Introduction to Analysis

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Course:	MATH 104
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Contents

1 Course Overview

Note

These notes compile key concepts, theorems, and insights from the Mathematical Analysis course. They serve as a comprehensive reference for understanding advanced mathematical principles.

2 Limits and Continuity

2.1 Fundamental Limit Concepts

Definition 2.1: Limit of a Function

For a function $f(x)$, we say $\lim_{x\to a} f(x) = L$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that:

$$0 < |x - a| < \delta \implies |f(x) - L| < \epsilon$$

Theorem 2.1: Limit Laws

If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then:

1. $\lim_{x\to a}[f(x) + g(x)] = L + M$
2. $\lim_{x\to a}[f(x) \cdot g(x)] = L \cdot M$
3. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$, if $M \neq 0$

2.2 Continuity Analysis

Example 2.1: Continuous Function

Consider $f(x) = x^2$. This function is continuous at every point $x \in \mathbb{R}$ because:

$$\lim_{h\to 0}[f(x+h) - f(x)] = \lim_{h\to 0}[(x+h)^2 - x^2] = 0$$

3 Differentiation Techniques

3.1 Product Rule Exploration

Exercise 3.1: Derivative Computation

Compute the derivative of $f(x) = x^3 \sin(x)$ using the product rule.

Solution

Using the product rule, $\frac{d}{dx}[f(x)] = \frac{d}{dx}[x^3] \cdot \sin(x) + x^3 \cdot \frac{d}{dx}[\sin(x)]$

$$f'(x) = 3x^2 \sin(x) + x^3 \cos(x)$$

4 Linear Algebra Foundations

4.1 Vector Space Fundamentals

Definition 4.1: Vector Space

A vector space V over a field $\mathbb{F}$ is a set with two operations:

- Vector addition: $+ : V \times V \to V$
- Scalar multiplication: $\cdot : \mathbb{F} \times V \to V$

satisfying specific axioms of vector spaces.

Example 4.1: Linear Transformation

Consider a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by:

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x - y \\ x + 3y \end{pmatrix}$$

This transformation preserves vector addition and scalar multiplication.

5 Numerical Methods

Algorithm 1: Selection Sort Algorithm

Data: Input array A of n elements
Result: Sorted array in ascending order

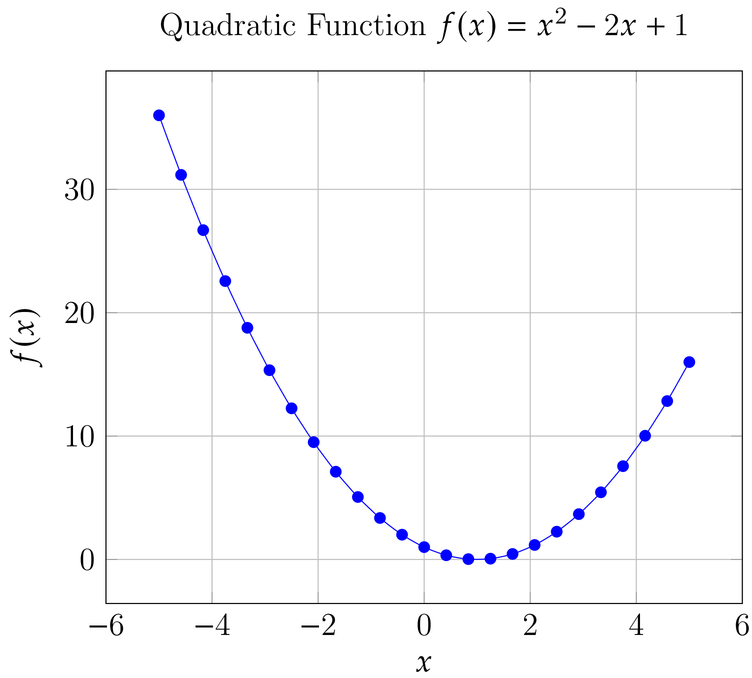
begin
    for $i \leftarrow 1$ **to** $n - 1$ **do**
        $\min\_idx \leftarrow i$;
        for $j \leftarrow i + 1$ **to** n **do**
            if $A[j] < A[\min\_idx]$ **then**
                $\min\_idx \leftarrow j$;
            end
        end
        Swap $A[i]$ and $A[\min\_idx]$;
    end
end

6 Graphical Representations

7 Supplementary Notes

Note

These notes are a personal compilation of key mathematical concepts, serving as a study aid and reference for Advanced Mathematical Analysis.



Problem 7.1: Research Challenge

Investigate the convergence properties of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for different values of p.