# Default with Policy – Randomness Overestimation (PRO)

## Pivoted Pricing, Deleveraging, and a Stability Illusion

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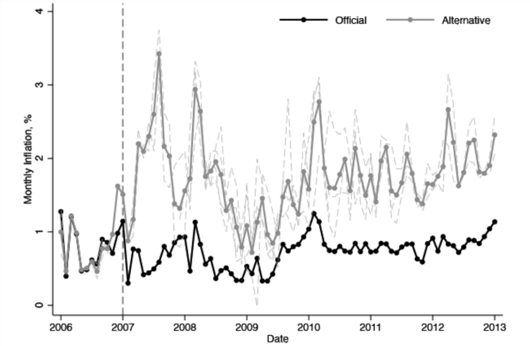
# Roadmap

# Motivation

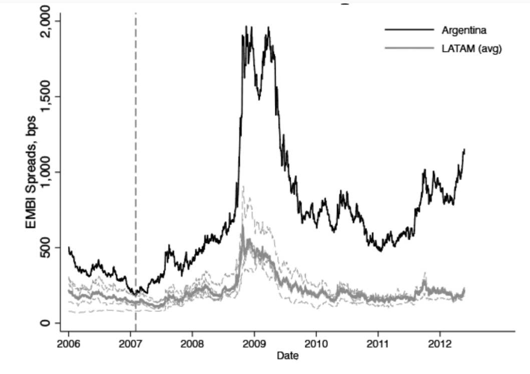
# A Persistent Puzzle

- Some sovereigns face persistently high spreads despite moderate debt and improving fundamentals.
- Event evidence (e.g., Argentina's inflation misreporting) shows spread decoupling beyond direct balance-sheet effects.
- Standard models struggle to match elevated average premia with lower volatility.
- This paper: a single pricing operator with a second-moment belief wedge (PRO) that *pivots* price/spread schedules.

# Argentina: Data Misreporting and Spread Decoupling




Official CPI vs. alternative measures




EMBI+ spreads: Argentina vs. LA peers

- **Interpretation**: reputational channel (type) + **PRO** (policy dispersion) both active.

# Model

# Environment

- **Endowment**: $\texttt{ln}\, y' = (1-\rho_y)\mu_y + \rho_y \texttt{ln}\, y + \sigma_y \varepsilon'$, $\varepsilon' \sim \mathcal{N}(0,1)$.
- **Debt**: long-term bond with coupon $\kappa$, decay $\delta$, risk-free rate $r$.
- **Default**: exclusion prob. $1-\gamma$; cost $h(y) = y - \texttt{max}\{0, \lambda_0 y + \lambda_1 y^2\}$.
- **Preferences**: $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$, discount $\beta$.

**Taste shocks (Gumbel)** yield smooth aggregator and logit rules.

$$\textbf{Ex-ante value:}\quad V(y,B)=\eta\ \mathtt{ln}\Big(e^{V^D(y)/\eta}+e^{V^R(y,B)/\eta}\Big),$$

$$\textbf{Default prob.:}\quad \mathbb{P}\{d{=}1\mid y,B\}=\mathsf{L}\Big(-\tfrac{\Delta V(y,B)}{\eta}\Big)=\tfrac{e^{V^D/\eta}}{e^{V^D/\eta}+e^{V^R/\eta}},$$

$$\textbf{Borrowing aggregator:}\quad V^R(y,B)=\rho\ \mathtt{ln}\sum_{B'\in\mathcal{B}}e^{W(y,B,B')/\rho},$$

$$\textbf{Borrowing policy:}\quad \mathbb{P}\{B'\mid y,B\}=\frac{e^{W(y,B,B')/\rho}}{\sum_{\tilde{B}'}e^{W(y,B,\tilde{B}')/\rho}},$$

where $\Delta V\equiv V^R{-}V^D$,
$W(y,B,B')=u(y-\kappa B+[B'-(1{-}\delta)B]q(y,B'))+\beta\mathbb{E}V(y',B')$.

# Lenders and Pricing Operator

**PRO** scales the *default logit* via tail weight $\theta \geq 001$:

$$\boxed{P_\theta(y, B') = \mathsf{L}\Big(-\tfrac{\Delta V(y,B')}{\theta\,\eta}\Big), \qquad \mathsf{L}(z) = \tfrac{1}{1+e^{-z}}}$$

Pricing operator (unique fixed point):

$$\boxed{(\mathcal{T}_\theta q)(B', y) = \tfrac{1}{1+r}\,\mathbb{E}_{y'|y}\Big[(1-P_\theta(y', B'))\big(\kappa + (1-\delta)\,\mathbb{E}_{B''|y',B'} q(y', B'')\big)\Big]}$$

Slope (joint contraction) condition:

$$L_{Jq} L_{TV} \; < \; (1-\beta)\Big(1 - \tfrac{1-\delta}{1+r}\Big) \quad \Rightarrow \quad \text{unique fixed point.}$$

# Pivot Intuition

# One-Line Schematic of Pivot

Compact schematic anchoring the single-crossing:

$$P_\theta(y,B') = \mathsf{L}\Big(-\tfrac{\Delta V(y,B')}{\theta\eta}\Big), \quad \Delta V \equiv V^R - V^D,$$
$$\Rightarrow \quad \mathtt{sign}(P_1 - P_\theta) = -\,\mathtt{sign}(\Delta V),$$
$$\Rightarrow \quad \mathtt{sign}(q_\theta - q_1) = \mathtt{sign}\,\mathbb{E}[(P_1 - P_\theta)\Pi] \;=\; -\,\mathtt{sign}(\Delta V),\ \Pi > 0.$$

**Define the threshold** $B^*(y): \ \Delta V(y, B^*(y)) = 0$. Then:

- $B' < B^*(y)$ (safe region, $\Delta V > 0$): $q_\theta < q_1$ (**PRO premium**).
- $B' > B^*(y)$ (near default, $\Delta V < 0$): $q_\theta > q_1$ (**softened doom**).

# Pivot: Proof Sketch and Comparative Statics

**Operator order**: If $P_\theta \gtrless P_1$ pointwise, positivity of payoff kernel implies $(\mathcal{T}_\theta q) \lessgtr (\mathcal{T}_1 q)$.

- **Sign**: $\texttt{sign}(P_1 - P_\theta) = -\texttt{sign}(\Delta V) \Rightarrow$ single crossing at $\Delta V = 0$.
- **Threshold monotonicity**: $B^*(y)$ increases in $y$ (rational schedule shifts out more than PRO).
- **Policies**: higher **default threshold**, **deleveraging**, **higher mean spreads**.

# Quantitative Results

# Calibration (Quarterly, EM stylized)

- Preferences and endowment: $\sigma = 2$, $\beta = 0.9775$, $\rho_y = 0.95$, $\sigma_y = 0.005$.
- Debt: $\delta = 0.04$ (5y duration), $\kappa = \delta + r$, $r = 1\%$/qtr, $\gamma = 0.125$.
- Default cost: $h(y) = y - \max\{0, \lambda_0 y + \lambda_1 y^2\}$ with $(\lambda_0, \lambda_1) = (-0.48, 0.525)$.
- Taste shocks small: $\eta = 5 \times 10^{-4}$, $\rho = 10^{-5}$; grids: $N_y$=201, $N_B$=600.
- Scenarios: $\theta \in \{1, 10, 100\}$.

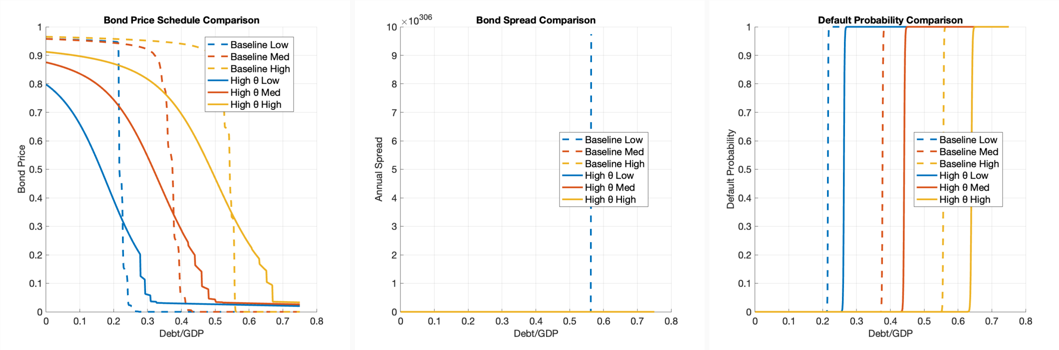
# Business Cycle Moments

Table 1: Simulation Moments Comparison

| Moment | Baseline ($\theta = 1$) | Med $\theta$ ($\theta = 10$) | High $\theta$ ($\theta = 100$) |
|---|---|---|---|
| Mean Debt/GDP | 7.646 | 5.520 | 2.695 |
| Std Debt/GDP | 1.301 | 0.864 | 0.754 |
| Mean Spread (ann.) | 2.028 | 2.762 | 4.153 |
| Std Spread (ann.) | 0.804 | 0.496 | 0.592 |
| Std log C | 3.580 | 3.586 | 3.464 |
| Std log GDP | 3.164 | 3.236 | 3.236 |
| Corr(Sp,GDP) | -0.336 | -0.802 | -0.894 |
| Corr(TB/GDP,GDP) | -0.003 | -0.284 | -0.259 |
| Mean TB/GDP | 0.268 | 0.320 | 0.177 |
| Std TB/GDP | 0.835 | 0.437 | 0.326 |
| Corr(Debt/GDP,GDP) | 0.697 | 0.858 | 0.839 |
| Default Rate | 3.947 | 0.000 | 0.000 |

- **Higher avg spreads** with **deleveraging** (pivot wedge dominates composition).
- Spreads **more countercyclical**; volatility of spreads/debt **falls** (**stability illusion**).
- Consumption volatility **nearly unchanged**; risk insurance **impaired**.

# Price, Spread, and Default Risk



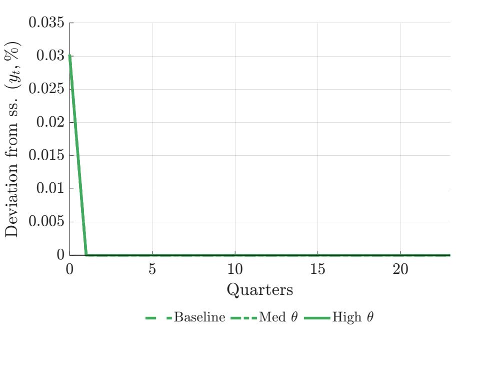

Bond prices

Spreads

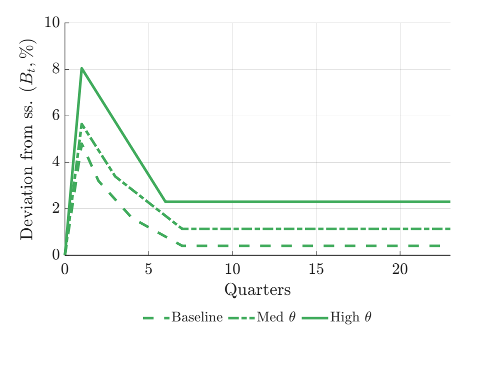
Default probabilities

**Single-crossing pivot** around $B^*(y)$; PRO discounts **safe region** and **softens near-doom**.

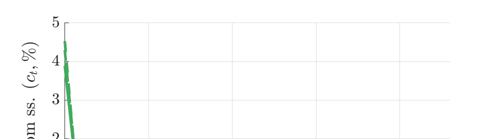
# Impulse Responses: Transitory Shock

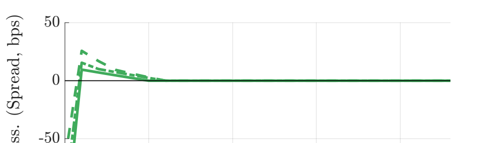



Output

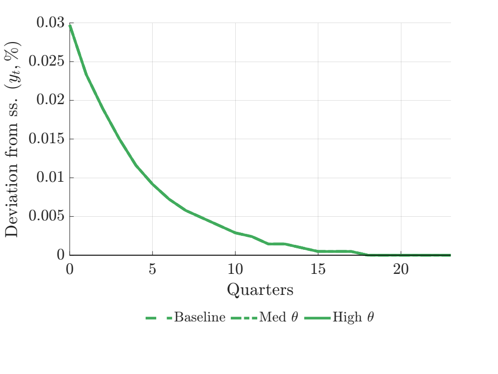


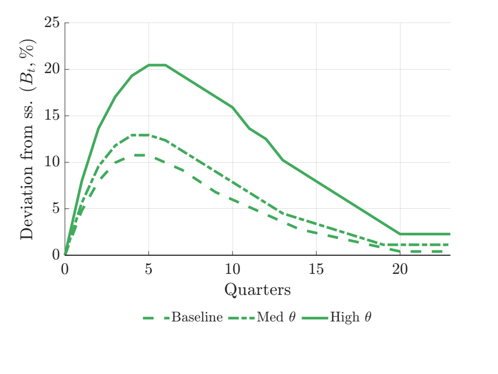

Debt



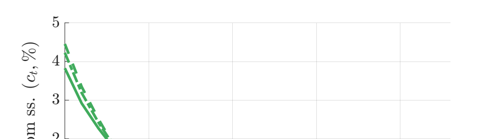
# Impulse Responses: Persistent Shock

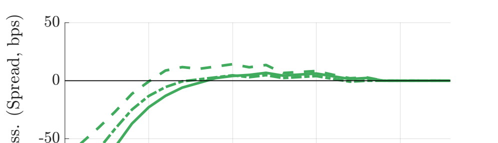



Output

Debt



# Microfoundation (RI)

# Rational Inattention: Tail Weight from Attention

- Lenders choose **precisions** $(a_\mu, a_\sigma)$ at convex cost $\Phi(a_\mu, a_\sigma)$.
- **FOC**: $\varphi\,\mathcal{S} = \kappa_\sigma a_\sigma \Rightarrow a_\sigma = \frac{\varphi}{\kappa_\sigma}\,\mathcal{S}$.
- **Tail weight**: $\theta_{\mathtt{RI}}(y, B') = \mathtt{min}\left\{1 + \dfrac{\varphi^2}{\kappa_\sigma}\,\mathcal{S}(y, B')\,,\ \bar{\theta}\right\}$.
- Pricing remains the **same operator** at $\theta_{\mathtt{RI}}(\cdot)$; comparative statics **inherit**.

$$q(B', y) = \mathcal{T}_{\theta_{\mathtt{RI}}(y,B')}[q](B', y), \qquad \mathcal{S} = \mathbb{E}\Big[\partial U / \partial \theta\Big] \geq 0.$$

# Empirical Hook: Misreporting $\Rightarrow$ Higher Dispersion Attention

- **Degraded mean-information** $(a_\mu)$ raises marginal value of dispersion info $\mathcal{S}$.
- $\uparrow \mathcal{S} \Rightarrow \uparrow a_\sigma \Rightarrow \uparrow \theta_{\texttt{RI}}$: **higher average spreads, steeper pivot, decoupling**.

# Policy & Information

# Ramsey with PRO: Transfers Cannot Undo Price Wedge

$$c_t + \kappa B_t + \tau_t = y_t + \big(B_{t+1} - (1{-}\delta)B_t\big)\, q_\theta(y_t, B_{t+1}),$$
$$\mathbb{E}_0 \sum_t \beta^t \tau_t = 0, \quad u'(\cdot) > 0,\ u''(\cdot) < 0.$$

- **Intertemporal trade price** distorted by PRO persists in implementability; **deadweight loss**.
- **Result**: $W_\theta^R < W_1^R$ even with optimal transfers.

# Endogenous Beliefs and Transparency

Belief dynamics with negativity bias:

$\theta_{t+1} = \lambda\,\theta_t + (1-\lambda)\,\hat{\theta}(\{d_s\}), \quad \xi(y,B) = \mathtt{max}\left\{0, \frac{P_1 - P_{\theta_t}}{P_1}\right\}, \quad$ defaults move beliefs more.

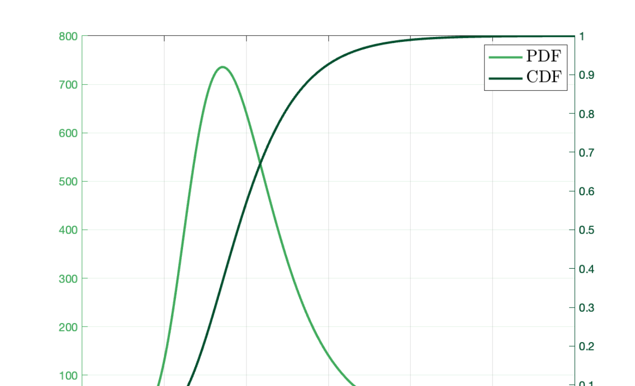
Effective transparency:

$$\theta_{\mathtt{eff}}(\alpha,\theta) = \alpha \cdot 1 + (1-\alpha)\cdot\theta, \qquad \alpha^* : \frac{\mathsf{d}}{\mathsf{d}\alpha} W(\alpha) = \gamma\alpha.$$

- **Persistent PRO** in invariant beliefs; **optimal transparency rises** with PRO severity.

# Computation

- **Value and price iteration** on $(N_y{=}201, N_B{=}600)$ grid; OpenMP parallel.
- **Stabilized log-sum-exp** for borrowing/default logits; **infeasible-consumption guard**.
- **Convergence** tolerances $10^{-6}$; long simulation for moments and IRFs.



# Conclusion

# Takeaways

- **Single operator + PRO** **pivot** in price/spread schedules.
- **Deleveraging** yet **higher average spreads**; volatility **falls** (**stability illusion**).
- **RI microfoundation** endogenizes tail tilt; **policy/info** extensions clarify limits and levers.
- Event hooks (Argentina) align with **pivot**, **threshold**, and **decoupling** predictions.

# Literature Review

# Sovereign Default: Canonical and Extensions

- **Canonical strategic default**: Eaton – Gersovitz (1981); Aguiar – Gopinath (2007); Arellano (2008); long-term debt: Hatchondo – Martinez (2009); Chatterjee – Eyigungor (2012); Mendoza – Yue (2012).
- **Empirics**: Tomz – Wright (2013); Meyer – Reinhart – Trebesch (2022); event studies (Argentina misreporting).
- **Reputation**: Cole – Dow – English (1995); Phelan (2006); Amador – Phelan (2021, 2023); learning via policy signals (Fourakis, 2024).

# Beliefs, Ambiguity, and Information

- **Ambiguity/robust control**: Hansen – Sargent (2001, 2008); smooth/variational prefs (Klibanoff et al., 2005; Maccheroni et al., 2006); applications to default (Pouzo – Presno, 2016; Roch – Roldan, 2023).
- **Behavioral beliefs**: Diagnostic expectations (Gennaioli – Shleifer, 2018; Bordalo et al., 2023); sentiment, noise trading, limits to arbitrage.
- **Information choice**: Rational inattention (Sims, 2003; Mačkowiak – Wiederholt, 2009; Matejka – McKay, 2015); macro finance (Veldkamp, 2011).

**This paper**: *second-moment* belief wedge (PRO) **pivot**, **deleveraging**, **stability illusion**; RI microfoundation embeds **in the same operator**.

# Deeper Math

# Bellman Aggregator and Sensitivities

$$J_\rho[V](y,B) = \rho \ln \sum_{B'} \exp \frac{W(y,B,B')}{\rho}, \quad V(y,B) = \eta \ln \left( e^{V^D/\eta} + e^{V^R/\eta} \right).$$

$$\frac{\partial V}{\partial q} \;=\; u'(c)\,[B' - (1{-}\delta)B] \cdot \mathsf{softmax}(W/\rho). \quad \text{(Envelope over } B'\text{)}$$

- Lipschitz: $\|J(V_1){-}J(V_2)\| \le \beta\|V_1{-}V_2\|$, $\|J(\cdot,q_1){-}J(\cdot,q_2)\| \le L_{Jq}\|q_1{-}q_2\|$.
- Pricing: $\|T(V_1,\cdot){-}T(V_2,\cdot)\| \le L_{TV}\|V_1{-}V_2\|$, $\|T(\cdot,q_1){-}T(\cdot,q_2)\| \le m_q\|q_1{-}q_2\|$.
- **Slope condition**: $L_{Jq}L_{TV} < (1-\beta)(1-m_q)$ with $m_q = (1-\delta)/(1+r)$.

# From $\partial_\theta P$ to $\partial_\theta q$

$$P_\theta = \mathsf{L}\Big(-\tfrac{\Delta V}{\theta\eta}\Big), \quad \partial_\theta P_\theta \;=\; L'(\cdot)\,\tfrac{\Delta V}{\theta^2\eta}.$$

$$(I-\mathcal{T}_\theta)'\,\partial_\theta q_\theta \;=\; \partial_\theta \mathcal{T}_\theta[q_\theta], \quad (I-\mathcal{T}_\theta)^{-1} \geq 0.$$

$\Rightarrow$ $\texttt{sign}(\partial_\theta q_\theta)$ follows from $\texttt{sign}(\partial_\theta P_\theta)$ and positivity of $(I - \mathcal{T}_\theta)^{-1}$.

**Implication**: **pivot** persists under smooth perturbations and state-dependent $\theta_{\texttt{RI}}(y, B')$.

# Backup

# Operator View (Sketch)

- $\mathcal{T}_\theta$ is positive and order-preserving; fixed point unique under slope condition.
- Fixed-point differentiation signs $\partial_\theta q_\theta$; monotone propagation yields pivot.