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CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Lecture #3: Light

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School of Computer Science and Statistics,
Trinity College Dublin

October 15, 2021

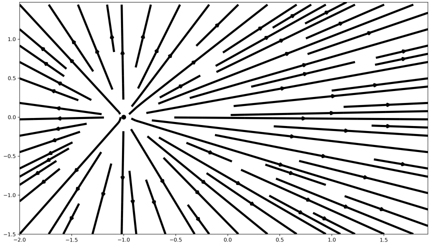
Fields

Electricity and magnetism are two of the four fundamental forces.

An electric field $\mathbf{E}$ exerts force on an electric charge.

$\mathbf{E}(\boldsymbol{p}, t)$ is a vector denoting its magnitude and direction at position $\boldsymbol{p}$ and time t.

Fields



Electric monopole field streamlines.

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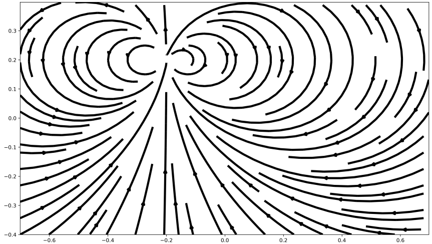

Magnetic dipole field streamlines.

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A magnetic field **B** exerts force on magnetic materials (and on electric charges in motion.)

$\mathbf{B}(\boldsymbol{p}, t)$ is a (pseudo)vector denoting its magnitude and direction at position $\boldsymbol{p}$ and time t.

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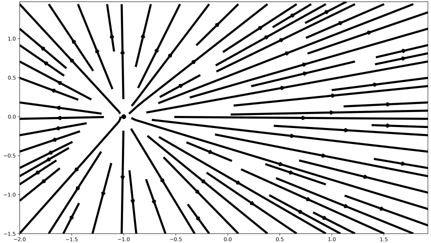

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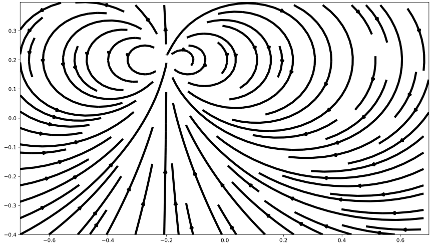

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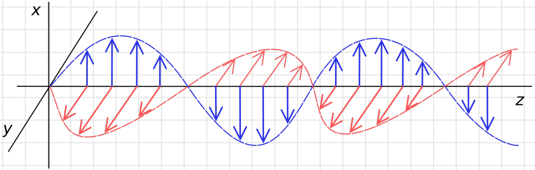
Electromagnetic radiation



Synchronised oscillations of electric and mag. fields propagating at max. speed $c \approx 300 \times 10^6\,\mathrm{m\,s^{-1}}$.

- "Light" is electromagnetic radiation with particular ranges of wavelength λ, Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.
- Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ.
- For example, $\lambda = 532\,\mathrm{nm}$ is a human-visible "green,"
$$\nu \approx \frac{300 \times 10^6\,\mathrm{m\,s^{-1}}}{532 \times 10^{-9}\,\mathrm{m}} = 0.564 \times 10^{15}\,\mathrm{s^{-1}} = 564 \times 10^{12}\,\mathrm{s^{-1}} = 564\,\mathrm{THz}.$$
- Light has much higher frequency (shorter wavelength) than the "radio" frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

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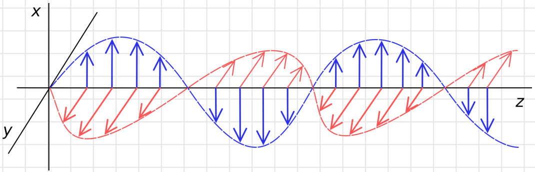

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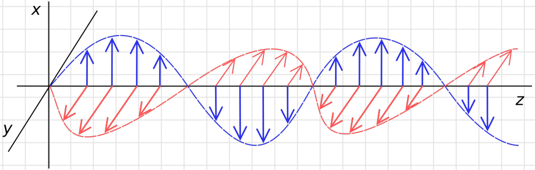

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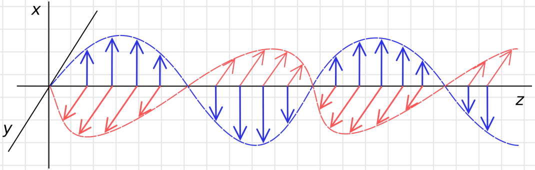

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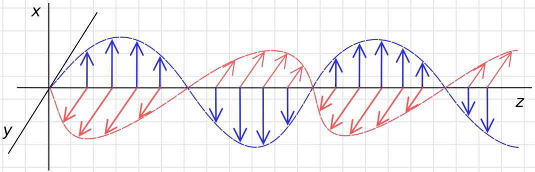

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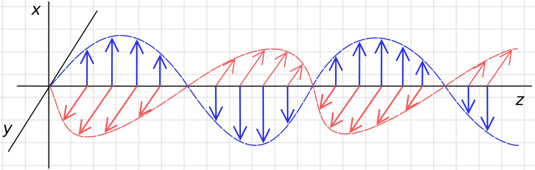

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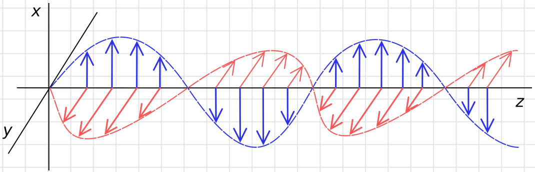

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Maxwell's equations

A *system* of equations that describes relationships between electromagnetic radiation field characteristics at a point and time $(\boldsymbol{p}, t)$.

Solving the system at a sequence of points and moments in time allows the *propagation* of radiation to be modelled.

Maxwell's equations

$$\begin{cases} \nabla \cdot \mathbf{E} = \rho/\epsilon \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon \dfrac{\partial \mathbf{E}}{\partial t}) \end{cases}$$

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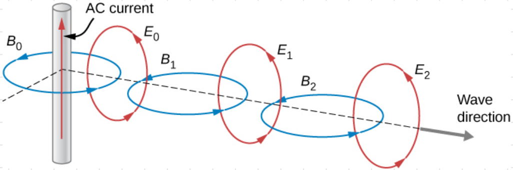

AC line generating EM radiation

Maxwell's equations

For very good explanations of electromagnetic radiation and Maxwell's equations, see:

`https://tinyurl.com/y6dbsrxj` (text)

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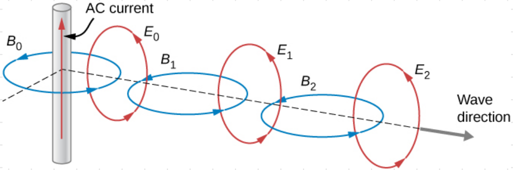

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Vectors

For $a, b, c \in \mathbb{R}$, a vector $\boldsymbol{v} \in \mathbb{R}^3$ can be written,

$$\boldsymbol{v} = a\,\boldsymbol{i} + b\,\boldsymbol{j} + c\,\boldsymbol{k} = (a, b, c)$$

with standard basis vectors,

$$\boldsymbol{i} = (1, 0, 0) \text{ and } \boldsymbol{j} = (0, 1, 0) \text{ and } \boldsymbol{k} = (0, 0, 1)$$

in a Euclidean coordinate system with axes X,Y,Z.

Vectors

Vectors$\leftarrow$Quaternions$\leftarrow$Hamilton$\leftarrow$TCD!

Magnitude of vector $\boldsymbol{w} = (x, y, z)$ is its length,

$$\|\boldsymbol{w}\| = \sqrt{x^2 + y^2 + z^2}.$$

Vectors

Scalar (or dot) product,

$$\boldsymbol{v} \cdot \boldsymbol{w} = ax + by + cz$$
$$= \|\boldsymbol{v}\|\|\boldsymbol{w}\| \cos\theta.$$

Vectors

Vector (or cross) product is $\perp \boldsymbol{v}$ and $\perp \boldsymbol{w}$,

$$\boldsymbol{v} \times \boldsymbol{w} = (bz - cy)\boldsymbol{i} + (cx - az)\boldsymbol{j} + (ay - bx)\boldsymbol{k}$$
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Vector fields and calculus

A vector-valued function of space, time, etc.,

$$\boldsymbol{F}(\boldsymbol{p}, t) = (F_x(\boldsymbol{p}, t),\ F_y(\boldsymbol{p}, t),\ F_z(\boldsymbol{p}, t))$$

with position vector $\boldsymbol{p} = (x, y, z)$ and time t.

Vector calculus is concerned with differentiation and integration of vector fields.

Vector fields and calculus

Space, time function parameters $(\boldsymbol{p}, t)$ can be omitted for improved readability but you have to remember this when looking at formulae! E.g.

$$\frac{\partial \boldsymbol{F}}{\partial t} = (\frac{\partial F_x}{\partial t}, \frac{\partial F_y}{\partial t}, \frac{\partial F_z}{\partial t}).$$

Vector fields and calculus

Divergence is,

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

A scalar denoting by how much, if at all, the field is like a point source at that position.

Vector fields and calculus

Curl is,

$$\nabla \times \boldsymbol{F} = (\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z})\,\boldsymbol{i} + (\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x})\,\boldsymbol{j} + (\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y})\,\boldsymbol{k}.$$

A vector denoting rotation axis and magnitude, for how much the field rotates, if at all, at that position.

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Some physical laws

Gauss's law for electricity: electric charges generate an electric field.

$$\nabla \cdot \mathbf{E} = \rho/\epsilon$$

where ρ is electric charge and ϵ is electric permittivity.

Some physical laws

Gauss's law for magnetism: there are no separate magnetic charges (no monopoles.)

$$\nabla \cdot \mathbf{B} = 0.$$

Some physical laws

Faraday's law of induction: A changing magnetic field creates a rotating electric field and vice-versa.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Some physical laws

Ampère-Maxwell's law: an electric current and a changing electric field create a magnetic field.

$$\nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon \frac{\partial \mathbf{E}}{\partial t})$$

where $\mathbf{J}$ is electric current density and μ is magnetic permeability.

Note that $c = 1/\sqrt{\epsilon\mu}$.

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Assignment # 4: Electromagnetic Wave Simulation

- Write a SciPy program to make an animation of the propagation of electriomagnetic fields, inspired by the plot shown on slide 4.
- Make it into a self-contained project repository in your personal account on gitlab.scss.tcd.ie.
- Work out how to make it into a Jupyter notebook so that I can view over the web.
- For this and every other assignment, feel free to collaborate with your classmates about the the non-mathematical parts like plotting.

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