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Notes

CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Lecture #1: Waves

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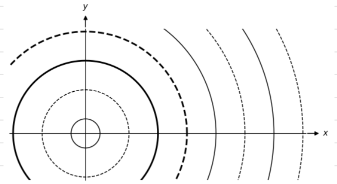
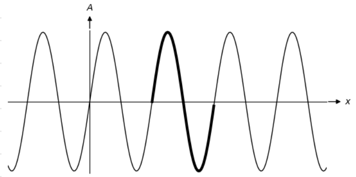
Notes

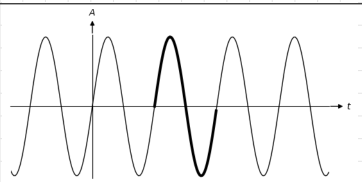
Physical waves



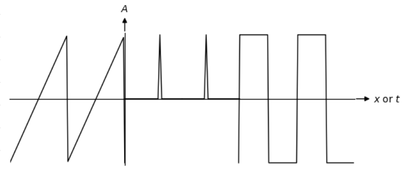
Emission from a point source in space. Amplitude $\propto r^{-1}$ for 2-D and $\propto r^{-2}$ for 3-D.



Approx. amplitude along a radial line through space. Spatial wavelength $\lambda := p\,\mathrm{m}$. Spatial freq. $\xi := (1\,\mathrm{m}/p\,\mathrm{m})\ \mathrm{c/m} = 1\,\mathrm{m}/\lambda\ \mathrm{c/m}$.



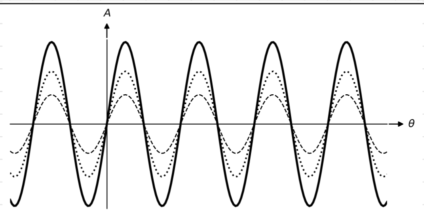
Approx. amplitude at any point x in space wrt time t. Temporal period $T := q\,\mathrm{s}$. Temporal freq. $\nu := (1\,\mathrm{s}/q\,\mathrm{s})\ \mathrm{c/s} = 1/q\,\mathrm{s} = 1\,\mathrm{s}/T\ \mathrm{c/s}$.



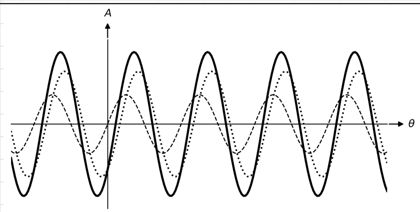
Non-smooth (instantaneous change) or non-changing wrt space and time is non-physical.

Notes

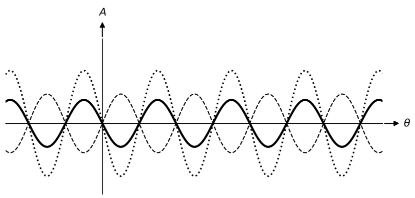
Wave summation, $A_{1+2} := A_1 + A_2$

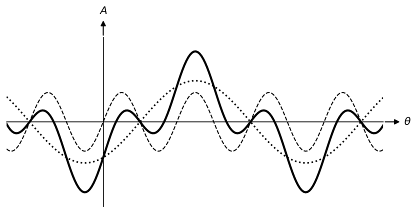


When $\lambda_1 = \lambda_2$ and $\phi_1 = \phi_2$, $\lambda_{1+2} = \lambda_1 = \lambda_2$ and $\phi_{1+2} = \phi_1 = \phi_2$.



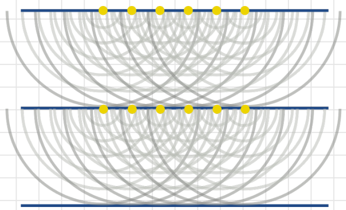
When $\lambda_1 = \lambda_2$ and $\phi_1 > \phi_2$ and $A_1 > A_2$, $\lambda_{1+2} = \lambda_1 = \lambda_2$ and $\phi_1 > \phi_{1+2} > \phi_2$.



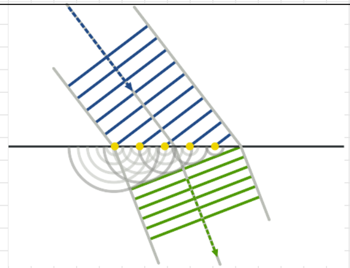


Notes

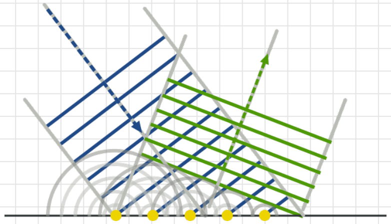
Wavefront propagation



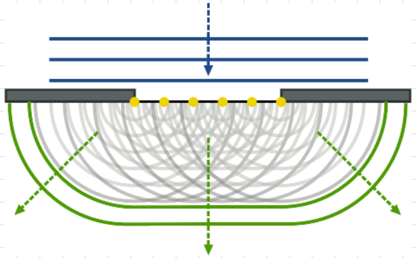
Huygens-Fresnel Construction.



Refraction.



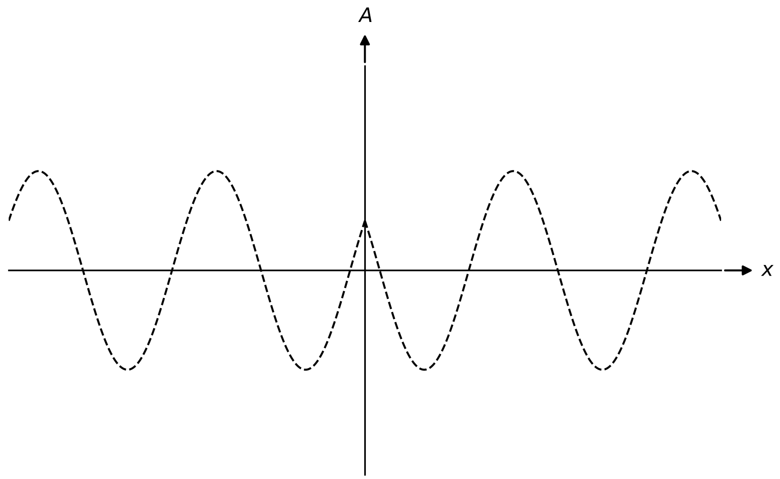
Reflection.



Diffraction.

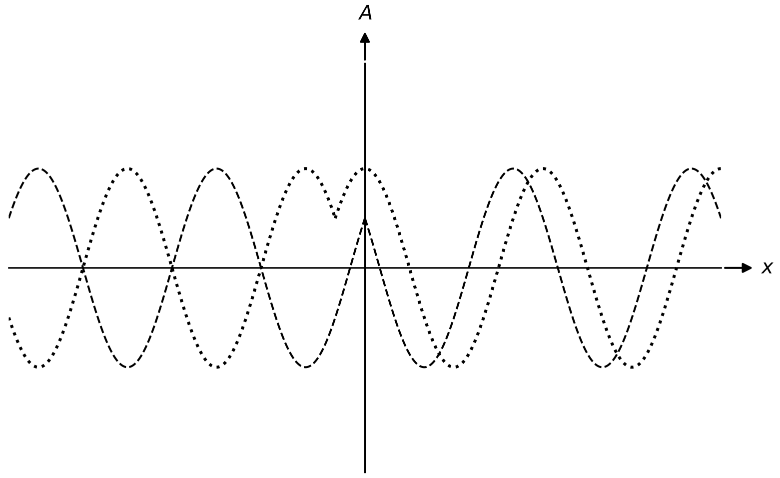
Notes

Propagation from a point where amplitude is increasing



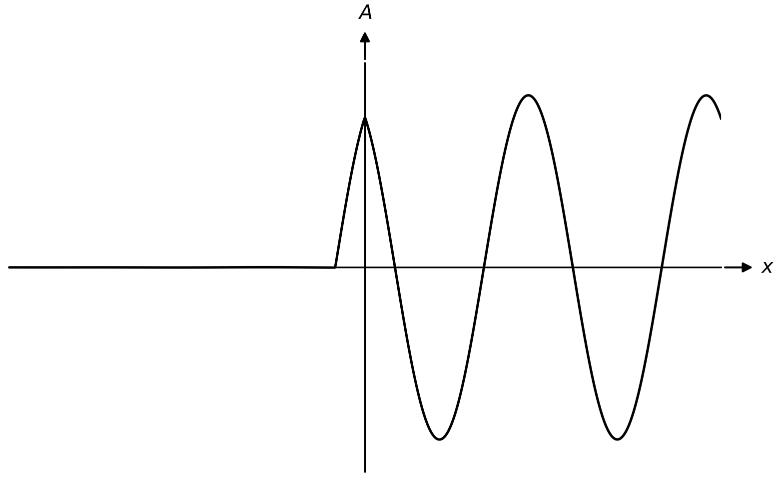
Notes

. . . and from another point where amplitude is decreasing



Notes

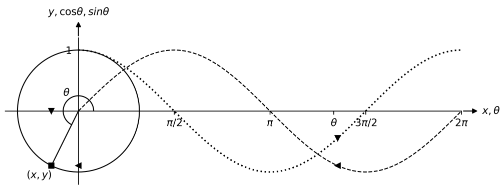
Radial propagation



Huygens-Miller construction.

Notes

Sinusoids



For any point (x, y) on unit circle, $\cos\theta := x$ and $\sin\theta := y$.

Point coordinates corresponding to angle θ are,

$$(\cos\theta, \sin\theta).$$

Angle θ corresponding to point coords (x, y) is,

$$\arccos x \text{ and } \arcsin y.$$

Geometric contruction is impractical and mathematical expression is complicated:

$$\sin\theta = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\,\theta^{2n+1}$$

so calculators with programmed buttons or printed tables are used.

Sinusoids with same λ but arbitrary ϕ and A sum to a sinusoid with same λ.

This is how physical waves behave.

Sinusoids are *only* periodic functions with this property.

Notes

Wave equation

How can waves be described so their behaviour can be analysed mathematically?

“They look like sinusoids” isn't rigorous enough.

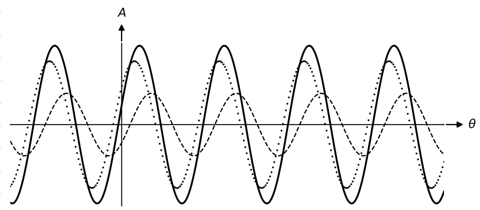
We will soon derive this constraint equation from Hooke's and Newton's Laws:

$$\frac{\partial^2 A}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = 0$$

One of many solutions can be found algebraically as,

$$\begin{aligned} A(x,t) =& R\cos(k\,x - \omega\,t) \\ &+ (1-R)\cos(k\,x + \omega\,t) \end{aligned}$$

where k and ω are constants related to (angular) wavelength and frequency and $|R| \leq 1$.



The sum of two weighted sinusoids travelling in opposite directions. Here for $R = 0.33$.

Notes

Assignment # 1: Huygens-Fresnel construction

- Write a SciPy program to make at least one plot similar to those shown the wavefront propagation slide.
- Use Huygens-Fresnel construction to determine where the wavefront should be at different times.
- Make it into a self-contained project repository in your personal account on gitlab.scss.tcd.ie.

Notes

Greek letters* often used as symbols in mathematics

α	alpha	θ	theta	o	omicron	τ	tau
β	beta	ϑ	caligr. theta	π	pi	υ	upsilon
γ	gamma	ι	iota	ϖ	caligr. pi	ϕ	phi
δ	delta	κ	kappa	ρ	rho	φ	caligr. phi
ϵ	epsilon	λ	lambda	ϱ	caligr. rho	χ	chi
ε	caligr. epsilon	μ	mu	σ	sigma	ψ	psi
ζ	zeta	ν	nu	ς	caligr. sigma	ω	omega
η	eta	ξ	xi				

Γ	big gamma	Λ	big lambda	Σ	big sigma	Ψ	big psi
Δ	big delta	Ξ	big xi	Υ	big upsilon	Ω	big omega
Θ	big theta	Π	big pi	Φ	big phi		

*With their anglophone pronunciations.

Notes