CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Lecture #7: Intensity

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Intensity

Forces exerted by electric and magnetic fields can move or heat matter and move charges. This means electromagnetic waves have *energy*. (Actually they *transport* energy.)

The energy required to accelerate an object over 1 m distance with 1 N force is,

$$1\,\mathrm{J} = 1\,\mathrm{N\,m} = 1\,\mathrm{kg\,m\,s^{-2}} \cdot 1\,\mathrm{m} = 1\,\mathrm{kg\,m^2\,s^{-2}}.$$

Intensity

Power is energy per unit time, $1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}}$. Power per unit area, $\mathrm{W\,m^{-2}}$, is energy flux or *intensity*.

Intensity of electromagnetic waves is what what our eyes see; and what is measured by the photosensitive elements in cameras.

Intensity

Electromagnetic wave intensity at a point in space at time t is $\propto$ to the product of field amplitudes,

$$^{1}/_{\mu}\|\mathbf{E}(t)\|\,\|\mathbf{B}(t)\|.$$

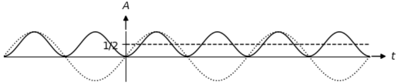
Average intensity over a wave time period is a more useful quantity,

$$I = \frac{1}{2}\frac{1}{\mu}EB = \frac{E^2}{2\mu c} = \frac{cB^2}{2\mu}$$

where E and B are max. amplitudes.

Intensity

Note that the average of the product of two equal sinusoids is $\propto$ average of $\cos^2$ or $\sin^2$ which is ½.



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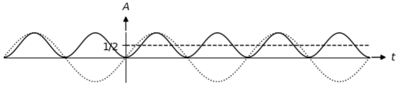

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Intensity PDF

$f_A(a)$ for amplitude A of a random phasor sum was found to follow a Rayleigh distribution. The *derived* PDF for intensity $I = A^2$ is,

$$f_I(i) = f_A(\sqrt{i}) \left| \frac{\mathrm{d}\sqrt{i}}{\mathrm{d}i} \right| = f_A(\sqrt{i}) \, \frac{1}{2\sqrt{i}}$$

$$\text{since } \frac{\mathrm{d}i^{\frac{1}{2}}}{\mathrm{d}i} = \frac{1}{2} \, i^{\frac{-1}{2}} = \frac{1}{2} \, \frac{1}{\sqrt{i}} = \frac{1}{2\sqrt{i}}.$$

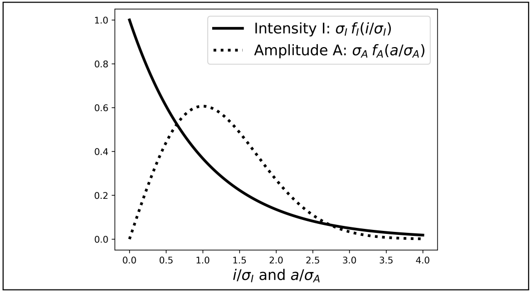
Intensity PDF

$$f_A(a) = \exp\left\{-\frac{a^2}{2\sigma_A^2}\right\}\frac{a}{\sigma_A^2}.$$

$$\begin{aligned} f_I(i) &= \exp\left\{-\frac{i}{2\sigma_A^2}\right\}\frac{\sqrt{i}}{\sigma_A^2}\,\frac{1}{2\sqrt{i}} \\ &= \exp\left\{-\frac{i}{2\sigma_A^2}\right\}\frac{1}{2\sigma_A^2}. \end{aligned}$$

So intensity PDF follows an *exponential* distribution, i.e. $f_X(x) = \lambda\, \mathrm{e}^{-\lambda\, x}$.

Intensity PDF



Intensity PDF

Mean intensity $\overline{I}$ can be found as $2\sigma_A^2$ so the PDF can be written,

$$f_I(i) = \exp\left\{-\frac{i}{\overline{I}}\right\}\frac{1}{\overline{I}}.$$

$$\text{Variance } \sigma_I^2 = \overline{I}^{\,2}, \quad \text{Std. dev. } \sigma_I = \overline{I}\,,$$

$$\text{Contrast } C = \sigma_I/\overline{I} = 1.0,$$

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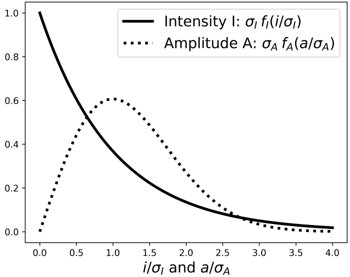

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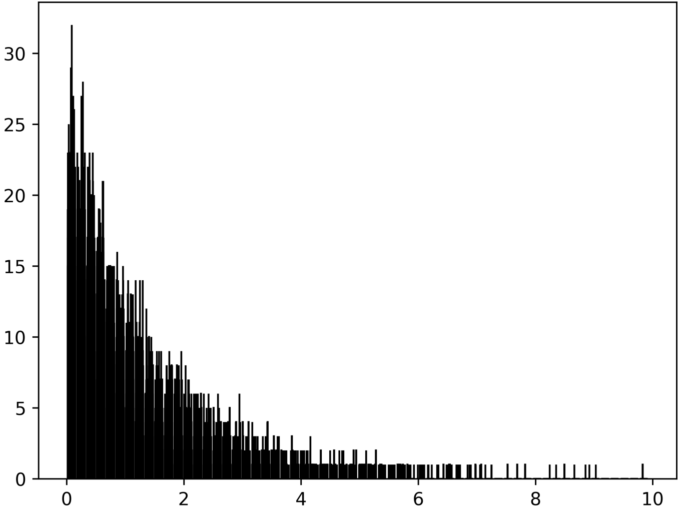
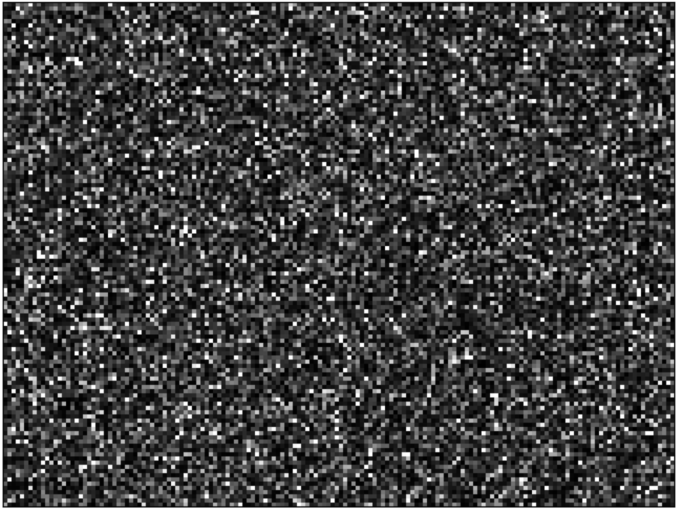
Histogram of simulated intensities



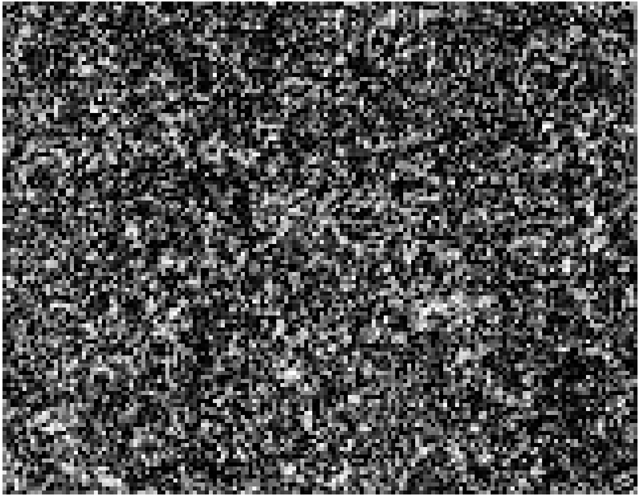
19,200 intensity values chosen randomly in accordance with an exponential PDF with $\overline{I} = 1.0$ and $\sigma_I = 1.0$. These could be many observations over time at a single point in space or many observations over space at a single point in time.

Image of simulated intensities



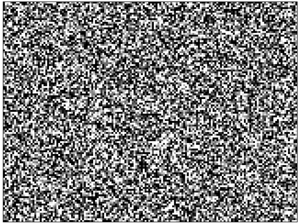
The same* intensity values i arranged as a 160×120 pixel image.
Contrast $C = \sigma_I / \overline{I} = 1.0$.

Image of actual intensities



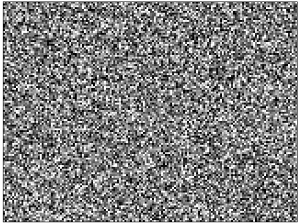
Material with diffuse reflectance characteristics illuminated evenly with *monochromatic* light with *no phase or amplitude change* during the observation time period. $C \approx 0.83$.

Averages of simulated intensity images



$N = 16$. $C \approx 0.251 = 1/\sqrt{N}$.

Averages of simulated intensity images



$N = 64$. $C \approx 0.125 = 1/\sqrt{N}$.

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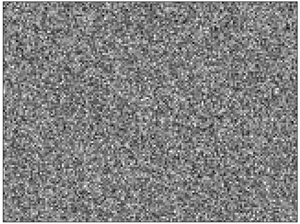

$N = 256$. $C \approx 0.061 = 1/\sqrt{N}$.

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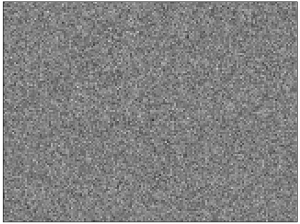

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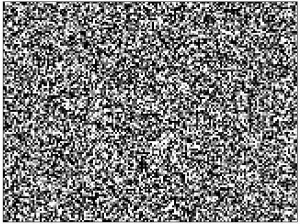

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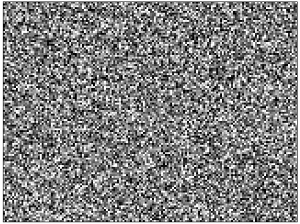

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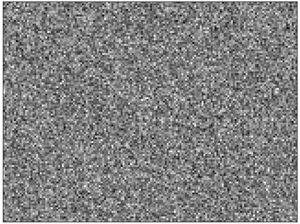

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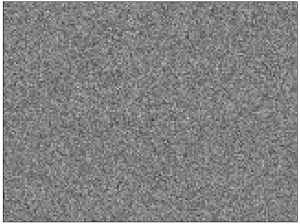

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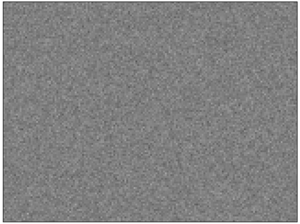

$N = 4,096$. $C \approx 0.015 = 1/\sqrt{N}$.

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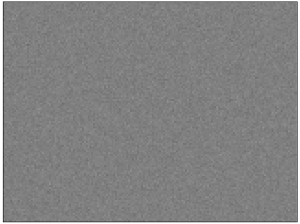

$N = 16,384$. $C \approx 0.007 = 1/\sqrt{N}$.

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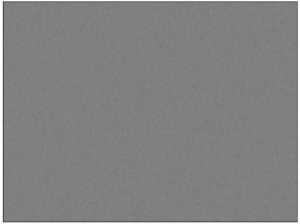

$N = 65,536$. $C \approx 0.003 = 1/\sqrt{N}$.

Averages of simulated intensity images

These simulations show, that with appropriate variation of phase and amplitude over a time period, variation of intensity can reduce* from $C = 1$ to $C = 1/\sqrt{N}$ where N is the number of intensities observed.

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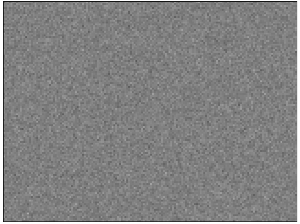

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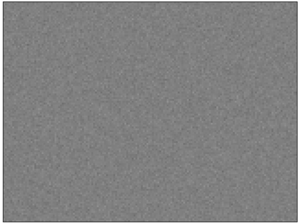

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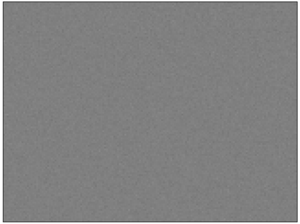

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