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Unique equilibrium in the Eaton–Gersovitz model of sovereign debt[☆]

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ABSTRACT

A common view of sovereign debt markets is that they are prone to multiple equilibria. We prove that, to the contrary, Markov perfect equilibrium is unique in the widely studied model of Eaton and Gersovitz (1981), and we discuss multiple extensions and limitations of this finding. Our results show that no improvement in a borrower's reputation for repayment can be self-sustaining, thereby strengthening the Bulow and Rogoff (1989) argument that debt cannot be sustained by reputation alone.

1. Introduction

A common view of sovereign debt markets is that they are prone to multiple equilibria: a market panic may inflate bond yields, deteriorate the sustainability of government debt and precipitate a default event, justifying investor fears. Indeed, Mario Draghi's speech in July 2012, announcing that the ECB was "ready to do whatever it takes" to preserve the single currency, and the subsequent creation of the Outright Monetary Transactions (OMT) program, are widely seen as having moved Eurozone sovereign debt markets out of an adverse equilibrium: since then, bond spreads have experienced dramatic falls as fears of default have receded.

At the same time, in the last decade, a booming quantitative literature in the line of Eaton and Gersovitz (1981)—initiated by Arellano (2008) and Aguiar and Gopinath (2006), and summarized in Aguiar and Amador (2015)—has studied sovereign debt markets using an infinite-horizon incomplete markets model for which no result on equilibrium multiplicity was known. Many researchers suspected that the model might feature multiple equilibria,[1] yet in numerical computations the literature had not found any explicit case of multiplicity. In this paper we explain why, by proving that equilibrium is *unique* in the benchmark infinite-horizon model with a Markov process for the exogenous driving state and exogenous value from default. Although we emphasize Markov perfect equilibrium—the usual equilibrium concept in the literature, and one for which our argument is especially direct—we show that our core uniqueness result extends to subgame perfect equilibria more generally. We also extend our proof to several modifications of the benchmark model, as described below.

[☆] This is a revised version of a chapter of our Ph.D. dissertations at MIT.
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[1] See Krusell and Smith (2003) for an example of multiple Markov perfect equilibria in an infinite-horizon economy.

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Why could multiplicity arise in the benchmark model we study? To build intuition, consider the simplest environment: one in which debt is restricted to be risk-free, as in Zhang (1997). A Markov perfect equilibrium of this model features a (constant) endogenous debt limit, which is the most that can be incurred today without the possibility that the government will want to default tomorrow. Suppose credit becomes tighter in the future—tomorrow's debt limit falls. Since the government is less able to smooth consumption fluctuations, its perceived benefit from access to credit is now lower, and so its willingness to repay today's debts falls. In response, investors lower today's debt limit as well.

Through this process, an equilibrium with loose credit and high willingness to repay debts could turn into one with tight credit and low willingness to repay. Similarly, in the full Eaton and Gersovitz (1981) model with risky debt, investors' pessimistic expectations about the likelihood of default could translate into higher risk premia on debt—which, by making debt service more costly and continued access to credit markets less valuable, would encourage default and validate the original pessimism. This mechanism sounds appealing, and in our view it captures an important part of the common intuition for equilibrium multiplicity in sovereign debt markets.

Our results rule it out. The intuition remains simplest in the Zhang (1997) environment. If there are two equilibria with distinct debt limits, we consider two governments that are each at the limit in their respective equilibria. We argue that the government with less debt must have a strictly higher value: starting from that point, it can follow a strategy that parallels the strategy of the higher-debt government, maintaining its liabilities at a uniform distance and achieving higher consumption at every point by economizing on interest payments. But this contradicts the assumption that both governments start at their debt limits, where each must obtain the (constant) value of default. In short, once both governments have exhausted their debt capacity, the one with a strictly lower level of debt is strictly better off—meaning that this government should be able to borrow slightly more without running the risk of default, and cannot have exhausted its capacity after all.

Interestingly, this proof strategy by replication has echoes of that used by Bulow and Rogoff (1989) to rule out reputational equilibria in a similar class of models where sovereign governments retain the ability to save after defaulting. The original Bulow–Rogoff result is cast in a complete markets setting. In a second modification of the benchmark model, we specify the only punishment from default as the loss of ability to borrow. As an immediate corollary to our Eaton–Gersovitz uniqueness result, we then obtain the *incomplete markets Bulow–Rogoff result*: under this specification of default costs, the no-borrowing equilibrium is the unique equilibrium. Hence our general uniqueness result nests a central impossibility result for the sovereign debt literature. Here, our paper complements parallel and independent work by Bloise et al. (2016), who explore the validity of the Bulow–Rogoff result in environments with general asset market structures.

We next explore the robustness of our uniqueness result to relaxing various model assumptions. We first consider a case where savings are exogenously bounded. Echoing a result of Passadore and Xandri (2014), we prove that multiple equilibria can exist when no savings is allowed. We also show, however, that uniqueness holds whenever the bound on savings is strictly positive even if arbitrarily small. Next, we consider a case where the value of default is endogenous because governments in default have a stochastic option to reenter markets (a typical assumption in the quantitative literature). In that case, we rule out multiplicity of the most widely suspected form—where bond prices in a favorable equilibrium dominate those in a self-fulfilling adverse one—and obtain complete uniqueness when shocks are independent and identically distributed. Finally, we discuss alternative assumptions that are known to generate multiple equilibria in related contexts, including modifying the timing and commitment assumptions, introducing long-term debt, or assuming low international interest rates.

Our objective is not to deny that sovereign debt markets can be prone to self-fulfilling crises, or that OMT may have ruled out a bad equilibrium. Instead, we hope that our results may help sharpen the literature's understanding of the assumptions that are needed for such multiple equilibria to exist. Our replication-based proof strategy may also be of independent interest, as a general technique for proving uniqueness of equilibrium in infinite-horizon games.

The rest of the paper is organized as follows. Section 2 lays out the benchmark Eaton–Gersovitz model, and establishes uniqueness of Markov perfect equilibrium and uniqueness of subgame perfect equilibrium. Section 3 adapts our main proof to two related models. Section 3.1 proves uniqueness in the Zhang (1997) model, where debt is restricted to be risk-free, and Section 3.2 derives the incomplete markets version of the Bulow and Rogoff (1989) result as a corollary of our main uniqueness result. Section 4 considers the robustness of our results as we relax various assumptions. Section 4.1 considers exogenous restrictions on savings. Section 4.2 considers the case where reentry is allowed after default. Section 4.3 discusses other extensions. Section 5 concludes. Proofs not included in the main text are collected in the online appendix.

2. Equilibrium uniqueness in the benchmark model

In this section we describe our benchmark environment, provide a proof of existence, and move on to establish the core uniqueness result of the paper.

2.1. Model description

We now describe what we call the *benchmark* infinite-horizon model with Markov income (see Aguiar and Amador, 2015). We focus first on Markov perfect equilibria, in which the current states b and s encode all the relevant history. In

Section 2.3 we will show that this is without loss of generality, since one can specify this model as a game whose only subgame perfect equilibria are Markov perfect equilibria.

An exogenous state s follows a discrete Markov chain with elements in $\mathcal{S}$, $|\mathcal{S}| = S \in \mathbb{N}$ and transition matrix $\pi(s'|s)$. Output $y(s)$ is a function of this underlying state.

At the beginning of each period, the government starts with some level of debt $b \in \mathcal{B}$. After observing the realization of s, it decides whether to repay b or default. If it does not repay, it receives an exogenous value $V^d(s)$, which encodes all the consequences of default. The assumption that $V^d(s)$ is exogenous and independent of b follows some of the quantitative sovereign debt literature. This assumption is important for the current result. When the literature has considered endogenous $V^d(s)$, it has typically been by including a stochastic reentry option; we will consider this possibility in Section 4.2.

If the government does not default, it receives $y(s)$ as endowment, pays b, and issues new bonds $b' \in \mathcal{B}$ that will be due next period, raising revenue $Q(b', s)$. Its (possibly state-dependent) flow utility from consumption is $u(c, s)$, so that the value V from repayment is given by

$$V(b,s) = \max_{b'} u(c,s) + \beta \mathbb{E}_{s'|s}\left[V^o(b',s')\right]$$
$$\text{s.t.} \quad c + b = y(s) + Q(b',s) \tag{1}$$

and the value V^o including the option to default at the beginning of a period is given by

$$V^o(b,s) = \max_{p \in \{0,1\}} pV(b,s) + (1-p)V^d(s) \tag{2}$$

where $p=1$ denotes the decision to repay and $p=0$ denotes the decision to default.

Debt is purchased by risk-neutral international investors that demand an expected return of R. For convenience, we assume that when a government is indifferent between repayment and default, it chooses to repay: $p(b,s)=1$ if and only if $V(b,s) \geq V^d(s)$. Since investors receive expected repayment $\mathbb{E}_{s'|s}[p(b',s')]$, if follows that the bond revenue schedule Q is

$$Q(b',s) = \frac{b'}{R}\mathbb{E}_{s'|s}\left[p\left(b',s'\right)\right] = \frac{b'}{R}\mathbb{P}_{s'|s}\left[V(b',s') \geq V^d(s')\right] \tag{3}$$

where $\mathbb{P}_{s'|s}[\cdot]$ is the conditional probability operator. We are now ready to define Markov perfect equilibrium, which is the typical focus in the literature.

Definition 1. A Markov perfect equilibrium is a set of policy functions $p(b,s)$, $c(b,s)$, $b'(b,s)$ for repayment, consumption and next period debt, value functions $V(b,s)$ and $V^o(b,s)$, and a bond revenue schedule $Q(b',s)$, all defined on the set $\mathcal{B} \times \mathcal{S}$, such that (1)–(3) are satisfied.

We first prove an existence result—to our knowledge, the first such formal result in the literature. For this we make the following four assumptions.

Assumption 1. $\beta \in (0,1)$, and for each s, $u(\cdot, s)$ is continuous and strictly increasing.

Assumption 2. There exist $\gamma > 0$ and $\kappa > 0$ such that $u(c,s) \leq \gamma c^\kappa$ for all c, s; and $\beta R^\kappa < 1$.

Assumption 3. $\lim_{c \to 0} u(c,s) = -\infty$.

Assumption 4. $\mathcal{B} = \left[\underline{b}, \overline{b}\right]$, where $-\infty \leq \underline{b} \leq 0 < \overline{b} < \infty$.

Assumption 1 is a standard restriction on preferences. Assumption 2 guarantees that the government cannot obtain unboundedly high utility by deferring consumption indefinitely and earning interest on the resulting savings. Assumption 3 ensures that the government is never at a corner of zero consumption, since such corner solutions can lead to analytically intractable discontinuities in V. Assumptions 2 and 3 are jointly satisfied by some common parametric specifications, including CRRA utility $u(c,s) = \frac{c^{1-\sigma}-1}{1-\sigma}$ for any $\sigma \geq 1$.

Assumption 4 includes several restrictions on allowable bond positions. The upper bound $\overline{b} < \infty$ rules out Ponzi schemes; it can be chosen high enough not to be binding. $\overline{b} > 0$ restricts our focus to cases where debt is allowed. Our assumption that $\underline{b} \leq 0$ allows the government to pay down all of its debt if it so desires, and possibly to save.[2] For now, $\underline{b}$ is left otherwise unrestricted. (Later, in Sections 2.2 and 4.1, we will establish separate uniqueness results for the cases of $\underline{b} = -\infty$ and $\underline{b} > -\infty$, respectively. In the latter case, Assumption 2 is superfluous.).

We also need an assumption to guarantee that default is never optimal when the government has positive assets. Define $V^{nb}(b,s)$ to be the value function for a government that can save at the risk-free rate but not borrow,

$$V^{nb}(b,s) = \max_{b'} u(c,s) + \beta \mathbb{E}_{s'|s}\left[V^{nb}(b',s')\right]$$

[2] In an environment with $\underline{b} > 0$, default frees the government from the otherwise inflexible requirement to borrow at least $\underline{b}$. This creates a reward for defaulting which can be difficult to interpret and is not the typical focus of the literature.

$$\text{s.t.} \quad c+b=y(s)+\frac{b'}{R}, \; \underline{b} \le b' \le 0 \tag{4}$$

Then we assume

Assumption 5. $-\infty < V^d(s) \le V^{nb}(0,s)$.

Assumption 5 is satisfied, for example, if default is punished by permanent autarky, with output also reduced exogenously. In that case, V^d is defined recursively by

$$V^d(s) = u(y(s)-\tau(s),s)+\beta\mathbb{E}_{s'|s}\left[V^d(s')\right] \tag{5}$$

for any exogenous cost of default $\tau(s) \in [0, y(s)]$.

Proposition 1. *Under Assumptions 1–5, a Markov perfect equilibrium exists. In any equilibrium, $V(b,s)$ is strictly decreasing in b for each s, and there exists a set of default thresholds $\{b^*(s)\}_{s \in \mathcal{S}}$ such that the government repays in state s if and only if $b \le b^*(s)$. Both V and Q are uniquely determined by the thresholds $\{b^*(s)\}_{s \in \mathcal{S}}$.*

The proof, developed in Appendix A.1, is constructive and relies on a fixed-point procedure similar to the one used by the quantitative literature to search for an equilibrium. As highlighted by Aguiar and Amador (2015), this procedure involves iterating on a monotone and bounded operator in the space of default thresholds. These iterations converge to a fixed point, and our proof verifies that this fixed point defines an equilibrium. Our assumptions ensure that value functions exist, are continuous and finite-valued, and that default thresholds $b^*(s)$ are uniquely defined[3] by the equalities

$$V\left(b^*(s),s\right) = V^d(s) \tag{6}$$

The set $\{b^*(s)\}_{s \in \mathcal{S}}$ then characterizes the bond revenue schedule Q: following (3),

$$Q(b',s) = \frac{b'}{R}\mathbb{P}_{s'|s}\left[b' \le b^*(s')\right] = \frac{b'}{R}\sum_{\{s':b' \le b^*(s')\}} \pi(s'|s) \tag{7}$$

In this environment, it is natural to conjecture that multiple equilibria could be present. Starting from an equilibrium with default thresholds $\{b^*(s)\}_{s \in \mathcal{S}}$, lowering default thresholds increases the cost of borrowing a given amount b', through (7). This, in turn, lowers the value to the government of repaying any amount b, shifting down the function $V(\cdot,s)$ for every s. Through (6), this lowers the levels of debt at which the government is tempted to default, reinforcing the initial impulse in a vicious cycle.

For a given application, one could in principle check such multiplicity directly using a variant of the procedure used in our existence proof. If the iterative procedure, starting from a set of minimal default thresholds, converges to the same fixed point as when starting from a set of maximal default thresholds, it follows from monotonicity that no other equilibrium exists.

In the next section, we provide an alternative argument that establishes uniqueness in every case. Our proof illuminates, in this environment, why the vicious cycle described above is never strong enough to create multiple equilibria, highlighting the role played by assumptions on R and $\underline{b}$.

2.2. Uniqueness of Markov perfect equilibrium

Suppose that we have two distinct revenue schedules Q and $\tilde{Q}$, each derived via (7) from anticipated default thresholds $\{b^*(s)\}_{s \in \mathcal{S}}$ and $\{\tilde{b}^*(s)\}_{s \in \mathcal{S}}$. Let V and $\tilde{V}$ be the value functions for a government facing these schedules. To prove uniqueness of equilibrium, we need to show that at most one of these value functions can be consistent with the default thresholds that generate it—in other words, that we cannot have both $V(b^*(s),s) = V^d(s)$ and $\tilde{V}(\tilde{b}^*(s),s) = V^d(s)$ for all s.

The key observation of this paper is that we can derive a simple inequality for the two value functions V and $\tilde{V}$, related to the maximum difference between the default thresholds. This inequality requires Assumptions 1–5 together with two crucial new assumptions:

Assumption 6. $R > 1$.

Assumption 7. $\underline{b} = -\infty$.

The basis of our inequality is a simple replication strategy we call *mimicking at a distance*. Suppose that $b^*(s)$ exceeds $\tilde{b}^*(s)$ by at most $M > 0$. Then we show that it is always weakly better to start with debt of $b-M$ when facing prices $\tilde{Q}$ than with debt of b when facing prices Q, and indeed strictly better whenever $V(b,s) \ge V^d(s)$. This observation, formalized in Lemma 1, will ultimately be the basis of the proof that distinct equilibria are impossible in Proposition 2.

[3] In the special case where $u(c,s)=u(c)$, income is i.i.d, and V^d is the expected value of autarky, it is possible to show that $b^*(s)$ is increasing in $y(s)$ (see Arellano, 2008), but such monotonicity is not needed for our proof.

Lemma 1 (*Mimicking at a distance.*). *Let Q and $\tilde{Q}$ be two distinct revenue schedules, with Q reflecting expected default thresholds $\{b^*(s)\}_{s\in\mathcal{S}}$ and $\tilde{Q}$ reflecting expected default thresholds $\{\tilde{b}^*(s)\}_{s\in\mathcal{S}}$. Let V and $\tilde{V}$ be the respective value functions for governments facing these revenue schedules. Define*

$$M = \max_s b^*(s) - \tilde{b}^*(s) \tag{8}$$

and assume without loss of generality that $M>0$. Then, for any s and b,

$$\tilde{V}(b-M,s) \geq V(b,s) \tag{9}$$

with strict inequality whenever $V(b,s) \geq V^d(s)$.

Proof. First, note that for any b' and s, applying (7) we have

$$\begin{aligned}\tilde{Q}(b'-M,s) &= \frac{(b'-M)}{R}\sum_{\left\{s': b'-M \leq \tilde{b}^*(s')\right\}} \pi(s'|s) \geq \frac{(b'-M)}{R}\sum_{\{s': b' \leq b^*(s')\}} \pi(s'|s) \\ &> \left(\frac{b'}{R}\sum_{\{s': b' \leq b^*(s')\}} \pi(s'|s)\right) - M = Q(b',s) - M\end{aligned} \tag{10}$$

Thus the amount that a government with schedule $\tilde{Q}$ can raise by issuing $b'-M$ of debt is always strictly larger than the amount that a government with schedule Q can raise by issuing b' of debt, minus M. The two intermediate inequalities in (10) reflect the two sources of this advantage. First, there are weakly more cases in which $b'-M \leq \tilde{b}^*(s')$ than in which $b' \leq b^*(s')$, and this higher chance of repayment makes it possible to raise more. Second, since Assumption 6 requires $R>1$, issuing M less debt costs strictly less than M in foregone revenue in the current period.

This inequality leads us to consider the following *mimicking at a distance policy* for a government starting with debt $b-M$ and facing prices $\tilde{Q}$. This government has the option to *mimic* the policy of the government with debt b facing prices Q—always defaulting at the same points, and otherwise choosing the same level of debt for the next period minus M. Such mimicking is possible, irrespective of the value of $M>0$, because savings is unrestricted (Assumption 7). Due to inequality (10), the former government will achieve strictly higher consumption than the latter government in every period prior to default, and therefore a strictly higher value overall.

More formally, for any states and debt levels s and b, let the history s^0 be such that the state and debt owed at $t=0$ are respectively s and b. The optimal strategy for a government facing schedule Q induces an allocation $\{c(s^t), b(s^{t-1}), p(s^t)\}s^t \succcurlyeq s_0$ at all histories following s^0.[4] We construct a policy for the government facing schedule $\tilde{Q}$ in state s and debt level $b-M$ as follows. For every history s^t following and including s^0, let

$$\tilde{p}(s^t) = p(s^t)$$

and provided that $p(s^t)=1$, choose consumption and next-period debt as

$$\begin{aligned}\tilde{b}(s^t) &= b(s^t) - M \\ \tilde{c}(s^t) &= c(s^t) + \tilde{Q}(b(s^t)-M, s_t) - (Q(b(s^t), s_t) - M)\end{aligned} \tag{11}$$

Note that $\tilde{b}(s^t) \in \mathcal{B}$ due to Assumption 7. This choice of $\tilde{b}$ and $\tilde{c}$ ensures that the budget constraint is satisfied at all histories s^t where repayment takes place:

$$\begin{aligned}\tilde{c}(s^t) + \tilde{b}(s^{t-1}) - \tilde{Q}\left(\tilde{b}(s^t), s_t\right) &= \tilde{c}(s^t) + b(s^{t-1}) - M - \tilde{Q}(b(s^t)-M, s_t) \\ &= c(s^t) + b(s^{t-1}) - Q(b(s^t), s_t) = y(s_t)\end{aligned}$$

Furthermore, using (10) we see that $\tilde{c}(s^t) > c(s^t)$: when there is repayment, the mimicking policy (11) sets consumption $\tilde{c}(s^t)$ equal to consumption $c(s^t)$ in the other equilibrium, plus a bonus $\tilde{Q}(b(s^t)-M, s_t) - (Q(b(s^t), s_t) - M) > 0$ from lower debt costs.

The mimicking policy, of course, need not be optimal; but since it is feasible, it serves as a lower bound for $\tilde{V}(b-M,s)$:

$$\begin{aligned}\tilde{V}(b-M,s) &\geq \sum_{\tilde{p}(s^t)=1} \beta^t \Pi(s^t) u(\tilde{c}(s^t), s_t) + \sum_{\tilde{p}(s^t)=0, \tilde{p}(s^{t-1})=1} \beta^t \Pi(s^t) V^d(s_t) \\ &\geq \sum_{p(s^t)=1} \beta^t \Pi(s^t) u(c(s^t), s_t) + \sum_{p(s^t)=0, p(s^{t-1})=1} \beta^t \Pi(s^t) V^d(s_t) = V(b,s)\end{aligned}$$

with strict inequality whenever $p(s^0)=1$ (or equivalently $b \leq b^*(s)$), since this implies $\tilde{c}(s^0) > c(s^0)$ and $u(c, s_0)$ is strictly increasing in c thanks to Assumption 1. □

An illustration of the mimicking policy used in Lemma 1 is given in Fig. 1a and b, which depict time paths in a hypothetical two-state case. In this case, debt starts relatively high and the high-income state $y(s_H)$ keeps recurring, leading the

[4] $b(s^t)$ is defined to be the amount of debt chosen at history s^t to be repaid in period $t+1$.

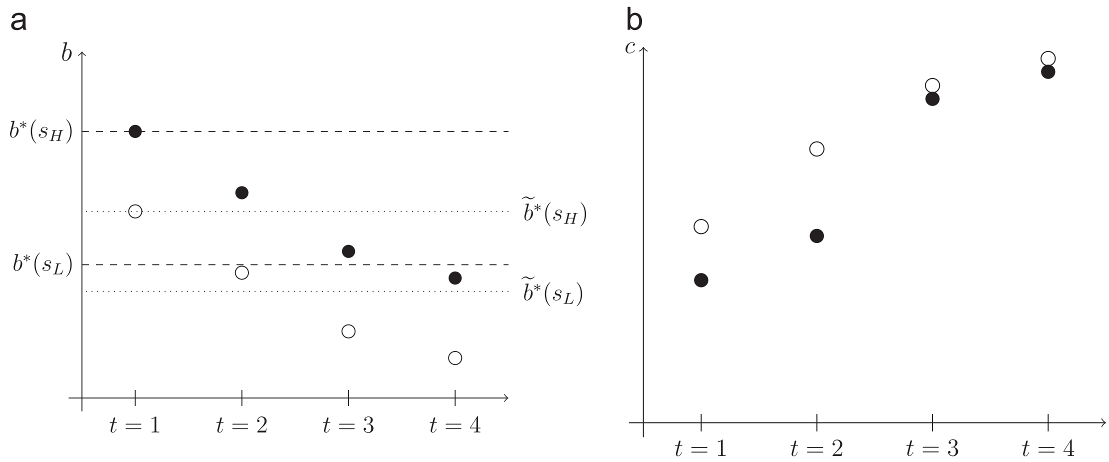

Fig. 1. (a) Example paths for b and $\tilde{b}$ and (b) example paths for c and $\tilde{c}$. *Notes*: Illustration of the mimicking policy used in Lemma 2. The filled circles represent the original policy (b, c), and the hollow circles the mimicking policy $(\tilde{b}, \tilde{c})$. 1a shows that the debt of the mimicking government $\tilde{b}$ is always at a constant distance M below b. 1b shows that consumption of the mimicking government $\tilde{c}$ is always strictly above c. However, the gap $\tilde{c}-c$ differs across periods. This reflects fluctuations in the two sources of $\tilde{c}-c$: differences in default premia, and the lower cost of servicing $\tilde{b}=b-M$ rather than b. First, since both debt levels at $t=2$ are above the respective default thresholds for $y(s_L)$, there is no difference at $t=1$ in the two default premia. At $t=3$, however, the mimicking policy achieves a debt level below $\tilde{b}^*(s_L)$, while the other policy has debt that remains above $b^*(s_L)$. Thus the default premium disappears at $t=2$ for the mimicking policy while still being paid for the other policy, leading to an expansion in the gap $\tilde{c}-c$. From $t=4$ onward both policies achieve debt levels below their s_L default thresholds, leading to the disappearance of all default premia. This causes the gap $\tilde{c}-c$ to compress starting at $t=3$.

government to deleverage in anticipation of lower incomes in the future. Fig. 1a shows the paths of b (filled circles) and the mimicking policy $\tilde{b}=b-M$ (hollow circles), while Fig. 1b shows the paths of c (filled circles) and the consumption $\tilde{c}=c+\tilde{Q}(b-M,s)-(Q(b,s)-M)$ induced by the mimicking policy (hollow circles). Observe that $\tilde{c}$ is always greater than c.

The central observation is that if it starts with debt $M=\max_s b^*(s)-\tilde{b}^*(s)$ below the other government, the mimicking government can keep itself at the fixed distance M, achieving higher consumption along the way.

We now turn to the main result, which uses Lemma 1 to rule out multiple equilibria (V,Q) and $(\tilde{V},\tilde{Q})$ altogether.

Proposition 2. In the benchmark model, Markov perfect equilibrium has a unique value function $V(b,s)$ and bond revenue schedule $Q(b,s)$.

Proof. Suppose to the contrary that there are distinct equilibria (V,Q) and $\left(\tilde{V},\tilde{Q}\right)$. Proposition 1 shows that these are characterized by their default thresholds $\{b^*(s)\}_{s\in\mathcal{S}}$ and $\{\tilde{b}^*(s)\}_{s\in\mathcal{S}}$. Therefore, it suffices for us to show that the thresholds are unique.

Without loss of generality, assume that the maximal difference between b^* and $\tilde{b}^*$ is positive and is attained in a state $\overline{s}\in\mathcal{S}$:

$$\max_s b^*(s)-\tilde{b}^*(s)=b^*(\overline{s})-\tilde{b}^*(\overline{s})=M>0$$

Applying Lemma 1 for $s=\overline{s}$ and $b=b^*(\overline{s})=\tilde{b}^*(\overline{s})+M$, we know that

$$\tilde{V}(\tilde{b}^*(\overline{s}),\overline{s})>V(b^*(\overline{s}),\overline{s})$$

But this contradicts the fact that $b^*(\overline{s})$ and $\tilde{b}^*(\overline{s})$ are default thresholds, which requires $\tilde{V}(\tilde{b}^*(\overline{s}),\overline{s})=V(b^*(\overline{s}),\overline{s})=V^d(\overline{s})$. Thus our premise of distinct equilibria cannot stand. □

The intuitive thrust of Lemma 1 and Proposition 2 is that distinct bond revenue schedules cannot both be self-sustaining. No two schedules Q and $\tilde{Q}$ can simultaneously rationalize their corresponding default thresholds $b^*(\overline{s})$ and $\tilde{b}^*(\overline{s})$ in the state $\overline{s}$ where these thresholds differ most. Instead, the argument of Lemma 1 shows that it is better for a government to start at the lower threshold $\tilde{b}^*(\overline{s})$ given schedule $\tilde{Q}$ than to start at the higher threshold $b^*(\overline{s})$ given schedule Q; at this point, any advantages of Q over $\tilde{Q}$ are outweighed by the heavier debt burden, and the former government can use a simple mimicking strategy to guarantee itself strictly higher consumption than the latter. It follows that these cannot both be default thresholds, which by definition must be equally desirable, with common value equal to the default value $V^d(\overline{s})$.

2.3. Uniqueness of subgame perfect equilibrium

The arguments used to prove Proposition 2 can be extended to show that this model admits a unique subgame perfect equilibrium. While the Markov perfect concept exogenously restricts equilibrium to depend on a limited set of states, subgame perfect equilibria allow an arbitrary dependence of strategies at time t on the history h^{t-1} of past states and actions. The following result shows that the current states s and b summarize this dependence, demonstrating that the Markov concept—which has been the focus of much of the quantitative literature—is not restrictive. Proving this formally requires defining the game played by the government and international investors more precisely. Crucially, in this game, the value from government default is still exogenous—endogenizing the default option as part of the game is outside of the scope of this paper (see Kletzer and Wright, 2000; Wright, 2002 or Krueger and Uhlig, 2006 for such an exercise, and our discussion in Footnote 10). Here we summarize our result, and relegate the description of the game and the proof to Appendix A.2. Let $V\left(h^{t-1}, s\right)$ be the value achieved by a government after history h^{t-1}, when the current exogenous state is s. Then the following result holds.

Proposition 3. Consider two subgame perfect equilibria A and B. For any (b,s), and any histories (h_A, h_B) such that $b(h_A) = b(h_B) = b$, we have $V_A(h_A, s) = V_B(h_B, s)$.

The key to the proof of Proposition 3 is to show that, conditional on the exogenous state s, a government with higher debt must have lower value, independently of the equilibrium that is played *or* the history of past actions. This in turn relies on another mimicking argument, whereby a government with lower debt can always choose a strategy that ensures it higher consumption and higher future value than its higher-debt counterpart.

3. Application to other models

The argument used to prove uniqueness of equilibrium in Section 2 is very general and can be used in other contexts, as the following applications illustrate.

3.1. Bewley models with endogenous debt limits

Consider a modification of the environment of Section 2, in which lenders are restricted to offer a price of $\frac{1}{R}$ for every unit of debt that they buy. Borrowing must therefore be risk-free: this is the equilibrium defined in Zhang (1997). This restriction can be captured within the framework of the previous section by specifying that the price of non-riskless debt is zero. Instead of (3), the bond revenue schedule becomes

$$Q^z(b', s) = \frac{b'}{R}\mathbf{1}\left[V^z(b', s') \geq V^d(s') \quad \forall s'|s\right] \tag{12}$$

Define $\phi(s)$ as the value that satisfies $V^z(\phi(s), s) = V^d(s)$, and assume that for all s and s', $\pi(s'|s) > 0$. Then, writing $\varphi \equiv \min_s\{\phi(s)\}$, (12) becomes

$$Q^z(b', s) = \frac{b'}{R}\mathbf{1}_{\{b' \leq \varphi\}} \tag{13}$$

In other words, the model is a standard incomplete markets model in the tradition of Bewley (1977), with a debt limit φ determined endogenously by the requirement that the government should never prefer default.

We can immediately prove analogs of Lemma 1 and Proposition 2 in this new environment.

Lemma 2. *Consider two distinct equilibria with value functions V and $\tilde{V}$ and debt limits $\tilde{\varphi} < \varphi$. Then, letting $M = \varphi - \tilde{\varphi}$, for any b and s we have*

$$\tilde{V}(b - M, s) \geq V(b, s) \tag{14}$$

with strict inequality whenever $b \leq \varphi$.

Proof. Same as the proof of Lemma 1, with (3) replaced by (13) and inequality (10) becoming

$$\tilde{Q}^z(b' - M, s) = \frac{b' - M}{R}\mathbf{1}_{\{b' - M \leq \tilde{\varphi}\}} = \frac{b' - M}{R}\mathbf{1}_{\{b' \leq \varphi\}} > \frac{b'}{R}\mathbf{1}_{\{b' \leq \varphi\}} - M = Q^z(b', s) - M \tag{15}$$

□

The intuition behind (14) and (15) is well known in this class of environment: an increase in the debt limit is equivalent to a translation of the value function, accompanied by a translation of the income process that reflects the interest costs of debt. Our earlier inequality (10) can be interpreted as a generalization of this result.

Proposition 4. In the model with riskless debt, Markov perfect equilibrium has a unique value function $V(b,s)$ and debt limit φ.

Proof. Same as the proof of Proposition 2, but using Lemma 2 rather than Lemma 1. □

As highlighted in the introduction, this particular application illustrates the intuition behind our main uniqueness result in Section 2: a deterioration in the terms of borrowing cannot be self-sustaining in this class of models since, once governments have exhausted their debt capacity, those with less debt are always better off.

3.2. Bulow and Rogoff

Our proof is also related to that used by Bulow and Rogoff (1989) to rule out reputational equilibria in sovereign debt models where saving is allowed after default. As originally written, the Bulow–Rogoff result only applies directly to environments with complete markets, but a similar result also holds in the incomplete markets framework we study: if a government can save at a strictly positive net risk-free rate after defaulting, and there are no other exogenous penalties for default, then no debt can be sustained. Though this result has not—to our knowledge—been written formally until now, it has informally motivated the ingredients of modern variations on the Eaton–Gersovitz model, which all specify some exclusion from international markets after default, together with additional costs of default such as output losses.

Recall from (4) that V^{nb} is the value of a government that is able to save at the risk-free rate but not borrow. The incomplete markets analog of Bulow and Rogoff (1989) corresponds to the special case where $V^d(s) \equiv V^{nb}(0,s)$. In other words, when the government defaults, its debt is reset at 0 and it can subsequently save but not borrow.

Proposition 5 (*Incomplete markets Bulow–Rogoff*). In the model with $V^d(s) = V^{nb}(0,s)$ (i.e. savings after default), no debt can be sustained: in the unique Markov perfect equilibrium, the default thresholds $b^*(s)$ equal 0 for all s, and $Q(b',s) = 0$ for all $b' \geq 0$. Hence $V(b,s) = V^{nb}(b,s)$.

The proof in Appendix A.3 has two steps: first, it verifies that there exists an equilibrium with no lending, and second, it applies Proposition 2 to show that no other equilibrium is possible. In particular, any equilibrium with debt is ruled out.

Going back to the proof of Proposition 2, the intuition behind this result is that once a government has already borrowed the maximum amount that can obtain a nonzero price, access to debt markets offers no benefits beyond access to a market for savings. It is impossible to borrow more until some debt is repaid—and rather than repay and reborrow, it is cheaper to default and then run savings up and down in a parallel way, achieving higher consumption by avoiding the costs of debt service. No amount of debt is sustainable: whenever a government has borrowed the maximum, it will default with certainty, and in anticipation creditors will never allow any debt.

This resembles the logic behind the original Bulow and Rogoff (1989) result, which observed that for a reputational debt contract in complete markets, there must always be some state of nature in which a government can default and use the amount demanded for repayment as collateral for a sequence of state-contingent "cash in advance" contracts that deliver strictly higher consumption in every future date and state. The main idea behind their proof carries over to our incomplete markets environment, once the cash in advance contracts are replaced with a simple, parallel savings strategy. Our contribution here is to show that this result is a special case of a much broader equilibrium uniqueness result: once the existence of a no-debt equilibrium is verified, the Bulow–Rogoff result follows immediately from Proposition 2.[5]

4. Extensions of the benchmark model

We now discuss some common variants of the benchmark Eaton–Gersovitz model covered in Section 2, showing when the uniqueness result does and does not carry through.[6]

4.1. Bound on savings

We first consider the case $\underline{b} > -\infty$, dropping Assumption 7 and replacing it with

Assumption 7′. $\underline{b} > -\infty$.

This is a case of practical interest, since applied work frequently restricts the space of allowable government savings.[7] Since such a restriction limits the ability of a government to carry out the parallel mimicking strategy discussed in Section 2.2, leading our proof to break down, it is also natural to ask whether it could be a source of multiplicity.

[5] In parallel and independent work, Bloise et al. (2016) have established a sufficient condition under which the Bulow and Rogoff (1989) result survives in incomplete market environments with a general asset market structure, and discuss examples where it fails (see also Pesendorfer, 1992). This sufficient condition is a "high implied interest rates" condition, as in Alvarez and Jermann (2000). When the only available asset is a risk-free bond and the endowment process is bounded, this condition simplifies to $R > 1$ (our Assumption 6). Our result in this section therefore complements theirs, by exhibiting an explicit replication strategy with risk-free bonds, and reinterpreting the no-lending result as a result about equilibrium uniqueness.

[6] While our focus is on sovereign debt, the benchmark model we study also constitutes the core of a literature that analyzes unsecured consumer credit (Chatterjee et al., 2007), and we conjecture that equilibrium is also unique in many of the models used in that literature.

[7] For example, Chatterjee and Eyigungor (2012) exclude savings from their grid, although they find in their numerical simulations that this restriction does not bind.

In fact we show, by means of a simple example, that when $\underline{b}=0$ and V^d is the value of autarky, there may be multiple equilibria. This turns out to be a special case, however, since we are able to extend the uniqueness result whenever either $\underline{b}<0$ or V^d is strictly worse than the value of autarky.

Potential multiplicity with no savings and autarky punishment: We first start from the observation that, when no government savings is allowed and default is punished by autarky, there is always an equilibrium without any debt.

Lemma 3. *When* $\underline{b}=0$ *and* $V^d(s)=V^{aut}(s)\equiv u(y(s),s)+\beta\mathbb{E}\left[V^{aut}(s')|s\right]$, *there exists an equilibrium where all default thresholds are identically equal to zero and the government never borrows*: $b^*(s)=0$ *for all* s *and* $b'(b,s)=0$ *for all* b,s.

The argument is as follows: consider a government with some outstanding debt that cannot save or borrow again. This government can either default now, achieving the value of autarky, or repay its debt today and live off its endowment in the future, which is strictly worse. It therefore always chooses to default. Anticipating this behavior, creditors do not lend.

The no-debt equilibrium is not necessarily the only one, however, as the following proposition illustrates.

Proposition 6. Suppose, as in Lemma 3, that $\underline{b}=0$ and $V^d(s)=V^{aut}(s)$. Suppose also that $\mathcal{S}=\{s_L,s_H\}$ with $y(s_L)<y(s_H)$, $\pi(s_L|s_H)=\pi(s_H|s_L)=1$, $u(c,s)=\upsilon(c)$ for strictly concave υ, and $R=1/\beta$. Then, for β sufficiently close to 1, there also exists an equilibrium where both default thresholds are strictly positive and the government sometimes borrows: $b^*(s_L),b^*(s_H)>0$ and $b'(b,s)>0$ for some b,s.

Why does an equilibrium with debt exist? Suppose that the government can borrow at the risk-free rate R. Then, in this example, it would like to achieve constant consumption across periods, and can do so by borrowing in state s_L and repaying in state s_H. If β is close enough to one, the present value of smoothing endowment fluctuations in this way exceeds the one-off return from neglecting to repay, and the government chooses not to default. Anticipating repayment, creditors lend at rate R within the relevant range of b.

This multiplicity is a contrast to our uniqueness result.[8] Indeed, it embodies the intuition for multiplicity discussed in the introduction—an intuition that we rejected in Section 2.2 for the $\underline{b}=-\infty$ case. In the new example, expectations *can* be self-fulfilling. The equilibrium in Lemma 3 has a vicious cycle where pessimistic creditors never lend and there is no incentive to repay, while the equilibrium in Proposition 6 has a virtuous cycle where optimistic creditors lend on favorable terms and there is a strong incentive to repay.

We will now show, however, that this multiplicity is a special case. Moving away from the assumptions in Lemma 3, either by allowing some savings or by adding a penalty for default, restores uniqueness.

Before proceeding to a uniqueness result, we must make two additional assumptions on the environment.

Assumption 8. For each s, $u(c,s)$ is concave in c.

Assumption 9. There is some function $v^d(s)\le u(y(s),s)$ such that $V^d(s)=v^d(s)+\beta\mathbb{E}\left[V^d(s')|s\right]$ for all s.

The concavity in Assumption 8 is satisfied by most standard specifications of u, and it will be crucial to the modified proof strategy. Assumption 9 states that the value of default is the present discounted value of some flow utility $v^d(s)$, weakly less desirable than autarky.[9] This is common in the literature, which often assumes an output cost $\tau(s)$ of default such that $v^d(s)=u(y(s)-\tau(s),s)$.

With these assumptions in hand, we can describe the new argument for uniqueness. This echoes the replication argument from Section 2 in some respects, but there are also substantial differences. Rather than *mimicking at a distance*, which is no longer feasible, we use *compressed mimicking*. Given distinct revenue schedules Q and $\tilde{Q}$ derived via (7) from default thresholds $\{b^*(s)\}_{s\in\mathcal{S}}$ and $\{\tilde{b}^*(s)\}_{s\in\mathcal{S}}$, Lemma 4 defines λ to be the minimum ratio between $\tilde{b}^*(s)-\underline{b}$ and $b^*(s)-\underline{b}$. For any s and $\tilde{b}-\underline{b}=\lambda(b-\underline{b})$, a government starting at $(\tilde{b},s)$ can *compress* by λ the optimal strategy for a government (which we call the *target*) starting at (b,s), choosing $\tilde{b}(s^t)-\underline{b}=\lambda(b(s^t)-\underline{b})$ whenever the target government repays and defaulting whenever the target government defaults.

As in Section 2, the mimicking government by construction obtains weakly better prices than the target government for its debt. Unlike in Section 2, the mimicking government need not achieve higher consumption than the target government. Instead, because it is compressing the target's debt issuance plan by λ, in each period it obtains consumption $\tilde{c}$ that is weakly higher than the convex combination $\lambda c+(1-\lambda)y(s)$ of the target's consumption c and state-s autarky income $y(s)$. This inequality is strict when $\underline{b}<0$, where the mimicking government can consume extra due to forgone financing costs. Concavity of u then implies that $u(\tilde{c},s)$ is strictly greater than $\lambda u(c,s)+(1-\lambda)u(y(s),s)$. Summing the expected value across all periods, we obtain (17), the analog of (9); when $\underline{b}=0$, the strict inequality can also follow from $v^d(s)<u(y(s),s)$.

Lemma 4. *Let* Q *and* $\tilde{Q}$ *be two distinct revenue schedules, with* Q *reflecting expected default thresholds* $\{b^*(s)\}_{s\in\mathcal{S}}$ *and* $\tilde{Q}$ *reflecting expected default thresholds* $\{\tilde{b}^*(s)\}_{s\in\mathcal{S}}$. *Let* V *and* $\tilde{V}$ *be the respective value functions for governments facing these*

[8] Passadore and Xandri (2014) were the first to identify this type of multiplicity in a sovereign debt model without savings. Proposition 6 reframes this finding within the framework of this paper, and provides a simplified example.

[9] Imposing this structure involves some loss of generality, since we can no longer make the value of defaulting depend on state s without also affecting the value of being excluded from markets in state s after originally defaulting in state $s'\neq s$.

revenue schedules. Define

$$\lambda \equiv \min_s \frac{\tilde{b}^*(s) - \underline{b}}{b^*(s) - \underline{b}} \tag{16}$$

and assume, without loss of generality, that $0 \le \lambda < 1$. *Assume also either that* $\underline{b} < 0$, *or that* $v^d(s) < u(y(s), s)$ *for all s. Then for any s and b such that* $V(b,s) \ge V^d(s)$, *we have*

$$\tilde{V}(\tilde{b}, s) > (1-\lambda)V^d(s) + \lambda V(b,s) \tag{17}$$

where $\tilde{b} - \underline{b} \equiv \lambda(b - \underline{b})$. *This can equivalently be written as*

$$\tilde{V}(\tilde{b}, s) - V^d(s) > \lambda\left(V(b,s) - V^d(s)\right) \tag{18}$$

In contrast to (9), inequality (17) in Lemma 4 does not show that $\tilde{V}(\tilde{b}, s)$ is higher than $V(b,s)$. Fortunately, this is not needed to establish uniqueness in Proposition 7. Instead, (18) suffices to obtain a contradiction. Inequality (18) shows that if a government facing Q weakly prefers not to default at (b,s) (so that $V(b,s) - V^d(s) \ge 0$), then a government facing $\tilde{Q}$ must strictly prefer not to default at $(\tilde{b}, s)$ (so that $\tilde{V}(\tilde{b}, s) - V^d(s) > 0$). It is therefore impossible for both b and $\tilde{b}$ to be default thresholds for their respective value functions.

Proposition 7. If either $\underline{b} < 0$, or $v^d(s) < u(y(s), s)$ for all s, Markov perfect equilibrium has a unique value function $V(b,s)$ and bond revenue schedule $Q(b,s)$.

Proof. If, to the contrary, we have distinct equilibria (V,Q) and $(\tilde{V}, \tilde{Q})$ with default thresholds $\{b^*(s)\}_{s \in \mathcal{S}}$ and $\{\tilde{b}^*(s)\}_{s \in \mathcal{S}}$, define λ as in (16) and assume without loss of generality that $0 \le \lambda < 1$.

Let $\bar{s}$ be the state where the minimum in (16) is obtained. Evaluating (18) at $\tilde{b} = \tilde{b}^*(\bar{s})$, $b = b^*(\bar{s})$, and $s = \bar{s}$, we obtain

$$0 = \tilde{V}(\tilde{b}^*(\bar{s}), \bar{s}) - V^d(\bar{s}) > \lambda\left(V(b^*(\bar{s}), \bar{s}) - V^d(\bar{s})\right) = 0$$

which is a contradiction. □

The crucial innovation of Proposition 7 is that it departs slightly from the conditions in Lemma 3, which turn out to be fragile. Rather than disallowing all savings and punishing default with autarky, the proposition assumes that either some saving is allowed or that the value from default is strictly worse than autarky. With either modification, the no-debt equilibrium in Lemma 3 is eliminated, because the value of default is now strictly worse than the value of remaining in the market with zero debt, and the government will always opt to repay a sufficiently small debt. Essentially, the government needs *some* reason not to default, however small—and it may be either a carrot (access to saving) or a stick (losses from default). Either way, the no-debt equilibrium is eliminated, and at that point Proposition 7 can establish uniqueness.

4.2. Stochastic market reentry

In the literature, a very common departure from the benchmark model of Section 2 is an assumption that market reaccess is possible after default (for example Aguiar and Gopinath, 2006; Arellano, 2008). This makes the value of default depend on the equilibrium value of borrowing, implying that Lemma 1 and Proposition 2 do not directly apply. Our argument can no longer completely establish uniqueness, but we are able to rule out the most commonly hypothesized form of multiplicity—the existence of distinct "favorable" and "adverse" equilibria, in which the favorable equilibrium offers uniformly better revenues Q. We also show uniqueness in the special case where states are independently and identically distributed.

To be concrete, suppose that it is possible to re-access markets with zero debt after a stochastic period of exclusion, which has independent probability $1-\lambda$ of ending in each period. That is, replace Assumption 9 with[10]

Assumption 9′. V^d satisfies

$$V^d(s) = v^d(s) + \beta\lambda \mathbb{E}_{s'}\left[V^d(s')\right] + \beta(1-\lambda)\mathbb{E}_{s'}\left[V^o(0, s')\right] \tag{19}$$

We also revert to Assumption 7 ($\underline{b} = -\infty$). In this framework, we can now prove the following specialized analog of Proposition 2.

Proposition 8. In the model with stochastic reentry, there do not exist two distinct equilibria (V,Q) and $(\tilde{V}, \tilde{Q})$ such that $Q(b,s) \ge \tilde{Q}(b,s)$ for all b and s.

[10] This formulation is the one used by Arellano (2008) and Aguiar and Gopinath (2006). It does not encompass the possibility of recovery on defaulted debt or debt renegotiation (see for example Pitchford and Wright, 2007; Yue, 2010, or Arellano and Bai, 2014). Multiple equilibria might result if multiple V^d are possible—for example as a result of coordination on different bargaining equilibria, or more generally if V^d was endogenized as part of the game.

In general, the endogeneity of $V^d(s)$ in (19) makes it difficult to analytically characterize equilibria. In the particular case examined by Proposition 8, however, the proof strategy from Proposition 2 still applies with some modification. The core insight is that if $Q \geq \tilde{Q}$, then $V^d \geq \tilde{V}^d$, because a government facing a uniformly better revenue schedule after reentry is better off. Furthermore, if Q and $\tilde{Q}$ are distinct and $Q \geq \tilde{Q}$, there must be some s for which $b^*(s) > \tilde{b}^*(s)$. We then can apply the argument from Lemma 1 and Proposition 2, having a government in the $(\tilde{V}, \tilde{Q})$ equilibrium mimic the strategy of a government in the (V,Q) equilibrium. The fact that $\tilde{V}^d \leq V^d$ only helps our argument, since it is further reason why the government in the $(\tilde{V}, \tilde{Q})$ equilibrium will prefer the mimicking strategy to default.

In short, when there is reentry, uniformly higher bond prices defeat themselves: they make default and eventual reentry more attractive, raising the probability of default and pushing bond prices back down.

Although we cannot prove uniqueness more generally, this result does rule out the popular hypothesis—as discussed in the introduction—that sovereign debt markets can vary between self-sustaining "favorable" and "adverse" equilibria. Instead, if multiplicity exists, we know that it must be a surprising kind of multiplicity: among any two equilibria, each must offer cheaper borrowing in some places and more expensive borrowing in others.

Special case with iid exogenous state: It is possible to demonstrate full uniqueness in one special case. Suppose now that s follows an iid process with probability $\pi(s)$. It follows that the expected value from reentry $\mathbb{E}_{s'}[V^o(0,s')]$ in (19) is independent of the states preceding s', and we can denote this expectation by V^{re}. The iid assumption also implies that the bond revenue schedule Q depends only on the debt amount b', not the current state s, as (7) reduces to

$$Q(b') = \frac{b'}{R} \sum_{\{s' : b' \leq b^*(s')\}} \pi(s') \tag{20}$$

Proposition 9. In the model with iid states and stochastic market reentry, Markov perfect equilibrium has a unique value function $V(b,s)$ and bond revenue schedule $Q(b)$.

Proposition 9 follows for reasons similar to Proposition 8. For any distinct equilibria (V,Q) and $(\tilde{V}, \tilde{Q})$, the only difference between the default value functions V^d and $\tilde{V}^d$ arises from the expected reentry value, which is now just a scalar V^{re}. Whichever equilibrium has the higher reentry value must have a more favorable bond revenue schedule, meaning that at least one of its default thresholds is higher. As with Proposition 8, we can then invoke a mimicking argument to show that the equilibrium with a higher default value cannot also have a higher default threshold for some s.

This result emphasizes that any multiplicity in the model with reentry, if it exists, must rely on the transition probabilities of the Markov process being non-iid.

4.3. Other variations on the model and multiplicity results

We have showed that the benchmark Eaton–Gersovitz model of sovereign debt with default does not admit multiple equilibria, and that this uniqueness result partly extends to the more complex environments of Sections 4.1 and 4.2. Nevertheless, multiplicity arises in several other sovereign debt models in the literature. This section reviews the ways in which these models differ from the benchmark framework analyzed in this paper.

Markov perfect equilibrium in the model we studied includes a revenue function $Q(b', s)$, which depends only on the current state s and the bond payment b' promised tomorrow. After observing s, in each period the government can choose either to default or to repay and sell some quantity b' of bonds for next period. Once the government chooses to repay and selects some b', there is no uncertainty about the amount $Q(b', s)$ that will be raised; no further choices are made until the next period, when the next state s' is realized and the process repeats itself. As presented in Appendix A.2, this process can be explicitly written as a game between governments and risk-neutral investors. It is possible to define subgame perfect equilibria in this game, and Proposition 3 shows that uniqueness still holds for these equilibria in the benchmark model.

Our uniqueness result can disappear if the timing and action space of the game are modified. For instance, in the model of Cole and Kehoe (2000), the government has the option to default after observing the outcome of the current period's bond auction. If it defaults, it can keep the proceeds of the auction but avoid repayment on its maturing debt. Given enough risk aversion, this option is preferable when the current period's auction yields little revenue, and the cost of repaying maturing debt out of current-period resources is prohibitively high. A coordination problem among creditors thus emerges, leading to multiple equilibria: they might either offer high prices, in which case the government will repay, or offer low prices, in which case the government will default and thereby justify the low prices. The literature sometimes refers to this phenomenon as "rollover multiplicity". It is absent in the model we study, which excludes the option to default after revenue from the auction comes in; but it captures an important intuition, which is that rolling over large amounts of short-term debt can be a source of fragility.

In the model of Calvo (1988), multiplicity arises because of the way the bond revenue-raising process works. In the Calvo model, a government borrows an exogenous amount b at date 0 and inherits a liability of $R_b b$ at date 1. It then uses a mix of distortionary taxation and debt repudiation to finance a given level of government spending. Since a higher interest rate R_b tilts the balance towards more repudiation at date 1, and since investors need to break even when lending to the government, there exist two rational expectations equilibria: one with high R_b and high repudiation, and one with low R_b and low repudiation. This is sometimes called "Laffer curve multiplicity" in reference to the shape of the bond revenue curve that arises in this model (the function that gives bond revenue b as a function of promised repayment $R_b b$ has an inverted-V shape). In the model we study, the government directly announces the amount it will owe tomorrow, allowing it to avoid the downward-sloping part of the bond revenue curve.[11] Lorenzoni and Werning (2014) make a forceful argument that such an assumption requires a form of commitment that governments are unlikely to have: in practice, if they raise less auction revenue than expected, they may auction additional debt rather than making the burdensome fiscal adjustments that are otherwise necessary.

In effect, both the rollover multiplicity of Cole and Kehoe (2000) and the Laffer curve multiplicity of Calvo (1988) emerge from a more elaborate game between governments and investors. They create self-fulfilling alternate equilibria by allowing governments to act in ways ruled out by the game-theoretic formulation of the benchmark Eaton–Gersovitz model: when auction revenue is insufficient, governments can either take the revenue and then default (as in Cole and Kehoe, 2000) or dilute investors by issuing more debt in the same period (as in Lorenzoni and Werning's interpretation of Calvo, 1988). Since the Eaton–Gersovitz model alone cannot produce multiplicity, these modifications to the game may prove important to interpreting any multiplicity we see in practice. More generally, they suggest that a detailed look at institutions, and the practical options available to sovereign debtors when they raise funds in debt markets, is necessary to understand when the Eaton–Gersovitz model succeeds and when it fails as a benchmark.

Another important strand of the literature considers long-term debt, as in Hatchondo and Martinez (2009). Here, our mimicking-based proof of uniqueness breaks down, since bond prices are influenced by the likelihood of endogenous default in the arbitrarily distant future. Indeed, in recent work, Aguiar and Amador (2016) have constructed an example of multiplicity with long-term debt. This provides a contrast to our result—and in a surprising direction, since informal discussions often suggest that multiplicity is more likely, not less, with short-term debt.

A final route to multiplicity is the possibility of dynamic inefficiency. When $R \leq 1$ and Assumption 6 is violated, classical results on the possibility of bubbles in dynamically inefficient economies lead us to expect the possibility of multiple equilibria. Indeed, in a model with complete markets, Hellwig and Lorenzoni (2009) exhibit an equilibrium with $R=1$ and self-sustaining debt in spite of a Bulow–Rogoff punishment for exclusion.[12]

5. Conclusion

The Eaton–Gersovitz model and several of its variants have a unique equilibrium. Our results settle an important outstanding question in the literature, making use of a replication-based proof that may be applicable more generally. By showing that no changes in a government's reputation for repayment can be self-sustaining, we rule out a widely suspected source of multiple equilibria in sovereign debt markets. We hope that future research will build on this result, exploring the extent to which alternative economic mechanisms—for instance, long-term debt, risk-averse lenders, a richer supply side, or partial recovery by creditors—might either reinforce uniqueness or generate multiplicity.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2016.10.013.

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[11] Interestingly, the setup of the original Eaton and Gersovitz (1981) model does not let the government choose on the bond revenue curve *a priori*, although their analysis focuses on equilibria in which it effectively does.

[12] Note that $R=1$ arises endogenously in their general equilibrium setting (see also Jeske, 2006; Wright, 2006).

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