The Intertemporal Keynesian Cross

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Cambridge
May 2023

This paper

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- what micro moments are important?

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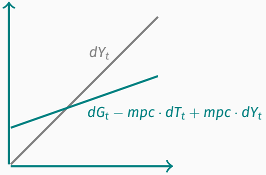

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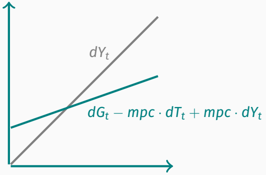

Textbook answer: the **Keynesian cross** [Keynes, Hicks, Haavelmo...]

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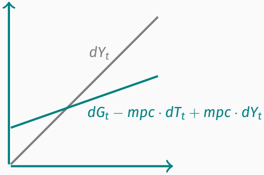

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Our answer: the **intertemporal Keynesian cross**

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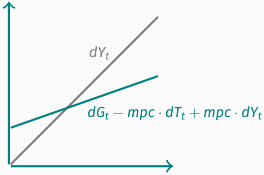

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- sufficient statistics now: "**intertemporal MPCs**" (**iMPCs**)

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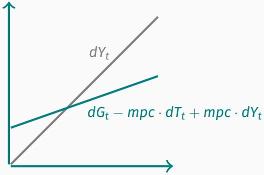

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- sufficient statistics now: "**intertemporal MPCs**" (**iMPCs**)
  - $\equiv$ dynamic responses of spending to income (**M**) and (in extension) capital gains

Application: when is the fiscal multiplier large?

- Lots of theory + empirical work. Two workhorse classes of modern models:

1. **Representative-agent (RA)** models [Eggertsson 2004; Christiano-Eichenbaum-Rebelo 2011]
    - **response of monetary policy** is key
    - large when at ZLB
2. **Two-agent (TA)** models [Galí-López-Salido-Vallés 2007; Coenen et al 2012; Bilbiie et al 2013]
    - aggregate **MPC** is key
    - large when deficit financed, effects not persistent

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New: **Heterogeneous-agent (HA)** models

$\rightarrow$ **iMPCs** are key, can be used for calibration

$\rightarrow$ large and persistent Y effect when deficit financed

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Also: Can **tractable models** replicate this behavior? (THANK, BU/WUNK, PRANK...)

$\rightarrow$ Existing ones miss out on iMPCs out of income, capital gains, or both

Our contribution: Interaction of iMPCs and deficit-financing

1. **IKC environment,** allows for RA, TA, HA, tractable models of consumption:
   - without capital & constant-real-rate monetary policy
   - multiplier $=$ function of **iMPCs** and **deficits** *only*

     $=$ **1** if zero deficits or flat iMPCs (RA)

     impact $>$ **1** if **deficit-financed** and **high MPCs** (TA, HA, tractable)

     cumulative $>$ **1** if **deficit-financed** and **realistic iMPCs** (HA, tractable?)

     why? "spending down of past savings" creates dynamic $C-Y$ feedback

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2. **Quantitative environment** with capital & Taylor rule
   - large & persistent Y effects, despite these extra elements
   - iMPCs (incl. capital gains) **still sufficient statistics** for household behavior

Related literature

- **Fiscal multipliers**
  - **Theory**: IS-LM [Keynes 1936, Gelting 1941, Haavelmo 1945, Blinder-Solow 1973, ...]
    Rep-agent (RA) [Aiyagari-Christiano-Eichenbaum 1992, Baxter-King 1993, Christiano-Eichenbaum-Rebelo 2011, ...]
    Two-agent (TA) [Bilbiie-Straub 2004, Galí-López-Salido-Vallés 2007, Coenen et al. 2012, Drautzburg-Uhlig 2015, Farhi-Werning 2017, ...]
    Heterogeneous-agent (HA) [Oh-Reis 2010, McKay-Reis 2016, Ferrière-Navarro 2017, Hagedorn-Manovski-Mitman 2017, ...]
    Tractable models [Bilbiie 2019, Cantore-Freund 2021, Hagedorn 2018, Michaillat-Saez 2021, Ravn-Sterk 2017, Acharya-Dogra 2020...]
  - **Empirics**: Aggregate evidence [Ramey-Shapiro 1998, Blanchard-Perotti 2002, Mountford-Uhlig 2009, Ramey 2011, Barro-Redlick 2011, Auerbach-Gorodnichenko 2012, Ramey-Zubairy 2018, ...]
    Cross-sectional multipliers [Shoag 2010, Chodorow-Reich et al. 2012, Nakamura-Steinsson 2014, Chodorow-Reich 2018, ...]
- **PE to GE** [Farhi-Werning ’17, Auclert-Rognlie ’18, Guren-McKay-Nakamura-Steinsson ’18, Wolf ’20...]

Outline

1. The intertemporal Keynesian Cross
2. Using iMPCs to discriminate across models
3. Fiscal policy in the IKC environment
4. Sufficient statistics beyond the baseline model
5. Fiscal policy in a quantitative environment

The intertemporal Keynesian Cross

Overview of environment

▸ Unions

- GE, discrete $t = 0 \ldots \infty$. Small aggregate MIT shocks ($\Leftrightarrow$1st order perturbation)
- Mass 1 of households:
  - idiosyncratic shocks to skills e_{it}, various market structures
  - real pre-tax labor income $y_{it} \equiv W_t/P_t e_{it} n_{it}$
  - **after tax labor income** $z_{it} \equiv y_{it} - T_t(y_{it}) \equiv \tau_t y_{it}^{1-\theta}$ [Bénabou, HSV]

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  - **tax revenue** $T_t = \int (y_{it} - z_{it})\, di$
  - **government spending** G_t
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- Supply side:
  - linear production function $Y_t = N_t$
  - flexible prices $\Rightarrow$ $P_t = W_t$
  - sticky $W_t \Rightarrow \pi_t^w = \kappa^w \int N_t (v'(n_{it}) - \frac{\partial z_{it}}{\partial n_{it}} u'(c_{it}))di + \beta \pi_{t+1}^w$

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labor allocation

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labor allocation

relax later

Nested models of consumption

Household i solves:

$$\max \mathbb{E}\left[\sum_{t\geq 0}\beta^t\left\{u\left(c_{it}\right)-v\left(n_{it}\right)\qquad\right\}\right]$$

$$c_{it}+a_{it}=z_{it}+(1+r)a_{it-1}$$

- RA: no risk in e (or complete markets)
- TA: share μ of agents with $c_{it}=z_{it}$
- HA-one: one account model, constraint $a_{it}\geq 0$

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$$\begin{aligned} c_{it} + a_{it}^{liq} &= z_{it} + (1+r)(1-\zeta)a_{it-1}^{liq} - d_{it} \cdot 1_{\{adj_{it}=1\}} \\ a_{it}^{illiq} &= (1+r)a_{it-1}^{illiq} + d_{it} \cdot 1_{\{adj_{it}=1\}} \end{aligned}$$

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- HA-one: one account model, constraint $a_{it} \geq 0$
- HA-two: two account model w. $\Pr(1_{\{adj_{it}=1\}}) = \nu < 1$, $\zeta > 0$, $a_{it}^{illiq} \geq 0$, $a_{it}^{liq} \geq 0$

Nested models of consumption

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$$\max \mathbb{E}\left[\sum_{t\geq 0} \beta^t \left\{u\left(c_{it}\right) - v\left(n_{it}\right) + \chi\left(a_{it}\right)\right\}\right]$$

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- BU/WU: no risk in e, extra additive term $\chi\left(a_{it}\right)$ in utility

The aggregate consumption function

▸ Eq definition

- Given $\{a_{i,-1}\}$ and r, **aggregate consumption function** is

$$C_t = \int c_{it} di + \zeta \int_i a_{it}^{liq} di = \mathcal{C}_t \left(\{Z_s\}_{s=0}^{\infty} \right)$$

[Kaplan Moll Violante 2018, Farhi Werning 2019, ...]

with $Z_t \equiv$aggregate after-tax labor income

$$Z_t \equiv \int z_{it} di = Y_t - T_t$$

- $\mathcal{C}$ summarizes the heterogeneity and market structure
- Equilibrium defined as usual

Intertemporal MPCs

- Consider $\{G_t, T_t\}$ such that $\sum_{t=0}^{\infty} (1+r)^{-t} (G_t - T_t) = 0$
- An output path $\{Y_t\}_{t=0}^{\infty}$ is part of equilibrium $\Leftrightarrow$

$$Y_t = G_t + \mathcal{C}_t (\{Y_s - T_s\}) \quad \forall t \geq 0$$

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- Impulse response to shock $\{dG_t, dT_t\}$ around steady state

$$dY_t = dG_t + \sum_{s=0}^{\infty} \underbrace{\frac{\partial \mathcal{C}_t}{\partial Z_s}}_{\equiv M_{t,s}} \cdot (dY_s - dT_s)$$

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$\rightarrow$ Response $\{dY_t\}$ entirely characterized by $\{M_{ts}\}$: **iMPCs out of income**

- this is a **sequence-space Jacobian**, made of *partial equilibrium* derivatives

  [Auclert, Bardoczy, Rognlie, Straub 2021]

- how much of income change at date s is spent at date t
- agent budget constraints $\Rightarrow \sum_{t=0}^{\infty}(1+r)^{s-t}M_{ts} = 1$

The intertemporal Keynesian cross

- Stack objects: $\mathbf{M} \equiv [M_{ts}] = \left[\frac{\partial \mathcal{C}_t}{\partial Z_s}\right]$, $d\mathbf{Y} \equiv \{dY_t\}$, $d\mathbf{G} \equiv \{dG_t\}$, $d\mathbf{T} \equiv \{dT_t\}$
  - $d\mathbf{Y}$, $d\mathbf{G}$, $d\mathbf{T}$ are sequences in ℓ^∞, $\mathbf{M}$ is bounded linear operator on ℓ^∞
  - $\mathbf{M}$ satisfies $\mathbf{q}'\mathbf{M} = \mathbf{q}'$ where $\mathbf{q}' \equiv \left\{1 \quad \frac{1}{1+r} \quad \left(\frac{1}{1+r}\right)^2 \cdots \right\}$ is the PDV sequence

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Proposition

Consider a shock $d\mathbf{G}, d\mathbf{T}$ s.t. $\mathbf{q}'d\mathbf{G} = \mathbf{q}'d\mathbf{T}$. Any output response $d\mathbf{Y}$ must satisfy

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y} \tag{IKC}$$

- This is an **intertemporal Keynesian cross**
  - entire complexity of model is in $\mathbf{M}$
  - with $\mathbf{M}$ from data, could get $d\mathbf{Y}$ without model!

Examples of **M**

- For simple models, analytical solution. eg RA $[\beta(1+r)=1]$

$$\mathbf{M}^{RA} = \begin{pmatrix} (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \\ \vdots & & & \ddots \end{pmatrix} = \frac{\mathbf{1q}'}{\mathbf{q}'\mathbf{1}}$$

- Harder: BU model parameterized by slope λ, β and r [any $\beta(1+r)$] ▸ full expression

$$M^{BU}_{to} = \left(1 - \frac{\lambda}{1+r}\right) \cdot \lambda^t \quad \text{all } t$$

- Adding fraction μ of hand to mouth agents, for any **M**, gives

$$\mathbf{M}^{\mu} = (1-\mu)\,\mathbf{M} + \mu\mathbf{I}$$

- Rewrite:

$$(\mathbf{I} - \mathbf{M})\, d\mathbf{Y} = d\mathbf{G} - \mathbf{M} d\mathbf{T}$$

problem: $\mathbf{I} - \mathbf{M}$ is never invertible!

Solving the intertemporal Keynesian cross

- Rewrite:

$$(\mathbf{I}-\mathbf{M})\, d\mathbf{Y} = d\mathbf{G} - \mathbf{M} d\mathbf{T}$$

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- But, let $\mathbf{K} = -\sum_{t=1}^{\infty}(1+r)^{-t}\mathbf{F}^t$ where $\mathbf{F}$ is the forward operator. Multiplying:

$$\mathbf{K}(\mathbf{I}-\mathbf{M})\, d\mathbf{Y} = \mathbf{K}(d\mathbf{G} - \mathbf{M} d\mathbf{T})$$

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Proposition

There exists a unique solution to (IKC) for any $d\mathbf{G}, d\mathbf{T}$ satisfying $\mathbf{q}' d\mathbf{G} = \mathbf{q}' d\mathbf{T}$, iff $\mathbf{K}\,(\mathbf{I}-\mathbf{M})$ is invertible. Then, the solution is:

$$d\mathbf{Y} = \mathcal{M}\,(d\mathbf{G} - \mathbf{M} d\mathbf{T})$$

where $\mathcal{M} \equiv (\mathbf{K}\,(\mathbf{I}-\mathbf{M}))^{-1}\,\mathbf{K}$ is a bounded linear operator (“multiplier”)

Baseline model takeaway

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    - Pin down both direct fiscal impulse $d\mathbf{G} - \mathbf{M}d\mathbf{T}$ and multiplier $\mathcal{M}$
- RA, TA, HA, tractable differ in their **M** matrices

Baseline model takeaway

- **iMPCs** $\mathbf{M} = \{\frac{\partial C_t}{\partial Z_s}\}$ are all that matters for macro response to fiscal policy
  - Pin down both direct fiscal impulse $d\mathbf{G} - \mathbf{M}d\mathbf{T}$ and multiplier $\mathcal{M}$
- RA, TA, HA, tractable differ in their $\mathbf{M}$ matrices
- **Next:**
  - look at $\mathbf{M}$'s in data and compare with RA, TA, HA, tractable
  - implications for $d\mathbf{Y}$

Using iMPCs to discriminate across models

Measuring aggregate iMPCs using individual iMPCs

- Object of interest: **(aggregate) iMPCs**

$$M_{ts} = \frac{\partial \mathcal{C}_t}{\partial Z_s}$$

where $C_t = \int c_{it} di$ and $Z_s = \int z_{is} di$

- Direct evidence on M_{ts} is hard to come by for general s
- More work on column $s = 0$ (unanticipated income shock)
    - Can write

$$M_{t0} = \int \underbrace{\frac{z_{i0}}{Z_0}}_{\text{income weight}} \cdot \underbrace{\frac{\partial \mathbb{E}_0 [c_{it}]}{\partial z_{i0}}}_{\text{individual iMPC}} di$$

$\rightarrow$ aggregate iMPCs are **weighted individual iMPCs**

Obtain date-0 iMPCs from cross-sectional microdata

- Two sources of evidence on $\frac{\partial \mathbb{E}_0[c_{it}]}{\partial z_{i0}}$:

1. Fagereng Holm Natvik (2021) measure in Norwegian data

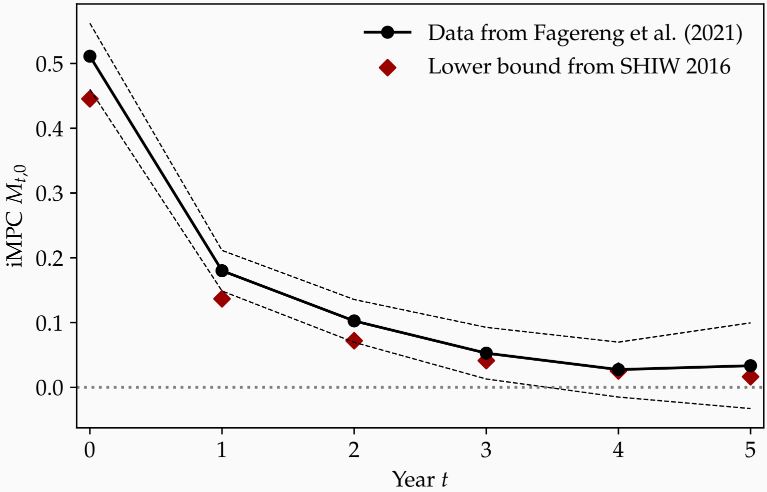
$$c_{it+k} = \alpha_i + \tau_{t+k} + \gamma_k \text{lottery}_{it} + \delta X_{it} + \epsilon_{it}$$

   - Weighting by income in year of lottery receipt $\Rightarrow M_{t,0}$

2. Italian survey data (SHIW 2016) on $\frac{\partial c_{i0}}{\partial z_{i0}}$
   - Lower bound for $M_{t,0}$ using distribution of MPCs
   - Example: income-weighted average of $(1 - MPC_i)MPC_i \Rightarrow$ lower bound for $M_{1,0}$

M1,0 is smallest when the households who save the most out of an income shock in year 0—i.e. the households with the lowest MPC, and the highest savings entering the next period—are again the households who have the lowest MPC out of their savings in year 1.

iMPCs in the data: first column $M_{t,0}$

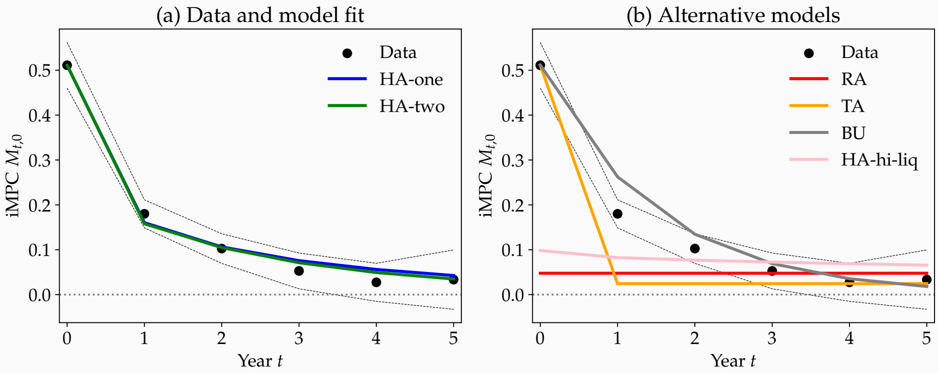


Compare iMPCs across models

▸ Calibration

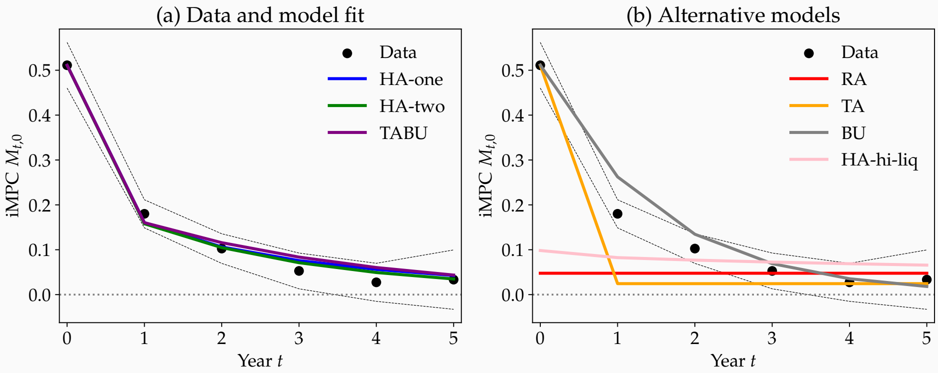
- Models with standard calibration $r = 5\%$, $EIS = 1$, $A/Y = 4.1$ [Kaplan-Violante]
  - RA, calibrated to $r = 5\%$
  - HA-hi-liq: one-asset HA, calibrated to $\frac{A}{Y}$
- Models with one free parameter, calibrated to match $M_{0,0}$
  - TA: adjusting share of hand-to-mouth μ
  - BU: adjusting curvature $\chi''(A)$
  - HA-one: adjusting level of assets A
  - HA-two: adjusting Calvo freq of adjustment ν

iMPCs across models



- Useful discrimination device! Rule in HA-one and HA-two, out others
- Can a tractable model fit too?

iMPCs across models including TABU



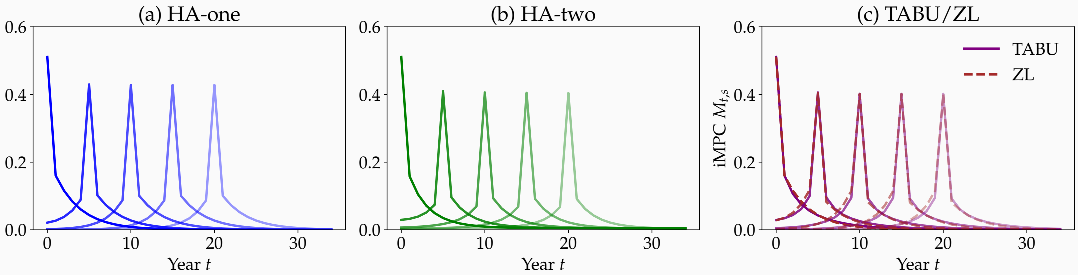
- Yes! TABU provides another great fit (parameterize with $M_{0,0}$ and $M_{1,0}$)
- Why? TABU has same first column as *zero* liquidity model (vs HA-one ~ 0)

What about non-date-0 iMPCs?

- Existing evidence useful for response to date-0 income shocks, $\{M_{t,0}\}$
- What about response to future shocks?

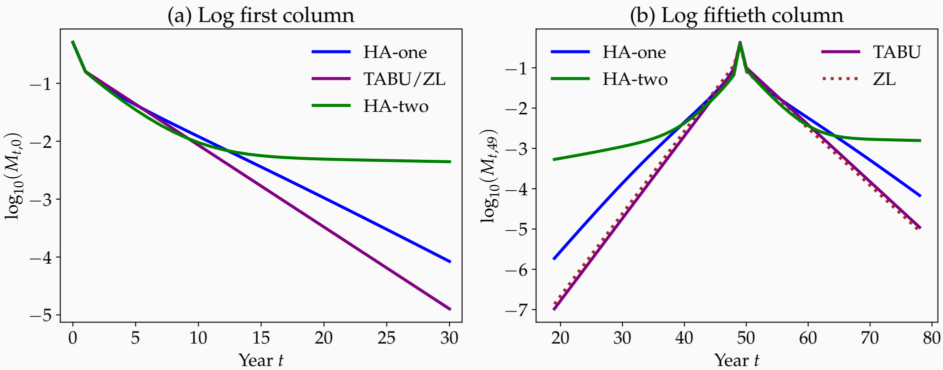
$\rightarrow$ rely on calibrated models fitting iMPCs to fill in the blanks

Response to other income shocks



- Reassuring: all models that fit first column imply similar later columns

Zooming in



most assets remaining from an income shock are in the illiquid account, and thereafter are depleted slowly

- Zooming in, some subtle differences appear...
- This will turn out to matter quantitatively (though not qualitatively)

Fiscal policy in the IKC environment

Fiscal policy in the baseline model

- Recall **intertemporal Keynesian cross:**

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M} \cdot d\mathbf{T} + \mathbf{M} \cdot d\mathbf{Y}$$

- $d\mathbf{Y}$ entirely determined by iMPCs $\mathbf{M}$ and fiscal policy $(d\mathbf{G}, d\mathbf{T})$

- Next: Characterize role of iMPCs for

    1. balanced budget policies, $d\mathbf{G} = d\mathbf{T}$
    2. deficit-financed policies

The balanced-budget multiplier

Proposition (Balanced-budget multiplier)

Under balanced-budget policy, $d\mathbf{G} = d\mathbf{T}$, the fiscal multiplier is 1 at all dates

$$d\mathbf{Y} = d\mathbf{G}$$

- Similar reasoning already in Haavelmo (1945)
- Generalizes Bilbiie (2009)'s and Woodford (2011)'s RA result
  - heterogeneity irrelevant for balanced budget fiscal policy
  - similar to Werning (2015)'s result for monetary policy
- Proof: $d\mathbf{Y} = d\mathbf{G}$ is unique solution to

$$d\mathbf{Y} = (I - \mathbf{M}) \cdot d\mathbf{G} + \mathbf{M} \cdot d\mathbf{Y}$$

Deficit-financed fiscal policy

Proposition (Deficit-financed multiplier)

When $d\mathbf{G} \neq d\mathbf{T}$, the consumption response $d\mathbf{C}$ depends on the primary deficit

$$d\mathbf{Y} = d\mathbf{G} + \underbrace{\mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T})}_{d\mathbf{C}}$$

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- Example 1: TA model with deficit financing

$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu}(d\mathbf{G} - d\mathbf{T})$$

- Here, consumption $d\mathbf{C}$ depends only on **current** deficit
  - **initial multiplier** can be large $\in \left[1, \frac{1}{1-\mu}\right]$ ...
  - but **cumulative multiplier** is $= 1$, $\mathbf{q}' d\mathbf{Y} = \mathbf{q}' d\mathbf{G} \Rightarrow \frac{\sum (1+r)^{-t} dY_t}{\sum (1+r)^{-t} dG_t} = 1$

[see also Bilbiie, Monacelli, Perotti 2013]

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- Example 2: TABU model with deficit financing

$$dY_t = dG_t + \frac{\mu}{1-\mu}(dG_t - dT_t) + (1+r)\frac{1-\frac{\lambda}{1+r}}{1-\mu}\left(\frac{1}{\lambda} - \beta(1+r)\right)\sum_{s=0}^{\infty}(\beta(1+r))^s dB_{t+s}$$

- Very powerful effect of deficits: discounted time path of *debt* dB_t matters

Deficit-financed fiscal policy

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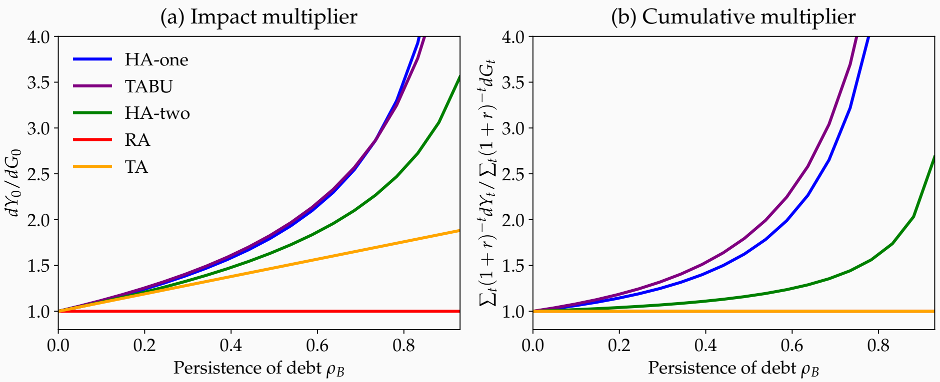
- Very powerful effect of deficits: discounted time path of *debt* dB_t matters
- Corollary: when $d\mathbf{B} > 0$, recalibrating μ and λ to match a higher M_{10} with the same M_{00} always strictly increases the **cumulative multiplier** !

Simulate model responses for more general shocks

- Parametrize: $dG_t = \rho_G dG_{t-1}$ and $dB_t = \rho_B (dB_{t-1} + dG_t)$, vary ρ_B [with $\rho_G = 0.76$]



- Data-consistent iMPCs $\Rightarrow$ cumulative multipliers above 1, rise with ρ_B
- HA-two multipliers below HA-one $\sim$ TABU (long-term savers more ricardian)

HA-two's higher long-term iMPCs make the model slightly more Ricardian, weakening the short-term intertemporal Keynesian cross.

Sufficient statistics beyond the baseline model

Broader models

- In broader models, (IKC) no longer enough to characterize equilibrium
- But if heterogeneity is all at household level, still 3 sufficient statistics:

$$d\mathbf{C} = \mathbf{M}d\mathbf{Z} + \mathbf{m}^{cap}dcap + \mathbf{M}^{r}d\mathbf{r}$$

- $\mathbf{m}^{cap}$ is response of C to capital gains, $\mathbf{M}^{r}$ to interest rate changes

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- $\mathbf{m}^{cap}$ is response of C to capital gains, $\mathbf{M}^{r}$ to interest rate changes
- Some data on former, not much on latter! Yet we prove: [cf Werning, 2015]

Proposition

For RA, TA, HA-one, and HA-two, if EIS $= 1$ and equal initial porfolio shares:

$$\mathbf{M}^{r} = -C\left(\mathbf{I} - \left(1 - \frac{rA}{C}\right)\mathbf{M}\right)\mathbf{U} + (1+r)A\,\mathbf{m}^{cap}\mathbf{1}'$$

where $\mathbf{U}$ *the matrix with ones on and above the diagonal*

- iMPCs still sufficient statistics, provided you include capital gains!

A and C are aggregate assets and consumption in the steady state.

Fiscal policy in a quantitative environment

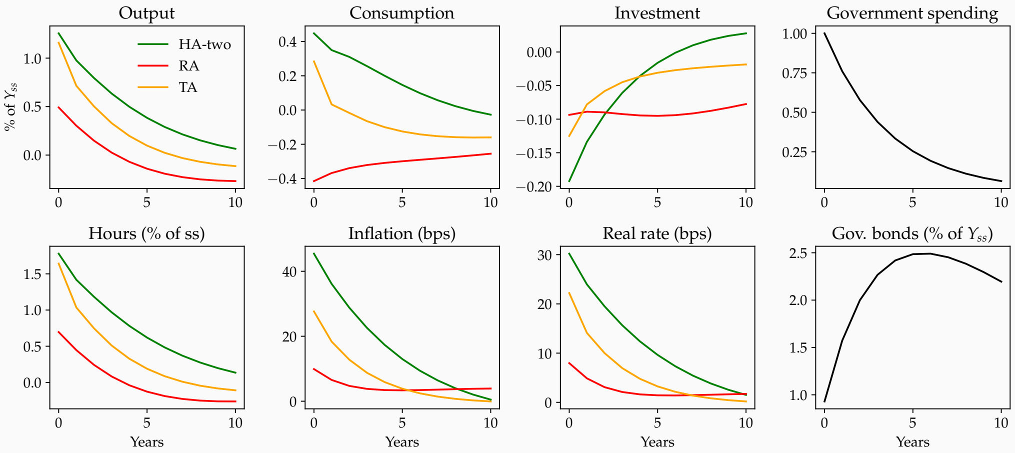
Additional elements relative to the IKC environment...

Calibration

- **Government:**
  - still has spending shock $dG_t = \rho_G dG_{t-1}$ and fiscal rule $dB_t = \rho_B \left(dB_{t-1} + dG_t\right)$
  - now follows Taylor rule $i_t = r + \phi\pi_t$, $\phi > 1$
- **Supply side:**
  - Cobb-Douglas production, $Y_t = K_t^{\alpha} N_t^{1-\alpha}$
  - K_t subject to quadratic capital adjustment costs
  - sticky prices à la Calvo, $\pi_t = \kappa^p mc_t + \frac{1}{1+r_t}\pi_{t+1}$
- **Two reasons for lower multipliers** relative to IKC environment:
  - distortionary taxation & crowding-out of consumption and investment via r

Sizable output response to deficit-financed G in HA-two

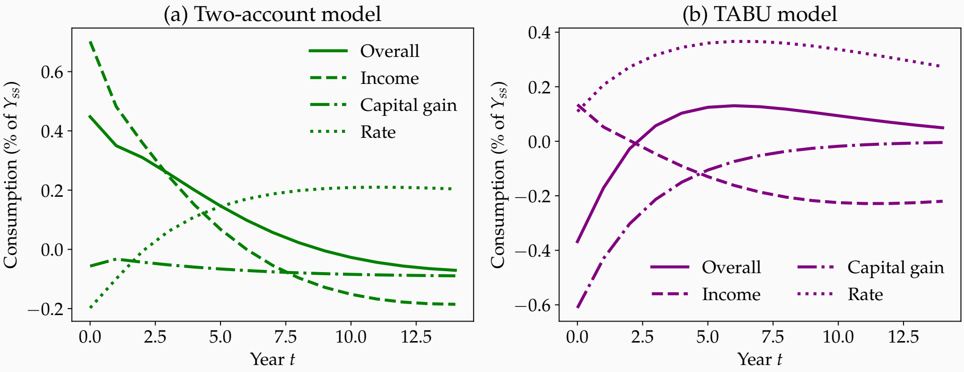
▸ Other



Calibration: $\rho_G = 0.76$, $\kappa^w = 0.03$, $\kappa^p = 0.23$, $\phi = 1.5$; vary ρ_B in $dB_t = \rho_B\,(dB_{t-1} + dG_t)$

Output response shaped by iMPCs from income and capital gains

Impulse



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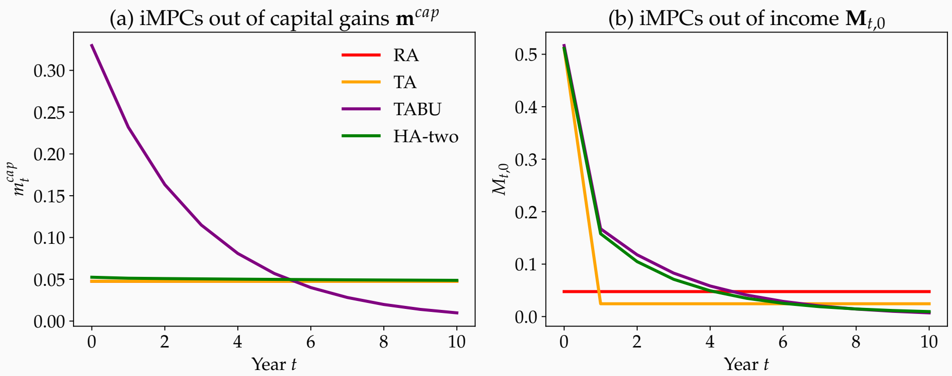
Tractable models calibrated to income MPCs imply implausibly large $\mathbf{m}^{cap}$!



- Empirical evidence suggests MPC out of capital gains small , $\sim$RA
  [di Maggio, Kermani and Majlesi 2018, Chodorow-Reich-Nenov-Simsek 2021, ...]

Summary: TA, HA-two have large **initial** deficit-financed multipliers

▸ graph

Initial multipliers $\frac{dY_0}{dG_0}$

Fiscal rule	Model	RA	TA	TABU	HA-one	HA-two
bal. budget	IKC	1.0	1.0	1.0	1.0	1.0
	quantitative	0.5	0.4	0.3	—	0.5
deficit-financed	IKC	1.0	1.9	5.6	6.9	3.6
	quantitative	0.5	1.2	0.4	—	1.3

Calibration: $\rho_G = 0.76$, $\kappa^w = 0.03$, $\kappa^p = 0.23$, $\phi = 1.5$

. . . but only HA-two has large **cumulative** multipliers in quantitative model

Cumulative multipliers $\frac{\sum_t (1+r)^{-t} dY_t}{\sum_t (1+r)^{-t} dG_t}$

Fiscal rule	Model	RA	TA	TABU	HA-one	HA-two
bal. budget	IKC	1.0	1.0	1.0	1.0	1.0
	quantitative	0.4	0.3	0.2	—	0.4
deficit-financed	IKC	1.0	1.0	15.5	16.6	2.7
	quantitative	-0.4	0.5	0.8	—	1.3

Calibration: $\rho_G = 0.76$, $\kappa^w = 0.03$, $\kappa^p = 0.23$, $\phi = 1.5$

Conclusion

M matters for **M**acro !

$\rightarrow$ crucial for GE propagation
$\rightarrow$ new insights for fiscal policy

New avenues for research:

$\rightarrow$ more evidence on **M** and $\mathbf{m}^{cap}$
$\rightarrow$ sequence-space Jacobians for aggregate implications of heterogeneity

Extra slides

 back

- Mass 1 of unions. Each union k
  - employs a representative sample of individuals n_{ik}
  - produces task $N_k = \int e_i n_{ik} di$ from member hours
  - pays common wage W_k per efficient unit of work e
  - requires that all members work $n_{ik} = N_k$
- Final good firms aggregate $N \equiv \left(\int_0^1 N_k^{\frac{\epsilon-1}{\epsilon}} dk\right)^{\frac{\epsilon}{\epsilon-1}}$
- Union k sets W_{kt} each period to maximize

$$\max_{W_{kt}} \sum_{\tau \geq 0} \beta^\tau \left\{ \int \left\{ u\left(c_{it+\tau}\right) - v\left(n_{it+\tau}\right) \right\} di - \frac{\psi}{2} \left(\frac{W_{kt+\tau}}{W_{kt+\tau-1}} - 1 \right)^2 \right\}$$

$\Rightarrow$ same $W_k = W$ for all; nonlinear wage Phillips curve

$$\left(1+\pi_t^w\right)\pi_t^w = \frac{\epsilon}{\psi} \int N_t \left(v'\left(n_{it}\right) - \frac{\epsilon-1}{\epsilon} \frac{\partial z_{it}}{\partial n_{it}} u'\left(c_{it}\right) \right) di$$
$$+ \beta \pi_{t+1}^w \left(1+\pi_{t+1}^w\right)$$

Complete expression for $\mathbf{M}^{BU}$

back

- Define $\mathbf{A}^{BU}$ as:

$$\mathbf{A}^{BU} = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda & 1 & 0 & \cdots \\ \lambda^2 & \lambda & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \frac{\lambda}{1+r} & -\left(1 - \frac{\lambda}{1+r}\right) \cdot \beta\lambda & -\left(1 - \frac{\lambda}{1+r}\right) \cdot (\beta\lambda)^2 & \cdots \\ 0 & \frac{\lambda}{1+r} & -\left(1 - \frac{\lambda}{1+r}\right) \cdot \beta\lambda & \cdots \\ 0 & 0 & \frac{\lambda}{1+r} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

and $\mathbf{L}$ as lag operator, then:

$$\mathbf{M}^{BU} = \mathbf{I} - (\mathbf{I} - (1+r)\,\mathbf{L})\mathbf{A}^{BU}$$

Why $\mathbf{K}$?

back

- Recall buget constraint:

$$a_{it} + c_{it} = z_{it} + (1 + r)\, a_{it-1}$$

Aggregating and defining $A_{ts} = \frac{\partial A_t}{\partial Z_s}$, we have:

$$(\mathbf{I} - (1 + r)\, \mathbf{L})\, \mathbf{A} = \mathbf{I} - \mathbf{M}$$

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Now, $\mathbf{K} = -\sum_{t=1}^{\infty} (1 + r)^{-t}\, \mathbf{F}^t$ is left inverse of $(\mathbf{I} - (1 + r)\,\mathbf{L})$ so

$$\mathbf{A} = \mathbf{K}\,(\mathbf{I} - \mathbf{M})$$

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- IKC rewrites:

$$\mathbf{A}\,(d\mathbf{Y} - d\mathbf{T}) = \mathbf{K}\,(d\mathbf{G} - d\mathbf{T}) = d\mathbf{B}$$

where $\{dB_t\}$ is the path of government debt

- This is the asset market clearing condition!

Why **K**?

back

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where $\{dB_t\}$ is the path of government debt

- This is the asset market clearing condition!
- Solving in the asset market is also numerically more stable

back

- We have:

$$\mathbf{A} = \mathbf{K}\left(\mathbf{I} - \mathbf{M}\right)$$

how do we know when **A** is invertible?

Winding number test

◂ back

- We have:

$$\mathbf{A} = \mathbf{K}\left(\mathbf{I} - \mathbf{M}\right)$$

  how do we know when **A** is invertible?

  1. Analytical models: **A** has closed form solution, can check directly
  2. Quantitative models: **A** has "quasi-Toeplitz" structure

$$A_{t,s} \underset{t,s\to\infty}{\to} a_{t-s}$$

- (Generic) invertibility iff "winding number" of $a\left(z\right) = \sum_{j=-\infty}^{\infty} a_j z^j$ is zero
- This is sequence-space equivalent of Blanchard-Kahn condition
  - For recursive (Toeplitz) models see Onatski (2010)
  - For stationary heterogeneous-agent (quasi-Toeplitz) models see Auclert, Rognlie, Straub (2023) "Determinacy and Existence in the Sequence Space"

Equilibrium definition

back

- Given $\{G_t, T_t\}$, a **general equilibrium** is a set of prices, household decision rules and quantities such that, at all t:

    1. firms optimize
    2. households optimize
    3. fiscal and monetary policy rules are satisfied
    4. the goods market clears

Calibration for baseline model

back

- Preferences: $u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$, $v(n) = b\frac{n^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}$. Income process: $\log e_t = \rho_e \log e_{t-1} + \sigma \epsilon_t$

Parameter	Parameter	HA-two	HA-hi-liq	HA-one
σ	EIS	1		
ϕ	Frisch	1		
(ρ_e, σ)	Log e persistence & s.dev	(0.91,0.92)		
λ	Tax progressivity	0.181		
G/Y	Spending-to-GDP	0.2		
A/Z	Wealth-to-aftertax income	6.29		0.21
A^{illiq}/Z	Iliquid assets-to-aftertax income	4.83	–	–
β	Discount factor	0.93	0.94	0.87
r	Real interest rate	0.05		
$r - r^l$	Illiquid-liquid spread	0.08	–	–
ν	Adjustment probability	0.089	–	–

Calibration for quantitative model

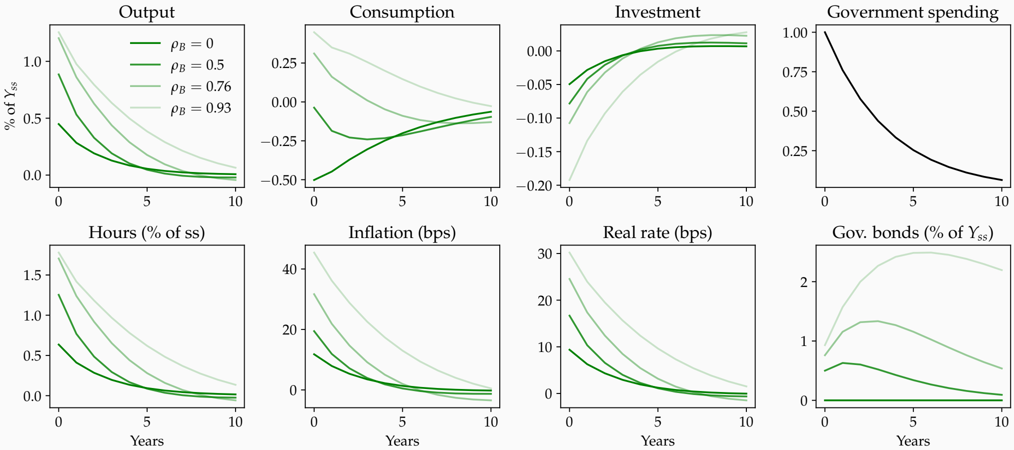
back

- As in baseline model, plus:

Parameter	Parameter	
α	Capital share	0.294
B/Y	Debt-to-GDP	0.7
K/Y	Capital-to-GDP	2.26
μ	SS markup	1
δ	Depreciation rate	0.08
ϵ_I	Invest elasticity to q	4
κ^p	Price flexibility	0.27
κ^w	Wage flexibility	0.03
ϕ	Taylor rule coefficient	1.5

Impulse responses in quantitative model

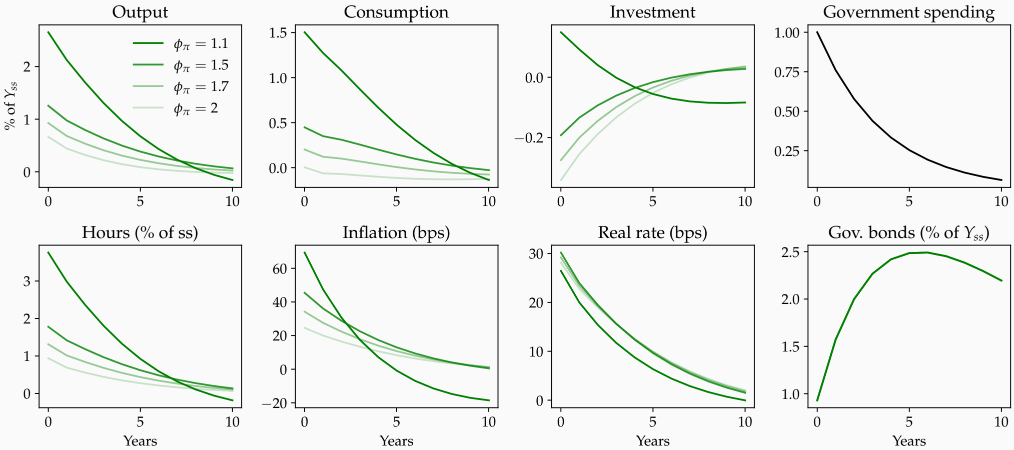
back



Calibration: $\rho_G = 0.76$, $\kappa^w = 0.03$, $\kappa^p = 0.23$, $\phi = 1.5$

True unless very responsive Taylor rule

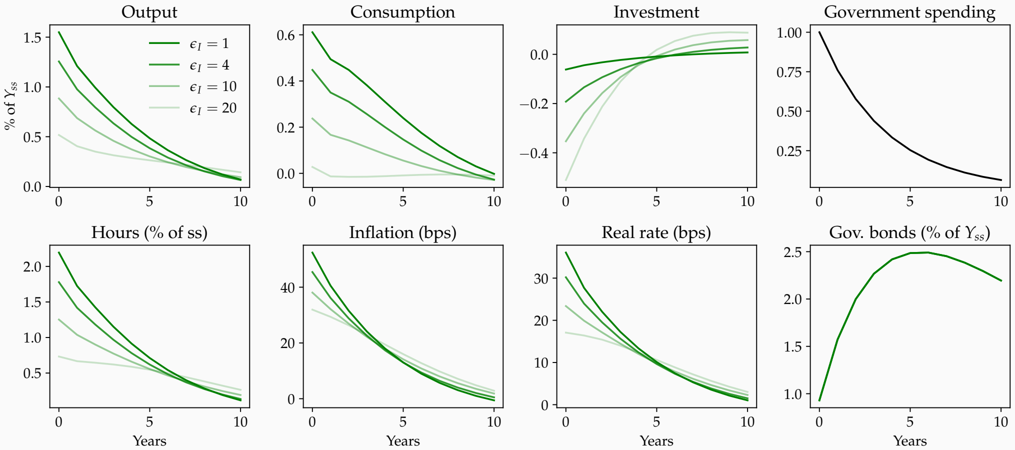
back



Calibration $\rho_G = 0.76$, $\rho_G = 0.93$, $\kappa^w = 0.03$, $\kappa^p = 0.23$, and vary ϕ in Taylor rule

True even with more elastic (unless *very* elastic)

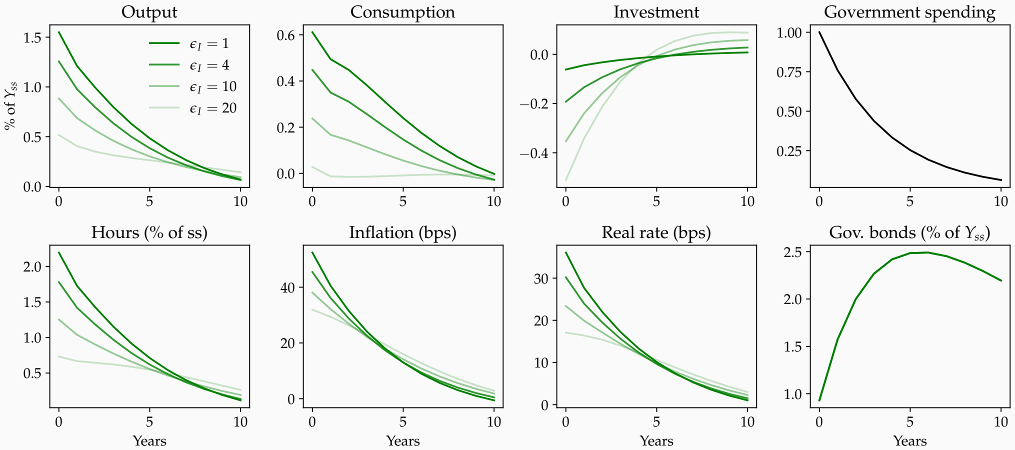
back



Calibration: $\rho_G = 0.76$, $\rho_G = 0.93$, $\kappa^w = 0.03$, $\phi = 1.5$ and vary κ^p in price Phillips curve

True even with more flexible wages (unless *very* flexible)

back



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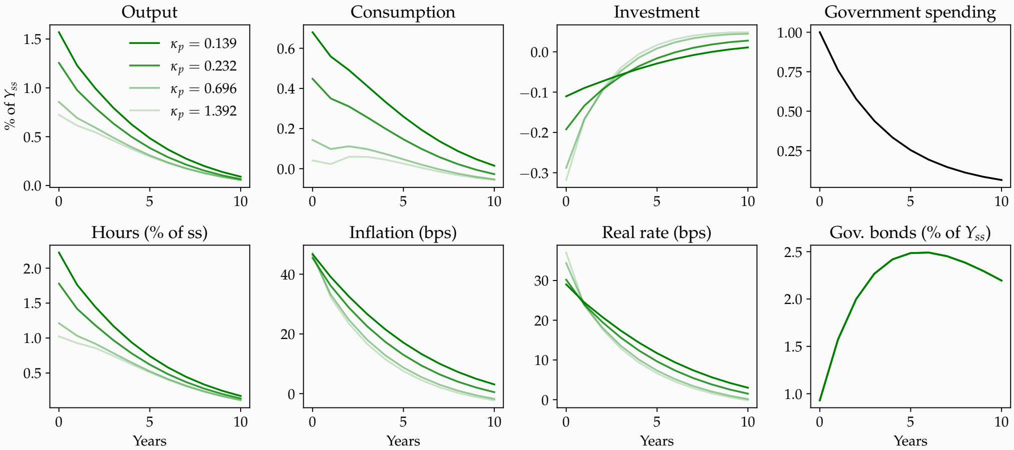
True even with more flexible prices (unless *very* flexible)

back

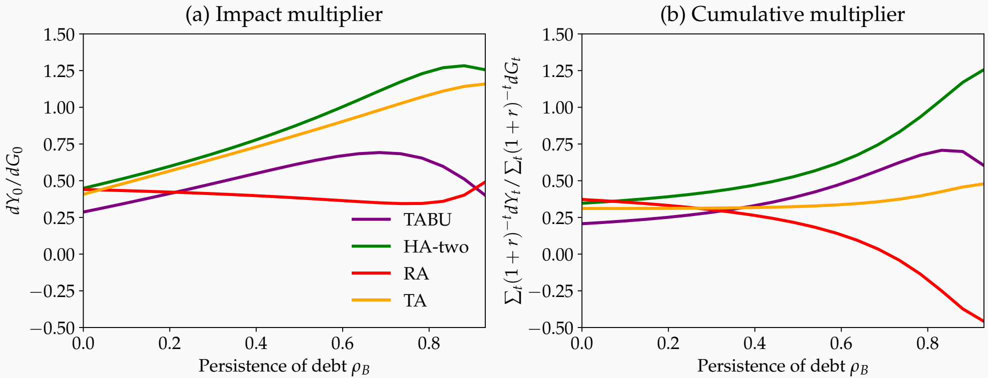


Calibration: $\rho_G = 0.76$, $\rho_G = 0.93$, $\kappa^w = 0.03$, $\phi = 1.5$, and vary κ^p in price Phillips curve

Quantitative models against deficit-financing

back

- Parametrize: $dG_t = \rho_G dG_{t-1}$ and $dB_t = \rho_B (dB_{t-1} + dG_t)$, vary ρ_B [with $\rho_G = 0.76$]



- Multipliers lower across the board due to neoclassical forces
- But role of iMPCs still very visible!