Model Order Reduction Techniques for Hyperbolic Partial Differential Equations

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Contents

Chapter 1

Introduction

- Partial differential equations describe lots of natural phenomena. Can be expensive to solve, yet sometimes this is necessary to determine outcomes for a parameter space
- for example, determining the likelihood function for a Bayesian parameter estimation problem (brief description of what this is)
- Model Order Reduction seeks to reduce the computational expense
- Reduced basis methods are one type of MOR
- Literature mostly focusses on elliptic problems
- Here we'll focus on hyperbolic problems
- First scalar- scalar wave equation eg radiation patterns
- Then vector- eg global wind patterns
- Finally tensor- gravitational waves
- Summarise results

Chapter 2

Hyperbolic PDEs and the wave equation

- classifying PDEs
- properties of the wave equation in up to 3 dimensions
- exact solutions in one and 3 dimensions

Chapter 3

Spectral Methods for Solving Hyperbolic PDEs

- spectral methods
- using Chebyshev polynomials and Spherical Harmonics
- collocation
- stability, accuracy and efficiency results for spectral methods for hyperbolic equations
- illustrative examples of 1D and 3D wave equation (3D using Spherical Harmonics)

Chapter 4

Reduced Basis Methods for Model Order Reduction

- RB methods, theory, accuracy results
- applied to examples above
- results about accuracy, stability and efficiency
- how to predict best parameters to perform high fidelity solution in order to get accurate RB solutons

Chapter 5

Other Methods?

- other MOR methods, I don't know what they are

Chapter 6

Vector valued PDEs

- Hyperbolic PDEs with vector-valued solutions (2 or 3 dimensional). Maybe something aerodynamics related- global wind patterns?
- using spin-weighted spherical harmonics

Chapter 7

Model Order Reduction for Vector-valued PDEs

- repeat the RB and other MOR work for vector valued pdes where high fidelity solution is performed using spectral methods with spin weighted spherical harmonics.
- accuracy, stability and efficiency results.

Chapter 8

2-Tensor valued PDEs

- numerical relativity with 2-tensor output (metric tensor)
- spin 2 weighted spherical harmonics
- results from GW catalogues

Chapter 9

Model Order Reduction for 2-Tensor valued PDEs

- repeat RB and MOR work again for NR results
- using GW catalogues

Chapter 10

Conclusion

- summarise results
- suggest where results could be used.