Sovereign Default with Bounded Rationality

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Abstract

Recent sovereign defaults are characterized by soaring interest rate spreads and deep recessions. This paper develops a small open economy model to study default risk and its interaction with output and foreign debt. I introduce a novel mechanism where the market is populated by both fully rational and boundedly rational lenders. The latter form expectations based on a simplified heuristic. I show that this heterogeneity of beliefs generates a discontinuous bond price schedule, creating a "crisis threshold" that can trigger sudden interest rate spikes. When calibrated to the Argentine economy, the model not only matches the high average sovereign spread observed in the data a challenge for the standard model but also endogenously generates excess volatility in interest rates and a more fragile financial environment characterized by higher equilibrium debt and default rates.

1 Introduction

Emerging market economies often exhibit more volatile business cycles than their developed counterparts and experience financial crises with painful frequency. These crises are typically associated with a sudden loss of access to international credit, soaring interest rate spreads, and deep contractions in output and consumption. The 2001 Argentine default serves as a stark example: the crisis was accompanied by a collapse in economic activity and a dramatic spike in sovereign risk premia,

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highlighting the tight linkage between sovereign default risk and macroeconomic outcomes. Understanding the mechanisms that drive these dynamics remains a priority for research in international macroeconomics.

The canonical quantitative model of sovereign default, pioneered by (Arellano, 2008), provides a powerful framework for analyzing these issues. In this class of models, default is an endogenous decision, and the risk premium is countercyclical, consistent with empirical evidence. However, a well-documented limitation of the standard model under risk-neutral lenders is its inability to quantitatively generate both the high average level and the high volatility of sovereign spreads observed in the data without resorting to counterfactually high levels of risk aversion or default costs.

In this paper, I propose and analyze a novel, behaviorally-grounded mechanism to bridge this gap. I depart from the standard full rationality assumption and develop a model where the credit market is populated by a mix of two types of lenders: a fraction λ of lenders are fully rational and form expectations over the entire distribution of future shocks, while the remaining fraction are "boundedly rational." These boundedly rational agents use a simple but plausible heuristic, forming expectations based only on the conditional mean of future output.

The introduction of this heterogeneity in lender beliefs has profound consequences for the functioning of the credit market. The central theoretical result of this paper is that the presence of boundedly rational lenders replaces the smooth bond price schedule of the standard model with one that features a sharp, endogenous discontinuity. I demonstrate that there exists a critical debt threshold, which I term a "crisis threshold," at which the market's perception of risk jumps, causing the bond price to drop discontinuously. This mechanism provides a clear theoretical foundation for sudden stops and interest rate spikes. I further show that this threshold is state-dependent, allowing the government to sustain more debt during economic expansions, which explains why crises are more likely to be ignited during recessions.

I then evaluate the quantitative implications of this mechanism by calibrating the model to the Argentine economy. The results are striking. By calibrating the fraction of rational lenders, λ, to match Argentina's high average interest rate spread, the model simultaneously generates a massive increase in spread volatility, bringing it much closer to the data. The analysis further reveals that the government, responding to the altered credit terms, optimally adopts a more aggressive borrowing policy. This leads to a higher equilibrium debt-to-GDP ratio and a significantly higher default probability, creating a state of endogenous financial fragility. This finding suggests that the high country risk premia observed in emerging markets may be intrinsically linked to a financial environment that encourages higher indebtedness and, consequently, a greater frequency of crises.

The remainder of the paper is organized as follows. Section 2 presents the model environment. Section 3 establishes the main theoretical results. Section 4 details the calibration and discusses the quantitative findings. Section 5 concludes.

2 Model

This section outlines the baseline model, which extends the framework of (Arellano, 2008). The decision maker is a government of a small open economy that borrows from risk-neutral foreign creditors.

Output A small open economy is endowed with an exogenous stochastically fluctuating potential output stream $\{y_t\}_{t=0}^{\infty}$. The logarithm of potential output, denoted $\ln(y_t)$, follows a first-order autoregressive (AR(1)) process:

$$\ln(y_{t+1}) = \rho \ln(y_t) + \epsilon_{t+1}, \quad \text{where } \epsilon_{t+1} \sim N(0, \sigma_\epsilon^2) \tag{1}$$

Here, y_t is the level of potential output, ρ is the persistence parameter (e.g., $|\rho| < 1$), and ϵ_{t+1} is an i.i.d. normal shock with mean zero and variance σ_ϵ^2. The process for $\ln(y_t)$ implies a transition probability kernel for the level of output, $p(y, y')$. Potential output y_t is realized only in periods in which the government honors its sovereign debt. The output good can be traded or consumed. Households within the country are identical and rank stochastic consumption streams according to

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

where $u(\cdot)$ is an increasing and strictly concave utility function. Household consumption $\{c_t\}_{t=0}^{\infty}$ is affected by the government's international borrowing and lending decisions. Because households are averse to consumption fluctuations, the government will try to smooth consumption by borrowing from (and lending to) foreign creditors.

Asset Market The only credit instrument available to the government is a one-period bond traded in international credit markets. The bond market has the following features: first, the bond matures in one period and is not state contingent. Second, a purchase of bonds with total face value B' (a claim to B' units of the consumption good next period) costs $q(B',y)B'$ today, where $q(B',y)$ is the price of a bond promising one unit of face value next period, given current state y and chosen next period assets B'. Third, if the government issues bonds (i.e., borrows, $B'<0$), it sells a promise to repay $-B'$ units next period (note that $-B'$ is a positive quantity). For this, it receives $q(B',y)(-B')$ units of the good today.

Earnings on the government portfolio are distributed (or, if negative, taxed) lump sum to households. When the government is not excluded from financial markets, the one-period national budget constraint is

$$c_t = y_t + B_t - q\left(B_{t+1}, y_t\right) B_{t+1}. \tag{2}$$

To rule out Ponzi schemes, we also require that $B_t \geq -Z$ in every period where Z is an exogenous borrowing upper bound. Z is chosen to be sufficiently large that the constraint never binds in equilibrium.

Financial Markets There are risk-neutral foreign creditors who know the domestic output stochastic process $\{y_t\}_{t=0}^{\infty}$and observe $\{y_s\}_{s=0}^{t}$ at time t. The creditors can borrow or lend without limit in an international credit market at a constant risk-free rate r. The creditors receive full payment if the government chooses to pay and receive 0 if the government defaults on its one-period debt due.

If the government is expected to default on obligations due at $t+1$ with probability $\delta(B',y_t)$ (where B' is the chosen bond position for $t+1$ and y_t is the output at time t), the expected value of a promise to pay one unit of consumption next period is $1-\delta(B',y_t)$. Therefore, the bond price $q(B',y)$ for a promise to pay one unit of consumption next period is given by its discounted expected value:

$$q\left(B', y\right) = \frac{1-\delta\left(B', y\right)}{1+r}. \tag{3}$$

where $\delta(B',y)$ is the probability of default next period, conditional on choosing B' today when current output is y. The key is to determine the default probability $\delta(B',y)$.

Decision on default At each time t, the government chooses between defaulting and meeting its current obligations and purchasing or selling an optimal quantity of one-period sovereign debt. Defaulting means declining to pay all of its current obligations. Default triggers two consequences: first, the output is decreased from y_t to $h\left(y_t\right)$, where $0 \leq h\left(y_t\right) \leq y_t$. For instance, $h(y_t)$ could take the form $\min(\psi\bar{y}, y_t)$, where ψ is a parameter and $\bar{y}$ represents a reference output level (such as the long-run average output).

Output returns to normal only after the sovereign regains access to international credit markets. Second, the sovereign loses access to foreign credit markets. While in a state of default, the economy regains access to foreign credit in each subsequent period with probability θ.

2.1 The Government's Problem

Default Let $V^D(y)$ be the value function of government in default, the government has only one state variable y_t and facing the recursion:

$$V^D(y) = u(h(y)) + \beta \mathbb{E}_{y'}\left[\theta V(0, y') + (1-\theta) V^D(y') \mid y\right]. \tag{4}$$

In the continuation value, the government either regain credit market access with 0 debt with probability θ, or stay in default and gain $V^D(y')$ next period with probability $1-\theta$.

Repay If the government stays in credit market at time t and chooses to repay, then it choose a control variable B_{t+1} to maximize the value of repayment. The Bellman equation is

$$\begin{aligned} V^R(B, y) &= \max_{B'}\left\{u(c) + \beta \mathbb{E}_{y'}\left[V(B', y') \mid y\right]\right\} \\ \text{s.t.} \quad & c = y + B - q(B', y) B' \\ & B \geq -Z \end{aligned}, \tag{5}$$

where $V(B', y')$ is the optimal value of the government's problem when facing the choice of whether to repay or default at the beginning of the next period with state (B', y'). The government chooses to default if and only if $V^D(y) > V^R(B, y)$ and thus $V(B, y)$ is given by the maximum of the two values:

$$V(B, y) = \max\left\{V^R(B, y), V^D(y)\right\}. \tag{6}$$

Default probability with λ- rationality Given the value function, global lenders can now evaluate the probability of default and therefore give the price function $q(B', y)$. Unlike Arellano (2008) where all lenders are rational in comparing the next period value, I introduce a parameter $\lambda \in [0, 1]$ to denote the fraction of "rational" lenders. For the rational lenders, they predict the default decision of all possible next period output level y' and then formulate expectation of default next period given B' as:

$$\begin{aligned}\delta_r\left(B',y\right) &= \mathbb{E}_{y'}\left[\mathbb{I}_{\{V^D(y')>V^R(B',y')\}}|y\right] \\ &= \int \mathbb{I}_{\{V^D(y')>V^R(B',y')\}}\, p\left(y,y'\right)\mathrm{d}y' \end{aligned}, \tag{7}$$

where $\mathbb{I}_{\{A\}}$ is the indicator function of condition A.

However, for the $(1-\lambda)$ fraction of "boundedly rational" lenders, they evaluate the default decision based on a point expectation of next period's output, conditional on current output y. Specifically, given that $\ln(y)$ follows the AR(1) process in (1), these lenders form their expectation of the *level* of next period's output, y', as $\hat{y}' = \exp(E[\ln(y')|\ln(y)]) = \exp(\rho\ln(y))$. If y denotes the current level of output, this forecast can be written as y^ρ. Thus, for a given next period bond position B', these lenders formulate the probability of default as:

$$\begin{aligned}\delta_{ir}\left(B',y\right) &= \mathbb{I}_{\{V^D(\mathbb{E}[y'|y])>V^R(B',\mathbb{E}[y'|y])\}} \\ &= \mathbb{I}_{\{V^D(y^\rho)>V^R(B',y^\rho)\}}\end{aligned}. \tag{8}$$

Given the fraction of rational lenders $\lambda \in [0,1]$, the total probability of default next period perceived by the global lenders is:

$$\begin{aligned}\delta\left(B',y;\lambda\right) &= \lambda\delta_r\left(B',y\right) + (1-\lambda)\,\delta_{ir}\left(B',y\right) \\ &= \lambda\int \mathbb{I}_{\{V^D(y')>V^R(B',y')\}}\, p\left(y,y'\right)\mathrm{d}y' + (1-\lambda)\,\mathbb{I}_{\{V^D(y^\rho)>V^R(B',y^\rho)\}}\end{aligned}. \tag{9}$$

Given zero profits for foreign creditors in equilibrium, we can combine (3) and (9) to pin down the bond price function.

2.2 Timeline

Timing of Events within Period t The sequence of events within each period t unfolds as follows:

1. The government begins the period with an inherited net asset position B_t from period $t-1$. The current period's output level y_t is realized.

2. Given the state (B_t, y_t), the government chooses whether to **default** on or **repay** its obligations associated with B_t.

3. If the Government Defaults:

   (a) Current output is reduced to $h(y_t)$, and consumption is $c_t = h(y_t)$.

   (b) The government is excluded from international credit markets.

   (c) Transition to period $t+1$: With probability θ, market access is regained, starting period $t+1$ with $B_{t+1} = 0$. With probability $1-\theta$, the government remains in default, also starting period $t+1$ with $B_{t+1} = 0$ but still excluded from borrowing.

4. If the Government Repays:

   (a) Obligations related to B_t are honored.

   (b) The government faces the bond price schedule $q(B', y_t)$. This price is determined by risk-neutral creditors as in (3) where the market-perceived default probability $\delta(B', y_t)$ is formed according to (9).

   (c) The government chooses an optimal new net asset position B_{t+1} subject to $B_{t+1} \geq -Z$ to maximize its continuation value, $V_R(B_t, y_t)$.

   (d) Consumption c_t is determined by the budget constraint (2).

   (e) The government carries the chosen B_{t+1} into period $t+1$.

2.3 Equilibrium

I seek for Markov perfect equilibrium for ease of computation. The equilibrium can be described as follows:

Definition 1. A Markov perfect equilibrium is: i) a pricing function $q\left(B', y\right)$, ii) a triple of value functions $\left(V^R\left(B, y\right), V^D\left(y\right), V\left(B, y\right)\right)$, iii) a default decision as a function of the state $\left(B, y\right)$ and iv) an asset accumulation rule that conditional on choosing repayment $B'\left(B, y\right)$ such that:

1. The 3 Bellman equations (4), (5) and (6) are satisfied,

2. given the price function $q\left(B', y\right)$, the decision rule and accumulation rule attain the optimal value function $V\left(B, y\right)$, and

3. The price function $q\left(B', y\right)$ satisfies (3).

2.4 Basic Theoretical Results

Before proceeding to the quantitative analysis, I briefly establish that several key theoretical properties from the standard sovereign default model of (Arellano, 2008) continue to hold in my framework with boundedly rational lenders. The primary purpose of this section is to verify the robustness of the model's basic mechanics. First, I define the default set, $\mathcal{D}(B)$, as the set of output realizations y for which default is the optimal policy given an initial asset level B. That is,

$$\mathcal{D}(B) = \{y \mid V^D(y) > V^R(B, y)\}.$$

The following propositions characterize the equilibrium default policy.

Proposition 1. *The incentive to default is non-increasing with the government's net asset position. For any two asset levels B_1 and B_2 such that $B_1 \le B_2$, if default is optimal with asset level B_2 at some output level y, then default is also optimal with asset level B_1 at the same output level y. Formally, $\mathcal{D}(B_2) \subseteq \mathcal{D}(B_1)$.*

Proof. See Appendix A.1. □

The proof is standard and follows from the fact that $V^R(B, y)$ is strictly increasing in B while $V^D(y)$ is independent of B. The introduction of λ-rationality affects the level of the bond price schedule $q(\cdot)$, but not the monotonicity of V^R with respect to current assets B. Proposition 1 establishes that default becomes more attractive as the government's balance sheet deteriorates. The next proposition shows that once debt reaches a level where default becomes a possibility, the government faces an endogenous borrowing constraint.

Proposition 2. *If, for some asset level B, the default set is non-empty, i.e., $\mathcal{D}(B) \neq \emptyset$, then there are no contracts $\{q(B', y), B'\}$ chosen in equilibrium such that the economy can experience a capital inflow. That is, any equilibrium choice must satisfy $B - q(B', y; \lambda)B' \le 0$.*

Proof. See Appendix A.2. □

The proof follows the logic in (Arellano, 2008), demonstrating that any equilibrium price offered to a risky sovereign must be inconsistent with the condition required for a net capital inflow. Our λ-rationality mechanism alters the precise equilibrium price but does not invalidate this fundamental no-free-lunch condition. Finally, we establish that default is a response to adverse economic conditions.

Proposition 3. *The incentive to default is stronger for lower levels of current endowment. For any given asset level B, if it is optimal to default at an output level y_2, it is also optimal to default at any output level $y_1 \leq y_2$. Formally, if $y_2 \in \mathscr{D}(B)$, then $y_1 \in \mathscr{D}(B)$.*

Proof. See Appendix A.3. □

This result holds because the marginal value of income is higher when the government maintains market access than when it is in default. Access to credit markets allows for better intertemporal smoothing, making the continuation value under repayment more sensitive to positive income shocks. Taken together, these propositions confirm that my model with boundedly rational lenders preserves the core theoretical structure of the canonical sovereign default model. Default remains a state-contingent decision that occurs when the country is highly indebted and experiences a negative output shock. This verification of the model's basic properties is crucial, as it provides a stable foundation upon which I can analyze the novel effects of my central mechanism.

3 Theoretical Analysis

Having established that the standard mechanics are intact, the analysis can now move beyond them. I establish the main theoretical contribution of this paper in this section, demonstrating how this heterogeneity in lender beliefs fundamentally reshapes the credit market by introducing sharp, endogenous discontinuities into the bond price schedule.

Price Drop The theoretical contribution of this paper stems from a novel non-linearity introduced by the assumption of boundedly rational lenders. I establish the model's main theoretical result: when a fraction of lenders form expectations irrationally ($\lambda < 1$), the otherwise smooth bond price schedule of the standard model is replaced by one that exhibits a sharp, endogenous discontinuity. This result is formalized in the following theorem.

Theorem 1. *With a fraction $(1-\lambda) > 0$ of boundedly rational lenders, the equilibrium bond price schedule $q(B', y; \lambda)$ exhibits a unique discontinuity at a critical debt threshold $\tilde{B}'(y)$. Specifically, the following properties hold:*

1. *The price function drops discontinuously at the threshold:*

$$\lim_{B' \to \tilde{B}'(y)^+} q(B', y; \lambda) - \lim_{B' \to \tilde{B}'(y)^-} q(B', y; \lambda) = \frac{1-\lambda}{1+r}$$

2. *For* $B' \in (\tilde{B}'(y), 0]$, *the price is **weakly higher** than the full rationality benchmark:* $q(B', y; \lambda) \geq q(B', y; 1)$.

3. *For* $B' < \tilde{B}'(y)$, *the price is **weakly lower** than the full rationality benchmark:* $q(B', y; \lambda) \leq q(B', y; 1)$.

Proof. See Appendix A.4. □

The economic intuition behind Theorem 1 lies in the abrupt shift in the beliefs of boundedly rational lenders. Their assessment of default risk (8), acts as a step function. For low levels of debt, i.e., $B' > \tilde{B}'(y)$), the value of repayment at the expected future output, $V^R(B', y^\rho)$, is high. Consequently, these lenders perceive zero default risk ($\delta_{ir} = 0$). Their presence dilutes the concerns of rational lenders over low-probability tail events, leading to an aggregate market perception of lower risk and thus a higher bond price compared to the fully rational benchmark.

This market optimism, however, is fragile. From Proposition 1, $V^R(B', y^\rho)$ is a decreasing function of the debt level. As the government proposes to take on more debt, it eventually crosses the critical threshold $\tilde{B}'(y)$ where the value of repayment falls below the value of default at the expected output level. At this precise point, the boundedly rational lenders' risk assessment flips immediately from 0 to 1. This causes a discrete upward jump in the aggregate default probability, which in turn triggers a discontinuous drop in the bond price and therefore leads to a price drop. For all debt levels beyond this threshold, the market becomes systematically more pessimistic than the fully rational benchmark, as the extreme pessimism of the boundedly rational agents (who now see default as certain) weighs down the average price. This discontinuity can be interpreted as a crisis threshold in the credit market, driven entirely by the heterogeneity of lender expectations.

Equilibrium Interest Rate This price discontinuity has a direct consequence on the interest rates the sovereign faces in equilibrium. I define the equilibrium interest rate, $r^c(B, y)$, as the rate implied by the government's optimal bond choice at state (B, y), which is $B'(B, y)$. Formally, it is given by:

$$r^c(B, y) = \frac{1}{q(B'(B, y), y; \lambda)} - 1 \tag{10}$$

Unlike the bond price *schedule* $q(B', y)$, which is a function of a potential choice B', the equilibrium interest rate is the single rate that is realized after the government has made its optimal decision. The following corollary describes its properties.

Corollary 1. *The presence of boundedly rational lenders ($\lambda < 1$) can lead to equilibrium interest rates that are significantly higher than those under full rationality ($\lambda = 1$). Specifically, for states (B, y) where the optimal new debt level $B'(B, y)$ is beyond the critical threshold ($B' < \tilde{B}'(y)$), the resulting equilibrium interest rate will be strictly higher than in the fully rational benchmark.*

Proof. See Appendix A.5. □

The intuition for Corollary 1 follows directly from Theorem 1. The equilibrium interest rate is determined not by the entire price schedule, but by the price of the specific bond $B'(B, y)$ that the government optimally chooses at state (B, y). The government's choice, in turn, depends on the trade-offs presented by the schedule. If the government is not heavily indebted and faces a good income shock, it may choose a safe asset position $B' > \tilde{B}'(y)$. In this case, it benefits from the "market optimism" region with a *lower* interest rate. However, in adverse states (low y and/or high initial debt B), the government has a strong incentive to borrow for consumption smoothing. This need may be so strong that it is optimal to choose a new debt level $B' < \tilde{B}'(y)$, effectively accepting the discontinuous price drop. In this pessimistic region, Theorem 1 mechanically leads to a higher equilibrium interest rate. Because the price drop can be substantial, my model predicts that bounded rationality can generate the kind of sharp interest rate spikes often observed during sovereign debt crises, a feature that is difficult to produce in the standard, smooth model.

Threshold The existence of the critical threshold $\tilde{B}'(y)$ is central to the model's dynamics. A natural question that follows is how this crisis threshold behaves with respect to output. The next proposition establishes that the threshold is state-dependent, showing that the government can sustain a higher level of debt during economic expansions before triggering the pessimistic shift in lender beliefs. This provides a clear theoretical channel for why crises are more likely to be triggered during recessions.

Proposition 4. *The critical debt threshold $\tilde{B}'(y)$ is a decreasing function of current output y. That is,* $\frac{d\tilde{B}'(y)}{dy} < 0$.

Proof. See Appendix A.6. □

The critical threshold $\tilde{B}'(y)$ is determined by the beliefs of the boundedly rational lenders, which are based on the expected future output y^{ρ}. Due to the persistence of the income process ($\rho > 0$), a higher current output y leads to a more optimistic forecast for future output. A better future economic outlook increases the value of both repayment and default, but as established in Proposition 3, it increases the value of maintaining market access (V^R) more significantly. Consequently, when today's outlook is good, the government can accumulate a larger amount of debt (a more negative $\tilde{B}'(y)$) before the boundedly rational lenders perceive the situation as unsustainable. In contrast, a low current income y generates pessimism about the future, causing the crisis threshold to be triggered at much lower levels of debt.

4 Quantative Analysis

In this section, I solve the model numerically to visualize and analyze the main theoretical predictions. The analysis highlights the quantitative impact of bounded rationality by comparing the benchmark case with fully rational lenders ($\lambda = 1$) to the case where half of the lenders are boundedly rational ($\lambda = 0.5$). The model is parameterized according to the benchmark calibration discussed in Section 4.1.

4.1 Functional Forms and Parameterization

To analyze the model's quantitative implications, I adopt the functional forms and benchmark parameterization directly from (Arellano, 2008). This approach allows me to isolate the effects of my novel bounded-rationality mechanism by grounding the analysis in a well-established quantitative framework calibrated to a typical emerging market economy (Argentina).

The utility function is of the standard Constant Relative Risk Aversion form,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

The output cost of default is specified as a threshold function, $h(y) = \min(\hat{y}, y)$, which makes default costs disproportionately larger during economic expansions. The parameter values are summarized in Table 1. These values constitute the full rationality benchmark in my model, corresponding to the case where $\lambda = 1$. I use $n_y = 201$ and $n_B = 401$ in the model and value function iteration that is standard in this literature.

Table 1: Benchmark Parameterization

Parameter	Symbol	Value
Risk Aversion	γ	2.0
Discount Factor	β	0.953
Risk-Free Rate (Quarterly)	r	0.017
Output Persistence	ρ	0.945
Output Shock Std. Dev.	η	0.025
Re-entry Probability	θ	0.282
Output Cost Threshold	$\hat{y}$	$0.969 \cdot \mathbb{E}[y]$

Note: All parameter values are adopted from the benchmark calibration in (Arellano, 2008), which was targeted to match business cycle moments for Argentina.

4.2 Main Results

The Discontinuous Price Schedule My main theoretical result, Theorem 1, posits that bounded rationality induces a sharp, endogenous discontinuity in the bond price schedule. Figure 1 provides a visualization of this phenomenon. The figure plots the bond price $q(B', y)$ as a function of the next period's asset choice B' for both high and low current income states.

As predicted, the price schedules for the boundedly rational economy (blue and orange lines, $\lambda = 0.5$) are fundamentally different from their smooth counterparts in the fully rational economy (green and purple lines, $\lambda = 1.0$). Consistent with Theorem 1, for low levels of debt (i.e., B' to the right of the drop), prices are higher under bounded rationality, reflecting the market optimism of lenders who ignore tail risks. At a critical debt threshold, $\tilde{B}'(y)$, the price drops discontinuously. For all debt levels beyond this threshold, prices are strictly lower than in the rational benchmark, reflecting the market pessimism.

Furthermore, the figure provides visual confirmation of Proposition 4. The critical threshold at which the price drops is state-dependent. The drop for the low-income state (blue line) occurs at a higher asset level (less debt) than for the high-income state (orange line). This illustrates that the government can sustain more debt in good times before *triggering the pessimistic shift* in lender beliefs, consistent with the fact that crises are more likely to be ignited during recessions.

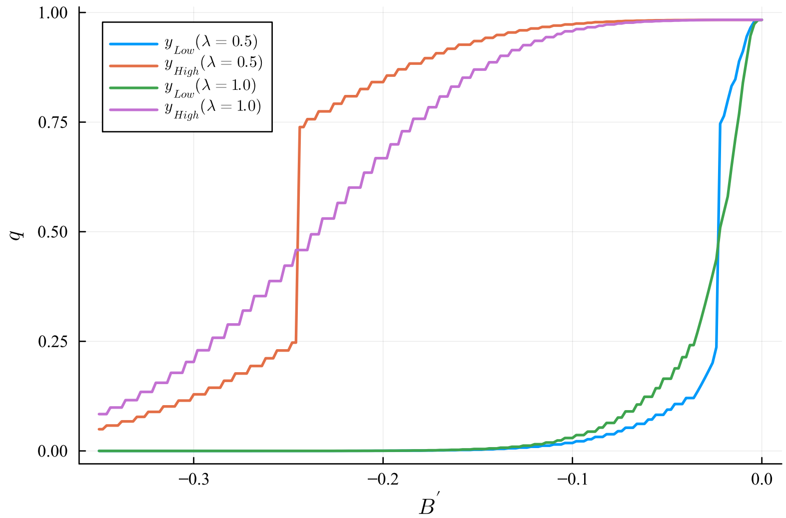


Figure 1: Bond Price Schedule $q(B', y)$

Note: The figure plots the price q as a function of next period's asset choice B'. The comparison is between the full rationality benchmark ($\lambda = 1.0$) and the bounded rationality model ($\lambda = 0.5$) for high (y_{High}) and low (y_{Low}) income states. The discontinuity predicted by Theorem 1 is clearly visible for the $\lambda = 0.5$ case.

Equilibrium Interest Rate Spikes The discontinuous price schedule has powerful implications for the interest rate the sovereign faces in equilibrium. Figure 2 plots the realized equilibrium interest rate, $r^c(B, y)$, which corresponds to the price of the optimally chosen bond $B'(B, y)$ at each state (B, y). Figure 2 provides quantitative support for Corollary 1. For the high-income state, default risk is minimal, and the government's borrowing needs do not compel it to cross the discontinuity. As a result, the equilibrium interest rates under bounded and full rationality (orange and purple lines) are nearly identical and low. In sharp contrast, for the low-income state (blue line), once the government's initial debt B is sufficiently high, its optimal policy $B'(B, y)$ falls into the "pessimistic" region of the price schedule. Consequently, the government is forced to pay a punitively high interest rate, far exceeding the rate paid in the fully rational benchmark (green line). This result highlights that my model, through the bounded rationality mechanism, can endogenously generate the kind of sudden interest rate spikes characteristic of sovereign debt crises, a feature that is difficult to produce in standard models with smooth price functions.

Equilibrium Borrowing and Financial Fragility I now turn to the analysis of the government's equilibrium asset policy, $B'(B, y)$, which reveals the ultimate behavioral consequence of the bounded ra-

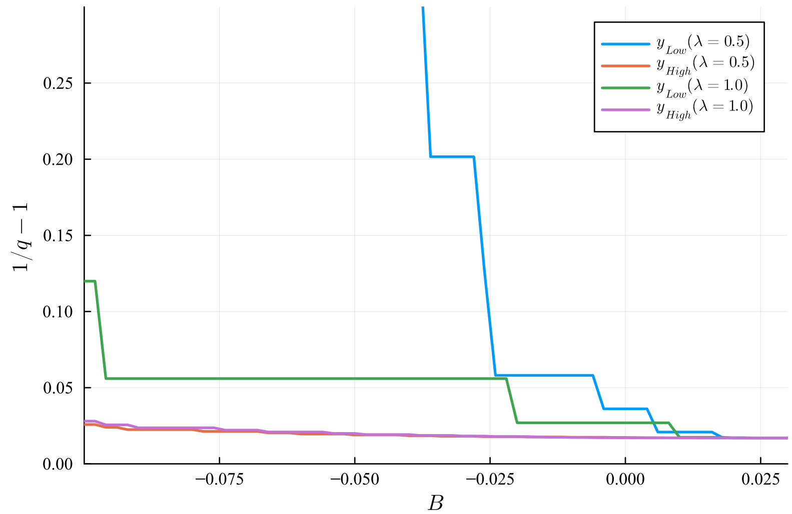


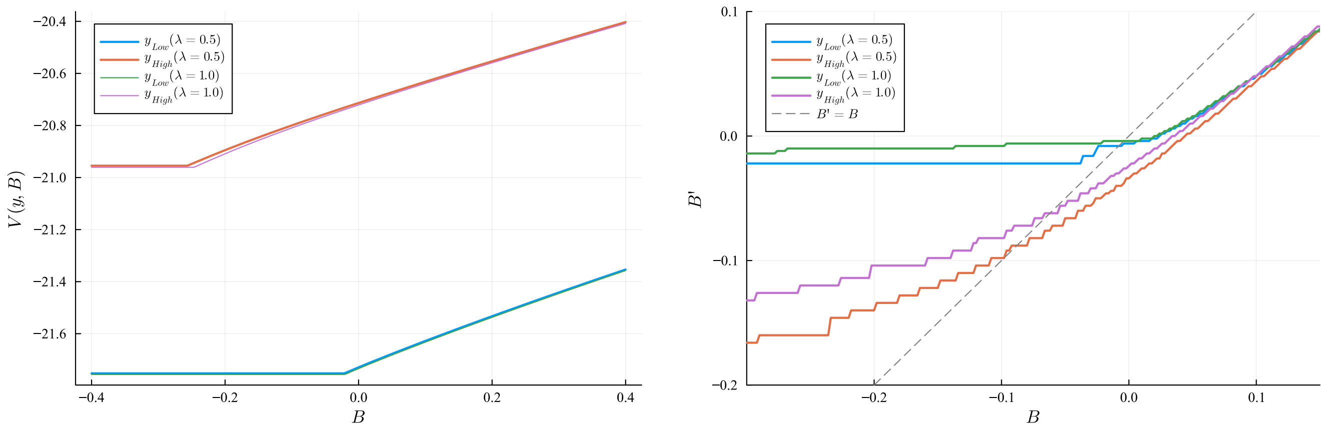
Figure 2: Equilibrium Interest Rate $r^c(B, y) = 1/q(B'(B, y), y) - 1$

Note: The figure plots the realized equilibrium interest rate as a function of the current asset position B. Each point reflects the rate corresponding to the government's optimal choice $B'(B, y)$ at that state. The comparison is between the full rationality benchmark ($\lambda = 1.0$) and the bounded rationality model ($\lambda = 0.5$).

tionality I introduce. While one might intuitively suspect that the threat of a price cliff would induce more cautious behavior, I find that the *opposite* is true. The presence of boundedly rational lenders incentivizes the government to adopt a more aggressive borrowing strategy, systematically increasing the economy's indebtedness.

This result is illustrated in Figure 3. The left panel shows that the ex-ante value function, $V(B, y)$, is remarkably similar across the two rationality regimes. The right panel plots the resulting optimal savings policy, $B'(B, y)$. For nearly all initial asset levels B, the government chooses a lower next-period asset position under bounded rationality.

The explanation for this counterintuitive result lies in the two opposing effects that bounded rationality has on the government's borrowing decision. On one hand, a "value effect" could encourage more saving: the prospect of better future borrowing terms might increase the continuation value, raising the incentive to save today. However, the left panel of Figure 3a reveals that this effect is quantitatively negligible, as the overall value functions are almost identical. On the other hand, a "price effect" encourages more borrowing: Theorem 1 shows that for "safe" levels of debt ($B' > \tilde{B}'(y)$), the interest rate is lower when $\lambda < 1$. With the precautionary saving motive (the value effect) effectively neutralized, the government's optimal response is to aggressively exploit the immediate "bargain" of-



(a) Value Function $V(B, y)$ (b) Savings Function $B'(B, y)$

Figure 3: Value and Savings Functions under Bounded Rationality

Note: The left panel plots the value function $V(B, y)$. The right panel plots the optimal asset policy function $B'(B, y)$. Both panels compare the full rationality benchmark ($\lambda = 1.0$) with the bounded rationality model ($\lambda = 0.5$) for high (y_{High}) and low (y_{Low}) income states.

fered by the market's optimism. It, therefore, takes on more debt than it would in the fully rational world.

In conclusion, the introduction of boundedly rational lenders creates a form of *moral hazard*. The government is induced to maintain a higher level of debt, pushing the economy closer to the crisis threshold $\tilde{B}'(y)$. This makes the economy endogenously more fragile and susceptible to the very type of sudden funding crises that the beliefs of these boundedly rational agents create.

4.3 Simulation and Business Cycle Statistics

To evaluate the model's quantitative performance, I compare its simulated business cycle moments to empirical data from Argentina. The main analysis contrasts two versions of my model: the full rationality benchmark ($\lambda = 1.0$) and the calibrated model with boundedly rational lenders ($\lambda \approx 0.518$), where this value of λ is chosen to match the average interest rate spread observed in the data.

To generate the model's statistics, I first perform a long-run simulation of the calibrated economy for 50,000 quarters, discarding an initial burn-in period. Following the methodology of Bornstein (2020), I then identify all default episodes from the simulation. For each of the first 100 default events (or fewer if not available), I create a data window consisting of the 74 quarters preceding the event.

Key business cycle statistics are calculated for each window, and the final reported moments in Table 2 are the average of these statistics across all sampled windows. Crucially, following this methodology, statistics for output and consumption are computed on their logarithmic values without prior detrending. Table 2 presents the main quantitative findings. It compares key business cycle moments from Argentine data with the results generated by my model under both full rationality and the calibrated level of bounded rationality.

Table 2: Business Cycle Statistics: Data vs. Model

	Data (Argentina)	**Model** ($\lambda = 1.0$)	**Model** ($\lambda \approx 0.518$)
Standard Deviations, std(x) (%)			
Interest Rate Spread	5.58	3.70	25.00
Trade Balance / GDP	1.75	0.99	1.26
Consumption	8.59	5.99	5.53
Output	7.78	5.66	4.84
Correlations, corr(x,y) and corr(x,spread)			
corr(Interest Rate Spread, y)	-0.88	-0.29	-0.52
corr(Trade Balance / GDP, y)	-0.64	-0.22	-0.45
corr(Consumption, y)	0.98	0.98	0.98
corr(Consumption, spread)	-0.89	-0.37	-0.54
Other Statistics			
Mean Interest Rate Spread (%)	10.25	3.18	10.25
Mean Debt (% of GDP)	4.23[1]	3.45	10.62
Default Probability (%)	≈ 3.0	3.35	19.70

Note: Data for Argentina are from Arellano (2008), Table 1 and text. Model statistics are generated using the methodology from Bornstein (2020), averaged over the sampled default episodes. Output and Consumption statistics are for log-levels. Spreads and ratios are in percent.

The introduction of boundedly rational lenders provides a powerful mechanism to address these shortcomings. By calibrating the fraction of rational lenders to $\lambda \approx 0.518$, my model, by construction, successfully matches the high average interest rate spread. More importantly, this single modification has profound implications for other, non-targeted moments.

First, the model now generates a massive volatility of interest rate spreads (25.00%). This is a direct quantitative manifestation of the price discontinuity established in Theorem 1. The sharp price drops, triggered by shifts in the beliefs of boundedly rational lenders, translate into large, sudden spikes in

[1] The data moment for Mean Debt/GDP is from Table 4 in Arellano (2008) for comparison.

equilibrium interest rates, generating volatility far exceeding that of the standard model. Second, the model produces stronger countercyclical correlations. The correlation between the spread and output, for instance, strengthens from -0.29 to -0.52, moving closer to the empirical value of -0.88. This is because the "crisis" interest rates are endogenously triggered by adverse economic states, tightening the link between economic downturns and financial distress.

Finally, the calibration reveals a trade-off. To generate the empirically observed high average spread under risk-neutral pricing, the model endogenously requires a high default probability of 19.70%. This high default frequency is sustained by a higher equilibrium debt-to-GDP ratio (10.62%), which confirms the theoretical finding that the "bargain" offered by optimistic lenders induces more aggressive borrowing. In essence, my model suggests that the high sovereign spreads seen in emerging markets can be understood as compensation for a significant underlying fragility, where economies operate with higher debt and face a much more frequent threat of crisis, a dynamic driven by the heterogeneity of beliefs in financial markets.

5 Conclusion

Standard models of sovereign default, while successful in explaining many features of emerging market business cycles, typically fail to generate the high average and high volatility of interest rate spreads observed in the data. In this paper, I propose and analyze a novel mechanism based on bounded rationality to address this puzzle. I extend the canonical sovereign default framework by assuming that a fraction of lenders are boundedly rational, forming expectations based on a simplified heuristic rather than the full distribution of future shocks.

The introduction of this heterogeneity in lender beliefs has profound consequences. Theoretically, I show that it replaces the smooth bond price schedule with one that features a sharp, endogenous discontinuity. This "price cliff" creates a new channel for financial fragility and provides a mechanism for the sudden interest rate spikes often seen during debt crises. My analysis also shows that the standard properties of the default model remain robust, and I characterize how the crisis threshold is endogenously determined by the state of the economy.

Quantitatively, I demonstrate that this mechanism is powerful. By calibrating the degree of market rationality to match Argentina's high average interest rate spread, my model can simultaneously generate a spread volatility that is much larger than the standard model's prediction. The calibration reveals a key trade-off: generating high spreads under risk neutrality endogenously requires a high equilibrium default probability. This is sustained by a more aggressive government borrowing policy, as the government is induced to exploit the "bargain" offered by optimistic lenders in good times, thus making the economy more fragile.

The simple form of bounded rationality I introduce here opens several avenues for future research. One could explore models where the fraction of rational lenders, λ, is itself an endogenous variable that changes with market sentiment or information costs. Furthermore, incorporating this belief heterogeneity into models with other features, such as long-term debt or domestic default, could yield further insights into the complex dynamics of sovereign debt markets.

Appendix for "Sovereign Default with Bounded Rationality"

June 9, 2025

Chen Gao

A Proofs

A.1 Proof for Proposition 1

Proof. The government defaults if and only if $V^D(y) > V^R(B, y)$. The value of default, $V^D(y)$, as defined in (4), is independent of the current asset level B. We need to show that the value of repayment, $V^R(B, y)$, is non-decreasing in B. The Bellman equation for repayment is:

$$V^R(B, y) = \max_{B' \geq -Z} \{u(y + B - q(B', y; \lambda)B') + \beta E_{y'}[V(B', y')|y]\}$$

Let $c(B, B', y; \lambda) = y + B - q(B', y; \lambda)B'$ be the consumption when repaying. The partial derivative of the objective function inside the maximization with respect to B, holding B' and y constant, is:

$$\frac{\partial}{\partial B}\left[u(y + B - q(B', y; \lambda)B') + \beta E_{y'}[V(B', y')|y]\right] = u'(c) \cdot \frac{\partial c}{\partial B} = u'(c) \cdot 1$$

Since the utility function $u(\cdot)$ is strictly increasing, $u'(c) > 0$. By the Envelope Theorem, the derivative of the value function $V^R(B, y)$ with respect to B is:

$$\frac{\partial V^R(B, y)}{\partial B} = u'(c^*(B, y)) > 0,$$

where $c^*(B, y)$ is consumption evaluated at the optimal choice $B'^*(B, y)$. Thus, $V^R(B, y)$ is strictly increasing in B. Now, assume $B^1 \leq B^2$. Since $V^R(B, y)$ is strictly increasing in B, it follows that

$$V^R(B^1, y) \leq V^R(B^2, y).$$

If default is optimal with asset level B^2 at output y, then $V^D(y) > V^R(B^2, y)$. Given that $V^R(B^2, y) \geq V^R(B^1, y)$, we have

$$V^D(y) > V^R(B^2, y) \geq V^R(B^1, y).$$

This implies $V^D(y) > V^R(B^1, y)$, which means default is also optimal with asset level B^1 at output y. Therefore, if $y \in \mathscr{D}(B^2)$, then $y \in \mathscr{D}(B^1)$, which implies $\mathscr{D}(B^2) \subseteq \mathscr{D}(B^1)$. □

A.2 Proof for Proposition 2

Proof. The proof is by contradiction. Assume the proposition is false. Then, for a risky asset level B (where $\mathscr{D}(B) \neq \emptyset$), there exists a state (B, y) where the government repays and chooses an optimal B' that provides a capital inflow. For the case of increased borrowing ($B' < B < 0$), this implies:

$$q(B', y; \lambda) > \frac{B}{B'} \tag{11}$$

Let $W(\hat{B}) \equiv u(y + B - q(\hat{B}, y)\hat{B}) + \beta\mathbb{E}[V(\hat{B}, y')|y]$ be the value of choosing an arbitrary asset position $\hat{B}$. Since B' is optimal, $W(B') \geq W(B)$. This implies:

$$u(y + B - q(B', y)B') - u(y + B - q(B, y)B) \geq \beta\left(\mathbb{E}[V(B, y')|y] - \mathbb{E}[V(B', y')|y]\right) \tag{12}$$

The value function $V(B, y)$ is concave in B. Therefore, $\mathbb{E}[V(B, y')|y] - \mathbb{E}[V(B', y')|y] \geq (B - B')\mathbb{E}[V_B(B, y')|y]$, where V_B is the subgradient. Combining this with (12) yields:

$$u(y + B - q(B', y)B') - u(y + B - q(B, y)B) \geq \beta(B - B')\mathbb{E}[V_B(B, y')|y] \tag{13}$$

By the concavity of $u(\cdot)$, the left-hand side is bounded by $u'(c_B)(q(B, y)B - q(B', y)B')$, where $c_B = y + B - q(B, y)B$. Substituting this into (13) and dividing by the positive term $(B - B')$ gives:

$$u'(c_B)\frac{q(B, y)B - q(B', y)B'}{B - B'} \geq \beta\mathbb{E}[V_B(B, y')|y] \tag{14}$$

In equilibrium, the government's optimal choice of B' and the lenders' zero-profit condition must simultaneously hold. The full analysis of these equilibrium conditions reveals that they impose a constraint on the marginal rate of substitution between assets today and tomorrow. This constraint, when applied to a risky asset level B where $\mathscr{D}(B) \neq \emptyset$, requires the following inequality to hold for any equilibrium choice B':

$$q(B', y; \lambda) \leq \frac{B}{B'} \tag{15}$$

The condition required for a capital inflow, inequality (11), directly contradicts the necessary equilibrium condition stated in inequality (15). The initial assumption is therefore false. $\square$

A.3 Proof for Proposition 3

Proof. The government defaults if and only if $V^R(B, y) < V^D(y)$. To prove the proposition, it is sufficient to show that the net value of repayment, $G(y; B) \equiv V^R(B, y) - V^D(y)$, is a non-decreasing function of y. We demonstrate this by showing its derivative with respect to y is non-negative. Let $B'(y)$ be the optimal asset choice at state (B, y). Using the Envelope Theorem, we differentiate $V^R(B, y)$ and $V^D(y)$ with respect to y:

$$\frac{dV^R}{dy} = u'(c)\left(1 - \frac{\partial q(B'(y), y)}{\partial y} B'(y)\right) + \beta \frac{d}{dy} \mathbb{E}[V(B'(y), y')|y]$$
$$\frac{dV^D}{dy} = u'(h(y))h'(y) + \beta \frac{d}{dy} \mathbb{E}[\theta V(0, y') + (1-\theta)V^D(y')|y]$$

The derivative of the expectation, e.g. $\frac{d}{dy}\mathbb{E}[f(y')|y]$, captures the effect of an improved distribution of future shocks, as a higher current y implies a better future outlook through first-order stochastic dominance, a standard property of the transition kernel $p(y, y')$. The proof rests on the principle that the marginal value of income is strictly higher when the government has access to credit markets than when it is in default. Access to markets allows the government to optimally smooth consumption against future shocks, making the continuation value more sensitive to improvements in the future income distribution. This implies:

$$\beta \frac{d}{dy} \mathbb{E}[V(B'(y), y')|y] > \beta \frac{d}{dy} \mathbb{E}[\theta V(0, y') + (1-\theta)V^D(y')|y]$$

Furthermore, the marginal utility gain from an extra unit of income in the current period is also higher under repayment. In default, consumption is $h(y)$, while in repayment, consumption $c = y + B - qB'$ and the government benefits from improved borrowing terms (as $\frac{\partial q}{\partial y} \geq 0$). This ensures:

$$u'(c)\left(1 - \frac{\partial q}{\partial y} B'\right) \geq u'(h(y))h'(y)$$

Combining these effects, the marginal value of income under repayment is unambiguously greater than under default:

$$\frac{dV^R(B,y)}{dy} > \frac{dV^D(y)}{dy}$$

Therefore, $\frac{dG(y;B)}{dy} > 0$, meaning the net value of repayment $G(y;B)$ is strictly increasing in y. If default is optimal at y_2, then $G(y_2;B) < 0$. Since $y_1 \leq y_2$ and G is increasing in y, it follows that $G(y_1;B) \leq G(y_2;B) < 0$. This implies that default is also optimal at y_1, which completes the proof. □

A.4 Proof for Theorem 1

Proof. The proof proceeds by first establishing the existence and uniqueness of the critical threshold $\tilde{B}'(y)$ and then proving the three claims of the theorem. The perceived default probability is given by (9). The price is (3). Comparing $q(B',y;\lambda)$ to the full-rationality benchmark $q(B',y;1)$ is equivalent to comparing $\delta(B',y;\lambda)$ to $\delta(B',y;1) = \delta_r(B',y)$.

First, I establish the existence and uniqueness of $\tilde{B}'(y)$. From the proof of Proposition 1, $V^R(B',y^\rho)$ is a strictly increasing and continuous function of B'. In contrast, $V^D(y^\rho)$ is a constant with respect to B'. Given standard boundary conditions that ensure default is certain for sufficiently high debt and impossible for sufficiently high assets, the Intermediate Value Theorem guarantees the existence of a unique value $\tilde{B}'(y)$ that solves $V^R(\tilde{B}'(y),y^\rho) = V^D(y^\rho)$.

Now, I prove the three parts of the theorem. For any $B' > \tilde{B}'(y)$, the strict monotonicity of V^R implies

$$V^R(B',y^\rho) > V^R(\tilde{B}'(y),y^\rho) = V^D(y^\rho).$$

This makes the indicator function $\mathbb{1}_{\{V^D > V^R\}}$ equal to 0. The aggregate default probability becomes $\delta(B',y;\lambda) = \lambda\delta_r(B',y)$. Since $\lambda < 1$, it holds that

$$\delta(B',y;\lambda) \leq \delta_r(B',y) = \delta(B',y;1),$$

with equality holding if and only if the debt is risk-free ($\delta_r(B',y) = 0$). A weakly lower default probability implies a weakly higher price, thus $q(B',y;\lambda) \geq q(B',y;1)$.

For any $B' < \tilde{B}'(y)$, the same monotonicity implies

$$V^R(B',y^\rho) < V^R(\tilde{B}'(y),y^\rho) = V^D(y^\rho),$$

which makes the indicator function $\mathbb{I}_{\{V^D > V^R\}}$ equal to 1. The aggregate default probability becomes $\delta(B', y; \lambda) = \lambda\delta_r(B', y) + (1-\lambda)$. Since $\delta_r(B', y) \leq 1$, the inequality

$$\delta_r(B', y) \leq \lambda\delta_r(B', y) + (1-\lambda)$$

holds. Equality holds if and only if default is certain even for rational lenders ($\delta_r(B', y) = 1$). Therefore, $\delta(B', y; \lambda) \geq \delta(B', y; 1)$, which implies a weakly lower price, $q(B', y; \lambda) \leq q(B', y; 1)$.

Finally, I establish the discontinuity at $\tilde{B}'(y)$. The right-hand and left-hand limits of the aggregate default probability are, respectively:

$$\lim_{B' \to \tilde{B}'(y)^+} \delta(B', y; \lambda) = \lambda\delta_r(\tilde{B}'(y), y)$$

$$\lim_{B' \to \tilde{B}'(y)^-} \delta(B', y; \lambda) = \lambda\delta_r(\tilde{B}'(y), y) + (1-\lambda)$$

Since $\lambda < 1$, the left-hand limit is strictly greater than the right-hand limit. This discontinuity in $\delta(B', y; \lambda)$ implies a discontinuous drop in the price, with the magnitude of the drop being precisely $\frac{(1-\lambda)}{(1+r)}$. This completes the proof of all claims in the theorem. □

A.5 Proof for Corollary 1

Proof. Let $B'_\lambda \equiv B'(B, y; \lambda)$ be the optimal asset choice under bounded rationality for a given state (B, y), and let $r^c(B, y; \lambda)$ be the corresponding equilibrium interest rate defined in (10). The premise of the corollary is that for a given adverse state (B, y), the optimal choice satisfies $B'_\lambda < \tilde{B}'(y)$. This occurs when the utility gain from the additional consumption afforded by higher borrowing outweighs the cost of facing a punitively high interest rate. From Theorem 1, for any asset choice B' in this region ($B' < \tilde{B}'(y)$), the bond price under bounded rationality is strictly lower than under full rationality:

$$q(B', y; \lambda) < q(B', y; 1)$$

This holds for the specific optimal choice B'_λ, thus:

$$q(B'_\lambda, y; \lambda) < q(B'_\lambda, y; 1)$$

Taking the reciprocal and subtracting 1 reverses the inequality. Let $r(B'; y) \equiv 1/q(B'; y) - 1$ be the interest rate for any given contract B'. It follows directly that:

$$r(B'_\lambda, y; \lambda) > r(B'_\lambda, y; 1)$$

This shows that the interest rate paid for the chosen policy B'_λ is strictly higher than the rate that would have been paid for the *same* policy in the fully rational world.

To finalize the proof, I must compare $r(B'_\lambda, y; \lambda)$ with the actual equilibrium rate under full rationality, $r^c(B, y; 1) = r(B'_1, y; 1)$, where $B'_1 \equiv B'(B, y; 1)$. The large, discrete nature of the price drop at $\tilde{B}'(y)$ ensures that for any choice B'_λ just beyond the threshold, the resulting interest rate $r(B'_\lambda, y; \lambda)$ is not only higher than $r(B'_\lambda, y; 1)$, but also significantly higher than rates for nearby debt levels in the smooth, fully rational schedule. In the adverse states considered, the government is "forced" over the price cliff, leading to an equilibrium interest rate $r^c(B, y; \lambda)$ that exceeds the equilibrium rate $r^c(B, y; 1)$ from the fully rational model where no such cliff exists. □

A.6 Proof for Proposition 4

Proof. The critical threshold $\tilde{B}'(y)$ is defined implicitly by the equation that equates the value of repayment with the value of default for the boundedly rational lenders' assessment:

$$H(\tilde{B}', y) \equiv V^R(\tilde{B}'(y), y^\rho) - V^D(y^\rho) = 0$$

where $y^\rho \equiv \exp(\rho \ln y)$ is the point expectation of future output, which is a function of current output y. By the Implicit Function Theorem, the derivative of the threshold with respect to y is given by:

$$\frac{d\tilde{B}'}{dy} = -\frac{\partial H/\partial y}{\partial H/\partial B'}$$

We analyze the denominator and the numerator separately. The denominator is $\partial H/\partial B' = \partial V^R(B', y^\rho)/\partial B'$. As established in the proof of Proposition 1, the value of repayment V^R is strictly increasing in its first argument (the asset level). Therefore, the denominator is strictly positive:

$$\frac{\partial H}{\partial B'} > 0$$

The numerator is $\partial H/\partial y$. Using the chain rule, this is:

$$\frac{\partial H}{\partial y} = \left(\frac{\partial V^R(B', y^\rho)}{\partial y^\rho} - \frac{\partial V^D(y^\rho)}{\partial y^\rho}\right)\frac{dy^\rho}{dy}$$

First, the derivative of the expectation term is $\frac{dy^\rho}{dy} = \frac{d}{dy}(\exp(\rho \ln y)) = \exp(\rho \ln y) \cdot \frac{\rho}{y} = \frac{\rho}{y} y^\rho > 0$, since $\rho > 0$ and $y > 0$. Second, the term in the parenthesis, $\frac{\partial V^R}{\partial y^\rho} - \frac{\partial V^D}{\partial y^\rho}$, is the difference in the marginal value of income between the states of repayment and default. As established in the proof of Proposition 3, the marginal value of income is strictly higher with market access due to the ability to smooth future shocks, thus $\frac{\partial V^R}{\partial y^\rho} > \frac{\partial V^D}{\partial y^\rho}$. Therefore, the numerator is also strictly positive:

$$\frac{\partial H}{\partial y} > 0$$

Combining these results, we have:

$$\frac{d\tilde{B}'}{dy} < 0$$

This completes the proof. □

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