A Review of Applied Probability Theory

Rules for Counting

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Rules for Counting - Summary

- Number of ways to put n distinct objects in order: $n!$
- Number of ways to choose k objects from n distinct objects, with replacement: n^k
- Number of ways to make ordered sequence of k objects chosen from n distinct objects, without replacement: ${}_nP_k = P_n^k = \frac{n!}{(n-k)!}$
- Number of ways to choose k objects from n distinct objects, without replacement: $\binom{n}{k} = {}_nC_k = C_n^k = \frac{n!}{k!\,(n-k)!}$

Rules for Counting - Summary

- Number of ways to make ordered sequence of n objects when n_1 of the are similar, n_2 of the are similar,…, n_k of the are similar

$$\binom{n}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1!\, n_2! \dots, n_k!} \quad , \qquad (n_1 + n_2 + \cdots + n_k = n)$$

- Number of ways to split n distinct objects into groups of $n_1, n_2, \dots, n_k$

$$\binom{n}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1!\, n_2! \dots, n_k!} \quad , \qquad (n_1 + n_2 + \cdots + n_k = n)$$

Rules for Counting - Summary

- Number of ways to split n similar objects into groups of $n_1, n_2, \ldots, n_k$

$$\binom{n+k-1}{k-1} = {}_{n+k-1}C_{k-1} = C_{n+k-1}^{k-1}$$

Rules for Counting - Summary

- Number of ways to split n similar objects into groups of $n_1, n_2, \ldots, n_k$ where $0 \leq n_1 \leq m$ equals

$$\sum_{s=0}^{k} (-1)^s \binom{k}{s} \binom{n + k - (m+1)s - 1}{k - 1}$$

Rules for Counting - Summary

- Within $n!$ Possible orders of n distinct objects, there are some orders in which none of objects are in their original locations. We call these orders as Dispersion. The number Dispersions for n distinct objects equals:

$$D(n) = n! \sum_{i=0}^{n} \frac{(-1)^i}{i!}$$