CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Lecture #1: Waves

Fergal Shevlin, Ph.D.

School of Computer Science and Statistics,
Trinity College Dublin

September 16, 2022

5 ECTS $\approx$ 125 hours!

125h / (11 + 2 + 1) w

9h/w

11 assignments

3 hour exam questions based on assignments

2 hours Lectures

3 hours follow-up

+ 4 hours Programming

9 hours

SciPy ~ Python

PLOTTING

CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

1

ANACONDA

SciPy

Install on your own pc.

Lecture #1: Waves

2

gitlab.scss.tcd.ie

CS7GV2-2022-23

Lecture-Notes

(a) pdfs

(b) SciPy code

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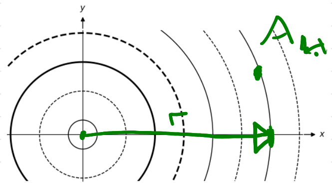
3

Make a project for each assignment

September 16, 2022

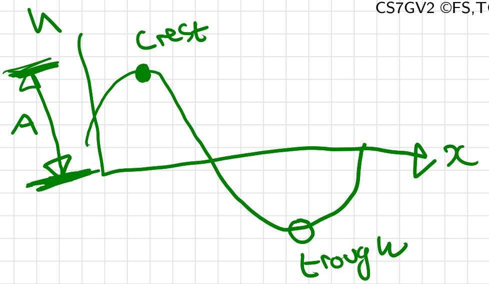
→ add me as a "reporter".

Physical waves

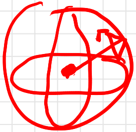


Emission from a point source in space.
Amplitude $\propto r^{-1}$ for 2-D and $\propto r^{-2}$ for 3-D.

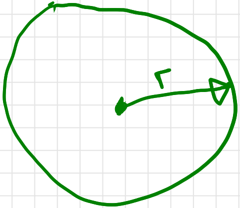
proportional to



$r^{-1} = \frac{1}{r}$



A_0



$\frac{2\pi r}{}$

$$\frac{A_0}{2\pi r} = A_0 \times \cancel{\frac{1}{2\pi}} \times \frac{1}{r}$$

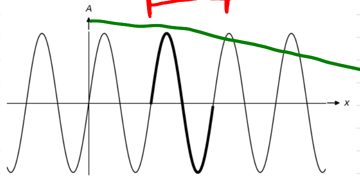
$\propto A_0 r^{-1}$

Physical waves

$\lambda = p$ m

Lambda

Xi



Approx. amplitude along a radial line through space. Spatial wavelength $\lambda := p\,\mathrm{m}$. Spatial freq. $\xi := (1\,\mathrm{m}/p\,\mathrm{m})\ \mathrm{c/m} = 1\,\mathrm{m}/\lambda\ \mathrm{c/m}$.

Physical waves

$\boxed{\nu \approx 17\ \mathrm{GHz}}$

A(x,y)

c means cycles

c/s → Hertz

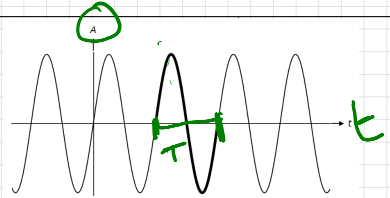
Hz

kHz AM

MHz FM

GHz WiFi

ν nu



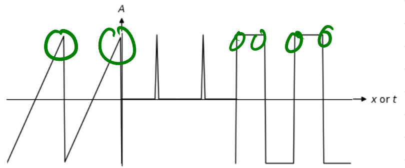
Approx. amplitude at any point x in space wrt time t. Temporal period $T := q\,\mathrm{s}$. Temporal freq. $\nu := (1\,\mathrm{s}/q\,\mathrm{s})\ \mathrm{c/s} = 1/q\,\mathrm{s} = 1\,\mathrm{s}/T\ \mathrm{c/s}$.

Physical waves

WAVEFORMS

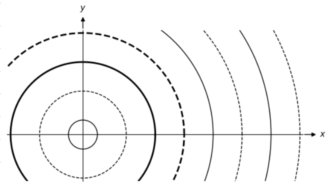
THESE ARE NOT PHYSICAL WAVES

sawtooth impulse square



Non-smooth (instantaneous change) or non-changing wrt space and time is non-physical.

Physical waves



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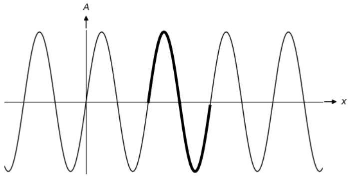

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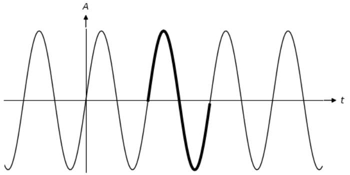

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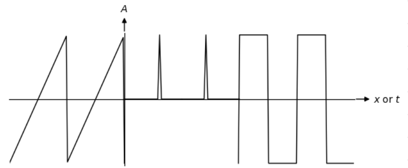
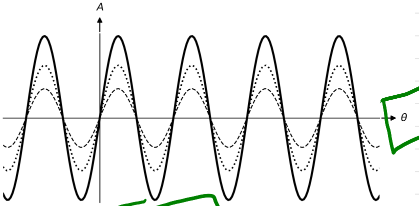


Non-smooth (instantaneous change) or non-changing wrt space and time is non-physical.

Wave summation, $A_{1+2} := \boxed{A_1 + A_2}$

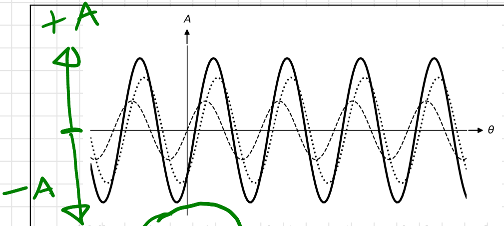


θ theta

x or t

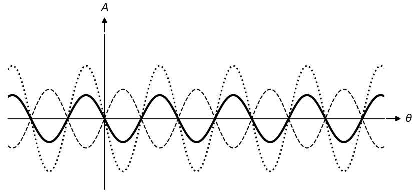
When $\boxed{\lambda_1 = \lambda_2}$ and $\phi_1 = \phi_2$,
$\lambda_{1+2} = \lambda_1 = \lambda_2$ and $\phi_{1+2} = \phi_1 = \phi_2$.

Wave summation, $A_{1+2} := A_1 + A_2$

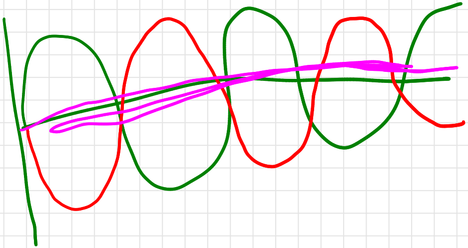


When $\lambda_1 = \lambda_2$ and $\phi_1 > \phi_2$ and $A_1 > A_2$, $\lambda_{1+2} = \lambda_1 = \lambda_2$ and $\phi_1 > \phi_{1+2} > \phi_2$.

Wave summation, $A_{1+2} := A_1 + A_2$



destructive interference



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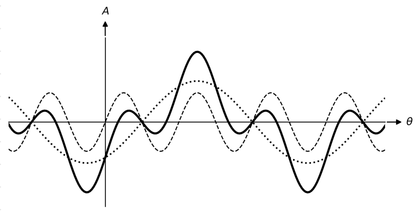

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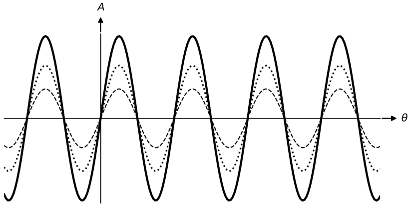

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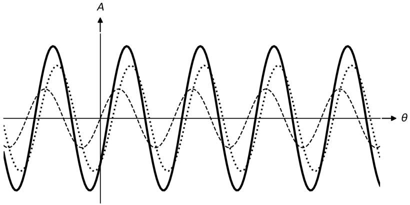

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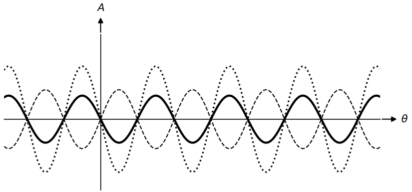
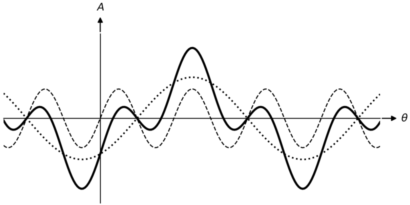
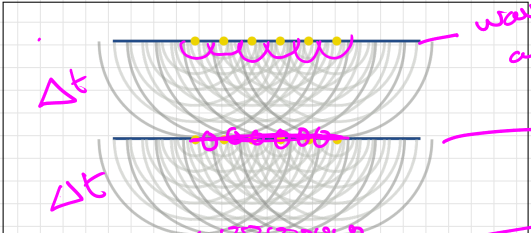




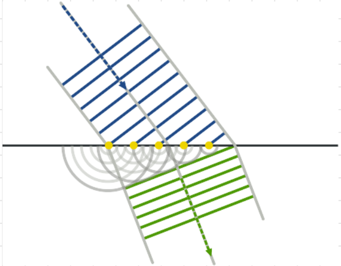
Wavefront propagation

Δ delta



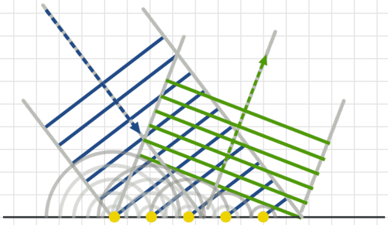
Huygens-Fresnel Construction.

Wavefront propagation



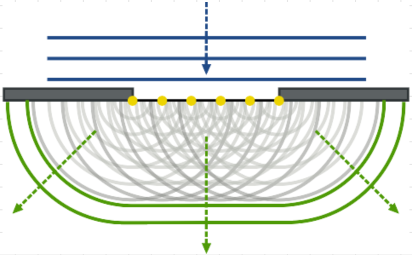
Refraction.

Wavefront propagation



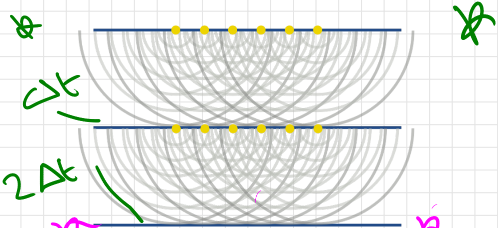
Reflection.

Wavefront propagation

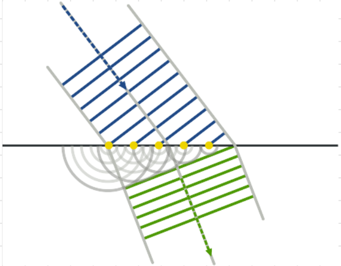


Diffraction.

Wavefront propagation



Huygens-Fresnel Construction.



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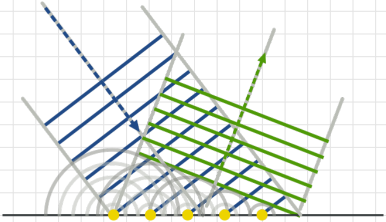

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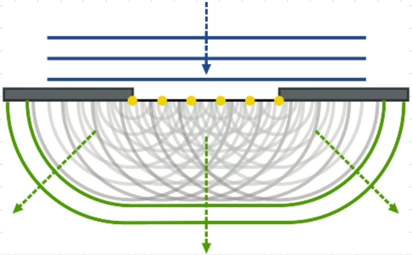
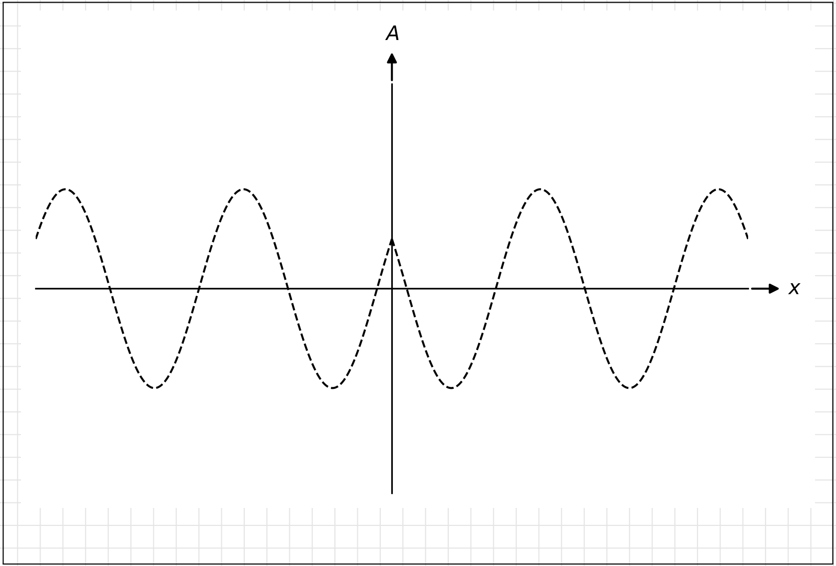
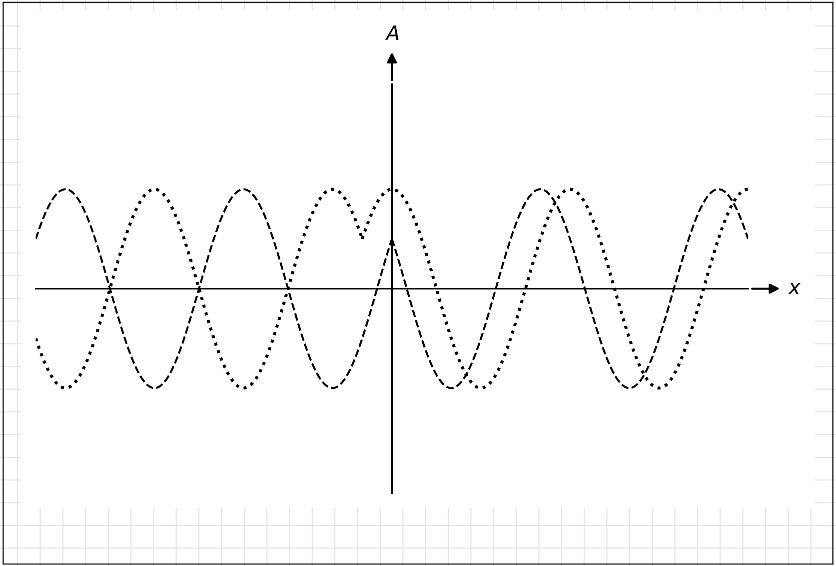


Diffraction.

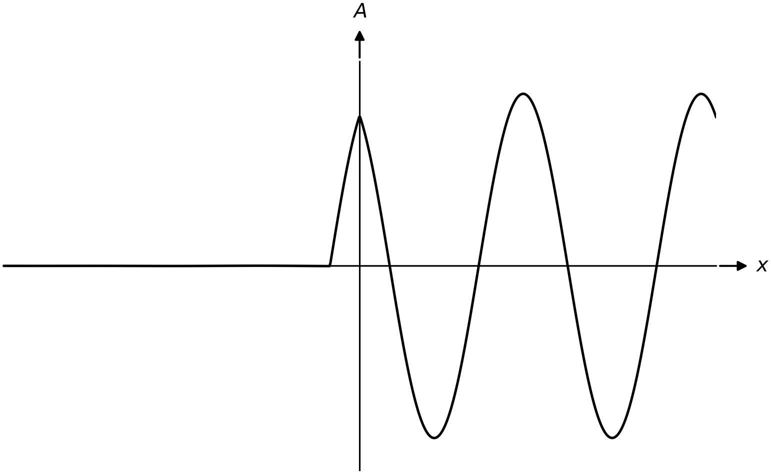
Propagation from a point where amplitude is increasing



. . . and from another point where amplitude is decreasing

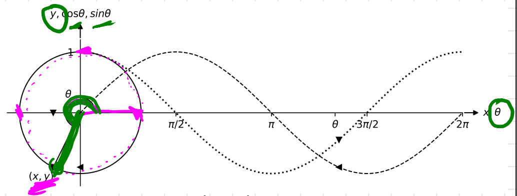


Radial propagation



Huygens-Miller construction.

Sinusoids



For any point (x, y) on unit circle,
$\cos\theta := x$ and $\sin\theta := y$.

$\cos(\theta)$

θ theta

Sinusoids

Point coordinates corresponding to angle θ are,

$$(\cos\theta, \sin\theta).$$

Angle θ corresponding to point coords (x, y) is,

$$\arccos x \text{ and } \arcsin y.$$

Sinusoids

Geometric contruction is impractical and mathematical expression is complicated:

$$\sin\theta = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\, \theta^{2n+1}$$

so calculators with programmed buttons or printed tables are used.

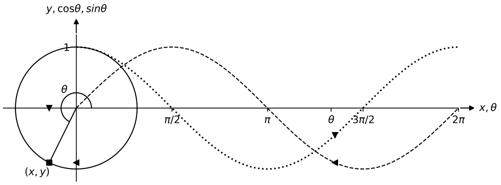
Sinusoids

Sinusoids with same λ but arbitrary ϕ and A sum to a sinusoid with same λ.

This is how physical waves behave.

Sinusoids are *only* periodic functions with this property.

Sinusoids



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Wave equation

How can waves be described so their behaviour can be analysed mathematically?

“They look like sinusoids” isn't rigorous enough.

Wave equation

$$\frac{\partial^2 A(x,t)}{\partial x^2} - \frac{\partial^2 A(x,t)}{\partial t^2}$$

DIFFERENTIAL EQUATION

We will soon derive this constraint equation from Hooke's and Newton's Laws:

$$\boxed{\frac{\partial^2 A}{\partial x^2}} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = 0$$

$$\frac{\partial^2 A}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}$$

Spatial behaviour

temporal behaviour

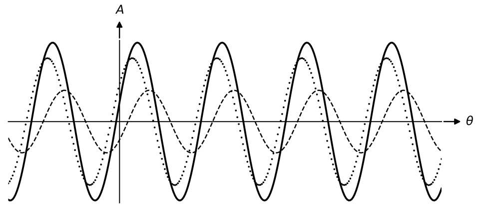
Wave equation

One of many solutions can be found algebraically as,

$$\begin{aligned} A(x,t) =& R\cos(k\,x - \omega\,t) \\ &+ (1-R)\cos(k\,x + \omega\,t) \end{aligned}$$

where k and ω are constants related to (angular) wavelength and frequency and $|R| \leq 1$.

Wave equation



The sum of two weighted sinusoids travelling in opposite directions. Here for $R = 0.33$.

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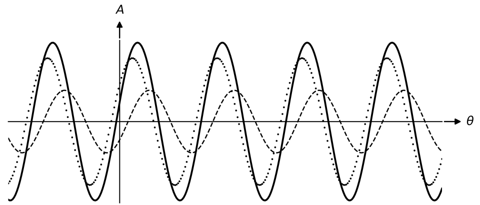

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Assignment # 1: Huygens-Fresnel construction

- Write a SciPy program to make at least one plot similar to those shown the wavefront propagation slide.
- Use Huygens-Fresnel construction to determine where the wavefront should be at different times.
- Make it into a self-contained project repository in your personal account on gitlab.scss.tcd.ie.
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Greek letters* often used as symbols in mathematics

α	alpha	θ	theta	o	omicron	τ	tau
β	beta	ϑ	caligr. theta	π	pi	υ	upsilon
γ	gamma	ι	iota	ϖ	caligr. pi	ϕ	phi
δ	delta	κ	kappa	ρ	rho	φ	caligr. phi
ϵ	epsilon	λ	lambda	ϱ	caligr. rho	χ	chi
ε	caligr. epsilon	μ	mu	σ	sigma	ψ	psi
ζ	zeta	ν	nu	ς	caligr. sigma	ω	omega
η	eta	ξ	xi				

Γ	big gamma	Λ	big lambda	Σ	big sigma	Ψ	big psi
Δ	big delta	Ξ	big xi	Υ	big upsilon	Ω	big omega
Θ	big theta	Π	big pi	Φ	big phi		

*With their anglophone pronunciations.