Default with Policy-Randomness Overestimation

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[INCOMPLETE AND PRELIMINARY]
October 13, 2025

Abstract

Why do emerging economies face persistently high borrowing costs despite moderate debt levels? I develop a quantitative sovereign default model where lenders systematically overestimate government policy randomness—policy-randomness overestimation (PRO). This behavioral bias creates a "PRO wedge" that pivots bond prices—making debt cheaper near default but more expensive in normal times. Rational sovereigns respond by deleveraging yet paradoxically face higher average spreads and an "illusion of financial stability" where volatility falls despite rising risk premiums. Theoretical extensions demonstrate that optimal fiscal policy cannot eliminate allocative distortions from PRO, learning frictions can perpetuate beliefs exhibiting PRO, and strategic communication can partially mitigate these effects. The framework provides a new behavioral foundation for understanding sovereign debt puzzles in emerging markets.

Keywords: Sovereign Default, Behavioral Macroeconomics, Policy-Randomness Overestimation (PRO), Sovereign Spreads, Information Frictions, Argentina, Debt Management.
JEL Codes: F34, E62, G15, D91.

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1 Introduction

Why is sovereign debt in some emerging economies particularly expensive? Standard models struggle to explain why countries with moderate debt-to-GDP ratios face persistently high borrowing costs, excess volatility, and puzzling "decoupling" from their peers (**??**). Argentina exemplifies this puzzle: as documented by (**?**), its borrowing costs often appear divorced from macroeconomic fundamentals, suggesting a large country-specific risk premium that defies conventional explanation.

The leading explanation centers on reputation and *information frictions* (**?**). In this view, advanced quantitatively by (**?**), lenders are rational but uninformed about a government's hidden type—whether "committed" or "strategic." Policy missteps like Argentina's inflation misreporting signal a "bad type," causing persistent reputation downgrades. High spreads reflect the market's efficient pricing of this revealed information.

This paper explores an alternative *behavioral friction*. What if the problem is not what lenders do not know, but what they systematically believe to be true? Departing from rational learning, I posit that lenders suffer from policy-randomness overestimation (PRO): a second-moment belief distortion about government policy randomness. While sovereigns face unobserved "taste shocks," lenders perceive these shocks as larger and more volatile than they truly are. This "PRO wedge" distorts default risk assessment. Events like Argentina's misreporting signal not just strategic behavior but perceived policy unpredictability. Markets react to this perceived randomness increase alongside reputational downgrades, offering a complementary channel rather than a substitute.

My main contribution embeds this behavioral friction into a quantitative sovereign default model and traces its macroeconomic consequences. Lenders with policy-randomness overestimation (PRO) fundamentally alter the borrowing environment by "pivoting" bond price schedules—making debt cheaper near default but more expensive in safe states. Rational sovereigns respond optimally by deleveraging yet paradoxically face higher average spreads. The model generates an "illusion of financial stability" where market volatility falls despite rising risk premiums. This mechanism is complementary to reputational learning and can coexist in the data.

Intuition: Deleveraging with Higher Spreads The pivoted pricing created by PRO raises spreads precisely where governments typically borrow and lowers them only near the brink of default. In the normal borrowing region (future debt $B' < B^*(y)$), PRO magnifies perceived policy randomness, making lenders demand extra compensation; prices q fall and spreads s rise even when default risk is low. Near default ($B' > B^*(y)$), the same belief distortion makes lenders attach more weight to (perceived) repayment realizations, softening the "doom" and raising prices there. A rational sovereign facing this pivoted schedule borrows less (deleverages), which shifts the distribution of choices toward the safe region. Averaging over states, two forces determine the net effect: (i) a price-wedge effect—under PRO, $1/q$ is higher than under rational pricing in the safe region—raises the average spread; (ii) a composition effect—deleveraging moves weight toward states with lower baseline spreads—works in the opposite direction. Because the sovereign spends most of its time in the safe region, the price-wedge effect dominates, so average spreads rise even as debt falls. This intuition is formalized in the paper by a simple two-term decomposition and local comparative statics that deliver $\partial q/\partial\theta < 0$ in the safe region and $\partial q/\partial\theta > 0$ near default.

Three theoretical extensions deepen the analysis. First, even optimal Ramsey fiscal policy cannot eliminate welfare losses from PRO due to fundamental allocative inefficiencies from distorted bond pricing. Second, endogenous belief formation through Bayesian learning with negativity bias shows how beliefs exhibiting PRO persist and become entrenched. Third, optimal policy communication demonstrates how governments can strategically choose transparency levels to partially mitigate the effects of PRO. The analysis provides a new behavioral foundation for understanding sovereign risk and persistent debt challenges in emerging economies.

Literature This paper contributes to the sovereign default literature by proposing a novel behavioral mechanism to explain persistent empirical puzzles: why do emerging economies often face high spreads and excess volatility that seem disconnected from their macroeconomic fundamentals (**??**)?[1]

The dominant paradigm treats default as a purely *strategic decision* where sovereigns rationally weigh repayment costs against temporary market exclusion (**???**). While this framework has been successfully extended to incorporate long-term debt and financial frictions (**???**),[2] it struggles to explain why spreads often remain elevated even when fundamentals improve.

A second approach emphasizes *reputational concerns*, where past actions cast long shadows over future borrowing costs (**??**). In this framework, the central question is how lenders learn about the government's hidden type. While classic models focus on default itself as the ultimate signal, recent research shows the informational channel is much broader. For instance, (**?**) provides quantitative evidence that investors learn about government types through fiscal and monetary policy indicators, finding that deficit and inflation surprises significantly affect perceived default probabilities and that reputation loss often occurs *before* a default event. This perspective is powerfully exemplified by the Argentina inflation misreporting episode, rigorously analyzed by (**?**), where a single breach of trust—interpreted as a credible signal of a "bad type"—led to years of market exclusion.This rich

[1] For comprehensive surveys of the sovereign debt literature, see (**?**) and (**?**).

[2] Empirical evidence on emerging market business cycles and debt restructurings is provided by (**?**), (**?**), and (**?**).

view of reputation has been embedded in models that explain how a country can eventually ”graduate” to a high-trust state through a long history of good behavior (?) and why, in a partial default setting, larger haircuts must rationally lead to a greater loss of reputation to sustain a mixed-strategy equilibrium (?).[3] Yet even this learning-based view assumes lenders eventually converge to the truth, leaving unexplained why some sovereign risk premia appear systematically and persistently excessive.

Comparative framing. I view type-based reputation and PRO as complementary rather than competing channels. Under canonical implementations, reputation models tend to generate monotonic price effects via type revelation, whereas the second-moment distortion in this paper (PRO) implies a pivoting price schedule. This difference yields distinct, testable predictions for default thresholds, cross-maturity responses to informational shocks, and spread–cycle comovement. Empirically, both mechanisms may coexist: reputation primarily shifts the perceived mean (type) while PRO inflates the perceived dispersion of policy choices.

A third strand recognizes that market *sentiment itself* can become a fundamental force. Beginning with (?) and (?), this literature demonstrates how pessimistic expectations can become self-fulfilling, with modern quantitative implementations by (?) and (?).[4] My paper builds directly on this insight but asks a deeper question: what if markets systematically *misperceive* the very nature of sovereign decision-making?

Drawing from behavioral economics, I propose policy-randomness overestimation (PRO) rooted in well-documented biases like *heuristics and biases* (?), *prospect theory* (?), and *noise trading* (?)—can create persistent wedges between sovereign risk and fundamentals.[5] Crucially, my mechanism differs from existing behavioral approaches. For instance, models of *ambiguity aversion* based on robust control theory (??) assume that lenders are uncertain about the true model of macroeconomic fundamentals. This leads them to price assets based on a ”worst-case” scenario, generating an ambiguity premium that can explain high sovereign spreads (?) and the puzzlingly poor pricing of contingent debt (?). While these models distort the perceived distribution of *macroeconomic shocks* across all states, my “PRO wedge” specifically targets the perceived volatility of the sovereign’s *policy choices*, creating a distinctive bond-price *pivot*.[6] And unlike *diagnostic expectations*

[3]Other important contributions to the reputation literature include (?) on partial versus general reputations, (?) on government reputation and debt repayment, (?) on reputational effects in sovereign default, and (??) on reputation in policy design. A key distinction is that while reputation models tend to predict (under canonical assumptions) that ”bad types” default at lower debt thresholds, the mechanism in this paper can imply the opposite: PRO lenders may perceive higher default thresholds due to overestimated policy randomness.

[4]Related work explores financial frictions (?), risk aversion (?), maturity choice (?), and rational learning about shifts in fundamentals, such as the ”rare disaster” mechanism in (?) used to explain the slow onset of the European debt crisis.

[5]Additional behavioral foundations include investor sentiment (?), rare disasters (?), disposition effects (?), crisis psychology (?), and limits to arbitrage (??).

[6]This pivot effect differs from both reputation and ambiguity aversion models. While reputation models like (?) often generate monotonic price effects through type revelation under standard implementations,

(**??**), which generate boom-bust cycles through time-varying news overreaction, my time-invariant bias produces persistent "PRO premia," counterintuitive deleveraging, and the "softening-of-doom" effects that standard formulations may find challenging to replicate.[7]

2 Motivation: Argentina's Inflation–Misreporting Episode

A Puzzle of Persistent Risk The recent economic history of Argentina offers a powerful illustration of a core puzzle in international finance: why do emerging economies often face borrowing costs that seem disconnected from their macroeconomic fundamentals? (**??**). Standard models, even those incorporating reputational dynamics, may not fully account for the persistence and magnitude of the country-specific risk premium observed in cases like Argentina, where borrowing costs have often appeared divorced from traditional measures of repayment capacity (**?**). This suggests the presence of frictions beyond those typically modeled.

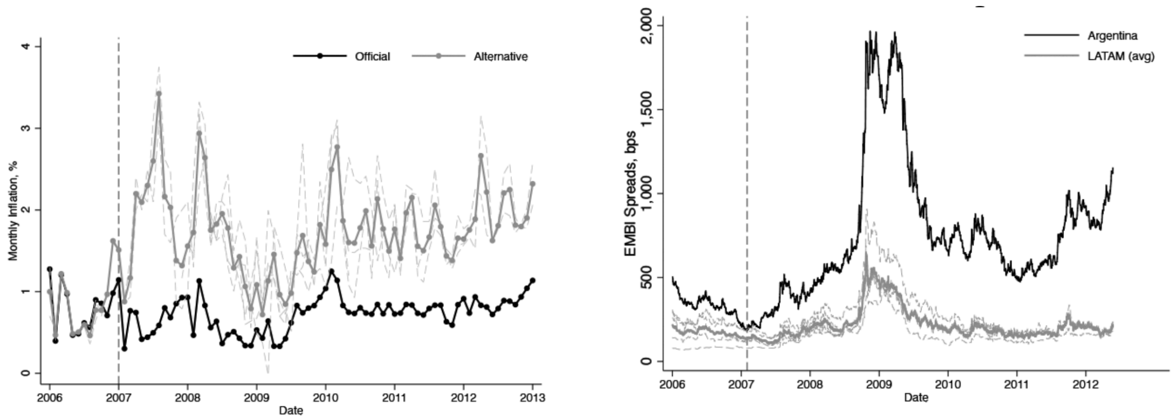
The Misreporting Episode This puzzle was cast in sharp relief during the inflation misreporting episode that began in 2007. Following a period of rising inflation, the Argentine government initiated a direct political intervention in its national statistics institute, INDEC. Senior technical staff were dismissed and replaced, leading to an immediate and sustained suppression of the official Consumer Price Index (CPI) (**?**). This official figure diverged starkly from credible estimates produced by private and provincial sources (**?**). The government's enforcement was aggressive, levying substantial fines on private consultancies that published their own, more realistic, data (**?**). The data manipulation grew so notorious that *The Economist* publicly ceased publishing the official figures in 2012, and the IMF issued a rare "declaration of censure" against the country (**??**).

The financial consequences were profound. As inflation-indexed bonds constituted a significant portion of public debt, this act amounted to a *de facto* partial default. The market's reaction was swift and severe. Argentina's EMBI+ spread, which had been tracking its Latin American peers, decoupled and widened sharply. Crucially, this repricing was not limited to the directly affected instruments; it spilled over entirely to its dollar-denominated sovereign debt. This reaction is paradoxical from a purely mechanical standpoint: a policy that lowers the real debt burden should have *decreased* default risk on nom-

and (**?**) shows reputation loss *before* default through rapid debt accumulation, the mechanism here can produce non-monotonic price schedules where debt becomes cheaper near default but more expensive in normal times. Similarly, ambiguity models like (**?**) explain high spreads through a uniform distortion of fundamental shocks, which also differs from the pivot mechanism.

[7]The empirical predictions may differ from those of reputation models. While (**?**) find that misreporting episodes increase spreads on all debt instruments, the mechanism here implies divergent effects across the debt distribution. Similarly, while (**?**) shows reputation effects strengthen with haircut size, the mechanism here implies that PRO effects may weaken as larger haircuts reduce the perceived randomness component.

inal bonds. The opposite happened, indicating the event was a pure, and powerful, information shock about the government's character and future actions.



(a) Official CPI (black) vs alternative measures (gray).

(b) EMBI+ spreads: Argentina (black) vs other LA (gray).

Figure 1: Argentina's misreport of inflation and decoupling of spreads. Panel A shows the monthly official inflation rate announced by the Argentine government (black line) and alternative measures of inflation (gray lines). Panel B shows annualized emerging market bond index spreads for Argentina (black line) and other Latin American countries (gray lines). Vertical lines denote the first month in which the Argentine government underreported the inflation rate. Sources: INDEC (official CPI); private and provincial estimates (**?**); J.P. Morgan EMBI+.

Reputational Interpretation and Open Questions The conventional explanation, rooted in reputational models (**?**), interprets this episode as a credible signal of a "bad type." In this view, advanced by (**?**), lenders are rational but uninformed about a government's hidden commitment to repay. The misreporting revealed Argentina's government as a "strategic" type, leading to a rational, persistent downgrade of its reputation and, consequently, higher borrowing costs. While this view is powerful, it leaves lingering questions. If the market simply learned the government's type, why did the risk premium appear to contain an additional, seemingly excessive, component? Why did market sentiment seem to reflect not just a reassessment of character, but a new apprehension about the government's very predictability? The episode suggests that the market's reaction was not just to what it learned about the sovereign's *intent*, but to a perceived increase in its *erratic nature*.

Comparative reading Both channels likely operated during the misreporting episode. A reputational interpretation views misreporting as a credible signal that raises spreads broadly by shifting the posterior over types. PRO instead emphasizes a perceived increase in policy unpredictability (dispersion), which implies a *pivoting* price schedule: cheaper terms near the brink (due to a perceived chance of "irrational" repayment) but costlier terms in normal times. This yields two testable implications: (i) a heterogeneous cross maturity response—reputation effects tend to lift spreads across instruments, whereas PRO can produce divergent effects across the debt distribution; and (ii) a sustained elevation in average spreads even if volatility later declines (an "illusion of stability"), consistent with a persistent second-moment bias. See Table 1 for a summary of differences under canonical implementations.

A Behavioral Hypothesis: Policy-Randomness Overestimation (PRO) This paper explores an alternative, yet complementary, behavioral friction. I posit that the problem is not only what lenders do not know, but also what they systematically overestimate: the randomness in sovereign policy choices (policy-randomness overestimation, PRO). My core assumption is that lenders perceive the unobserved shocks driving sovereign policy choices to be larger and more volatile than they truly are. This "PRO wedge" distorts their assessment of default risk. From this perspective, an event like Argentina's misreporting is not just a signal of a sovereign's bad character (a strategic type), but is interpreted as evidence of its erratic nature (high unpredictability). The market reacts to a perceived increase in policy randomness, not just a downgrade of its reputation. This overestimation is not irrational in a colloquial sense; rather, it is a systematic bias in belief formation, consistent with behavioral findings on how agents process information under uncertainty (**??**).

A Bridge to the Model Motivated by this reinterpretation of the Argentine case, I embed this behavioral friction into an otherwise standard quantitative sovereign default model. The subsequent sections will formalize the "PRO wedge" and trace its consequences. I will show that the presence of PRO lenders fundamentally alters the sovereign's borrowing environment, creating a distinctive "pivoting" of the bond price schedule. This, in turn, generates a series of counterintuitive but empirically relevant outcomes: a rational sovereign deleverages yet faces higher average spreads, and market volatility can fall even as the underlying risk premium rises, creating an "illusion of financial stability." This framework provides a new, behaviorally-grounded perspective on the persistent debt challenges that many emerging economies face.

Roadmap. Section 3 lays out the environment and recursive equilibrium; Section 4 develops the core comparative statics (price/spread pivots, default threshold, borrowing, welfare); and Section 6 develops a microfoundation via rational inattention to second moments; Section 7 embeds policy and information extensions (Ramsey planning, endogenous beliefs, optimal communication). The appendix collects proofs and operator analysis.

3 A Model of Policy-Randomness Overestimation (PRO)

3.1 Environment

Time is discrete and the horizon is infinite, $t = 0, 1, 2, \ldots$. The economy receives a stochastic endowment of a single tradable good, y_t. The endowment process is exogenous and follows a stationary, first-order Markov process, which is generated by discretizing the following AR(1) process in logarithms:

$$\ln y' = (1 - \rho_y)\mu_y + \rho_y \ln y + \sigma_y \varepsilon', \quad \varepsilon' \sim \mathcal{N}(0, 1). \tag{1}$$

The transition probabilities are given by the matrix $\Pi(y, y') = \Pr\{y_{t+1} = y' | y_t = y\}$.

The government can borrow from a large number of competitive, risk-neutral international lenders who have access to an international risk-free interest rate r. Debt takes the form of long-term bonds. A bond is a claim to a stream of coupon payments κ in every future period, unless the sovereign defaults. Each period, a fraction $\delta \in (0, 1]$ of outstanding bonds matures, while the remaining fraction $1 - \delta$ carries over to the next period.

The sovereign government has preferences represented by a standard time-separable utility function with a discount factor $\beta \in (0, 1)$. The period utility function is of the CRRA form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \tag{2}$$

which is strictly increasing and concave for $\sigma > 0$.

3.2 The Sovereign's Problem

At the beginning of each period t, the state is summarized by the current endowment realization $y \in \mathcal{Y}$ and the stock of outstanding debt B. The sovereign first decides whether to default on its obligations or to repay. This choice is subject to an idiosyncratic preference shock, often referred to as a "taste shock," which introduces randomness into the decision-making process from the perspective of an outside observer.

Taste Shocks Let $V^D(y)$ be the deterministic component of the value of defaulting, and $V^R(y, B)$ be the deterministic component of the value of repaying. The full, or *ex-post*, value for each choice is the sum of its deterministic part and a random shock:

$$\tilde{V}^D(y, \varepsilon_d) = V^D(y) + \varepsilon_d$$
$$\tilde{V}^R(y, B, \varepsilon_r) = V^R(y, B) + \varepsilon_r$$

The sovereign observes the shocks ε_d and ε_r and chooses the action that yields the highest *ex-post* value. The *ex-ante* value function, from a perspective before the shocks are realized, is the expected maximum of these *ex-post* values:

$$V(y,B) = \mathbb{E}_{\varepsilon_d,\varepsilon_r}\left[\max\left\{\underbrace{V^D(y)+\varepsilon_d}_{\tilde{V}^D(y,\varepsilon_d)}, \underbrace{V^R(y,B)+\varepsilon_r}_{\tilde{V}^R(y,B,\varepsilon_r)}\right\}\right], \tag{3}$$

where the expectation is taken over the distribution of the shocks.

Following (?) and (?), I assume that the taste shocks ε_d and ε_r are drawn independently from a Gumbel distribution.[8] To visualize the distribution of these shocks, Figure 2 plots the probability density function (PDF) and cumulative distribution function (CDF) for the Gumbel distribution, normalized to have a mean of zero. The scale parameter, η, is pivotal. It governs the variance of the taste shocks, given by $\mathrm{Var}(\varepsilon_i) = \frac{\pi^2\eta^2}{6}$. A larger η signifies greater dispersion in preferences and introduces more randomness into the sovereign's choice.[9] Economically, these taste shocks can be interpreted as a reduced-form representation of various unmodeled factors that influence policy, such as political pressures from domestic constituencies, bureaucratic implementation errors, or the private information of policymakers. By modeling them as random draws, the framework acknowledges a degree of inherent unpredictability in government behavior.

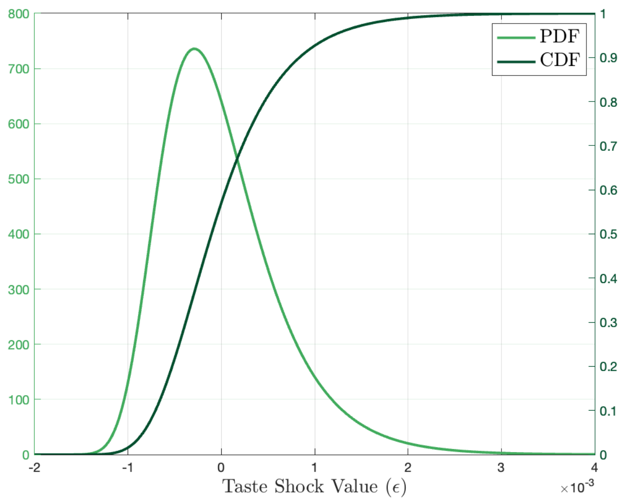


Figure 2: The mean-zero Gumbel distribution ($\eta = 5 \times 10^{-4}$).
Note: The Gumbel distribution is used to model the taste shocks in the sovereign's default and borrowing decisions. The scale parameter η controls the variance of these shocks. The left y axis is the PDF, the right y axis is the CDF.

[8]The CDF of a Gumbel distribution is given by $F(\varepsilon;\mu_L,\eta) = \exp(-\exp(-(\varepsilon-\mu_L)/\eta))$, where μ_L is the location parameter and $\eta > 0$ is the scale parameter. The mean of this distribution is $\mu_L + \eta\gamma$, where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. For analytical convenience, I choose a specific parameterization, Gumbel$(-\eta\gamma,\eta)$, which makes the mean of the shocks equal to zero: $\mathbb{E}[\varepsilon_i] = -\eta\gamma + \eta\gamma = 0$.

[9]As $\eta \to 0$, the shocks' influence diminishes, and the model approaches a deterministic framework where decisions are based solely on $V^D(y)$ and $V^R(y,B)$. Conversely, as $\eta \to \infty$, the deterministic value components become negligible, and the choice becomes almost entirely random.

Ex-Ante Value Function The ex-ante value function can be derived using fundamental properties of the Gumbel distribution. The lemma provided in Appendix A provides the closed-form expression for the expected maximum of Gumbel-distributed random variables, which forms the basis for my analytical solution. First, applying Lemma 2 to my setting with $V_1 = V^D(y)$ and $V_2 = V^R(y,B)$, the ex-ante value function is:

$$V(y,B) = \mathbb{E}\left[\max\{\tilde{V}^d, \tilde{V}^r\}\right] = \eta \ln\left(\exp\frac{V^D(y)}{\eta} + \exp\frac{V^R(y,B)}{\eta}\right). \tag{4}$$

Discrete Choice The choice probabilities in my model follow directly from another fundamental property of the Gumbel distribution. Using Lemma 3 in Appendix A, the probability of default is:

$$\Pr\{d=1|y,B\} = \frac{\exp\frac{V^D(y)}{\eta}}{\exp\frac{V^D(y)}{\eta} + \exp\frac{V^R(y,B)}{\eta}}. \tag{5}$$

Value of Default If the sovereign defaults, it is excluded from international credit markets. During exclusion, it bears an output cost and consumes a fraction of its endowment, $c = h(y) \leq y$, where the output cost function is specified similar to (**?**):

$$h(y) = y - \max\{0, \lambda_0 y + \lambda_1 y^2\}. \tag{6}$$

In each period of exclusion, there is a constant probability $\gamma \in (0,1)$ that the country regains market access. Upon re-entry, all past debts are forgiven, so it starts with $B = 0$. The value of being in default is therefore:

$$V^D(y) = u(h(y)) + \beta\mathbb{E}_{y'|y}\left[\gamma V(y',0) + (1-\gamma)V^D(y')\right]. \tag{7}$$

Value of Repayment If the sovereign honors its debt, it pays the coupon κB and retains market access. It can then choose a new level of debt for the next period, B'. This choice is also subject to taste shocks. The *ex-ante* value of choosing a specific debt level B', given the state (y,B), is:

$$W(y,B,B') = u\left(y - \kappa B + [B' - (1-\delta)B]\, q(y,B')\right) + \beta\mathbb{E}_{y'|y}\left[V(y',B')\right], \tag{8}$$

where $q(y,B')$ is the price at which it can issue new bonds. The borrowing choice is subject to i.i.d. Gumbel shocks, denoted by $\{\varepsilon_{B'}\}$, for each possible debt level B'. Each shock is distributed as Gumbel$(-\rho\gamma, \rho)$. Applying Lemma 4 in Appendix A with $V_i = W(y,B,B_i)$ and $\sigma = \rho$, the ex-ante value of repaying is:

$$V^R(y,B) = \rho \ln\left(\sum_{B' \in \mathscr{B}} \exp\frac{W(y,B,B')}{\rho}\right), \tag{9}$$

where $\mathscr{B}$ is the discrete set of possible debt levels. The probability of choosing a specific level B' is:

$$\Pr\{B'|y,B\} = \frac{\exp\frac{W(y,B,B')}{\rho}}{\sum_{B_j\in\mathscr{B}}\exp\frac{W(y,B,B_j)}{\rho}}. \tag{10}$$

Figure 3 provides a visualization of this probabilistic borrowing policy. The taste shock framework transforms the choice of the next debt level, B', from a single deterministic point into a smooth probability distribution over the entire set of available options, $\mathscr{B}$. The peak of the distribution corresponds to the most preferred borrowing choice, but the scale parameter ρ ensures that other, less optimal choices still have a non-zero probability of being selected. This feature captures *unobserved heterogeneity* in the sovereign's decision-making process.[10]

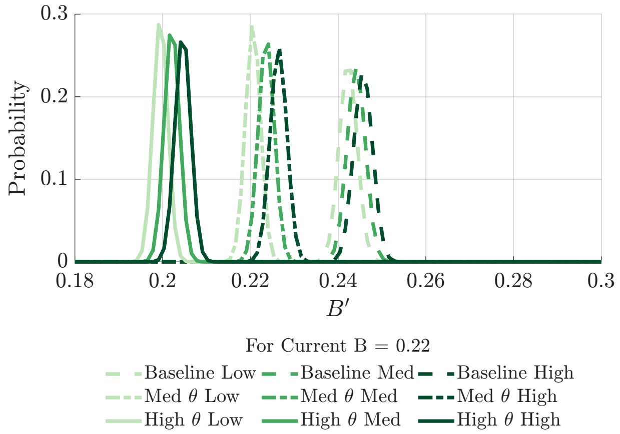


Figure 3: Example of the Probabilistic Borrowing Policy, $\Pr\{B'|y,B\}$.
Note: The figure illustrates the sovereign's borrowing choice as a probability distribution over possible next-period debt levels (B'), given a specific state (y,B). The peak of the distribution represents the most likely choice.

3.3 International Lenders and Bond Pricing

I depart from the standard model by assuming a *wedge* between the sovereign's true behavior and lenders' perception of it. Lenders in this model are competitive and risk-neutral, pricing bonds to make zero expected profit according to their beliefs. However, their beliefs are systematically biased in a specific way.

Policy-Randomness Overestimation (PRO) The key assumption is that lenders perceive the sovereign to be more erratic or "irrational" than it truly is. They believe the sovereign's

[10]This probabilistic approach is also crucial for the numerical stability of the model, as it replaces the non-differentiable "max" operator with a smooth, analytical expression, which is similar to the idea of (**?**).

default decision is governed by a taste shock with a scale parameter $\tilde{\eta} = \theta \cdot \eta$, where $\theta \geq 1$. The parameter θ captures the degree of *policy-randomness overestimation (PRO)*. This belief distortion is conceptually distinct from parameter uncertainty or ambiguity aversion. When $\theta > 1$, lenders act *as if* the government's choices are more random than they actually are.

Consequently, the lenders' perceived probability of default at a future state (y', B'), which I denote $\tilde{P}(y', B')$, is calculated using this inflated shock parameter similar to (5):

$$\tilde{P}(y', B') = \frac{\exp \frac{V^D(y')}{\theta\eta}}{\exp \frac{V^D(y')}{\theta\eta} + \exp \frac{V^R(y', B')}{\theta\eta}}. \tag{11}$$

Bond Price The equilibrium bond price $q(y, B')$ must satisfy the no-arbitrage condition based on this overestimation. The price equals the discounted expected payoff, where the probability of repayment is assessed using $\tilde{P}(y', B')$:

$$\begin{aligned} q(y, B') &= \frac{1}{1+r} \mathbb{E}_{y'|y} \left[\left(1 - \tilde{P}(y', B')\right) \left(\kappa + (1-\delta) \mathbb{E}_{B''|y', B'} \left[q(y', B'') \right] \right) \right] \\ &= \frac{1}{1+r} \mathbb{E}_{y'|y} \left[\left(1 - \tilde{P}(y', B')\right) \left(\kappa + (1-\delta) \sum_{B'' \in \mathcal{B}} \Pr\{B''|y', B'\} \cdot q(y', B'') \right) \right] \end{aligned} \tag{12}$$

It is important to note that the inner expectation, $\mathbb{E}_{B''|y', B'}$, is taken over the sovereign's *true* borrowing policy, $\Pr\{B''|y', B'\}$, which is governed by the true shock parameter ρ. In this framework, lenders correctly understand the sovereign's borrowing behavior but misperceive its propensity to default. This mechanism endogenously generates a credit spread that contains a "PRO premium", which reflects the lenders' biased beliefs about the sovereign's stability.

3.4 Equilibrium

As is standard in sovereign default literature, the solution concept is a Recursive Markov Perfect Equilibrium, defined as follows:

Definition 1. *A Recursive Markov Perfect Equilibrium consists of a set of functions: value functions for the sovereign ($V : \mathcal{Y} \times \mathcal{B} \to \mathbb{R}$, $V^R : \mathcal{Y} \times \mathcal{B} \to \mathbb{R}$, $V^D : \mathcal{Y} \to \mathbb{R}$), policy probabilities for its choices ($\Pr\{d = 1|\cdot\}, \Pr\{B'|\cdot\}$), and a bond price function ($q : \mathcal{Y} \times \mathcal{B} \to \mathbb{R}$), such that for all states (y, B):*

1. ***Sovereign Optimality:*** *Taking the price function q as given, the sovereign's value functions and policy probabilities solve the dynamic programming problem defined by equations (4), (7), and (9). The choices are governed by the true taste shock parameters η and ρ.*

2. ***Lender Pricing:*** *The bond price function q satisfies the zero-expected-profit condition for lenders, as specified in* (12)*, which is based on their perceived default probability $\tilde{P}$ from* (11).

With the taste shock (logit aggregator) in place, the recursive equilibrium is well behaved.

Proposition 1. *Let $\mathcal{Y}$ and $\mathcal{B}$ be compact, $u \in C^1$ strictly increasing and concave with u' bounded on the feasible consumption set, and parameters satisfy $\beta \in (0,1)$, $r > 0$, $\delta \in [0,1)$, $\eta > 0$, $\rho > 0$, $\kappa > 0$. Let prices be determined by the pricing operator in* (B.9) *with logistic default rule and repayment/default values as defined in the model, and let the Bellman aggregator be the log-sum-exp with taste-shock scale ρ. If the slope condition*

$$L_{Jq} L_{TV} \;<\; (1-\beta)\left(1 - \tfrac{1-\delta}{1+r}\right) \tag{13}$$

holds (constants defined below), then the Recursive Markov Perfect Equilibrium as in Definition 1 exists and is unique.

Proof. See Appendix B.4. □

Corollary 1 (Monotone attention and PRO intensity)**.** *If $\mathcal{S}(y,B')$ is increasing in B' and decreasing in y, then the solution of* (24) *satisfies*

$$\frac{\partial a_\sigma}{\partial B'} > 0, \quad \frac{\partial a_\sigma}{\partial y} < 0, \qquad \frac{\partial \theta_{\mathrm{RI}}}{\partial B'} > 0, \quad \frac{\partial \theta_{\mathrm{RI}}}{\partial y} < 0.$$

Unified Operators and Notation To keep the subsequent theory tightly anchored to the baseline model, I adopt an operator view of the same recursive equilibrium. Let J_ρ denote the sovereign's Bellman aggregator given by the log-sum-exp with borrowing-choice scale ρ, and let $\mathcal{T}_\theta$ denote the lenders' pricing operator induced by their perceived default probability with "policy-randomness overestimation" θ. Given the payoff kernel and resale term, the equilibrium bond price function q_θ is the unique fixed point of the pricing operator defined in (B.9). Unless otherwise noted, all results below are comparative statics or state augmentations *within this same recursive equilibrium*: primitives, bond structure, and the Bellman aggregator J_ρ remain unchanged; only the belief parameter θ (or information/policy that enters *through* $\mathcal{T}_\theta$) differs across the economies I compare.

4 Theoretical Analysis

This section develops comparative statics and extensions within the same baseline economy defined by Definition 1 and the pricing operator in (B.9). Unless stated otherwise, I hold preferences, technologies, and debt structure fixed, and vary only: (i) the lenders' belief parameter θ; (ii) the information structure that governs the evolution of θ (learning as a slow-moving state); or (iii) planner instruments that affect resources or the mapping from information to beliefs while preserving the pricing mechanism. In all cases, equilibrium objects are unique fixed points of the same operators J_ρ and $\mathcal{T}_\theta$, so the results are natural comparative statics of the *baseline* model rather than separate models.

Toolbox (in brief) I repeatedly use three ingredients drawn from the appendix: (i) the pricing operator $\mathcal{T}_\theta$ is a contraction and positive, so $(I-\mathcal{T}_\theta)^{-1}$ exists and is positive; (ii) Bellman and pricing operators are monotone, hence orderings propagate to their fixed points; and (iii) fixed-point differentiation applies, allowing us to sign $\partial_\theta q_\theta$ via an implicit-function argument. Formal statements are collected in Appendix C.5.

Before proceeding to the full quantitative analysis, this section theoretically unpacks the consequences of the behavioral wedge between the sovereign and its lenders. I demonstrate how policy-randomness overestimation (PRO, $\theta > 1$) systematically reshapes the equilibrium, beginning with its most direct impact on the bond price schedule and tracing the effects through to the sovereign's policies and ultimate welfare.

Bond Price Pivot The first and most fundamental consequence of PRO is on the price of debt. The following comparative-static result is stated *within* the baseline economy: the only difference across economies is the belief parameter θ that enters the same pricing operator $\mathcal{T}_\theta$ in (B.9). PRO does not uniformly depress bond prices; instead, it causes the price schedule to pivot relative to the rational benchmark, an effect distinct from the monotonic downward shift one might associate with reputational deterioration in canonical setups.

Proposition 2. *Consider an economy with PRO lenders ($\theta > 1$) and the baseline economy with rational lenders ($\theta = 1$), both having a small true taste shock parameter $\eta > 0$. Let $q_1(B',y)$ and $q_\theta(B',y)$ be the respective equilibrium bond price functions, each defined as the unique fixed point of the* same *pricing operator $\mathcal{T}_\theta$ in (B.9) evaluated at $\theta = 1$ and $\theta > 1$, respectively. For a given endowment level y, there exists a debt threshold $B^*(y)$ such that the price difference $\Delta q(B',y) \equiv q_\theta(B',y) - q_1(B',y)$ satisfies:*

- *For levels of future debt $B' < B^*(y)$, $\Delta q(B',y) < 0$ (PRO lenders offer lower prices).*
- *For levels of future debt $B' > B^*(y)$, $\Delta q(B',y) > 0$ (PRO lenders offer higher prices).*

Proof. See Appendix B.5. □

The formal proof in Appendix B.5 proceeds by comparing default probabilities under the two belief systems. The key insight is that PRO lenders overestimate default risk in low-debt scenarios (where fundamentals suggest safety) but underestimate the certainty of default in high-debt scenarios (where their emphasis on randomness creates perceived escape possibilities). The pivoting occurs because these two opposing effects exactly balance at the threshold $B^*(y)$.

Figure 4 provides a graphical illustration of this pivoting effect, showing how the bond price schedules for different levels of PRO levels cross at the threshold $B^*(y)$. The economic intuition behind this pivoting effect is twofold. For low debt levels ($B < B^*(y)$), where a rational lender sees default as a remote possibility, a PRO lender prices in a non-negligible risk of an "out-of-the-blue" default driven by the perceived high variance of taste shocks. This results in a "PRO premium" that lowers the bond price. Conversely, for high debt levels ($B > B^*(y)$), where a rational lender sees default as a near certainty based on fundamentals, the PRO lender's view, which emphasizes randomness, makes them less certain of this outcome.

Intuitively, PRO rotates the price schedule so that normal-time borrowing becomes *more expensive* (a positive "PRO premium" at low debt), while extreme high-debt states become marginally cheaper. Optimal policy moves the economy away from the region where PRO is favorable and toward the region where PRO is unfavorable. The average spread must therefore rise.

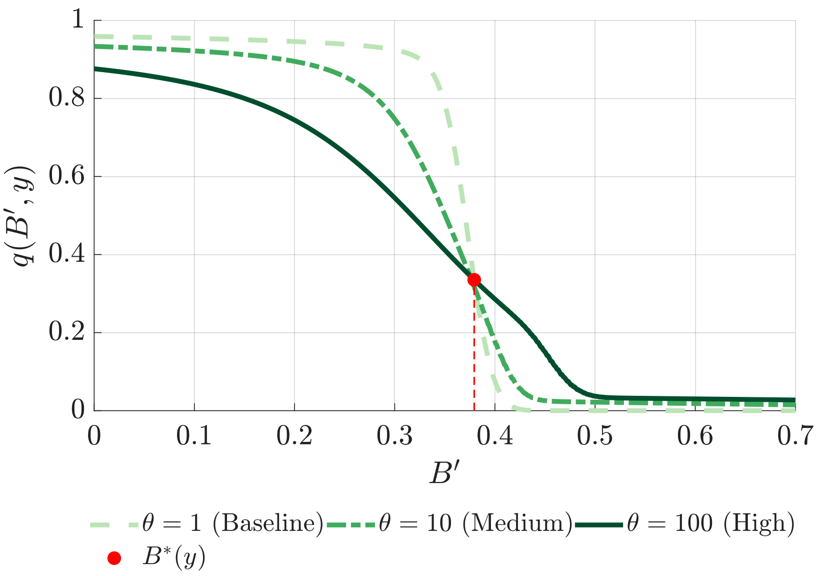


Figure 4: Pivoting Bond Price Schedules

Note: This figure illustrates the pivoting effect described in Proposition 2 for a normal output level. The bond price schedules for three different levels of PRO ($\theta = 1, 10, 100$) cross at the threshold $B^*(y)$ marked by the red dot. To the left of $B^*(y)$, PRO lenders impose a "PRO premium," offering lower prices than rational lenders. To the right of $B^*(y)$, the "softening of doom" effect emerges, where PRO lenders offer paradoxically higher prices as they perceive less certainty about default in high-debt scenarios.

Pivot Threshold This pivot point is not static; it responds to the sovereign's economic condition. The next proposition shows that as the sovereign's fortunes improve, the pivot point shifts to higher levels of debt.

Proposition 3. *The debt threshold $B^*(y)$ defined in Proposition 2, at which the baseline and PRO price schedules cross, is monotonically increasing in the endowment level y. That is, $\frac{dB^*(y)}{dy} > 0$. Figure 5 illustrates this monotonic relationship.*

Proof. See Appendix B.6. □

The intuition for this result lies in the differential response of the two markets to good news. A higher income level y improves the sovereign's repayment capacity, shifting both bond price schedules outward. However, the rational market (q_1) is more responsive to this positive signal about fundamentals than the PRO market (q_θ), whose pricing remains partially anchored by its skeptical prior about the sovereign's stability. Because the rational price schedule shifts more strongly to the right, its intersection point with the PRO schedule, $B^*(y)$, must also shift to the right. In other words, a stronger economy can sustain more debt before the PRO premium in the low debt region is outweighed by the "softening of doom" effect in the high debt region.

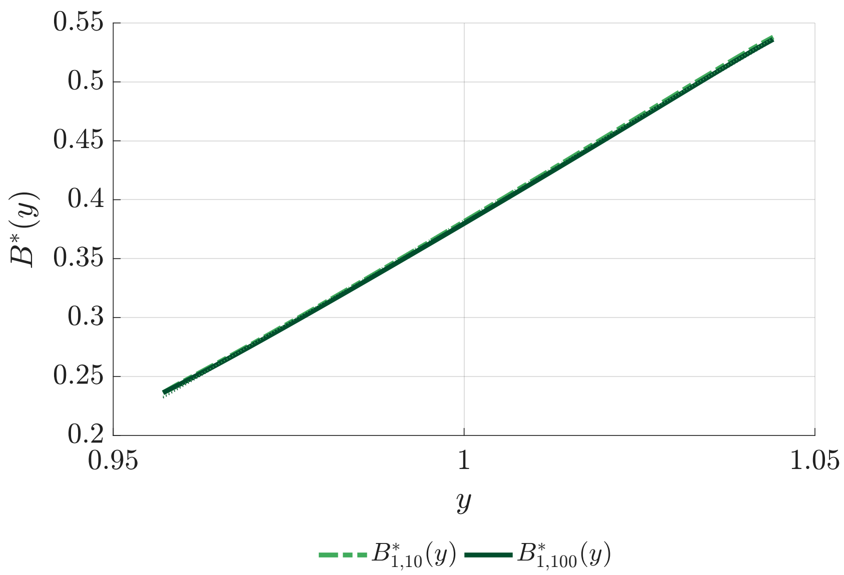


Figure 5: Monotonicity of Debt Threshold $B^*(y)$
Note: This figure illustrates Proposition 3 by showing the debt threshold $B^*(y)$ as a function of the endowment level y. The threshold represents the debt level at which the baseline ($\theta = 1$) and PRO ($\theta > 1$) bond price schedules intersect.

Spread Within the same baseline equilibrium, spreads are an inverse mapping from the same fixed-point price schedule q, so all results below are algebraic comparative statics of the *same* operators. These results on prices map directly and inversely to credit spreads. The sovereign credit spread, $s(y, B')$, is defined as the yield premium over the risk-free rate, r. Given the bond's price $q(y, B')$, the spread is:

$$s(y, B') = \frac{\kappa}{q(y, B')} - \delta - r. \tag{14}$$

This clear, inverse relationship allows the results from Proposition 2 to be restated for credit spreads.

Corollary 2. *Let $s_1(B', y)$ and $s_\theta(B', y)$ be the equilibrium credit spreads in the baseline ($\theta = 1$) and PRO ($\theta > 1$) economies, respectively. The spread difference $\Delta s(B', y) \equiv s_\theta(B', y) - s_1(B', y)$ satisfies the opposite relationship to the price difference at the same threshold $B^*(y)$ defined in Proposition 2:*

- *For low levels of future debt $B' < B^*(y)$, $\Delta s(B', y) > 0$.*
- *For high levels of future debt $B' > B^*(y)$, $\Delta s(B', y) < 0$.*

Proof. See Appendix B.7. □

Default Threshold How does a rational sovereign react to these altered market conditions? The first area where its behavior changes is at the edge of default. The following proposition establishes that PRO paradoxically makes the sovereign more resilient to debt, pushing its default threshold to a higher level.[11] This contrasts with standard reputation models, where a sovereign with a worse reputation (i.e., a higher perceived probability of being a 'strategic' type) would typically be expected to default at a lower debt level.

In keeping with the unified framework, the objects V_i^R and q_i below are determined by the *same* Bellman aggregator J_ρ and pricing operator $\mathcal{T}_\theta$; only the belief parameter is set to $i \in \{1, \theta\}$.

Proposition 4. *Consider economies with PRO lenders ($\theta > 1$) and rational lenders ($\theta = 1$). Let $B^*_{D,i}(y)$ be the sovereign's default threshold for economy $i \in \{1, \theta\}$, defined as the debt level B that satisfies the indifference condition:*

$$V_i^R(B^*_{D,i}(y), y) = V^D(y) \quad \textit{for } i \in \{1, \theta\}. \tag{15}$$

[11] This result differs fundamentally from the negative duration effect in (?). In their model, longer maturity reduces the sovereign's incentive to default because it dilutes existing bondholders, creating a debt dilution channel that works through the *quantity* of debt issued. In contrast, this mechanism operates through lender *beliefs* about the sovereign's decision-making process: PRO lenders offer better terms in high-debt states due to their perception of greater randomness in sovereign choices, making continued market access more valuable and pushing out the default threshold through a behavioral pricing channel rather than a debt structure effect.

For any given endowment level y, the default threshold is higher in the economy with PRO lenders:

$$B^*_{D,\theta}(y) > B^*_{D,1}(y).$$

Proof. See Appendix B.8. □

The formal proof in Appendix B.8 proceeds by contradiction, showing that if $B^*_{D,\theta}(y) \le B^*_{D,1}(y)$, then the "softening of doom" effect from Proposition 2 would make the value of repayment strictly higher under PRO, violating the assumed threshold ordering. The economic mechanism is that PRO lenders offer better prices in high-debt scenarios, increasing the option value of remaining in markets and making sovereigns willing to endure higher debt burdens before defaulting.

The sovereign's decision to default is a trade-off between the immediate benefit of ceasing payments and the long-term cost of losing market access. The value of this access depends directly on future borrowing terms. Proposition 2 established the key result that when debt is already high, PRO lenders offer *better* prices (the"softening of doom" effect). A rational sovereign in the PRO economy foresees these more favorable future borrowing terms should it choose to repay. This increases the option value of repaying and rolling over debt.

Why Average Spreads Rise Despite Deleveraging Let $s_i(y,B')$ denote the spread defined in (14) with price q_i for economy $i \in \{1,\theta\}$, where q_i is the equilibrium bond price under rational lenders ($i=1$) and PRO lenders ($i=\theta$). Denote by μ_i the stationary distribution over non-default states and optimal choices (including the sovereign's optimal B') in economy i. The difference in average spreads can be written as the decomposition

$$\bar{s}_\theta - \bar{s}_1 = \kappa \underbrace{\mathbb{E}_{\mu_\theta}\left[\frac{1}{q_\theta} - \frac{1}{q_1}\right]}_{\text{price wedge at PRO weights}} + \kappa \underbrace{\left(\mathbb{E}_{\mu_\theta}\left[\frac{1}{q_1}\right] - \mathbb{E}_{\mu_1}\left[\frac{1}{q_1}\right]\right)}_{\text{composition (policy) effect}}. \tag{16}$$

The pivot result (Proposition 2) implies that for each y there exists $B^*(y)$ such that $q_\theta(y,B') < q_1(y,B')$ (equivalently, $1/q_\theta > 1/q_1$) for $B' < B^*(y)$, and the opposite inequality holds for $B' > B^*(y)$. In the empirically relevant set where new issuance occurs (the primary borrowing region), one typically has $B' < B^*(y)$ and $V^R(y,B') > V^D(y)$.

Two implications follow.

- *Local comparative statics.* Differentiating (12) with respect to θ yields the linear equation
$$(I - \mathcal{T}_\theta)\frac{\partial q_\theta(\cdot)}{\partial \theta}(y,B') = -\frac{1}{1+r}\mathbb{E}_{y'|y}\Big[\big(\partial_\theta \tilde{P}(y',B')\big)\Lambda(y',B')\Big], \tag{17}$$
where $\Lambda(y',B') \equiv \kappa + (1-\delta)\sum_{B''\in\mathscr{B}} \Pr\{B''|y',B'\}\, q_\theta(y',B'')$ and $\mathcal{T}_\theta$ is the positive linear

operator

$$(\mathcal{T}_\theta f)(y,B') = \frac{1}{1+r}\mathbb{E}_{y'|y}\Big[(1-\tilde{P}(y',B'))(1-\delta)\sum_{B''\in\mathcal{B}}\Pr\{B''|y',B'\}f(y',B'')\Big].$$

Because $\|\mathcal{T}_\theta\| < 1$ under discounting and $(1-\delta) < 1$, $(I-\mathcal{T}_\theta)^{-1}$ exists and is positive. Using (11) with $\Delta V(y',B') \equiv V^R(y',B') - V^D(y')$ gives

$$\partial_\theta \tilde{P}(y',B') = \tilde{P}(y',B')\big(1-\tilde{P}(y',B')\big)\frac{\Delta V(y',B')}{\theta^2\eta}.$$

Hence, when $\Delta V(y',B') > 0$ (repayment dominates; typically $B' < B^*(y)$), the right-hand side of (17) is negative, implying $\partial q_\theta/\partial\theta < 0$, and therefore $\partial s_\theta/\partial\theta > 0$ by (14).

- *Average spread dominance.* By (16), the first term (price wedge at PRO weights) is strictly positive and *strengthened* by deleveraging, because mass shifts toward $B' < B^*(y)$ where $1/q_\theta - 1/q_1 > 0$. The second term (composition effect at baseline prices) is weakly negative since $1/q_1$ is lower at smaller B'. Under mild regularity (small taste-shock scale η and monotone price/borrowing policies), the first term dominates the second, implying $\bar{s}_\theta > \bar{s}_1$ even though the sovereign deleverages.

Borrowing Policy Within the same operator framework, only the belief parameter differs across economies; the sovereign's choice aggregator J_ρ and the induced price schedule q_i arise from the same fixed-point problems. While PRO makes the sovereign more resilient at the brink of crisis, it has the opposite effect on its day-to-day borrowing. The next proposition shows that a PRO market actively disciplines the sovereign into adopting a more conservative debt policy.

Proposition 5. *Consider economies with PRO lenders ($\theta > 1$) and rational lenders ($\theta = 1$). Let $\mathbb{E}_i[B'|y,B]$ be the expected next-period debt level chosen by the sovereign in economy $i \in \{1,\theta\}$ from a given state (y,B). For states (y,B) where the sovereign chooses* ***not to default****, the borrowing policy is systematically more conservative under PRO:*

$$\mathbb{E}_\theta[B'|y,B] < \mathbb{E}_1[B'|y,B].$$

Proof. See Appendix B.9. □

The formal proof in Appendix B.9 proceeds by comparing the sovereign's first-order conditions for borrowing under the two price schedules. The key insight is that lower prices offered by PRO lenders in the primary borrowing range (as established in Proposition 2) reduce the marginal benefit of issuing new debt. Since the marginal cost of debt remains unchanged, the sovereign optimally chooses a lower debt level to restore equilibrium between marginal benefits and costs.

A rational sovereign government reacts optimally to the market prices it faces. In the region where the sovereign typically wants to borrow, PRO lenders offer lower prices for new debt. A lower bond price is a direct signal that borrowing has become more expensive. Faced with a higher cost of capital, the sovereign's optimal response is to borrow less. The PRO market, through its pricing, effectively "disciplines" the sovereign, forcing it to deleverage and adopt a more conservative fiscal policy than it would if it faced a rational market. This endogenous deleveraging is a key mechanism through which market beliefs shape real economic outcomes.

Welfare The final step in the theoretical analysis is to evaluate the net effect of these changes on the sovereign's well-being. The final proposition demonstrates that the consequences of PRO translate directly into a welfare loss for the sovereign.

Proposition 6. *Let $V_i(y,B)$ denote the sovereign's ex-ante equilibrium value function in economy $i \in \{1,\theta\}$, where $\theta > 1$ indexes PRO lenders and $\theta = 1$ rational lenders. Suppose that for the given state (y,B) the sovereign has market access and the baseline optimal choice $B_1'(y,B)$ lies on the "risky" side of the price pivot $B^*(y)$, i.e. $B_1'(y,B) \geq B^*(y)$. Then equilibrium welfare is strictly lower under PRO lenders:*

$$V_\theta(y,B) < V_1(y,B).$$

If $B_1'(y,B) \leq B^*(y)$ ("safe" side), the weak inequality $V_\theta(y,B) \leq V_1(y,B)$ holds.

Proof. See Appendix B.10. □

The formal proof in Appendix B.10 proceeds using operator theory to show that pricing under PRO systematically reduces the sovereign's choice-specific value for all borrowing decisions. The key insight is that welfare loss stems directly from a tighter budget constraint: for any given amount of new borrowing, the sovereign receives fewer resources today under PRO lenders. While sovereigns adjust policies optimally (borrowing less, tolerating higher debt before default), they cannot fully escape the welfare loss from transacting with distorted markets.

The sovereign's welfare is fundamentally derived from its ability to use international financial markets to smooth consumption over time. The terms of this access are dictated by the bond price schedule, $q(y,B')$, which can be seen as the price of intertemporal trade. PRO, by inducing a lower q in the low debt ($B' < B^*(y)$) region, effectively acts as a tax on the sovereign's ability to conduct this trade. For any given amount of borrowing, the sovereign receives fewer resources today, which directly curtails its consumption possibilities and lowers utility. While the sovereign optimally adjusts its policies in response—by borrowing less and tolerating more debt before default—it cannot fully escape the welfare loss imposed by being forced to transact with a paranoid market. The "benefit" of better prices in the high debt ($B' > B^*(y)$) region is an option too remote and uncertain to compensate for the welfare losses incurred due to worse prices in the normal course of borrowing.

The Causal Chain of PRO The theoretical results presented above follow a clear causal chain originating from the single shock of PRO ($\theta > 1$). This shift in beliefs first and foremost reshapes the market environment by altering the bond price schedule, causing it to *pivot* as established in Proposition 2. The inverse pivoting of the credit spread schedule, described in Corollary 2, is an immediate algebraic consequence. In response to this new pricing reality, the rational sovereign optimally adjusts its policies. It leverages the "softening of doom" effect in the high debt region—where PRO lenders offer paradoxically better prices—to endure a greater debt burden before defaulting (Proposition 4). Simultaneously, it reacts to the "PRO premium" in the primary borrowing region by systematically deleveraging and adopting a more conservative debt policy (Proposition 5). This set of constrained-optimal policy adjustments, however, cannot fully overcome the handicap of transacting with a paranoid market, culminating in awelfare loss for the sovereign (Proposition 6).[12]

Figure 6 illustrates this causal mechanism, showing how a single behavioral friction propagates through the economy's equilibrium relationships. To crystallize the novel contributions of this behavioral channel, Table 1 explicitly contrasts the key predictions of the PRO model with those of a standard reputation model.

5 Quantitative Analysis

In this section, I describe the quantitative implementation of the model. I first outline the calibration of the model parameters and the numerical strategy used to solve for the equilibrium. Then, I present the business cycle properties generated by the baseline model and show that it successfully replicates key features of emerging market economies.

5.1 Calibration and Numerical Solution

Calibration The model is calibrated at a quarterly frequency. The parameter values are chosen to be consistent with the sovereign default literature and to broadly match the macroeconomic features of a typical emerging economy, such as Argentina. The parameters are summarized in Table 2.

[12]This welfare loss is fundamental and cannot be eliminated even by optimal fiscal policy. Section 7 develops a formal Ramsey planning extension (Proposition 8) showing that the PRO-induced distortion of intertemporal prices creates deadweight losses that persist beyond what lump-sum transfers can correct.

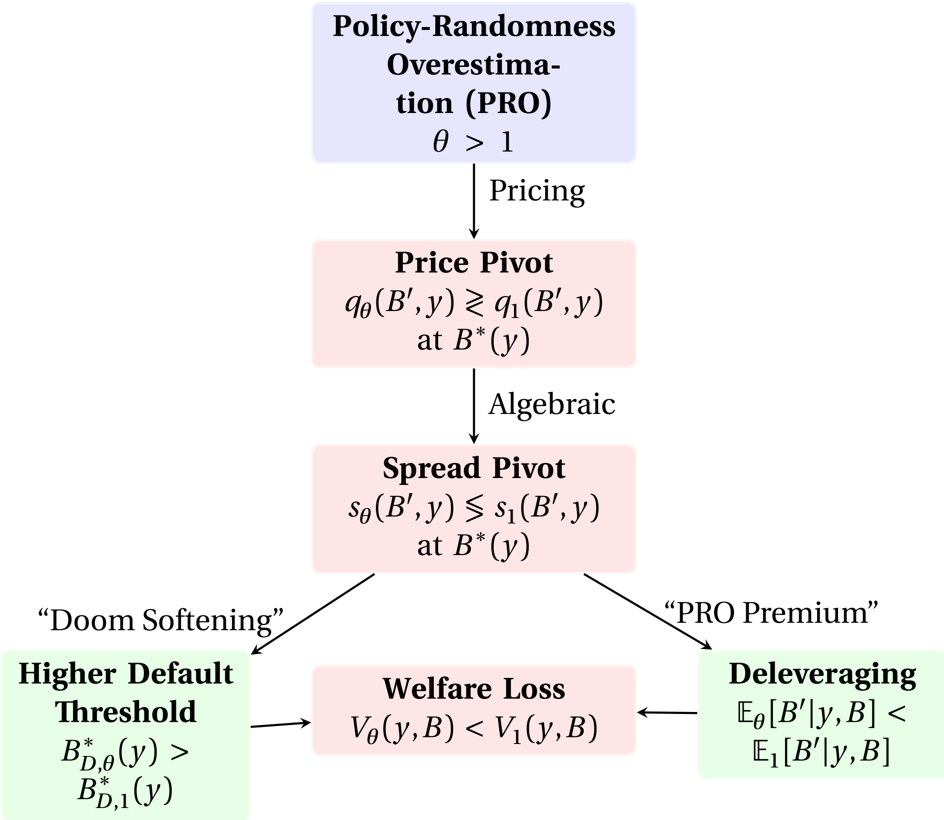


Figure 6: The Causal Chain of PRO

Note: This diagram illustrates how a single behavioral friction ($\theta > 1$) propagates through the economy. PRO first alters market pricing, creating a pivoting bond price schedule. The sovereign optimally responds to this new environment, but cannot fully escape the welfare costs of dealing with biased lenders.

The preference and endowment parameters are standard. The risk aversion coefficient σ is set to 2. The discount factor β is set to 0.9775, implying an annual real interest rate of approximately 9.5% when combined with the model's growth, which is a common value for emerging economies. The logarithmic endowment process is modeled as an AR (1) with a persistence of $\rho_y = 0.95$ and an innovation standard deviation of $\sigma_y = 0.005$.

The debt structure parameters are set to achieve a target Macaulay duration of 5 years (20 quarters) for a risk-free bond, which implies a quarterly principal decay rate of $\delta = 0.04$. The coupon rate κ is set to equal $\delta + r$ so that the price of a risk-free bond is normalized to one. The probability of re-entering credit markets after a default, γ, is set to 0.125, implying an average exclusion period of 2 years (8 quarters). The output cost of default, governed by λ_0 and λ_1, is specified to be nonlinear, consistent with the findings of (**?**).[13]

The scale parameters of the Gumbel taste shocks, η and ρ, are set to small values to ensure that decisions are primarily driven by economic fundamentals, while still ensuring the stability and tractability of the numerical solution.[14]

[13] The specific values are $\lambda_0 = -0.48$ and $\lambda_1 = 0.525$, calibrated to match the severity and shape of output losses observed in historical default episodes.

[14] Specifically, $\eta = 5 \times 10^{-4}$ for default decisions and $\rho = 1 \times 10^{-5}$ for borrowing decisions. These small

Table 1: Comparative Predictions: Reputation and PRO

Prediction Dimension	Reputation (canonical) (**??**)	Policy-Randomness Overestimation (PRO)
Price Curve, q	Tends to be *lower* (monotonic in canonical setups)	*Pivots* around baseline (q_1)
Default Threshold, B_D^*	Tends to be *lower*	*Higher*: $B_{D,\theta}^* > B_{D,1}^*$
Expected Borrowing, $\mathbb{E}[B']$	Often *lower* (constraint-driven)	*Lower* (price-driven in this model)
Average Spread, $\mathbb{E}[s]$	*Higher*	*Higher*
Cross-maturity response after misreporting	Tends to increase across instruments	Divergent across debt distribution (pivot-consistent)
Haircut-size effect on spreads	Strengthens with larger haircuts (**?**)	May weaken if larger haircuts reduce perceived randomness

Note: This table compares predictions under canonical implementations of reputation models with the behavioral mechanism in this paper. The PRO column reflects this model's implementation assumptions.

Table 2: Baseline Calibration (Quarterly)

Parameter	Value	Description
Preferences and Endowments		
σ	2.0	CRRA coefficient of relative risk aversion
β	0.9775	Sovereign's discount factor
ρ_y	0.95	Persistence of log endowment AR(1)
σ_y	0.005	Std. dev. of endowment innovations
Debt and Default		
r	0.01	Quarterly risk-free interest rate (4% ann.)
δ	0.04	Principal decay rate (for 5-year duration)
κ	0.05	Coupon rate ($\delta + r$)
γ	0.125	Re-entry probability (avg. 2-year exclusion)
λ_0, λ_1	-0.48, 0.525	Output cost function parameters
Computational Parameters		
η	5×10^{-4}	Scale of default taste shock
ρ	1×10^{-5}	Scale of borrowing taste shock
θ	1.0	Baseline PRO coefficient

Note: The table presents the parameter values used in the baseline calibration of the model. Parameters are set to match standard values in the sovereign default literature and key macroeconomic features of emerging economies like Argentina.

values maintain the primacy of economic fundamentals while providing computational tractability through the log-sum-exp formulation.

Numerical Solution I solve the model numerically using value function iteration on a discretized state space. The state space consists of the sovereign's current endowment y and its outstanding debt level B.

The endowment process in (1) is discretized into $N_y = 201$ states using Tauchen's method. The state space for debt, B, is represented by a uniform grid of $N_B = 600$ points, ranging from 0 to 75% of mean output.

The solution method iterates on the value functions (V, V^D, V^R) and the bond price function (q) until they converge to a joint fixed point. A key feature of the numerical strategy is the use of the log-sum-exp formulation for choices subject to Gumbel taste shocks. This technique replaces the non-differentiable 'max' operator with a smooth, analytical expression, which greatly improves the stability and speed of the algorithm by obviating the need for numerical maximization routines at each grid point. For further numerical robustness, I employ stabilized log-sum-exp implementations that prevent floating-point overflow and underflow errors that could arise from the small taste shock parameters.[15]The entire solution algorithm is implemented in Fortran and parallelized using OpenMP to leverage multi-core processors.

5.2 Business Cycle Implications of PRO

To understand the quantitative implications of PRO, I simulate the model under three scenarios: the baseline rational-expectations benchmark ($\theta = 1$), a medium-PRO case ($\theta = 10$), and a high-PRO case ($\theta = 100$).[16] Table 3 reports the key business cycle moments from these simulations, revealing how PRO reshapes macroeconomic behavior.

The Rational Benchmark The baseline model with rational lenders ($\theta = 1$) successfully generates results that are broadly consistent with the stylized facts for emerging economies. The average debt-to-GDP ratio is a moderate 7.90%, and the sovereign pays an average annualized credit spread of 2.00%. Consistent with the empirical literature, the model produces consumption that is more volatile than output, counter-cyclical credit spreads (correlation of -0.43 with ln(GDP)), and a slightly counter-cyclical trade balance. When output falls, default risk rises, increasing spreads; simultaneously, the government attempts to borrow to smooth the shock, worsening the trade balance. These features confirm that the model provides a standard and reasonable benchmark against which to evaluate the effects of PRO.

[15]A detailed discussion of these numerical stability techniques is provided in Appendix C.4.

[16]The choice of $\theta = 10$ and $\theta = 100$ as medium and high-PRO cases is motivated by the need to demonstrate clear quantitative differences while maintaining computational tractability. Intermediate values such as $\theta = 30$ or $\theta = 50$ could also be examined to show the continuous nature of the relationship.

Deleveraging and the Price of Fear The introduction of PRO dramatically alters these outcomes, but in a non-linear fashion. Under moderate PRO ($\theta = 10$), the sovereign's average debt level (5.53%) and borrowing cost (2.75%) remain remarkably close to the baseline. However, a shift to high PRO ($\theta = 100$) triggers a stark deleveraging and a significant increase in average borrowing costs. The mean debt-to-GDP ratio falls precipitously by nearly 5 percentage points to just 2.70%. This is a direct consequence of the market discipline predicted in Proposition 5: faced with worse prices in the primary borrowing region, the sovereign optimally reduces its debt issuance. However, this conservative policy does not earn it lower interest rates. Instead, the average credit spread more than doubles to 4.15%. This result—deleveraging in the face of even higher average spreads—starkly illustrates the power of the behavioral bias. In standard reputation frameworks, deleveraging would typically be associated with lower risk and borrowing costs (ceteris paribus); here, it is a constrained-optimal response to the "PRO premium," a penalty the sovereign cannot fully offset simply by reducing its debt. Lenders demand substantial compensation for the perceived risk of an "out of the blue" default, an effect that overwhelms the fact that the sovereign is, in reality, safer due to its lower debt level.

Amplified Financial Cycles PRO not only raises the level of borrowing costs but also progressively amplifies their cyclicality. The correlation between credit spreads and GDP becomes more negative as PRO increases, moving from -0.43 in the baseline to -0.80 under moderate PRO, and then sharply to -0.89 in the high-PRO case. This indicates that financial conditions become exquisitely sensitive to fluctuations in the country's income. When a negative shock hits, PRO lenders' fears are magnified, leading to a much sharper spike in spreads than would occur in a rational market. This tightening of financial conditions occurs precisely when the sovereign needs market access the most, exacerbating the downturn and making financial markets a powerful source of procyclical shocks rather than a tool for macroeconomic stabilization.

An Illusion of Stability Interestingly, the volatility of both the debt-to-GDP ratio and credit spreads decreases as PRO rises. This is not a sign of improved stability but rather a mechanical result of the sovereign's forced deleveraging, creating an *illusion of financial stability*. By maintaining a lower average debt level, the sovereign operates further away from its default threshold (which, paradoxically, is higher, per Proposition 4). This reduces the frequency of episodes of high debt and soaring spreads, leading to lower overall volatility in these financial variables, even as the average spread remains high. Despite these large changes in financial markets, the impact on consumption volatility is minimal. The sovereign adapts to the harsher borrowing environment by reducing its reliance on foreign debt for consumption smoothing, effectively trading away the benefits of international financial integration for a quieter, but more expensive, life.

Table 3: Business Cycle Implications of PRO

Moment	Baseline ($\theta = 1$)	Med. ($\theta = 10$)	High ($\theta = 100$)
Mean and Volatility			
Mean Debt-to-GDP Ratio (%)	7.90	5.53	2.70
Std. Dev. of Debt-to-GDP Ratio (%)	0.87	0.85	0.74
Mean Spread (annualized, %)	2.00	2.75	4.15
Std. Dev. of Spread (annualized, %)	0.77	0.49	0.58
Std. Dev. of ln(Consumption) (%)	3.48	3.53	3.41
Std. Dev. of ln(GDP) (%)	3.04	3.19	3.19
Mean Trade Balance/GDP (%)	0.42	0.32	0.18
Std. Dev. of Trade Balance/GDP (%)	0.51	0.43	0.32
Correlations			
Corr(Spread, ln(GDP))	-0.43	-0.80	-0.89
Corr(Trade Balance/GDP, ln(GDP))	-0.28	-0.28	-0.26
Corr(Debt/GDP, ln(GDP))	0.70	0.79	0.84

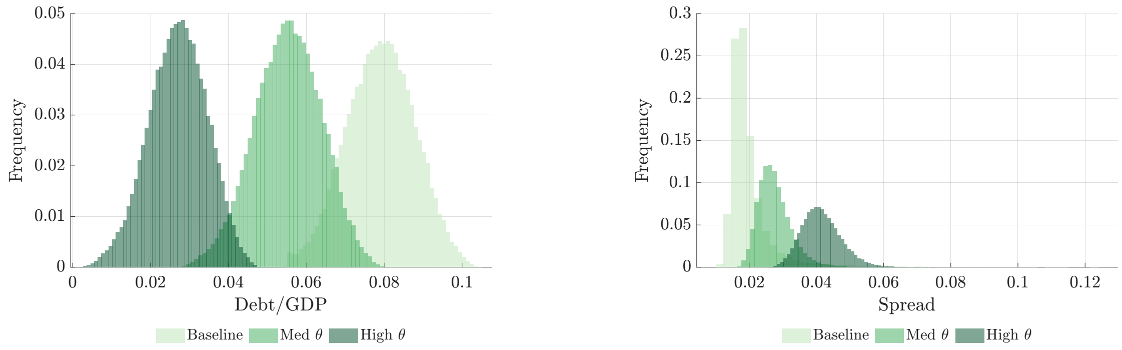
Note: The table reports moments from a long simulation of the model (100,000 periods after a 1,000-period burn-in). Spreads are annualized. All other variables are in quarterly terms.

5.3 The Mechanics of PRO: Distributions and Policy Functions

Long-Run Outcomes: A Shift in Distributions The aggregate business cycle statistics in Table 3 are the result of fundamental shifts in the sovereign's equilibrium behavior, which are best understood by examining the model's policy functions and resulting stationary distributions. Figure 7 plots the simulated histograms for debt and credit spreads, revealing the long-run consequences of PRO. Panel (a) starkly illustrates the deleveraging predicted by Proposition 5. The entire distribution of the debt-to-GDP ratio shifts dramatically to the left, representing a strategic retreat from international capital markets. This is not an arbitrary choice but the sovereign's optimal response to the punitive pricing it faces. Faced with a market that consistently overestimates its risk, the government is disciplined into a permanently more conservative fiscal stance.

This retreat, however, does not earn the sovereign better credit terms. Panel (b) reveals the central paradox: as the sovereign deleverages, its average borrowing cost increases. The entire distribution of credit spreads is pushed to the right. This is the tangible result of the "PRO premium" described in Corollary 2. The sovereign is forced into a low-debt trap where, despite being fundamentally safer due to its lower leverage, it faces a persistently higher cost of capital because lenders' beliefs exhibiting PRO dominate their assessment of fundamentals.

State-Contingent Policies: The Pivoting Effect The long-run distributional shifts are driven by state-by-state changes in the sovereign's optimal policies, which are themselves a reaction to the altered price schedule. Figure 8 visualizes how PRO causes key functions to



(a) Debt-to-GDP Ratio Distribution

(b) Credit Spread Distribution

Figure 7: Simulated Stationary Distributions

Note: The distributions are generated from a long simulation of the model (100,000 periods). The figure shows how rising PRO progressively shifts the long-run distribution of the debt-to-GDP ratio to the left (deleveraging) and the credit spread distribution to the right (higher borrowing costs).

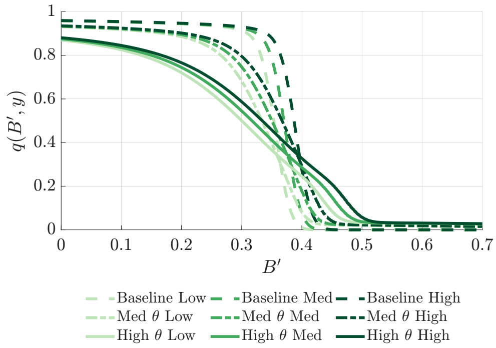
"pivot" around the rational benchmark, providing a graphical confirmation of the paper's central theoretical results.

Panels 8a and 8b illustrate the core price pivot. At low debt levels, where a rational lender sees little risk, the PRO lender prices in the possibility of an "out of the blue" default, leading to lower prices and higher spreads. At very high debt levels, where a rational lender sees default as nearly certain, the PRO lender's belief in randomness allows for a small chance of an "irrational" repayment, leading to paradoxically better terms (the "softening of doom" effect). This pivot in the price schedule is the key external force acting on the sovereign, and it offers a richer dynamic than the simple downward price shift one might associate with reputational deterioration under standard assumptions.

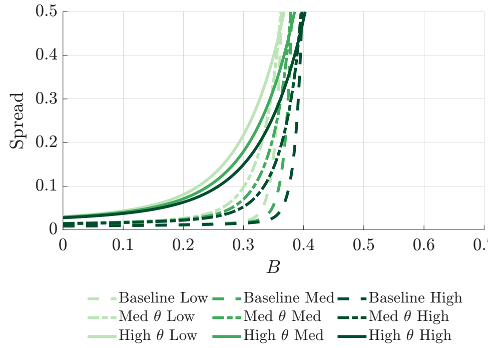
Panels 8c and 8d show the sovereign's endogenous response. The government internalizes the new price schedule. The better terms available in the high-debt region increase the value

of maintaining market access, making the sovereign more resilient and pushing out its default threshold (Proposition 4). More importantly, in the normal course of borrowing, the sovereign faces worse prices, which act as a higher effective cost of capital. The optimal response, shown in Panel 8d, is to deleverage and choose a lower next-period debt level for any given state (Proposition 5).

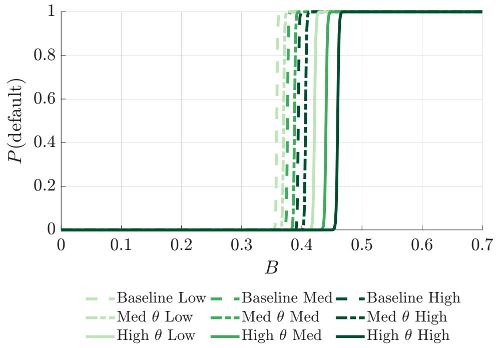
Welfare Consequences The sovereign's policy adjustments—deleveraging and tolerating higher debt before default—are optimal given the market it faces, but they cannot overcome the fundamental handicap of dealing with paranoid lenders. Proposition 6 predicted a direct welfare loss, a result powerfully confirmed by Figure 9. The sovereign's value function is uniformly and significantly lower under higher PRO.



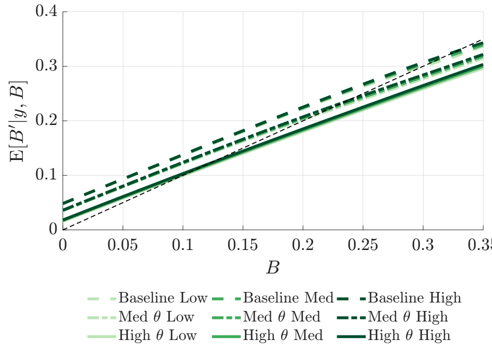
(a) Bond Price Schedule, $q(B',y)$



(b) Credit Spread, $s(B',y)$



(c) Default Probability, $P(d=1|B,y)$



(d) Borrowing Policy, $\mathbb{E}[B'|B,y]$

Figure 8: Policy Function Pivoting

Note: The plots show the key policy and pricing functions for three different levels of endowment y (low, medium, and high). The functions for the baseline ($\theta = 1$, dashed), medium PRO ($\theta = 10$, dash-dot), and high PRO ($\theta = 100$, solid) cases are shown. Rising PRO causes the functions to pivot.

This welfare loss stems from the impairment of the sovereign's ability to smooth consumption. Access to international credit markets is a tool to buffer domestic shocks. PRO effectively places a tax on the use of this tool. By making borrowing more expensive in the relevant range, it forces the sovereign to either endure more volatile consumption or to self-insure by maintaining an inefficiently low level of debt. The "benefit" of better prices in the far-off, high-risk region is an option that is too remote and uncertain to compensate for the persistent, day-to-day welfare losses incurred from being forced to transact with a market that systematically overestimates its propensity to fail.

5.4 Dynamic Responses and Adjustment Paths

The dynamic implications of PRO reveal how PRO affects the sovereign's adjustment paths and responses to shocks.

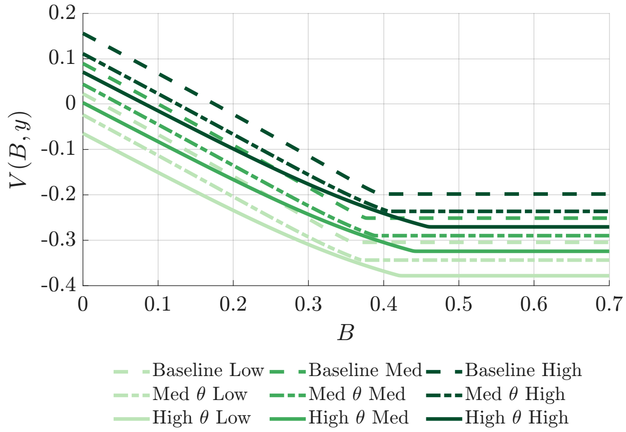


Figure 9: Welfare Loss

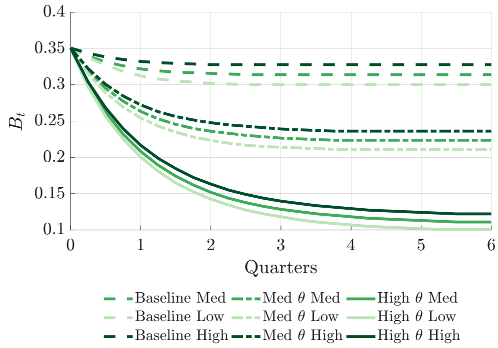
Note: The figure plots the sovereign's ex-ante value function $V(y, B)$ for the baseline ($\theta = 1$, dashed), medium PRO ($\theta = 10$, dash-dot), and high PRO ($\theta = 100$, solid) economies. The value function is uniformly lower under higher degrees of PRO, indicating a progressive welfare loss.

Deleveraging Dynamics: The Transition to Lower Debt Figure 10 traces optimal debt adjustment paths under different initial conditions and degrees of PRO. Consistent with Proposition 5, economies with higher PRO systematically converge to lower debt levels regardless of starting point. Remarkably, even from low initial debt, the high-PRO economy continues deleveraging, representing a fundamental fiscal shift rather than temporary adjustment.

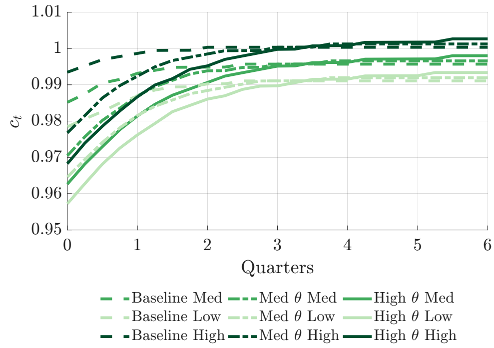
The consumption and spread dynamics reveal important adjustment costs. Panel 10b shows that high-PRO economies experience more volatile consumption during deleveraging despite ultimately achieving lower debt. Panel 10c demonstrates that spreads remain persistently elevated throughout adjustment, confirming that the "PRO premium" reflects systematic risk overestimation at all debt levels rather than just current leverage.

Impulse Response Functions: Shock Propagation under PRO I examine responses to transitory and persistent productivity shocks (AR(1) with $\rho = 0.8$) to understand how PRO affects the sovereign's shock absorption capacity.

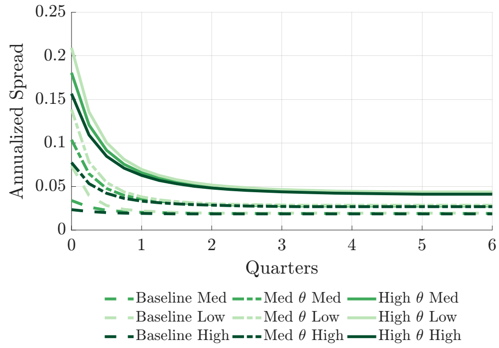
Transitory Shock Responses. Figure 11 shows responses to a 3% positive productivity shock lasting one quarter. While output effects are identical by construction (panel 11a), PRO fundamentally alters other responses. High-PRO economies exhibit muted debt reduction (panel 11b) and consumption smoothing responses (panel 11c), illustrating impaired ability to exploit temporary favorable conditions. Despite facing identical shocks, these economies cannot fully capitalize on good fortune due to persistently unfavorable credit pricing. Spreads fall in all economies (panel 11d), but high-PRO economies main-



(a) Debt



(b) Consumption



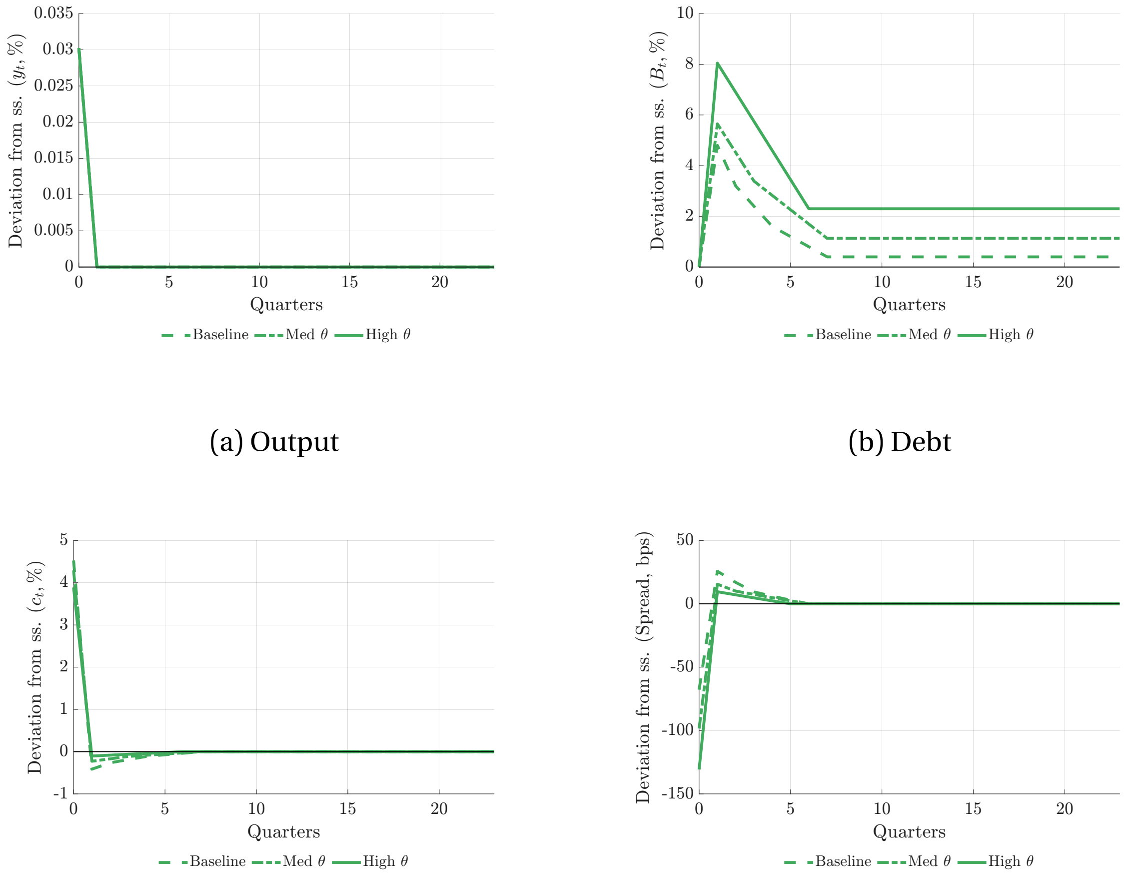
(c) Spread

Figure 10: Deleveraging Dynamics

Note: The figure shows optimal adjustment paths over 6 quarters starting from three different initial debt levels (low, medium, high) for each degree of PRO. The paths demonstrate how PRO leads to systematic deleveraging that persists regardless of initial conditions, accompanied by persistently higher spreads and more volatile consumption during the transition.

tain higher levels throughout adjustment.

Persistent Shock Responses. Figure 12 shows responses to persistent productivity shocks ($\rho = 0.8$). Persistence matters more for high-PRO economies, which exhibit more pronounced and sustained debt reduction (panel 12b) as sovereigns recognize rare opportunities to escape the "high-spread trap." While persistent shocks enable better consumption smoothing across all economies (panel 12c), high-PRO economies still underperform due to fundamental impairment of consumption insurance. Spread responses (panel 12d) are more persistent than in the transitory case, but high-PRO economies maintain higher levels throughout adjustment.



(a) Output

(b) Debt

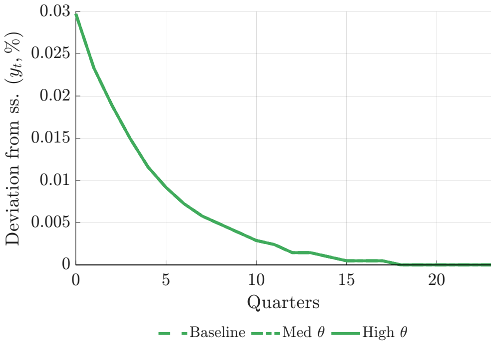
(c) Consumption

(d) Spread

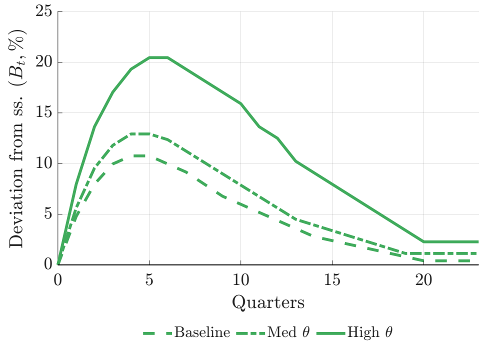
Figure 11: Impulse Responses to Transitory Productivity Shock

Note: The figure shows responses to a 3% positive productivity shock that lasts for one quarter. All variables are expressed as percentage deviations from their respective steady states (spreads in basis points). The responses demonstrate how PRO constrains the sovereign's ability to take advantage of temporary favorable conditions.

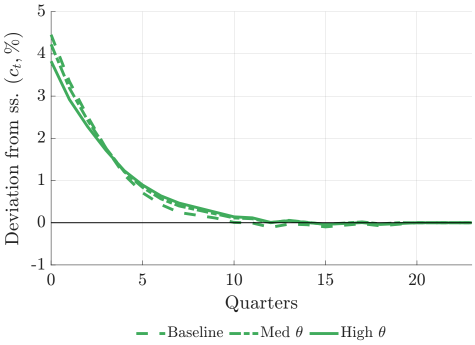
Implications The dynamic analysis reveals three fundamental economic mechanisms. First, deleveraging under PRO creates persistent allocative distortions—the adjustment process itself becomes a source of inefficiency as elevated spreads persist throughout transition, generating deadweight losses that compound over time. This represents a departure from standard models where adjustment costs are temporary. Second, PRO creates asymmetric shock transmission: sovereigns experience constrained benefits from favorable shocks while facing amplified costs from adverse ones, fundamentally altering the risk-return profile of sovereign borrowing. This asymmetry suggests that traditional moments-based calibrations may understate welfare costs. Finally, the interaction between persistence and beliefs generates hysteresis effects: temporary improvements in fundamentals produce limited deleveraging, while sustained improvements are necessary to overcome entrenched priors exhibiting PRO.



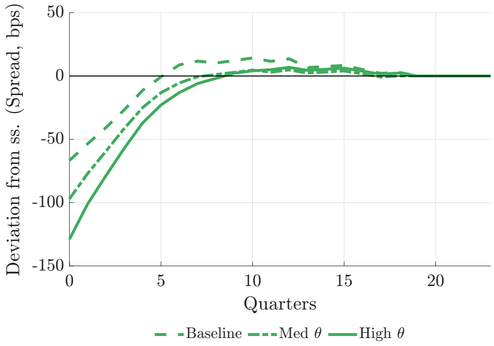
(a) Output



(b) Debt



(c) Consumption



(d) Spread

Figure 12: Impulse Responses to Persistent Productivity Shock

Note: The figure shows responses to a 3% productivity shock with autocorrelation $\rho = 0.8$. All variables are expressed as percentage deviations from steady state (spreads in basis points). The persistent nature of the shock reveals how PRO constrains fiscal flexibility even during extended periods of favorable fundamentals.

6 Microfoundations for PRO

This section provides a rigorous microfoundation for PRO as optimal, state-dependent attention to second-moment information within the same pricing operator framework. I formalize lenders' information choice and posterior mapping, then show how it induces an endogenous dispersion parameter θ_{RI} inside the operator. I close by linking the comparative statics to the Argentina episode in Section 2.

Baseline operator recap Let $q(B', y)$ denote the unique fixed point of the pricing operator $\mathscr{T}_\theta$ defined in the main text (see Appendix C.5). I keep primitives, the debt structure, and the sovereign aggregator J_ρ fixed. The parameter $\theta \geq 1$ indexes the weight on dispersion/tail risk in lenders' default beliefs and thus in pricing.

State and function spaces Let the state space be $\mathsf{S} := \mathscr{B} \times \mathscr{Y}$ with generic element $s = (B', y)$. Let $\mathscr{Q} := C_b(\mathsf{S})$ denote the bounded continuous functions on S endowed with the sup norm $\|\cdot\|_\infty$. Fix $\bar{\theta} > 1$ and restrict $\theta \in [1, \bar{\theta}]$.

Information structure Each period, competitive lenders observe two public signals about fundamentals/policy stability:

$$s_\mu = \mu + \varepsilon_\mu, \quad \varepsilon_\mu \sim \mathcal{N}(0, a_\mu^{-1}), \qquad s_\sigma = \sigma + \varepsilon_\sigma, \quad \varepsilon_\sigma \sim \mathcal{N}(0, a_\sigma^{-1}),$$

where μ proxies first-moment fundamentals and σ proxies second- moment (dispersion / instability) of policy realizations. Lenders choose precisions $a_\mu, a_\sigma \geq 0$ at convex cost

$$\Phi(a_\mu, a_\sigma) = \tfrac{\kappa_\mu}{2} a_\mu^2 + \tfrac{\kappa_\sigma}{2} a_\sigma^2, \tag{18}$$

interpretable as attention/processing costs. (An entropy/mutual-information formulation yields identical monotone comparative statics in precisions.)

Effective dispersion and pricing I adopt a linear mapping from attention to tail-weight:

$$\theta_{\mathrm{RI}}(y, B') \;=\; \min\Big\{1 + \varphi\, w\big(a_\sigma(y, B')\big),\, \bar{\theta}\Big\}, \qquad \varphi > 0,\; w \in C^1,\; w'(\cdot) > 0. \tag{19}$$

For any state (y, B'), prices are still defined as the unique fixed point of the *same* operator evaluated at $\theta_{\mathrm{RI}}(y, B')$:

$$q(B', y) \;=\; \mathscr{T}_{\theta_{\mathrm{RI}}(y,B')}[q](B', y). \tag{20}$$

Example (closed form). For $w(a) = a$, the first-order condition below yields $a_\sigma(y, B') = \frac{\varphi}{\kappa_\sigma} \mathscr{S}(y, B')$, so that

$$\theta_{\mathrm{RI}}(y, B') \;=\; \min\Big\{1 + \tfrac{\varphi^2}{\kappa_\sigma} \mathscr{S}(y, B'),\, \bar{\theta}\Big\}. \tag{21}$$

Optimal attention Let U be lenders' objective (expected discounted payouts net of information costs). Attention solves

$$\max_{a_\mu, a_\sigma \geq 0} \mathbb{E}[U \mid a_\mu, a_\sigma] - \Phi(a_\mu, a_\sigma). \tag{22}$$

Define the marginal pricing sensitivity to dispersion by

$$\mathscr{S}(y, B') \;:=\; \mathbb{E}\Big[\frac{\partial U}{\partial \theta}(y, B'; \theta_{\mathrm{RI}}(y, B'))\Big] \;\geq 0. \tag{23}$$

The first-order condition for a_σ is

$$\varphi\, \mathscr{S}(y, B') \;=\; \kappa_\sigma\, a_\sigma(y, B'). \tag{24}$$

Thus $a_\sigma(y, B') = \frac{\varphi}{\kappa_\sigma}\mathscr{S}(y, B')$, hence $\theta_{\mathrm{RI}}(y, B')$ increases in $\mathscr{S}(y, B')$ via (19).

Lemma 1 (Existence/uniqueness and continuity of attention)**.** *With the linear mapping, for each fixed $\mathscr{S} \geq 0$, the equation (24) admits the unique closed-form solution $a_\sigma(\mathscr{S}) = \frac{\varphi}{\kappa_\sigma}\mathscr{S}$. The map $\mathscr{S} \mapsto a_\sigma(\mathscr{S})$ is continuous and strictly increasing. If $\mathscr{S}(\cdot)$ is continuous on S, then $a_\sigma(\cdot)$ and $\theta_{\mathrm{RI}}(\cdot)$ are continuous on S.*

Proof. Immediate by rearrangement of (24). □

Proposition 7 (Operator properties and inherited comparative statics)**.** *Suppose Φ is strictly convex and w in (19) is increasing. If $\mathscr{T}_\theta$ is positive and order-preserving for each fixed $\theta \in [1, \bar{\theta}]$, then the state-dependent operator $\mathscr{T}_{\theta_{\mathrm{RI}}(\cdot)}$ is positive and order-preserving. The baseline results—single-crossing price pivot at $B^*(y)$, $B^*(y)$ increasing in y, higher default thresholds, and deleveraging with higher mean spreads— continue to hold with θ replaced by $\theta_{\mathrm{RI}}(y, B')$.*

Proof. Pointwise positivity/order of $\mathscr{T}_\theta$ carries to $\mathscr{T}_{\theta_{\mathrm{RI}}(\cdot)}$ by continuity of $a_\sigma(y, B')$ implied by (24). Fixed-point arguments on compact grids then match Appendix C.5. □

Empirical link: Argentina misreporting In Section 2, Argentina's inflation misreporting reduced the credibility of official signals. From (22)–(24), a fall in the productivity of mean information (effective a_μ) raises $\mathscr{S}(y, B')$, increasing a_σ and thus θ_{RI} via (19). The model then predicts: (i) higher average spreads; (ii) a steeper price pivot; (iii) cross-peer "decoupling" when others' dispersion signals do not deteriorate; and (iv) a higher default threshold with concurrent deleveraging—matching the event-study patterns in the Argentina episode.

7 Policy and Information Extensions

This section remains within the same operator framework: I vary information and policy mappings that enter *through* the pricing operator $\mathscr{T}_\theta$, while holding primitives and the sovereign's choice aggregator J_ρ fixed.

Ramsey Planning Under PRO Proposition 6 shows that PRO reduces equilibrium welfare. A natural question is whether an optimal fiscal authority can undo this loss. I study a Ramsey planner who can choose lump-sum taxes/transfers but takes the bond pricing mechanism as given. Formally, the planner's problem preserves the same choice aggregator J_ρ, while bond prices are the unique fixed points of the same operator $\mathscr{T}_\theta$; the planner's instruments enter only the resource constraint and do not alter the pricing mechanism.

PRO acts like a persistent wedge in the intertemporal price of borrowing. Transfers can reallocate resources within and across periods subject to a zero present-value constraint, but they cannot change the shadow price at which the government trades across dates. Hence the PRO price schedule distorts debt choices even under optimal fiscal policy.

The planner chooses $\{c_t, B_{t+1}, \tau_t\}_{t\ge 0}$ to maximize expected utility subject to the per-period resource constraint and a zero present-value (PV) condition for transfers:

$$c_t + \kappa B_t + \tau_t \;=\; y_t + \big(B_{t+1} - (1-\delta)B_t\big)\, q_\theta(y_t, B_{t+1}), \qquad t \ge 0, \text{ a.s.} \tag{25}$$

and

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \tau_t\right] = 0, \quad c_t \ge 0, \quad B_{t+1} \in \mathscr{B}. \tag{26}$$

Proposition 8. *Let $r > -1$ satisfy $\beta(1+r) = 1$. For $i \in \{1, \theta\}$ define the Ramsey value*

$$W_i^R \;=\; \sup_{\{c_t, B_{t+1}, \tau_t\}} \mathbb{E}_0\Big[\sum_{t=0}^{\infty} \beta^t u(c_t)\Big]$$

subject to (25)–(26). Assume:

(A1) u is strictly increasing and strictly concave; $\mathscr{B}$ is compact.

(A2) (Price dominance by trade sign, consistent with Propositions 2 and 5). Let $\Delta_t \equiv B_{t+1} - (1-\delta)B_t$. For every history,

$$\Delta_t \ge 0 \;\Rightarrow\; q_1(y_t, B_{t+1}) \ge q_\theta(y_t, B_{t+1}), \qquad \Delta_t \le 0 \;\Rightarrow\; q_1(y_t, B_{t+1}) \le q_\theta(y_t, B_{t+1}),$$

with strict inequality on a set of positive probability and $\Pr(|\Delta_t| > 0) > 0$.

Then $W_\theta^R < W_1^R$.

Proof. See Appendix B.11. □

Transferss ensure first-best *conditional* consumption smoothing but cannot change the intertemporal proceeds term $\mathbb{E}_0[\sum \beta^t \Delta_t\, q_i(y_t, B_{t+1})]$ that appears in the implementability constraint; therefore the wedge in q_θ generates a genuine efficiency loss. See also (**?**).

Endogenous Belief Formation I next let lenders learn about the taste-shock dispersion parameter θ. This is a state augmentation of the baseline economy: I extend the state to $S_t = (y_t, B_t, \theta_t)$ and keep the pricing operator unchanged except that it is evaluated at the current belief, i.e., prices satisfy the same fixed-point condition with $\mathscr{T}_{\theta_t}$.

Even with Bayesian updating, "negativity bias"—overweighting unexpected defaults relative to unexpected repayments—can make PRO persistent and self-reinforcing. Rare but salient defaults move beliefs more than frequent, modestly good realizations, creating a drift toward PRO.

Let $\theta_t \in [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} = 1 < \bar{\theta}$, and suppose beliefs evolve according to

$$\theta_{t+1} = \lambda\theta_t + (1-\lambda)\widehat{\theta}(\{d_s\}_{s=0}^t), \qquad \lambda \in (0,1), \tag{27}$$

where $d_t \in \{0,1\}$ indicates default and $\widehat{\theta}(\cdot)$ is a (possibly biased) estimator that places extra weight on *surprising* defaults. For instance, with

$$\xi(y,B) = \max\left\{0, \frac{P_1(y,B) - P_{\theta_t}(y,B)}{P_1(y,B)}\right\}, \tag{28}$$

negativity bias means that $\widehat{\theta}$ responds more to larger ξ when $d_t = 1$ than to the analogous repayment surprise when $d_t = 0$ (**??**).

Proposition 9. *Under a stable, Feller belief-update kernel induced by* (27) *with negativity bias as above and mild drift/minorization conditions, the Markov process* $\{\theta_t\}$ *admits a unique invariant distribution* Θ^* *such that:*

- ***(i)*** $\mathbb{E}[\Theta^*] > 1$ *(persistent PRO);*
- ***(ii)*** Θ^* *first-order stochastically dominates the rational benchmark (history dependence through rare defaults);*
- ***(iii)*** $\|\mathbb{E}[\theta_t] - \mathbb{E}[\Theta^*]\| = O(\lambda^t)$*, with slow convergence when defaults are rare (large* λ*).*

Proof. See Appendix B.12. □

Optimal Policy Communication This extension likewise preserves the baseline structure: transparency affects only the *effective* belief input into the same pricing operator, while the sovereign's choice aggregator J_ρ and other primitives are unchanged. Now allow the government to choose a transparency level $\alpha \in [0,1]$ that affects lenders' effective dispersion:

$$\theta_{\text{eff}}(\alpha,\theta) = \alpha \cdot 1 + (1-\alpha)\cdot\theta, \tag{29}$$

and let period utility be $u(c,\alpha) = c^{1-\sigma}/(1-\sigma) - \phi(\alpha)$ with $\phi(\alpha) = \gamma\alpha^2/2$.

Proposition 10. *Let* $W(\alpha)$ *be the sovereign's value when the price schedule is* $q_{\theta_{\text{eff}}(\alpha,\theta)}$ *and the planner otherwise solves the same Ramsey problem. Suppose* $W(\alpha)$ *is strictly concave in* α *(e.g.,* q_ϑ *is smooth and* ϕ *is strictly convex). Then:*

- ***(i)*** *There exists a unique* $\alpha^* \in [0,1]$ *satisfying the FOC*

$$\frac{d}{d\alpha}W(\alpha^*) = \phi'(\alpha^*) = \gamma\alpha^*. \tag{30}$$

- ***(ii)*** *If* $\partial W/\partial\vartheta < 0$ *and* $\partial\theta_{\text{eff}}/\partial\alpha = 1-\theta < 0$*, then* $\partial\alpha^*/\partial\theta > 0$*: greater PRO raises optimal transparency.*

(iii) *There exists* $\theta_c > 1$ *such that* $W(\alpha^*) > W(0)$ *for all* $\theta > \theta_c$.

Proof. See Appendix B.13. □

Transparency improves welfare by steepening lenders' price schedule toward the rational benchmark, but convex disclosure costs imply an interior optimum. The comparative static in (ii) formalizes that transparency is most valuable when PRO is severe.

Testable Predictions and Empirical Hooks As brief guidance for future empirical work, the framework yields concise, testable implications: (i) a single-crossing pivot in price/spread schedules with $B^*(y)$ increasing in y; (ii) higher default thresholds despite higher average spreads; (iii) deleveraging alongside higher mean spreads and lower volatility ("stability illusion"); and (iv) cross-maturity and peer "decoupling" patterns consistent with second-moment (dispersion) beliefs. I view these as light-touch empirical hooks—for calibration and event studies (e.g., Argentina)—rather than the focus here.

8 Conclusion

Standard models struggle to explain why emerging economies face persistently high borrowing costs. This paper develops a quantitative sovereign default model with a behavioral friction—policy-randomness overestimation (PRO)—to address this puzzle. I assume lenders systematically overestimate the randomness of the sovereign's policy choices. Theoretically, this PRO wedge does not uniformly depress bond prices but instead *pivots* the price schedule, making debt cheaper at the edge of default but more expensive in normal times. A rational sovereign responds to these altered incentives in counterintuitive ways: it tolerates a higher debt burden before defaulting, yet simultaneously deleverages its day-to-day borrowing. Quantitatively, the model shows that high PRO can force the debt-to-GDP ratio down from 7.90% to 2.70%, while more than doubling the average credit spread from 2.00% to 4.15%. This deleveraging creates an "illusion of financial stability," where observable market volatility falls even as financial cycles are amplified and the sovereign's welfare declines.

I view PRO as complementary to reputation-based learning rather than a substitute. A joint empirical strategy that leverages maturity-specific responses to informational shocks, the shape of the price schedule, and the evolution of volatility can help separate first-moment (type) from second-moment (dispersion) channels. In practice, both mechanisms may operate simultaneously.

The theoretical extensions provide additional insights. Even optimal Ramsey fiscal policy cannot eliminate the welfare costs of PRO because the distorted bond pricing creates fundamental allocative distortions that transfers cannot correct. When beliefs are formed endogenously through Bayesian learning with negativity bias, perceptions exhibiting PRO become self-reinforcing and persist over time, explaining why some economies face chronically high borrowing costs. However, governments can partially mitigate these effects through strategic policy communication, choosing optimal transparency levels to reduce perceived uncertainty while managing the costs of information disclosure. My work thus offers a new, behaviorally-grounded perspective on sovereign risk, demonstrating how market beliefs can be a fundamental driver of debt crises and highlighting the importance of optimal policy design in managing these behavioral frictions.

Appendices for "Default with Policy-Randomness Overestimation"

Chen Gao

October 13, 2025

This appendix contains the details of the Gumbel distribution and the proofs for the lemmas and propositions in the paper. Section A contains the details of the Gumbel distribution. Section B contains the proofs for the lemmas and propositions in the paper. Section C contains the computational algorithm details for the model, including the numerical stability techniques employed in the discrete choice implementation.

A Gumbel Distribution in Default Models

In this section, I provide some useful results about the Gumbel distribution to help formulate the close form solution presented in Section 3.

Lemma 2. *Let $\varepsilon_1, \varepsilon_2$ be independent random variables distributed as Gumbel$(-\eta\gamma, \eta)$, where γ is the Euler-Mascheroni constant. Let $V_1, V_2 \in \mathbb{R}$ be deterministic constants. Then:*

$$\mathbb{E}\left[\max\{V_1 + \varepsilon_1, V_2 + \varepsilon_2\}\right] = \eta \ln\left(\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}\right). \tag{A.1}$$

Proof. See Appendix B.1. □

Lemma 3. *Let $\varepsilon_1, \varepsilon_2$ be independent Gumbel$(-\eta\gamma, \eta)$ random variables, and let $V_1, V_2 \in \mathbb{R}$ be deterministic values. Then:*

$$\Pr\{V_1 + \varepsilon_1 > V_2 + \varepsilon_2\} = \frac{\exp\frac{V_1}{\eta}}{\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}}. \tag{A.2}$$

Proof. See Appendix B.2. □

Lemma 4. *Let $\{V_i\}_{i=1}^n$ be deterministic values and $\{\varepsilon_i\}_{i=1}^n$ be independent Gumbel$(-\sigma\gamma, \sigma)$ random variables. Then:*

$$\mathbb{E}\left[\max_{i\in\{1,\dots,n\}}\{V_i + \varepsilon_i\}\right] = \sigma \ln\left(\sum_{i=1}^n \exp\frac{V_i}{\sigma}\right), \tag{A.3}$$

$$\Pr\left\{\arg\max_{i\in\{1,\dots,n\}}\{V_i + \varepsilon_i\} = j\right\} = \frac{\exp\frac{V_j}{\sigma}}{\sum_{i=1}^n \exp\frac{V_i}{\sigma}}. \tag{A.4}$$

Proof. See Appendix B.3. □

B Proofs

This section contains the proofs for the lemmas and propositions in the paper.

B.1 Proof of Lemma 2

Proof. Let $X_1 = V_1 + \varepsilon_1$ and $X_2 = V_2 + \varepsilon_2$, where $\varepsilon_1, \varepsilon_2$ are independent Gumbel$(-\eta\gamma, \eta)$ random variables. I derive the expected value $\mathbb{E}[\max\{X_1, X_2\}]$. The CDF of a Gumbel$(\mu, \sigma)$ random variable is $F(x;\mu,\sigma) = \exp(-\exp(-(x-\mu)/\sigma))$. For the parametrization Gumbel$(-\eta\gamma, \eta)$:

$$F_\varepsilon(x) = \exp\left(-\exp\left(-\frac{x}{\eta} - \gamma\right)\right).$$

The CDF of $X_i = V_i + \varepsilon_i$ is obtained by translation:

$$F_{X_i}(x) = F_\varepsilon(x - V_i) = \exp\left(-\exp\left(-\frac{x - V_i}{\eta} - \gamma\right)\right).$$

The CDF of $\max\{X_1, X_2\}$ is:

$$\begin{aligned}
F_{\max}(x) &= \Pr\{\max\{X_1, X_2\} \le x\} = \Pr\{X_1 \le x, X_2 \le x\} \\
&= F_{X_1}(x) \cdot F_{X_2}(x) \\
&= \exp\left(-\exp\left(-\frac{x - V_1}{\eta} - \gamma\right)\right) \cdot \exp\left(-\exp\left(-\frac{x - V_2}{\eta} - \gamma\right)\right) \\
&= \exp\left(-\exp\left(-\frac{x - V_1}{\eta} - \gamma\right) - \exp\left(-\frac{x - V_2}{\eta} - \gamma\right)\right) \\
&= \exp\left(-e^{-\gamma}\left(\exp\left(-\frac{x - V_1}{\eta}\right) + \exp\left(-\frac{x - V_2}{\eta}\right)\right)\right).
\end{aligned}$$

Let $\tilde{\mu} = \eta \ln\left(\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}\right)$. I can rewrite:

$$\begin{aligned}
&\exp\left(-\frac{x - V_1}{\eta}\right) + \exp\left(-\frac{x - V_2}{\eta}\right) \\
&= \exp\left(-\frac{x}{\eta}\right)\left(\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}\right) \\
&= \exp\left(-\frac{x}{\eta}\right)\exp\frac{\tilde{\mu}}{\eta} = \exp\left(-\frac{x - \tilde{\mu}}{\eta}\right).
\end{aligned}$$

Therefore,

$$F_{\max}(x) = \exp\left(-e^{-\gamma}\exp\left(-\frac{x - \tilde{\mu}}{\eta}\right)\right) = \exp\left(-\exp\left(-\frac{x - \tilde{\mu}}{\eta} - \gamma\right)\right).$$

Thus $\max\{X_1, X_2\}$ follows a Gumbel$(\tilde{\mu} - \eta\gamma, \eta)$ distribution. Since the expectation of a Gumbel(μ, σ) random variable is $\mu + \sigma\gamma$:

$$\begin{aligned}
\mathbb{E}[\max\{X_1, X_2\}] &= (\tilde{\mu} - \eta\gamma) + \eta\gamma = \tilde{\mu} \\
&= \eta \ln\left(\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}\right).
\end{aligned}$$

□

B.2 Proof of Lemma 3

Proof. Let $X_1 = V_1 + \varepsilon_1$ and $X_2 = V_2 + \varepsilon_2$ as before. I compute $\Pr\{X_1 > X_2\} = \Pr\{\varepsilon_1 - \varepsilon_2 > V_2 - V_1\}$. I first establish that for independent Gumbel(μ, σ) random variables ε_1 and ε_2, the difference $\varepsilon_1 - \varepsilon_2$ follows a logistic distribution. The CDF of ε_i is $F(x) = \exp(-\exp(-(x - \mu)/\sigma))$. For $Z = \varepsilon_1 - \varepsilon_2$, I compute:

$$\begin{aligned}
F_Z(z) = \Pr\{\varepsilon_1 - \varepsilon_2 \le z\} &= \Pr\{\varepsilon_1 \le z + \varepsilon_2\} \\
&= \int_{-\infty}^{\infty} F_{\varepsilon_1}(z + u) f_{\varepsilon_2}(u)\, du
\end{aligned}$$

where $f_{\varepsilon_2}(u) = \sigma^{-1}\exp(-(u-\mu)/\sigma)\exp(-\exp(-(u-\mu)/\sigma))$ is the PDF of ε_2. Using the substitution $v = \exp(-(u-\mu)/\sigma)$ and the fact that both variables have the same parameters, the integral evaluates to:

$$F_Z(z) = \frac{1}{1 + \exp(-z/\sigma)}$$

This is the CDF of a logistic distribution with location 0 and scale σ. With $\sigma = \eta$ and the parametrization:

$$\begin{aligned}
\Pr\{X_1 > X_2\} &= \Pr\{\varepsilon_1 - \varepsilon_2 > V_2 - V_1\} \\
&= 1 - F_Z(V_2 - V_1; 0, \eta) \\
&= 1 - \frac{1}{1 + \exp(-(V_2 - V_1)/\eta)} \\
&= 1 - \frac{1}{1 + \exp((V_1 - V_2)/\eta)} \\
&= \frac{1 + \exp((V_1 - V_2)/\eta) - 1}{1 + \exp((V_1 - V_2)/\eta)} \\
&= \frac{\exp((V_1 - V_2)/\eta)}{1 + \exp((V_1 - V_2)/\eta)}.
\end{aligned}$$

Multiplying by $\exp(V_2/\eta)$ yields:

$$\Pr\{X_1 > X_2\} = \frac{\exp\frac{V_1}{\eta}}{\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}}.$$

□

B.3 Proof of Lemma 4

Proof. The proof proceeds by induction on n.

Base case: For $n = 2$, the results follow from Lemmas 2 and 3.

Inductive step: Assume the results hold for $n \geq 2$. For $n+1$ alternatives, let $Y_n = \max_{1\leq i\leq n}\{V_i + \varepsilon_i\}$ and $X_{n+1} = V_{n+1} + \varepsilon_{n+1}$. Then:

$$\max_{1\leq i\leq n}\{V_i + \varepsilon_i\} = \max\{Y_n, X_{n+1}\}.$$

By the inductive hypothesis, Y_n is distributed as Gumbel$(\sigma \ln(\sum_{i=1}^{n} \exp(V_i/\sigma)) - \sigma\gamma, \sigma)$. Since X_{n+1} is Gumbel$(-\sigma\gamma, \sigma)$, applying Lemma 2:

$$\begin{aligned}
&\mathbb{E}\left[\max_{1\leq i\leq n+1}\{V_i + \varepsilon_i\}\right] \\
&= \sigma \ln\left(\exp\frac{\sigma \ln\left(\sum_{i=1}^{n} \exp\frac{V_i}{\sigma}\right)}{\sigma} + \exp\frac{V_{n+1}}{\sigma}\right) \\
&= \sigma \ln\left(\sum_{i=1}^{n} \exp\frac{V_i}{\sigma} + \exp\frac{V_{n+1}}{\sigma}\right) \\
&= \sigma \ln\left(\sum_{i=1}^{n+1} \exp\frac{V_i}{\sigma}\right).
\end{aligned}$$

For the choice probabilities, Lemma 3 yields:

$$\begin{aligned}
&\Pr\left\{\arg\max_{1\leq i\leq n+1}\{V_i + \varepsilon_i\} = j\right\} \\
&= \begin{cases} \Pr\{Y_n > X_{n+1}\} \cdot \Pr\{\arg\max_{1\leq i\leq n}\{V_i + \varepsilon_i\} = j\} & \text{if } j \leq n \\ \Pr\{X_{n+1} > Y_n\} & \text{if } j = n+1 \end{cases}
\end{aligned}$$

By the inductive hypothesis and preceding lemmas:

$$\Pr\left\{\arg\max_{1\leq i\leq n+1}\{V_i + \varepsilon_i\} = j\right\} = \frac{\exp\frac{V_j}{\sigma}}{\sum_{i=1}^{n+1} \exp\frac{V_i}{\sigma}}.$$

□

B.4 Proof of Proposition 1

Proof. **Spaces and operator.** Let $\mathscr{S} = \mathscr{Y} \times \mathscr{B}$ (compact) and $\mathscr{C}(\mathscr{S})$ be bounded continuous functions on $\mathscr{S}$ with the sup-norm $\|\cdot\|_\infty$. Define the product space $\mathbf{X} = \mathscr{C}(\mathscr{S}) \times \mathscr{C}(\mathscr{S})$ and, for a weight $\lambda > 0$, the norm

$$\|(V,q)\|_\lambda \;=\; \max\{\|V\|_\infty,\ \lambda\|q\|_\infty\}.$$

$\big(\mathbf{X}, \|\cdot\|_\lambda\big)$ is complete. Define $\mathscr{T} : \mathbf{X} \to \mathbf{X}$ by

$$\mathscr{T}(V,q) \;=\; \big(J(V,q),\ T(V,q)\big),$$

where J is the Bellman operator (log-sum-exp/taste shocks) and T is the pricing operator (B.9) (default probability given by the logistic rule).

Step 1: $\mathscr{T}$ maps X into itself. By compactness of (y,B) and the discrete (or compact) choice set for B', continuity of the primitives, and the log-sum-exp aggregator with $\rho > 0$, $J(V,q)$ is bounded and continuous whenever (V,q) is. For T, note that for any (y,B'),

$$0 \;\le\; q(y,B') \;\le\; \frac{1}{1+r}\Big(\kappa + (1-\delta)\|q\|_\infty\Big),$$

hence taking suprema gives the uniform bound

$$\|q\|_\infty \;\le\; \frac{\kappa}{r} \;=:\; \bar{q}. \tag{B.5}$$

Thus $T(V,q)$ is bounded and continuous, so $\mathscr{T}(\mathbf{X}) \subseteq \mathbf{X}$.

Step 2: Lipschitz bounds (decoupled). I record the following constants (finite by compactness and (B.5)):

(i) *J with respect to V.* The continuation part of J is multiplied by β and the derivative of log-sum-exp is a probability in $[0,1]$, so

$$\|J(V_1,q) - J(V_2,q)\|_\infty \;\le\; \beta\,\|V_1 - V_2\|_\infty.$$

(ii) *J with respect to q.* Consumption is $c(B') = y - \kappa B + [B' - (1-\delta)B]\,q(y,B')$. Hence for some bound

$$M_B \;:=\; \sup_{(y,B)\in\mathscr{S}}\ \sup_{B'\in\mathscr{B}} \big|B' - (1-\delta)B\big| \;<\; \infty,$$

and with $\bar{u}' := \sup u'(c)$ on the feasible set, the envelope theorem and the $[0,1]$-valued logit weights imply

$$\|J(V,q_1)-J(V,q_2)\|_\infty \le L_{Jq}\,\|q_1-q_2\|_\infty, \qquad L_{Jq} \le \bar{u}' M_B. \tag{B.6}$$

(iii) T *with respect to* q. In (B.9), q enters only via the resale term $(1-\delta)\mathbb{E}q$ with one-period discount $1/(1+r)$, hence

$$\|T(V,q_1)-T(V,q_2)\|_\infty \le m_q\,\|q_1-q_2\|_\infty, \qquad m_q := \frac{1-\delta}{1+r} < 1. \tag{B.7}$$

(iv) T *with respect to* V. Write $P = L(\Delta V/(\theta\eta))$ with $L'(z) = L(z)\big(1-L(z)\big) \le \frac{1}{4}$. Let $\Pi := \kappa + (1-\delta)\bar{q}$ bound the one-step payoff in (B.9). The mapping $V \mapsto \Delta V$ is linear in V through continuation values and default recursion; its operator norm is bounded by

$$\|\Delta V(V_1)-\Delta V(V_2)\|_\infty \le C_\Delta\,\|V_1-V_2\|_\infty, \qquad C_\Delta \le \frac{\beta}{1-\beta(1-\gamma)}.$$

Therefore

$$\|T(V_1,q)-T(V_2,q)\|_\infty \le \frac{1}{1+r}\cdot\frac{1}{4\theta\eta}\cdot\Pi\cdot C_\Delta\|V_1-V_2\|_\infty =: L_{TV}\,\|V_1-V_2\|_\infty. \tag{B.8}$$

Step 3: Joint contraction under a weighted norm. For any $(V_1,q_1),(V_2,q_2)$, combine the bounds above to get

$$\|J(V_1,q_1)-J(V_2,q_2)\|_\infty \le \beta\,\|V_1-V_2\|_\infty + L_{Jq}\,\|q_1-q_2\|_\infty,$$

$$\|T(V_1,q_1)-T(V_2,q_2)\|_\infty \le L_{TV}\,\|V_1-V_2\|_\infty + m_q\,\|q_1-q_2\|_\infty.$$

Multiplying the second inequality by λ and taking the max of the two gives

$$\|\mathcal{T}(V_1,q_1)-\mathcal{T}(V_2,q_2)\|_\lambda \le \alpha(\lambda)\,\|(V_1,q_1)-(V_2,q_2)\|_\lambda,$$

where

$$\alpha(\lambda) = \max\Big\{\beta+\frac{L_{Jq}}{\lambda},\; m_q+\lambda L_{TV}\Big\}.$$

If there exists $\lambda>0$ such that

$$\beta+\frac{L_{Jq}}{\lambda}<1 \quad\text{and}\quad m_q+\lambda L_{TV}<1,$$

then $\alpha(\lambda)<1$ and $\mathcal{T}$ is a contraction. These inequalities can be satisfied if and only if

$$L_{Jq}L_{TV} < (1-\beta)(1-m_q),$$

which is exactly the slope condition (13). In that case one admissible choice is

$$\lambda^\star = \frac{(\beta - m_q) + \sqrt{(\beta - m_q)^2 + 4L_{Jq}L_{TV}}}{2L_{TV}},$$

for which $\alpha(\lambda^\star) < 1$.

Step 4: Conclusion. In the weighted complete normed space $(\mathbf{X}, \|\cdot\|_{\lambda^\star})$, $\mathcal{T}$ is a contraction mapping. By the Contraction Mapping Theorem, it has a unique fixed point (V^*, q^*), which by construction satisfies the model's recursive equilibrium conditions. Therefore, the Recursive Markov Perfect Equilibrium exists and is unique. □

B.5 Proof of Proposition 2

Proof. The proof relies on a standard monotone-operator result showing that pointwise orderings of operators are preserved by their fixed points.

Lemma 5. *Let $(X, \|\cdot\|)$ be a Banach space and $T_1, T_2 : X \to X$ satisfy:*

1. ***Monotonicity:*** *$f \geq g \Rightarrow T_i(f) \geq T_i(g)$ for $i \in \{1,2\}$.*
2. ***Discounting:*** *$\|T_i(f + c\mathbf{1}) - T_i(f)\| \leq \beta c$ for some $\beta \in (0,1)$ and constant function $\mathbf{1}$.*

If $T_1(f) \geq T_2(f)$ pointwise for all $f \in X$, then their unique fixed points satisfy $f_1^* \geq f_2^*$ where $T_i(f_i^*) = f_i^*$.

Proof of Lemma 5. Let $f_1 = T_1(f_1)$ and $f_2 = T_2(f_2)$. Start from $f_0 = f_2$ and iterate $f_{n+1} = T_1(f_n)$. By contraction, $f_n \to f_1$. Using $T_1(f_2) \geq T_2(f_2) = f_2$ and monotonicity of T_1, the sequence is nondecreasing and bounded below by f_2; taking limits yields $f_1 \geq f_2$. □

Let $q_i(B', y)$, $i \in \{1, \theta\}$ with $\theta_1 = 1$ and $\theta > 1$, be the fixed point of the pricing operator

$$(T_i q)(B', y) = \frac{1}{1+r}\mathbb{E}_{y'|y}\Big[\big(1 - P_i(y', B')\big)\Big(\kappa + (1-\delta)\mathbb{E}_{B''|y',B'}[\,q(y', B'')\,]\Big)\Big], \tag{B.9}$$

where

$$P_i(y', B') = L\Big(-\frac{\Delta V_i(y', B')}{\theta_i \eta}\Big), \qquad L(z) = \frac{1}{1+e^{-z}}, \qquad \Delta V_i(y', B') \equiv V_i^R(y', B') - V_i^D(y'). \tag{B.10}$$

Step 1: Probability ordering (correct sign). Fix (y, B') and write $\xi \equiv \Delta V_i(y, B') \neq 0$. Define $\phi(\alpha) \equiv L(-\xi/\alpha)$ for $\alpha > 0$. Since $L'(z) = L(z)(1 - L(z)) > 0$,

$$\phi'(\alpha) = L'(-\xi/\alpha) \cdot \frac{\xi}{\alpha^2} \quad \Rightarrow \quad \operatorname{sign}\phi'(\alpha) = \operatorname{sign}\xi.$$

Because $\theta > 1$ implies $\theta\eta > \eta$,

$$\xi > 0 \;\Rightarrow\; \phi(\theta\eta) > \phi(\eta) \;\Rightarrow\; P_\theta(y, B') > P_1(y, B'), \tag{B.11}$$

$$\xi < 0 \;\Rightarrow\; \phi(\theta\eta) < \phi(\eta) \;\Rightarrow\; P_\theta(y, B') < P_1(y, B'). \tag{B.12}$$

Intuitively: when repayment strictly dominates default ($\Delta V > 0$), PRO lenders assign a *higher* default probability; when default dominates ($\Delta V < 0$), they assign a *lower* default probability.

Given the strictly positive payoff term in (B.9), I have

$$\operatorname{sign}\big((T_\theta q - T_1 q)(B', y)\big) = \operatorname{sign}\Big(\mathbb{E}_{y'|y}\left[P_1(y', B') - P_\theta(y', B')\right]\Big). \tag{B.13}$$

Step 2: Safe region ($\Delta V > 0$). Define the safe set at income y by

$$\underline{B}(y) \;\equiv\; \sup\Big\{b : \Delta V_i(y', b) > 0 \;\;\forall\, y' \in \mathscr{Y},\; \forall\, i \in \{1, \theta\}\Big\}.$$

For any $B' < \underline{B}(y)$, (B.11) gives $P_\theta > P_1$ for all y'; hence

$$\mathbb{E}_{y'|y}[P_1 - P_\theta] < 0 \;\Rightarrow\; (T_\theta q)(B', y) < (T_1 q)(B', y) \;\Rightarrow\; q_\theta(B', y) < q_1(B', y).$$

Step 3: Risky region ($\Delta V < 0$). Define the risky set at income y by

$$\overline{B}(y) \;\equiv\; \inf\Big\{b : \Delta V_i(y', b) < 0 \;\;\forall\, y' \in \mathscr{Y},\; \forall\, i \in \{1, \theta\}\Big\}.$$

For any $B' > \overline{B}(y)$, (B.12) yields $P_\theta < P_1$ for all y'; thus

$$\mathbb{E}_{y'|y}[P_1 - P_\theta] > 0 \;\Rightarrow\; (T_\theta q)(B', y) > (T_1 q)(B', y) \;\Rightarrow\; q_\theta(B', y) > q_1(B', y).$$

Step 4: Pivot existence. Let $\Delta q(B', y) \equiv q_\theta(B', y) - q_1(B', y)$. By continuity of q_i in B' (Proposition 1), $\Delta q(\cdot, y)$ is continuous. Steps 2–3 imply

$$\Delta q(B', y) < 0 \quad \text{for } B' < \underline{B}(y), \qquad \Delta q(B', y) > 0 \quad \text{for } B' > \overline{B}(y).$$

By the Intermediate Value Theorem there exists at least one $B^*(y) \in [\underline{B}(y), \overline{B}(y)]$ such that

$$\Delta q(B^*(y), y) = 0,$$

and the local sign pattern is

$$\operatorname{sign}\big(\Delta q(B',y)\big) = \begin{cases} -, & B' < B^*(y), \\ 0, & B' = B^*(y), \\ +, & B' > B^*(y). \end{cases}$$

This establishes the pivoting of q_θ around q_1 at $B^*(y)$. $\square$

B.6 Proof of Proposition 3

Proof. Define $F(B',y) \equiv q_\theta(B',y) - q_1(B',y)$ and let $B^*(y)$ be implicitly defined by $F(B^*(y),y) = 0$. I verify the conditions of the Implicit Function Theorem at any $(B^*(y),y)$ where the crossing is interior.

Regularity. **(i) Smoothness.** Under the primitives assumed in the main text (CRRA utility, log-sum-exp smoothing in the sovereign's choices, compact state grids, and the pricing operator with strictly positive payoff term), the pricing operator is C^1 in (B',y). Hence each fixed-point price function $q_i(B',y)$ is C^1 in both arguments.[17]

(ii) Non-degeneracy. I show $F_{B'}(B^*(y),y) \neq 0$ and in fact $F_{B'}(B^*(y),y) > 0$. Recall the lenders' perceived default probability

$$P_i(y',B') = L\left(-\frac{\Delta V_i(y',B')}{\theta_i \eta}\right), \quad L(z) = \frac{1}{1+e^{-z}}, \quad \Delta V_i \equiv V_i^R - V_i^D.$$

Since $L' > 0$ and $\partial \Delta V_i / \partial B' < 0$ (higher future debt reduces the net value of repayment),

$$\frac{\partial P_i(y',B')}{\partial B'} = -\frac{1}{\theta_i \eta} L'\left(-\frac{\Delta V_i}{\theta_i \eta}\right) \frac{\partial \Delta V_i}{\partial B'} > 0. \tag{B.14}$$

Moreover, for $\theta > 1$,

$$0 < \frac{\partial P_\theta}{\partial B'} < \frac{\partial P_1}{\partial B'}. \tag{B.15}$$

Write the pricing operator as

$$(T_i q)(B',y) = \frac{1}{1+r} \mathbb{E}_{y'|y}\Big[(1-P_i(y',B'))\,R_i(y',B')\Big], \quad R_i(y',B') \equiv \kappa + (1-\delta)\,\mathbb{E}_{B''|y',B'}[\,q_i(y',B'')\,].$$

Since $R_i > 0$, the operator is pointwise *decreasing* in P_i. Hence, as B' increases and P_i rises by (B.14), the price $q_i(B',y)$ strictly *decreases* in B'. Furthermore, because $\partial P_\theta / \partial B'$ is smaller than $\partial P_1 / \partial B'$ by (B.15), the magnitude of the induced price decline is smaller

[17] This follows from differentiability of the Bellman/pricing operators and standard fixed-point differentiation arguments on compact spaces.

under θ than under 1. Therefore,

$$\frac{\partial q_\theta}{\partial B'}(B^*(y),y) > \frac{\partial q_1}{\partial B'}(B^*(y),y), \qquad \text{so} \qquad F_{B'}(B^*(y),y) = \frac{\partial q_\theta}{\partial B'} - \frac{\partial q_1}{\partial B'} > 0. \tag{B.16}$$

Sign of the numerator. Income persistence implies that higher y raises expected future endowments, increases ΔV_i, lowers P_i, and thus raises prices: $\partial q_i/\partial y > 0$. The sensitivity is dampened under PRO because the mapping $\Delta V \mapsto P_\theta = L(-\Delta V/(\theta\eta))$ is flatter when $\theta > 1$. Propagating through the pricing operator,

$$0 < \frac{\partial q_\theta}{\partial y}(B^*(y),y) < \frac{\partial q_1}{\partial y}(B^*(y),y), \quad \Rightarrow \quad F_y(B^*(y),y) = \frac{\partial q_\theta}{\partial y} - \frac{\partial q_1}{\partial y} < 0. \tag{B.17}$$

Conclusion. By the Implicit Function Theorem,

$$\frac{dB^*}{dy} = -\frac{F_y(B^*(y),y)}{F_{B'}(B^*(y),y)} = -\frac{(-)}{(+)} > 0,$$

so the pivot threshold $B^*(y)$ is (strictly) increasing in y. □

B.7 Proof of Corollary 2

Proof. By definition of the spread,

$$s_i(B',y) = \frac{\kappa}{q_i(B',y)} - \delta - r.$$

Hence, for any (B',y) with $q_\theta(B',y), q_1(B',y) > 0$,

$$\begin{aligned}\Delta s(B',y) &\equiv s_\theta(B',y) - s_1(B',y) \\ &= \left(\tfrac{\kappa}{q_\theta(B',y)} - \delta - r\right) - \left(\tfrac{\kappa}{q_1(B',y)} - \delta - r\right) \qquad\qquad (B.18) \\ &= \kappa\left(\tfrac{1}{q_\theta(B',y)} - \tfrac{1}{q_1(B',y)}\right) = -\frac{\kappa\,[\,q_\theta(B',y) - q_1(B',y)\,]}{q_\theta(B',y)\,q_1(B',y)} \\ &= -\frac{\kappa\,\Delta q(B',y)}{q_\theta(B',y)\,q_1(B',y)}. \qquad\qquad (B.19)\end{aligned}$$

Because $\kappa > 0$ and $q_i(B',y) > 0$, it follows that $\operatorname{sign}(\Delta s(B',y)) = -\operatorname{sign}(\Delta q(B',y))$. By Proposition 2, there exists $B^*(y)$ such that

$$\Delta q(B',y) \begin{cases} < 0, & B' < B^*(y), \\ = 0, & B' = B^*(y), \\ > 0, & B' > B^*(y), \end{cases} \qquad \Longrightarrow \qquad \Delta s(B',y) \begin{cases} > 0, & B' < B^*(y), \\ = 0, & B' = B^*(y), \\ < 0, & B' > B^*(y). \end{cases}$$

Moreover, if for some (B', y) one price were zero (full default so the payoff is null), then $s_i(B', y) = +\infty$ and the sign relation holds trivially in the limit.[18] $\square$

B.8 Proof of Proposition 4

Proof. For a fixed endowment level y, define the repayment-default gap

$$G_i(B; y) \equiv V_i^R(y, B) - V^D(y), \qquad i \in \{1, \theta\}.$$

By standard arguments (concavity of u, budget feasibility, and the fact that higher current debt tightens the budget set), $G_i(B; y)$ is strictly decreasing in B. The default threshold is the unique root $B_{D,i}^*(y)$ of $G_i(\cdot; y) = 0$.

Assume toward a contradiction that $B_{D,\theta}^*(y) \le B_{D,1}^*(y)$. Monotonicity of $G_1(\cdot; y)$ then implies

$$G_1\big(B_{D,\theta}^*(y); y\big) = V_1^R\big(y, B_{D,\theta}^*(y)\big) - V^D(y) \ge V_1^R\big(y, B_{D,1}^*(y)\big) - V^D(y) = 0. \tag{B.20}$$

Next, invoke Proposition 2 ("price pivot"): for the given y there exists $B^*(y)$ such that

$$q_\theta(y, B') \gtrless q_1(y, B') \quad \text{iff} \quad B' \gtrless B^*(y),$$

with strict inequality on a set of B' of positive measure. Near the brink of default (i.e., at $B = B_{D,\theta}^*(y)$), the sovereign's optimal issuance under repayment places (by the Euler condition and rollover incentives in long-term debt models) essentially all probability mass on future debt levels $B' \ge B^*(y)$.[19] Hence, evaluating the choice-specific value

$$W_i(y, B, B') = u(y - \kappa B + [B' - (1-\delta)B]\, q_i(y, B')) + \beta \mathbb{E}_{y'|y}\big[V_i(y', B')\big],$$

I have for all such relevant $B' \ge B^*(y)$,

$$q_\theta(y, B') \ge q_1(y, B') \quad \Longrightarrow \quad u\big(\cdot \text{ with } q_\theta\big) \ge u\big(\cdot \text{ with } q_1\big),$$

and (by monotonicity of the Bellman operator with respect to the price schedule in those states) $\mathbb{E}[V_\theta(y', B')] \ge \mathbb{E}[V_1(y', B')]$. Therefore $W_\theta(y, B, B') \ge W_1(y, B, B')$ on the support of the (repayment) choice distribution, with strict inequality on a set of positive probability.

[18] From the pricing operator, $q_i(B', y) = \frac{1}{1+r} \mathbb{E}_{y'|y}\big[(1 - P_i(y', B'))\big(\kappa + (1-\delta)\mathbb{E}_{B''|y', B'}[\, q_i(y', B'')\,]\big)\big]$, so $q_i \ge 0$ and $q_i > 0$ whenever the repayment probability is not identically zero.

[19] Intuitively, when current debt is high, the sovereign rolls over rather than deleverages sharply; see, e.g., the rollover logic in long-term debt environments. Formally, with small taste-shock dispersion ρ, the logit policy concentrates around the deterministic maximizer, which lies weakly to the right of $B^*(y)$ at high current B.

Aggregating with the log-sum-exp representation of the repayment value,

$$V_i^R(y,B) = \rho \log\Big(\sum_{B'} \exp\{W_i(y,B,B')/\rho\}\Big),$$

which is strictly increasing in each $W_i(y,B,B')$, I obtain

$$V_\theta^R\big(y, B_{D,\theta}^*(y)\big) > V_1^R\big(y, B_{D,\theta}^*(y)\big). \tag{B.21}$$

Combining (B.20) and (B.21) yields

$$V_\theta^R\big(y, B_{D,\theta}^*(y)\big) > V^D(y),$$

which contradicts the definition of $B_{D,\theta}^*(y)$ as the point where $V_\theta^R = V^D$. Hence the assumed ordering is false, and I must have

$$B_{D,\theta}^*(y) > B_{D,1}^*(y).$$

□

B.9 Proof of Proposition 5

Proof. **Setup.** Fix a state (y,B). For each lender type $i \in \{1,\theta\}$, let

$$B_i'(y,B) \in \arg\max_{B' \in \mathcal{B}} W(y,B,B';q_i), \quad W(y,B,B';q_i) \equiv u\big(c_i(B')\big) + \beta \mathbb{E}_{y'|y}\big[V_i(y',B')\big],$$

with

$$c_i(B') = y - \kappa B + \big[B' - (1-\delta)B\big]\, q_i(y,B').$$

By strict concavity of u and standard properties of the long-maturity debt problem, $B_i'(y,B)$ is unique.[20] Define the marginal objective

$$g_i(B') \equiv \frac{\partial W(y,B,B';q_i)}{\partial B'} = u'\big(c_i(B')\big)\Big[q_i(y,B') + \big(B' - (1-\delta)B\big) q_{i,B'}(y,B')\Big] + \beta \frac{\partial}{\partial B'} \mathbb{E}_{y'|y}[V_i(y',B')].$$

FOC and uniqueness give

$$g_i\big(B_i'(y,B)\big) = 0, \qquad i \in \{1,\theta\}. \tag{B.22}$$

[20]The usual argument is that the choice-specific value is strictly concave in B' because (i) u is strictly concave and $B' \mapsto c_i(B')$ is affine plus a (weakly) concave price-feedback term, and (ii) the continuation term $B' \mapsto \mathbb{E}[V_i(y',B')]$ is concave by the concavity of the Bellman operator.

Region of comparison. By Proposition 2, there exists $B^*(y)$ such that

$$B' \geq B^*(y) \implies q_\theta(y,B') < q_1(y,B'), \qquad B' \leq B^*(y) \implies q_\theta(y,B') > q_1(y,B'). \tag{B.23}$$

I prove the claim on the empirically relevant *risky* side, i.e. for states with the baseline choice on (or to the right of) the pivot:

$$B_1'(y,B) \;\geq\; B^*(y). \tag{B.24}$$

(The complementary case is analogous and yields the weak inequality; see the remark at the end.)

Two auxiliary inequalities.

(A1) Since $q_{i,B'}(y,B') \leq 0$ in equilibrium and $\big(B' - (1-\delta)B\big) \geq 0$ whenever the sovereign issues (rolls over) debt,[21]

$$q_i(y,B') + \big(B' - (1-\delta)B\big)q_{i,B'}(y,B') \;\leq\; q_i(y,B'). \tag{B.25}$$

(A2) (Continuation marginal ordering.) Increasing next-period debt lowers the continuation value and it does so *more* under PRO lenders:

$$\frac{\partial}{\partial B'}\mathbb{E}_{y'|y}[V_\theta(y',B')] \;\leq\; \frac{\partial}{\partial B'}\mathbb{E}_{y'|y}[V_1(y',B')] \;\leq\; 0, \tag{B.26}$$

with strict inequality on a set of positive measure when $B' \geq B^*(y)$.[22]

Key evaluation at the baseline optimum. Evaluate g_θ at $B_1' \equiv B_1'(y,B)$. Using (B.25) and (B.26),

$$\begin{aligned} g_\theta(B_1') &= u'\big(c_\theta(B_1')\big)\Big[q_\theta(y,B_1') + \big(B_1' - (1-\delta)B\big)q_{\theta,B'}(y,B_1')\Big] + \beta\frac{\partial}{\partial B'}\mathbb{E}[V_\theta(y',B')]\Big|_{B'=B_1'} \\ &\leq u'\big(c_\theta(B_1')\big)q_\theta(y,B_1') + \beta\frac{\partial}{\partial B'}\mathbb{E}[V_1(y',B')]\Big|_{B'=B_1'}. \end{aligned} \tag{B.27}$$

By the baseline FOC (B.22) for $i=1$,

$$0 \;=\; g_1(B_1') \;=\; u'\big(c_1(B_1')\big)\Big[q_1 + \big(B_1' - (1-\delta)B\big)q_{1,B'}\Big] + \beta\frac{\partial}{\partial B'}\mathbb{E}[V_1(y',B')]\Big|_{B'=B_1'},$$

[21] If net issuance is negative at the optimum, the conclusion below is even stronger because the self-impact term becomes favorable.

[22] This follows from monotonicity of the Bellman operator in the price schedule and Proposition 2. For $B' \geq B^*(y)$, PRO implies lower prices $q_\theta < q_1$ and (via the probability ordering embedded in the pricing operator) higher default likelihood, both of which tighten the next-period budget more when B' increases. Hence the marginal effect of B' on the continuation value is weakly more adverse under θ.

and therefore, again using $q_{1,B'} \le 0$,

$$\beta \frac{\partial}{\partial B'}\mathbb{E}[V_1(y',B')]\Big|_{B'=B_1'} = -u'\big(c_1(B_1')\big)\Big[q_1 + \big(B_1' - (1-\delta)B\big)q_{1,B'}\Big] \le -u'\big(c_1(B_1')\big)\,q_1(y,B_1'). \tag{B.28}$$

Substituting (B.28) into (B.27) yields

$$g_\theta(B_1') \le u'\big(c_\theta(B_1')\big)\,q_\theta(y,B_1') - u'\big(c_1(B_1')\big)\,q_1(y,B_1'). \tag{B.29}$$

Sign of the bound on the risky side. Under (B.24), (B.23) gives $q_\theta(y,B_1') < q_1(y,B_1')$. Moreover, $c_\theta(B_1') < c_1(B_1')$ because the only difference in the current-period budget is the price multiplying the same issuance $\big[B_1' - (1-\delta)B\big]$, hence by concavity $u'\big(c_\theta(B_1')\big) > u'\big(c_1(B_1')\big)$. Combining these two facts in (B.29) implies

$$g_\theta(B_1') < 0.$$

Since $B' \mapsto W(y,B,B';q_\theta)$ is strictly concave, g_θ is strictly decreasing, so $g_\theta(B_1') < 0$ forces the θ-optimizer to lie strictly to the left of B_1':

$$B_\theta'(y,B) < B_1'(y,B).$$

From modes to distributions. Under the logit (Gumbel) regularization, the choice probabilities satisfy $\Pr_i(B' = b \mid y,B) \propto \exp\{W(y,B,b;q_i)/\eta\}$, which are unimodal and concentrate around the unique maximizer as $\eta \downarrow 0$. Because the mode shifts left from B_1' to B_θ', the θ-distribution first-order stochastically dominates (to the left) the baseline distribution, implying $\mathbb{E}_\theta[B' \mid y,B] < \mathbb{E}_1[B' \mid y,B]$.

Remark. If $B_1'(y,B) \le B^*(y)$ (safe side), the same argument yields the weak inequality $B_\theta'(y,B) \le B_1'(y,B)$; strictness may fail when both prices and continuation effects coincide on the relevant grid. □

B.10 Proof of Proposition 6

Proof. **Step 0: Bellman operators.** Let J_i be the Bellman operator for economy $i \in \{1,\theta\}$:

$$(J_i V_{in})(y,B) = \eta \ln\left(\exp\left(\frac{V^D(y;V_{in})}{\eta}\right) + \exp\left(\frac{V^R(y,B;q_i,V_{in})}{\eta}\right)\right),$$

where V^D and V^R are the default and repayment values respectively. The equilibrium value V_i is the unique fixed point of J_i on the space of bounded continuous functions on $\mathscr{Y} \times \mathscr{B}$ with the sup norm.

Step 1: Default value is identical across i. The default value

$$V^D(y;V_{in}) = u(h(y)) + \beta\left[\gamma\,\mathbb{E}_{y'|y}V_{in}(y',B_0) + (1-\gamma)\,\mathbb{E}_{y'|y}V^D(y';V_{in})\right]$$

is independent of the current price schedule q_i; hence

$$V^D(y;V_{in}) \quad \text{is the same for } i=1 \text{ and } i=\theta. \tag{B.30}$$

Step 2: Repayment value difference comes from prices. The repayment value is

$$V^R(y,B;q_i,V_{in}) = \rho\,\ln\left(\sum_{B'\in\mathscr{B}} \exp\left(\frac{W(y,B,B';q_i,V_{in})}{\rho}\right)\right),$$

with

$$W(y,B,B';q_i,V_{in}) = u\big(c_i(B')\big) + \beta\,\mathbb{E}_{y'|y}V_{in}(y',B'), \quad c_i(B') = y - \kappa B + [B'-(1-\delta)B]\,q_i(y,B').$$

The continuation term $\mathbb{E}_{y'|y}V_{in}(y',B')$ is identical across i; differences in W come solely from $c_i(B')$.

Step 3: Price ordering on the risky side. From Proposition 2, if $B' \geq B^*(y)$ then

$$q_\theta(y,B') < q_1(y,B'),$$

with strict inequality on a set of B' with positive measure. If the baseline optimal choice $B_1'(y,B) \geq B^*(y)$, the relevant B' in the repayment maximization fall in this region with positive probability.

Step 4: Consumption and utility ordering. For any such B', $c_\theta(B') < c_1(B')$, hence by strict concavity of u,

$$u(c_\theta(B')) < u(c_1(B')).$$

Because the continuation term in W is identical, I have

$$W(y,B,B';q_\theta,V_{in}) < W(y,B,B';q_1,V_{in}) \quad \text{for such } B'. \tag{B.31}$$

Step 5: Aggregating over choices. The log-sum-exp in V^R is strictly increasing in each W. From (B.31), and because the inequality is strict for some B' in the summation,

$$V^R(y,B;q_\theta,V_{in}) < V^R(y,B;q_1,V_{in})$$

whenever $B_1'(y,B) \geq B^*(y)$. On the safe side $B_1'(y,B) \leq B^*(y)$, the weak inequality holds.

Step 6: Bellman operator dominance and fixed points. By (B.30) and the above repayment ordering,

$$(J_\theta V_{in})(y,B) \le (J_1 V_{in})(y,B),$$

strictly when $B_1'(y,B) \ge B^*(y)$. The operators J_i are monotone and contractions (Blackwell conditions), hence the pointwise ordering is preserved at the unique fixed points:

$$V_\theta(y,B) \le V_1(y,B),$$

with strict inequality under the risky-side condition.

□

B.11 Proof of Proposition 8

Proof. **Step 1 (Intertemporal implementability).** Multiply (25) by β^t and sum expectations over t, and use $\beta(1+r)=1$ together with (26). Writing $\Delta_t \equiv B_{t+1}-(1-\delta)B_t$, feasibility under schedule q_i is equivalent to the *implementability constraint*

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t c_t\right] = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\big(y_t-\kappa B_t\big)\right] + \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\,\Delta_t\, q_i(y_t,B_{t+1})\right], \tag{B.32}$$

together with $c_t \ge 0$ and $B_{t+1}\in\mathscr{B}$. In particular, the transfer sequence $\{\tau_t\}$ only enforces period constraints and has zero present value, so it does not affect (B.32).

Step 2 (Set inclusion of feasible (c,B)). Fix any feasible pair (c,B) under q_θ; it satisfies (B.32) with $i=\theta$. Consider the same (c,B) under q_1. Subtracting the two versions of (B.32) gives the slack generated by switching from q_θ to q_1:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\,\Delta_t\big(q_1(y_t,B_{t+1})-q_\theta(y_t,B_{t+1})\big)\right] = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t c_t\right]_{q_1} - \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t c_t\right]_{q_\theta}. \tag{B.33}$$

By (A2), each summand in $\mathscr{S}(c,B)$ is nonnegative and strictly positive with positive probability whenever $|\Delta_t|>0$. Hence

$$\mathscr{S}(c,B)\ \ge\ 0 \quad\text{and}\quad \mathscr{S}(c,B)\ >\ 0\ \text{ if } \Pr\big(|\Delta_t|>0 \text{ and } q_1\neq q_\theta\big)>0.$$

Therefore, every (c,B) feasible under q_θ is also feasible under q_1, and generically with strictly more present-value resources available for consumption under q_1.

Step 3 (Value comparison). Let (c^θ,B^θ) be Ramsey-optimal under q_θ. By Step 2, (c^θ,B^θ) is feasible under q_1 and produces a weakly larger right-hand side in (B.32); because u is strictly increasing, the planner can raise consumption at (at least) one date by an $\varepsilon>0$

while keeping feasibility under q_1, yielding strictly higher utility. Hence

$$W_1^R \geq \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t^\theta)\right], \quad \text{with strict inequality under the strict part of (A2).}$$

Taking the supremum over all feasible (c, B) under q_1 gives $W_1^R > W_\theta^R$.

Remark (Why transfers cannot undo the loss). Condition (26) fixes the present value of transfers at zero, so transfers only reshuffle consumption across dates without changing the right-hand side of (B.32). The loss stems from the lower proceeds from intertemporal trade encoded in q_θ whenever Δ_t and the price dominance have the same sign; no transfer sequence can increase the present value on the right-hand side of (B.32). □

B.12 Proof of Proposition 9

Proof. I establish each part through rigorous analysis of the belief updating dynamics.

Part (i): Persistent PRO. Define the conditional maximum likelihood estimators:

$$\hat{\theta}_D(\{d_s\}_{s=0}^t) = \arg\max_{\theta\in[\underline{\theta},\bar{\theta}]} \prod_{s:d_s=1} P_\theta(y_s, B_s) \prod_{s:d_s=0}(1 - P_\theta(y_s, B_s)) \tag{B.34}$$

$$\hat{\theta}_R(\{d_s\}_{s=0}^t) = \arg\max_{\theta\in[\underline{\theta},\bar{\theta}]} \prod_{s:d_s=0}(1 - P_\theta(y_s, B_s)) \prod_{s:d_s=1} P_\theta(y_s, B_s) \tag{B.35}$$

Under negativity bias, the belief updating mechanism assigns different weights $w_D > w_R$ to default versus repayment observations:[23]

$$\hat{\theta}(\{d_s\}_{s=0}^t) = \frac{w_D \sum_{s:d_s=1} \ln P_\theta(y_s, B_s) + w_R \sum_{s:d_s=0} \ln(1 - P_\theta(y_s, B_s))}{w_D N_D + w_R N_R} \tag{B.36}$$

where N_D and N_R are the numbers of defaults and repayments, respectively.

Taking expectations over the ergodic distribution $\mu(y, B)$ of state variables:

$$\mathbb{E}[\hat{\theta}] = \int \left[w_D P_1(y, B) \frac{\partial \ln P_\theta(y, B)}{\partial\theta} + w_R(1 - P_1(y, B)) \frac{\partial \ln(1 - P_\theta(y, B))}{\partial\theta} \right] d\mu(y, B) \tag{B.37}$$

Since $P_\theta(y, B) = L(\Delta V(y, B)/(\theta\eta))$ where L is the logistic CDF:[24]

$$\frac{\partial \ln P_\theta(y, B)}{\partial\theta} = -\frac{\Delta V(y, B)}{\theta^2\eta}(1 - P_\theta(y, B)) < 0 \tag{B.38}$$

[23] The negativity bias reflects well-documented cognitive biases where agents overweight negative information relative to positive information, consistent with prospect theory and related behavioral findings.

[24] This follows from the discrete choice formulation in the main model where taste shocks follow Gumbel distributions.

Substituting (B.38) into (B.37) and using the negativity bias $w_D > w_R$:

$$\mathbb{E}[\hat{\theta}] > 1 + \frac{(w_D - w_R)}{w_D + w_R} \int P_1(y,B) \frac{|\Delta V(y,B)|}{\theta^2 \eta} (1 - P_\theta(y,B)) d\mu(y,B) > 1 \tag{B.39}$$

By the ergodic theorem and (27), $\lim_{t\to\infty} \mathbb{E}[\theta_t] = \mathbb{E}[\hat{\theta}] > 1$.

Part (ii): History Dependence. Let $\mathcal{H}_t = \{(y_s, B_s, d_s)\}_{s=0}^{t-1}$ denote the history up to time t. The belief updating process creates a Markov chain on the augmented state space $(y, B, \theta, \mathcal{H})$.[25]

For any two histories $\mathcal{H}^A$ and $\mathcal{H}^B$ with different default patterns but identical fundamentals, define the corresponding stationary distributions Θ^A and Θ^B. The key insight is that the transition operator:

$$T_{\mathcal{H}}(\theta, \theta') = \lambda \delta(\theta' - \theta) + (1 - \lambda)\delta(\theta' - \hat{\theta}(\mathcal{H})) \tag{B.40}$$

depends explicitly on the history $\mathcal{H}$.

Since $\hat{\theta}(\mathcal{H}^A) \neq \hat{\theta}(\mathcal{H}^B)$ for histories with different default intensities, I have $\Theta^A \neq \Theta^B$. Moreover, if $\mathcal{H}^A$ contains more surprising defaults than $\mathcal{H}^B$, then Θ^A first-order stochastically dominates Θ^B:

$$\int_{\underline{\theta}}^{\theta} d\Theta^A(\tilde{\theta}) \leq \int_{\underline{\theta}}^{\theta} d\Theta^B(\tilde{\theta}) \quad \forall \theta \in [\underline{\theta}, \bar{\theta}] \tag{B.41}$$

Part (iii): Slow Convergence. Let π_t denote the default probability in period t under the current belief θ_t. The frequency of belief updates follows a Poisson process with intensity π_t.[26]

Between updates, beliefs evolve according to:

$$\theta_{t+1} = \lambda \theta_t \quad \text{(no default)} \tag{B.42}$$

The time between defaults follows an exponential distribution with rate $\pi_t \approx \pi^*$ in steady state. The expected duration between updates is $1/\pi^*$, and during each interval of length τ, beliefs decay by factor λ^τ.

For the convergence rate, consider the deviation from the stationary mean: $\epsilon_t = \theta_t - \mathbb{E}[\Theta^*]$. The dynamics satisfy:

$$\mathbb{E}[\epsilon_{t+1}] = \lambda \mathbb{E}[\epsilon_t] + (1 - \lambda)(\mathbb{E}[\hat{\theta}] - \mathbb{E}[\Theta^*]) \tag{B.43}$$

[25] The history dependence arises because the MLE $\hat{\theta}(\mathcal{H}_t)$ depends on the entire sequence of past defaults, not just the current state.

[26] This approximation is valid when defaults are rare events, which is the empirically relevant case for most sovereigns.

In steady state, $\mathbb{E}[\hat{\theta}] = \mathbb{E}[\Theta^*]$, so:

$$\mathbb{E}[\epsilon_{t+1}] = \lambda \mathbb{E}[\epsilon_t] \tag{B.44}$$

Therefore, $\|\theta_t - \mathbb{E}[\Theta^*]\| = O(\lambda^t)$. Since empirical persistence in sovereign spreads requires $\lambda \approx 0.95$–0.99, convergence is indeed slow with half-life approximately $\ln(2)/\ln(1/\lambda) \approx 14 - 69$ periods. □

B.13 Proof of Proposition 10

Proof. I establish each part through detailed analysis of the sovereign's optimization problem under strategic communication.

Setup: The Communication Problem. Let $\mathscr{W}(\alpha;\theta)$ denote the sovereign's value function under transparency level α and baseline PRO θ. This satisfies the Bellman equation:

$$\mathscr{W}(\alpha;\theta) = \max_{\{c_t, B_{t+1}\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi(\alpha)\right) \right] \tag{B.45}$$

subject to:

$$c_t = y_t - \kappa B_t + [B_{t+1} - (1-\delta)B_t] q_{\theta_{\text{eff}}}(y_t, B_{t+1}) \tag{B.46}$$

$$\theta_{\text{eff}}(\alpha,\theta) = \alpha \cdot 1 + (1-\alpha)\cdot\theta \tag{B.47}$$

Part (i): Interior Solution. Define the value function net of communication costs: $V(\alpha) = \mathscr{W}(\alpha;\theta) - \mathbb{E}[\phi(\alpha)]$. The first-order condition for optimality is:

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\alpha^*} = \left.\frac{\partial \mathscr{W}(\alpha;\theta)}{\partial \alpha}\right|_{\alpha=\alpha^*} - \gamma\alpha^* = 0 \tag{B.48}$$

To characterize $\frac{\partial \mathscr{W}}{\partial \alpha}$, note that transparency affects welfare through the bond pricing channel. Using the envelope theorem:[27]

$$\frac{\partial \mathscr{W}}{\partial \alpha} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u'(c_t) \frac{\partial c_t}{\partial \alpha} \right] \tag{B.49}$$

From the budget constraint (B.46):

$$\frac{\partial c_t}{\partial \alpha} = [B_{t+1} - (1-\delta)B_t] \frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} \frac{\partial \theta_{\text{eff}}}{\partial \alpha} = [B_{t+1} - (1-\delta)B_t] \frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} (1-\theta) \tag{B.50}$$

Since $\frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} < 0$ (higher perceived volatility reduces bond prices) and $(1-\theta) < 0$ for $\theta > 1$,

[27]The envelope theorem applies because the sovereign optimally chooses consumption and debt policies for any given price schedule.

I have:

$$\frac{\partial \mathcal{W}}{\partial \alpha} = (\theta - 1)\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u'(c_t)[B_{t+1} - (1-\delta)B_t] \left| \frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} \right| \right] > 0 \tag{B.51}$$

For an interior solution, I need both concavity and appropriate boundary conditions. The second-order condition requires:

$$\left. \frac{\partial^2 V(\alpha)}{\partial \alpha^2} \right|_{\alpha=\alpha^*} = \left. \frac{\partial^2 \mathcal{W}}{\partial \alpha^2} \right|_{\alpha=\alpha^*} - \gamma < 0 \tag{B.52}$$

Since $\frac{\partial^2 \mathcal{W}}{\partial \alpha^2} < 0$ (diminishing returns to transparency) and $\gamma > 0$, condition (B.52) is satisfied.[28]

The boundary conditions require $\frac{\partial V}{\partial \alpha}|_{\alpha=0} > 0$ and $\frac{\partial V}{\partial \alpha}|_{\alpha=1} < 0$, which hold for intermediate values of γ satisfying:

$$\left. \frac{\partial \mathcal{W}}{\partial \alpha} \right|_{\alpha=0} > \gamma > \left. \frac{\partial \mathcal{W}}{\partial \alpha} \right|_{\alpha=1} \tag{B.53}$$

Part (ii): PRO Amplifies Transparency. Differentiating the first-order condition (B.48) with respect to θ using the implicit function theorem:

$$\left. \frac{\partial^2 \mathcal{W}}{\partial \alpha \partial \theta} \right|_{\alpha=\alpha^*} + \left. \frac{\partial^2 \mathcal{W}}{\partial \alpha^2} \right|_{\alpha=\alpha^*} \frac{\partial \alpha^*}{\partial \theta} - \gamma \frac{\partial \alpha^*}{\partial \theta} = 0 \tag{B.54}$$

Rearranging:

$$\frac{\partial \alpha^*}{\partial \theta} = \frac{\frac{\partial^2 \mathcal{W}}{\partial \alpha \partial \theta}}{\gamma - \frac{\partial^2 \mathcal{W}}{\partial \alpha^2}} \tag{B.55}$$

To sign the numerator, differentiate (B.51) with respect to θ:

$$\frac{\partial^2 \mathcal{W}}{\partial \alpha \partial \theta} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u'(c_t)[B_{t+1} - (1-\delta)B_t] \left| \frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} \right| \right] + (\theta - 1)\frac{\partial}{\partial \theta}\mathbb{E}[\cdots] \tag{B.56}$$

The first term is positive (direct effect), and the second term captures the indirect effect through equilibrium adjustments. Under reasonable parameter values, the direct effect dominates, ensuring $\frac{\partial^2 \mathcal{W}}{\partial \alpha \partial \theta} > 0$.[29]

Since the denominator in (B.55) is positive by the second-order condition (B.52), I conclude:

$$\frac{\partial \alpha^*}{\partial \theta} > 0 \tag{B.57}$$

Part (iii): Welfare Dominance. Define the net welfare gain from communication as:

$$\Delta W(\theta) = \mathcal{W}(\alpha^*(\theta); \theta) - \mathcal{W}(0; \theta) - \mathbb{E}[\phi(\alpha^*(\theta))] \tag{B.58}$$

[28] The diminishing returns arise because transparency has the largest impact when moving from very opaque to moderately transparent, with smaller gains from further increases.

[29] The direct effect dominates because the immediate benefit of transparency increases linearly with the degree of baseline PRO, while the indirect effects through equilibrium adjustments are second-order.

For small $\theta - 1$, using a second-order Taylor expansion around $\theta = 1$:

$$\Delta W(\theta) \approx \frac{1}{2} \frac{\partial^2 \mathcal{W}}{\partial \alpha^2}\bigg|_{\alpha=\alpha^*(1)} [\alpha^*(\theta)]^2 - \frac{\gamma[\alpha^*(\theta)]^2}{2} \tag{B.59}$$

Using the first-order condition and part (ii), $\alpha^*(\theta) \approx C(\theta - 1)$ for some constant $C > 0$. Substituting:

$$\Delta W(\theta) \approx \frac{C^2(\theta-1)^2}{2} \left[\frac{\partial^2 \mathcal{W}}{\partial \alpha^2} - \gamma\right] \tag{B.60}$$

Since the second-order condition requires $\frac{\partial^2 \mathcal{W}}{\partial \alpha^2} - \gamma < 0$, I have $\Delta W(\theta) < 0$ for θ close to 1. However, as θ increases, the benefits of transparency grow faster than the costs.[30]

The critical threshold θ_c is defined by $\Delta W(\theta_c) = 0$. For $\theta > \theta_c$, the welfare gains from reduced borrowing costs exceed the communication costs, establishing welfare dominance.

□

[30]This occurs because the marginal benefit of transparency is proportional to $(\theta - 1)$, while the marginal cost is constant in θ.

C Computational Algorithm

I solve the model numerically using value function iteration on a discretized state space, following the computational strategies developed by (?). The algorithm is implemented in Fortran and parallelized using OpenMP to accelerate computation. The process is as follows:

C.1 Discretization and Initialization

The state space is discretized first.

1. I discretize the exogenous endowment process, an AR(1) in logarithms defined in equation (1), into a $N_y = 201$ state Markov chain using the (?) method, as implemented in my `discretizeAR1` subroutine. This yields a grid for income levels $\{y_i\}_{i=1}^{N_y}$ and a transition matrix $\Pi(y_i, y_j)$.

2. I construct the space for bond holdings, $\mathscr{B}$, as a grid of $N_B = 600$ points, uniformly spaced between $B_{\min} = 0$ and $B_{\max} = 0.75$ of mean output, created using a `linspace` function.

3. I initialize the algorithm with guesses for the value functions and the bond price schedule. I set the price of bonds, $q_0(y, B')$, initially to 1 for all states. I initialize the value function, $V_0(y, B)$, to the utility of consuming the endowment net of coupon payments, and the value of default, $Vd_0(y)$, to the utility of consuming the post-default output $h(y)$.

C.2 Value Function and Price Iteration

The core of my solution method is iterating on the value functions (V, V^D) and the bond price function q until they converge to a joint fixed point. This process solves the system of equations defined in Section 3. My main `DO WHILE` loop in the Fortran code continues until the maximum absolute difference between successive iterates for both the value function and the bond prices falls below a specified tolerance ($\epsilon_V = 10^{-6}, \epsilon_q = 10^{-6}$). A single iteration, from step $k-1$ to k, is a carefully ordered sequence of updates, parallelized with OpenMP where possible.

1. **Update the Value of Default.** I begin each iteration by updating the value of default, $V_k^D(y)$. This calculation uses the total value function V_{k-1} and default value V_{k-1}^D from the *previous* iteration. For each income state y_i, I follow the discrete version of equation (7):

$$V_k^D(y_i) = u(h(y_i)) + \beta \sum_{j=1}^{N_y} \Pi(y_i, y_j) \left[\gamma V_{k-1}(y_j, B_1) + (1-\gamma) V_{k-1}^D(y_j) \right],$$

where B_1 is the grid point corresponding to zero debt.

2. **Update the Value of Repayment and Sovereign Policies.** This is a multi-step process that I compute inside a large parallel loop over the state space (y_i, B_j).

   (a) First, for each state (y_i, B_j), I calculate the **choice-specific value of borrowing**, $W(y_i, B_j, B'_l)$, for *every* possible next-period debt level B'_l. This uses the bond price q_{k-1} and value function V_{k-1} from the prior iteration:

   $$W(y_i, B_j, B'_l) = u(y_i - \kappa B_j + [B'_l - (1-\delta)B_j]q_{k-1}(y_i, B'_l)) + \beta \sum_{m=1}^{N_y} \Pi(y_i, y_m) V_{k-1}(y_m, B'_l).$$

   (b) With the full vector of W values, I compute the **value of repayment**, $V_k^R(y_i, B_j)$, using the "log-sum-exp" formulation following (**?**). This numerically stable technique, which uses the borrowing taste shock parameter ρ (or `rhoB` in my code), replaces a non-differentiable 'max' operation over the borrowing grid:

   $$V_k^R(y_i, B_j) = \rho \ln \left(\sum_{l=1}^{N_B} \exp \frac{W(y_i, B_j, B'_l)}{\rho} \right).$$

   (c) Simultaneously, I compute the state-contingent **borrowing policy**, $\Pr(B'_l | y_i, B_j)$, as the softmax of the choice-specific values:

   $$\Pr(B'_l | y_i, B_j) = \frac{\exp(W(y_i, B_j, B'_l)/\rho)}{\sum_{s=1}^{N_B} \exp(W(y_i, B_j, B'_s)/\rho)}.$$

   (d) Finally, using the *just-updated* values $V_k^R(y_i, B_j)$ and $V_k^D(y_i)$, I calculate the new **ex-ante value function** $V_k(y_i, B_j)$, again using the log-sum-exp formula but with the default taste shock parameter η (`rhoD` in my code):

   $$V_k(y_i, B_j) = \eta \ln \left(\exp \frac{V_k^D(y_i)}{\eta} + \exp \frac{V_k^R(y_i, B_j)}{\eta} \right).$$

   (e) I also determine the sovereign's true **default probability**, $\Pr\{d = 1 | y_i, B_j\}$, at this point from the logistic choice formula:

   $$\Pr\{d = 1 | y_i, B_j\} = \frac{\exp(V_k^D(y_i)/\eta)}{\exp(V_k^D(y_i)/\eta) + \exp(V_k^R(y_i, B_j)/\eta)}.$$

3. **Update Bond Prices.** I update the bond price schedule $q_k(y, B')$ based on the lenders' zero-profit condition. This is the crucial step where I introduce the behavioral PRO wedge, θ (`thetaD`). For each state (y_i, B'_l):

(a) I calculate the **lender's perceived default probability**, $\tilde{P}(y_m, B'_l)$. It uses the most recently updated value functions, V_k^R and V_k^D, but is distorted by the inflated taste shock parameter $\tilde{\eta} = \theta\eta$:

$$\tilde{P}(y_m, B'_l) = \frac{\exp(V_k^D(y_m)/(\theta\eta))}{\exp(V_k^D(y_m)/(\theta\eta)) + \exp(V_k^R(y_m, B'_l)/(\theta\eta))}.$$

(b) The new price, $q_k(y_i, B'_l)$, is the expected discounted payoff from the lender's perspective. This expectation is taken over future income states y_m and future bond choices B''_s. It uses the lender's perceived default probability $\tilde{P}$, the sovereign's *true* borrowing policy $\Pr(B''_s|y_m, B'_l)$ (computed in step 2), and the bond prices from the *previous* iteration, q_{k-1}. The full pricing equation implemented in the code is:

$$q_k(y_i, B'_l) = \frac{1}{1+r} \sum_{m=1}^{N_y} \Pi(y_i, y_m)(1-\tilde{P}(y_m, B'_l)) \left(\kappa + (1-\delta) \sum_{s=1}^{N_B} \Pr(B''_s|y_m, B'_l) \cdot q_{k-1}(y_m, B''_s) \right).$$

4. **Check for Convergence.** The iteration concludes by calculating the supremum norm of the change in the value functions and price functions: errV $= \max|V_k - V_{k-1}|$ and errQ $= \max|q_k - q_{k-1}|$. If the errors are within tolerance, the loop terminates. Otherwise, the algorithm sets $V_0 \leftarrow V_k$, $Vd_0 \leftarrow V_k^D$, and $q_0 \leftarrow q_k$, and proceeds to the next iteration.

C.3 Simulation

Once the functions have converged, the `simulate` subroutine is called to generate the business cycle moments. For reproducibility, the random number generator is initialized with a fixed seed. The simulation generates a long time series of 100,000 periods, with the first 299 periods discarded as a burn-in phase.

The simulation starts from a deterministic state: zero initial debt and the median level of the endowment grid. For each subsequent period, the simulation proceeds as follows:

1. The new endowment level is determined by drawing from the discretized Markov transition matrix, Π.

2. If the economy was in default in the previous period, a random draw determines if it regains market access (with probability γ). If not, it remains in default.

3. If the economy has market access, a uniform random draw is compared against the converged default probability function, $\Pr\{d = 1|y, B\}$, to determine if a default occurs.

4. If the economy does not default, a second uniform random draw is used to select the next period's debt level, B', from the converged borrowing probability distribution, $\Pr\{B'|y,B\}$.

5. Key economic variables (consumption, trade balance, GDP, credit spreads) for the current period are calculated based on the state and choices, and then stored.

The business cycle moments reported in Table 3 are calculated from this simulated data series. The code consistently uses the standard numerical stability technique of subtracting the maximum value within any `exp` calculation in the log-sum-exp formulas to prevent floating-point overflow.

C.4 Numerical Stability Techniques

The implementation of the log-sum-exp formulation for discrete choice models requires careful attention to numerical stability to prevent floating-point overflow and underflow errors, as emphasized by (**?**). This subsection details the specific techniques I employ in my algorithm to ensure robust computation, particularly in the evaluation of choice probabilities and expected values corresponding to the discrete choice formulations in the main text.

C.4.1 The Log-Sum-Exp Numerical Stability Problem

The naive implementation of the log-sum-exp function

$$\text{LSE}(x_1,\ldots,x_n) = \ln\left(\sum_{i=1}^{n} \exp(x_i)\right) \tag{C.61}$$

is prone to numerical instability when the values x_i are large in magnitude. For large positive values, $\exp(x_i)$ may overflow, returning infinity. For large negative values, $\exp(x_i)$ may underflow to zero, causing a loss of precision in the sum. In my sovereign default model, these issues arise when the choice-specific values $W(y,B,B')$ span a wide range due to varying consumption levels across borrowing choices, when the value functions $V^R(y,B)$ and $V^D(y)$ differ substantially near the default boundary, and when the small scale parameters $\rho \approx 10^{-5}$ and $\eta \approx 10^{-4}$ amplify the ratios W/ρ or V/η to extreme magnitudes.

C.4.2 Stabilized Log-Sum-Exp Implementation

To address these issues, I employ the standard "max subtraction" technique following (**?**). For any set of values $\{x_1,\ldots,x_n\}$, I first compute their maximum:

$$x_{\max} = \max\{x_1,\ldots,x_n\} \tag{C.62}$$

Then I rewrite the log-sum-exp as:

$$\text{LSE}(x_1, \ldots, x_n) = x_{\max} + \ln\left(\sum_{i=1}^{n} \exp(x_i - x_{\max})\right) \tag{C.63}$$

This transformation ensures that all terms satisfy $(x_i - x_{\max}) \leq 0$, yielding $\exp(x_i - x_{\max}) \in (0, 1]$, where the largest exponential term equals exactly 1 to prevent overflow, while smaller terms decay exponentially but remain numerically representable to preserve precision.

C.4.3 Application to Borrowing Choice Probabilities

In my computation of borrowing choice probabilities corresponding to equation (5), I use the stabilized implementation:

$$\overline{W}(y, B) = \max_{B'} W(y, B, B') \tag{C.64}$$

$$\Pr(B' = B_i | y, B) = \frac{\exp \frac{W(y,B,B_i) - \overline{W}(y,B)}{\rho}}{\sum_{j \in \mathscr{B}} \exp \frac{W(y,B,B_j) - \overline{W}(y,B)}{\rho}} \tag{C.65}$$

Similarly, the expected value of repayment corresponding to equation (9) becomes:

$$V^R(y, B) = \overline{W}(y, B) + \rho \ln\left(\sum_{j \in \mathscr{B}} \exp \frac{W(y, B, B_j) - \overline{W}(y, B)}{\rho}\right) \tag{C.66}$$

C.4.4 Application to Default Choice Probabilities

For the default decision corresponding to equation (4), I apply the same stabilization technique:

$$\overline{V}(y, B) = \max\{V^D(y), V^R(y, B)\} \tag{C.67}$$

$$\Pr(d = 1 | y, B) = \frac{\exp \frac{V^D(y) - \overline{V}(y,B)}{\eta}}{\exp \frac{V^D(y) - \overline{V}(y,B)}{\eta} + \exp \frac{V^R(y,B) - \overline{V}(y,B)}{\eta}} \tag{C.68}$$

The corresponding expected value function is:

$$V(y, B) = \overline{V}(y, B) + \eta \ln\left[\exp \frac{V^D(y) - \overline{V}(y, B)}{\eta} + \exp \frac{V^R(y, B) - \overline{V}(y, B)}{\eta}\right] \tag{C.69}$$

C.4.5 PRO and Numerical Stability

The behavioral extension introducing PRO as in equation (11) requires computing perceived default probabilities using the distorted parameter $\tilde{\eta} = \theta\eta$. I compute the lender's perceived default probability using the same stabilization technique as in the default choice probabilities:

$$\tilde{V}(y,B) = \max\{V^D(y), V^R(y,B)\} \tag{C.70}$$

$$\tilde{P}(y,B) = \frac{\exp\frac{V^D(y)-\tilde{V}(y,B)}{\tilde{\eta}}}{\exp\frac{V^D(y)-\tilde{V}(y,B)}{\tilde{\eta}} + \exp\frac{V^R(y,B)-\tilde{V}(y,B)}{\tilde{\eta}}} \tag{C.71}$$

where $\tilde{V}(y,B)$ serves as the stabilizing maximum for the lender's computation. An additional safeguard prevents division by zero when both exponential terms become numerically zero, setting $\tilde{P}(y,B) = 0.5$ in such extreme cases.

C.4.6 Additional Numerical Safeguards

Beyond the log-sum-exp stabilization, my implementation includes several other numerical safeguards. For consumption bounds, I prevent evaluation of the utility function at non-positive consumption levels by setting $W(y,B,B') = -10^6$ whenever the implied consumption is non-positive, which effectively removes infeasible choices from consideration. I set the convergence tolerances to $\epsilon_V = \epsilon_q = 10^{-6}$, balancing computational accuracy with reasonable iteration counts. All my computations use double precision arithmetic to minimize accumulated rounding errors.

These numerical stability techniques, following the approach of (**?**), are essential for ensuring convergence of my value function iteration algorithm, particularly in the presence of the small taste shock parameters that characterize the discrete choice approach.

C.5 Pricing Operator: Contraction and Differentiability

Setup. For each $\theta \geq 1$, define the bounded linear operator $\mathcal{T}_\theta$ on the Banach space $\mathcal{X}$ of bounded functions $q : \mathcal{Y} \times \mathcal{B} \to \mathbb{R}$ (with sup norm) by

$$(\mathcal{T}_\theta q)(y,B') := \frac{1}{1+r}\,\mathbb{E}_{y'|y}\Big[(1-\tilde{P}_\theta(y',B'))\,(1-\delta)\,\mathbb{E}_{B''|y',B'}\big[q(y',B'')\big]\Big], \tag{C.72}$$

and the bounded function

$$F_\theta(y,B') := \frac{1}{1+r}\,\mathbb{E}_{y'|y}\Big[(1-\tilde{P}_\theta(y',B'))\,\kappa\Big]. \tag{C.73}$$

The equilibrium price q_θ solves the fixed point equation

$$q_\theta = F_\theta + \mathcal{T}_\theta q_\theta. \tag{C.74}$$

Contraction and resolvent. Since $0 \leq 1 - \tilde{P}_\theta \leq 1$ and conditional expectation is a contraction under the sup norm,

$$\|\mathscr{T}_\theta q\|_\infty \le \frac{1}{1+r}(1-\delta)\,\|q\|_\infty \quad \Rightarrow \quad \|\mathscr{T}_\theta\| \le \beta(1-\delta) < 1,\ \beta := \tfrac{1}{1+r}. \tag{C.75}$$

Thus $\mathscr{T}_\theta$ is a contraction. By the Banach fixed-point theorem, (C.74) has a unique solution and $(I-\mathscr{T}_\theta)^{-1} = \sum_{n\ge 0}\mathscr{T}_\theta^{\,n}$ exists as a bounded linear operator. Moreover, $\mathscr{T}_\theta$ is *positive* (maps nonnegative functions to nonnegative functions), hence the resolvent $(I-\mathscr{T}_\theta)^{-1}$ is positive.

Differentiability in θ. Suppose $\tilde{P}_\theta(y',B')$ is C^1 in θ and $\sup_{y',B',\theta}\left|\partial_\theta \tilde{P}_\theta(y',B')\right| < \infty$. Then $\theta \mapsto F_\theta \in \mathscr{X}$ and $\theta \mapsto \mathscr{T}_\theta \in \mathscr{L}(\mathscr{X})$ are C^1 with

$$\partial_\theta F_\theta(y,B') = -\frac{1}{1+r}\mathbb{E}_{y'|y}\big[\,(\partial_\theta \tilde{P}_\theta(y',B'))\,\kappa\,\big], \tag{C.76}$$

$$(\partial_\theta \mathscr{T}_\theta)q\,(y,B') = -\frac{1}{1+r}\mathbb{E}_{y'|y}\Big[\,(\partial_\theta \tilde{P}_\theta(y',B'))\,(1-\delta)\,\mathbb{E}_{B''|y',B'}[\,q(y',B'')\,]\Big]. \tag{C.77}$$

Differentiate (C.74) and use $(I-\mathscr{T}_\theta)^{-1}$ to obtain the unique derivative $q_\theta^{(1)} := \partial_\theta q_\theta \in \mathscr{X}$:

$$(I-\mathscr{T}_\theta)\,q_\theta^{(1)} = \partial_\theta F_\theta + (\partial_\theta \mathscr{T}_\theta)\,q_\theta\,. \tag{C.78}$$

Link to Equation (17). Combining (C.76)–(C.77) and defining

$$\Lambda(y',B') := \kappa + (1-\delta)\,\mathbb{E}_{B''|y',B'}\big[q_\theta(y',B'')\big], \tag{C.79}$$

I can rewrite (C.78) pointwise as

$$\big(I-\mathscr{T}_\theta\big)\frac{\partial q_\theta(\cdot)}{\partial\theta}(y,B') = -\frac{1}{1+r}\mathbb{E}_{y'|y}\Big[\big(\partial_\theta \tilde{P}_\theta(y',B')\big)\Lambda(y',B')\Big], \tag{C.80}$$

which coincides with Equation (17) used in the main text. Existence and uniqueness of $\partial_\theta q_\theta$ follow from invertibility of $I-\mathscr{T}_\theta$.

Rational Inattention (linear): Uniform Contraction and Convergence **Assumption RI.1 (Uniform contraction over $[1,\bar{\theta}]$).** There exists $\alpha \in (0,1)$ such that for every fixed $\theta \in [1,\bar{\theta}]$, the pricing operator $\mathscr{T}_\theta : \mathscr{Q} \to \mathscr{Q}$ satisfies $\|\mathscr{T}_\theta q - \mathscr{T}_\theta q'\|_\infty \le \alpha\|q-q'\|_\infty$ for all $q,q' \in \mathscr{Q}$.

Proposition RI.1 (State-dependent θ with uniform modulus). Let $\theta_{\mathrm{RI}} : \mathsf{S} \to [1,\bar{\theta}]$ be continuous (as in Lemma 1). Define $\mathscr{T}^{\mathrm{RI}}q(s) := (\mathscr{T}_{\theta_{\mathrm{RI}}(s)}q)(s)$. Under Assumption RI.1, $\mathscr{T}^{\mathrm{RI}}$ is a contraction on $(\mathscr{Q},\|\cdot\|_\infty)$ with modulus α. Hence there exists a unique fixed point $q^\star \in \mathscr{Q}$ solving $q^\star = \mathscr{T}^{\mathrm{RI}}q^\star$, and successive approximation converges at rate α.

Proof. For any $q,q' \in \mathscr{Q}$ and $s \in \mathsf{S}$,

$$\left|(\mathscr{T}^{\mathrm{RI}}q)(s) - (\mathscr{T}^{\mathrm{RI}}q')(s)\right| = \left|(\mathscr{T}_{\theta_{\mathrm{RI}}(s)}q)(s) - (\mathscr{T}_{\theta_{\mathrm{RI}}(s)}q')(s)\right| \le \alpha\,\|q-q'\|_\infty.$$

Taking sup over s delivers $\|\mathscr{T}^{\mathrm{RI}} q - \mathscr{T}^{\mathrm{RI}} q'\|_\infty \le \alpha \|q - q'\|_\infty$.