

CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Lecture #3: Light

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September 30, 2022

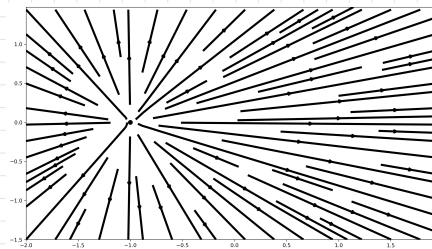
Fields

Electricity and magnetism (when considered together) comprise one of the four fundamental forces in nature.

An electric field \mathbf{E} exerts force on an electric charge.

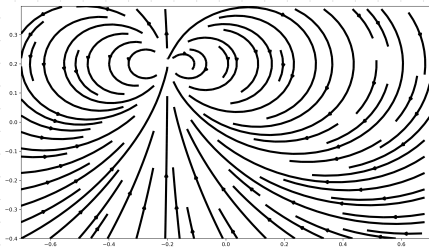
$\mathbf{E}(\mathbf{p}, t)$ is a vector denoting its magnitude and direction at position \mathbf{p} and time t .

Fields



Electric monopole field streamlines.

Fields



Magnetic dipole field streamlines.

Fields

A magnetic field \mathbf{B} exerts force on magnetic materials (and on electric charges in motion.)

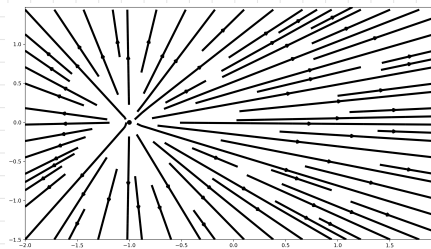
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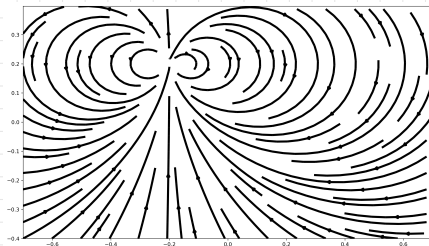
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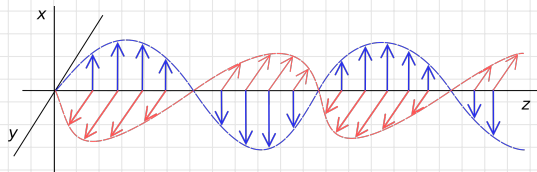


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Electromagnetic radiation



Synchronised oscillations of electric and mag. fields propagating at max. speed $c \approx 300 \times 10^6 \text{ m s}^{-1}$.

- ▶ “Light” is electromagnetic radiation with particular ranges of wavelength λ ,
Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.

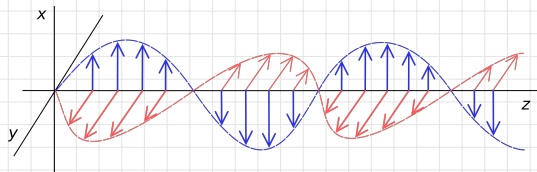
- ▶ Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ .

- ▶ For example, $\lambda = 532 \text{ nm}$ is a human-visible “green,”

$$\nu \approx \frac{300 \times 10^6 \text{ m s}^{-1}}{532 \times 10^{-9} \text{ m}} = 0.564 \times 10^{15} \text{ s}^{-1} = 564 \times 10^{12} \text{ s}^{-1} = 564 \text{ THz.}$$

- ▶ Light has much higher frequency (shorter wavelength) than the “radio” frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

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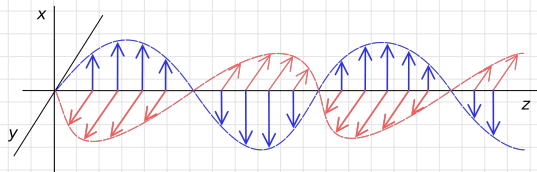
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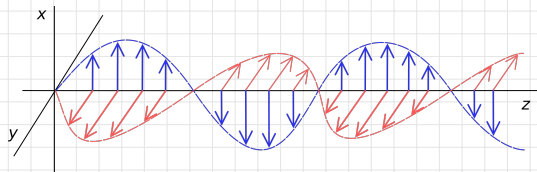
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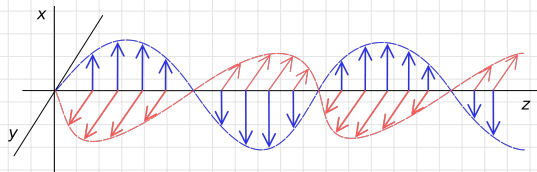
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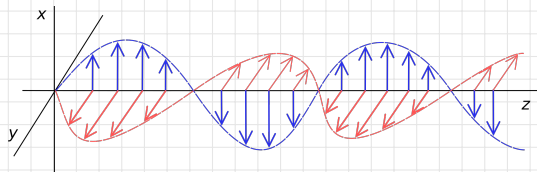
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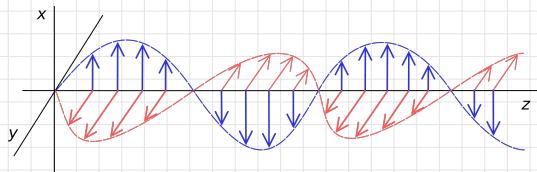
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Maxwell's equations

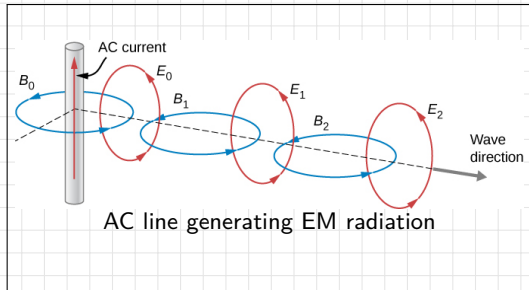
A *system* of equations that describes relationships between electromagnetic radiation field characteristics at a point and time (\mathbf{p}, t) .

Solving the system at a sequence of points and moments in time allows the *propagation* of radiation to be modelled.

Maxwell's equations

$$\left\{ \begin{array}{l} \nabla \cdot \mathbf{E} = \rho/\epsilon \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon \frac{\partial \mathbf{E}}{\partial t}) \end{array} \right.$$

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Maxwell's equations

For very good explanations of electromagnetic radiation and Maxwell's equations, see:

<https://tinyurl.com/y6dbsrxj> (text)

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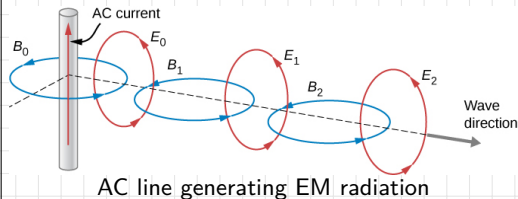
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Vectors

For $a, b, c \in \mathbb{R}$, a vector $\mathbf{v} \in \mathbb{R}^3$ can be written,

$$\mathbf{v} = a \mathbf{i} + b \mathbf{j} + c \mathbf{k} = (a, b, c)$$

with standard basis vectors,

$$\mathbf{i} = (1, 0, 0) \text{ and } \mathbf{j} = (0, 1, 0) \text{ and } \mathbf{k} = (0, 0, 1)$$

in a Euclidean coordinate system with axes X,Y,Z.

Vectors

Vectors ← Quaternions ← Hamilton ← TCD!

Magnitude of vector $\mathbf{w} = (x, y, z)$ is its length,

$$\|\mathbf{w}\| = \sqrt{x^2 + y^2 + z^2}.$$

Vectors

Scalar (or dot) product,

$$\begin{aligned}\mathbf{v} \cdot \mathbf{w} &= ax + by + cz \\ &= \|\mathbf{v}\| \|\mathbf{w}\| \cos \theta.\end{aligned}$$

Vectors

Vector (or cross) product is $\perp \mathbf{v}$ and $\perp \mathbf{w}$,

$$\mathbf{v} \times \mathbf{w} = (bz - cy)\mathbf{i} + (cx - az)\mathbf{j} + (ay - bx)\mathbf{k}$$

$$\|\mathbf{v} \times \mathbf{w}\| = \|\mathbf{v}\|\|\mathbf{w}\|\sin \theta|.$$

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Vector fields and calculus

A vector-valued function of space, time, etc.,

$$\mathbf{F}(\mathbf{p}, t) = (F_x(\mathbf{p}, t), F_y(\mathbf{p}, t), F_z(\mathbf{p}, t))$$

with position vector $\mathbf{p} = (x, y, z)$ and time t .

Vector calculus is concerned with differentiation and integration of vector fields.

Vector fields and calculus

Space, time function parameters (\mathbf{p}, t) can be omitted for improved readability but you have to remember this when looking at formulae! E.g.

$$\frac{\partial \mathbf{F}}{\partial t} = \left(\frac{\partial F_x}{\partial t}, \frac{\partial F_y}{\partial t}, \frac{\partial F_z}{\partial t} \right).$$

Vector fields and calculus

Divergence is,

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

A scalar denoting by how much, if at all, the field is like a point source at that position.

Vector fields and calculus

Curl is,

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \mathbf{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \mathbf{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \mathbf{k}.$$

A vector denoting rotation axis and magnitude, for how much the field rotates, if at all, at that position.

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Some physical laws

Gauss's law for electricity: electric charges generate an electric field.

$$\nabla \cdot \mathbf{E} = \rho/\epsilon$$

where ρ is electric charge and ϵ is electric permittivity.

Some physical laws

Gauss's law for magnetism: there are no separate magnetic charges (no monopoles.)

$$\nabla \cdot \mathbf{B} = 0.$$

Some physical laws

Faraday's law of induction: A changing magnetic field creates a rotating electric field and vice-versa.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Some physical laws

Ampère-Maxwell's law: an electric current and a changing electric field create a magnetic field.

$$\nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon \frac{\partial \mathbf{E}}{\partial t})$$

where \mathbf{J} is electric current density and μ is magnetic permeability.

Note that $c = 1/\sqrt{\epsilon\mu}$.

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Assignment # 4: Electromagnetic Wave Simulation

- ▶ Write a SciPy program to make an animation of the propagation of electromagnetic fields, inspired by the plot shown on slide 4.
- ▶ Make it into a self-contained project repository in your personal account on gitlab.scss.tcd.ie.
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