Lecture 9

Dampening general equilibrium: models of imperfect coordination

Chen Lian[1]

[1]UC Berkeley and NBER

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Outline

GE and Game Theory

- General equilibrium (GE) effects define the field of macro
- **GE effects**: the impact of others' decisions on an agent's decision
  - **strategic interactions** in the game theory language
- Complementary interactions
  - Keynesian multiplier (income-spending feedback)
  - Knowledge spillover
  - Currency attack, debt run, etc.
- Substitutable interactions
  - Competing for limited resources
  - RBC and real interest rate adjustments (Barro-King)

Perfect Coordination Embedded in FIRE

Implicit assumption in FIRE:

- Common knowledge about everyone's current information/belief
- Common knowledge about everyone's current action

They imply:

$\Longrightarrow$ **Perfect coordination across consumers and firms**

$\Longrightarrow$ General equilibrium effects are "maximized"

Moreover, perfect dynamic coordination **across periods**

$\Longrightarrow$ Law of iterated expectations hold for average expectations

$\Longrightarrow$ Perfectly know how future agents respond to current shocks

$\Longrightarrow$ Unintuitive puzzles (e.g., forward guidance)

Roadmap

- This lecture: **tools** to model imperfect coordinations
  - Noisy/incomplete info as a model of imperfect coordination
  - Level-k thinking
  - How does it translate into GE dampening
- Next lecture: **Macro applications:**
  - RBC responses to TFP shocks
  - Inertia in inflation
  - NK and forward guidance puzzles

Pause for Questions

Outline

PE and GE in a Nutshell

- A continuum of consumers $i \in [0,1]$ with optimal spending

$$c_i = \theta_i + \alpha E_i[c], \tag{1}$$

  - θ_i : individual-specific fundamental θ_i
  - $E_i[c]$: expectation of the aggregate spending (**Keynesian multiplier**)
  - $\alpha \in (0,1)$ strategic complements; GE amplifies PE
  - $\alpha \in (-1,0)$ strategic substitutes; GE attenuates PE
  - Equivalent to the best response in a beauty contest game (Morris-Shin, 2002)

- The aggregate counterpart

$$c = \underbrace{\theta}_{\text{PE}} + \underbrace{\alpha \bar{E}[c]}_{\text{GE}}. \tag{2}$$

where $\theta = \int \theta_i di$ and $\bar{E}[c] = \int E_i[c]\, di$.

A NK Micro-foundation (Angeletos & Lian, 2022, HB)

- Optimal consumption of any consumer i (permanent income hypothesis)

$$c_{i,t} = (1-\beta)a_{i,t} - \beta\sigma\left\{\sum_{k=0}^{+\infty}\beta^k E_{i,t}\left[i_{t+k} - \pi_{t+k+1}\right]\right\} + (1-\beta)\left\{\sum_{k=0}^{+\infty}\beta^k E_{i,t}\left[y_{t+k}\right]\right\} + \sigma\beta\rho_{i,t}$$

- NKPC

$$\pi_t = \kappa c_t + \psi_{-1}\pi_{t-1} + \psi_{+1}\mathbb{E}_t\left[\pi_{t+1}\right]$$

- Monetary policy
  - Replicates flexible-price outcomes for all $t \geq 1$ ($c_t = 0$ for all $t \geq 1$)
  - Taylor rule at $t = 0$:

$$i_0 = \phi_c c_0 + \phi_\pi \pi_0. \tag{3}$$

The FIRE Benchmark Info Case

- Substitute NKPC + MP into optimal consumption

$$c_{i,0} = \left(1 - \beta - \beta\sigma\left(\phi_c + \frac{\kappa}{1-\psi_{+1}\chi}\left(\phi_\pi - \chi\right)\right)\right) E_{i,0}\left[c_0\right] + \sigma\beta\rho_{i,0}, \tag{4}$$

where $\chi \equiv \frac{1-\sqrt{1-4\psi_{+1}\psi_{-1}}}{2\psi_{+1}} \in (0,1)$.

- This is readily nested in (1) with

$$\theta_i \equiv \sigma\beta\rho_{i,0} \qquad \text{and} \qquad c_i \equiv c_{i,0},$$

$$\alpha \equiv \underbrace{1-\beta}_{\text{Keynesian cross}} + \underbrace{\kappa\frac{\beta\sigma\chi}{1-\psi_{+1}\chi}}_{\text{inflation-spending spiral}} \underbrace{-\beta\sigma\left(\phi_c + \frac{\kappa}{1-\psi_{+1}\chi}\phi_\pi\right)}_{\text{monetary policy}}.$$

The FIRE Benchmark Case

- The FIRE Benchmark (common knowledge of θ):
  $\Longrightarrow$ common knowledge about actions & **perfect coordination**

$$E_i[c] = \mathbb{E}[c] = c, \tag{5}$$

- Equilibrium output:

$$c = \underbrace{\theta}_{\text{PE}} + \underbrace{\frac{\alpha}{1-\alpha}\theta}_{\text{GE}} = \underbrace{\frac{1}{1-\alpha}}_{\text{GE multiplier}} \theta. \tag{6}$$

Pause for Questions

Outline

Overview: What do Incomplete Information Imply?

- Noisy private signals: $\theta \sim \mathcal{N}\left(0, \sigma_\theta^2\right)$ and $\varepsilon_i \overset{i.i.d.}{\sim} \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$
- Imperfect knowledge about the **fundamental** ("first-order uncertainty")
- Imperfect knowledge about others' **information/signals** ("higher-order uncertainty")
- Imperfect knowledge about others' equilibrium **actions**
- Capture **frictions in coordination**

The Incomplete Information Case

Fundamental and information:

- Nature draws θ from $\mathcal{N}\left(0, \sigma_\theta^2\right)$.
- Let s_i be a sufficient statistic of the agent's information about θ (and others' information about θ)

$$s_i = \theta + \varepsilon_i, \tag{7}$$

  where $\varepsilon_i \sim \mathcal{N}\left(0, \sigma^2\right)$ is orthogonal to θ and i.i.d. across i.
  - This embeds the information about θ contained in θ_i

Solution concept: Noisy REE (Lucas 72; Grossman-Stiglitz 80)

- Each consumer's decision is given by (1) based on info (7)
- The decision rule and the info structure is common knowledge

Solution method: guess and verify the equilibrium $c = \mu\theta$

- "methods of undetermined coefficients"

Belief Anchoring and Imperfect Coordination

Lemma. In any equilibrium, the average expectation satisfies

$$\bar{E}[\theta] = \lambda\theta \quad \text{and} \quad \bar{E}[c] = \lambda c,$$

where $\lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} \in [0,1]$.

$\Longrightarrow$ imperfect knowledge about others' information

$\Longrightarrow$ **imperfect knowledge about others' actions**

$\Longrightarrow$ imperfect coordination

Dampening General Equilibrium

Proposition. There is a unique equilibrium such that

$$c = \underbrace{\theta}_{\text{PE}} + \underbrace{\frac{\alpha\lambda}{1-\alpha\lambda}\theta}_{\text{GE}} = \underbrace{\frac{1}{1-\alpha\lambda}}_{\text{GE multiplier}} \theta, \tag{8}$$

- Equivalent to a "twin" FIRE economy where the GE parameter is $\lambda\alpha$.
- No matter $\alpha < 0$ or $\alpha > 0$, the **absolute size** of the GE effect is **reduced**
- When the GE feedback is positive ($\alpha > 0$), c **under-reacts** to θ relative to FIRE
- When the GE feedback is negative ($\alpha < 0$), c **over-reacts** to θ relative to FIRE

Through the Lens of Higher-Order Beliefs (HOBs)

- Iterating:

$$c = \theta + \alpha \bar{E}[c]$$
$$= \sum_{h=1}^{\infty} \alpha^{h-1} \bar{E}^h[\theta],$$

where $\bar{E}^h[\theta] = \int E_i\left[\bar{E}^{h-1}[\theta]\right] di$ capture **higher-order beliefs (HOBs)**

  - beliefs about other agents' beliefs about other agents' beliefs $\cdots$
  - holds no matter how beliefs are formed (noisy info, level-k thinking, sticky info)

- Incomplete info anchors HOBs

$$\bar{E}^h[\theta] = \lambda^h \theta + (1-\lambda^h)\mu_\theta$$

  - comes from imperfect knowledge about others' information with $\mu_\theta = 0$.

- Translates into **anchoring of beliefs about others' actions**

$$\bar{E}[c] = \lambda c$$

Pause for Questions

Public Signals: Morris & Shin (02)

- A common information structure: private + public signals

$$s_i = \theta + \varepsilon_i \quad \text{and} \quad y = \theta + \eta,$$

where $\theta \sim \mathcal{N}\left(0, \sigma_\theta^2\right)$, $\varepsilon_i \overset{i.i.d.}{\sim} \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$, and $\eta \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$ are independent

- **Public signals** capture public news/statistics
  - "coordination device" when private signals are noisy
- Morris & Shin (02): a more precise public signal is not necessarily welfare improving
- Caveat: are public signals really common knowledge?
  - most households inattentive to Fed announcement

Public Signals in Morris & Shin (2002)

- Morris & Shin (02): $\sigma_\theta^2 = +\infty$, uninformed prior
- First-order beliefs

$$\bar{E}[\theta] = \lambda\theta + (1-\lambda)y,$$

where now $\lambda = \frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_\varepsilon^2} \in [0,1]$.

- Higher-order beliefs

$$\bar{E}^h[\theta] = \lambda^h\theta + \left(1-\lambda^h\right)y$$

  - "coordination" on the commonly known public signal
  - essentially plays the role of common prior

- Endogenous actions

$$c = \frac{(1-\alpha)\lambda}{1-\alpha\lambda}\theta + \frac{1-\lambda}{1-\alpha\lambda}y$$
$$\bar{E}[c] = \frac{(1-\alpha)\lambda^2}{1-\alpha\lambda}\theta + (1+(1-\alpha)\lambda)\frac{1-\lambda}{1-\alpha\lambda}y$$

Welfare in Morris & Shin (2002)

- Individual utility:

$$u_i(\vec{c}, \theta) = -(1-\alpha)(c_i - \theta)^2 - \alpha(L_i - L),$$

where $L_i \equiv \int_0^1 (c_j - c_i)^2 dj$ and $L = \int_0^1 L_j dj$.

- Individual response as above:

$$c_i = (1-\alpha) E_i[\theta] + \alpha E_i[c]$$

- Social welfare:

$$W(\vec{c}, \theta) = \frac{1}{1-\alpha} \int_0^1 u_i(\vec{c}, \theta)\, di = -\int_0^1 (c_i - \theta)^2\, di$$

A More Precise Public Signal is not Necessarily Welfare Improving

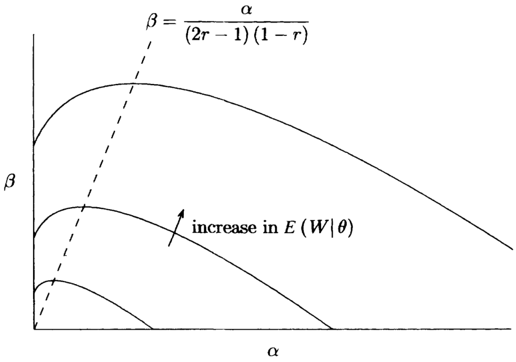


FIGURE 1. SOCIAL WELFARE CONTOURS

- In there notation $\alpha = \sigma_\eta^{-2}$ is the precision of public signal while $\beta = \sigma_\varepsilon^{-2}$ is the precision of private signals

Thoughts

- One way or another, it is **private signals** “driving” the imperfect strategic interactions
- Public signals serve as a coordination device
- But for a model of imperfect strategic interaction
  - **private signals** are essential
  - public signals are not essential
- Noises in private signals can be more broadly interpreted as
  - idiosyncratic inattention
  - idiosyncratic perception errors
  - etc....

Pause for Questions

Outline

Keynes on Beauty Contests

- Professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, **the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole**

- It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest.

- We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.

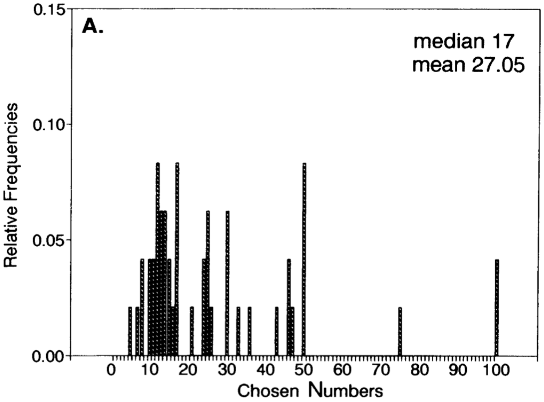
Nagel (1995): Unraveling in Guessing Games: An Experimental Study.

- **Guessing game** with the broad features of Keynes' beauty contest.
- A large number of players state simultaneously a number in $[0, 100]$
- The winner is the person whose chosen number is the closest to the mean of all chosen numbers multiplied by a commonly known parameter, p.
- For $0 \leq p < 1$, there is **one Nash equilibrium: all announce zero.**
  - Arrived at through iterative dominance.

Experimental Design

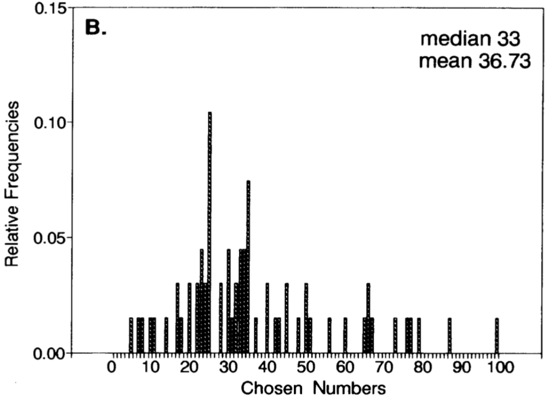
- 15-18 subjects seated far apart in large classroom (no communication).
- Same group played same game for 4 periods (no surprises) in one session.
- The number closest to optimal number announced and resulting payoff announced.
- Winner received around $13.

First Period Choices ($p = 1/2$)



$p = 1/2$

First Period Choices ($p = 2/3$)



$p = 2/3$

Level-k Thinking

- Behavior deviates strongly from the Nash Equilibrium
- **Anchored, naive iterated best response with learning presents a possible rationalization for the data.**
  - Stahl and Wilson (94,95); Nagel (95)
- "Level-0" (naive) player chooses actions without regard to the actions of other players

$$x^0 \sim U[0,100] \quad \text{or} \quad x^0 = 50$$

- "Level-1" player believes the population consists of all "Level-0" types.

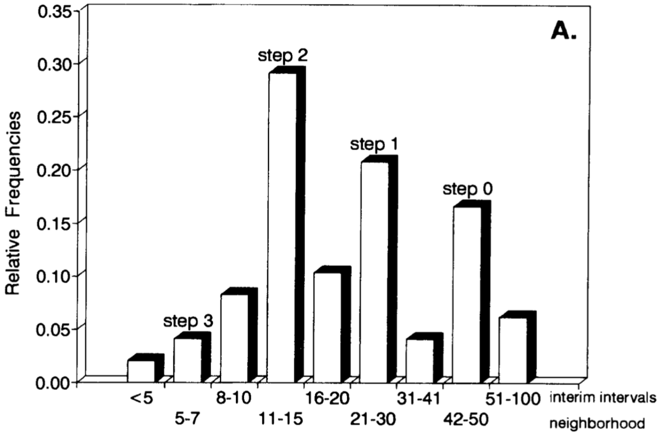
$$x^1 = px^0 = 50p$$

- "Level-2" player believes the population consists of all "Level-1" types.

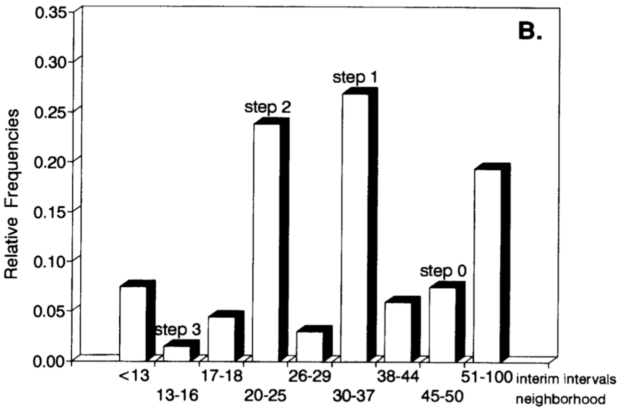
$$x^2 = px^1 = 50p^2$$

- ......

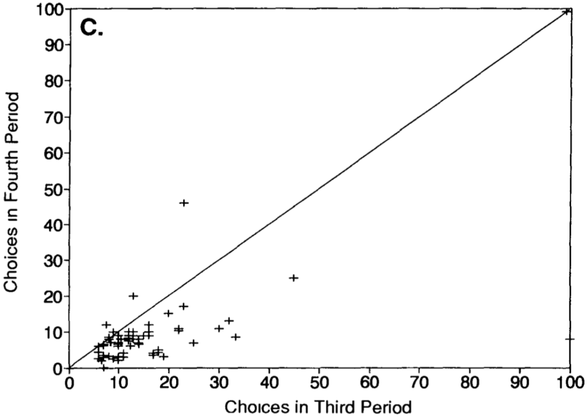
Level-k Thinking and Experimental Results ($p = 1/2$)



Level-k Thinking and Experimental Results ($p = 2/3$)



Learning ($p = 2/3$, Round 4)



3 to 4

Pause for Questions

Level-k Thinking in the Simple Beauty Contest

$$c^k = \theta + \alpha E^k[c] = \theta + \alpha c^{k-1}$$

- "Level-0" (naive) player:

$$c^0 \sim U[-\infty, \infty] \quad \text{or} \quad c^0 = 0$$

- "Level-1" player believes the population consists of all "Level-0" types.

$$E^1[\theta] = \theta, \quad E^1[c] = c^0, \text{ and } \quad c^1 = \theta$$

- "Level-2" player believes the population consists of all "Level-1" types.

$$E^2[\theta] = \theta, \quad E^2[c] = c^1, \text{ and } \quad c^2 = (1+\alpha)\theta$$

- "Level-k" player

$$c^k = \frac{1-\alpha^k}{1-\alpha}\theta$$

General Equilibrium with Level-k Thinking

$$c = \underbrace{\theta}_{\text{PE}} + \underbrace{\frac{\alpha - \alpha^k}{1-\alpha}\theta}_{\text{GE}} = \underbrace{\frac{1-\alpha^k}{1-\alpha}}_{\text{GE multiplier}} \theta.$$

Let GE^k denote the GE effect with level-k and GE^{FIRE} its FIRE counterpart

- When $\alpha > 0$, $|GE^k|$ is strictly increasing in k and bounded from above by $|GE^{\text{FIRE}}|$.
- When instead $\alpha < 0$, the above statement holds **only for k odd**. For k even, the opposite is true: $|GE^k|$ is strictly decreasing in k and bounded from below by $|GE^{\text{FIRE}}|$.

General Equilibrium with Level-k Thinking

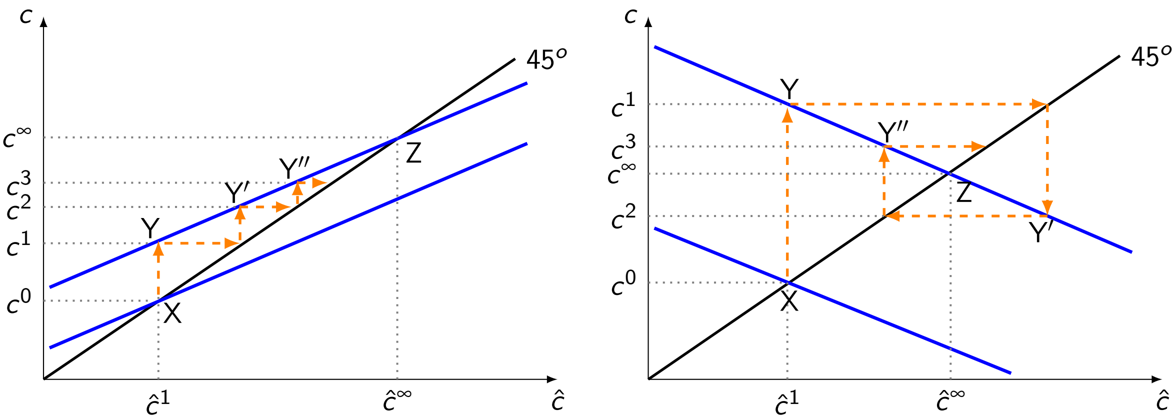


Figure: Level-k Thinking

Pause for Questions

Cognitive Hierarchy (Camerer et al., 04)

- A variant (improvement?) of level-k thinking

Cognitive hierarchy:

- "Level-k" players best-respond, assuming that other players **are distributed over level 0 through level $k-1$.**

The original level-k thinking:

- "Level-k" players best-respond, assuming that other players **are all level k-1.**

Cognitive Hierarchy (Camerer et al., 04)

- Actual distribution of types: Poisson (one free parameter τ)

$$f(k) = e^{-\tau}\tau^k / k!$$

  - $\tau = 1.5$ is a good calibration

- Level-k player's belief about the proportion of level-h player

$$g_k(h) = f(h) / \sum_{l=0}^{k-1} f(l) \quad \forall h \in \{0, \cdots, k-1\}$$

- Better empirical fits across different types of strategic games

- Can also avoid the "oscillating" property discussed before

Reflective Equilibrium (García-Schmidt & Woodford, 2019)

- Depth of thinking k is now treated as a **continuous** variable in $(0, \infty)$
- Consumption under reflective equilibrium

$$c(k) = \theta + \alpha \hat{c}(k). \tag{9}$$

And the conjecture is given by as the solution to the following ODE:

$$\frac{d\hat{c}(h)}{dh} = c(h) - \hat{c}(h) \quad \forall h \in [0, k] \tag{10}$$

with the initial condition $\hat{c}(0) = 0$.

General Equilibrium under Reflective Equilibrium

$$c = \underbrace{\theta}_{\text{PE}} + \underbrace{\frac{\delta(k,\alpha)\alpha}{1-\delta(k,\alpha)\alpha}\theta}_{\text{GE}} = \underbrace{\frac{1}{1-\delta(k,\alpha)\alpha}}_{\text{GE multiplier}}\theta,$$

where $\delta : [0,+\infty) \times (-1,1) \to [0,1)$. Regardless of the sign of α.

- $\delta(k,\alpha)$ is strictly increasing in k
- Starting from 0 at $k = 0$ and converging to 1 as $k \to \infty$

Pause for Questions

Outline

Market Signals

- Above: exogenous information structure
  - Exogenous private signals about the fundamentals
- But endogenous market prices can be informative about the agg. fundamentals and actions
  - facilitate coordination?
- Grossman & Stiglitz (80) on the information role of prices
  - "Unravels" private noisy signals
- **Cursed equilibrium** (Eyster, Rabin, and Vayanos, 19)
  - Neglect the Informational Content of Prices
  - Maintain imperfect coordination

Grossman & Stiglitz (80)

- There are two periods, 0 and 1.
- There is one risky asset that pays $\theta \sim \mathcal{N}\left(\mu, \sigma_\theta^2\right)$ in period 1.
- The risky asset is traded in period 0 at a price p. The supply of the risky asset is $S > 0$.
- The risk-less rate between periods 0 and 1 is r.
- There are N agents with CARA utility

$$U(w) = -e^{-\alpha W}$$

- Each agent i observes a private signal ($\varepsilon_i \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$) and the price p

$$s_i = \theta + \varepsilon_i.$$

Market Signals

The noisy REE:

- A price function $p\left(s^1, \cdots, s^N\right)$ and a demand vector $x^1(p), \cdots, x^N(p)$
- Each agent i's demand $x^i(p)$ solves

$$\max_x E\left[U\left(\left(W_0 - xp\right)(1+r) + x\theta\right) | s_i, p\right]$$

- Market clearing

$$\sum_{i=1}^{N} x^i(p) = S.$$

Proposition: when $N \to +\infty$, the noisy REE is the same as the full-info eq

- Price, as a function of θ, fully reveals θ

Avoiding the Full Revelation of the Information

Noisy traders:

- Stochastic supply of asset $S+u$, where $u \sim \mathcal{N}\left(0, \sigma_u^2\right)$
- Interpretation: "noisy traders" trading for liquidity motives
- Equilibrium price, $p = A + B\theta + Cu$, will not fully reveal θ
- Solution technique: methods of undetermined coefficients

Cursed equilibrium: (Eyster & Rabin, 05; Eyster et al., 19)

- Underestimate the information content of prices

An Example Based on Akerlof (1970)

General concept of **cursed equilibrium**:

- Players underestimate the correlation between the other players' actions and their private information

Example:

- A buyer might purchase a car from a seller at a predetermined price of $\$1,000$
- With 50% prob, the car is a lemon, worth \$0 to both seller and buyer.
- With 50% prob, the car is a peach, worth \$3000 to buyer and \$2000 to seller.

Seller's optimality: only lemon gets sold

- No incentives for the seller of a peach to pool

Rational buyer: does not buy, market unravels

An Example Based on Akerlof (1970)

- A **χ-cursed buyer** believes that, with prob. χ,
  - seller sells with prob. 50% **irrespective of the type of car**
  - neglect the information content of "selling"
- She believes the car being sold is a peach with prob

$$0.5\chi + 0(1-\chi) = 0.5\chi$$

and worth

$$0.5\chi \cdot 3000 = 1500\chi$$

- A buyer with $\chi > 2/3$ will buy the car

Pause for Questions

Cursed Equilibrium Meets Grossman & Stiglitz

- Eyster, E., Rabin, M., & Vayanos, D. (2019). Financial markets where traders neglect the informational content of prices. The Journal of Finance.

- Grossman-Stiglitz setting with χ_i-cursed buyer

$$\max_x E\left[U\left((W_0 - xp)(1+r) + x\theta\right) | s_i, p\right]^{1-\chi_i} \times E\left[U\left((W_0 - xp)(1+r) + x\theta\right) | s_i\right]^{\chi_i}$$

  - neglect informational content of prices
  - no need for noise traders

- Not surprising, price under-reacts to the fundamental θ

Key Predictions: Excessive Trading Volume

- More interesting, about trading volume

$$V \equiv \sum_{i=1}^{N} |x_i|$$

- Aggregate volume approaching infinity as the number of traders grows large

  $\Longrightarrow$ neglect the informational content of prices

  $\Longrightarrow$ "more" reliance on private signals

  $\Longrightarrow$ "more" trade

- Per-trader volume increases with the number of traders
  - the discrepancy between each private signal and the average of all signals increases with the number of traders

Comparisons with Other Models of Volume

Overconfidence:

- Odean (1998); Scheinkman and Xiong (2003)
- Exaggerating the precision of their own private signal
- While per-trader volume increases with the number of traders under cursedness, it converges to zero under overconfidence.
- Even though each overconfident trader thinks that he knows more than he does,
  - he understands that the total amount of "valid" information revealed by the price in a large market swamps his own information.

General Definitions (Eyster and Rabin, 05)

Standard BNE (t_k is type/information of player k):

$$a_k^* \in \arg\max_{a_k \in A_k} \sum_{t_{-k} \in T_{-k}} p_k(t_{-k}|t_k) \cdot \sum_{a_{-k} \in A_{-k}} \sigma_{-k}(a_{-k}|t_{-k}) u_k(a_k, a_{-k}; t_k, t_{-k})$$

$\chi-$**cursed equilibrium:**

$$a_k^* \in \arg\max_{a_k \in A_k} \sum_{t_{-k} \in T_{-k}} p_k(t_{-k}|t_k) \cdot \sum_{a_{-k} \in A_{-k}} [\chi \bar{\sigma}_{-k}(a_{-k}|t_k) + (1-\chi)\sigma_{-k}(a_{-k}|t_{-k})] u_k(a_k, a_{-k}; t_k, t_{-k}),$$

where $\bar{\sigma}_{-k}(a_{-k}|t_k)$ neglects the correlation between t_{-k} and a_{-k}

$$\bar{\sigma}_{-k}(a_{-k}|t_k) \equiv \sum_{t_{-k} \in T_{-k}} p_k(t_{-k}|t_k) \sigma_{-k}(a_{-k}|t_{-k})$$

Application: winner's curse

Cursed Equilibrium and Imperfect Strategic Interactions

Going back to the main theme of this lecture

- **Imperfect strategic interactions** comes from imperfect common knowledge about each other's actions
- **Cursed equilibrium**: market prices will not achieve full common knowledge about each other's actions

Pause for Questions