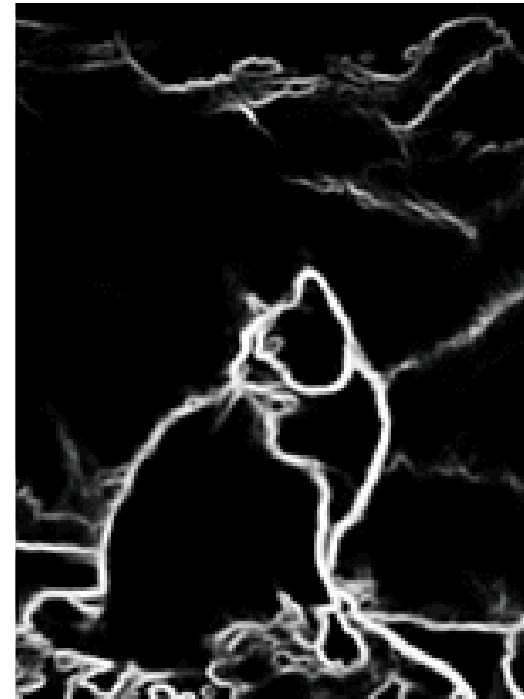


CS7GV6: Computer Vision

Edge Detection

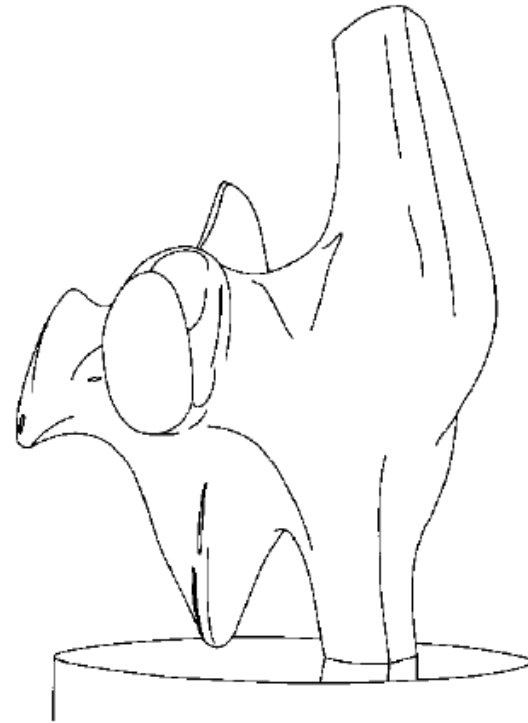
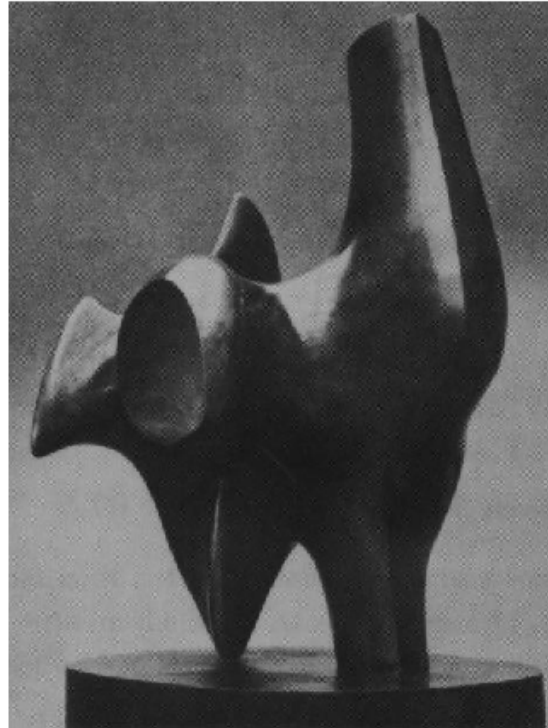


Read: Szeliski, Ch. 7.2

Credits: Some slides from Noah Snavely & others

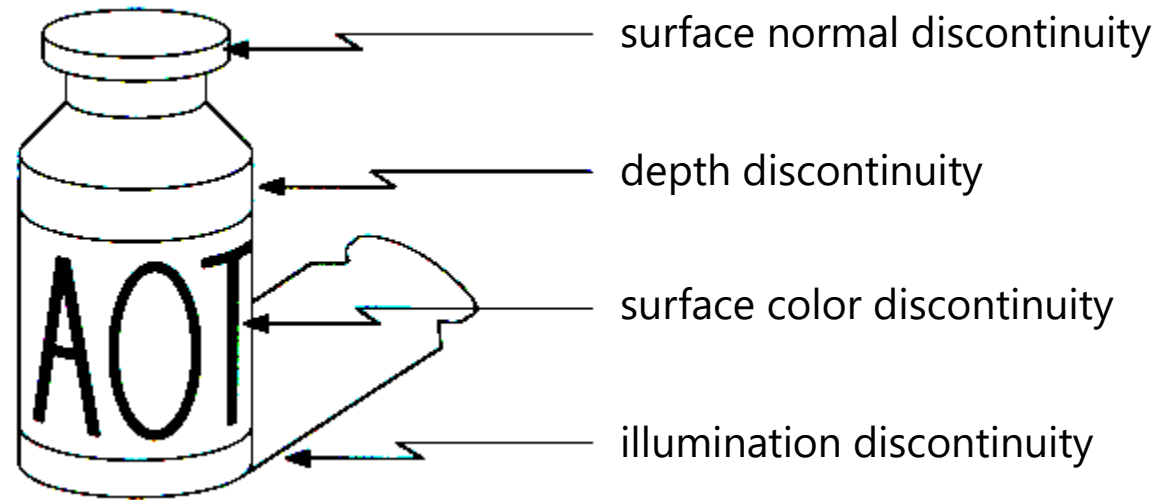
Source: pyimagesearch.com

Edge detection



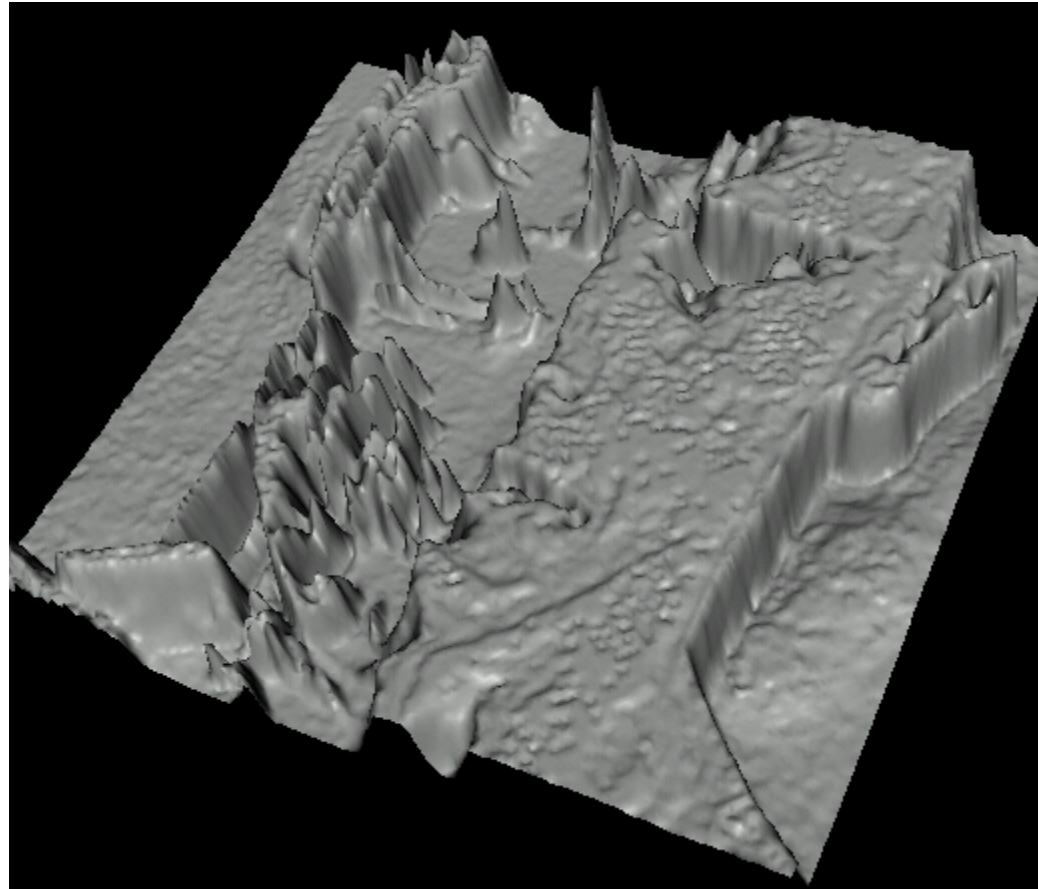
- Convert a 2D image into a set of curves
 - Extracts salient features of the scene
 - More compact than pixels

Origin of edges



- Edges are caused by a variety of factors

Images as functions...



- Edges look like steep cliffs

Characterizing edges

- An edge is a place of *rapid change* in the image intensity function

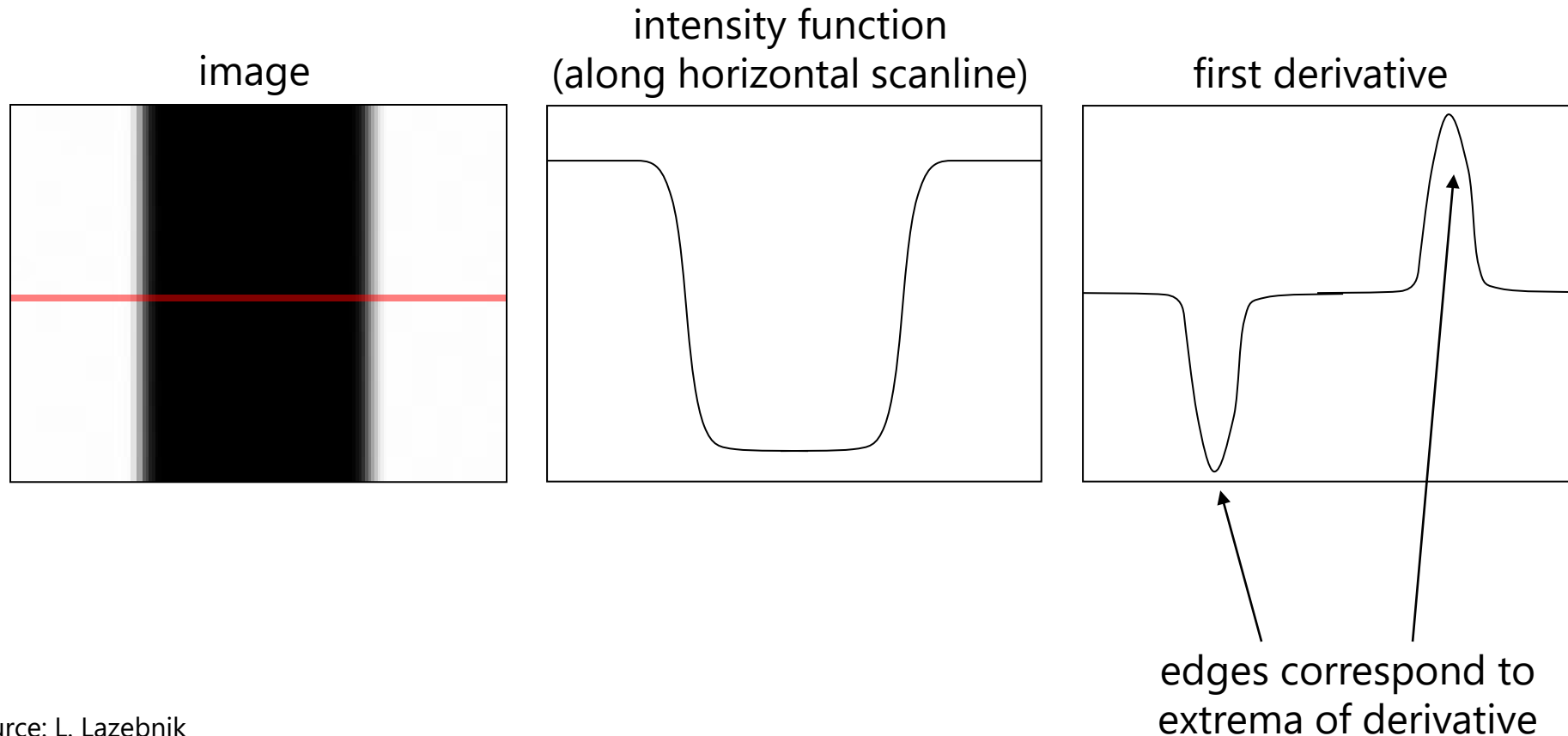


Image derivatives

- How can we differentiate a *digital* image $F[x,y]$?
 - Option 1: reconstruct a continuous image, f , then compute the derivative
 - Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x, y] \approx F[x + 1, y] - F[x, y]$$

How would you implement this as a linear filter?

$$\frac{\partial f}{\partial x} : \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

H_x

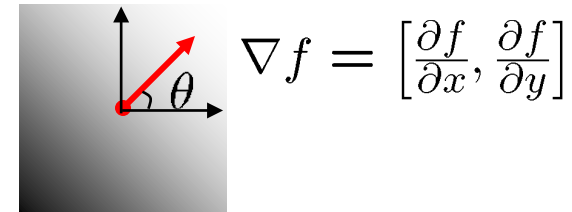
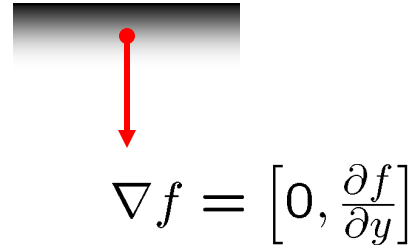
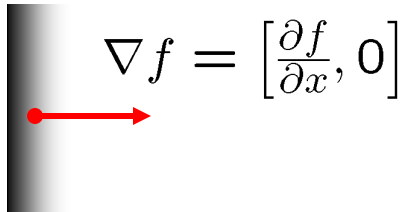
$$\frac{\partial f}{\partial y} : \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

H_y

Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$

The gradient points in the direction of most rapid increase in intensity



The *edge strength* is given by the gradient magnitude:

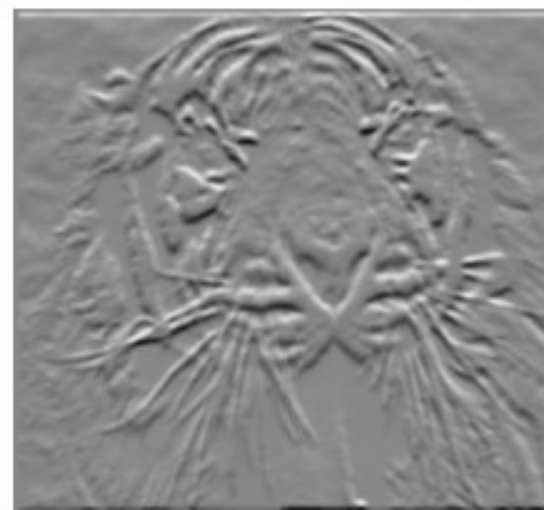
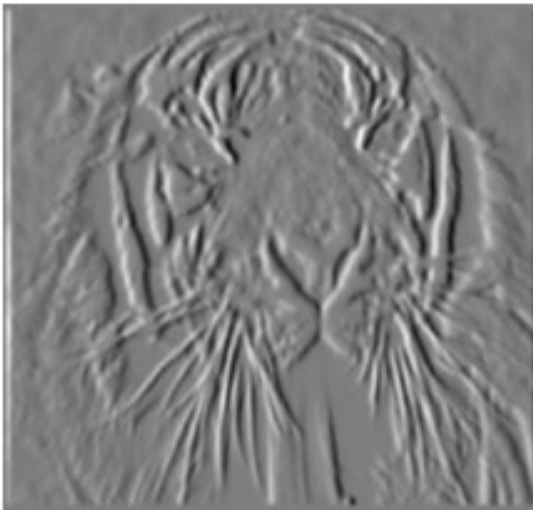
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The gradient direction is given by:

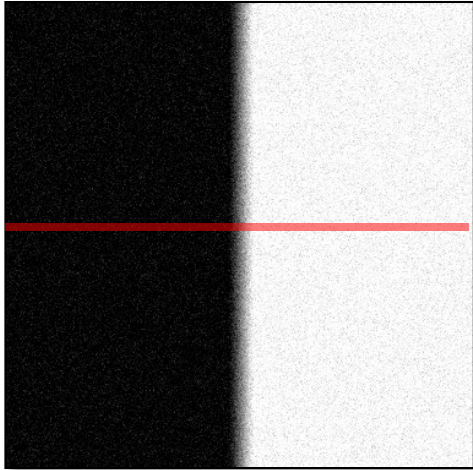
$$\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

- how does this relate to the direction of the edge?

Image gradient

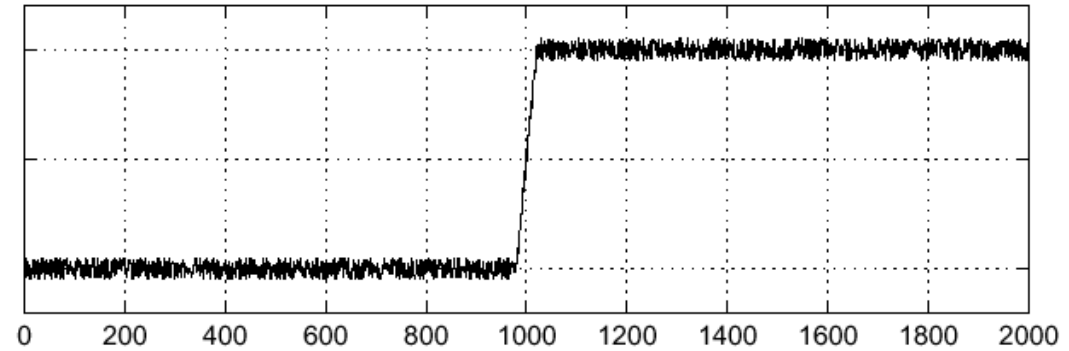


Effects of noise

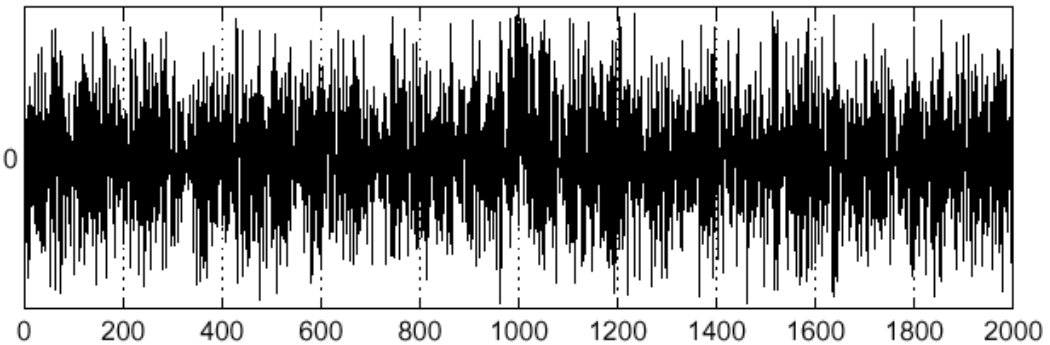


Noisy input image

$$f(x)$$

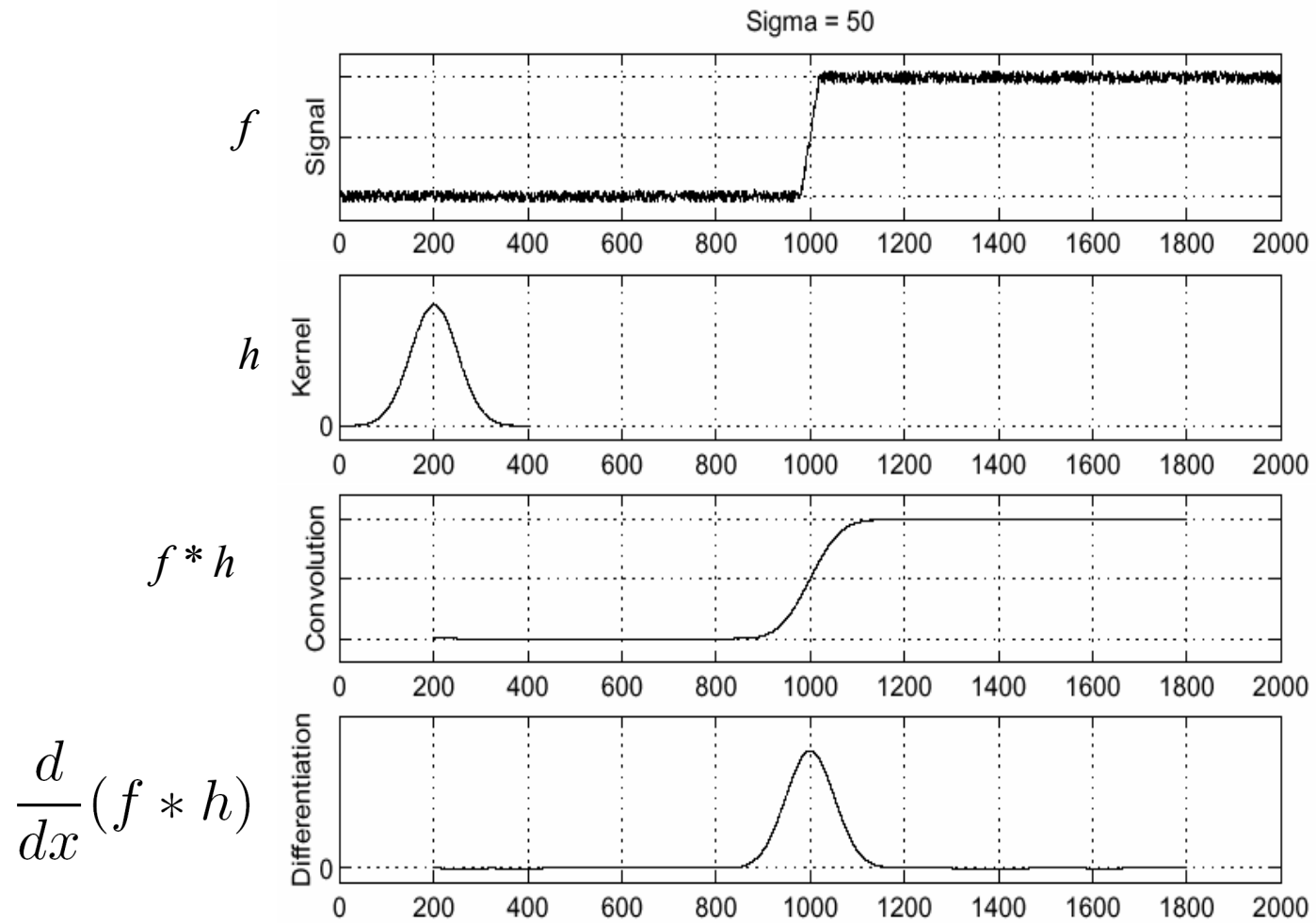


$$\frac{d}{dx}f(x)$$



Where is the edge?

Solution: smooth first

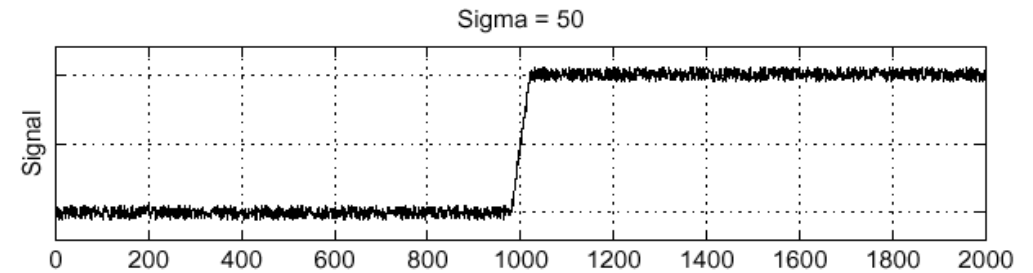


To find edges, look for peaks in $\frac{d}{dx}(f * h)$

Associative property of convolution

- Differentiation is convolution, and convolution is associative: $\frac{d}{dx}(f * h) = f * \frac{d}{dx}h$

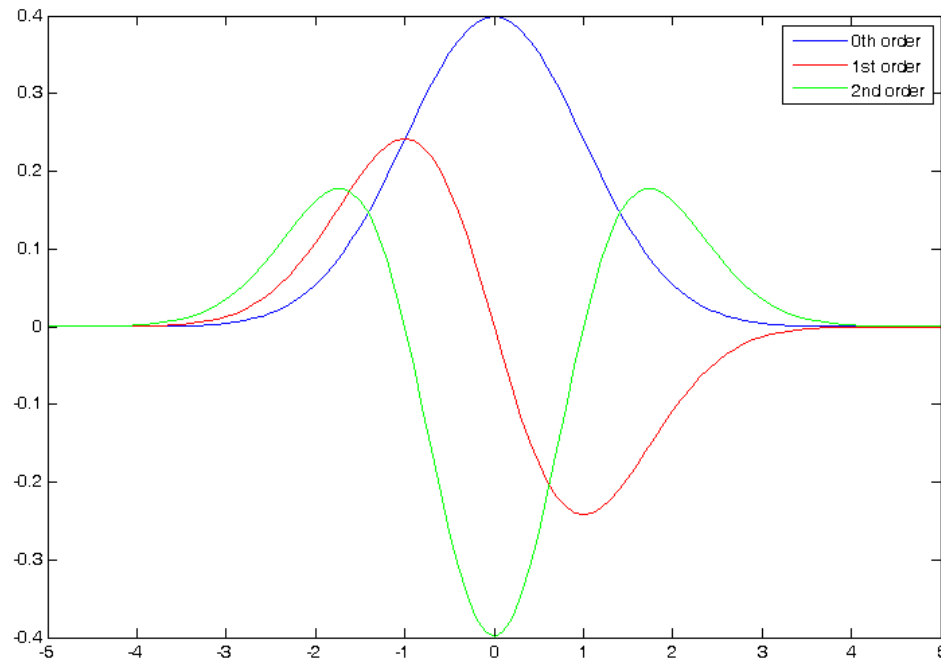
- This saves us one operation: f



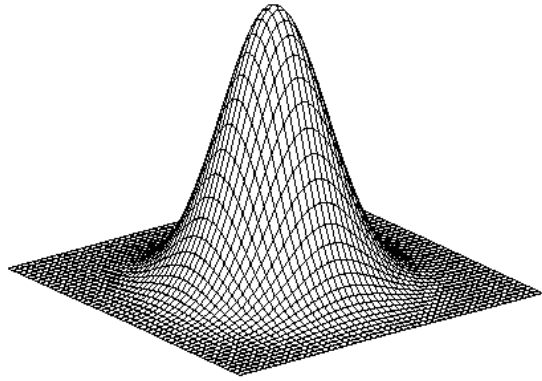
The 1D Gaussian and its derivatives

$$G_{\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$G'_{\sigma}(x) = \frac{d}{dx} G_{\sigma}(x) = -\frac{1}{\sigma} \left(\frac{x}{\sigma} \right) G_{\sigma}(x)$$

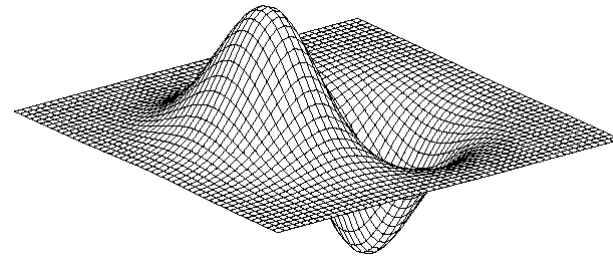


2D edge detection filters



Gaussian

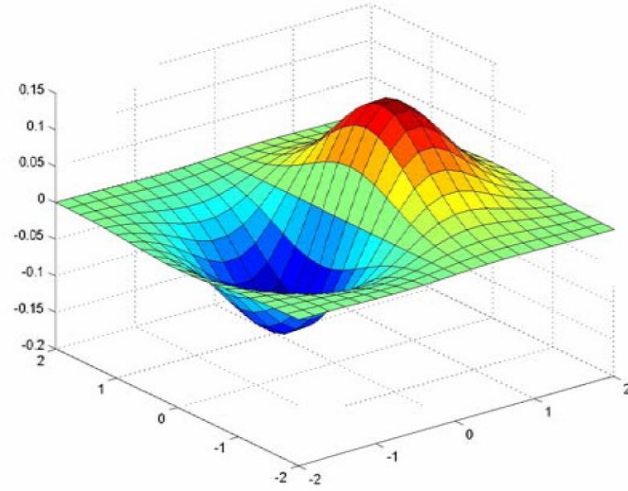
$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



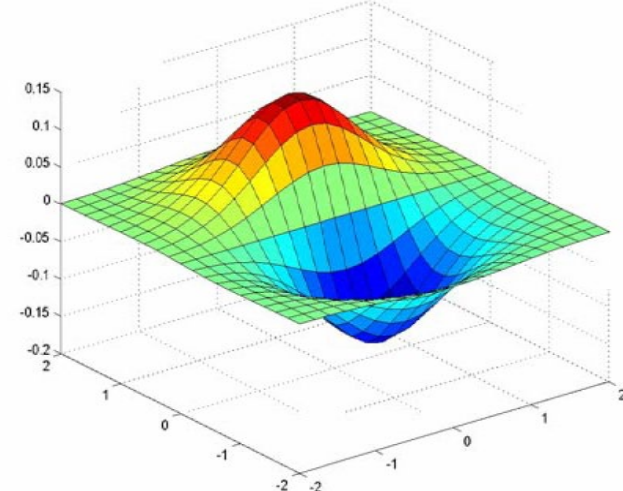
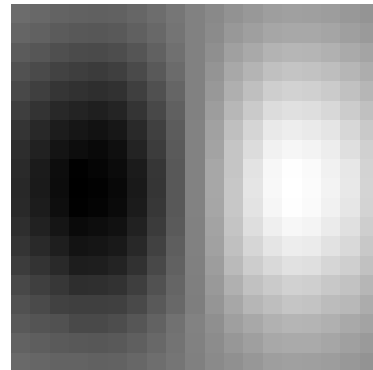
derivative of Gaussian (x)

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$

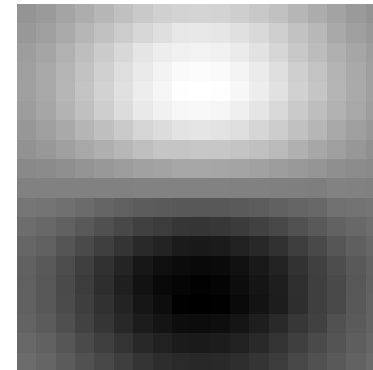
Derivative of Gaussian filter



x-direction



y-direction



The Sobel operator

- Common approximation of derivative of Gaussian

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

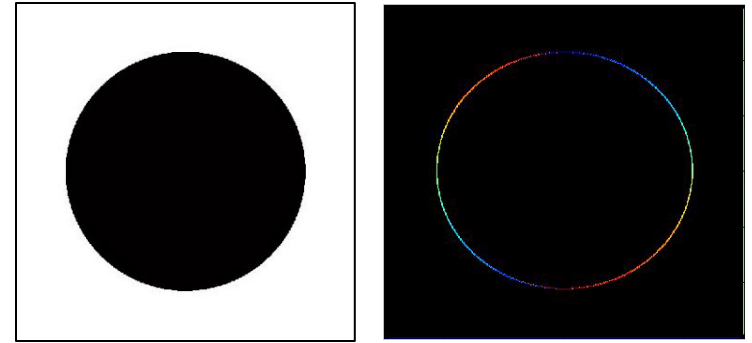
s_x

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

s_y

- The standard definition of the Sobel operator omits the $1/8$ term
 - doesn't make a difference for edge detection
 - the $1/8$ term **is** needed to get the right gradient magnitude

The Sobel operator



The operator uses two 3×3 kernels s_x and s_y

- these are convolved with the original image to calculate approximations of the derivatives, i.e., – one for horizontal changes, and one for vertical.

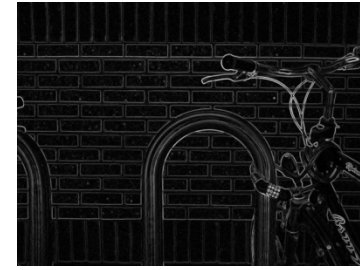
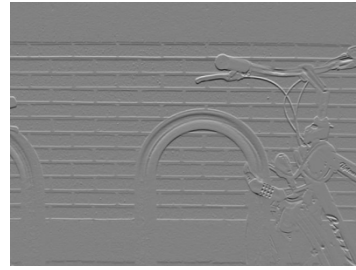
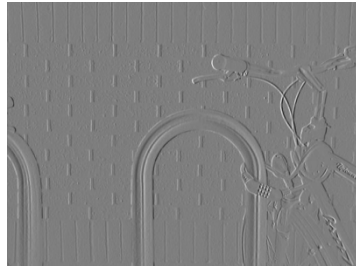
This gives two images, G_x and G_y which at each point contain the horizontal and vertical derivative approximations respectively

- At each point in the image, the resulting gradient approximations can be combined to give the gradient magnitude, using $G = \sqrt{G_x^2 + G_y^2}$

The gradient's direction at each point is $\theta = \text{atan2}(G_y, G_x)$

- For example, θ is 0 for a vertical edge which is lighter on the right side

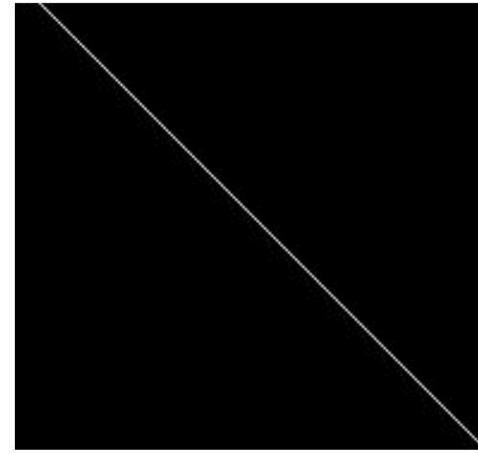
Sobel operator: example



Source: Wikipedia



Image with Edge



Edge Location

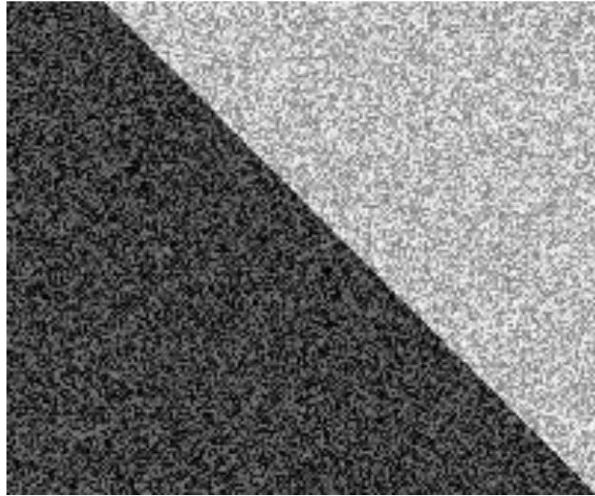
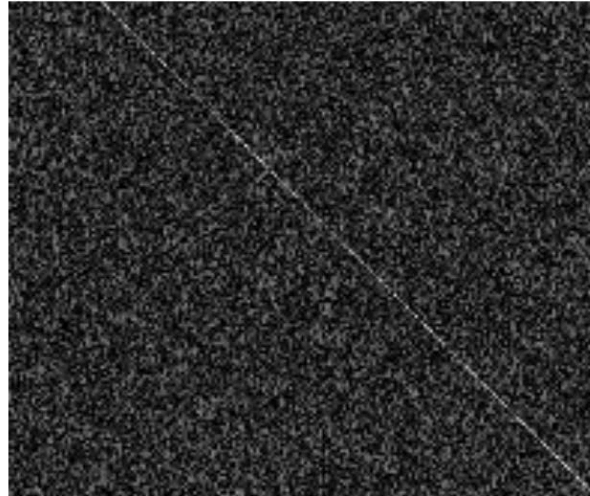
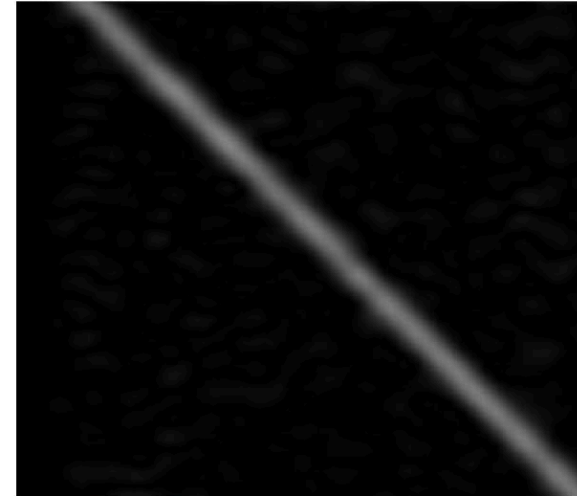


Image + Noise



Derivatives detect
edge *and* noise



Smoothed derivative removes
noise, but blurs edge

Example



original image

Finding edges



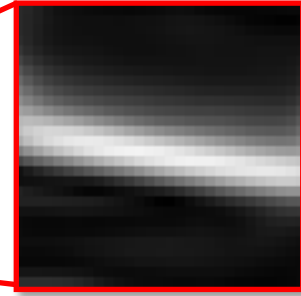
smoothed gradient magnitude

Finding edges



smoothed gradient magnitude

Finding edges

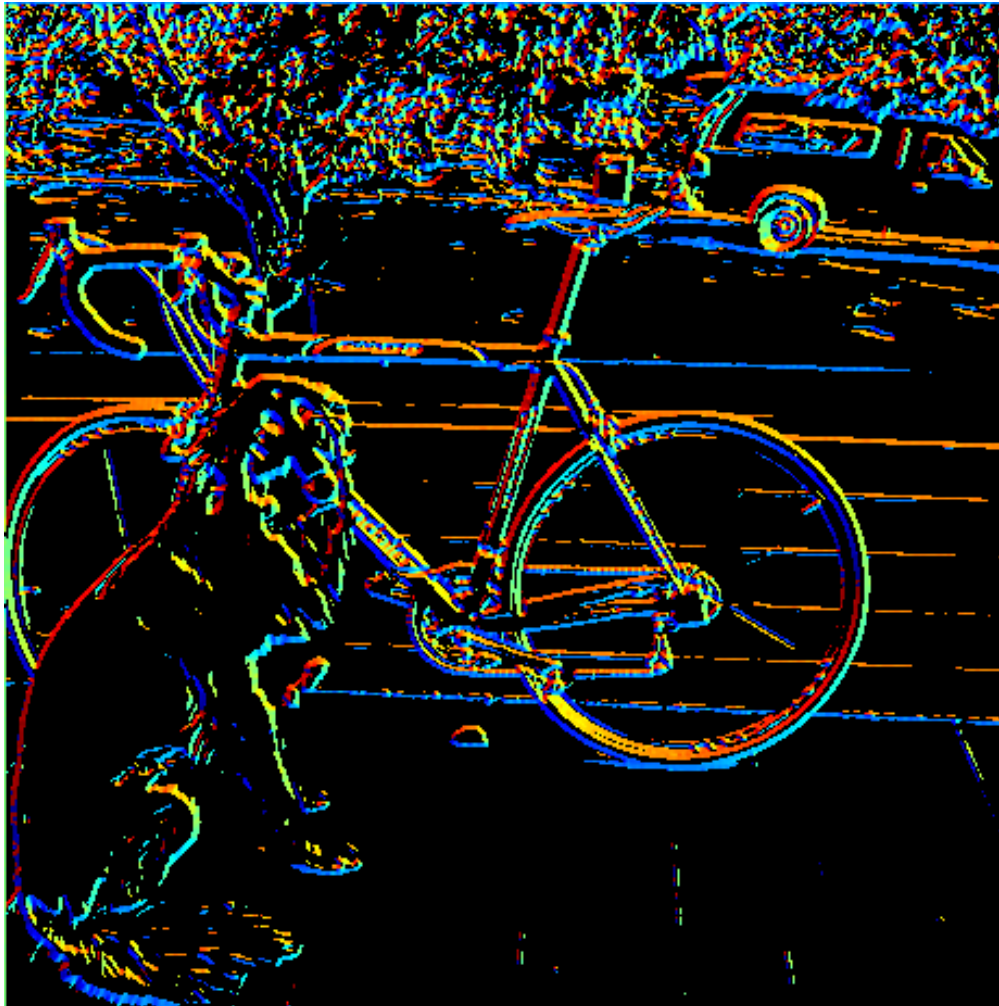


where is the edge?

thresholding

Get Orientation at Each Pixel

- Get orientation (below, threshold at minimum gradient magnitude)



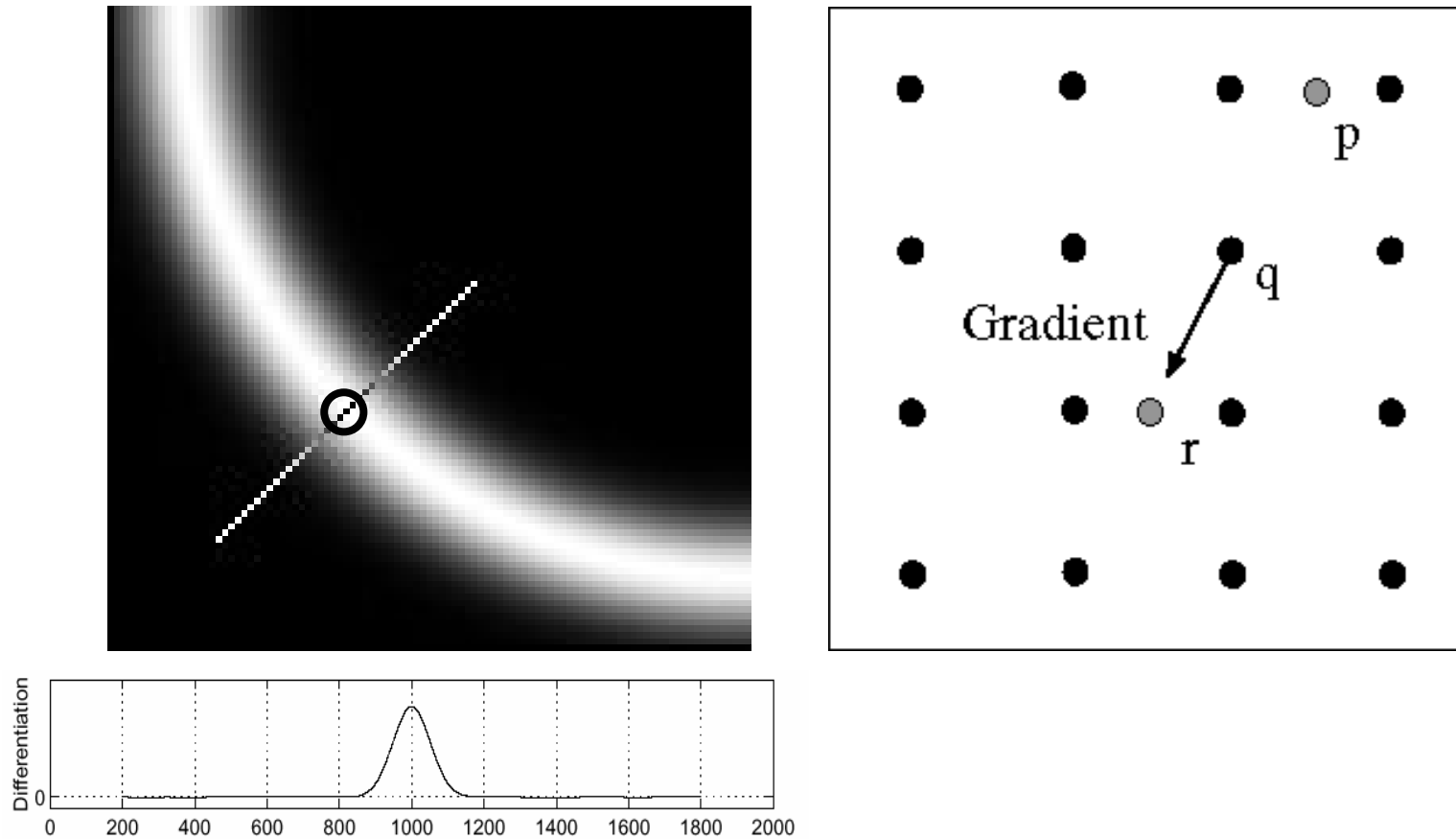
$$\text{theta} = \text{atan2}(\text{gy}, \text{gx})$$

360

Gradient orientation angle

0

Non-maximum suppression



- Check if pixel is local maximum along gradient direction
 - requires *interpolating* pixels p and r

Before Non-max Suppression

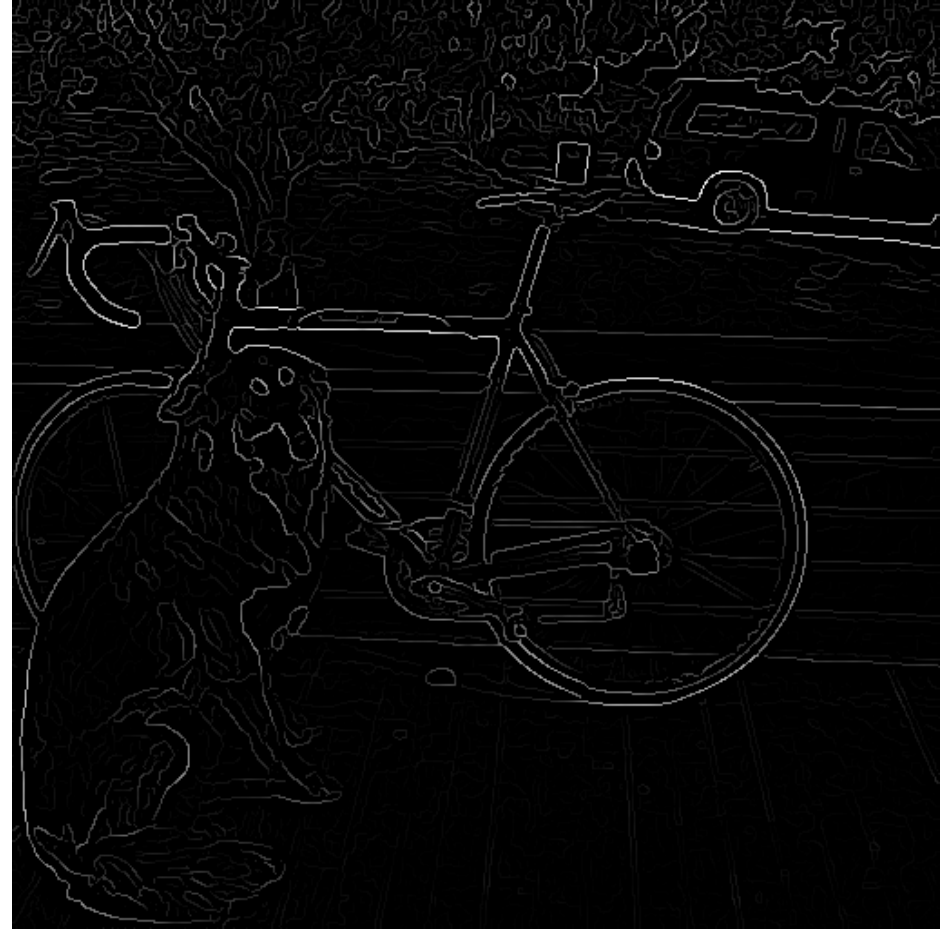


After Non-max Suppression



Thresholding edges

- Still some noise
- Only want strong edges
- 2 thresholds, 3 cases
 - $R > T$: strong edge
 - $R < T$ but $R > t$: weak edge
 - $R < t$: no edge
- Why two thresholds?



Connecting edges

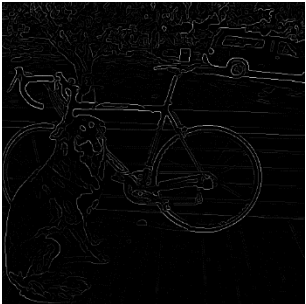
- Strong edges are edges!
- Weak edges are edges iff they connect to strong
- Look in some neighborhood (usually 8 closest)



Canny edge detector



MATLAB: `edge(image, 'canny')`



1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression
4. Linking and thresholding (hysteresis):
 - Define two thresholds: low and high
 - Use the high threshold to start edge curves and the low threshold to continue them

Canny edge detector

- Our first computer vision pipeline!
- Still a widely used edge detector in computer vision

J. Canny, [*A Computational Approach To Edge Detection*](#), IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

- Depends on several parameters:
 - high threshold
 - low threshold
 - σ : width of the Gaussian blur

Canny edge detector



original



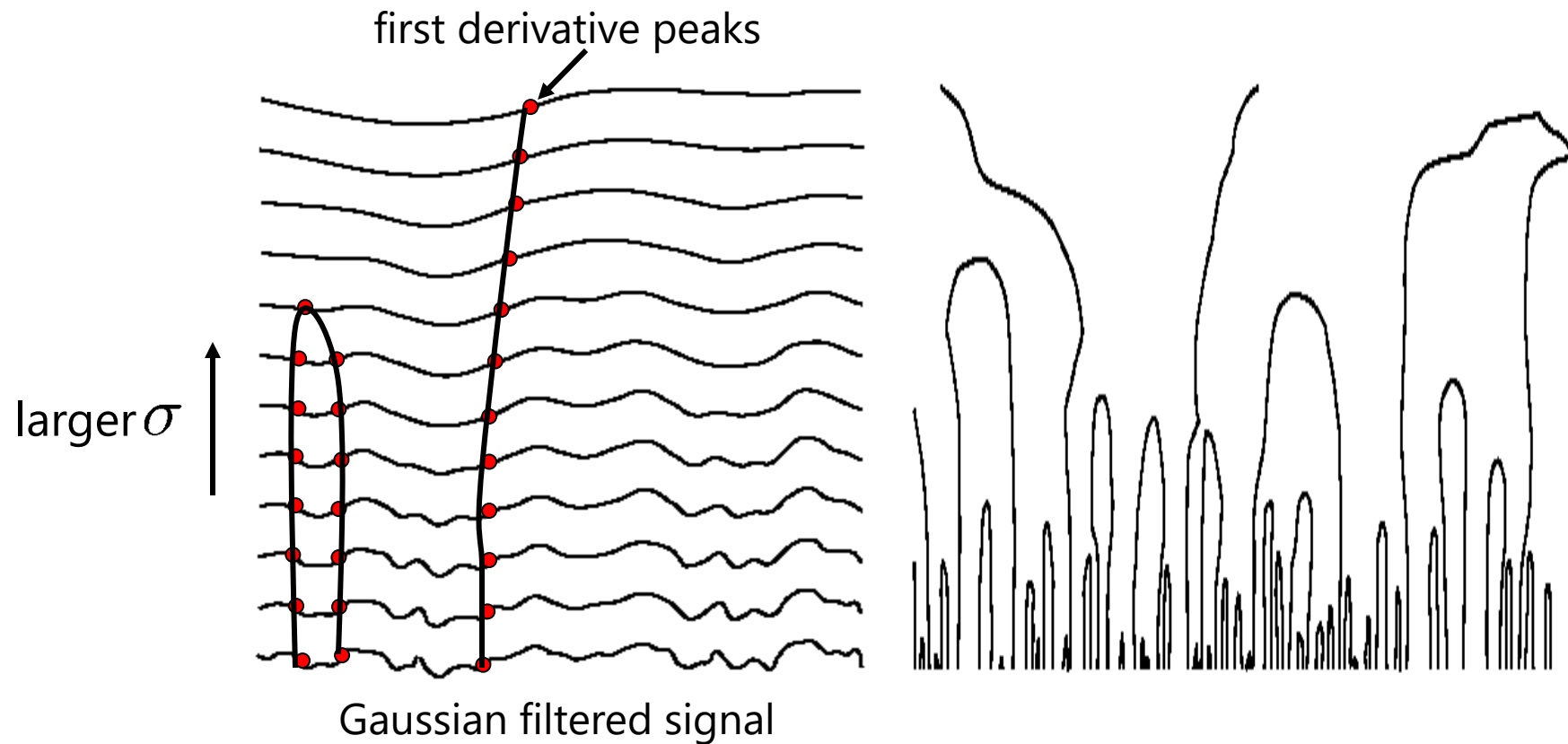
Canny with $\sigma = 1$



Canny with $\sigma = 2$

- The choice of σ depends on desired behavior
 - large σ detects "large-scale" edges
 - small σ detects fine edges

Scale space [Witkin 83]



- Properties of scale space (w/ Gaussian smoothing)
 - edge position may shift with increasing scale (σ)
 - two edges may merge with increasing scale
 - an edge may **not** split into two with increasing scale