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Physics, Engineering
Maths

CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Python
SciPy
Matlab
Mathematica
Plotting + Visualisation
Anaconda
① ②
gitlab.scss.tcd.ie

5 ECTS → 125 hrs

125 hrs / (11+1+1+1) wks

125/14 ≃ 9 hrs/week

2 hrs lectures

7 hrs follow-up

(a) re-read notes + investigate!

(b) programming assignment

not marked

10 questions

TAKE-AT-HOME

3 hr

EXAM!

Lecture #1: Waves

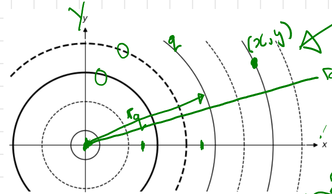
Fergal Shevlin, Ph.D.

School of Computer Science and Statistics,
Trinity College Dublin

September 24, 2021

Physical waves

SciPy plot

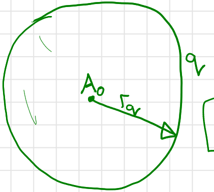


Emission from a point source in space.
Amplitude $\propto r^{-1}$ for 2-D and $\propto r^{-2}$ for 3-D.

Proportional to

$r^{-1} \equiv \frac{1}{r}$

$A_0 = 10\text{ cm}$



$A_q \propto \frac{1}{r_q}$

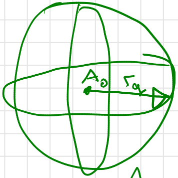
$2\pi r_q$

$A_q = A_0 / (2\pi r_q)$

$\propto \frac{1}{r_q}$

$\propto r_q^{-1}$

$A_0 / (4\pi r_q^2)$



$A_0 \frac{1}{4\pi} \frac{1}{r_q^2}$

$A_0 \frac{1}{2\pi r_q}$ r_q^{-2}

$= A_0 \frac{1}{2\pi} \frac{1}{r_q}$

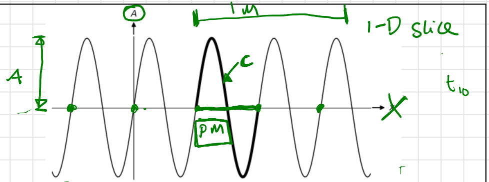
Physical waves

Spatial Characteristics

$A \propto \frac{1}{r} = r^{-1}$

Wavelength λ lambda

Spatial frequency ξ Xi



$\boxed{\text{Approx.}}$ amplitude along a radial line through space. Spatial wavelength $\lambda := \boxed{p\,\mathrm{m}}$. Spatial freq. $\xi := (1\,\mathrm{m}/p\,\mathrm{m})\,\boxed{\mathrm{c/m}} = 1\,\mathrm{m}/\lambda\,\boxed{\mathrm{c/m}}$.

$\lambda = p\,m$

$\lambda :=$

$1/\lambda$

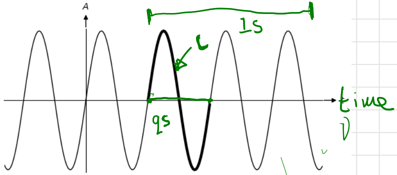
Physical waves

temporal freq ν - nu

$\nu = 2.5$ c/s

Hertz
KHz
MHz
GHz

Temporal Characteristics of a single point (x,y)



Approx. amplitude at any point x in space wrt time t. Temporal period $T := \boxed{q\,\mathrm{s}}$. Temporal freq. $\nu := (1\,\mathrm{s}/q\,\mathrm{s})\ \mathrm{c/s} = 1/q\,\mathrm{s} = 1\,\mathrm{s}/T\ \mathrm{c/s}$.

Hz

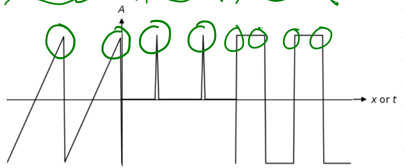
Physical waves

Waveforms

Saw-tooth
triangular

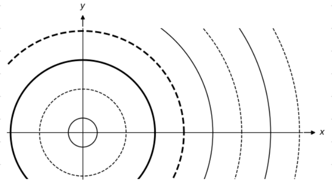
impulse

square



Non-smooth (instantaneous change) or non-changing wrt space and time is non-physical.

Physical waves



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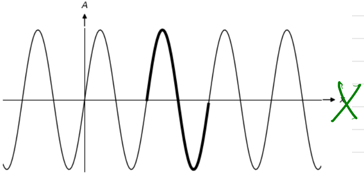

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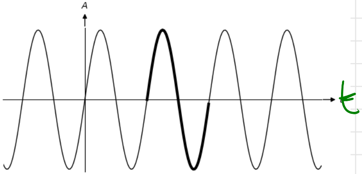

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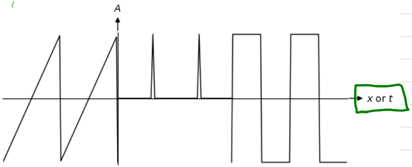
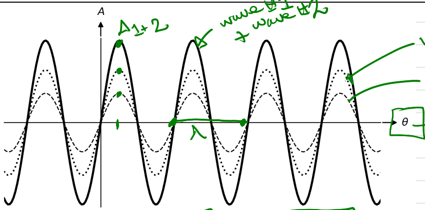


Non-smooth (instantaneous change) or non-changing wrt space and time is non-physical.

Wave summation, $\boxed{A_{1+2}} := A_1 + A_2$



When $\boxed{\lambda_1 = \lambda_2}$ and $\boxed{\phi_1 = \phi_2}$,
$\boxed{\lambda_{1+2} = \lambda_1 = \lambda_2}$ and $\boxed{\phi_{1+2}} = \boxed{\phi_1 = \phi_2}$.

phi ϕ

angles?

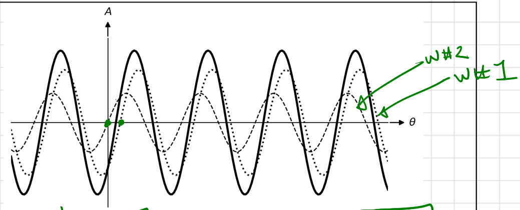
Wave characteristics
- λ - wavelength
- A - amplitude
- ϕ - phase

Wave summation, $A_{1+2} := A_1 + A_2$

$\phi_1 = \frac{1}{2}$ $\phi_2 = 0$

" Out of phase "

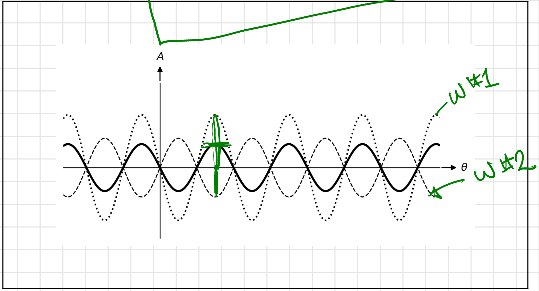
→ different phases.

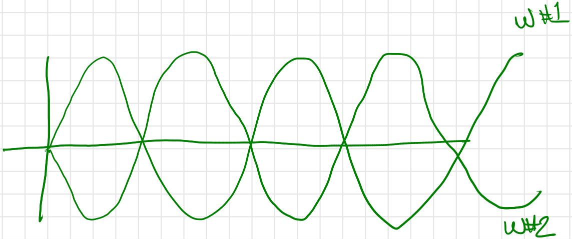


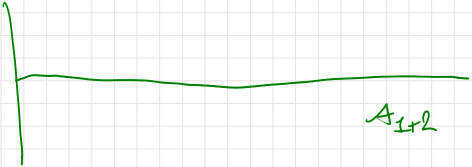
When $\boxed{\lambda_1 = \lambda_2}$ and $\boxed{\phi_1 > \phi_2}$ and $\boxed{A_1 > A_2}$,
$\boxed{\lambda_{1+2}}\boxed{= \lambda_1 = \lambda_2}$ and $\phi_1 > \boxed{\phi_{1+2}} > \phi_2$.

Wave summation, $\boxed{A_{1+2} := A_1 + A_2}$

"destructive interference"







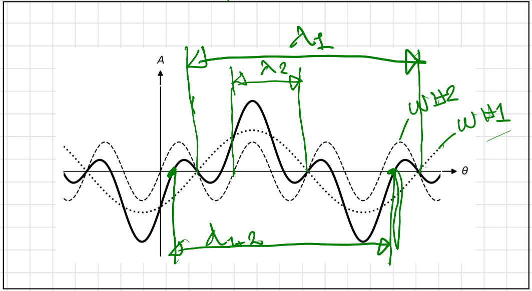
Wave summation, $A_{1+2} := A_1 + A_2$

$\lambda_1 \neq \lambda_2$

$\phi_1 \neq \phi_2$

$\Big\} \; \lambda_{1+2} \neq \lambda_1$ or λ_2

Will see more of this later as part of fourier transform for image processing + analysis.



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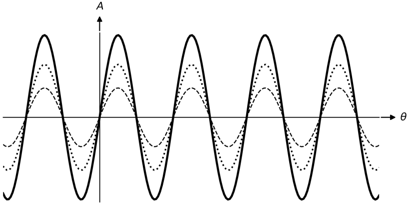

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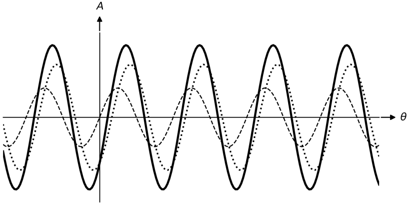

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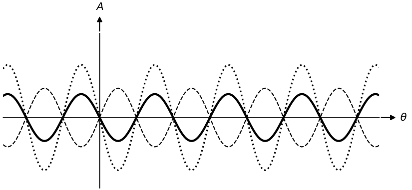
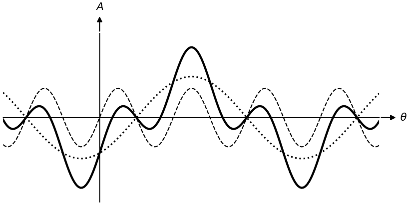
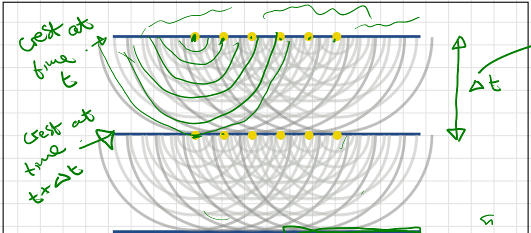


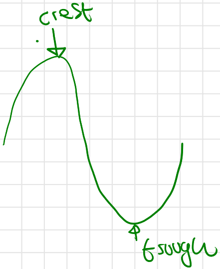


Wavefront propagation → movement through space

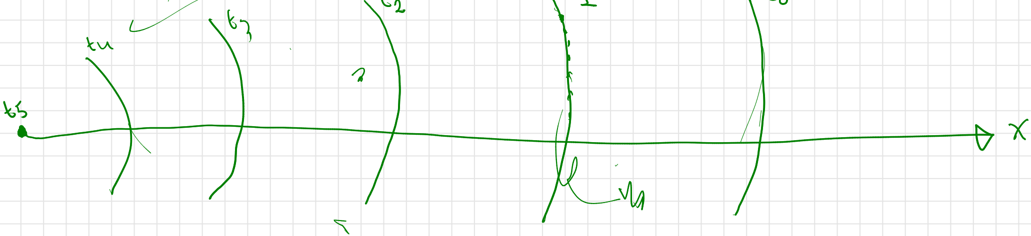


Huygens-Fresnel Construction.

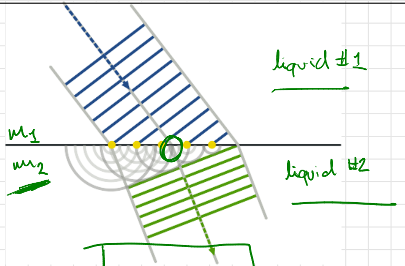
→ delta Δ often used for an interval.



a line through all the points that were emitted from the source at the same moment in time

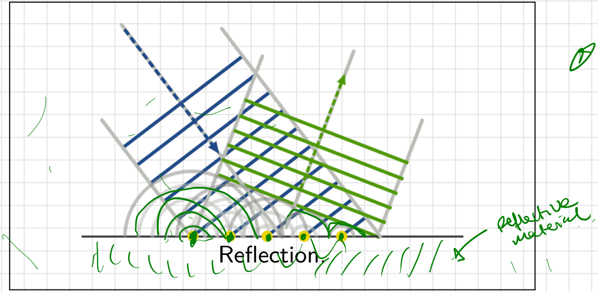


Wavefront propagation

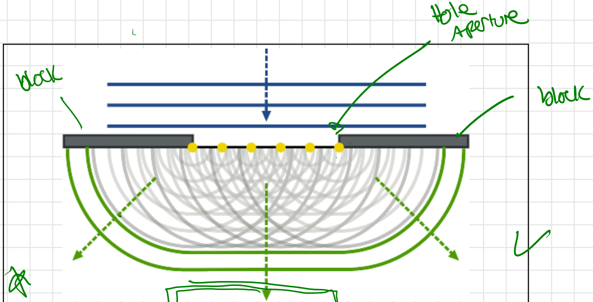


Refraction.

Wavefront propagation



Wavefront propagation

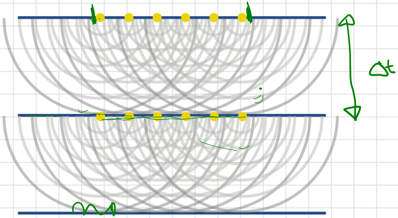


Diffraction.

Wavefront propagation

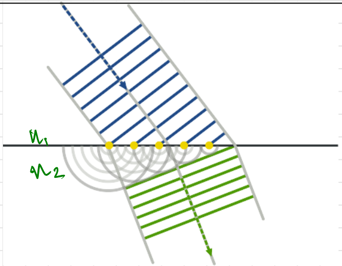
#1

FREE SPACE



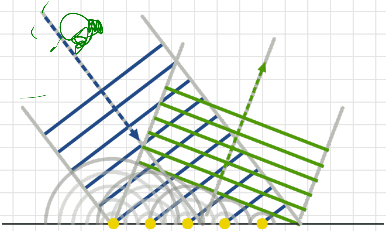
Huygens-Fresnel Construction.

#2



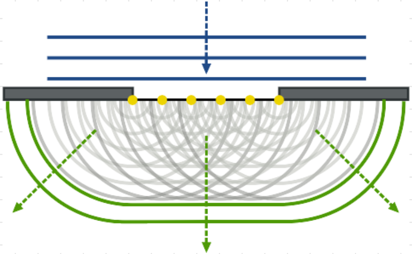
Refraction.

#3



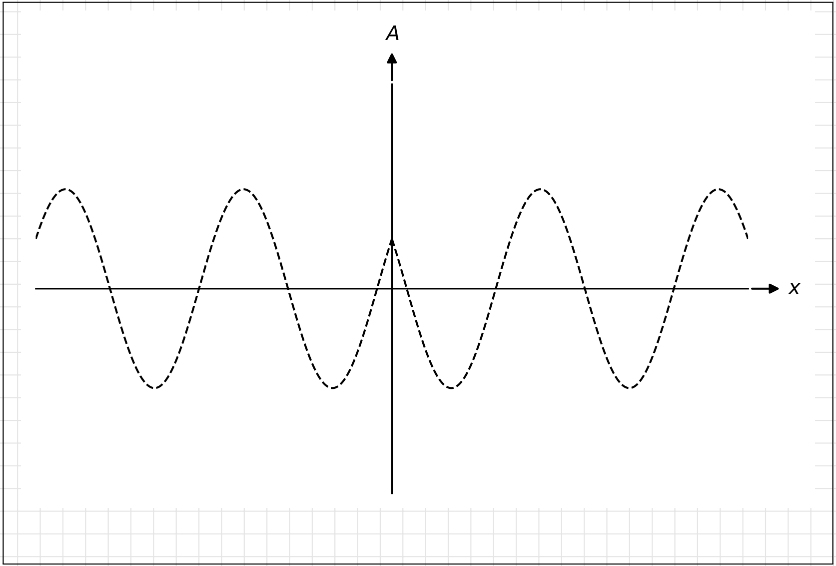
Reflection.

#4

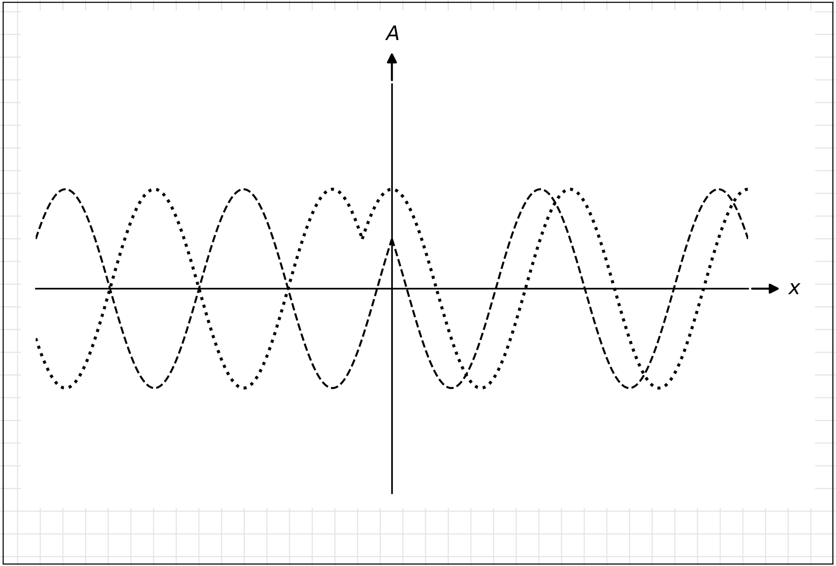


Diffraction.

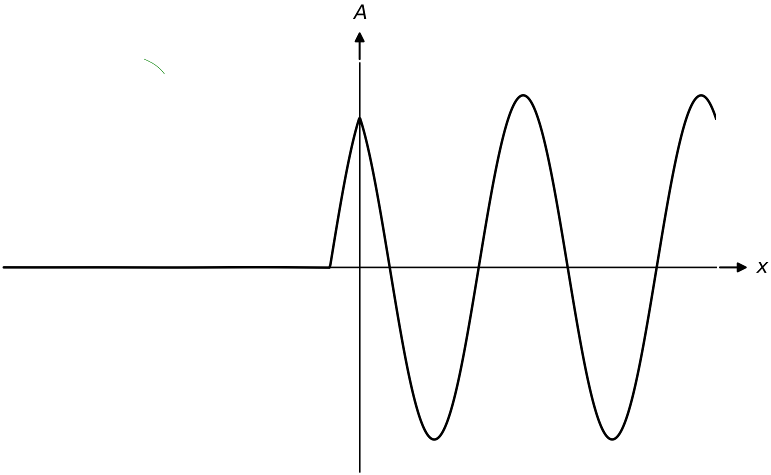
Propagation from a point where amplitude is increasing



. . . and from another point where amplitude is decreasing

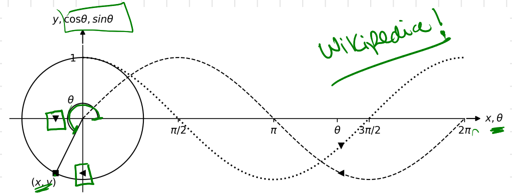


Radial propagation



Huygens-Miller construction.

Sinusoids → Sines and Cosines



For any point (x, y) on unit circle,

$\cos\theta \boxed{:=} x$ and $\sin\theta \boxed{:=} y$.

cos(θ) sin(θ)

Sinusoids

Point coordinates corresponding to angle θ are,

$$(\cos\theta, \sin\theta).$$

Angle θ corresponding to point coords (x, y) is,

$$\boxed{\arccos x} \text{ and } \boxed{\arcsin y}.$$

$\cos^{-1} x$ $\sin^{-1} y$

"depricated"

Sinusoids

Geometric contruction is impractical and mathematical expression is complicated:

$$\sin\boxed{\theta} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\boxed{\theta}^{2n+1}}$$

so calculators with programmed buttons or printed tables are used.

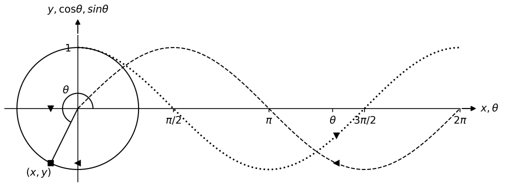
Sinusoids

Sinusoids with same λ but arbitrary $\boxed{\phi}$ and $\boxed{A}$ sum to a sinusoid with same λ.

This is how physical waves behave.

Sinusoids are *only* periodic functions with this property.

Sinusoids



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Wave equation

How can waves be described so their behaviour can be analysed mathematically?

"They look like sinusoids" isn't rigorous enough.

Wave equation

We will soon derive this constraint equation from Hooke's and Newton's Laws:

$$\frac{\partial^2 A}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = 0$$

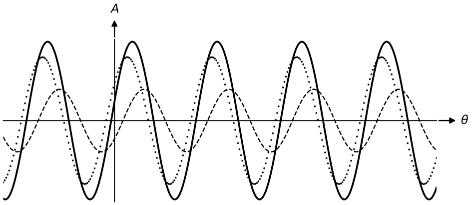
Wave equation

One of many solutions can be found algebraically as,

$$\begin{aligned} A(x,t) = &R\cos(k\,x - \omega\,t) \\ &+ (1-R)\cos(k\,x + \omega\,t) \end{aligned}$$

where k and ω are constants related to (angular) wavelength and frequency and $|R| \leq 1$.

Wave equation



The sum of two weighted sinusoids travelling in opposite directions. Here for $R = 0.33$.

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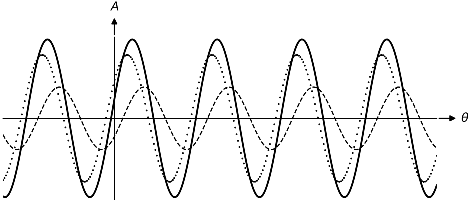

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Assignment # 1: Huygens-Fresnel construction

- Write a SciPy program to make at least one plot similar to those shown the wavefront propagation slide.
- Use Huygens-Fresnel construction to determine where the wavefront should be at different times.
- Make it into a self-contained project repository in your personal account on gitlab.scss.tcd.ie.

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Greek letters* often used as symbols in mathematics

α	alpha	θ	theta	o	omicron	τ	tau
β	beta	ϑ	caligr. theta	π	pi	υ	upsilon
γ	gamma	ι	iota	ϖ	caligr. pi	ϕ	phi
δ	delta	κ	kappa	ρ	rho	φ	caligr. phi
ϵ	epsilon	λ	lambda	ϱ	caligr. rho	χ	chi
ε	caligr. epsilon	μ	mu	σ	sigma	ψ	psi
ζ	zeta	ν	nu	ς	caligr. sigma	ω	omega
η	eta	ξ	xi				
Γ	big gamma	Λ	big lambda	Σ	big sigma	Ψ	big psi
Δ	big delta	Ξ	big xi	Υ	big upsilon	Ω	big omega
Θ	big theta	Π	big pi	Φ	big phi		

*With their anglophone pronunciations.