Note

Please write your solutions clearly and show all work. Use complete sentences where appropriate. You may use any theorem proved in class.

Problem 0.1: Limit Evaluation

Evaluate the following limit and prove your answer:

$$\lim_{x \to 0} \frac{\sin(x)}{x}$$

Solution

We will prove that $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$ using the squeeze theorem.
First, recall that for $x \neq 0$:

$$\cos(x) \leq \frac{\sin(x)}{x} \leq 1$$

Since $\lim_{x \to 0} \cos(x) = 1$, by the squeeze theorem:

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

Problem 0.2: Sequence Convergence

Determine whether the sequence $a_n = \frac{n^2+1}{n^2-1}$ converges as $n \to \infty$. If it converges, find its limit.

Solution

Let's analyze $\lim_{n \to \infty} \frac{n^2+1}{n^2-1}$

$$\lim_{n \to \infty} \frac{n^2 + 1}{n^2 - 1} = \lim_{n \to \infty} \frac{1 + \frac{1}{n^2}}{1 - \frac{1}{n^2}}$$
$$= \frac{1 + 0}{1 - 0} = 1$$

Therefore, the sequence converges to 1.