Lecture 10

Macroeconomics with imperfect coordination

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Outline

Macro Applications with Static Best Responses

Imperfect Common Knowledge and the Effects of Monetary Policy (Woodford, 03)

$$p_{i,t} = (1-\alpha)\, E_{i,t}\,[m_t] + \alpha E_{i,t}\,[p_t]$$

- Imperfect coordination as source of **nominal rigidity**
- **Inertia in price and inflation responses**

The RBC Model and Responses to Technology Shocks (Angeletos & La'O, 10)

$$y_{i,t} = (1-\alpha)\chi A_{i,t} + \alpha \mathbb{E}_{i,t}\,[y_t]$$

- Imperfect coordination as source of real rigidity
- **Negative short-run response of employment to productivity shocks** (Gali, 99)
- Inertia in output responses

Dynamic Macro Applications

- Dynamics I: **Learning** (inertia even with static best responses, as in the previous slides)
- Dynamics II: **Forward-looking behavior/best responses**

$$y_t = -\sigma \left\{ \sum_{k=0}^{+\infty} \beta^k \bar{E}_t[r_{t+k}] \right\} + (1-\beta) \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t\left[y_{t+k}\right] \right\}$$

- Q: How does the economy respond to news about the future?
  - e.g., news about future interest rates
- Imperfect intertemporal coordination and forward guidance puzzle
  - Angeletos & Lian (18, noisy/incomplete info)
  - Farhi & Werning (19, level-k)

Pause for Questions

Outline

Woodford (2003): Imperfect Common Knowledge and the Effects of Monetary Policy

- Optimal price by firm $i \in [0,1]$:

$$p_{i,t} = (1-\alpha)\, E_{i,t}[m_t] + \alpha E_{i,t}[p_t],$$

where $p_t = \int p_{i,t} di$ and $m_t = p_t + y_t$ is the exogenous nominal GDP
  - exogenous money supply (central bank) & constant velocity of money

- Δm_t follows an AR(1) process with innovations v_t:

$$\Delta m_t = \rho \Delta m_{t-1} + v_t$$

- Private signal about m_t

$$x_{i,t} = m_t + \varepsilon_{i,t}$$

Inertia in Higher-order Beliefs

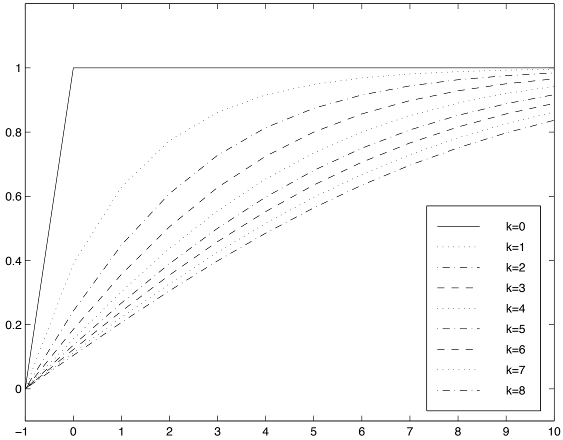
- As in the previous lecture, iterating

$$p_t = (1-\alpha)\sum_{k=1}^{\infty}\alpha^{k-1}\bar{E}_t^k[m_t]$$

- Here, beliefs will adjust over time because of learning
- But beliefs of higher order $\bar{E}_t^k[m_t]$ adjust more sluggishly
    - **with incomplete info, harder to know how much others have learned**

Inertia in Higher-order Beliefs

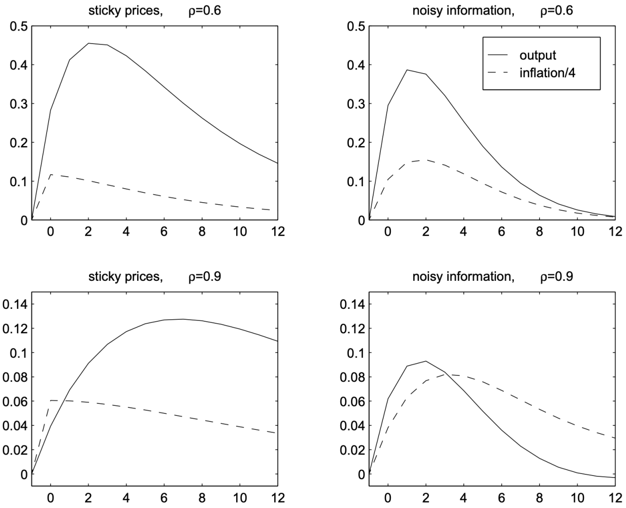
- Use $\rho = 0$ case as an example



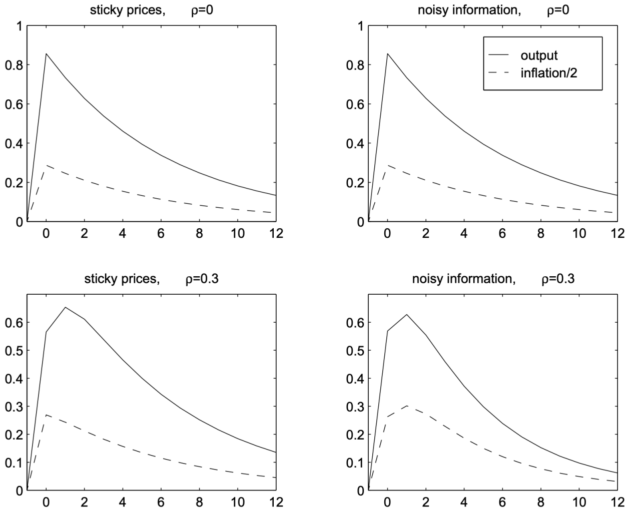
Inertia in the Price Level and Inflation

- The inertia in HOB translates to **inertia in the price level**
  - the more so the stronger the complementarity
- The price level can **adjust very slowly to the monetary shock**
  - even if every agent learns fast about the shock
- When ρ is high enough, one can get empirically desirable property of **inflation inertia**
  - "sticky inflation"
  - it is *impossible* to get this from the Calvo sticky-price

Inertia in Inflation



Inertia in Inflation



Inertia in Medium-Scale DSGE models

Quantitative NK models such as Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) generate such empirically relevant **inertia in inflation (and output)** by

- (i) adding adjustment costs of investment and habit in consumption
- (ii) replacing the standard NKPC with Hybrid NKPC with "indexing"

But micro-level empirical support of those elements controversial

Imperfect coordination with strong strategic complementarity offers a alternative

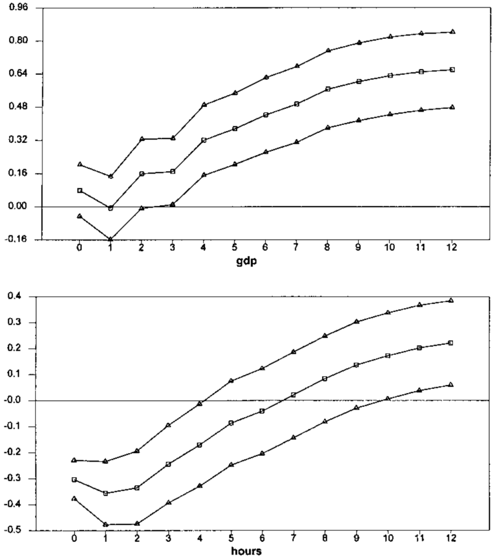
Pause for Questions

Outline

The Gali (1999) Puzzle for RBC Models

- A structural VAR method to estimate IRFs to an identified technology shock in US data
  - the technology shock as the only shock that drives labor productivity in the long run
- **Inertia in the response of output to productivity shocks**
- **Employment may actually *decrease* on impact**
  - completely opposite to RBC models
  - consistent with NK models (with contractionary monetary policy responses)
- Similar finding for Basu, Fernald, Kimball (2006)

The Gali (1999) Puzzles for RBC Models



An Alternative Flexible-Price Model Based on Imperfect Coordination

- Angeletos & La'O (2010). Noisy business cycles. NBER Macroeconomics Annual.
- Baseline RBC model (without investment) + **incomplete info about TFP shocks**
- Inertia in the response of aggregate output
- Even a negative initial response in employment

Decisions and Information

- Optimal production decisions:

$$y_{i,t} = (1-\alpha)\chi A_{i,t} + \alpha \mathbb{E}_{i,t}[y_t],$$

  where $y_t = \int y_{i,t} di$.

- Island structure:
  - knowledge of local TFP $A_{i,t} = A_t + \xi_{i,t}$ serves as a noisy private signal about aggregate TFP
  - also allows a public signal

- Solution: methods of undetermined coefficients + Kalman filter

Predictions (recall employment $n_t = \frac{1}{\theta}(y_t - A_t)$)

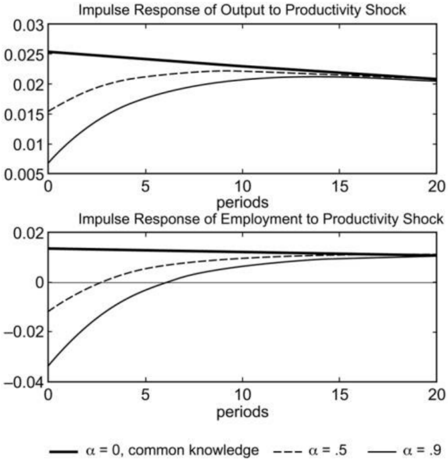


Fig. 1. Impulse responses to a positive innovation in productivity

Pause for Questions

Outline

Forward Guidance without Common Knowledge (Angeletos & Lian, 2018)

Context: A NK Economy at the ZLB

Forward guidance (FG): the central bank attempts to stimulate AD by committing to keep interest rates low after the economy exits the trap and the ZLB

Forward guidance puzzle: under FIRE, forward guidance is extremely powerful

- Explosive dynamic general-equilibrium effects (y_t and π_t depend on π_{t+k} and y_{t+k})
  - Keynesian multiplier, $\pi - y$ feedback
- Perfect dynamic coordination across **periods**

Main Findings

Key insight:

- Removing common knowledge of the FG news $\Longrightarrow$ imperfect dynamic coordination
- **Anchors expectations** of future y and π
- **Attenuates dynamic GE feedback loops**
- **Attenuation larger the longer these loops (horizon effect)**

Implications:

- Lessen forward guidance puzzle
- Offer rationale for front-loading fiscal stimuli

A More General IS Robust to Incomplete Info

- Individual rationality + individual budget constraint + aggregation:

$$c_t = y_t = -\sigma \left\{ \sum_{k=0}^{+\infty} \beta^k \bar{E}_t[r_{t+k}] \right\} + \underbrace{(1-\beta)\left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[y_{t+k}] \right\}}_{\text{Dynamic Keynesian Multiplier}}$$

- **Dynamic beauty contest** among consumers
  - follows from PIH and $c = y$
  - dynamic GE: intertemporal Keynesian income multiplier

- FIRE benchmark $E_{i,t}[\cdot] = \mathbb{E}_t[\cdot]$, where $\mathbb{E}_t[\cdot]$ is FIRE expectation

$$y_t = -\sigma \mathbb{E}_t[r_t] + \mathbb{E}_t[y_{t+1}],$$

where $r_t = i_t - \pi_{t+1}$ is the real rate between t and $t+1$.

- Why no recursive without FIRE?
  - Law of iterated expectation **do not hold** for $\bar{E}_t[\cdots]$

$$\bar{E}_t\left[\cdots \bar{E}_{t_1}\left[\cdots \bar{E}_{t_2}[\cdot]\right]\right] \neq \bar{E}_t[\cdot]$$

Question of Interest

- To develop intuition, focus on the demand block first
  - treat **real interest rate** $\{r_t\}_{t=0}^{+\infty}$ path exogenous
  - e.g., rigid price or CB directly controls real rate path
- Q: How does y_0 responds to news about $\bar{E}_0[r_T]$?
  - Isolate the effect of **frictional intertemporal coordination**
  - On top of any mechanical effect of first order informational friction

FIRE Benchmark

- FIRE benchmark:

$$E_{i,t}\left[r_{t+k}\right] = r_{t+k} \quad \text{and} \quad E_{i,t}\left[y_{t+k}\right] = y_{t+k}$$

- **Proposition**. Under FIRE,

$$\frac{\partial y_0}{\partial r_T} = \underbrace{-\sigma\beta^T}_{\text{PE}} + \underbrace{(1-\beta)\left\{\sum_{k=1}^{T}\beta^{k-1}\frac{\partial y_k}{\partial r_T}\right\}}_{\text{GE}} = -\sigma$$

  - PE effect of r_T on c_0 *decreases* with T
  - GE effect of r_T on c_0 *increases* with T
  - Total effect independent of T despite declining *PE*

Incomplete Information

- **Information Structure:**
  - noisy private signals about r_T at $t = 0$, $x_i = r_T + \varepsilon_i$
  - no learning

- **Belief anchoring:**

$$\bar{E}_t\left[r_{t+k}\right] = \lambda r_{t+k} \quad \text{and} \quad \bar{E}_t\left[y_{t+k}\right] = \lambda y_{t+k}$$

  - imperfect knowledge about **future aggregate action**

- **GE attenuation** due to imperfect intertemporal coordination:

$$\frac{\partial y_0}{\partial \bar{E}_0\left[r_T\right]} = \underbrace{-\sigma\beta^{T}}_{\text{PE}} + \underbrace{\lambda\left(1-\beta\right)\left\{\sum_{k=1}^{T}\beta^{k-1}\frac{\partial y_k}{\partial \bar{E}_0\left[r_T\right]}\right\}}_{\text{GE}}$$

Results

1. **Attenuation at any horizon**
   - $\phi_T = -\frac{dy_0}{d\bar{E}_0[r_T]}$ bounded between PE effect and CK counterpart:

$$\sigma\beta^T \;<\; \phi_T \;<\; \phi_T^* \equiv \sigma$$

   - "CK maximizes GE effect"

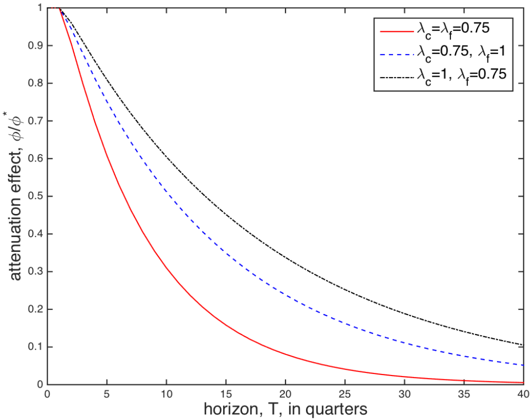
2. **Attenuation increases with the horizon**
   - ϕ_T/ϕ_T^* decreases in T
   - the distant future enters through multiple rounds of GE effects

3. **Attenuation grows without limit**
   - $\phi_T/\phi_T^* \to 0$ as $T \to \infty$ even if noise is tiny

Going Back to the Full NK model

- Demand block (IS):
  - attenuate GE feedback b/w c and y (Keynesian multiplier)
  - anchor **income expectations**
  - arrest response of c to news about future real rates
- Supply block (NKPC):
  - attenuate GE feedback from future to current π
  - anchor **inflation expectations**
  - arrest response of π to news about future marginal costs
- GE feedback b/w demand (IS) and supply (NKPC)
  - joint endogeneity of real rates and real marginal cost
  - attenuate **GE feedback between two blocks**

A Numerical Illustration (based on Gali, 2008)



- Modest info friction: $\lambda_c = \lambda_f = 0.75$ (25% prob that others failed to hear announcement)
- On **top** of any mechanical effect that first order informational friction

Pause for Questions

Outline

Farhi & Werning (2019)

- As illustrated in the static case in Lecture 1
  - incomplete information and level-k thinking both capture imperfect coordination
  - generate similar predictions for the complementarity case
- Here: how to apply level-k thinking in dynamic NK models (dynamic complementarity)
  - similar predictions regarding the impact of forward guidance

Back to the Demand Block in the NK Model

$$y_t = -\sigma \left\{ \sum_{l=0}^{+\infty} \beta^l \bar{E}_t \left[r_{t+l} \right] \right\} + (1-\beta) \left\{ \sum_{l=1}^{+\infty} \beta^{l-1} \bar{E}_t \left[y_{t+l} \right] \right\}$$

- Follow Farhi and Werning (2019)
  - treat **real interest rate** path exogenous
  - e.g. rigid price or CB directly controls real rate path

- Level-0 outcomes (no shock, steady state outcomes)

$$y_t^0 = 0$$

Level-1 Outcomes

- Level-1 outcomes (expect all **future** endogenous outcomes are at level 0)

$$
\begin{aligned}
y_t^1 &= -\sigma \sum_{l=0}^{+\infty} \beta^l r_{t+l} + (1-\beta) \left\{ \sum_{l=1}^{+\infty} \beta^{l-1} y_{t+l}^0 \right\} \\
&= -\sigma \sum_{l=0}^{+\infty} \beta^l r_{t+l}
\end{aligned}
$$

- Captures PE effects of interest rate changes

Level-k Outcomes

- Level-k outcomes (expect all **future** endogenous outcomes are at level $k-1$)

$$y_t^k = -\sigma \sum_{l=0}^{+\infty} \beta^l r_{t+l} + (1-\beta)\left\{\sum_{l=1}^{+\infty} \beta^{l-1} y_{t+l}^{k-1}\right\}$$

- Define ϕ_T^k : macro impact of forward guidance at level-k

$$\phi_T^k = -\frac{\partial y_t^k}{\partial r_{t+T}}$$

Results

- Attenuation for any level and any horizon:

$$\underbrace{\phi_T^1}_{\text{PE only}} < \phi_T^k < \underbrace{\phi_T^*}_{\text{Frictionless}},$$

where $\phi_T^* = \lim_{k\to+\infty} \phi_T^k = \sigma$.

- **Attenuation increases with the horizon**

$$\phi_T^k / \phi_T^* \quad \text{decreases} \quad \text{in} \quad T$$

- **Attenuation decreases with the depth of reasoning**

$$\phi_T^k / \phi_T^* \quad \text{increases} \quad \text{in} \quad k$$

Pause for Questions

Outline

Gabaix (20): Cognitive Discounting

A more "reduced-form" method to model "anchored" forward looking expectations (**"cognitive discounting"**)

$$E_{i,t}\left[\mathrm{X_{t+k}}\right] = \bar{m}^{k} E_{t}\left[\mathrm{X_{t+k}}\right], \tag{1}$$

no matter whether X_{t+k} is an exogenous or endogenous aggregate state.

- Extremely tractable and generalizable
- Sharp and empirically relevant predictions
- But micro-foundation delicate
- "The main phenomenon I want to capture is that the world is not fully understood by the agent, especially events that are far into the future."
    - expectations anchored towards the prior (0, i.e. the non-shock SS)

Optimal Consumption

- Individual optimal consumption:

$$
\begin{aligned}
c_{i,t} = -\sigma \left\{ \sum_{k=0}^{+\infty} \beta^{k+1} E_{i,t}\left[r_{t+k}\left(\mathrm{X}_{\mathrm{t+k}}\right)\right] \right\} \\
(1-\beta) \left\{ a_{i,t} + \sum_{k=0}^{+\infty} \beta^{k} E_{i,t}\left[y_{i,t+k}\left(\mathrm{X}_{\mathrm{t+k}}\right)\right] \right\}
\end{aligned}
$$

- Apply cognitive discounting in (1) and with linearity

$$
\begin{aligned}
E_{i,t}\left[r_{t+k}\left(\mathrm{X}_{\mathrm{t+k}}\right)\right] &= \bar{m}^{k} E_{t}\left[r_{t+k}\left(\mathrm{X}_{\mathrm{t+k}}\right)\right], \\
E_{i,t}\left[y_{i,t+k}\left(\mathrm{X}_{\mathrm{t+k}}\right)\right] &= \bar{m}^{k} E_{t}\left[y_{t+k}\left(\mathrm{X}_{\mathrm{t+k}}\right)\right]
\end{aligned}
$$

A Behavioral IS Curve

- Aggregate and using market clearing $y_t = c_t$ and $a_t = 0$

$$y_t = -\sigma \left\{ \sum_{k=0}^{+\infty} \beta^k \bar{m}^k E_t[r_{t+k}] \right\} + \frac{1-\beta}{\beta} \left\{ \sum_{k=1}^{+\infty} \beta^k \bar{m}^k E_t\left[y_{t+k}\right] \right\}$$

- Recursively, **a discounted aggregate Euler equation**

$$\begin{aligned} y_t &= -\sigma E_t[r_t] + \bar{m} E_t\left[y_{t+1}\right] \\ &= -\sigma \sum_{k=0}^{+\infty} \bar{m}^k E_t[r_{t+k}], \end{aligned}$$

where $M = \bar{m}$.

- Can directly see that the impact of forward guidance attenuated

A Behavioral NKPC

- Optimal reset price

$$p_{j,t}^{*} = p_t + (1-\beta\theta)\left\{\sum_{k=0}^{+\infty}(\beta\theta)^k E_{j,t}\left[mc_{t+k}\left(\mathrm{X_{t+k}}\right) + \pi_{t,t+k}\left(\mathrm{X_{t+k}}\right)\right]\right\},$$

  where $\pi_{t,t+k} = p_{t+k} - p_t$ is the inflation between t and $t+k$
- Cognitive discounting

$$E_{j,t}\left[mc_{t+k}\left(\mathrm{X_{t+k}}\right)\right] = \bar{m}^k E_t\left[mc_{t+k}\left(\mathrm{X_{t+k}}\right)\right]$$
$$E_{j,t}\left[\pi_{t,t+k}\left(\mathrm{X_{t+k}}\right)\right] = \bar{m}^k E_t\left[\pi_{t,t+k}\left(\mathrm{X_{t+k}}\right)\right].$$

- Important/sensitive to put cognitive discounting on $\pi_{t,t+k}$
  - but note that $\pi_{t,t+k}$ should depend on past aggregate states at $t+1,\cdots,t+k$ too
- Aggregate

$$p_t^{*} = p_t + (1-\beta\theta)\left\{\sum_{k=0}^{+\infty}(\beta\theta\bar{m})^k E_t\left[mc_{t+k} + p_{t+k} - p_t\right]\right\}$$

A Behavioral NKPC

- Note that inflation $\pi_t = (1-\theta)(p_t^* - p_{t-1})$

$$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} \left\{ \sum_{k=0}^{+\infty} (\beta\theta\bar{m})^k E_t[mc_{t+k}] + \sum_{k=1}^{+\infty} \frac{(\beta\theta\bar{m})^k}{1-\beta\theta\bar{m}} E_t[\pi_{t+k}] \right\}$$

- As a result, (note mc_t is linear in y_t)

$$\pi_t = \kappa y_t + \beta M^f E_t[\pi_{t+1}],$$

where

$$M^f = \bar{m}\left(\frac{(1-\theta)(1-\beta\theta)}{(1-\beta\theta\bar{m})} + \theta\right) \in [0,1].$$

Forward Guidance is Less Powerful

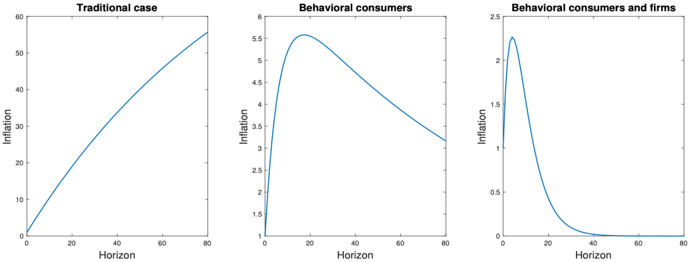


Figure 2. This Figure shows the response of current inflation to forward guidance about a one-period interest rate cut in T quarters, compared to an immediate rate change of the same magnitude. Left panel: traditional New Keynesian model. Middle panel: model with behavioral consumers and rational firms. Right panel: model with behavioral consumers and firms. Parameters are the same in both models, except for the attention parameters M, M^f which are equal to 1 in the rational model.

Zero Lower Bound is Less Costly

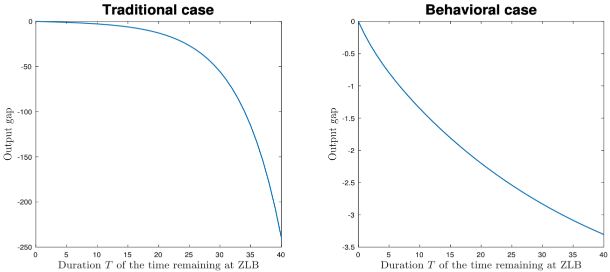


Figure 1. This figure shows the output gap $x_0(T)$ at time 0, given that the economy will be at the ZLB for T more periods. The left panel is the traditional New Keynesian model, the right panel the present behavioral model. Parameters are the same in both models, except for the attention parameters M, M^f which are equal to 1 in the rational model. The natural rate at the ZLB is -1%. Output gap units are percentage point. Time units are quarters.

Implications for Equilibrium Determinacy

- **Prop.** There is a unique non-explosive equilibrium if and only if

$$\phi_\pi + \frac{1-\beta M^f}{\kappa}\phi_y + \frac{\left(1-\beta M^f\right)(1-M)}{\kappa\sigma} > 1$$

- Easier to satisfy than the frictionless case ($M^f = M = 1$)

- If $\frac{\left(1-\beta M^f\right)(1-M)}{\kappa\sigma} > 1$, unique equilibrium even with passive monetary policy ($\phi_\pi = \phi_y = 0$)

Ricardian Equivalence no Longer Holds

- Introducing public debt, the IS equation becomes

$$y_t = ME_t[y_{t+1}] + b_d d_t - \sigma E_t[r_{t+1}]$$

- Spending depends positively on public debt
- Because of cognitive discounting, agents only partially incorporate the associated future taxes because of the deficit

Pause for Questions

Outline

Angeletos & Huo (2021): Myopia and Anchoring

- Two key insights from imperfect strategic interactions
  - "inertia"/"anchoring" in the responses of aggregate outcomes to current shocks
  - "discounting"/"myopia" in the responses of aggregate outcomes to news about future
- Summarize these two insights in a succinct framework (with dynamic best responses)
  - using noisy/incomplete-info as modeling devices
  - representative agent

$$a_t = \varphi \xi_t + \delta E_t [a_{t+1}]$$

  - imperfect strategic interactions, as if

$$a_t = \varphi \xi_t + \omega_f \delta E_t [a_{t+1}] + \omega_b a_{t-1}$$

Framework

- Optimal individual action:

$$a_{i,t} = E_{i,t}\left[\varphi \xi_t + \beta a_{i,t+1} + \gamma a_{t+1}\right]$$

- Aggregate outcome:

$$a_t = \bar{E}_t\left[\varphi \sum_{k\geq 0} \beta^k \xi_{t+k}\right] + \gamma \bar{E}_t\left[\varphi \sum_{k\geq 0} \beta^k a_{t+k+1}\right]$$

  - ξ_t : fundamental
  - $\gamma > 0$: regulates the degree of strategic complementarity/size of GE effects

Framework

- Aggregate outcome:

$$a_t = \bar{E}_t \left[\varphi \sum_{k \geq 0} \beta^k \xi_{t+k} \right] + \gamma \bar{E}_t \left[\varphi \sum_{k \geq 0} \beta^k a_{t+k+1} \right]$$

- Example: dynamic IS in NK models

$$y_t = -\sigma \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t \left[r_{t+k} \right] \right\} + (1-\beta) \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t \left[y_{t+k} \right] \right\}$$

- Example: dynamic NKPC ($1-\theta$ is the prob. of changing prices)

$$\pi_t = \varkappa \left\{ mc_t + \sum_{k=1}^{+\infty} (\beta\theta)^k \bar{E}_t \left[mc_{t+k} \right] \right\} + \tfrac{1-\theta}{\theta} \left\{ \sum_{k=1}^{+\infty} (\beta\theta)^k \bar{E}_t \left[\pi_{t+k} \right] \right\},$$

Information

- Fundamentals follow AR(1):
$$\xi_t = \rho \xi_{t-1} + \eta_t$$
- Information (noisy private signals):
$$x_{i,t} = \xi_t + u_{i,t}$$
- Differences from Woodford (03)
  - before: static best responses
  - here: dynamic best responses
- Differences from Angeletos & Lian (18)
  - before: one-time private signals about one-time news
  - here: recurring shocks with private signals each period
- Not easy to handle using the standard method of undetermined coefficients
- Technical contribution: use frequency-domain, which becomes easy to handle

Results

- Complete info $\Rightarrow$ representative agent ($\delta = \beta + \gamma$)

$$a_t = \varphi \xi_t + \delta E_t [a_{t+1}],$$

- Incomplete info, as if a representative agent model with

$$a_t = \varphi \xi_t + \omega_f \delta E_t [a_{t+1}] + \omega_b a_{t-1},$$

  where $\omega_f < 1$ and $\omega_b > 0$.

- "Anchoring" ($\omega_b > 0$) in the responses of aggregate outcomes
  - Woodford (03), Angeletos & La'O (10)

- "Myopia" ($\omega_f < 1$) in the responses of aggregate outcomes
  - Angeletos & Lian (18), Werning and Farhi (19)

- Both come from slow adjustments of HOBs

- Both distortions intensify ($\omega_f \downarrow$, $\omega_b \uparrow$) with stronger complementarity/GE γ

Results

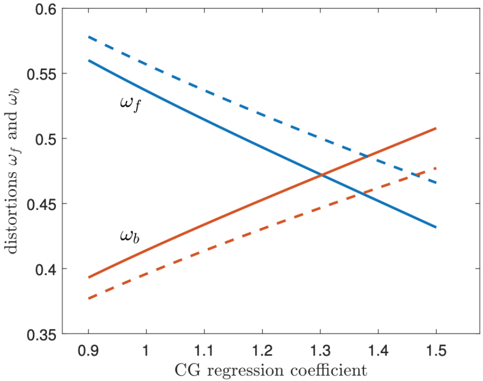


FIGURE 1. MYOPIA AND ANCHORING

Note: The distortions as functions of the proxy offered in Coibion and Gorodnichenko (2015). The solid lines correspond to a stronger degree of strategic complementarity, or GE feedback, than the dashed one.

Testing the Theory

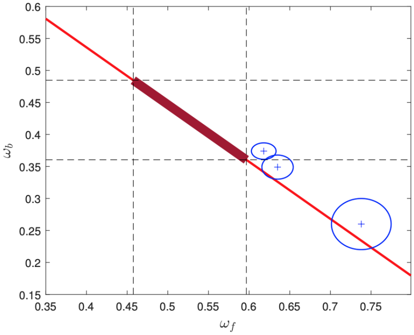


FIGURE 2. TESTING THE THEORY

Note: The straight line represents the relation between ω_f and ω_b implied by the theory. Raising the level of noise maps to moving northwest along this line. The darker, thicker segment of this line corresponds to the confidence interval of K_{CG}, the relevant moment of the inflation forecasts, as reported in column (1) of Table 1 Coibion and Gorodnichenko (2015). The three crosses represent the three estimates of the pair (ω_f, ω_b) provided in Table 1 of Galí, Gertler and Lopez-Salido (2005), and the surrounding disks give the corresponding confidence regions.

Applications

- Substitute for several different ad hoc sources of sluggishness
  - habit in consumption
  - adjustment cost to investment (IAC)
  - hybrid NKPC and indexation
- But distortions endogenous to the environment

Pause for Questions