

# Default with Policy – Randomness Overestimation (PRO)

Pivoted Pricing, Deleveraging, and a Stability Illusion

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Chen Gao

October 15

National School of Development, Peking University

# Roadmap

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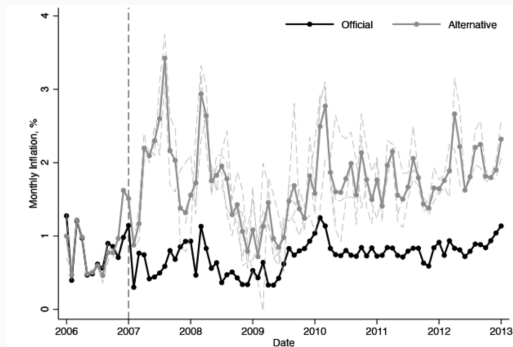
Conclusion

Motivation

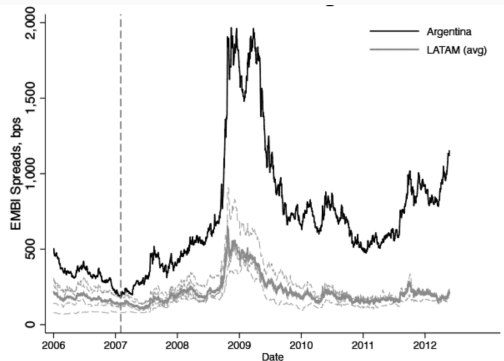
## A Persistent Puzzle

- Some sovereigns face persistently high spreads despite moderate debt and improving fundamentals.
- Event evidence (e.g., Argentina's inflation misreporting) shows spread decoupling beyond direct balance-sheet effects.
- Standard models struggle to match elevated average premia with lower volatility.
- This paper: a single pricing operator with a second-moment belief wedge (PRO) that *pivots* price/spread schedules.

# Argentina: Data Misreporting and Spread Decoupling



Official CPI vs. alternative measures



EMBI+ spreads: Argentina vs. LA peers

- Interpretation: reputational channel (type) + PRO (policy dispersion) likely both active.

Model

- Time:  $t = 0, 1, 2, \dots$ . Endowment  $\ln y' = (1 - \rho_y)\mu_y + \rho_y \ln y + \sigma_y \varepsilon'$ ,  $\varepsilon' \sim \mathcal{N}(0, 1)$ .
- Long-term debt with coupon  $\kappa$ , decay  $\delta$ , risk-free rate  $r$ .
- Default exclusion probability  $1 - \gamma$ ; output cost  $h(y) = y - \max\{0, \lambda_0 y + \lambda_1 y^2\}$ .
- Sovereign utility  $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$ , discount  $\beta$ .

## Discrete Choice: Default and Borrowing

Taste shocks (Gumbel) deliver closed-form aggregator and logit probabilities.

$$\begin{aligned} V(y, B) &= \eta \ln \left( e^{V^D(y)/\eta} + e^{V^R(y, B)/\eta} \right), \\ \mathbb{P}\{d=1 \mid y, B\} &= \frac{e^{V^D(y)/\eta}}{e^{V^D(y)/\eta} + e^{V^R(y, B)/\eta}} = \mathbb{L} \left( - \frac{\Delta V(y, B)}{\eta} \right), \\ V^R(y, B) &= \rho \ln \sum_{B' \in \mathcal{B}} e^{W(y, B, B')/\rho}, \quad \mathbb{P}\{B' \mid y, B\} = \frac{e^{W/\rho}}{\sum e^{W/\rho}}, \end{aligned}$$

where  $\Delta V \equiv V^R - V^D$ ,

$W(y, B, B') = u(y - \kappa B + [B' - (1-\delta)B]q(y, B')) + \beta \mathbb{E}V(y', B')$ .



PRO scales the taste-shock parameter in the *default logit* via a tail weight  $\theta \geq 1$ :

$$P_{\theta}(y, B') = \mathsf{L}\left(-\frac{\Delta V(y, B')}{\theta \eta}\right), \quad \mathsf{L}(z) = \frac{1}{1+e^{-z}}.$$

Pricing operator (unique fixed point):

$$(\mathcal{T}_{\theta} q)(B', y) = \frac{1}{1+r} \mathbb{E}_{y'|y} \left[ (1 - P_{\theta}(y', B')) (\kappa + (1-\delta) \mathbb{E}_{B''|y', B'} q(y', B'')) \right].$$

## Pivot Intuition

Compact schematic anchoring the single-crossing:

$$\begin{aligned} P_\theta(y, B') &= \mathbb{L}\left(-\frac{\Delta V(y, B')}{\theta\eta}\right), \quad \Delta V \equiv V^R - V^D, \\ \Rightarrow \quad \text{sign}(P_1 - P_\theta) &= -\text{sign}(\Delta V), \\ \Rightarrow \quad \text{sign}(q_\theta - q_1) &= \text{sign} \mathbb{E}[(P_1 - P_\theta)\Pi] = -\text{sign}(\Delta V), \quad \Pi > 0. \end{aligned}$$

Define the state-dependent threshold  $B^*(y) : \Delta V(y, B^*(y)) = 0$ . Then:

- $B' < B^*(y)$  (safe region,  $\Delta V > 0$ ):  $q_\theta < q_1$  (costlier borrowing).
- $B' > B^*(y)$  (near default,  $\Delta V < 0$ ):  $q_\theta > q_1$  (softened doom).

## Quantitative Results

- Preferences and endowment:  $\sigma = 2$ ,  $\beta = 0.9775$ ,  $\rho_y = 0.95$ ,  $\sigma_y = 0.005$ .
- Debt:  $\delta = 0.04$  (5y duration),  $\kappa = \delta + r$ ,  $r = 1\%/qtr$ ,  $\gamma = 0.125$ .
- Default cost:  $h(y) = y - \max\{0, \lambda_0 y + \lambda_1 y^2\}$  with  $(\lambda_0, \lambda_1) = (-0.48, 0.525)$ .
- Taste shocks small:  $\eta = 5 \times 10^{-4}$ ,  $\rho = 10^{-5}$ ; grids:  $N_y=201$ ,  $N_B=600$ .
- Scenarios:  $\theta \in \{1, 10, 100\}$ .

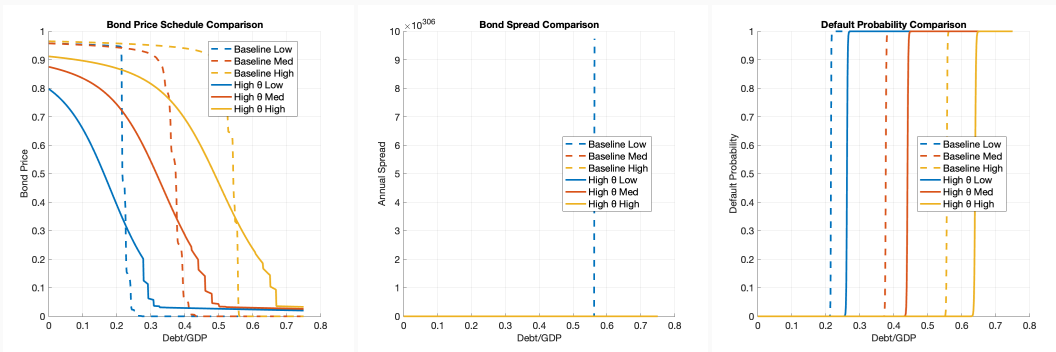
# Business Cycle Moments

Table 1: Simulation Moments Comparison

Moment	Baseline ( $\theta = 1$ )	Med $\theta$ ( $\theta = 10$ )	High $\theta$ ( $\theta = 100$ )
Mean Debt/GDP	7.646	5.520	2.695
Std Debt/GDP	1.301	0.864	0.754
Mean Spread (ann.)	2.028	2.762	4.153
Std Spread (ann.)	0.804	0.496	0.592
Std log C	3.580	3.586	3.464
Std log GDP	3.164	3.236	3.236
Corr(Sp,GDP)	-0.336	-0.802	-0.894
Corr(TB/GDP,GDP)	-0.003	-0.284	-0.259
Mean TB/GDP	0.268	0.320	0.177
Std TB/GDP	0.835	0.437	0.326
Corr(Debt/GDP,GDP)	0.697	0.858	0.839
Default Rate	3.947	0.000	0.000

- Higher avg spreads with deleveraging (pivot wedge dominates composition).
- Spreads more countercyclical; volatility of spreads/debt falls (illusion of stability).
- Consumption volatility nearly unchanged; risk insurance impaired.

# Price, Spread, and Default Risk



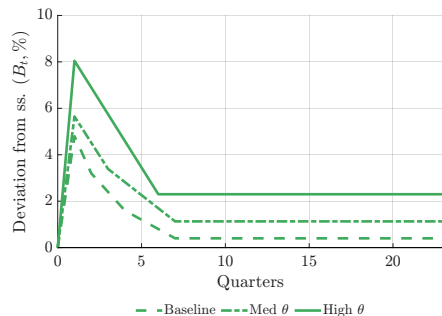
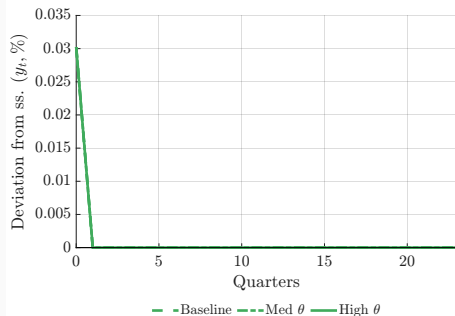
Bond prices

Spreads

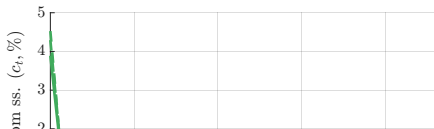
Default probabilities

Single-crossing pivot around  $B^*(y)$ ; PRO discounts safe region and softens near-doom.

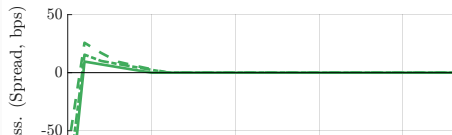
# Impulse Responses: Transitory Shock



Output

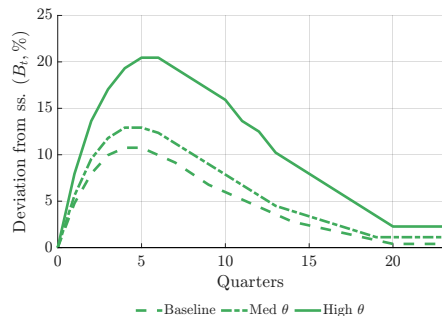
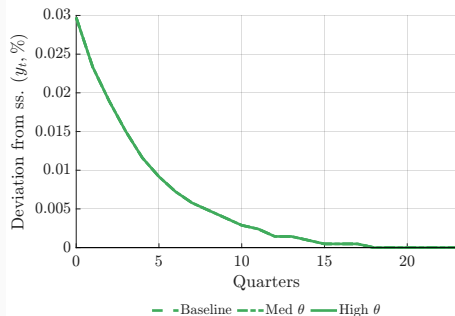


Debt

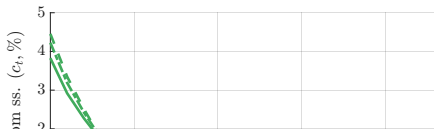




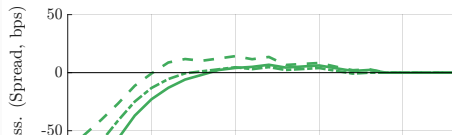
# Impulse Responses: Persistent Shock



Output



Debt



Microfoundation (RI)

## Rational Inattention: Tail Weight from Attention

- Lenders choose precisions  $(a_\mu, a_\sigma)$  at convex cost  $\Phi(a_\mu, a_\sigma)$ .
- Closed-form linear map:  $\theta_{\text{RI}}(y, B') = \min \left\{ 1 + \frac{\varphi^2}{\kappa_\sigma} \mathcal{S}(y, B'), \bar{\theta} \right\}$ .
- Pricing remains the same operator evaluated at  $\theta_{\text{RI}}(\cdot)$ ; comparative statics inherit.

$$q(B', y) = \mathcal{T}_{\theta_{\text{RI}}(y, B')}[q](B', y), \quad \mathcal{S} = \mathbb{E}[\partial U / \partial \theta] \geq 0.$$

## Empirical Hook: Misreporting $\Rightarrow$ Higher Dispersion Attention

- Degraded mean-information ( $a_\mu$ ) raises marginal value of dispersion info  $\mathcal{S}$ .
- $\uparrow \mathcal{S} \Rightarrow \uparrow a_\sigma \Rightarrow \uparrow \theta_{\text{RI}}$ : higher average spreads, steeper pivot, decoupling.

## Policy & Information

$$c_t + \kappa B_t + \tau_t = y_t + (B_{t+1} - (1-\delta)B_t) q_\theta(y_t, B_{t+1}),$$
$$\mathbb{E}_0 \sum_t \beta^t \tau_t = 0, \quad u'(\cdot) > 0, \quad u''(\cdot) < 0.$$

- Intertemporal trade price distorted by PRO persists in implementability; wedge creates deadweight loss.
- Result:  $W_\theta^R < W_1^R$  even with optimal transfers.

# Endogenous Beliefs and Transparency

Belief dynamics with negativity bias:

$$\theta_{t+1} = \lambda \theta_t + (1-\lambda) \hat{\theta}(\{d_s\}), \quad \xi(y, B) = \max \left\{ 0, \frac{P_1 - P_{\theta_t}}{P_1} \right\}, \quad \text{defaults move beliefs more.}$$

Effective transparency:

$$\theta_{\text{eff}}(\alpha, \theta) = \alpha \cdot 1 + (1-\alpha) \cdot \theta, \quad \alpha^* : \frac{d}{d\alpha} W(\alpha) = \gamma \alpha.$$

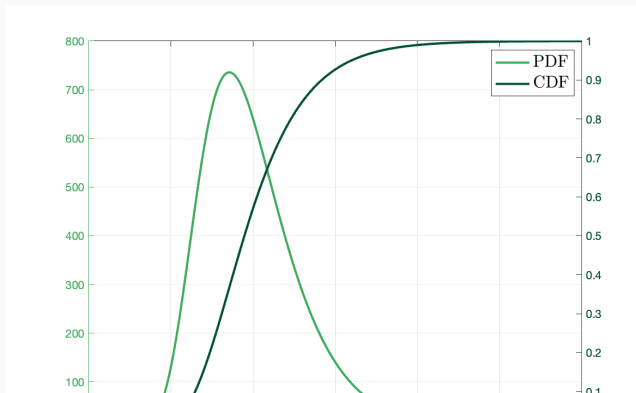
- Persistent PRO in invariant beliefs; optimal transparency rises with PRO severity.

Computation



# Computation and Stability

- Value and price iteration on  $(N_y=201, N_B=600)$  grid; OpenMP parallel.
- Stabilized log-sum-exp for borrowing/default logits; infeasible-consumption guard.
- Convergence tolerances  $10^{-6}$ ; long simulation for moments and IRFs.



## Conclusion

- Single operator with PRO produces a *pivot* in price/spread schedules.
- Sovereigns deleverage yet face higher average spreads; volatility falls (stability illusion).
- RI microfoundation endogenizes the tail tilt; policy/info extensions clarify limits and levers.
- Event hooks (Argentina) align with pivot, threshold, and decoupling predictions.

Backup

## Operator View (Sketch)

- $\mathcal{T}_\theta$  is positive and order-preserving; fixed point unique under slope condition.
- Fixed-point differentiation signs  $\partial_\theta q_\theta$ ; monotone propagation yields pivot.