Lecture 8

Survey evidence on expectations formation

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Outline

1. Under-reaction in Average Macroeconomic Expectations
2. Overreaction in Individual Macroeconomic Expectations
3. Models of Overreaction in Individual Expectations
4. Reconcile CG with BGMS

Under-reaction in Average Macroeconomic Expectations

- **Coibion & Gorodnichenko (JPE 2012):**
  - What can survey forecasts tell us about information rigidities?
- Findings: *in response to aggregate shocks*, **avg forecasts** fail to fully adjust on impact
- Statistically and economically significant deviations from FIRE
- Consistent with **common predictions** of info rigidities **(noisy info & sticky info)**
  - Avg forecast of a macro variable responds more gradually to the aggregate shock than the forecasted variable itself.
  - The conditional response of the avg forecast error is serially correlated and of the same sign as the forecasted variable.

Review: Mankiw & Reis (2002)

- In each period
  - **a fraction λ of firm obtains new info and re-optimizes**
  - **the other firms continue to set prices based on outdated info**
- Firm sets its price every period
  - can change prices even based on outdated info
  - e.g., anticipated news & steady-state inflation
- Connection with noisy-info/rational-inattention
  - a fraction λ of firm is "attentive" this period
  - the rest $1-\lambda$ of firm is "inattentive" this period

Sticky-Information Predictions

- Consider an AR(1) underlying fundamental

$$x_t = \rho x_{t-1} + w_t$$

- A fraction λ of agents obtain new info each period

$$\frac{d\bar{E}_{t+j}\left[x_{t+j+h}\right]}{dw_t} = \rho^{j+h}\left(1-(1-\lambda)^{j+1}\right),$$

  where $1-(1-\lambda)^{j+1}$ is **fraction of agents who have adjusted info about w_t at $t+j$.**
  - note that I follow Mankiw-Reis's notation for λ instead of CG's

- Define the average **forecast error**: $FE_{t,t+h} = E_t\left[x_{t+h}\right] - \bar{E}_t\left[x_{t+h}\right]$

$$\frac{dFE_{t+j,t+j+h}}{dw_t} = \rho^{j+h}(1-\lambda)^{j+1} = (1-\lambda)^{j+1}\frac{dE_{t+j}[\pi_{t+j+h}]}{dw_t}$$

- Average forecast error serially correlated & the same sign as the forecasted variables

Noisy-Information Predictions

An AR(1) underlying fundamental:

$$x_t = \rho x_{t-1} + w_t$$

Private signal about x_t each period:

$$s_{i,t} = x_t + \varepsilon_{i,t}$$

- Can be extended to the case with public signals (see the paper)

Notable differences from the static inference problem in the previous lecture

- **Dynamic learning** problems inferring x_t subject to recurring shocks

Technique: **Kalman filter**

Kalman Filter

- Predict x_t based on current and past signals
- Step 1: The Filtering Step
- Step 2: The Forecast Step
- Step 3: The Recursive Procedure and The Convergence
- Here: 1-dimensional fundamental and signal
  - can be easily extended to multi-dimensional cases

The Filtering Step

- Period t prior (based on all info at $t-1$): $x_t \sim \mathcal{N}\left(\hat{x}_t, \sigma_t^2\right)$
- Receive a signal: $s_{i,t} = x_t + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$
- Based on the signal, from Bayesian rule, **filtering distribution**
    - period t posterior, given by $\mathcal{N}\left(\hat{x}_t^F, \left(\sigma_t^F\right)^2\right)$

$$\hat{x}_t^F = \hat{x}_t + K_t\left(s_{i,t} - \hat{x}_t\right) = \left(1 - K_t\right)\hat{x}_t + K_t s_{i,t}$$

$$\left(\sigma_t^F\right)^2 = \frac{\sigma_t^2 \sigma_\varepsilon^2}{\sigma_t^2 + \sigma_\varepsilon^2},$$

where $K_t = \frac{\sigma_t^2}{\sigma_t^2 + \sigma_\varepsilon^2}$ is the "Kalman Gain."

The Forecast Step

- Posterior $\mathcal{N}\left(\hat{x}_t^F, \left(\sigma_t^F\right)^2\right)$ for current state x_t given prior and current information
- Now: predict the future state x_{t+1}
- State evolution: $x_{t+1} = \rho x_t + w_{t+1}$, $w_{t+1} \sim \mathcal{N}\left(0, \sigma_w^2\right)$
- Given $\mathcal{N}\left(\hat{x}_t^F, \left(\sigma_t^F\right)^2\right)$, we have the **predictive distribution** $x_{t+1} \sim \mathcal{N}\left(\hat{x}_{t+1}, \sigma_{t+1}^2\right)$, which is period $t+1$ prior

$$\hat{x}_{t+1} = \rho \hat{x}_t^F = \rho \hat{x}_t + \rho K_t \left(s_{i,t} - \hat{x}_t\right)$$

$$\sigma_{t+1}^2 = \rho^2 \left(\sigma_t^F\right)^2 + \sigma_w^2 = \rho^2 \frac{\sigma_t^2 \sigma_\varepsilon^2}{\sigma_t^2 + \sigma_\varepsilon^2} + \sigma_w^2.$$

The Recursive Procedure and the Convergence

- Increment t by one and go to step 1

- Variance of prior at period period σ_t^2 converges to $(\sigma^*)^2$

$$(\sigma^*)^2 = \rho^2 \frac{(\sigma^*)^2 \sigma_\varepsilon^2}{(\sigma^*)^2 + \sigma_\varepsilon^2} + \sigma_w^2$$

- Often just focus on this steady state, denote Kalman gain K^*

- Question of interest: how average period t posterior responds to the shock w_t (in x_t)

$$\frac{d\bar{E}_t[x_t]}{dw_t} = \lambda = K^*,$$

where K^*, "Kalman Gain," captures how informative $s_{i,t}$ is about w_t.

Noisy Information Predictions

- Moreover,
$$\frac{d\bar{E}_{t+j}\left[x_{t+j+h}\right]}{dw_t} = \rho^{j+h}\left(1-(1-\lambda)^{j+1}\right)$$
  - each period "$1-\lambda$" of the remaining uncertainty about w_t is still not learned

- Define **forecast error**: $FE_{t,t+h} = E_t\left[x_{t+h}\right] - \bar{E}_t\left[x_{t+h}\right]$
$$\frac{dFE_{t+j,t+j+h}}{dw_t} = \rho^{j+h}(1-\lambda)^{j+1} = (1-\lambda)^{j+1}\frac{dE_{t+j}[\pi_{t+j+h}]}{dw_t}$$

- Same formula as the stick info case with a different foundation
  - here: informative of $s_{i,t}$ about x_t ($\lambda = K^*$, Kalman gain)
  - sticky info: λ is the fraction of agent who updates

Common Predictions

Common predictions between noisy info and sticky info

- Avg forecast of a macro variable responds more gradually to the aggregate shock than the forecasted variable itself.
- The conditional response of the avg forecast error is serially correlated and of the same sign as the forecasted variable.

Pause for Questions

Empirical Implications

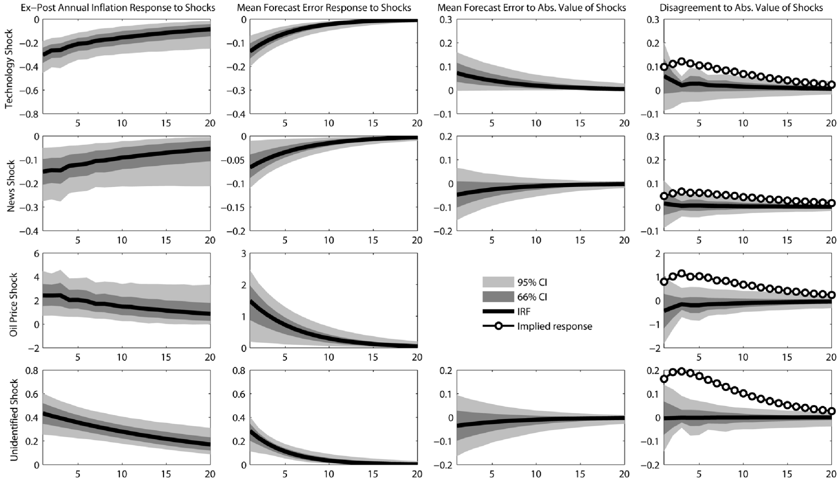
Inflation expectations:

- Survey of Professional Forecasters (SPF);
- Michigan Survey of Consumers (MSC);
- Livingston Survey (firm's expectations);
- FOMC inflation forecasts;

Shocks:

- (a) technology shocks, identified using long-run restrictions as in Gali (1999);
- (b) oil shocks, identified as in Hamilton (1996);
- (c) news shocks, identified as in Barsky and Sims (2011);
- (d) residuals in inflation;

Main Results: SPF



Translate into A Measure of Information Friction

- From
$$\frac{dFE_{t+j,t+j+h}}{dw_t} = \rho^{j+h}(1-\lambda)^{j+1} = (1-\lambda)^{j+1}\frac{dE_{t+j}[\pi_{t+j+h}]}{dw_t}$$

- Can use the **persistence of forecast error** to back out
$$\lambda = 0.2$$
  - e.g. 0.2 fraction of agents receive new info in sticky info each quarter
  - quite large information frictions even for professional forecasters

- Results hold for consumers, firms, and FOMCs
  - perhaps surprisingly, they have similar λs

Pause for Questions

Coibion and Gorodnichenko (AER 2015)

- Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts
- Instead of studying the IRFs of forecasts to identified aggregate shock
- Studies the overall properties of the forecasts and the forecasts errors
- Shifting attention from conditional to **unconditional moments**

Theoretical Frameworks

- Theoretical frameworks for information frictions
  - nesting both sticky-info and noisy-info

- Consider an AR(1) underlying fundamental

$$x_t = \rho x_{t-1} + w_t$$

- Preamble: **full-information rational expectation**

$$x_{t+h} - E_t\left[x_{t+h}\right] = {\color{red}0} \cdot \left(E_t\left[x_{t+h}\right] - E_{t-1}\left[x_{t+h}\right]\right) + v_{t+h,t},$$

where $v_{t+h,t}$ is unpredictable at period t.

- **Forecast errors unpredictable based on current and past variables**
  - incorporated in the information set

Sticky Information Predictions

- A fraction λ of agents obtain new info each period

$$\bar{E}_t\left[x_{t+h}\right] = \lambda E_t\left[x_{t+h}\right] + (1-\lambda)\,\bar{E}_{t-1}\left[x_{t+h}\right],$$

  where I follow Mankiw-Reis's notation for λ instead of CG's

- **Forecast error:**

$$x_{t+h} - \bar{E}_t\left[x_{t+h}\right] = \frac{1-\lambda}{\lambda}\left(\bar{E}_t\left[x_{t+h}\right] - \bar{E}_{t-1}\left[x_{t+h}\right]\right) + v_{t+h,t},$$

  where $v_{t+h,t}$ is unpredictable at period t.

- Predicted relationship between the **ex post mean forecast error** across agents and the **ex ante mean forecast revision**

Noisy Information Predictions

- Each period t receive a private signal: $s_{i,t} = x_t + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$
- Let $\lambda = K^*$ denote the Kalman Gain as above

$$\bar{E}_t\left[x_t\right] = \lambda x_t + (1-\lambda)\,\bar{E}_{t-1}\left[x_t\right]$$

- Similarly

$$\bar{E}_t\left[x_{t+h}\right] = \lambda E_t\left[x_{t+h}\right] + (1-\lambda)\,\bar{E}_{t-1}\left[x_{t+h}\right]$$

- **Forecast error:**

$$x_{t+h} - \bar{E}_t\left[x_{t+h}\right] = \frac{1-\lambda}{\lambda}\left(\bar{E}_t\left[x_{t+h}\right] - \bar{E}_{t-1}\left[x_{t+h}\right]\right) + v_{t+h,t},$$

where $v_{t+h,t}$ is unpredictable at period t.

- Predicted relationship between the **ex post mean forecast error** across agents and the **ex ante mean forecast revision**

Empirical Specifications

- Main specifications

$$x_{t+h} - \bar{E}_t\left[x_{t+h}\right] = c + \beta\left(\bar{E}_t\left[x_{t+h}\right] - \bar{E}_{t-1}\left[x_{t+h}\right]\right) + error_{t,h}$$

- Map to the degree of **information frictions**

$$\beta = \frac{1-\lambda}{\lambda}$$

- Main survey measures:
  - Forecasts of US annual inflation from SPF

Empirical Results ($\lambda \approx \frac{1}{1+\beta} = 0.46$)

TABLE 1—TESTS OF THE INFLATION EXPECTATIONS PROCESS

Forecast error $\pi_{t+3,t} - F_t\pi_{t+3,t}$	None (1)	Inflation (2)	Average quarterly 3-month Tbill rate (3)	Quarterly change in the log of the oil price (4)	Average unemployment rate (5)
	Additional control: z_{t-1}				
Panel A. $\pi_{t+3,t} - F_t\pi_{t+3,t} = c + \gamma F_t\pi_{t+3,t} + \delta z_{t-1} + error_t$					
Constant	−0.181	−0.045	−0.091	−0.181	1.449**
	(0.248)	(0.223)	(0.236)	(0.221)	(0.676)
$F_t\pi_{t+3,t}$	0.059	−0.299**	0.210*	0.045	0.095
	(0.085)	(0.148)	(0.111)	(0.078)	(0.085)
Additional control: z_{t-1}		0.318**	−0.125*	1.603**	−0.281**
		(0.147)	(0.066)	(0.763)	(0.117)
Observations	178	178	178	178	178
R^2	0.010	0.109	0.054	0.046	0.148
Panel B. $\pi_{t+3,t} - F_t\pi_{t+3,t} = c + \beta(F_t\pi_{t+3,t} - F_{t-1}\pi_{t+3,t}) + \delta z_{t-1} + error_t$					
Constant	0.002	−0.074	0.151	−0.021	1.134**
	(0.144)	(0.174)	(0.175)	(0.146)	(0.546)
$F_t\pi_{t+3,t} - F_{t-1}\pi_{t+3,t}$	1.193**	1.141**	1.196**	1.125**	1.062**
	(0.497)	(0.458)	(0.504)	(0.499)	(0.465)
Additional control: z_{t-1}		0.021	−0.029	0.576	−0.178**
		(0.050)	(0.031)	(0.608)	(0.076)
Observations	173	173	173	173	173
R^2	0.195	0.197	0.201	0.200	0.249

Robustness

Information rigidity across agents

- The Livingston Survey (academic institutions, commercial banks, and non-financial firms)
- Michigan Survey of Consumers
- Similar results

Information rigidity across horizons and variables

- SPF forecasts of real GDP, industrial production, housing starts, and unemployment rates
- Information rigidity exists for all variables

Pause for Questions

Outline

What Happens at the Individual Level?

A test of rational expectations at the **individual level**

$$x_{t+h} - E_{i,t}\left[x_{t+h}\right] = \beta\left(E_{i,t}\left[x_{t+h}\right] - E_{i,t-1}\left[x_{t+h}\right]\right) + v_{i,t+h,t}$$

Rational expectations (even with limited info): $\beta = 0$

- Forecast errors unpredictable from forecast revisions
- True even in the case of noisy info/sticky info
  - forecast revisions always in your information set

Empirical evidence: over-reaction at individual level, $\beta < 0$!

Empirical Evidence on Individual Macro Expectations

Let us just run this regression! (BGMS, 2020 AER)

$$x_{t+h} - E_{i,t}[x_{t+h}] = \beta\left(E_{i,t}[x_{t+h}] - E_{i,t-1}[x_{t+h}]\right) + v_{i,t+h,t}$$

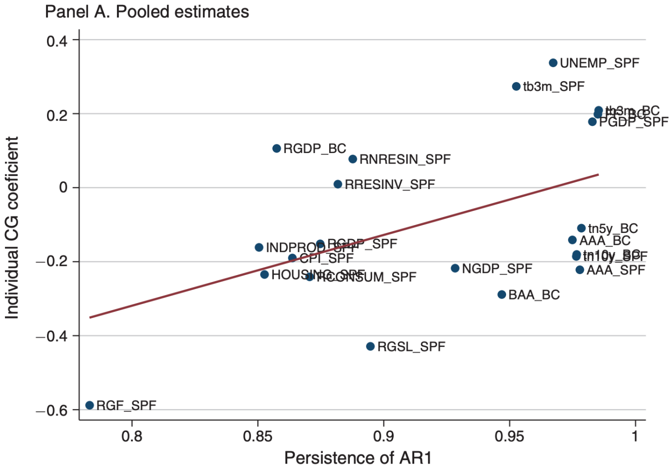
Data:

- Survey of Professional Forecasters as in CG
- Blue Chip
  - a survey of panelists from around 40 major financial institutions
- Focus on the *growth* rate of
  - macro outcomes: GDP, price indices, consumption, investment, unemployment
  - financial variables: yields on government bonds and corporate bonds
- Annual forecast horizon ($h = 4$)

Results

	Consensus			Individual			
	β_1	SE	p-value	β_1^p	SE	p-value	Median
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Nominal GDP (SPF)	0.56	0.21	0.01	−0.22	0.07	0.00	−0.20
Real GDP (SPF)	0.44	0.23	0.06	−0.15	0.09	0.09	−0.08
Real GDP (BC)	0.57	0.33	0.08	0.11	0.19	0.58	−0.03
GDP price index inflation (SPF)	1.41	0.21	0.00	0.18	0.13	0.18	−0.11
CPI (SPF)	0.29	0.22	0.17	−0.19	0.12	0.10	−0.25
Real consumption (SPF)	0.24	0.25	0.33	−0.24	0.11	0.02	−0.26
Industrial production (SPF)	0.71	0.30	0.02	−0.16	0.09	0.09	−0.19
Real nonresidential investment (SPF)	1.06	0.36	0.00	0.08	0.15	0.60	0.09
Real residential investment (SPF)	1.22	0.33	0.00	0.01	0.10	0.92	−0.09
Real federal government consumption (SPF)	−0.43	0.23	0.06	−0.59	0.07	0.00	−0.52
Real state and local government consumption (SPF)	0.63	0.34	0.06	−0.43	0.04	0.00	−0.44
Housing start (SPF)	0.40	0.29	0.18	−0.23	0.09	0.01	−0.27
Unemployment (SPF)	0.82	0.2	0.00	0.34	0.12	0.00	0.23
Fed funds rate (BC)	0.61	0.23	0.01	0.20	0.09	0.03	0.22
Three-month Treasury rate (SPF)	0.60	0.25	0.01	0.27	0.10	0.01	0.28
Three-month Treasury rate (BC)	0.64	0.25	0.01	0.21	0.09	0.02	0.17
Five-year Treasury rate (BC)	0.03	0.22	0.88	−0.11	0.10	0.29	−0.17
Ten-year Treasury rate (SPF)	−0.02	0.27	0.95	−0.19	0.10	0.06	−0.24
Ten-year Treasury rate (BC)	−0.08	0.24	0.73	−0.18	0.11	0.11	−0.29
AAA corporate bond rate (SPF)	−0.01	0.23	0.95	−0.22	0.07	0.00	−0.32
AAA corporate bond rate (BC)	0.21	0.20	0.29	−0.14	0.06	0.02	−0.27
BAA corporate bond rate (BC)	−0.18	0.27	0.50	−0.29	0.09	0.00	−0.32

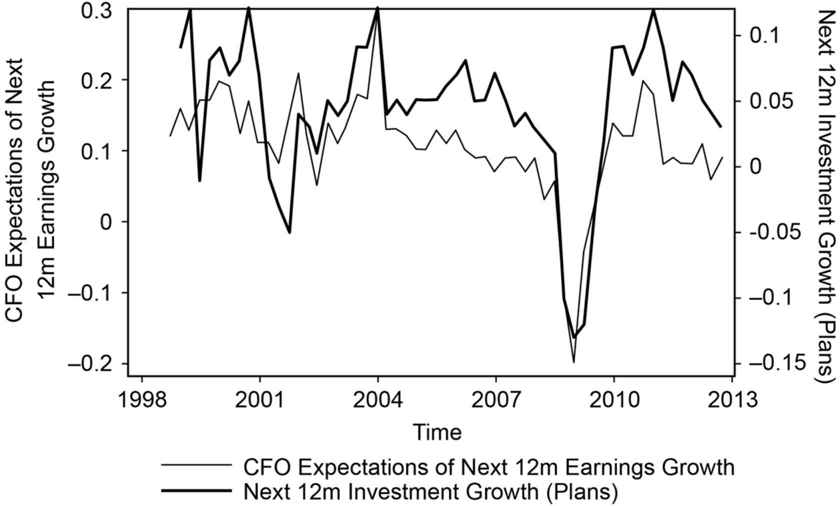
Stronger Persistence Reduces Individual Overreaction



Expectations and Investment

- Gennaioli, Ma, and Shleifer (2016, Macro Annual)
- Main data: Duke University quarterly survey of Chief Financial Officers
- Expectations of earnings growth of ***own*** firms
  - difference: overreaction in macro expectations (BGMS, 2020)
- Complementary data: Institutional Brokers' Estimate System (IBES)
  - Equity analysts' expectations of future firm earnings growth

Expectations Matter for Investment



Overreaction in Expectations

Individual level:

$$x_{i,t+h} - E_{i,t}\left[x_{i,t+h}\right] = \beta Z_{i,t} + v_{i,t+h,t},$$

where x_{t+h} is earning growth, $x_{t+h} - E_{i,t}\left[x_{t+h}\right]$ is forecast errors, and $Z_{i,t}$ are information available at period t.

Aggregate level:

$$\int \left(x_{i,t+h} - E_{i,t}\left[x_{i,t+h}\right]\right) di = \beta \int Z_{i,t} di + v_{t+h,t}.$$

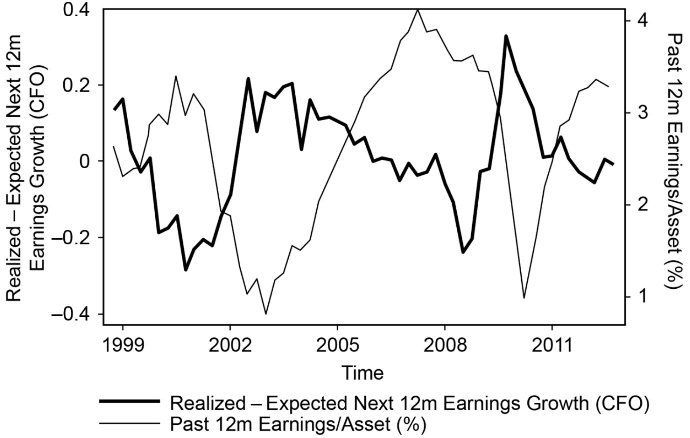
- Note average forecast errors in individual outcomes is not average forecasts errors in macro outcomes

  $$\int \left(x_{i,t+h} - E_{i,t}\left[x_{i,t+h}\right]\right) di \neq x_{t+h} - \int E_{i,t}\left[x_{t+h}\right] di,$$

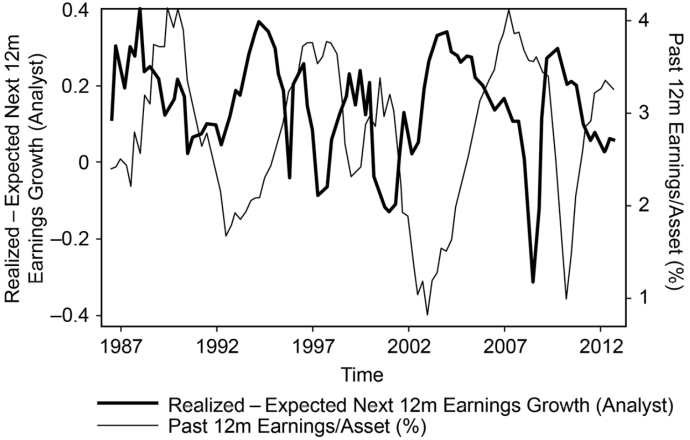
  where $x_{t+h} = \int x_{i,t+h} di$
- Fundamentally different from CG regressions

FIRE means $\beta = 0$ in both regressions

Aggregate Level



Aggregate Level



Individual Level

B. Firm-Level Evidence						
	Realized – CFO Expected Next 12m Earnings Growth					
	(1)	(2)	(3)	(4)	(5)	(6)
Past 12m firm earnings/asset (%)	−0.0511		−0.0500		−0.0324	−0.0353
	(−5.14)		(−5.22)		(−3.40)	(−3.56)
Past 12m GDP growth		−4.1472		−2.811		
		(−2.44)		(−1.75)		
Firm stock vol.			0.3959	0.2229		0.5299
			(1.74)	(0.94)		(1.13)
Firm fixed effects	Y	Y	Y	Y	Y	Y
Time fixed effects	No				Yes	
Observations	606	651	594	638	606	594
R-squared	0.082	0.032	0.103	0.033	0.037	0.050
Number of firms	142	147	139	144	142	139

Individual Level

B. Firm-Level Evidence						
	Realized – Analyst Expected Next 12m Earnings Growth					
	(1)	(2)	(3)	(4)	(5)	(6)
Past 12m firm earnings/asset (%)	−0.0080		−0.0081		−0.0061	−0.0062
	(−7.43)		(−7.36)		(−6.71)	(−6.63)
Past 12m GDP growth		−1.6167		−1.6235		
		(−3.83)		(−3.72)		
Firm stock vol.			0.0158	−0.0256		−0.0123
			(0.26)	(−0.50)		(−0.40)
Firm fixed effects	Y	Y	Y	Y	Y	Y
Time fixed effects	No				Yes	
Observations	103,930	123,430	100,451	115,120	103,930	100,451
R-squared	0.005	0.004	0.006	0.004	0.003	0.003
Number of firms	4,432	5,080	4,227	4,606	4,432	4,227

Questions

- How to model **overreaction in individual expectations**?

- How to reconcile **under-reaction in average expectations** with **over-reaction in individual expectations**?

- Hint: two elements (relax both RE and FI)
  - deviations from individual rational expectations (e.g., over-reaction to signals)
  - noisy-info/inattention about aggregate shocks

Pause for Questions

Outline

Models of Over-reaction: Overview

- Classical models of over-reaction
  - Adaptive expectations/Constant gain learning
  - Extrapolation
  - Misperception of the persistence
  - Natural expectations

- Lucas critique

- Modern models of over-reaction immune to Lucas critique
  - Diagnostic expectations
  - Bounded recall based models

Adaptive Expectations

- Specification

$$
\begin{aligned}
E_{i,t}[x_{t+1}] &= E_{i,t-1}[x_{t+1}] + \omega\left(x_t - E_{i,t-1}[x_t]\right) \\
&= \omega x_t + (1-\omega)\, E_{i,t-1}[x_{t+1}]
\end{aligned}
$$

- Lead to over-reaction if the shock is transitory
- Cagan (56): adaptive expectations can lead to hyperinflation
- Nerlove (58): adaptive expectations and Cobweb dynamics
- Plain-vanilla version: ω does not depend on the underlying environment
  - e.g. the process of x_t

Constant Gain Learning

- A micro-foundation of adaptive expectations
- Suppose the data generating process is $x_t = \mu + \eta_t$, where
  - μ is an unknown state
  - η_t is i.i.d. with mean zero
- **Least-square learning:** use all past realizations $(x_0, \cdots, x_t)$ equally to forecast μ

$$E_{i,t}[x_{t+1}] = E_{i,t}[\mu] = E_{i,t-1}[\mu] + \omega_t \left(x_t - E_{i,t-1}[\mu]\right),$$

where $\omega_t = \frac{1}{1+t}$ decreases with t.

Constant Gain Learning

- **Constant gain learning:**

$$
\begin{aligned}
E_{i,t}[x_{t+1}] = E_{i,t}[\mu] &= E_{i,t-1}[\mu] + \omega\left(x_t - E_{i,t-1}[\mu]\right) \\
&= E_{i,t-1}[x_{t+1}] + \omega\left(x_t - E_{i,t-1}[x_{t+1}]\right)
\end{aligned}
$$

  - Essentially weight past realizations less ("fading memory")
  - Effectively adaptive expectations
  - Evans and Honkapohja (01); Nagel and Xu (19)

Extrapolation

- Specifications (Metzler, 41)

$$E_{i,t}[x_{t+1} - x_t] = \omega\,(x_t - x_{t-1})$$
$$E_{i,t}[x_{t+1}] = (1+\omega)\,x_t - \omega x_{t-1}$$

- Depending on current and past realized outcomes
  - but not past beliefs

- A continuous time generalization (Barberis et al., 15)

$$E_{i,t}\,[dP_t] = \left(\lambda_0 + \lambda_1 \beta \int_{-\infty}^{t} e^{-\beta(t-s)}\,dP_{s-dt}\right),$$

where $dP_{s-dt} = P_s - P_{s-dt}$.

Misperception of the Persistence

- For example, for an AR (1) process with true persistence ρ

$$\rho^s = \lambda \rho + (1-\lambda)\rho^d,$$

  where ρ^s is the perceived preference and ρ^d is the default preference
  - Gabaix (20); Angeletos, Huo, Sastry (20)

- Essentially inattention to the persistence of the stochastic process

- Plausible explanations of over-reaction for transitory processes

Natural Expectations

- Fundamentals follow a relatively complicated DGP
  - e.g., with hump shape
- Agents use a simpler DGP to fit the data and form expectations
  - e.g., with AR (1)
- Fuster, Laibson, Mendel (10); Fuster, Hebert, Laibson (12)
- Micro foundation (Molavi, Tahbaz-Salehi, & Vedolin, 21)
- Cannot explain the experimental data in Afrouzi et al. (23) based on AR (1) underlying DGP.

Issues: Lucas Critique

- In above models, agents are essentially passive
- Expectations **unresponsive to policy changes** (Lucas, 76)
- Volcker disinflation happened
- Rational expectations revolution (Muth, 61)

Pause for Questions

Diagnostic Expectations

- Bordalo et al. (18, JF); Bordalo et al. (20, AER)
- **Overweight future outcomes** that become **more likely in light of current data**
  - based on Kahneman and Tversky's representativeness heuristic
- Forward looking and depends on the underlying stochastic process
  - immune to the Lucas critique

Representativeness Heuristic

Kahneman and Tversky's **representativeness heuristic**

- A certain attribute is judged to be **excessively common** in a population, when that attribute is
  - **diagnostic** for the population, meaning that it occurs more frequently in the given population than in a relevant reference population

- For example, $\frac{Pr(red\ \ hair|Irish)}{Pr(red\ \ hair|World)} = 10$

  $\Longrightarrow$ exaggerate the frequency of redhair among Irish $Pr(red\ \ hair|Irish)$

  $\Longrightarrow$ red hair is more likely to be Irish, so people think all Irish people have red hair

- Math:

$$\text{High} \quad \frac{Pr(t|G)}{Pr(t|-G)} \Longrightarrow \text{overweight} \quad Pr(t|G)$$

Representativeness Heuristic and Expectations Formation

Analogy in expectation formation

- Agents **overweight** those **future states whose likelihood increases in light of current news relative to what they know already**

  $\Longrightarrow$ good future is more likely given good news, so people overweight good future given good news

- For an AR (1) $x_{t+1} = \rho x_t + \varepsilon_{t+1}$, the diagnostic pdf is given by

$$h_t^\theta\left(\hat{x}_{t+1}\right) = h\left(\hat{x}_{t+1} | x_t = \hat{x}_t\right)\left[\frac{h\left(\hat{x}_{t+1} | x_t = \hat{x}_t\right)}{h\left(\hat{x}_{t+1} | x_t = \rho \hat{x}_{t-1}\right)}\right]^\theta \frac{1}{Z},$$

  where Z makes sure h_t^θ integrates to one and θ captures the degree of diagnostic expectations

Diagnostic Expectations

- Forecasts under diagnostic expectations:

$$E_t^{\theta}[x_{t+1}] = \int_{\mathbb{R}} x_{t+1} h_t^{\theta}(x_{t+1})\, dx_{t+1} = E_t[x_{t+1}] + \theta\left[E_t[x_{t+1}] - E_{t-1}[x_{t+1}]\right]$$

$$= E_t[x_{t+1}] + \rho\,\theta\left[x_t - E_{t-1}(x_t)\right]$$

- Similar for longer-term forecasts:

$$E_t^{\theta}[x_{t+h}] = E_t[x_{t+h}] + \theta\left(E_t[x_{t+h}] - E_{t-1}[x_{t+h}]\right)$$

Pause for Questions

Outline

Overview

CG (12, 15): evidence against "**FI**" in "FIRE"

- Disperse information or rational inattention
- Underreaction in average expectations to macro shocks

BGMS (20): evidence against "**RE**" in "FIRE"

- Overreaction in individual expectations

Not inconsistent

- Noisy/disperse info (or inattention to) aggregate shocks
- Violations of RE at the individual level given the info set

The Model of BGMS (20)

Data generating process, still :

$$x_{t+1} = \rho x_t + w_{t+1}$$

Each agent $i \in [0,1]$ receives a **noisy signal**:

$$s_t^i = x_t + \varepsilon_t^i$$

Diagnostic expectations:

$$f^{\theta}\left(x_t | S_t^i\right) = f\left(x_t | S_t^i\right) \left(\frac{f\left(x_t | S_t^i\right)}{f\left(x_t | S_{t-1}^i \cup \left\{ x_{t|t-1}^i \right\}\right)} \right)^{\theta} \frac{1}{Z_t},$$

- S_t^i denote full history of signals received by agent i
- **Representativeness**: if the signal s_t^i raises the probability of that state relative to the case where the signal equals the ex ante forecast $s_t^i = x_{t|t-1}^i$

Predictions

- Kalman-filter style of solution methods

- Distorted individual forecast:

$$E^{\theta}_{i,t}[x_{t+h}] = E_t[x_{t+h}|S^i_{t-1}] + (1+\theta)\frac{\sum}{\sum + \sigma^2_{\varepsilon}}\rho^h \cdot \left(s^i_t - E_t[x_{t+h}|S^i_{t-1}]\right)$$

- **CG coefficient** (average expectation):

$$\begin{aligned}\beta_{CG} &= \frac{Cov\left(x_{t+h} - \bar{E}^{\theta}_t\left[x_{t+h}\right], \bar{E}^{\theta}_t\left[x_{t+h}\right] - \bar{E}^{\theta}_{t-1}\left[x_{t+h}\right]\right)}{Var\left(\bar{E}^{\theta}_t\left[x_{t+h}\right] - \bar{E}^{\theta}_{t-1}\left[x_{t+h}\right]\right)}, \\ &= \left(\sigma^2_{\varepsilon} - \theta \sum\right) g\left(\sigma^2_{\varepsilon}, \sum, \rho, \theta\right)\end{aligned}$$

which is positive if **noise** σ^2_{ε} is large enough.

Predictions

- **BGMS regression coefficient** (individual expectation):

$$\beta_{BGMS} = \frac{Cov\left(x_{t+h} - E^{\theta}_{i,t}\left[x_{t+h}\right], E^{\theta}_{i,t}\left[x_{t+h}\right] - E^{\theta}_{i,t-1}\left[x_{t+h}\right]\right)}{Var\left(E^{\theta}_{i,t}\left[x_{t+h}\right] - E^{\theta}_{i,t-1}\left[x_{t+h}\right]\right)},$$
$$= -\frac{\theta\left(1+\theta\right)}{\left(1+\theta\right)^2 + \theta^2\rho^2}$$

- **Negative**

- Absolute value decreases with the persistence of DGP
    - consistent with additional evidence in BGMS

Pause for Questions

Angeletos, Huo, Sastry (20, Macro Annual)

- Noisy info + misperception of the persistence/precision
- DGS, still

$$x_{t+1} = \rho x_t + w_{t+1}$$

- Each agent $i \in [0,1]$ receives a noisy signal:

$$s_t^i = x_t + \varepsilon_t^i,$$

  where $\varepsilon_t^i \sim \mathcal{N}\left(0, \tau^{-1}\right)$

- Perceived persistence $\hat{\rho}$ instead of ρ
  - over-extrapolation if $\hat{\rho} > \rho$
- Perceived precision $\hat{\tau}$ instead of τ
  - overconfidence if $\hat{\tau} > \tau$

Predictions

- CG coefficient (average expectation):

$$\beta_{CG} = \kappa_1 \hat{\tau}^{-1} - \kappa_2(\hat{\rho} - \rho),$$

which is positive if perceived precision of signal is not too high

- BGMS coefficient (individual expectation):

$$\beta_{BGMS} = -\kappa_3\left(\hat{\tau} - \tau\right) - \kappa_4\left(\hat{\rho} - \rho\right),$$

which is positive if over-extrapolation and/or overconfidence
    - conflate over-extrapolation with overconfidence

Additional Predictions and Evidence

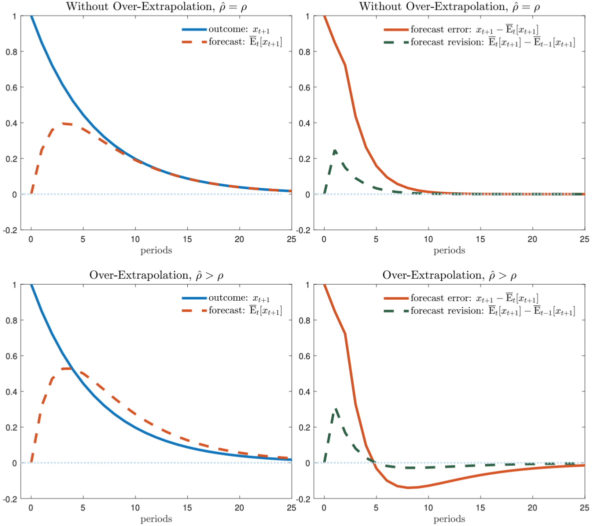
- How to disentangle misperception of persistence and overconfidence?
- Study IRFs of average forecast errors to aggregate shocks

$$\zeta_k = \frac{\partial \left(x_{t+k} - \bar{E}_{t+k-1}\left[x_{t+k}\right]\right)}{\partial w_t}$$

- Evidence: average forecasts initially under-react before over-shooting later on
- To explain this need a large enough information friction (perceived noise $\hat{\tau}^{-1}$) and $\hat{\rho} > \rho$

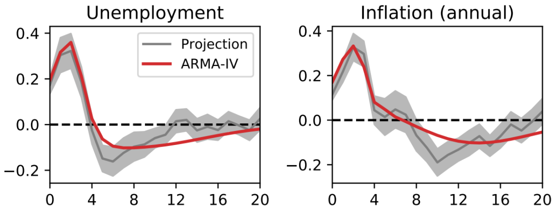
Theoretical Predictions: IRFs

Figure 2: IRFs of Aggregate Forecasts and Errors in the Theory



Empirical Evidence: IRFs

Figure 4: Dynamic Responses: Forecast Errors



Notes: The sample period is Q1 1968 to Q4 2017. The shaded areas are 68% confidence intervals based on HAC standard errors with a Bartlett kernel and 4 lags. The x-axis denotes quarters from the shock, starting at 0. In the first column the outcome is u_t and the forecast is $\bar{\mathbb{E}}_{t-3}[u_t]$; in the second column the outcome is $\pi_{t,t-4}$, or annual inflation, and the forecast is $\bar{\mathbb{E}}_{t-3}[\pi_{t,t-4}]$.

- SPF unemployment and inflation forecasts
- "Main business cycle" shocks: maximizing its contribution to the business cycle variation in unemployment/inflation

Pause for Questions