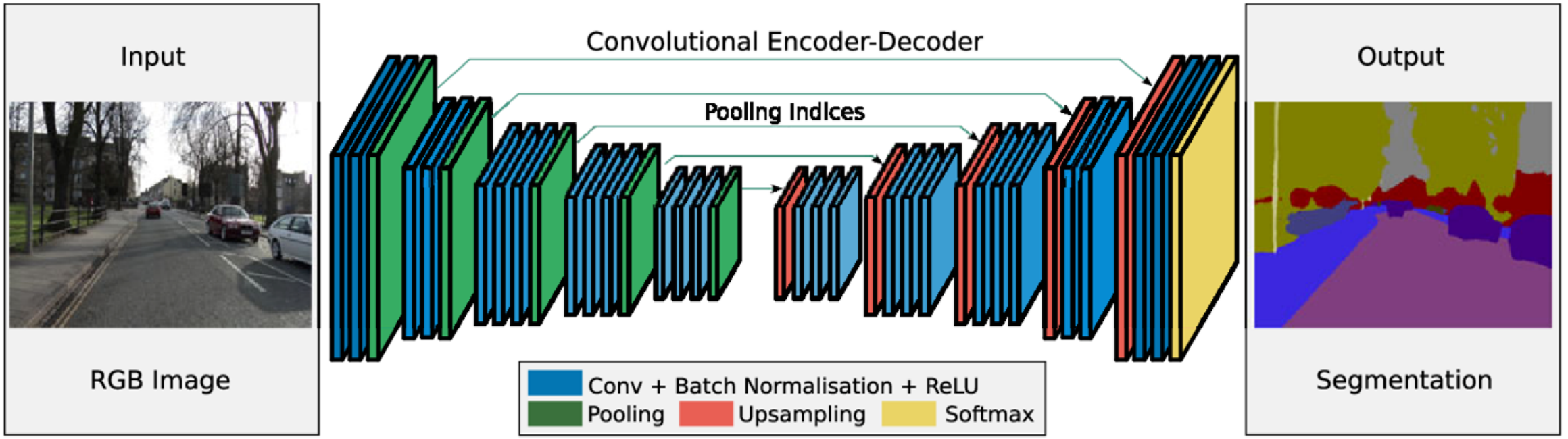
Architecture: Classification networks



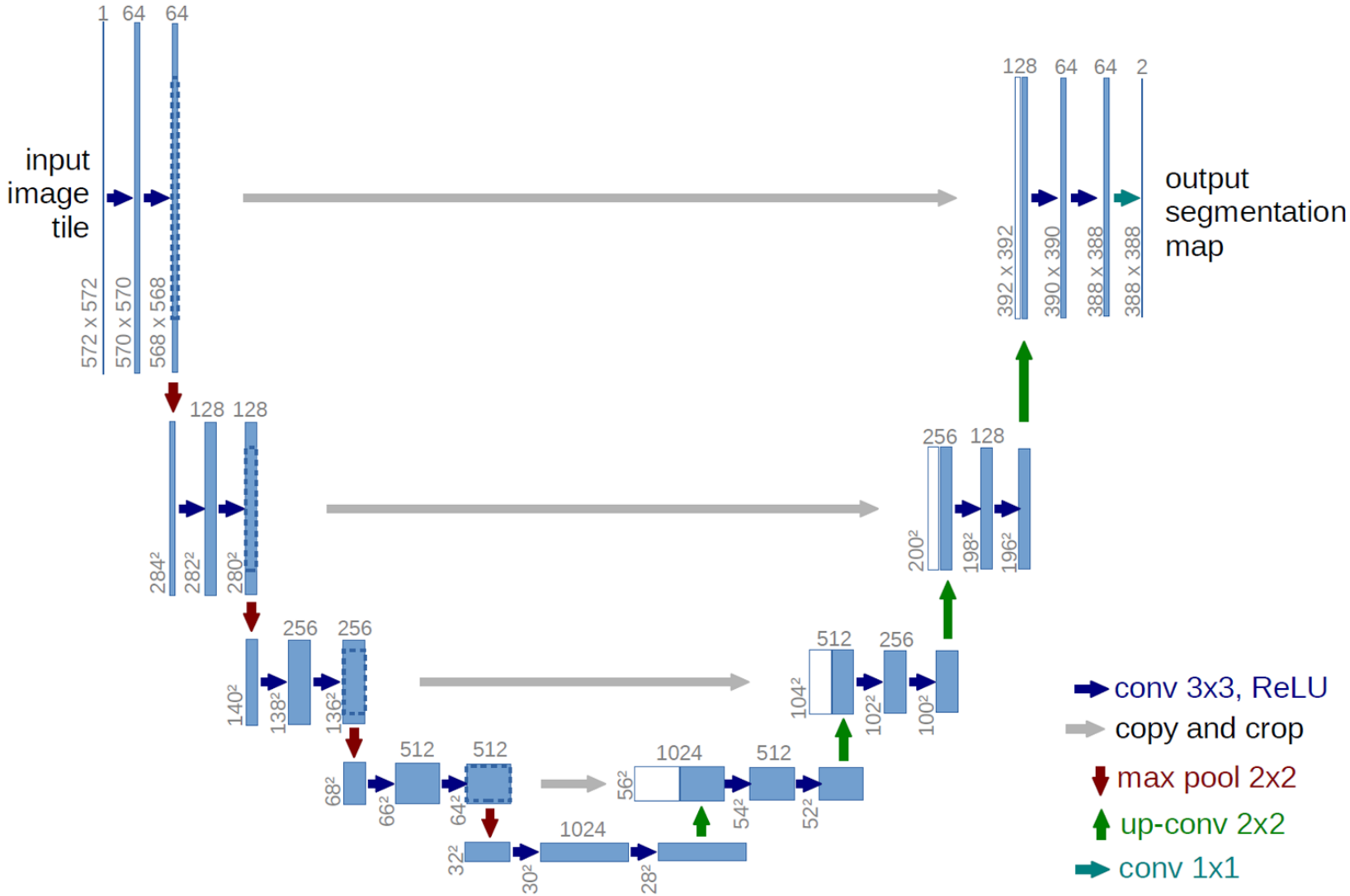
Architecture: Fully convolutional



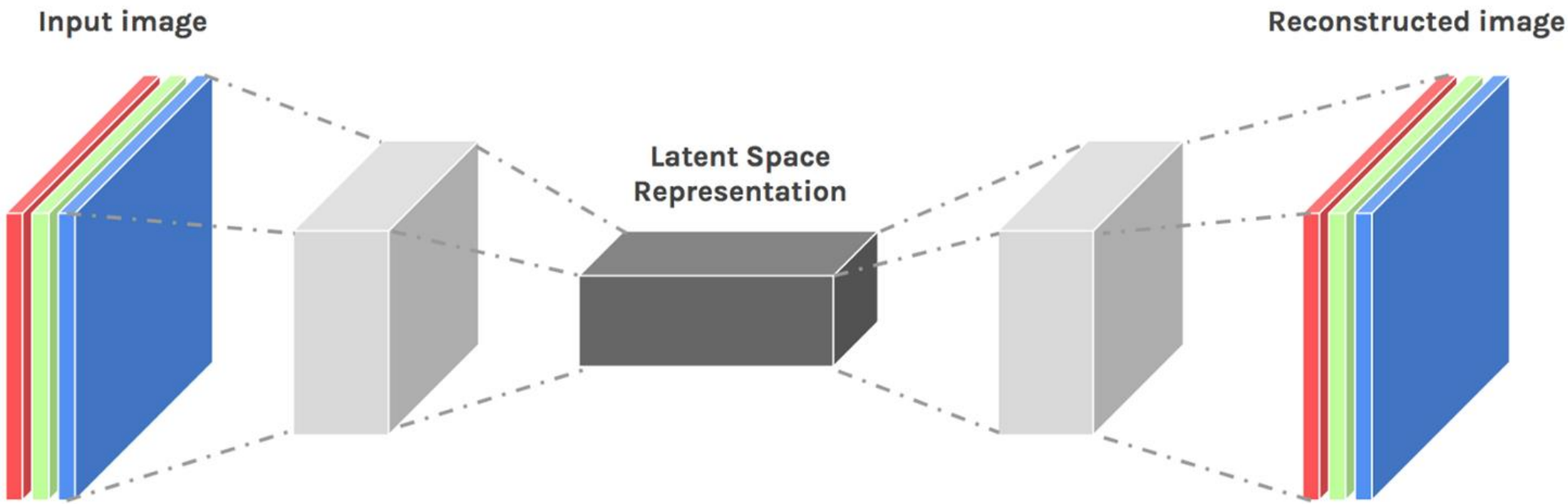
Architecture: Encoder/Decoder



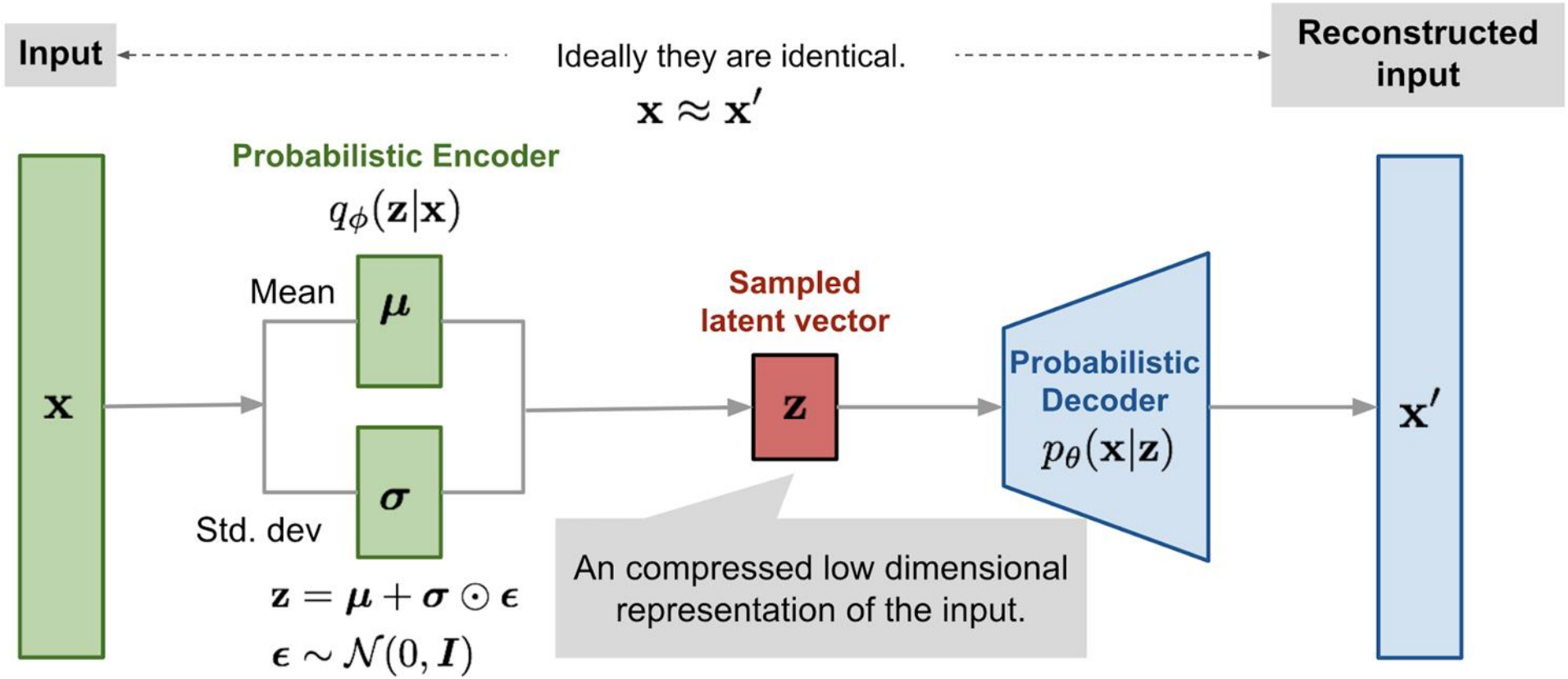
Architecture: U-Net



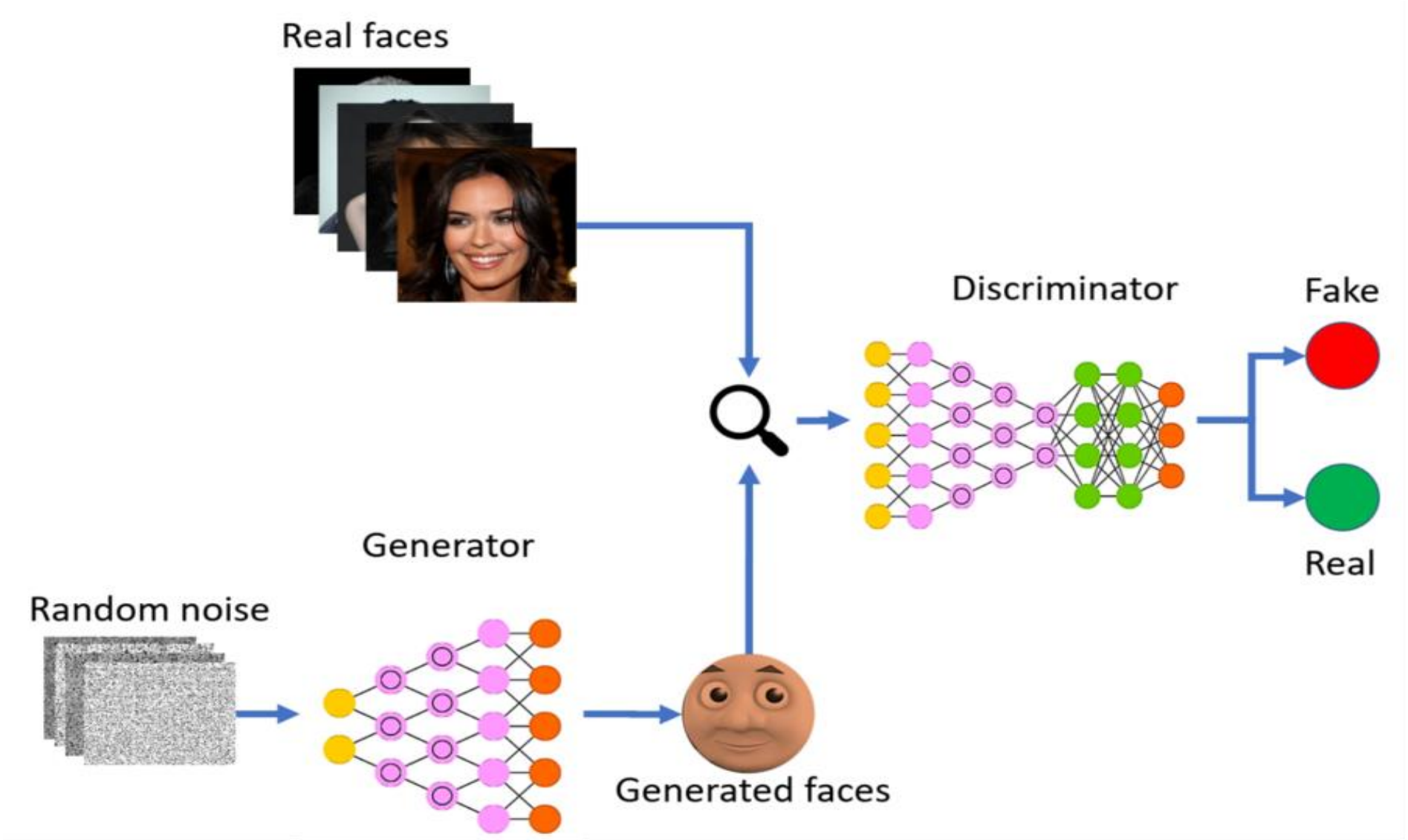
Architecture: Autoencoder



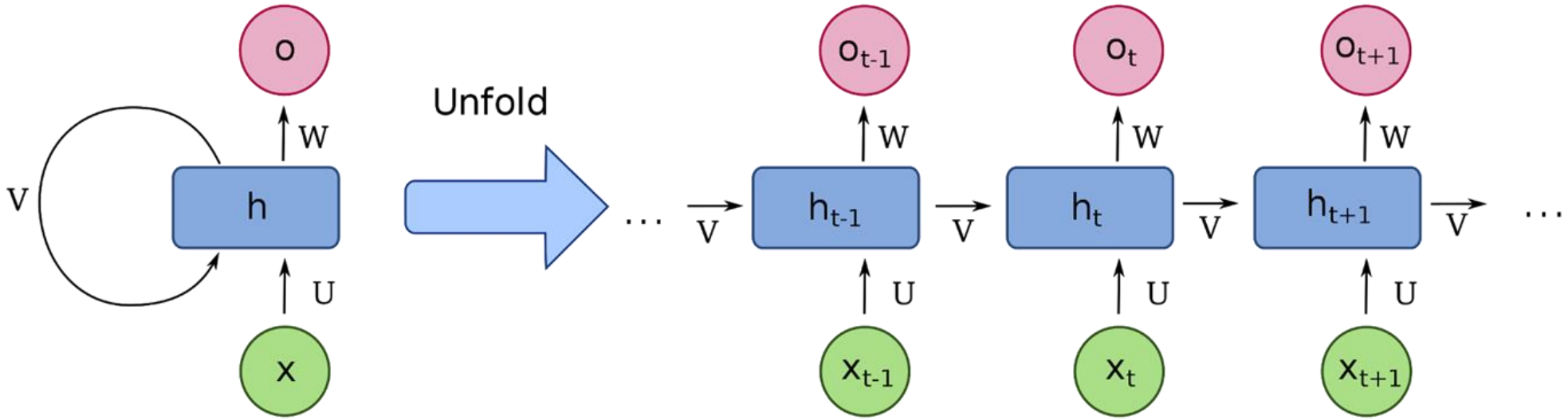
Architecture: Variational Autoencoder



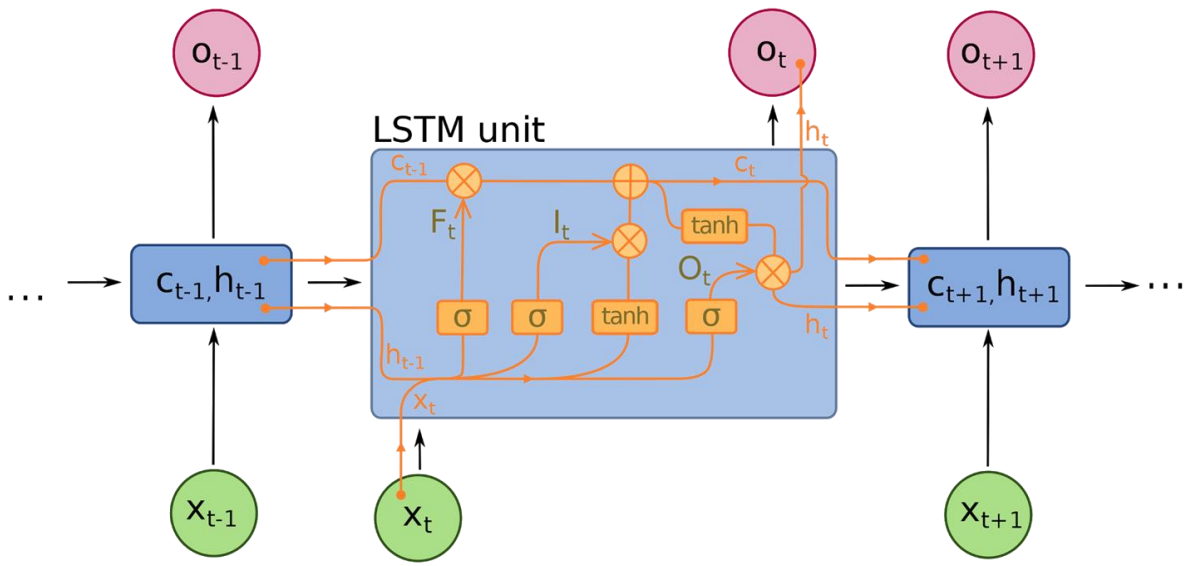
Architecture: Generative adversarial network

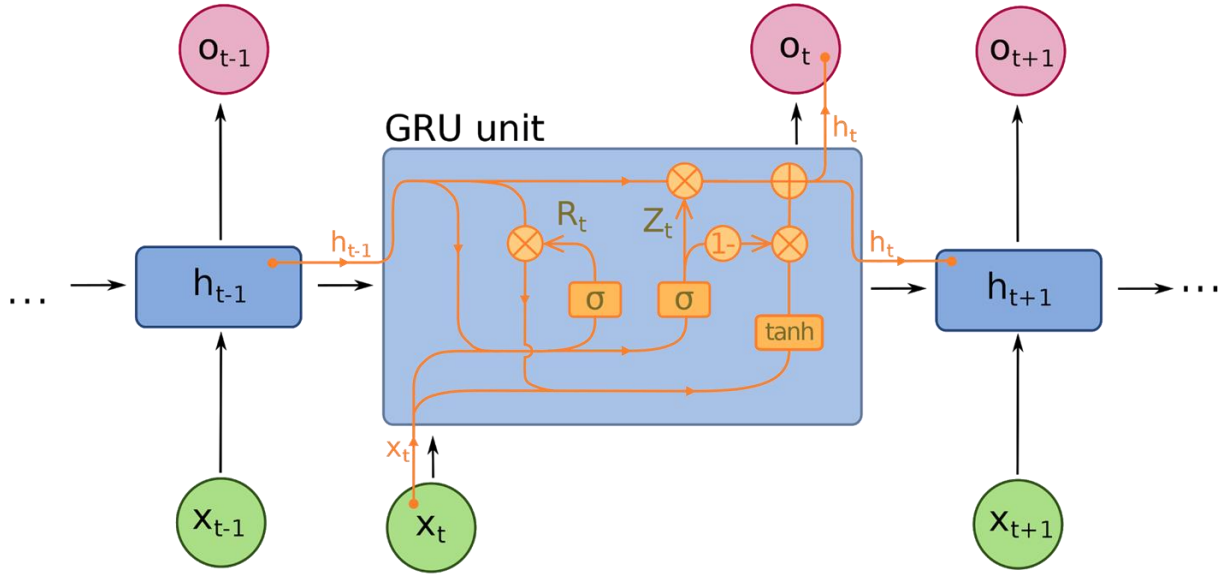


Architecture: Recurrent neural network



Architecture: LSTM/GRU





Architecture: Transformers

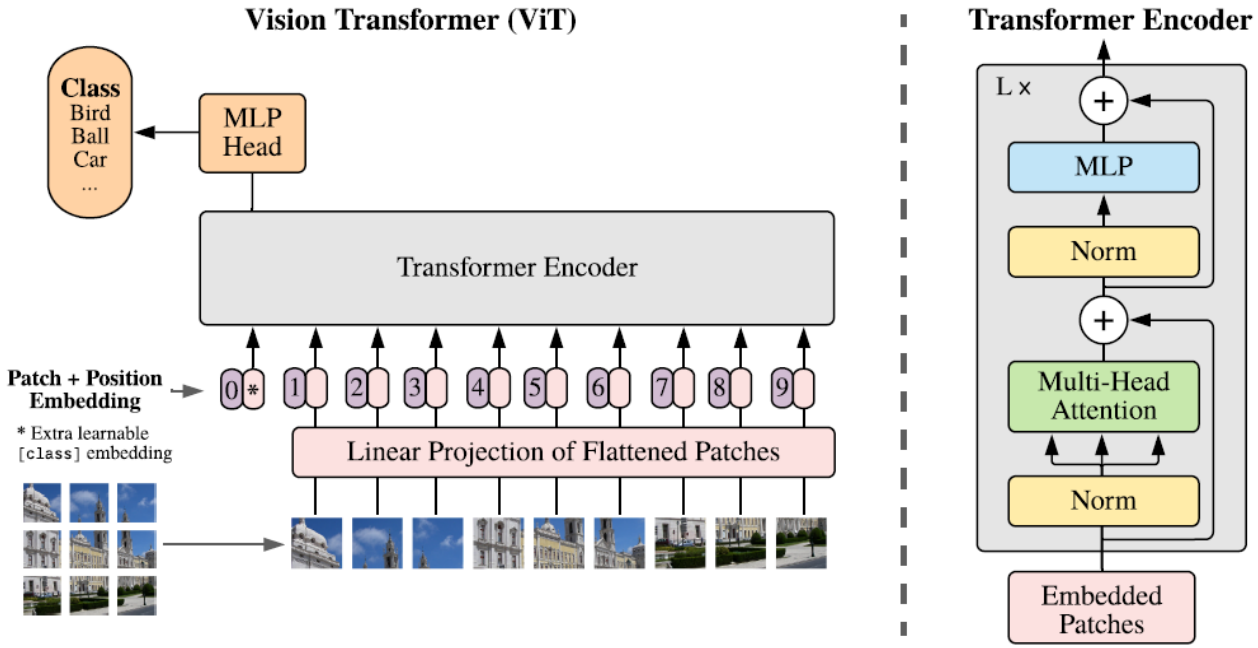


Figure 1: Model overview. We split an image into fixed-size patches, linearly embed each of them, add position embeddings, and feed the resulting sequence of vectors to a standard Transformer encoder. In order to perform classification, we use the standard approach of adding an extra learnable "classification token" to the sequence. The illustration of the Transformer encoder was inspired by Vaswani et al. (2017).

An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale, ICLR 2021