CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Lecture #3: Light

Fergal Shevlin, Ph.D.

School of Computer Science and Statistics,
Trinity College Dublin

September 30, 2022

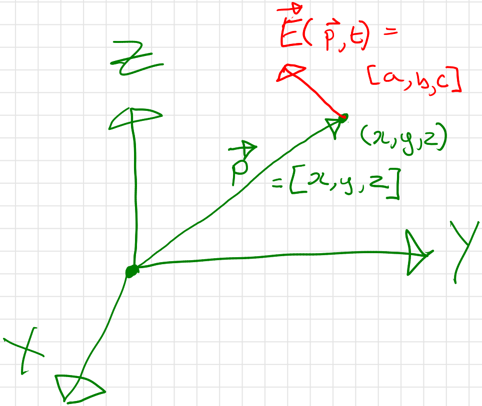
Fields

Electricity and magnetism (when considered together) comprise one of the four fundamental forces in nature.

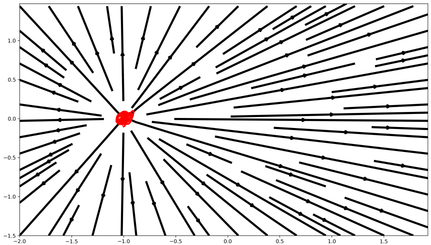
An electric field **E** exerts force on an electric charge.

$\mathbf{E}(\boldsymbol{p}, t)$ is a vector denoting its magnitude and direction at position $\boldsymbol{p}$ and time t.

$E(x, y, z, t)$



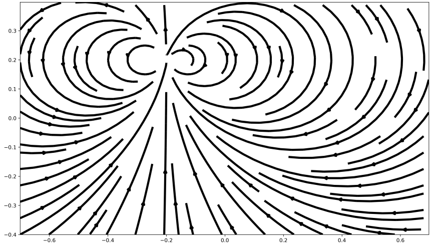
Fields



Electric monopole field streamlines.

Visualisation

Fields



Magnetic dipole field streamlines.

Fields

A magnetic field $\mathbf{B}$ exerts force on magnetic materials (and on electric charges in motion.)

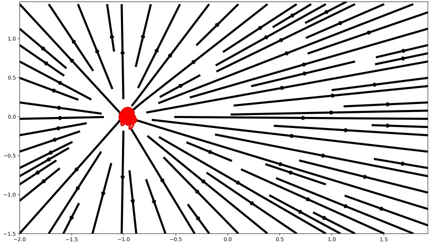
$\mathbf{B}(\boldsymbol{p}, t)$ is a (pseudo)vector denoting its magnitude and direction at position $\boldsymbol{p}$ and time t.

Fields

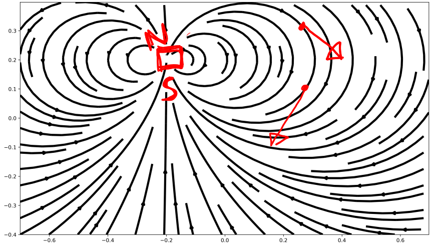
Electricity and magnetism (when considered together) comprise one of the four fundamental forces in nature.

An electric field **E** exerts force on an electric charge.

$\mathbf{E}(\boldsymbol{p}, t)$ is a vector denoting its magnitude and direction at position $\boldsymbol{p}$ and time t.



Electric monopole field streamlines.

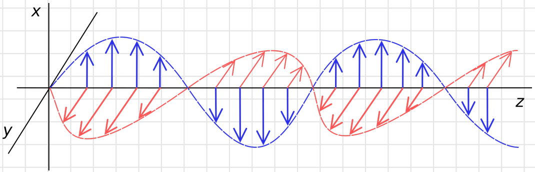


Magnetic dipole field streamlines.

A magnetic field **B** exerts force on magnetic materials (and on electric charges in motion.)

$\mathbf{B}(\boldsymbol{p}, t)$ is a (pseudo)vector denoting its magnitude and direction at position $\boldsymbol{p}$ and time t.

Electromagnetic radiation

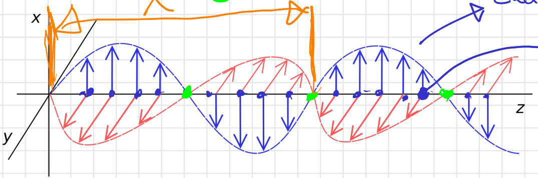


Synchronised oscillations of electric and mag. fields propagating at max. speed $c \approx 300 \times 10^6\,\mathrm{m\,s^{-1}}$.

- "Light" is electromagnetic radiation with particular ranges of wavelength λ, Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.
- Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ.
- For example, $\lambda = 532$ nm is a human-visible "green,"
$$\nu \approx \frac{300 \times 10^6\,\mathrm{m\,s^{-1}}}{532 \times 10^{-9}\,\mathrm{m}} = 0.564 \times 10^{15}\,\mathrm{s^{-1}} = 564 \times 10^{12}\,\mathrm{s^{-1}} = 564\,\mathrm{THz}.$$
- Light has much higher frequency (shorter wavelength) than the "radio" frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

Electromagnetic radiation

$t = 7s$



Electric field vectors

$\vec{p}_{14} = [0,0,14]$

$\vec{E}(\vec{p}_{14}, t) = [e_x, 0, 0]$

Synchronised oscillations of electric and mag. fields propagating at max. speed $c \approx 300 \times 10^6\,\mathrm{m\,s^{-1}}$.

$\vec{B}(\vec{p}_{14}, t) = [0, b_y, 0]$

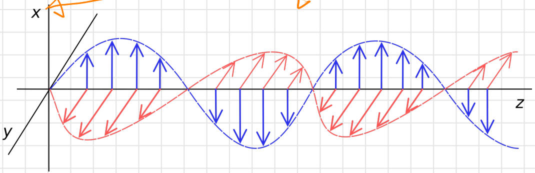
- “Light” is electromagnetic radiation with particular ranges of wavelength λ,
  Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.
- Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ.
- For example, $\lambda = 532$ nm is a human-visible “green,”

$$\nu \approx \frac{300 \times 10^6\,\mathrm{m\,s^{-1}}}{532 \times 10^{-9}\,\mathrm{m}} = 0.564 \times 10^{15}\,\mathrm{s^{-1}} = 564 \times 10^{12}\,\mathrm{s^{-1}} = 564\,\mathrm{THz}.$$

- Light has much higher frequency (shorter wavelength) than the “radio” frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

Electromagnetic radiation

$\lambda = 532\,\text{nm}$



Synchronised oscillations of electric and mag. fields propagating at max. speed $c \approx 300 \times 10^6\,\text{m}\,\text{s}^{-1}$.

- "Light" is electromagnetic radiation with particular ranges of wavelength λ, Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.

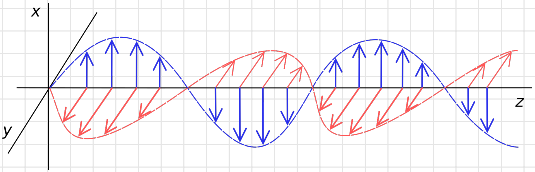
V I B G Y O R

- Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ.
- For example, $\lambda = 532\,\text{nm}$ is a human-visible "green,"

$$\nu \approx \frac{300 \times 10^6\,\text{m}\,\text{s}^{-1}}{532 \times 10^{-9}\,\text{m}} = 0.564 \times 10^{15}\,\text{s}^{-1} = 564 \times 10^{12}\,\text{s}^{-1} = 564\,\text{THz}.$$

- Light has much higher frequency (shorter wavelength) than the "radio" frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

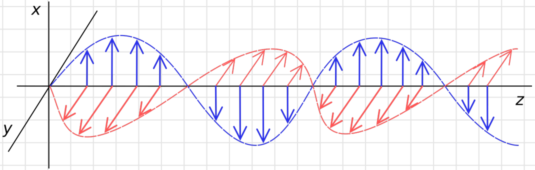
Electromagnetic radiation



Synchronised oscillations of electric and mag. fields propagating at max. speed $c \approx 300 \times 10^6\,\mathrm{m\,s^{-1}}$.

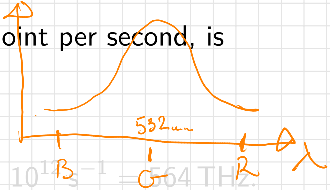
- "Light" is electromagnetic radiation with particular ranges of wavelength λ, Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.
- nu
  Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ.
- For example, $\lambda = 532$ nm is a human-visible "green,"
$$\nu \approx \frac{300 \times 10^6\,\mathrm{m\,s^{-1}}}{532 \times 10^{-9}\,\mathrm{m}} = 0.564 \times 10^{15}\,\mathrm{s^{-1}} = 564 \times 10^{12}\,\mathrm{s^{-1}} = 564\,\mathrm{THz}.$$
- Light has much higher frequency (shorter wavelength) than the "radio" frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

Electromagnetic radiation



Synchronised oscillations of electric and mag. fields propagating at max. speed $c \approx 300 \times 10^{6}\,\mathrm{m\,s^{-1}}$.

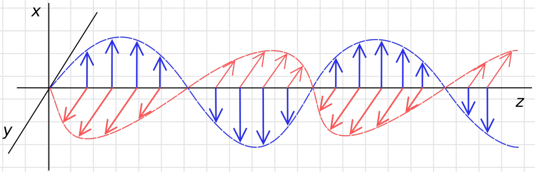
- "Light" is electromagnetic radiation with particular ranges of wavelength λ, Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.
- Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ.
- For example, $\lambda = 532\,\mathrm{nm}$ is a human-visible "green,"



$$\nu \approx \frac{300 \times 10^{6}\,\mathrm{m\,s^{-1}}}{532 \times 10^{-9}\,\mathrm{m}} = 0.564 \times 10^{15}\,\mathrm{s^{-1}} = 564 \times 10^{12}\,\mathrm{s^{-1}} = 564\,\mathrm{THz}.$$

- Light has much higher frequency (shorter wavelength) than the "radio" frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

Electromagnetic radiation



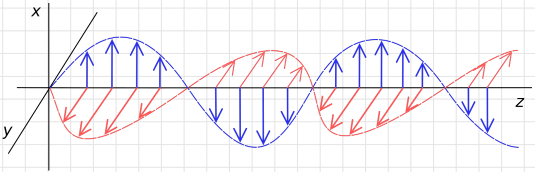
Synchronised oscillations of electric and mag. fields propagating at max. speed $c \approx 300 \times 10^6\,\mathrm{m\,s^{-1}}$.

- "Light" is electromagnetic radiation with particular ranges of wavelength λ, Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.
- Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ.
- For example, $\lambda = 532\,\mathrm{nm}$ is a human-visible "green,"

$$\nu \approx \frac{300 \times 10^6\,\mathrm{m\,s^{-1}}}{532 \times 10^{-9}\,\mathrm{m}} = 0.564 \times 10^{15}\,\mathrm{s^{-1}} = 564 \times 10^{12}\,\mathrm{s^{-1}} = 564\,\mathrm{THz}.$$

- Light has much higher frequency (shorter wavelength) than the "radio" frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

Electromagnetic radiation



Synchronised oscillations of electric and mag. fields propagating at max. speed $c \approx 300 \times 10^6\,\mathrm{m\,s^{-1}}$.

- "Light" is electromagnetic radiation with particular ranges of wavelength λ, Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.
- Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ.
- For example, $\lambda = 532\,\mathrm{nm}$ is a human-visible "green,"
$$\nu \approx \frac{300 \times 10^6\,\mathrm{m\,s^{-1}}}{532 \times 10^{-9}\,\mathrm{m}} = 0.564 \times 10^{15}\,\mathrm{s^{-1}} = 564 \times 10^{12}\,\mathrm{s^{-1}} = 564\,\mathrm{THz}.$$
- Light has much higher frequency (shorter wavelength) than the "radio" frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

Maxwell's equations

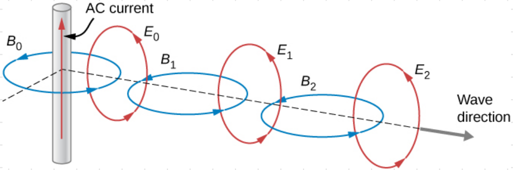
A *system* of equations that describes relationships between electromagnetic radiation field characteristics at a point and time $(\boldsymbol{p}, t)$.

Solving the system at a sequence of points and moments in time allows the *propagation* of radiation to be modelled.

Maxwell's equations

$$\begin{cases} \nabla \cdot \mathbf{E} = \rho/\epsilon \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon\dfrac{\partial \mathbf{E}}{\partial t}) \end{cases}$$

Maxwell's equations



AC line generating EM radiation

Maxwell's equations

For very good explanations of electromagnetic radiation and Maxwell's equations, see:

`https://tinyurl.com/y6dbsrxj` (text)

and

`https://tinyurl.com/y4uyma7u` (video.)

Maxwell's equations

System of equations

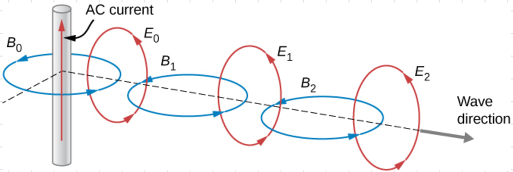
A *system* of equations that describes relationships between electromagnetic radiation field characteristics at a point and time $(\boldsymbol{p}, t)$.

Solving the system at a sequence of points and moments in time allows the *propagation* of radiation to be modelled.

Solution for E and B at position p at time t has to satisfy these constraints.

$$\begin{cases} \nabla \cdot \mathbf{E} = \rho / \epsilon \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon \dfrac{\partial \mathbf{E}}{\partial t}) \end{cases}$$

Simultaneous equations



AC line generating EM radiation

For very good explanations of electromagnetic radiation and Maxwell's equations, see:

`https://tinyurl.com/y6dbsrxj` (text)

and

`https://tinyurl.com/y4uyma7u` (video.)

Vectors

For $a, b, c \in \mathbb{R}$, a vector $\boldsymbol{v} \in \mathbb{R}^3$ can be written,

$$\boldsymbol{v} = a\,\boldsymbol{i} + b\,\boldsymbol{j} + c\,\boldsymbol{k} = (a, b, c)$$

with standard basis vectors,

$$\boldsymbol{i} = (1, 0, 0) \text{ and } \boldsymbol{j} = (0, 1, 0) \text{ and } \boldsymbol{k} = (0, 0, 1)$$

in a Euclidean coordinate system with axes X,Y,Z.

Vectors

Vectors$\leftarrow$Quaternions$\leftarrow$Hamilton$\leftarrow$TCD!

Magnitude of vector $\boldsymbol{w} = (x, y, z)$ is its length,

$$\|\boldsymbol{w}\| = \sqrt{x^2 + y^2 + z^2}.$$

Vectors

Scalar (or dot) product,

$$\boldsymbol{v} \cdot \boldsymbol{w} = ax + by + cz$$
$$= \|\boldsymbol{v}\| \|\boldsymbol{w}\| \cos\theta.$$

Vectors

Vector (or cross) product is $\perp \boldsymbol{v}$ and $\perp \boldsymbol{w}$,

$$\boldsymbol{v} \times \boldsymbol{w} = (bz - cy)\boldsymbol{i} + (cx - az)\boldsymbol{j} + (ay - bx)\boldsymbol{k}$$
$$\|\boldsymbol{v} \times \boldsymbol{w}\| = \|\boldsymbol{v}\|\|\boldsymbol{w}\||\sin\theta|.$$

Vectors

For $a, b, c \in \mathbb{R}$, a vector $\boldsymbol{v} \in \mathbb{R}^3$ can be written,

$$\boldsymbol{v} = a\,\boldsymbol{i} + b\,\boldsymbol{j} + c\,\boldsymbol{k} = (a, b, c)$$

with standard basis vectors,

$$\boldsymbol{i} = (1, 0, 0) \text{ and } \boldsymbol{j} = (0, 1, 0) \text{ and } \boldsymbol{k} = (0, 0, 1)$$

in a Euclidean coordinate system with axes X,Y,Z.

Vectors$\leftarrow$Quaternions$\leftarrow$Hamilton$\leftarrow$TCD!

Magnitude of vector $\boldsymbol{w} = (x, y, z)$ is its length,

$$\|\boldsymbol{w}\| = \sqrt{x^2 + y^2 + z^2}.$$

Scalar (or dot) product,

$$\begin{aligned} \boldsymbol{v} \cdot \boldsymbol{w} &= ax + by + cz \\ &= \|\boldsymbol{v}\|\|\boldsymbol{w}\| \cos\theta. \end{aligned}$$

Vector (or cross) product is $\perp \boldsymbol{v}$ and $\perp \boldsymbol{w}$,

$$\boldsymbol{v} \times \boldsymbol{w} = (bz - cy)\boldsymbol{i} + (cx - az)\boldsymbol{j} + (ay - bx)\boldsymbol{k}$$

$$\|\boldsymbol{v} \times \boldsymbol{w}\| = \|\boldsymbol{v}\|\|\boldsymbol{w}\||\sin\theta|.$$

Vector fields and calculus

A vector-valued function of space, time, etc.,

$$\boldsymbol{F}(\boldsymbol{p}, t) = (F_x(\boldsymbol{p}, t),\ F_y(\boldsymbol{p}, t),\ F_z(\boldsymbol{p}, t))$$

with position vector $\boldsymbol{p} = (x, y, z)$ and time t.

Vector calculus is concerned with differentiation and integration of vector fields.

Vector fields and calculus

Space, time function parameters $(\boldsymbol{p}, t)$ can be omitted for improved readability but you have to remember this when looking at formulae! E.g.

$$\frac{\partial \boldsymbol{F}}{\partial t} = (\frac{\partial F_x}{\partial t}, \frac{\partial F_y}{\partial t}, \frac{\partial F_z}{\partial t}).$$

Vector fields and calculus

Divergence is,

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

A scalar denoting by how much, if at all, the field is like a point source at that position.

Vector fields and calculus

Curl is,

$$\nabla \times \boldsymbol{F} = (\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z})\,\boldsymbol{i} + (\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x})\,\boldsymbol{j} + (\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y})\,\boldsymbol{k}.$$

A vector denoting rotation axis and magnitude, for how much the field rotates, if at all, at that position.

Vector fields and calculus

A vector-valued function of space, time, etc.,

$$\boldsymbol{F}(\boldsymbol{p}, t) = (F_x(\boldsymbol{p}, t),\ F_y(\boldsymbol{p}, t),\ F_z(\boldsymbol{p}, t))$$

with position vector $\boldsymbol{p} = (x, y, z)$ and time t.

Vector calculus is concerned with differentiation and integration of vector fields.

Space, time function parameters $(\boldsymbol{p}, t)$ can be omitted for improved readability but you have to remember this when looking at formulae! E.g.

$$\frac{\partial \boldsymbol{F}}{\partial t} = (\frac{\partial F_x}{\partial t}, \frac{\partial F_y}{\partial t}, \frac{\partial F_z}{\partial t}).$$

Divergence is,

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

A scalar denoting by how much, if at all, the field is like a point source at that position.

Curl is,

$$\nabla \times \boldsymbol{F} = (\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z})\,\boldsymbol{i} + (\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x})\,\boldsymbol{j} + (\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y})\,\boldsymbol{k}.$$

A vector denoting rotation axis and magnitude, for how much the field rotates, if at all, at that position.

Some physical laws

Gauss's law for electricity: electric charges generate an electric field.

$$\nabla \cdot \mathbf{E} = \rho/\epsilon$$

where ρ is electric charge and ϵ is electric permittivity.

Some physical laws

Gauss's law for magnetism: there are no separate magnetic charges (no monopoles.)

$$\nabla \cdot \mathbf{B} = 0.$$

Some physical laws

Faraday's law of induction: A changing magnetic field creates a rotating electric field and vice-versa.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Some physical laws

Ampère-Maxwell's law: an electric current and a changing electric field create a magnetic field.

$$\nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon \frac{\partial \mathbf{E}}{\partial t})$$

where $\mathbf{J}$ is electric current density and μ is magnetic permeability.

Note that $c = 1/\sqrt{\epsilon\mu}$.

Some physical laws

Gauss's law for electricity: electric charges generate an electric field.

$$\nabla \cdot \mathbf{E} = \rho/\epsilon$$

where ρ is electric charge and ϵ is electric permittivity.

Gauss's law for magnetism: there are no separate magnetic charges (no monopoles.)

$$\nabla \cdot \mathbf{B} = 0.$$

Faraday's law of induction: A changing magnetic field creates a rotating electric field and vice-versa.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Ampère-Maxwell's law: an electric current and a changing electric field create a magnetic field.

$$\nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon \frac{\partial \mathbf{E}}{\partial t})$$

where $\mathbf{J}$ is electric current density and μ is magnetic permeability.

Note that $c = 1/\sqrt{\epsilon\mu}$.

Assignment # 4: Electromagnetic Wave Simulation

Look at L1
Solution to wave equation
Sum of 2 (weighted) sinusoids.

- Write a SciPy program to make an animation of the propagation of electriomagnetic fields, inspired by the plot shown on slide 4.
- Make it into a self-contained project repository in your personal account on gitlab.scss.tcd.ie.
- For this and every other assignment, feel free to collaborate with your classmates about the the non-mathematical parts like plotting.

Assignment # 4: Electromagnetic Wave Simulation

- Write a SciPy program to make an animation of the propagation of electriomagnetic fields, inspired by the plot shown on slide 4.
- Make it into a self-contained project repository in your personal account on gitlab.scss.tcd.ie.
- For this and every other assignment, feel free to collaborate with your classmates about the the non-mathematical parts like plotting.

Assignment # 4: Electromagnetic Wave Simulation

- Write a SciPy program to make an animation of the propagation of electriomagnetic fields, inspired by the plot shown on slide 4.
- Make it into a self-contained project repository in your personal account on gitlab.scss.tcd.ie.
- For this and every other assignment, feel free to collaborate with your classmates about the the non-mathematical parts like plotting.