Solow

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1 Solow Model

This notebook writes a simple class to implement the Solow model.

1.1 What's a Class?

In Python, a class is a blueprint for creating objects. It encapsulates data for the object and methods to manipulate that data. Here's a simple example to illustrate what a class looks like in Python:

[11]:
```
class MyClass:
    def __init__(self, name):
        self.name = name  # Instance variable

    def greet(self):
        print(f"Welcome to {self.name}")

# Creating an instance of MyClass
my_object = MyClass("Intermediate Macroeconomics")

# Calling a method on the instance
my_object.greet()
```

```
Welcome to  Intermediate Macroeconomics
```

In this example: - `MyClass` is the name of the class. - The `__init__` method is a special method called a constructor, which is automatically called when a new object is created. It initializes the object's attributes. - `self` represents the instance of the class and is used to access the object's attributes and methods. - `name` is a parameter passed to the constructor, which is then set as an instance variable.

1.2 What's a solow model

The Solow growth model is a neoclassical growth model in which the per capita capital stock evolves according to the rule (**law of motion**)

$$k_{t+1} = \frac{szk_t^{\alpha} + (1-\delta)k_t}{1+n}$$

1.2.1 Here:

- s is an exogenously given saving rate
- z is a productivity parameter
- α is capital's share of income
- n is the population growth
- δ is the depreciation rate

1.2.2 Steady state

A steady state here is a level of capital k such that

$$k_t = k_{t+1} = k$$

1.3 Build the model

Here's a class that implements this model.

Some points of interest in the code are:

- An instance maintains a record of its current capital stock in the variable `self.k`.
- The `h` method implements **the right-hand side of law of motion**.
- The `update` method uses `h` to update capital
- Notice how inside `update`, the reference to the local method `h` is `self.h`.
- The methods `steady_state` and `generate_sequence` are fairly self-explanatory.

[12]:
```
class Solow:
    """
    Implements the Solow growth model with the update rule

        k_{t+1} = [(s z k^_t) + (1 - )k_t] /(1 + n)

    """
    def __init__(self, n=0.05,  # population growth rate
                       s=0.25,  # savings rate
                        =0.1,   # depreciation rate
                        =0.3,   # share of labor
                       z=2.0,   # productivity
                       k=1.0):  # current capital stock
        self.n, self.s, self. , self. , self.z = n, s,  ,  , z
        self.k = k

    def h(self):
        "Evaluate the right hand side of law of motion for capital"
        # Unpack parameters (get rid of self to simplify notation)
        n, s,  ,  , z = self.n, self.s, self. , self. , self.z
        # Apply the update rule
        return (s * z * self.k**  + (1 -  ) * self.k) / (1 + n)

    def update(self):
```

```
        "Update the current state (i.e., the capital stock)."
        self.k =  self.h()

    def steady_state(self):
        "Compute the steady state value of capital."
        # Unpack parameters (get rid of self to simplify notation)
        n, s,  ,  , z = self.n, self.s, self. , self. , self.z
        # Compute and return steady state
        return ((s * z) / (n +  ))**(1 / (1 -  ))

    def generate_sequence(self, t):
        "Generate and return a time series of length t"
        path = []
        for i in range(t):
            path.append(self.k)
            self.update()
        return path
```

Here's a little program that uses the class to compute time series from two different initial conditions.

The common steady state is also plotted for comparison

[13]:
```
import matplotlib.pyplot as plt
```

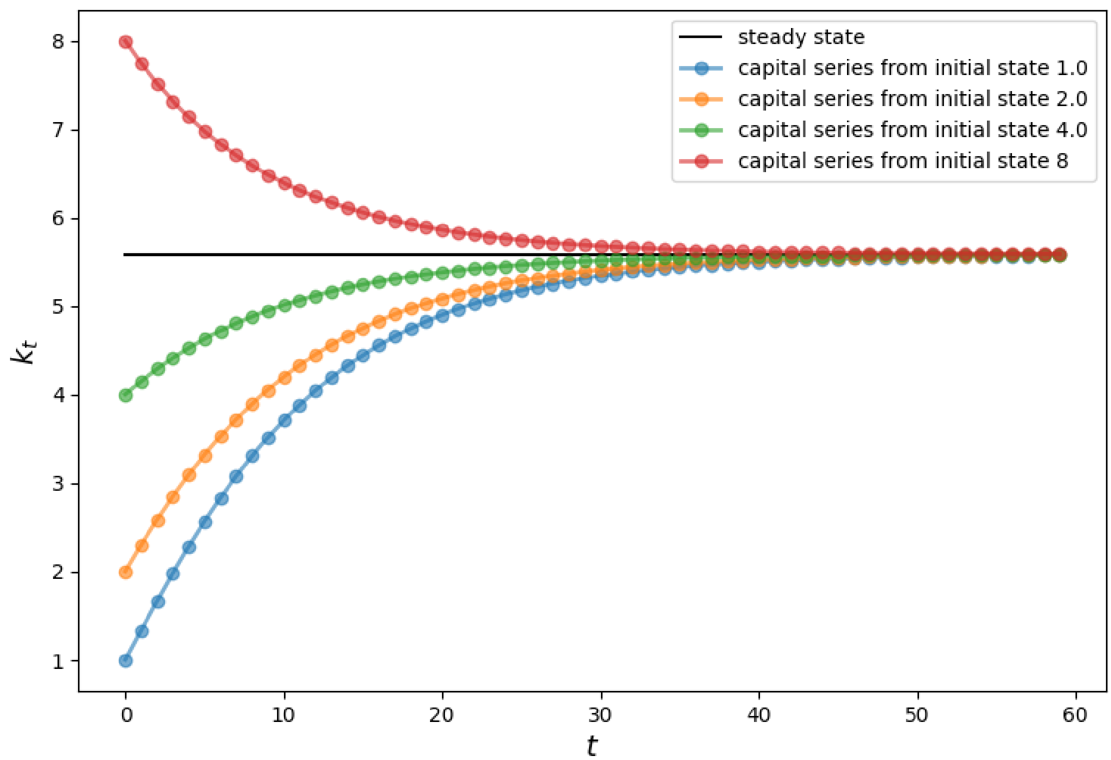
[14]:
```
s1 = Solow()
s2 = Solow(k=2.0)
s3= Solow(k=4.0)
s4=Solow(k=8)

T = 60
fig, ax = plt.subplots(figsize=(9, 6))

# Plot the common steady state value of capital
ax.plot([s1.steady_state()]*T, 'k-', label='steady state')

# Plot time series for each economy
for s in s1, s2, s3, s4:
    lb = f'capital series from initial state {s.k}'
    ax.plot(s.generate_sequence(T), 'o-', lw=2, alpha=0.6, label=lb)

ax.set_xlabel('$t$', fontsize=14)
ax.set_ylabel('$k_t$', fontsize=14)
ax.legend()
plt.show()
```


steady state
capital series from initial state 1.0
capital series from initial state 2.0
capital series from initial state 4.0
capital series from initial state 8
k_t
t