Default with Pessimism

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[INCOMPLETE AND PRELIMINARY]
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Abstract

Why do emerging economies face persistently high borrowing costs despite moderate debt levels? I develop a quantitative sovereign default model where lenders systematically overestimate government policy randomness. This behavioral bias creates a "pessimism wedge" that pivots bond prices—making debt cheaper near default but more expensive in normal times. Rational sovereigns respond by deleveraging yet paradoxically face higher average spreads and an "illusion of financial stability" where volatility falls despite higher risk premiums. Theoretical extensions demonstrate that optimal fiscal policy cannot eliminate allocative distortions from pessimism, negative learning bias perpetuates pessimistic beliefs, and strategic communication can partially mitigate these effects. The framework provides a new behavioral foundation for understanding sovereign debt puzzles in emerging markets.

Keywords: Sovereign Default, Behavioral Macroeconomics, Lender Pessimism, Sovereign Spreads, Information Frictions, Argentina, Debt Management.
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1 Introduction

Why is sovereign debt in some emerging economies particularly expensive? Standard models struggle to explain why countries with moderate debt-to-GDP ratios face persistently high borrowing costs, excess volatility, and puzzling "decoupling" from their peers (Tomz and Wright, 2013; Meyer et al., 2022). Argentina exemplifies this puzzle: as documented by (Morelli and Moretti, 2023), its borrowing costs often appear divorced from macroeconomic fundamentals, suggesting a large country-specific risk premium that defies conventional explanation.

The leading explanation centers on reputation and *information frictions* (Cole et al., 1995). In this view, advanced quantitatively by (Morelli and Moretti, 2023), lenders are rational but uninformed about a government's hidden type—whether "committed" or "strategic." Policy missteps like Argentina's inflation misreporting signal a "bad type," causing persistent reputation downgrades. High spreads reflect the market's efficient pricing of this revealed information.

This paper explores an alternative *behavioral friction*. What if the problem is not what lenders do not know, but what they systematically believe to be true? Departing from rational learning, I posit that lenders suffer from pessimistic bias about government policy randomness. While sovereigns face unobserved "taste shocks," lenders perceive these shocks as larger and more volatile than they truly are. This "pessimism wedge" distorts default risk assessment. Events like Argentina's misreporting signal not just strategic behavior but perceived policy unpredictability. Markets react to this perceived randomness increase beyond mere reputation downgrades.

My main contribution embeds this behavioral friction into a quantitative sovereign default model and traces its macroeconomic consequences. Pessimistic lenders fundamentally alter the borrowing environment by "pivoting" bond price schedules—making debt cheaper near default but more expensive in safe states. Rational sovereigns respond optimally by deleveraging yet paradoxically face higher average spreads. The model generates an "illusion of financial stability" where market volatility falls despite rising risk premiums.

Three theoretical extensions deepen the analysis. First, even optimal Ramsey fiscal policy cannot eliminate welfare losses from pessimism due to fundamental allocative inefficiencies from distorted bond pricing. Second, endogenous belief formation through Bayesian learning with negativity bias shows how pessimistic beliefs persist and become entrenched. Third, optimal policy communication demonstrates how governments can strategically choose transparency levels to partially mitigate lender pessimism. The analysis provides a new behavioral foundation for understanding sovereign risk and persistent debt challenges in emerging economies.

Literature This paper contributes to the sovereign default literature by proposing a novel behavioral mechanism to explain persistent empirical puzzles: why do emerging economies often face high spreads and excess volatility that seem disconnected from their macroeconomic fundamentals (Tomz and Wright, 2013; Mitchener and Trebesch, 2023)?[1]

The dominant paradigm treats default as a purely *strategic decision* where sovereigns rationally weigh repayment costs against temporary market exclusion (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2007; Arellano, 2008). While this framework has been successfully extended to incorporate long-term debt and financial frictions (Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012; Mendoza and Yue, 2012),[2] it struggles to explain why spreads often remain elevated even when fundamentals improve.

A second approach emphasizes *reputational concerns*, where past actions cast long shadows over future borrowing costs (Cole et al., 1995; Phelan, 2006). In this framework, the central question is how lenders learn about the government's hidden type. While classic models focus on default itself as the ultimate signal, recent research shows the informational channel is much broader. For instance, (Fourakis, 2024) provides quantitative evidence that investors learn about government types through fiscal and monetary policy indicators, finding that deficit and inflation surprises significantly affect perceived default probabilities and that reputation loss often occurs *before* a default event. This perspective is powerfully exemplified by the Argentina inflation misreporting episode, rigorously analyzed by (Morelli and Moretti, 2023), where a single breach of trust—interpreted as a credible signal of a "bad type"—led to years of market exclusion.This rich view of reputation has been embedded in models that explain how a country can eventually "graduate" to a high-trust state through a long history of good behavior (Amador and Phelan, 2021) and why, in a partial default setting, larger haircuts must rationally lead to a greater loss of reputation to sustain a mixed-strategy equilibrium (Amador and Phelan, 2023).[3] Yet even this learning-based view assumes lenders eventually converge to the truth, leaving unexplained why some sovereign risk premia appear systematically and persistently excessive.

A third strand recognizes that market *sentiment itself* can become a fundamental force. Beginning with (Calvo et al., 1996) and (Cole and Kehoe, 2000), this literature demonstrates how pessimistic expectations can become self-fulfilling, with modern quantitative implementations by (Gennaioli et al., 2014) and (Bocola and Dovis, 2019).[4] My paper builds di-

[1] For comprehensive surveys of the sovereign debt literature, see (Meyer et al., 2022) and (Abbas et al., eds, 2019).

[2] Empirical evidence on emerging market business cycles and debt restructurings is provided by (Neumeyer and Perri, 2005), (Cruces and Trebesch, 2013), and (Arellano and Ramanarayanan, 2012).

[3] Other important contributions to the reputation literature include (Cole and Kehoe, 1998) on partial versus general reputations, (D'Erasmo, 2011) on government reputation and debt repayment, (Egorov and Fabinger, 2016) on reputational effects in sovereign default, and (Dovis and Kirpalani, 2020, 2021) on reputation in policy design. A key distinction is that while reputation models predict "bad types" default at lower debt thresholds, my pessimism model counterintuitively predicts the opposite: pessimistic lenders perceive higher default thresholds due to overestimated policy randomness.

[4] Related work explores financial frictions (Longstaff et al., 2011), risk aversion (Lizarazo, 2013), matu-

rectly on this insight but asks a deeper question: what if markets systematically *misperceive* the very nature of sovereign decision-making?

Drawing from behavioral economics, I propose that lender pessimism—rooted in well-documented biases like *heuristics and biases* (Tversky and Kahneman, 1974), *prospect theory* (Kahneman and Tversky, 1979), and *noise trading* (De Long et al., 1990)—can create persistent wedges between sovereign risk and fundamentals.[5] Crucially, my mechanism differs from existing behavioral approaches. For instance, models of *ambiguity aversion* based on robust control theory (Gilboa and Schmeidler, 1989; Hansen and Sargent, 2008) assume that lenders are uncertain about the true model of macroeconomic fundamentals. This leads them to price assets based on a "worst-case" scenario, generating an ambiguity premium that can explain high sovereign spreads (Pouzo and Presno, 2016) and the puzzlingly poor pricing of contingent debt (Roch and Roldán, 2023). While these models distort the perceived distribution of *macroeconomic shocks* across all states, my "pessimism wedge" specifically targets the perceived volatility of the sovereign's *policy choices*, creating a distinctive bond-price *pivot*.[6] And unlike *diagnostic expectations* (Gennaioli and Shleifer, 2018; Bordalo et al., 2023), which generate boom-bust cycles through time-varying news overreaction, my time-invariant bias produces persistent "pessimism premia," counter-intuitive deleveraging, and the "softening-of-doom" effects that standard models cannot replicate.[7]

2 Motivation: Argentina's Inflation–Misreporting Episode

A Puzzle of Persistent Risk The recent economic history of Argentina offers a powerful illustration of a core puzzle in international finance: why do emerging economies often face borrowing costs that seem disconnected from their macroeconomic fundamentals? (Tomz and Wright, 2013; Meyer et al., 2022). Standard models, even those incorporating reputational dynamics, struggle to fully account for the persistence and magnitude of the

rity choice (Stangebye, 2020), and rational learning about shifts in fundamentals, such as the "rare disaster" mechanism in (Paluszynski, 2023) used to explain the slow onset of the European debt crisis.

[5]Additional behavioral foundations include investor sentiment (Baker and Wurgler, 2006), rare disasters (Gabaix, 2012), disposition effects (Shefrin and Statman, 1985), crisis psychology (Kindleberger, 1978), and limits to arbitrage (Brunnermeier and Nagel, 2004; Barberis and Thaler, 2003).

[6]This pivot effect contrasts sharply with both reputation and ambiguity aversion models. While reputation models like (Amador and Phelan, 2021) generate monotonic price effects through type revelation and (Fourakis, 2024) shows reputation loss *before* default through rapid debt accumulation, my model produces non-monotonic price schedules where debt becomes cheaper near default but more expensive in normal times. Similarly, ambiguity models like (Roch and Roldán, 2023) explain high spreads through a uniform distortion of fundamental shocks, which also differs from the pivot mechanism.

[7]The empirical predictions also differ markedly from reputation models. While (Morelli and Moretti, 2023) find that misreporting episodes increase spreads on all debt instruments, my model predicts divergent effects across the debt distribution. Similarly, while (Amador and Phelan, 2023) shows reputation effects strengthen with haircut size, my pessimism model predicts that pessimism effects would weaken as larger haircuts reduce the perceived randomness component.

country-specific risk premium observed in cases like Argentina, where borrowing costs have often appeared divorced from traditional measures of repayment capacity (Morelli and Moretti, 2023). This suggests the presence of frictions beyond those typically modeled. [TODO: Add comparison table showing Argentina vs Brazil/Chile core indicators (debt ratio, GDP growth, spreads) to more intuitively highlight the "anomaly" mentioned in reviewer feedback]

The Misreporting Episode This puzzle was cast in sharp relief during the inflation misreporting episode that began in 2007. Following a period of rising inflation, the Argentine government initiated a direct political intervention in its national statistics institute, INDEC. Senior technical staff were dismissed and replaced, leading to an immediate and sustained suppression of the official Consumer Price Index (CPI) (Morelli and Moretti, 2023). This official figure diverged starkly from credible estimates produced by private and provincial sources (Cavallo, 2013). The government's enforcement was aggressive, levying substantial fines on private consultancies that published their own, more realistic, data (Grajewski and Raszewski, 2013). The data manipulation grew so notorious that *The Economist* publicly ceased publishing the official figures in 2012, and the IMF issued a rare "declaration of censure" against the country (The Economist, 2012; International Monetary Fund, 2013).

The financial consequences were profound. As inflation-indexed bonds constituted a significant portion of public debt, this act amounted to a *de facto* partial default. The market's reaction was swift and severe. Argentina's EMBI+ spread, which had been tracking its Latin American peers, decoupled and widened sharply. Crucially, this repricing was not limited to the directly affected instruments; it spilled over entirely to its dollar-denominated sovereign debt. This reaction is paradoxical from a purely mechanical standpoint: a policy that lowers the real debt burden should have *decreased* default risk on nominal bonds. The opposite happened, indicating the event was a pure, and powerful, information shock about the government's character and future actions.

[Figure placeholder: Argentina's EMBI+ spread data - TODO: Download from Bloomberg terminal]

Limits of Reputational Models The conventional explanation, rooted in reputational models (Cole et al., 1995), interprets this episode as a credible signal of a "bad type." In this view, advanced by (Morelli and Moretti, 2023), lenders are rational but uninformed about a government's hidden commitment to repay. The misreporting revealed Argentina's government as a "strategic" type, leading to a rational, persistent downgrade of its reputation and, consequently, higher borrowing costs. While this view is powerful, it leaves lingering questions. If the market simply learned the government's type, why did the risk premium appear to contain an additional, seemingly excessive, component? Why did market sentiment seem to reflect not just a reassessment of character, but a new apprehension about the government's very predictability? The episode suggests that the market's reaction was not just to what it learned about the sovereign's *intent*, but to a perceived increase in its *erratic nature*.

A Behavioral Hypothesis: Lender Pessimism This paper explores an alternative, yet complementary, behavioral friction. I posit that the problem is not only what lenders do not know, but also what they systematically, and pessimistically, believe to be true. My core assumption is that lenders perceive the unobserved shocks driving sovereign policy choices to be larger and more volatile than they truly are. This "pessimism wedge" distorts their assessment of default risk. From this perspective, an event like Argentina's misreporting is not just a signal of a sovereign's bad character (a strategic type), but is interpreted as evidence of its erratic nature (high unpredictability). The market reacts to a perceived increase in policy randomness, not just a downgrade of its reputation. This pessimism is not irrational in a colloquial sense; rather, it is a systematic bias in belief formation, consistent with behavioral findings on how agents process information under uncertainty (Tversky and Kahneman, 1974; Barberis and Thaler, 2003).

A Bridge to the Model Motivated by this reinterpretation of the Argentine case, I embed this behavioral friction into an otherwise standard quantitative sovereign default model. The subsequent sections will formalize the "pessimism wedge" and trace its consequences. I will show that the presence of pessimistic lenders fundamentally alters the sovereign's borrowing environment, creating a distinctive "pivoting" of the bond price schedule. This, in turn, generates a series of counterintuitive but empirically relevant outcomes: a rational sovereign deleverages yet faces higher average spreads, and market volatility can fall even as the underlying risk premium rises, creating an "illusion of financial stability." This framework provides a new, behaviorally-grounded perspective on the persistent debt challenges that many emerging economies face.

3 A Model of Pessimism

3.1 Environment

Time is discrete and the horizon is infinite, $t = 0, 1, 2, \ldots$. The economy receives a stochastic endowment of a single tradable good, y_t. The endowment process is exogenous and follows a stationary, first-order Markov process, which is generated by discretizing the following AR(1) process in logarithms:

$$\ln y' = (1 - \rho_y)\mu_y + \rho_y \ln y + \sigma_y \varepsilon', \quad \varepsilon' \sim \mathcal{N}(0, 1). \tag{1}$$

The transition probabilities are given by the matrix $\Pi(y, y') = \Pr\{y_{t+1} = y' | y_t = y\}$.

The government can borrow from a large number of competitive, risk-neutral international lenders who have access to an international risk-free interest rate r. Debt takes the form of long-term bonds. A bond is a claim to a stream of coupon payments κ in every future period, unless the sovereign defaults. Each period, a fraction $\delta \in (0,1]$ of outstanding bonds matures, while the remaining fraction $1-\delta$ carries over to the next period.

The sovereign government has preferences represented by a standard time-separable utility function with a discount factor $\beta \in (0,1)$. The period utility function is of the CRRA form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \tag{2}$$

which is strictly increasing and concave for $\sigma > 0$.

3.2 The Sovereign's Problem

At the beginning of each period t, the state is summarized by the current endowment realization $y \in \mathcal{Y}$ and the stock of outstanding debt B. The sovereign first decides whether to default on its obligations or to repay. This choice is subject to an idiosyncratic preference shock, often referred to as a "taste shock," which introduces randomness into the decision-making process from the perspective of an outside observer.

Taste Shocks Let $V^D(y)$ be the deterministic component of the value of defaulting, and $V^R(y,B)$ be the deterministic component of the value of repaying. The full, or *ex-post*, value for each choice is the sum of its deterministic part and a random shock:

$$\tilde{V}^D(y, \varepsilon_d) = V^D(y) + \varepsilon_d$$
$$\tilde{V}^R(y, B, \varepsilon_r) = V^R(y, B) + \varepsilon_r$$

The sovereign observes the shocks ε_d and ε_r and chooses the action that yields the highest *ex-post* value. The *ex-ante* value function, from a perspective before the shocks are realized, is the expected maximum of these *ex-post* values:

$$V(y,B) = \mathbb{E}_{\varepsilon_d, \varepsilon_r}\left[\max\left\{\underbrace{V^D(y) + \varepsilon_d}_{\tilde{V}^D(y,\varepsilon_d)}, \underbrace{V^R(y,B) + \varepsilon_r}_{\tilde{V}^R(y,B,\varepsilon_r)}\right\}\right], \tag{3}$$

where the expectation is taken over the distribution of the shocks.

Following (Dvorkin et al., 2021) and (Mihalache, 2024), I assume that the taste shocks ε_d and ε_r are drawn independently from a Gumbel distribution.[8] To visualize the distribu-

[8]The CDF of a Gumbel distribution is given by $F(\varepsilon; \mu_L, \eta) = \exp(-\exp(-(\varepsilon - \mu_L)/\eta))$, where μ_L is the location parameter and $\eta > 0$ is the scale parameter. The mean of this distribution is $\mu_L + \eta\gamma$, where $\gamma \approx$

tion of these shocks, Figure 1 plots the probability density function (PDF) and cumulative distribution function (CDF) for the Gumbel distribution, normalized to have a mean of zero. The scale parameter, η, is pivotal. It governs the variance of the taste shocks, given by $\text{Var}(\varepsilon_i) = \frac{\pi^2\eta^2}{6}$. A larger η signifies greater dispersion in preferences and introduces more randomness into the sovereign's choice.[9] Economically, these taste shocks can be interpreted as a reduced-form representation of various unmodeled factors that influence policy, such as political pressures from domestic constituencies, bureaucratic implementation errors, or the private information of policymakers. By modeling them as random draws, the framework acknowledges a degree of inherent unpredictability in government behavior.

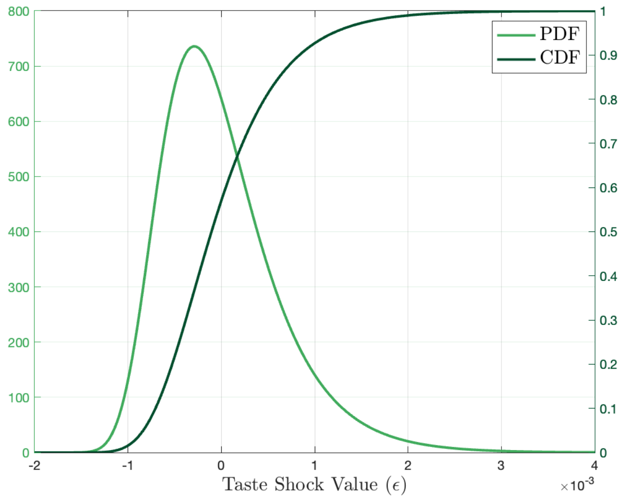


Figure 1: The mean-zero Gumbel distribution ($\eta = 5 \times 10^{-4}$).
Note: The Gumbel distribution is used to model the taste shocks in the sovereign's default and borrowing decisions. The scale parameter η controls the variance of these shocks. The left y axis is the PDF, the right y axis is the CDF.

Ex-Ante Value Function The ex-ante value function can be derived using fundamental properties of the Gumbel distribution. The lemma provided in Appendix A provides the closed-form expression for the expected maximum of Gumbel-distributed random variables, which forms the basis for my analytical solution. First, applying Lemma 1 to my setting with $V_1 = V^D(y)$ and $V_2 = V^R(y, B)$, the ex-ante value function is:

$$V(y,B) = \mathbb{E}\left[\max\{\tilde{V}^d, \tilde{V}^r\}\right] = \eta \ln\left(\exp\frac{V^D(y)}{\eta} + \exp\frac{V^R(y,B)}{\eta}\right). \tag{4}$$

0.5772 is the Euler-Mascheroni constant. For analytical convenience, I choose a specific parameterization, $\text{Gumbel}(-\eta\gamma, \eta)$, which makes the mean of the shocks equal to zero: $\mathbb{E}[\varepsilon_i] = -\eta\gamma + \eta\gamma = 0$.

[9] As $\eta \to 0$, the shocks' influence diminishes, and the model approaches a deterministic framework where decisions are based solely on $V^D(y)$ and $V^R(y,B)$. Conversely, as $\eta \to \infty$, the deterministic value components become negligible, and the choice becomes almost entirely random.

Discrete Choice The choice probabilities in my model follow directly from another fundamental property of the Gumbel distribution. Using Lemma 2 in Appendix A, the probability of default is:

$$\Pr\{d=1|y,B\} = \frac{\exp\frac{V^D(y)}{\eta}}{\exp\frac{V^D(y)}{\eta}+\exp\frac{V^R(y,B)}{\eta}}. \tag{5}$$

Value of Default If the sovereign defaults, it is excluded from international credit markets. During exclusion, it bears an output cost and consumes a fraction of its endowment, $c = h(y) \leq y$, where the output cost function is specified similar to (Chatterjee and Eyigungor, 2012):

$$h(y) = y - \max\{0, \lambda_0 y + \lambda_1 y^2\}. \tag{6}$$

In each period of exclusion, there is a constant probability $\gamma \in (0,1)$ that the country regains market access. Upon re-entry, all past debts are forgiven, so it starts with $B = 0$. The value of being in default is therefore:

$$V^D(y) = u(h(y)) + \beta \mathbb{E}_{y'|y}\left[\gamma V(y',0) + (1-\gamma)V^D(y')\right]. \tag{7}$$

Value of Repayment If the sovereign honors its debt, it pays the coupon κB and retains market access. It can then choose a new level of debt for the next period, B'. This choice is also subject to taste shocks. The *ex-ante* value of choosing a specific debt level B', given the state (y,B), is:

$$W(y,B,B') = u\left(y - \kappa B + [B' - (1-\delta)B]\, q(y,B')\right) + \beta \mathbb{E}_{y'|y}\left[V(y',B')\right], \tag{8}$$

where $q(y,B')$ is the price at which it can issue new bonds. The borrowing choice is subject to i.i.d. Gumbel shocks, denoted by $\{\varepsilon_{B'}\}$, for each possible debt level B'. Each shock is distributed as Gumbel$(-\rho\gamma,\rho)$. Applying Lemma 3 in Appendix A with $V_i = W(y,B,B_i)$ and $\sigma = \rho$, the ex-ante value of repaying is:

$$V^R(y,B) = \rho \ln\left(\sum_{B'\in\mathscr{B}} \exp\frac{W(y,B,B')}{\rho}\right), \tag{9}$$

where $\mathscr{B}$ is the discrete set of possible debt levels. The probability of choosing a specific level B' is:

$$\Pr\{B'|y,B\} = \frac{\exp\frac{W(y,B,B')}{\rho}}{\sum_{B_j\in\mathscr{B}}\exp\frac{W(y,B,B_j)}{\rho}}. \tag{10}$$

Figure 2 provides a visualization of this probabilistic borrowing policy. The taste shock framework transforms the choice of the next debt level, B', from a single deterministic point into a smooth probability distribution over the entire set of available options, $\mathscr{B}$. The peak of the distribution corresponds to the most preferred borrowing choice, but the scale parameter ρ ensures that other, less optimal choices still have a non-zero probability of being selected. This feature captures *unobserved heterogeneity* in the sovereign's decision-making process.[10]

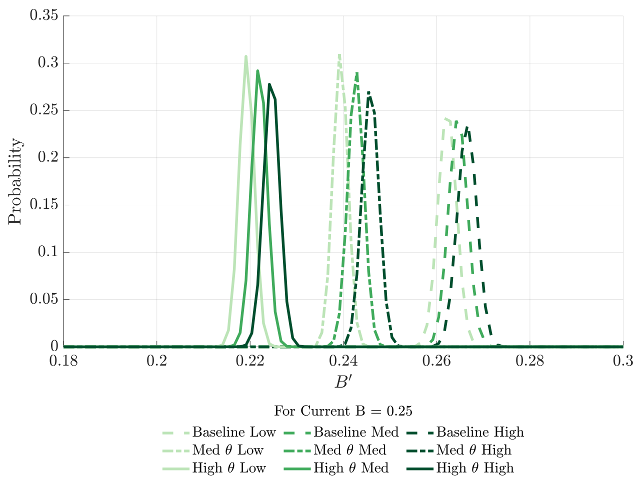


Figure 2: Example of the Probabilistic Borrowing Policy, $\Pr\{B'|y,B\}$.
Note: The figure illustrates the sovereign's borrowing choice as a probability distribution over possible next-period debt levels (B'), given a specific state (y,B). The peak of the distribution represents the most likely choice.

3.3 International Lenders and Bond Pricing

I depart from the standard model by assuming a *wedge* between the sovereign's true behavior and lenders' perception of it. Lenders in this model are competitive and risk-neutral, pricing bonds to make zero expected profit according to their beliefs. However, their beliefs are systematically biased in a specific way.

Pessimism The key assumption is that lenders perceive the sovereign to be more erratic or "irrational" than it truly is. They believe the sovereign's default decision is governed by a taste shock with a scale parameter $\tilde{\eta} = \theta \cdot \eta$, where $\theta \geq 1$. The parameter θ captures the degree of *lender pessimism, parameter uncertainty, or ambiguity aversion.* When $\theta > 1$, lenders act *as if* the government's choices are more random than they actually are.

Consequently, the lenders' perceived probability of default at a future state (y',B'), which I denote $\tilde{P}(y',B')$, is calculated using this inflated shock parameter similar to (5):

$$\tilde{P}(y',B') = \frac{\exp\frac{V^D(y')}{\theta\eta}}{\exp\frac{V^D(y')}{\theta\eta} + \exp\frac{V^R(y',B')}{\theta\eta}}. \tag{11}$$

[10]This probabilistic approach is also crucial for the numerical stability of the model, as it replaces the non-differentiable "max" operator with a smooth, analytical expression, which is similar to the idea of (Chatterjee and Eyigungor, 2012).

Bond Price The equilibrium bond price $q(y, B')$ must satisfy the no-arbitrage condition based on these pessimistic beliefs. The price equals the discounted expected payoff, where the probability of repayment is assessed using $\tilde{P}(y', B')$:

$$
\begin{aligned}
q(y,B') &= \frac{1}{1+r}\mathbb{E}_{y'|y}\left[\left(1-\tilde{P}(y',B')\right)\left(\kappa+(1-\delta)\mathbb{E}_{B''|y',B'}\left[q(y',B'')\right]\right)\right] \\
&= \frac{1}{1+r}\mathbb{E}_{y'|y}\left[\left(1-\tilde{P}(y',B')\right)\left(\kappa+(1-\delta)\sum_{B''\in\mathscr{B}}\Pr\{B''|y',B'\}\cdot q(y',B'')\right)\right]
\end{aligned}
\tag{12}
$$

It is important to note that the inner expectation, $\mathbb{E}_{B''|y',B'}$, is taken over the sovereign's *true* borrowing policy, $\Pr\{B''|y', B'\}$, which is governed by the true shock parameter ρ. In this framework, lenders correctly understand the sovereign's borrowing behavior but misperceive its propensity to default. This mechanism endogenously generates a credit spread that contains a "pessimism premium", which reflects the lenders' biased beliefs about the sovereign's stability.

3.4 Equilibrium

As is standard in sovereign default literature, the solution concept is a Recursive Markov Perfect Equilibrium, defined as follows:

Definition 1. *A Recursive Markov Perfect Equilibrium consists of a set of functions: value functions for the sovereign ($V : \mathscr{Y} \times \mathscr{B} \to \mathbb{R}$, $V^R : \mathscr{Y} \times \mathscr{B} \to \mathbb{R}$, $V^D : \mathscr{Y} \to \mathbb{R}$), policy probabilities for its choices ($\Pr\{d = 1|\cdot\}, \Pr\{B'|\cdot\}$), and a bond price function ($q : \mathscr{Y} \times \mathscr{B} \to \mathbb{R}$), such that for all states (y, B):*

1. ***Sovereign Optimality:*** *Taking the price function q as given, the sovereign's value functions and policy probabilities solve the dynamic programming problem defined by equations (4), (7), and (9). The choices are governed by the true taste shock parameters η and ρ.*

2. ***Lender Pricing:*** *The bond price function q satisfies the zero-expected-profit condition for lenders, as specified in (12), which is based on their perceived default probability $\tilde{P}$ from (11).*

With the taste shock (logit aggregator) in place, the recursive equilibrium is well behaved.

Proposition 1. *Let $\mathscr{Y}$ and $\mathscr{B}$ be compact, $u \in C^1$ strictly increasing and concave with u' bounded on the feasible consumption set, and parameters satisfy $\beta \in (0, 1)$, $r > 0$, $\delta \in [0, 1)$, $\eta > 0$, $\rho > 0$, $\kappa > 0$. Let prices be determined by the pricing operator in (B.9) with logistic default rule and repayment/default values as defined in the model, and let the Bellman* *aggregator be the log-sum-exp with taste-shock scale ρ. If the slope condition*

$$L_{Jq} L_{TV} < (1-\beta)\left(1 - \frac{1-\delta}{1+r}\right) \tag{13}$$

holds (constants defined below), then the Recursive Markov Perfect Equilibrium as in Definition 1 exists and is unique.

Proof. See Appendix B.4. □

4 Theoretical Analysis

Before proceeding to the full quantitative analysis, this section theoretically unpacks the consequences of the behavioral wedge between the sovereign and its lenders. I demonstrate how lender pessimism ($\theta > 1$) systematically reshapes the equilibrium, beginning with its most direct impact on the bond price schedule and tracing the effects through to the sovereign's policies and ultimate welfare.

Bond Price Pivot The first and most fundamental consequence of lender pessimism is on the price of debt. The following proposition establishes that pessimism does not uniformly depress bond prices. Instead, it causes the price schedule to pivot relative to the rational benchmark, an effect distinct from the monotonic downward shift one might expect from a simple reputation downgrade.

Proposition 2. *Consider an economy with pessimistic lenders ($\theta > 1$) and a baseline economy with rational lenders ($\theta = 1$), both having a small true taste shock parameter $\eta > 0$. Let $q_1(B', y)$ and $q_\theta(B', y)$ be the respective equilibrium bond price functions. For a given endowment level y, there exists a debt threshold $B^*(y)$ such that the price difference $\Delta q(B', y) \equiv q_\theta(B', y) - q_1(B', y)$ satisfies:*

- *For levels of future debt $B' < B^*(y)$, $\Delta q(B', y) < 0$ (pessimistic lenders offer lower prices).*
- *For levels of future debt $B' > B^*(y)$, $\Delta q(B', y) > 0$ (pessimistic lenders offer higher prices).*

Proof. See Appendix B.5. □

The formal proof in Appendix B.5 proceeds by comparing default probabilities under the two belief systems. The key insight is that pessimistic lenders overestimate default risk in low-debt scenarios (where fundamentals suggest safety) but underestimate the certainty of default in high-debt scenarios (where their emphasis on randomness creates perceived escape possibilities). The pivoting occurs because these two opposing effects exactly balance at the threshold $B^*(y)$.

Figure 3 provides a graphical illustration of this pivoting effect, showing how the bond price schedules for different levels of lender pessimism cross at the threshold $B^*(y)$. The economic intuition behind this pivoting effect is twofold. For low debt levels ($B < B^*(y)$), where a rational lender sees default as a remote possibility, a pessimistic lender prices in a non-negligible risk of an "out-of-the-blue" default driven by the perceived high variance of taste shocks. This results in a "pessimism premium" that lowers the bond price. Conversely, for high debt levels ($B > B^*(y)$), where a rational lender sees default as a near certainty based on fundamentals, the pessimistic lender's view, which emphasizes randomness, makes them less certain of this outcome. Their belief allows for a higher chance of an "irrational" repayment, which paradoxically supports a higher bond price. The threshold $B^*(y)$ marks the debt level where these two competing effects exactly offset each other.

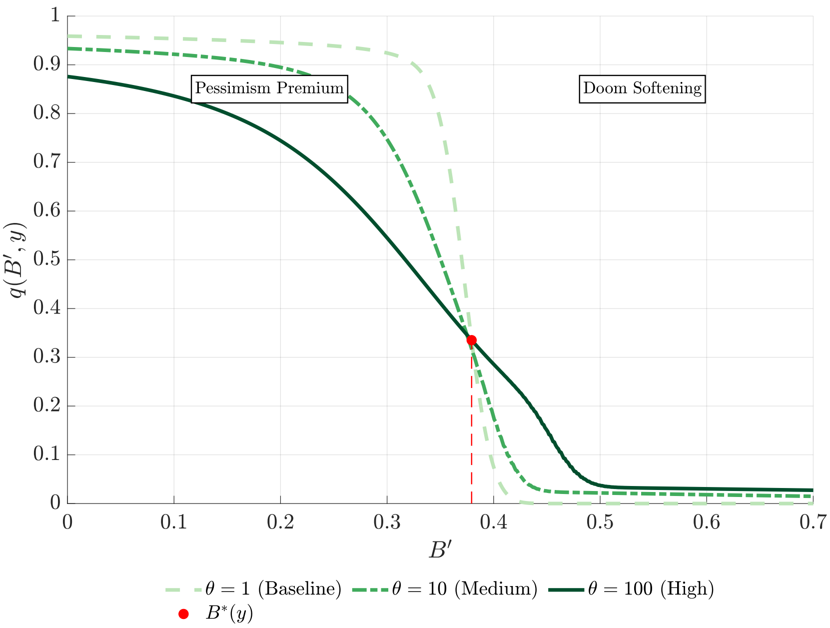


Figure 3: Pivoting Bond Price Schedules

Note: This figure illustrates the pivoting effect described in Proposition 2 for a normal output level. The bond price schedules for three different levels of lender pessimism ($\theta = 1, 10, 100$) cross at the threshold $B^*(y)$ marked by the red dot. To the left of $B^*(y)$, pessimistic lenders impose a "pessimism premium," offering lower prices than rational lenders. To the right of $B^*(y)$, the "softening of doom" effect emerges, where pessimistic lenders offer paradoxically higher prices as they perceive less certainty about default in high-debt scenarios.

Pivot Threshold This pivot point is not static; it responds to the sovereign's economic condition. The next proposition shows that as the sovereign's fortunes improve, the pivot point shifts to higher levels of debt.

Proposition 3. *The debt threshold $B^*(y)$ defined in Proposition 2, at which the baseline and* *pessimistic price schedules cross, is monotonically increasing in the endowment level y. That is, $\frac{dB^*(y)}{dy} > 0$. Figure 4 illustrates this monotonic relationship.*

Proof. See Appendix B.6. □

The intuition for this result lies in the differential response of the two markets to good news. A higher income level y improves the sovereign's repayment capacity, shifting both bond price schedules outward. However, the rational market (q_1) is more responsive to this positive signal about fundamentals than the pessimistic market (q_θ), whose pricing remains partially anchored by its skeptical prior about the sovereign's stability. Because the rational price schedule shifts more strongly to the right, its intersection point with the pessimistic schedule, $B^*(y)$, must also shift to the right. In other words, a stronger economy can sustain more debt before the pessimism premium in the low debt level is outweighed by the "softening of doom" effect in the high debt level.

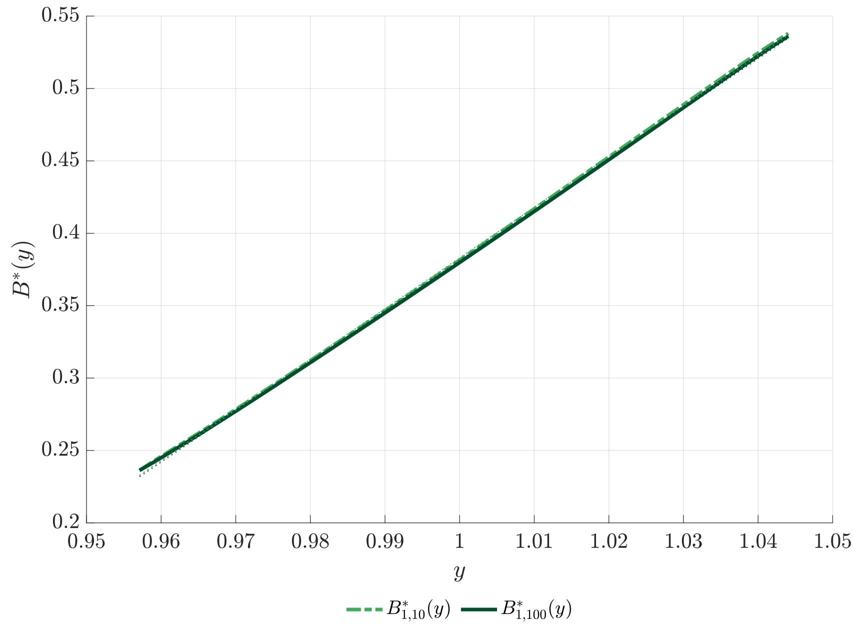


Figure 4: Monotonicity of Debt Threshold $B^*(y)$
Note: This figure illustrates Proposition 3 by showing the debt threshold $B^*(y)$ as a function of the endowment level y. The threshold represents the debt level at which the baseline ($\theta = 1$) and pessimistic ($\theta > 1$) bond price schedules intersect.

Spread These results on prices map directly and inversely to credit spreads. The sovereign credit spread, $s(y, B')$, is defined as the yield premium over the risk-free rate, r. Given the bond's price $q(y, B')$, the spread is:

$$s(y, B') = \frac{\kappa}{q(y, B')} - \delta - r. \tag{14}$$

This clear, inverse relationship allows the results from Proposition 2 to be restated for credit spreads.

Corollary 1. *Let $s_1(B', y)$ and $s_\theta(B', y)$ be the equilibrium credit spreads in the baseline ($\theta = 1$) and pessimistic ($\theta > 1$) economies, respectively. The spread difference $\Delta s(B', y) \equiv s_\theta(B', y) - s_1(B', y)$ satisfies the opposite relationship to the price difference at the same threshold $B^*(y)$ defined in Proposition 2:*

- *For low levels of future debt $B' < B^*(y)$, $\Delta s(B', y) > 0$.*
- *For high levels of future debt $B' > B^*(y)$, $\Delta s(B', y) < 0$.*

Proof. See Appendix B.7. □

Default Threshold How does a rational sovereign react to these altered market conditions? The first area where its behavior changes is at the edge of default. The following proposition establishes that lender pessimism paradoxically makes the sovereign more resilient to debt, pushing its default threshold to a higher level.[11] This contrasts with standard reputation models, where a sovereign with a worse reputation (i.e., a higher perceived probability of being a 'strategic' type) would typically be expected to default at a lower debt level.

Proposition 4. *Consider economies with pessimistic lenders ($\theta > 1$) and rational lenders ($\theta = 1$). Let $B^*_{D,i}(y)$ be the sovereign's default threshold for economy $i \in \{1, \theta\}$, defined as the debt level B that satisfies the indifference condition:*

$$V_i^R(B^*_{D,i}(y), y) = V^D(y) \quad \text{for } i \in \{1, \theta\}. \tag{15}$$

For any given endowment level y, the default threshold is higher in the economy with pessimistic lenders:

$$B^*_{D,\theta}(y) > B^*_{D,1}(y).$$

Proof. See Appendix B.8. □

The formal proof in Appendix B.8 proceeds by contradiction, showing that if $B^*_{D,\theta}(y) \leq B^*_{D,1}(y)$, then the "softening of doom" effect from Proposition 2 would make the value of repayment strictly higher under pessimism, violating the assumed threshold ordering. The

[11] This result differs fundamentally from the negative duration effect in (Chatterjee and Eyigungor, 2012). In their model, longer maturity reduces the sovereign's incentive to default because it dilutes existing bondholders, creating a debt dilution channel that works through the *quantity* of debt issued. In contrast, our mechanism operates through lender *beliefs* about the sovereign's decision-making process: pessimistic lenders offer better terms in high-debt states due to their perception of greater randomness in sovereign choices, making continued market access more valuable and pushing out the default threshold through a behavioral pricing channel rather than a debt structure effect.

economic mechanism is that pessimistic lenders offer better prices in high-debt scenarios, increasing the option value of remaining in markets and making sovereigns willing to endure higher debt burdens before defaulting.

The sovereign's decision to default is a trade-off between the immediate benefit of ceasing payments and the long-term cost of losing market access. The value of this access depends directly on future borrowing terms. Proposition 2 established the key result that when debt is already high, pessimistic lenders offer *better* prices (the"softening of doom" effect). A rational sovereign in the pessimistic economy foresees these more favorable future borrowing terms should it choose to repay. This increases the option value of repaying and rolling over debt. Because the prospect o is more attractive, the sovereign is willing to endure a higher debt burden before finally choosing to default, pushing its default threshold further out.

Borrowing Policy While pessimism makes the sovereign more resilient at the brink of crisis, it has the opposite effect on its day-to-day borrowing. The next proposition shows that a pessimistic market actively disciplines the sovereign into adopting a more conservative debt policy.

Proposition 5. *Consider economies with pessimistic lenders ($\theta > 1$) and rational lenders ($\theta = 1$). Let $\mathbb{E}_i[B'|y,B]$ be the expected next-period debt level chosen by the sovereign in economy $i \in \{1,\theta\}$ from a given state (y,B). For states (y,B) where the sovereign chooses **not to default**, the borrowing policy is systematically more conservative under lender pessimism:*

$$\mathbb{E}_\theta[B'|y,B] < \mathbb{E}_1[B'|y,B].$$

Proof. See Appendix B.9. □

The formal proof in Appendix B.9 proceeds by comparing the sovereign's first-order conditions for borrowing under the two price schedules. The key insight is that lower prices offered by pessimistic lenders in the primary borrowing range (as established in Proposition 2) reduce the marginal benefit of issuing new debt. Since the marginal cost of debt remains unchanged, the sovereign optimally chooses a lower debt level to restore equilibrium between marginal benefits and costs.

A rational sovereign government reacts optimally to the market prices it faces. In the region where the sovereign typically wants to borrow, pessimistic lenders offer lower prices for new debt. A lower bond price is a direct signal that borrowing has become more expensive. Faced with a higher cost of capital, the sovereign's optimal response is to borrow less. The pessimistic market, through its pricing, effectively "disciplines" the sovereign, forcing it to deleverage and adopt a more conservative fiscal policy than it would if it faced a rational market. This endogenous deleveraging is a key mechanism through which market beliefs shape real economic outcomes.

Welfare The final step in the theoretical analysis is to evaluate the net effect of these changes on the sovereign's well-being. The final proposition demonstrates that the consequences of lender pessimism translate directly into a welfare loss for the sovereign.

Proposition 6. *Let $V_i(y,B)$ denote the sovereign's ex-ante equilibrium value function in economy $i \in \{1,\theta\}$, where $\theta > 1$ indexes pessimistic lenders and $\theta = 1$ rational lenders. Suppose that for the given state (y,B) the sovereign has market access and the baseline optimal choice $B'_1(y,B)$ lies on the "risky" side of the price pivot $B^*(y)$, i.e. $B'_1(y,B) \geq B^*(y)$. Then equilibrium welfare is strictly lower under pessimistic lenders:*

$$V_\theta(y,B) < V_1(y,B).$$

If $B'_1(y,B) \leq B^*(y)$ ("safe" side), the weak inequality $V_\theta(y,B) \leq V_1(y,B)$ holds.

Proof. See Appendix B.10. □

The formal proof in Appendix B.10 proceeds using operator theory to show that pessimistic pricing systematically reduces the sovereign's choice-specific value for all borrowing decisions. The key insight is that welfare loss stems directly from a tighter budget constraint: for any given amount of new borrowing, the sovereign receives fewer resources today under pessimistic lenders. While sovereigns adjust policies optimally (borrowing less, tolerating higher debt before default), they cannot fully escape the welfare loss from transacting with distorted markets.

The sovereign's welfare is fundamentally derived from its ability to use international financial markets to smooth consumption over time. The terms of this access are dictated by the bond price schedule, $q(y,B')$, which can be seen as the price of intertemporal trade. Lender pessimism, by inducing a lower q in the low debt ($B' < B^*(y)$) region, effectively acts as a tax on the sovereign's ability to conduct this trade. For any given amount of borrowing, the sovereign receives fewer resources today, which directly curtails its consumption possibilities and lowers utility. While the sovereign optimally adjusts its policies in response—by borrowing less and tolerating more debt before default—it cannot fully escape the welfare loss imposed by being forced to transact with a paranoid market. The "benefit" of better prices in the high debt ($B' > B^*(y)$) region is an option too remote and uncertain to compensate for the welfare losses incurred due to worse prices in the normal course of borrowing.

The Causal Chain of Pessimism The theoretical results presented above follow a clear causal chain originating from the single shock of lender pessimism ($\theta > 1$). This shift in beliefs first and foremost reshapes the market environment by altering the bond price schedule, causing it to *pivot* as established in Proposition 2. The inverse pivoting of the credit spread schedule, described in Corollary 1, is an immediate algebraic consequence.

In response to this new pricing reality, the rational sovereign optimally adjusts its policies. It leverages the "softening of doom" effect in the high debt region—where pessimistic lenders offer paradoxically better prices—to endure a greater debt burden before defaulting (Proposition 4). Simultaneously, it reacts to the "pessimism premium" in the primary borrowing region by systematically deleveraging and adopting a more conservative debt policy (Proposition 5). This set of constrained-optimal policy adjustments, however, cannot fully overcome the handicap of transacting with a paranoid market, culminating in awelfare loss for the sovereign (Proposition 6).[12]

Figure 5 illustrates this causal mechanism, showing how a single behavioral friction propagates through the economy's equilibrium relationships. To crystallize the novel con-

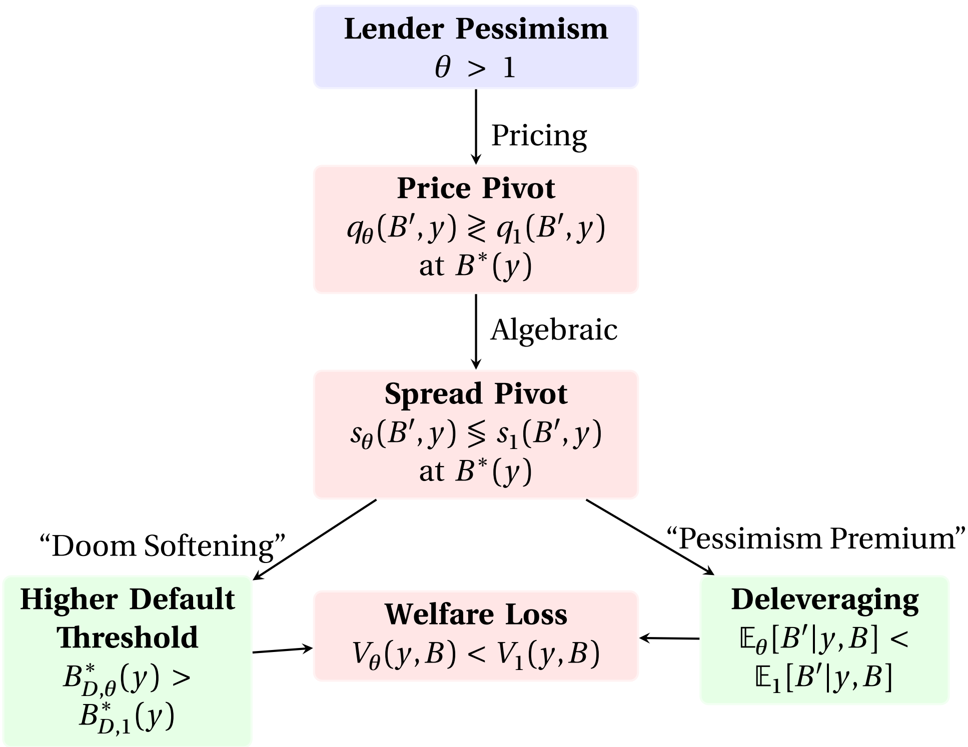


Figure 5: The Causal Chain of Lender Pessimism

Note: This diagram illustrates how a single behavioral friction ($\theta > 1$) propagates through the economy. Lender pessimism first alters market pricing, creating a pivoting bond price schedule. The sovereign optimally responds to this new environment, but cannot fully escape the welfare costs of dealing with biased lenders.

tributions of this behavioral channel, Table 1 explicitly contrasts the key predictions of the pessimism model with those of a standard reputation model.

[12] This welfare loss is fundamental and cannot be eliminated even by optimal fiscal policy. Section 6.1 develops a formal Ramsey planning extension (Proposition 7) showing that the pessimism-induced distortion of intertemporal prices creates deadweight losses that persist beyond what lump-sum transfers can correct.

Table 1: Theoretical Predictions: Pessimism vs. Reputation

Prediction Dimension	Reputation Model (Cole et al., 1995; Morelli and Moretti, 2023)	Pessimism Model (This Paper)
Price Curve, q	Monotonically *lower*	*Pivots* around baseline (q_1)
Default Threshold, B_D^*	*Lower*	*Higher*: $B_{D,\theta}^* > B_{D,1}^*$
Expected Borrowing, $\mathbb{E}[B']$	*Lower* (Constraint-driven)	*Lower* (Price-driven)
Average Spread, $\mathbb{E}[s]$	*Higher*	*Higher*

Note: This table compares the main theoretical predictions of a standard model where lenders learn about the sovereign's hidden type (reputation) with the behavioral model in this paper where lenders are systematically pessimistic about policy randomness. The bolded items highlight the unique and often counter-intuitive predictions of the pessimism model.

5 Quantitative Analysis

In this section, I describe the quantitative implementation of the model. I first outline the calibration of the model parameters and the numerical strategy used to solve for the equilibrium. Then, I present the business cycle properties generated by the baseline model and show that it successfully replicates key features of emerging market economies.

5.1 Calibration and Numerical Solution

Calibration The model is calibrated at a quarterly frequency. The parameter values are chosen to be consistent with the sovereign default literature and to broadly match the macroeconomic features of a typical emerging economy, such as Argentina. The parameters are summarized in Table 2.

The preference and endowment parameters are standard. The risk aversion coefficient σ is set to 2. The discount factor β is set to 0.9775, implying an annual real interest rate of approximately 9.5% when combined with the model's growth, which is a common value for emerging economies. The logarithmic endowment process is modeled as an AR (1) with a persistence of $\rho_y = 0.95$ and an innovation standard deviation of $\sigma_y = 0.005$.

The debt structure parameters are set to achieve a target Macaulay duration of 5 years (20 quarters) for a risk-free bond, which implies a quarterly principal decay rate of $\delta = 0.04$. The coupon rate κ is set to equal $\delta + r$ so that the price of a risk-free bond is normalized to one. The probability of re-entering credit markets after a default, γ, is set to 0.125, implying an average exclusion period of 2 years (8 quarters). The output cost of default, governed by λ_0 and λ_1, is specified to be nonlinear, consistent with the findings of (Chatterjee and Eyigungor, 2012).[13]

The scale parameters of the Gumbel taste shocks, η and ρ, are set to small values to

[13]The specific values are $\lambda_0 = -0.48$ and $\lambda_1 = 0.525$, calibrated to match the severity and shape of output losses observed in historical default episodes.

ensure that decisions are primarily driven by economic fundamentals, while still ensuring the stability and tractability of the numerical solution.[14]

Table 2: Baseline Calibration (Quarterly)

Parameter	Value	Description
Preferences and Endowments		
σ	2.0	CRRA coefficient of relative risk aversion
β	0.9775	Sovereign's discount factor
ρ_y	0.95	Persistence of log endowment AR(1)
σ_y	0.005	Std. dev. of endowment innovations
Debt and Default		
r	0.01	Quarterly risk-free interest rate (4% ann.)
δ	0.04	Principal decay rate (for 5-year duration)
κ	0.05	Coupon rate ($\delta + r$)
γ	0.125	Re-entry probability (avg. 2-year exclusion)
λ_0, λ_1	-0.48, 0.525	Output cost function parameters
Computational Parameters		
η	5×10^{-4}	Scale of default taste shock
ρ	1×10^{-5}	Scale of borrowing taste shock
θ	1.0	Baseline lender pessimism coefficient

Note: The table presents the parameter values used in the baseline calibration of the model. Parameters are set to match standard values in the sovereign default literature and key macroeconomic features of emerging economies like Argentina.

[TODO: Add parameter-target correspondence table showing which parameter targets which moment/statistic, as suggested by reviewer to improve transparency of calibration strategy]

Numerical Solution I solve the model numerically using value function iteration on a discretized state space. The state space consists of the sovereign's current endowment y and its outstanding debt level B.

The endowment process in (1) is discretized into $N_y = 201$ states using Tauchen's method. The state space for debt, B, is represented by a uniform grid of $N_B = 600$ points, ranging from 0 to 75% of mean output.

The solution method iterates on the value functions (V, V^D, V^R) and the bond price function (q) until they converge to a joint fixed point. A key feature of the numerical strategy is the use of the log-sum-exp formulation for choices subject to Gumbel taste shocks. This technique replaces the non-differentiable ‘max‘ operator with a smooth, analytical expression, which greatly improves the stability and speed of the algorithm by obviating the need for numerical maximization routines at each grid point. For further numerical

[14]Specifically, $\eta = 5 \times 10^{-4}$ for default decisions and $\rho = 1 \times 10^{-5}$ for borrowing decisions. These small values maintain the primacy of economic fundamentals while providing computational tractability through the log-sum-exp formulation.

robustness, I employ stabilized log-sum-exp implementations that prevent floating-point overflow and underflow errors that could arise from the small taste shock parameters.[15] The entire solution algorithm is implemented in Fortran and parallelized using OpenMP to leverage multi-core processors.

5.2 Business Cycle Implications of Lender Pessimism

To understand the quantitative implications of lender pessimism, I simulate the model under three scenarios: the baseline rational-expectations benchmark ($\theta = 1$), a medium-pessimism case ($\theta = 10$), and a high-pessimism case ($\theta = 100$).[16] Table 3 reports the key business cycle moments from these simulations, revealing how pessimism reshapes macroeconomic behavior.

The Rational Benchmark The baseline model with rational lenders ($\theta = 1$) successfully generates results that are broadly consistent with the stylized facts for emerging economies. The average debt-to-GDP ratio is a moderate 7.90%, and the sovereign pays an average annualized credit spread of 2.00%. Consistent with the empirical literature, the model produces consumption that is more volatile than output, counter-cyclical credit spreads (correlation of -0.43 with ln(GDP)), and a slightly counter-cyclical trade balance. When output falls, default risk rises, increasing spreads; simultaneously, the government attempts to borrow to smooth the shock, worsening the trade balance. These features confirm that the model provides a standard and reasonable benchmark against which to evaluate the effects of pessimism.

Deleveraging and the Price of Fear The introduction of lender pessimism dramatically alters these outcomes, but in a non-linear fashion. Under moderate pessimism ($\theta = 10$), the sovereign's average debt level (5.53%) and borrowing cost (2.75%) remain remarkably close to the baseline. However, a shift to high pessimism ($\theta = 100$) triggers a stark deleveraging and a significant increase in average borrowing costs. The mean debt-to-GDP ratio falls precipitously by nearly 5 percentage points to just 2.70%. This is a direct consequence of the market discipline predicted in Proposition 5: faced with worse prices in the primary borrowing region, the sovereign optimally reduces its debt issuance. However, this conservative policy does not earn it lower interest rates. Instead, the average credit spread more than doubles to 4.15%. This result—deleveraging in the face of even higher average spreads—starkly illustrates the power of the behavioral bias. In a pure reputation model,

[15] A detailed discussion of these numerical stability techniques is provided in Appendix C.4.

[16] The choice of $\theta = 10$ and $\theta = 100$ as medium and high pessimism cases is motivated by the need to demonstrate clear quantitative differences while maintaining computational tractability. Intermediate values such as $\theta = 30$ or $\theta = 50$ could also be examined to show the continuous nature of the relationship.

deleveraging should lower risk and borrowing costs; here, it is a constrained-optimal response to the "pessimism premium," a penalty the sovereign can never fully escape simply by reducing its debt. Lenders demand substantial compensation for the perceived risk of an "out of the blue" default, an effect that overwhelms the fact that the sovereign is, in reality, safer due to its lower debt level.

Amplified Financial Cycles Pessimism not only raises the level of borrowing costs but also progressively amplifies their cyclicality. The correlation between credit spreads and GDP becomes more negative as pessimism increases, moving from -0.43 in the baseline to -0.80 under moderate pessimism, and then sharply to -0.89 in the high-pessimism case. This indicates that financial conditions become exquisitely sensitive to fluctuations in the country's income. When a negative shock hits, pessimistic lenders' fears are magnified, leading to a much sharper spike in spreads than would occur in a rational market. This tightening of financial conditions occurs precisely when the sovereign needs market access the most, exacerbating the downturn and making financial markets a powerful source of procyclical shocks rather than a tool for macroeconomic stabilization.

An Illusion of Stability Interestingly, the volatility of both the debt-to-GDP ratio and credit spreads decreases as pessimism rises. This is not a sign of improved stability but rather a mechanical result of the sovereign's forced deleveraging, creating an *illusion of financial stability.* By maintaining a lower average debt level, the sovereign operates further away from its default threshold (which, paradoxically, is higher, per Proposition 4). This reduces the frequency of episodes of high debt and soaring spreads, leading to lower overall volatility in these financial variables, even as the average spread remains high. Despite these large changes in financial markets, the impact on consumption volatility is minimal. The sovereign adapts to the harsher borrowing environment by reducing its reliance on foreign debt for consumption smoothing, effectively trading away the benefits of international financial integration for a quieter, but more expensive, life.

5.3 The Mechanics of Pessimism: Distributions and Policy Functions

Long-Run Outcomes: A Shift in Distributions The aggregate business cycle statistics in Table 3 are the result of fundamental shifts in the sovereign's equilibrium behavior, which are best understood by examining the model's policy functions and resulting stationary distributions. Figure 6 plots the simulated histograms for debt and credit spreads, revealing the long-run consequences of lender pessimism. Panel (a) starkly illustrates the deleveraging predicted by Proposition 5. The entire distribution of the debt-to-GDP ratio shifts dramatically to the left, representing a strategic retreat from international capital markets. This is not an arbitrary choice but the sovereign's optimal response to the

Table 3: Business Cycle Implications of Lender Pessimism

Moment	Baseline ($\theta = 1$)	Med. ($\theta = 10$)	High ($\theta = 100$)
Mean and Volatility			
Mean Debt-to-GDP Ratio (%)	7.90	5.53	2.70
Std. Dev. of Debt-to-GDP Ratio (%)	0.87	0.85	0.74
Mean Spread (annualized, %)	2.00	2.75	4.15
Std. Dev. of Spread (annualized, %)	0.77	0.49	0.58
Std. Dev. of ln(Consumption) (%)	3.48	3.53	3.41
Std. Dev. of ln(GDP) (%)	3.04	3.19	3.19
Mean Trade Balance/GDP (%)	0.42	0.32	0.18
Std. Dev. of Trade Balance/GDP (%)	0.51	0.43	0.32
Correlations			
Corr(Spread, ln(GDP))	-0.43	-0.80	-0.89
Corr(Trade Balance/GDP, ln(GDP))	-0.28	-0.28	-0.26
Corr(Debt/GDP, ln(GDP))	0.70	0.79	0.84

Note: The table reports moments from a long simulation of the model (100,000 periods after a 1,000-period burn-in). Spreads are annualized. All other variables are in quarterly terms.

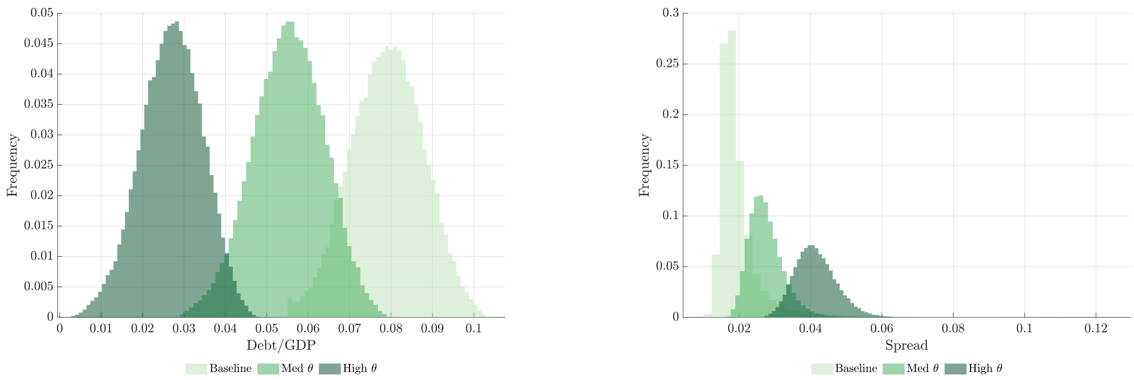
[TODO: Add robustness checks showing results remain consistent under longer simulations and different random seeds]

punitive pricing it faces. Faced with a market that consistently overestimates its risk, the government is disciplined into a permanently more conservative fiscal stance.

This retreat, however, does not earn the sovereign better credit terms. Panel (b) reveals the central paradox: as the sovereign deleverages, its average borrowing cost increases. The entire distribution of credit spreads is pushed to the right. This is the tangible result of the ”pessimism premium” described in Corollary 1. The sovereign is forced into a low-debt trap where, despite being fundamentally safer due to its lower leverage, it faces a persistently higher cost of capital because lenders' pessimistic beliefs dominate their assessment of fundamentals.

State-Contingent Policies: The Pivoting Effect The long-run distributional shifts are driven by state-by-state changes in the sovereign's optimal policies, which are themselves a reaction to the altered price schedule. Figure 7 visualizes how pessimism causes key functions to ”pivot” around the rational benchmark, providing a graphical confirmation of the paper's central theoretical results.

Panels 7a and 7b illustrate the core price pivot. At low debt levels, where a rational lender sees little risk, the pessimistic lender prices in the possibility of an ”out of the blue” default, leading to lower prices and higher spreads. At very high debt levels, where a rational lender sees default as nearly certain, the pessimistic lender's belief in randomness allows for a small chance of an ”irrational” repayment, leading to paradoxically better terms (the ”softening of doom” effect). This pivot in the price schedule is the key external force



(a) Debt-to-GDP Ratio Distribution

(b) Credit Spread Distribution

Figure 6: Simulated Stationary Distributions

Note: The distributions are generated from a long simulation of the model (100,000 periods). The figure shows how rising lender pessimism progressively shifts the long-run distribution of the debt-to-GDP ratio to the left (deleveraging) and the credit spread distribution to the right (higher borrowing costs).

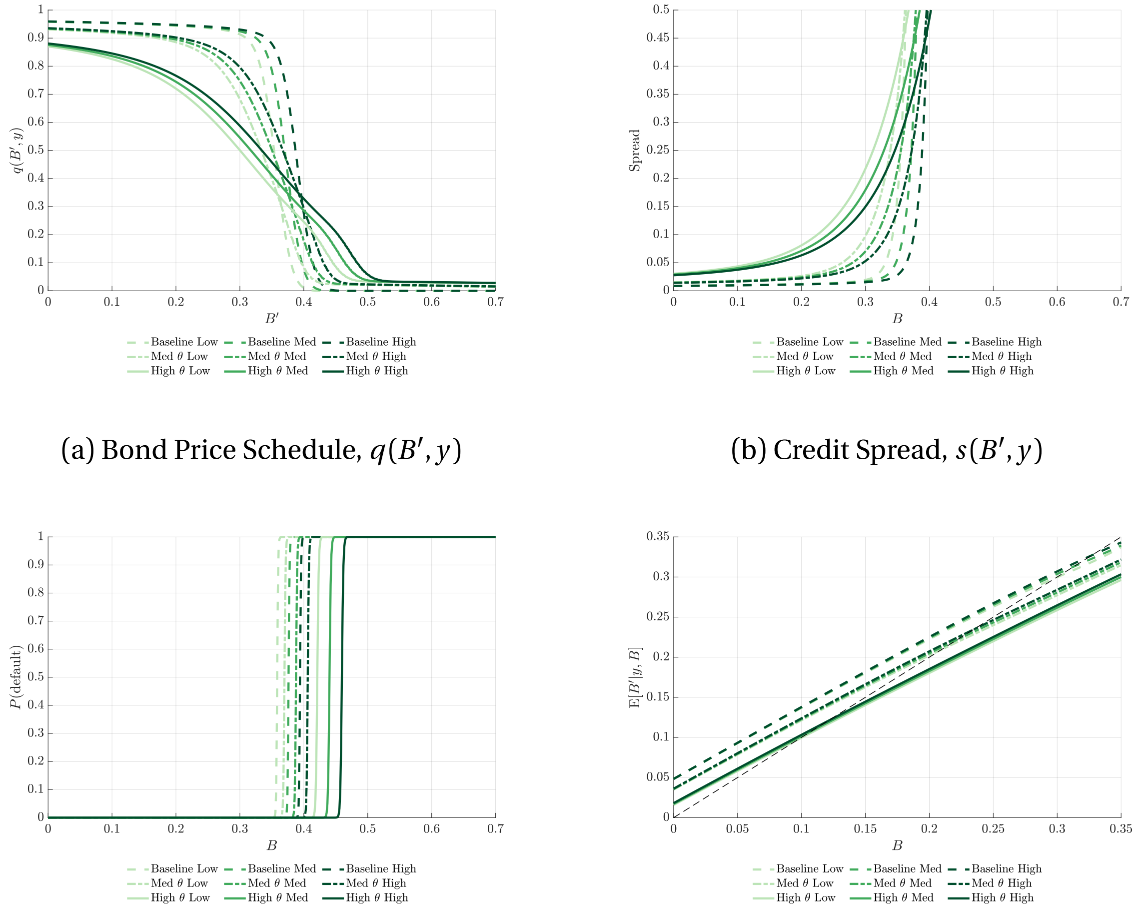
acting on the sovereign, and it offers a richer dynamic than the simple downward price shift one would expect from a pure reputation loss.

Panels 7c and 7d show the sovereign's endogenous response. The government internalizes the new price schedule. The better terms available in the high-debt region increase the value

of maintaining market access, making the sovereign more resilient and pushing out its default threshold (Proposition 4). More importantly, in the normal course of borrowing, the sovereign faces worse prices, which act as a higher effective cost of capital. The optimal response, shown in Panel 7d, is to deleverage and choose a lower next-period debt level for any given state (Proposition 5).

Welfare Consequences The sovereign's policy adjustments—deleveraging and tolerating higher debt before default—are optimal given the market it faces, but they cannot overcome the fundamental handicap of dealing with paranoid lenders. Proposition 6 predicted a direct welfare loss, a result powerfully confirmed by Figure 8. The sovereign's value function is uniformly and significantly lower in the pessimistic economy.

This welfare loss stems from the impairment of the sovereign's ability to smooth consumption. Access to international credit markets is a tool to buffer domestic shocks. Pessimism effectively places a tax on the use of this tool. By making borrowing more expensive in the relevant range, it forces the sovereign to either endure more volatile consumption or to self-insure by maintaining an inefficiently low level of debt. The "benefit" of better prices in the far-off, high-risk region is an option that is too remote and uncertain to compensate for the persistent, day-to-day welfare losses incurred from being forced to transact



(a) Bond Price Schedule, $q(B', y)$

(b) Credit Spread, $s(B', y)$

(c) Default Probability, $P(d = 1|B, y)$

(d) Borrowing Policy, $\mathbb{E}[B'|B, y]$

Figure 7: Policy Function Pivoting

Note: The plots show the key policy and pricing functions for three different levels of endowment y (low, medium, and high). The functions for the baseline ($\theta = 1$, dashed), medium pessimism ($\theta = 10$, dash-dot), and high pessimism ($\theta = 100$, solid) cases are shown. Rising pessimism causes the functions to pivot.

with a market that systematically overestimates its propensity to fail.

5.4 Dynamic Responses and Adjustment Paths

The dynamic implications of lender pessimism reveal how pessimism affects the sovereign's adjustment paths and responses to shocks.

Deleveraging Dynamics: The Transition to Lower Debt Figure 9 traces optimal debt adjustment paths under different initial conditions and degrees of lender pessimism. Consistent with Proposition 5, economies with higher pessimism systematically converge to lower debt levels regardless of starting point. Remarkably, even from low initial debt, the high-pessimism economy continues deleveraging, representing a fundamental fiscal shift rather than temporary adjustment.

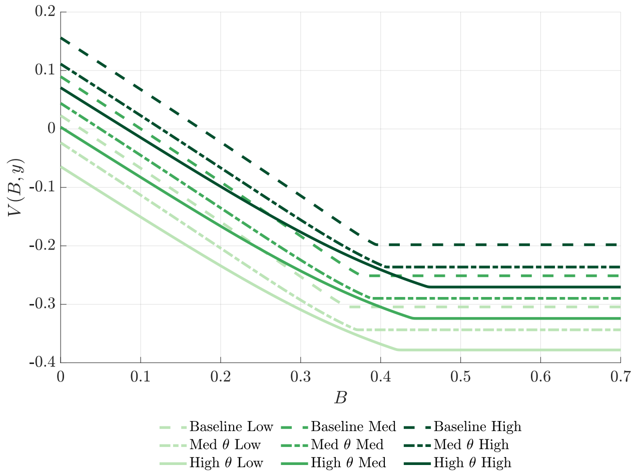


Figure 8: Welfare Loss

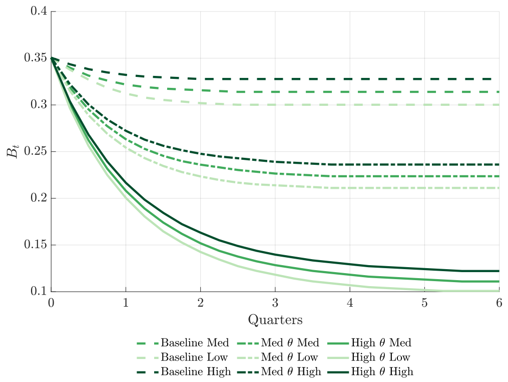
Note: The figure plots the sovereign's ex-ante value function $V(y, B)$ for the baseline ($\theta = 1$, dashed), medium pessimism ($\theta = 10$, dash-dot), and high pessimism ($\theta = 100$, solid) economies. The value function is uniformly lower under higher degrees of pessimism, indicating a progressive welfare loss.

The consumption and spread dynamics reveal important adjustment costs. Panel 9b shows that high-pessimism economies experience more volatile consumption during deleveraging despite ultimately achieving lower debt. Panel 9c demonstrates that spreads remain persistently elevated throughout adjustment, confirming that the "pessimism premium" reflects systematic risk overestimation at all debt levels rather than just current leverage.

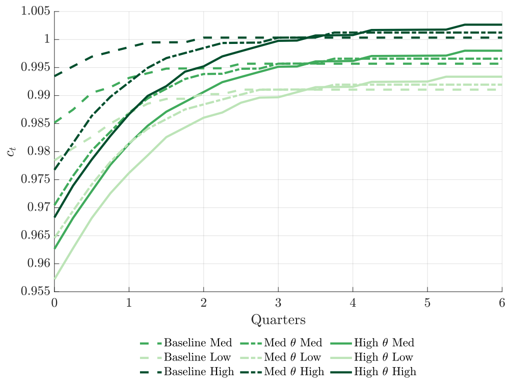
Impulse Response Functions: Shock Propagation under Pessimism I examine responses to transitory and persistent productivity shocks (AR(1) with $\rho = 0.8$) to understand how lender pessimism affects the sovereign's shock absorption capacity.

Transitory Shock Responses. Figure 10 shows responses to a 3% positive productivity shock lasting one quarter. While output effects are identical by construction (panel 10a), pessimism fundamentally alters other responses. High-pessimism economies exhibit muted debt reduction (panel 10b) and consumption smoothing responses (panel 10c), illustrating impaired ability to exploit temporary favorable conditions. Despite facing identical shocks, these economies cannot fully capitalize on good fortune due to persistently unfavorable credit pricing. Spreads fall in all economies (panel 10d), but high-pessimism economies maintain higher levels throughout adjustment.

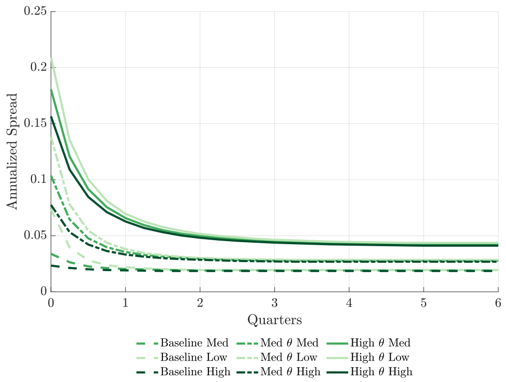
Persistent Shock Responses. Figure 11 shows responses to persistent productivity shocks ($\rho = 0.8$). Persistence matters more for high-pessimism economies, which exhibit more pronounced and sustained debt reduction (panel 11b) as sovereigns recognize rare



(a) Debt



(b) Consumption



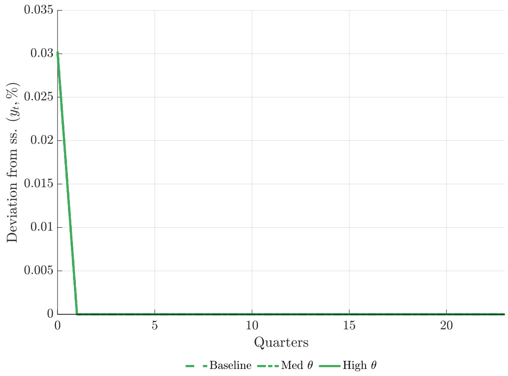
(c) Spread

Figure 9: Deleveraging Dynamics

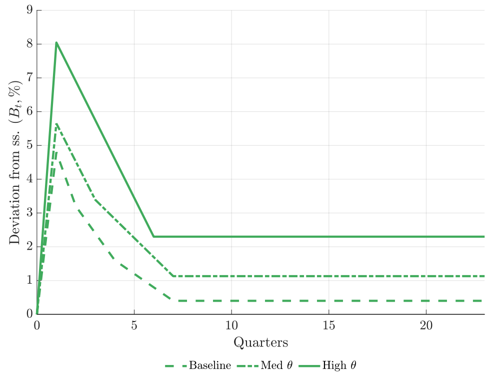
Note: The figure shows optimal adjustment paths over 6 quarters starting from three different initial debt levels (low, medium, high) for each degree of lender pessimism. The paths demonstrate how pessimism leads to systematic deleveraging that persists regardless of initial conditions, accompanied by persistently higher spreads and more volatile consumption during the transition.

opportunities to escape the "high-spread trap." While persistent shocks enable better consumption smoothing across all economies (panel 11c), high-pessimism economies still underperform due to fundamental impairment of consumption insurance. Spread responses (panel 11d) are more persistent than in the transitory case, but high-pessimism economies maintain higher levels throughout adjustment.

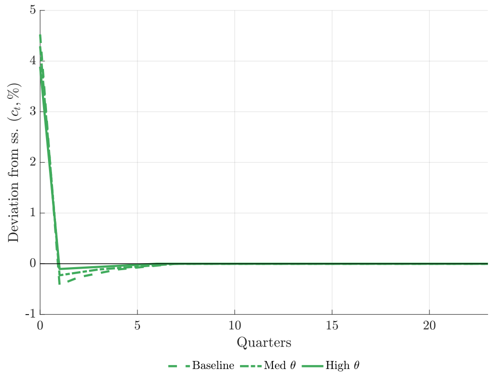
Implications The dynamic analysis reveals three fundamental economic mechanisms. First, deleveraging under pessimism creates persistent allocative distortions—the adjustment process itself becomes a source of inefficiency as elevated spreads persist throughout transition, generating deadweight losses that compound over time. This represents a departure from standard models where adjustment costs are temporary. Second, pessimism creates asymmetric shock transmission: sovereigns experience constrained bene-



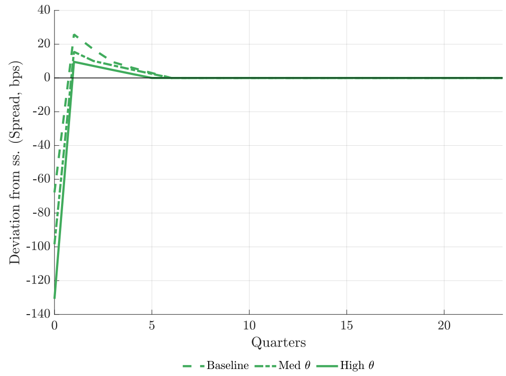
(a) Output



(b) Debt



(c) Consumption

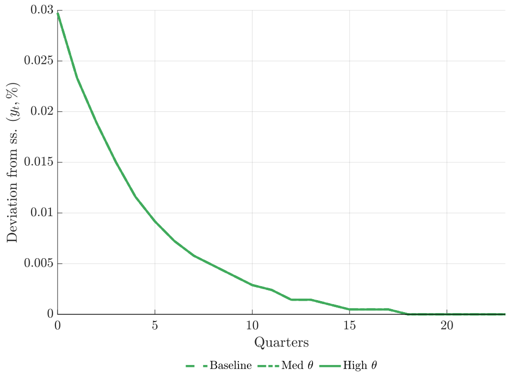


(d) Spread

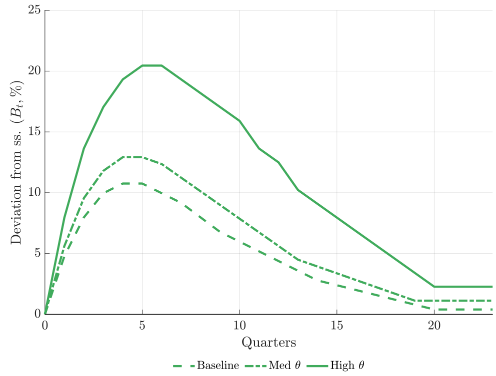
Figure 10: Impulse Responses to Transitory Productivity Shock

Note: The figure shows responses to a 3% positive productivity shock that lasts for one quarter. All variables are expressed as percentage deviations from their respective steady states (spreads in basis points). The responses demonstrate how lender pessimism constrains the sovereign's ability to take advantage of temporary favorable conditions.

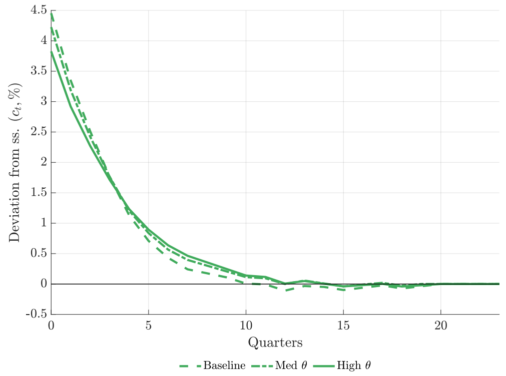
fits from favorable shocks while facing amplified costs from adverse ones, fundamentally altering the risk-return profile of sovereign borrowing. This asymmetry suggests that traditional moments-based calibrations may understate welfare costs. Finally, the interaction between persistence and beliefs generates hysteresis effects: temporary improvements in fundamentals produce limited deleveraging, while sustained improvements are necessary to overcome entrenched pessimistic priors.



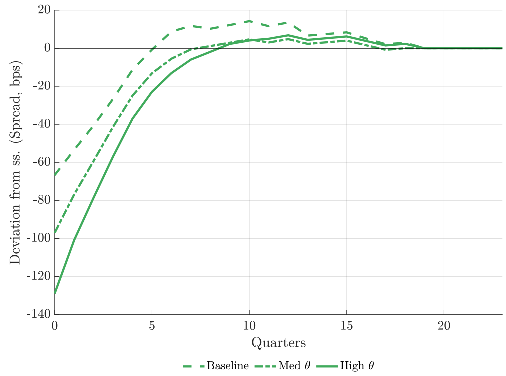
(a) Output



(b) Debt



(c) Consumption



(d) Spread

Figure 11: Impulse Responses to Persistent Productivity Shock

Note: The figure shows responses to a 3% productivity shock with autocorrelation $\rho = 0.8$. All variables are expressed as percentage deviations from steady state (spreads in basis points). The persistent nature of the shock reveals how lender pessimism constrains fiscal flexibility even during extended periods of favorable fundamentals.

6 Extensions

6.1 Theoretical Extensions

Ramsey Planning Under Pessimism Proposition 6 shows that lender pessimism reduces equilibrium welfare. A natural question is whether an optimal fiscal authority can undo this loss. We study a Ramsey planner who can choose lump-sum taxes/transfers but takes the bond pricing mechanism as given.

Lender pessimism acts like a persistent wedge in the intertemporal price of borrowing. Transfers can reallocate resources within and across periods subject to a zero present-value constraint, but they cannot change the shadow price at which the government trades across dates. Hence the pessimistic price schedule distorts debt choices even under optimal fiscal policy.

The planner chooses $\{c_t, B_{t+1}, \tau_t\}_{t\geq 0}$ to maximize expected utility subject to the per-period resource constraint and a zero present-value (PV) condition for transfers:

$$c_t + \kappa B_t + \tau_t \;=\; y_t + \big(B_{t+1} - (1-\delta)B_t\big)\, q_\theta(y_t, B_{t+1}), \qquad t \geq 0, \text{ a.s.} \tag{16}$$

and

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \tau_t\right] = 0, \quad c_t \geq 0, \quad B_{t+1} \in \mathscr{B}. \tag{17}$$

Proposition 7. *Let $r > -1$ satisfy $\beta(1+r) = 1$. For $i \in \{1, \theta\}$ define the Ramsey value*

$$W_i^R \;=\; \sup_{\{c_t, B_{t+1}, \tau_t\}} \mathbb{E}_0\Big[\sum_{t=0}^{\infty} \beta^t u(c_t)\Big]$$

subject to (16)–(17). Assume:

(A1) u is strictly increasing and strictly concave; $\mathscr{B}$ is compact.

(A2) (Price dominance by trade sign, consistent with Propositions 2 and 5). Let $\Delta_t \equiv B_{t+1} - (1-\delta)B_t$. For every history,

$$\Delta_t \geq 0 \;\Rightarrow\; q_1(y_t, B_{t+1}) \geq q_\theta(y_t, B_{t+1}), \qquad \Delta_t \leq 0 \;\Rightarrow\; q_1(y_t, B_{t+1}) \leq q_\theta(y_t, B_{t+1}),$$

with strict inequality on a set of positive probability and $\Pr(|\Delta_t| > 0) > 0$.

Then $W_\theta^R < W_1^R$.

Proof. See Appendix B.11. □

Transfers ensure first-best *conditional* consumption smoothing but cannot change the intertemporal proceeds term $\mathbb{E}_0[\sum \beta^t \Delta_t\, q_i(y_t, B_{t+1})]$ that appears in the implementability constraint; therefore the wedge in q_θ generates a genuine efficiency loss. See also (Aiyagari et al., 2002).

Endogenous Belief Formation We next let lenders learn about the taste-shock dispersion parameter θ.

Even with Bayesian updating, "negativity bias"—overweighting unexpected defaults relative to unexpected repayments—can make pessimism persistent and self-reinforcing. Rare but salient defaults move beliefs more than frequent, modestly good realizations, creating a drift toward pessimism.

Let $\theta_t \in [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} = 1 < \bar{\theta}$, and suppose beliefs evolve according to

$$\theta_{t+1} \;=\; \lambda\, \theta_t + (1-\lambda)\, \widehat{\theta}(\{d_s\}_{s=0}^{t}), \qquad \lambda \in (0,1), \tag{18}$$

where $d_t \in \{0,1\}$ indicates default and $\widehat{\theta}(\cdot)$ is a (possibly biased) estimator that places extra weight on *surprising* defaults. For instance, with

$$\xi(y,B) = \max\left\{0,\ \frac{P_1(y,B) - P_{\theta_t}(y,B)}{P_1(y,B)}\right\}, \tag{19}$$

negativity bias means that $\widehat{\theta}$ responds more to larger ξ when $d_t = 1$ than to the analogous repayment surprise when $d_t = 0$ (Baumeister et al., 2001; Bordalo et al., 2018).

Proposition 8. *Under a stable, Feller belief-update kernel induced by (18) with negativity bias as above and mild drift/minorization conditions, the Markov process $\{\theta_t\}$ admits a unique invariant distribution Θ^* such that:*

- ***(i)*** $\mathbb{E}[\Theta^*] > 1$ *(persistent pessimism);*
- ***(ii)*** Θ^* *first-order stochastically dominates the rational benchmark (history dependence through rare defaults);*
- ***(iii)*** $\|\mathbb{E}[\theta_t] - \mathbb{E}[\Theta^*]\| = O(\lambda^t)$*, with slow convergence when defaults are rare (large λ).*

Proof. See Appendix B.12. □

Optimal Policy Communication Now allow the government to choose a transparency level $\alpha \in [0,1]$ that affects lenders' effective dispersion:

$$\theta_{\text{eff}}(\alpha,\theta) = \alpha \cdot 1 + (1-\alpha)\cdot\theta, \tag{20}$$

and let period utility be $u(c,\alpha) = c^{1-\sigma}/(1-\sigma) - \phi(\alpha)$ with $\phi(\alpha) = \gamma\alpha^2/2$.

Proposition 9. *Let $W(\alpha)$ be the sovereign's value when the price schedule is $q_{\theta_{\text{eff}}(\alpha,\theta)}$ and the planner otherwise solves the same Ramsey problem. Suppose $W(\alpha)$ is strictly concave in α (e.g., q_ϑ is smooth and ϕ is strictly convex). Then:*

- ***(i)*** *There exists a unique $\alpha^* \in [0,1]$ satisfying the FOC*

$$\frac{d}{d\alpha}W(\alpha^*) = \phi'(\alpha^*) = \gamma\,\alpha^*. \tag{21}$$

- ***(ii)*** *If $\partial W/\partial\vartheta < 0$ and $\partial\theta_{\text{eff}}/\partial\alpha = 1-\theta < 0$, then $\partial\alpha^*/\partial\theta > 0$: more pessimism raises optimal transparency.*
- ***(iii)*** *There exists $\theta_c > 1$ such that $W(\alpha^*) > W(0)$ for all $\theta > \theta_c$.*

Proof. See Appendix B.13. □

Transparency improves welfare by steepening lenders' price schedule toward the rational benchmark, but convex disclosure costs imply an interior optimum. The comparative static in (ii) formalizes that transparency is most valuable when pessimism is severe.

7 Conclusion

Standard models struggle to explain why emerging economies face persistently high borrowing costs. This paper develops a quantitative sovereign default model with a behavioral friction—lender pessimism—to address this puzzle. I assume lenders systematically overestimate the randomness of the sovereign's policy choices. Theoretically, this pessimism wedge does not uniformly depress bond prices but instead *pivots* the price schedule, making debt cheaper at the edge of default but more expensive in normal times. A rational sovereign responds to these altered incentives in counterintuitive ways: it tolerates a higher debt burden before defaulting, yet simultaneously deleverages its day-to-day borrowing. Quantitatively, the model shows that high pessimism can force the debt-to-GDP ratio down from 7.90% to 2.70%, while more than doubling the average credit spread from 2.00% to 4.15%. This deleveraging creates an "illusion of financial stability," where observable market volatility falls even as financial cycles are amplified and the sovereign's welfare declines.

The theoretical extensions provide additional insights. Even optimal Ramsey fiscal policy cannot eliminate the welfare costs of pessimism because the distorted bond pricing creates fundamental allocative distortions that transfers cannot correct. When beliefs are formed endogenously through Bayesian learning with negativity bias, pessimistic perceptions become self-reinforcing and persist over time, explaining why some economies face chronically high borrowing costs. However, governments can partially mitigate these effects through strategic policy communication, choosing optimal transparency levels to reduce perceived uncertainty while managing the costs of information disclosure. My work thus offers a new, behaviorally-grounded perspective on sovereign risk, demonstrating how market beliefs can be a fundamental driver of debt crises and highlighting the importance of optimal policy design in managing these behavioral frictions.

Appendices for “Default with Pessimism”

Chen Gao

September 2, 2025

This appendix contains the details of the Gumbel distribution and the proofs for the lemmas and propositions in the paper. Section A contains the details of the Gumbel distribution. Section B contains the proofs for the lemmas and propositions in the paper. Section C contains the computational algorithm details for the model, including the numerical stability techniques employed in the discrete choice implementation.

A Gumbel Distribution in Default Models

In this section, I provide some useful results about the Gumbel distribution to help formulate the close form solution presented in Section 3.

Lemma 1. *Let $\varepsilon_1, \varepsilon_2$ be independent random variables distributed as Gumbel$(-\eta\gamma, \eta)$, where γ is the Euler-Mascheroni constant. Let $V_1, V_2 \in \mathbb{R}$ be deterministic constants. Then:*

$$\mathbb{E}\left[\max\{V_1+\varepsilon_1, V_2+\varepsilon_2\}\right] = \eta \ln\left(\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}\right). \tag{A.1}$$

Proof. See Appendix B.1. □

Lemma 2. *Let $\varepsilon_1, \varepsilon_2$ be independent Gumbel$(-\eta\gamma, \eta)$ random variables, and let $V_1, V_2 \in \mathbb{R}$ be deterministic values. Then:*

$$\Pr\{V_1+\varepsilon_1 > V_2+\varepsilon_2\} = \frac{\exp\frac{V_1}{\eta}}{\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}}. \tag{A.2}$$

Proof. See Appendix B.2. □

Lemma 3. *Let $\{V_i\}_{i=1}^n$ be deterministic values and $\{\varepsilon_i\}_{i=1}^n$ be independent Gumbel$(-\sigma\gamma, \sigma)$ random variables. Then:*

$$\mathbb{E}\left[\max_{i\in\{1,\dots,n\}}\{V_i+\varepsilon_i\}\right] = \sigma \ln\left(\sum_{i=1}^n \exp\frac{V_i}{\sigma}\right), \tag{A.3}$$

$$\Pr\left\{\arg\max_{i\in\{1,\dots,n\}}\{V_i+\varepsilon_i\} = j\right\} = \frac{\exp\frac{V_j}{\sigma}}{\sum_{i=1}^n \exp\frac{V_i}{\sigma}}. \tag{A.4}$$

Proof. See Appendix B.3. □

B Proofs

This section contains the proofs for the lemmas and propositions in the paper.

B.1 Proof of Lemma 1

Proof. Let $X_1 = V_1 + \varepsilon_1$ and $X_2 = V_2 + \varepsilon_2$, where $\varepsilon_1, \varepsilon_2$ are independent Gumbel$(-\eta\gamma, \eta)$ random variables. I derive the expected value $\mathbb{E}[\max\{X_1, X_2\}]$. The CDF of a Gumbel$(\mu, \sigma)$ random variable is $F(x;\mu,\sigma) = \exp(-\exp(-(x-\mu)/\sigma))$. For the parametrization Gumbel$(-\eta\gamma, \eta)$:

$$F_\varepsilon(x) = \exp\left(-\exp\left(-\frac{x}{\eta} - \gamma\right)\right).$$

The CDF of $X_i = V_i + \varepsilon_i$ is obtained by translation:

$$F_{X_i}(x) = F_\varepsilon(x - V_i) = \exp\left(-\exp\left(-\frac{x - V_i}{\eta} - \gamma\right)\right).$$

The CDF of $\max\{X_1, X_2\}$ is:

$$\begin{aligned}
F_{\max}(x) &= \Pr\{\max\{X_1, X_2\} \le x\} = \Pr\{X_1 \le x, X_2 \le x\} \\
&= F_{X_1}(x) \cdot F_{X_2}(x) \\
&= \exp\left(-\exp\left(-\frac{x - V_1}{\eta} - \gamma\right)\right) \cdot \exp\left(-\exp\left(-\frac{x - V_2}{\eta} - \gamma\right)\right) \\
&= \exp\left(-\exp\left(-\frac{x - V_1}{\eta} - \gamma\right) - \exp\left(-\frac{x - V_2}{\eta} - \gamma\right)\right) \\
&= \exp\left(-e^{-\gamma}\left(\exp\left(-\frac{x - V_1}{\eta}\right) + \exp\left(-\frac{x - V_2}{\eta}\right)\right)\right).
\end{aligned}$$

Let $\tilde{\mu} = \eta \ln\left(\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}\right)$. We can rewrite:

$$\begin{aligned}
&\exp\left(-\frac{x - V_1}{\eta}\right) + \exp\left(-\frac{x - V_2}{\eta}\right) \\
&= \exp\left(-\frac{x}{\eta}\right)\left(\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}\right) \\
&= \exp\left(-\frac{x}{\eta}\right)\exp\frac{\tilde{\mu}}{\eta} = \exp\left(-\frac{x - \tilde{\mu}}{\eta}\right).
\end{aligned}$$

Therefore,

$$F_{\max}(x) = \exp\left(-e^{-\gamma}\exp\left(-\frac{x - \tilde{\mu}}{\eta}\right)\right) = \exp\left(-\exp\left(-\frac{x - \tilde{\mu}}{\eta} - \gamma\right)\right).$$

Thus $\max\{X_1, X_2\}$ follows a Gumbel$(\tilde{\mu}-\eta\gamma, \eta)$ distribution. Since the expectation of a Gumbel(μ, σ) random variable is $\mu + \sigma\gamma$:

$$\begin{aligned}\mathbb{E}[\max\{X_1, X_2\}] &= (\tilde{\mu} - \eta\gamma) + \eta\gamma = \tilde{\mu} \\ &= \eta \ln\left(\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}\right).\end{aligned}$$

□

B.2 Proof of Lemma 2

Proof. Let $X_1 = V_1 + \varepsilon_1$ and $X_2 = V_2 + \varepsilon_2$ as before. I compute $\Pr\{X_1 > X_2\} = \Pr\{\varepsilon_1 - \varepsilon_2 > V_2 - V_1\}$. I first establish that for independent Gumbel(μ, σ) random variables ε_1 and ε_2, the difference $\varepsilon_1 - \varepsilon_2$ follows a logistic distribution. The CDF of ε_i is $F(x) = \exp(-\exp(-(x - \mu)/\sigma))$. For $Z = \varepsilon_1 - \varepsilon_2$, I compute:

$$\begin{aligned}F_Z(z) &= \Pr\{\varepsilon_1 - \varepsilon_2 \leq z\} = \Pr\{\varepsilon_1 \leq z + \varepsilon_2\} \\ &= \int_{-\infty}^{\infty} F_{\varepsilon_1}(z+u) f_{\varepsilon_2}(u) du\end{aligned}$$

where $f_{\varepsilon_2}(u) = \sigma^{-1}\exp(-(u-\mu)/\sigma)\exp(-\exp(-(u-\mu)/\sigma))$ is the PDF of ε_2. Using the substitution $v = \exp(-(u-\mu)/\sigma)$ and the fact that both variables have the same parameters, the integral evaluates to:

$$F_Z(z) = \frac{1}{1+\exp(-z/\sigma)}$$

This is the CDF of a logistic distribution with location 0 and scale σ. With $\sigma = \eta$ and our parametrization:

$$\begin{aligned}\Pr\{X_1 > X_2\} &= \Pr\{\varepsilon_1 - \varepsilon_2 > V_2 - V_1\} \\ &= 1 - F_Z(V_2 - V_1; 0, \eta) \\ &= 1 - \frac{1}{1+\exp(-(V_2 - V_1)/\eta)} \\ &= 1 - \frac{1}{1+\exp((V_1 - V_2)/\eta)} \\ &= \frac{1+\exp((V_1 - V_2)/\eta) - 1}{1+\exp((V_1 - V_2)/\eta)} \\ &= \frac{\exp((V_1 - V_2)/\eta)}{1+\exp((V_1 - V_2)/\eta)}.\end{aligned}$$

Multiplying by $\exp(V_2/\eta)$ yields:

$$\Pr\{X_1 > X_2\} = \frac{\exp\frac{V_1}{\eta}}{\exp\frac{V_1}{\eta} + \exp\frac{V_2}{\eta}}.$$

□

B.3 Proof of Lemma 3

Proof. The proof proceeds by induction on n.

Base case: For $n = 2$, the results follow from Lemmas 1 and 2.

Inductive step: Assume the results hold for $n \geq 2$. For $n+1$ alternatives, let $Y_n = \max_{1\leq i\leq n}\{V_i + \varepsilon_i\}$ and $X_{n+1} = V_{n+1} + \varepsilon_{n+1}$. Then:

$$\max_{1\leq i\leq n}\{V_i + \varepsilon_i\} = \max\{Y_n, X_{n+1}\}.$$

By the inductive hypothesis, Y_n is distributed as Gumbel$(\sigma \ln(\sum_{i=1}^{n} \exp(V_i/\sigma)) - \sigma\gamma, \sigma)$. Since X_{n+1} is Gumbel$(-\sigma\gamma, \sigma)$, applying Lemma 1:

$$\begin{aligned}
&\mathbb{E}\left[\max_{1\leq i\leq n+1}\{V_i + \varepsilon_i\}\right] \\
&= \sigma \ln\left(\exp\frac{\sigma \ln\left(\sum_{i=1}^{n} \exp\frac{V_i}{\sigma}\right)}{\sigma} + \exp\frac{V_{n+1}}{\sigma}\right) \\
&= \sigma \ln\left(\sum_{i=1}^{n} \exp\frac{V_i}{\sigma} + \exp\frac{V_{n+1}}{\sigma}\right) \\
&= \sigma \ln\left(\sum_{i=1}^{n+1} \exp\frac{V_i}{\sigma}\right).
\end{aligned}$$

For the choice probabilities, Lemma 2 yields:

$$\begin{aligned}
&\Pr\left\{\arg\max_{1\leq i\leq n+1}\{V_i + \varepsilon_i\} = j\right\} \\
&= \begin{cases} \Pr\{Y_n > X_{n+1}\} \cdot \Pr\{\arg\max_{1\leq i\leq n}\{V_i + \varepsilon_i\} = j\} & \text{if } j \leq n \\ \Pr\{X_{n+1} > Y_n\} & \text{if } j = n+1 \end{cases}
\end{aligned}$$

By the inductive hypothesis and preceding lemmas:

$$\Pr\left\{\arg\max_{1\leq i\leq n+1}\{V_i + \varepsilon_i\} = j\right\} = \frac{\exp\frac{V_j}{\sigma}}{\sum_{i=1}^{n+1} \exp\frac{V_i}{\sigma}}.$$

□

B.4 Proof of Proposition 1

Proof. **Spaces and operator.** Let $\mathscr{S} = \mathscr{Y} \times \mathscr{B}$ (compact) and $\mathscr{C}(\mathscr{S})$ be bounded continuous functions on $\mathscr{S}$ with the sup-norm $\|\cdot\|_\infty$. Define the product space $\mathbf{X} = \mathscr{C}(\mathscr{S}) \times \mathscr{C}(\mathscr{S})$ and, for a weight $\lambda > 0$, the norm

$$\|(V, q)\|_\lambda = \max\{\|V\|_\infty, \lambda\|q\|_\infty\}.$$

$(\mathbf{X}, \|\cdot\|_\lambda)$ is complete. Define $\mathscr{T} : \mathbf{X} \to \mathbf{X}$ by

$$\mathscr{T}(V, q) = \big(J(V, q), T(V, q)\big),$$

where J is the Bellman operator (log-sum-exp/taste shocks) and T is the pricing operator (B.9) (default probability given by the logistic rule).

Step 1: $\mathscr{T}$ maps X into itself. By compactness of (y, B) and the discrete (or compact) choice set for B', continuity of the primitives, and the log-sum-exp aggregator with $\rho > 0$, $J(V, q)$ is bounded and continuous whenever (V, q) is. For T, note that for any (y, B'),

$$0 \le q(y, B') \le \frac{1}{1+r}\big(\kappa + (1-\delta)\|q\|_\infty\big),$$

hence taking suprema gives the uniform bound

$$\|q\|_\infty \le \frac{\kappa}{r} =: \bar{q}. \tag{B.5}$$

Thus $T(V, q)$ is bounded and continuous, so $\mathscr{T}(\mathbf{X}) \subseteq \mathbf{X}$.

Step 2: Lipschitz bounds (decoupled). We record the following constants (finite by compactness and (B.5)):

(i) *J with respect to V.* The continuation part of J is multiplied by β and the derivative of log-sum-exp is a probability in $[0, 1]$, so

$$\|J(V_1, q) - J(V_2, q)\|_\infty \le \beta\|V_1 - V_2\|_\infty.$$

(ii) *J with respect to q.* Consumption is $c(B') = y - \kappa B + [B' - (1-\delta)B]\, q(y, B')$. Hence for some bound

$$M_B := \sup_{(y,B)\in\mathscr{S}} \sup_{B'\in\mathscr{B}} \big|B' - (1-\delta)B\big| < \infty,$$

and with $\bar{u}' := \sup u'(c)$ on the feasible set, the envelope theorem and the $[0, 1]$-valued logit weights imply

$$\|J(V,q_1)-J(V,q_2)\|_\infty \le L_{Jq}\,\|q_1-q_2\|_\infty, \qquad L_{Jq} \le \bar{u}' M_B. \tag{B.6}$$

(iii) *T with respect to q.* In (B.9), q enters only via the resale term $(1-\delta)\mathbb{E}q$ with one-period discount $1/(1+r)$, hence

$$\|T(V,q_1)-T(V,q_2)\|_\infty \le m_q\,\|q_1-q_2\|_\infty, \qquad m_q := \frac{1-\delta}{1+r} < 1. \tag{B.7}$$

(iv) *T with respect to V.* Write $P = L(\Delta V/(\theta\eta))$ with $L'(z) = L(z)\big(1-L(z)\big) \le \frac{1}{4}$. Let $\Pi := \kappa + (1-\delta)\bar{q}$ bound the one-step payoff in (B.9). The mapping $V \mapsto \Delta V$ is linear in V through continuation values and default recursion; its operator norm is bounded by

$$\|\Delta V(V_1)-\Delta V(V_2)\|_\infty \le C_\Delta\,\|V_1-V_2\|_\infty, \qquad C_\Delta \le \frac{\beta}{1-\beta(1-\gamma)}.$$

Therefore

$$\|T(V_1,q)-T(V_2,q)\|_\infty \le \frac{1}{1+r}\cdot\frac{1}{4\theta\eta}\cdot\Pi\cdot C_\Delta\|V_1-V_2\|_\infty =: L_{TV}\,\|V_1-V_2\|_\infty. \tag{B.8}$$

Step 3: Joint contraction under a weighted norm. For any $(V_1,q_1),(V_2,q_2)$, combine the bounds above to get

$$\|J(V_1,q_1)-J(V_2,q_2)\|_\infty \le \beta\,\|V_1-V_2\|_\infty + L_{Jq}\,\|q_1-q_2\|_\infty,$$

$$\|T(V_1,q_1)-T(V_2,q_2)\|_\infty \le L_{TV}\,\|V_1-V_2\|_\infty + m_q\,\|q_1-q_2\|_\infty.$$

Multiplying the second inequality by λ and taking the max of the two gives

$$\|\mathscr{T}(V_1,q_1)-\mathscr{T}(V_2,q_2)\|_\lambda \le \alpha(\lambda)\,\|(V_1,q_1)-(V_2,q_2)\|_\lambda,$$

where

$$\alpha(\lambda) = \max\Big\{\beta+\frac{L_{Jq}}{\lambda},\ m_q+\lambda L_{TV}\Big\}.$$

If there exists $\lambda>0$ such that

$$\beta+\frac{L_{Jq}}{\lambda}<1 \quad\text{and}\quad m_q+\lambda L_{TV}<1,$$

then $\alpha(\lambda)<1$ and $\mathscr{T}$ is a contraction. These inequalities can be satisfied if and only if

$$L_{Jq}L_{TV} < (1-\beta)(1-m_q),$$

which is exactly the slope condition (13). In that case one admissible choice is

$$\lambda^\star = \frac{(\beta - m_q) + \sqrt{(\beta - m_q)^2 + 4L_{Jq}L_{TV}}}{2L_{TV}},$$

for which $\alpha(\lambda^\star) < 1$.

Step 4: Conclusion. In the weighted complete normed space $\big(\mathbf{X}, \|\cdot\|_{\lambda^\star}\big)$, $\mathcal{T}$ is a contraction mapping. By the Contraction Mapping Theorem, it has a unique fixed point (V^*, q^*), which by construction satisfies the model's recursive equilibrium conditions. Therefore, the Recursive Markov Perfect Equilibrium exists and is unique. □

B.5 Proof of Proposition 2

Proof. The proof relies on a standard monotone-operator result showing that pointwise orderings of operators are preserved by their fixed points.

Lemma 4. *Let $(X, \|\cdot\|)$ be a Banach space and $T_1, T_2 : X \to X$ satisfy:*

1. ***Monotonicity:*** *$f \geq g \Rightarrow T_i(f) \geq T_i(g)$ for $i \in \{1,2\}$.*
2. ***Discounting:*** *$\|T_i(f + c\mathbf{1}) - T_i(f)\| \leq \beta c$ for some $\beta \in (0,1)$ and constant function $\mathbf{1}$.*

If $T_1(f) \geq T_2(f)$ pointwise for all $f \in X$, then their unique fixed points satisfy $f_1^* \geq f_2^*$ where $T_i(f_i^*) = f_i^*$.

Proof of Lemma 4. Let $f_1 = T_1(f_1)$ and $f_2 = T_2(f_2)$. Start from $f_0 = f_2$ and iterate $f_{n+1} = T_1(f_n)$. By contraction, $f_n \to f_1$. Using $T_1(f_2) \geq T_2(f_2) = f_2$ and monotonicity of T_1, the sequence is nondecreasing and bounded below by f_2; taking limits yields $f_1 \geq f_2$. □

Let $q_i(B', y)$, $i \in \{1, \theta\}$ with $\theta_1 = 1$ and $\theta > 1$, be the fixed point of the pricing operator

$$(T_i q)(B', y) = \frac{1}{1+r}\mathbb{E}_{y'|y}\Big[\big(1 - P_i(y', B')\big)\Big(\kappa + (1-\delta)\mathbb{E}_{B''|y',B'}[\,q(y', B'')\,]\Big)\Big], \tag{B.9}$$

where

$$P_i(y', B') = L\Big(-\frac{\Delta V_i(y', B')}{\theta_i \eta}\Big), \qquad L(z) = \frac{1}{1+e^{-z}}, \qquad \Delta V_i(y', B') \equiv V_i^R(y', B') - V_i^D(y'). \tag{B.10}$$

Step 1: Probability ordering (correct sign). Fix (y, B') and write $\xi \equiv \Delta V_i(y, B') \neq 0$. Define $\phi(\alpha) \equiv L(-\xi/\alpha)$ for $\alpha > 0$. Since $L'(z) = L(z)(1 - L(z)) > 0$,

$$\phi'(\alpha) = L'(-\xi/\alpha) \cdot \frac{\xi}{\alpha^2} \quad \Rightarrow \quad \operatorname{sign}\phi'(\alpha) = \operatorname{sign}\xi.$$

Because $\theta > 1$ implies $\theta\eta > \eta$,

$$\xi > 0 \;\Rightarrow\; \phi(\theta\eta) > \phi(\eta) \;\Rightarrow\; P_\theta(y, B') > P_1(y, B'), \tag{B.11}$$

$$\xi < 0 \;\Rightarrow\; \phi(\theta\eta) < \phi(\eta) \;\Rightarrow\; P_\theta(y, B') < P_1(y, B'). \tag{B.12}$$

Intuitively: when repayment strictly dominates default ($\Delta V > 0$), pessimistic lenders assign a *higher* default probability; when default dominates ($\Delta V < 0$), they assign a *lower* default probability.

Given the strictly positive payoff term in (B.9), we have

$$\operatorname{sign}\big((T_\theta q - T_1 q)(B', y)\big) = \operatorname{sign}\Big(\mathbb{E}_{y'|y}\left[P_1(y', B') - P_\theta(y', B')\right]\Big). \tag{B.13}$$

Step 2: Safe region ($\Delta V > 0$). Define the safe set at income y by

$$\underline{B}(y) \;\equiv\; \sup\Big\{b : \Delta V_i(y', b) > 0 \;\; \forall\, y' \in \mathscr{Y},\; \forall\, i \in \{1, \theta\}\Big\}.$$

For any $B' < \underline{B}(y)$, (B.11) gives $P_\theta > P_1$ for all y'; hence

$$\mathbb{E}_{y'|y}[P_1 - P_\theta] < 0 \;\Rightarrow\; (T_\theta q)(B', y) < (T_1 q)(B', y) \;\Rightarrow\; q_\theta(B', y) < q_1(B', y).$$

Step 3: Risky region ($\Delta V < 0$). Define the risky set at income y by

$$\overline{B}(y) \;\equiv\; \inf\Big\{b : \Delta V_i(y', b) < 0 \;\; \forall\, y' \in \mathscr{Y},\; \forall\, i \in \{1, \theta\}\Big\}.$$

For any $B' > \overline{B}(y)$, (B.12) yields $P_\theta < P_1$ for all y'; thus

$$\mathbb{E}_{y'|y}[P_1 - P_\theta] > 0 \;\Rightarrow\; (T_\theta q)(B', y) > (T_1 q)(B', y) \;\Rightarrow\; q_\theta(B', y) > q_1(B', y).$$

Step 4: Pivot existence. Let $\Delta q(B', y) \equiv q_\theta(B', y) - q_1(B', y)$. By continuity of q_i in B' (Proposition 1), $\Delta q(\cdot, y)$ is continuous. Steps 2–3 imply

$$\Delta q(B', y) < 0 \quad \text{for } B' < \underline{B}(y), \qquad \Delta q(B', y) > 0 \quad \text{for } B' > \overline{B}(y).$$

By the Intermediate Value Theorem there exists at least one $B^*(y) \in [\underline{B}(y), \overline{B}(y)]$ such that

$$\Delta q(B^*(y), y) = 0,$$

and the local sign pattern is

$$\operatorname{sign}\big(\Delta q(B',y)\big) = \begin{cases} -, & B' < B^*(y), \\ 0, & B' = B^*(y), \\ +, & B' > B^*(y). \end{cases}$$

This establishes the pivoting of q_θ around q_1 at $B^*(y)$. $\square$

B.6 Proof of Proposition 3

Proof. Define $F(B',y) \equiv q_\theta(B',y) - q_1(B',y)$ and let $B^*(y)$ be implicitly defined by $F(B^*(y),y) = 0$. We verify the conditions of the Implicit Function Theorem at any $(B^*(y),y)$ where the crossing is interior.

Regularity. **(i) Smoothness.** Under the primitives assumed in the main text (CRRA utility, log-sum-exp smoothing in the sovereign's choices, compact state grids, and the pricing operator with strictly positive payoff term), the pricing operator is C^1 in (B',y). Hence each fixed-point price function $q_i(B',y)$ is C^1 in both arguments.[17]

(ii) Non-degeneracy. We show $F_{B'}(B^*(y),y) \neq 0$ and in fact $F_{B'}(B^*(y),y) > 0$. Recall the lenders' perceived default probability

$$P_i(y',B') \;=\; L\!\left(-\frac{\Delta V_i(y',B')}{\theta_i \eta}\right), \qquad L(z) = \frac{1}{1+e^{-z}}, \qquad \Delta V_i \equiv V_i^R - V_i^D.$$

Since $L' > 0$ and $\partial \Delta V_i / \partial B' < 0$ (higher future debt reduces the net value of repayment),

$$\frac{\partial P_i(y',B')}{\partial B'} = -\frac{1}{\theta_i \eta} L'\!\left(-\frac{\Delta V_i}{\theta_i \eta}\right) \frac{\partial \Delta V_i}{\partial B'} \;>\; 0. \tag{B.14}$$

Moreover, for $\theta > 1$,

$$0 \;<\; \frac{\partial P_\theta}{\partial B'} \;<\; \frac{\partial P_1}{\partial B'}. \tag{B.15}$$

Write the pricing operator as

$$(T_i q)(B',y) = \frac{1}{1+r} \mathbb{E}_{y'|y}\Big[(1 - P_i(y',B'))\, R_i(y',B')\Big], \qquad R_i(y',B') \equiv \kappa + (1-\delta)\, \mathbb{E}_{B''|y',B'}[\, q_i(y',B'')\,].$$

Since $R_i > 0$, the operator is pointwise *decreasing* in P_i. Hence, as B' increases and P_i rises by (B.14), the price $q_i(B',y)$ strictly *decreases* in B'. Furthermore, because $\partial P_\theta / \partial B'$ is smaller than $\partial P_1 / \partial B'$ by (B.15), the magnitude of the induced price decline is smaller

[17] This follows from differentiability of the Bellman/pricing operators and standard fixed-point differentiation arguments on compact spaces.

under θ than under 1. Therefore,

$$\frac{\partial q_\theta}{\partial B'}(B^*(y),y) > \frac{\partial q_1}{\partial B'}(B^*(y),y), \qquad \text{so} \qquad F_{B'}(B^*(y),y) = \frac{\partial q_\theta}{\partial B'} - \frac{\partial q_1}{\partial B'} > 0. \tag{B.16}$$

Sign of the numerator. Income persistence implies that higher y raises expected future endowments, increases ΔV_i, lowers P_i, and thus raises prices: $\partial q_i/\partial y > 0$. The sensitivity is dampened under pessimism because the mapping $\Delta V \mapsto P_\theta = L(-\Delta V/(\theta\eta))$ is flatter when $\theta > 1$. Propagating through the pricing operator,

$$0 < \frac{\partial q_\theta}{\partial y}(B^*(y),y) < \frac{\partial q_1}{\partial y}(B^*(y),y), \quad \Rightarrow \quad F_y(B^*(y),y) = \frac{\partial q_\theta}{\partial y} - \frac{\partial q_1}{\partial y} < 0. \tag{B.17}$$

Conclusion. By the Implicit Function Theorem,

$$\frac{dB^*}{dy} = -\frac{F_y(B^*(y),y)}{F_{B'}(B^*(y),y)} = -\frac{(-)}{(+)} > 0,$$

so the pivot threshold $B^*(y)$ is (strictly) increasing in y. □

B.7 Proof of Corollary 1

Proof. By definition of the spread,

$$s_i(B',y) = \frac{\kappa}{q_i(B',y)} - \delta - r.$$

Hence, for any (B',y) with $q_\theta(B',y), q_1(B',y) > 0$,

$$\begin{aligned}
\Delta s(B',y) &\equiv s_\theta(B',y) - s_1(B',y) \\
&= \left(\tfrac{\kappa}{q_\theta(B',y)} - \delta - r\right) - \left(\tfrac{\kappa}{q_1(B',y)} - \delta - r\right) \qquad (\text{B.18}) \\
&= \kappa\left(\tfrac{1}{q_\theta(B',y)} - \tfrac{1}{q_1(B',y)}\right) = -\frac{\kappa\,[\,q_\theta(B',y) - q_1(B',y)\,]}{q_\theta(B',y)\,q_1(B',y)} \\
&= -\frac{\kappa\,\Delta q(B',y)}{q_\theta(B',y)\,q_1(B',y)}. \qquad (\text{B.19})
\end{aligned}$$

Because $\kappa > 0$ and $q_i(B',y) > 0$, it follows that $\operatorname{sign}(\Delta s(B',y)) = -\operatorname{sign}(\Delta q(B',y))$. By Proposition 2, there exists $B^*(y)$ such that

$$\Delta q(B',y) \begin{cases} < 0, & B' < B^*(y), \\ = 0, & B' = B^*(y), \\ > 0, & B' > B^*(y), \end{cases} \qquad \Longrightarrow \qquad \Delta s(B',y) \begin{cases} > 0, & B' < B^*(y), \\ = 0, & B' = B^*(y), \\ < 0, & B' > B^*(y). \end{cases}$$

Moreover, if for some (B', y) one price were zero (full default so the payoff is null), then $s_i(B', y) = +\infty$ and the sign relation holds trivially in the limit.[18] $\square$

B.8 Proof of Proposition 4

Proof. For a fixed endowment level y, define the repayment-default gap

$$G_i(B; y) \equiv V_i^R(y, B) - V^D(y), \qquad i \in \{1, \theta\}.$$

By standard arguments (concavity of u, budget feasibility, and the fact that higher current debt tightens the budget set), $G_i(B; y)$ is strictly decreasing in B. The default threshold is the unique root $B_{D,i}^*(y)$ of $G_i(\cdot; y) = 0$.

Assume toward a contradiction that $B_{D,\theta}^*(y) \le B_{D,1}^*(y)$. Monotonicity of $G_1(\cdot; y)$ then implies

$$G_1\big(B_{D,\theta}^*(y); y\big) = V_1^R\big(y, B_{D,\theta}^*(y)\big) - V^D(y) \ge V_1^R\big(y, B_{D,1}^*(y)\big) - V^D(y) = 0. \tag{B.20}$$

Next, invoke Proposition 2 ("price pivot"): for the given y there exists $B^*(y)$ such that

$$q_\theta(y, B') \gtrless q_1(y, B') \quad \text{iff} \quad B' \gtrless B^*(y),$$

with strict inequality on a set of B' of positive measure. Near the brink of default (i.e., at $B = B_{D,\theta}^*(y)$), the sovereign's optimal issuance under repayment places (by the Euler condition and rollover incentives in long-term debt models) essentially all probability mass on future debt levels $B' \ge B^*(y)$.[19] Hence, evaluating the choice-specific value

$$W_i(y, B, B') = u(y - \kappa B + [B' - (1-\delta)B]\, q_i(y, B')) + \beta \mathbb{E}_{y'|y}\big[V_i(y', B')\big],$$

we have for all such relevant $B' \ge B^*(y)$,

$$q_\theta(y, B') \ge q_1(y, B') \quad \Longrightarrow \quad u\big(\cdot \text{ with } q_\theta\big) \ge u\big(\cdot \text{ with } q_1\big),$$

and (by monotonicity of the Bellman operator with respect to the price schedule in those states) $\mathbb{E}[V_\theta(y', B')] \ge \mathbb{E}[V_1(y', B')]$. Therefore $W_\theta(y, B, B') \ge W_1(y, B, B')$ on the support of the (repayment) choice distribution, with strict inequality on a set of positive probability.

[18]From the pricing operator, $q_i(B', y) = \frac{1}{1+r}\mathbb{E}_{y'|y}\big[(1 - P_i(y', B'))\big(\kappa + (1-\delta)\mathbb{E}_{B''|y',B'}[q_i(y', B'')]\big)\big]$, so $q_i \ge 0$ and $q_i > 0$ whenever the repayment probability is not identically zero.

[19]Intuitively, when current debt is high, the sovereign rolls over rather than deleverages sharply; see, e.g., the rollover logic in long-term debt environments. Formally, with small taste-shock dispersion ρ, the logit policy concentrates around the deterministic maximizer, which lies weakly to the right of $B^*(y)$ at high current B.

Aggregating with the log-sum-exp representation of the repayment value,

$$V_i^R(y,B) \;=\; \rho \log\Bigl(\sum_{B'} \exp\{W_i(y,B,B')/\rho\}\Bigr),$$

which is strictly increasing in each $W_i(y,B,B')$, we obtain

$$V_\theta^R\bigl(y, B_{D,\theta}^*(y)\bigr) \;>\; V_1^R\bigl(y, B_{D,\theta}^*(y)\bigr). \tag{B.21}$$

Combining (B.20) and (B.21) yields

$$V_\theta^R\bigl(y, B_{D,\theta}^*(y)\bigr) \;>\; V^D(y),$$

which contradicts the definition of $B_{D,\theta}^*(y)$ as the point where $V_\theta^R = V^D$. Hence the assumed ordering is false, and we must have

$$B_{D,\theta}^*(y) \;>\; B_{D,1}^*(y).$$

□

B.9 Proof of Proposition 5

Proof. **Setup.** Fix a state (y,B). For each lender type $i \in \{1,\theta\}$, let

$$B_i'(y,B) \in \arg\max_{B' \in \mathcal{B}} W(y,B,B';q_i), \qquad W(y,B,B';q_i) \equiv u\bigl(c_i(B')\bigr) + \beta\, \mathbb{E}_{y'|y}\bigl[V_i(y',B')\bigr],$$

with

$$c_i(B') \;=\; y - \kappa B + \bigl[B' - (1-\delta)B\bigr]\, q_i(y,B').$$

By strict concavity of u and standard properties of the long-maturity debt problem, $B_i'(y,B)$ is unique.[20] Define the marginal objective

$$g_i(B') \;\equiv\; \frac{\partial W(y,B,B';q_i)}{\partial B'} \;=\; u'\bigl(c_i(B')\bigr)\Bigl[q_i(y,B') + \bigl(B' - (1-\delta)B\bigr) q_{i,B'}(y,B')\Bigr] \;+\; \beta \frac{\partial}{\partial B'} \mathbb{E}_{y'|y}[V_i(y',B')].$$

FOC and uniqueness give

$$g_i\bigl(B_i'(y,B)\bigr) = 0, \qquad i \in \{1,\theta\}. \tag{B.22}$$

[20] The usual argument is that the choice-specific value is strictly concave in B' because (i) u is strictly concave and $B' \mapsto c_i(B')$ is affine plus a (weakly) concave price-feedback term, and (ii) the continuation term $B' \mapsto \mathbb{E}[V_i(y',B')]$ is concave by the concavity of the Bellman operator.

Region of comparison. By Proposition 2, there exists $B^*(y)$ such that

$$B' \geq B^*(y) \implies q_\theta(y,B') < q_1(y,B'), \qquad B' \leq B^*(y) \implies q_\theta(y,B') > q_1(y,B'). \tag{B.23}$$

We prove the claim on the empirically relevant *risky* side, i.e. for states with the baseline choice on (or to the right of) the pivot:

$$B_1'(y,B) \;\geq\; B^*(y). \tag{B.24}$$

(The complementary case is analogous and yields the weak inequality; see the remark at the end.)

Two auxiliary inequalities.

(A1) Since $q_{i,B'}(y,B') \leq 0$ in equilibrium and $\big(B' - (1-\delta)B\big) \geq 0$ whenever the sovereign issues (rolls over) debt,[21]

$$q_i(y,B') + \big(B' - (1-\delta)B\big) q_{i,B'}(y,B') \;\leq\; q_i(y,B'). \tag{B.25}$$

(A2) (Continuation marginal ordering.) Increasing next-period debt lowers the continuation value and it does so *more* under pessimistic lenders:

$$\frac{\partial}{\partial B'} \mathbb{E}_{y'|y}[V_\theta(y',B')] \;\leq\; \frac{\partial}{\partial B'} \mathbb{E}_{y'|y}[V_1(y',B')] \;\leq\; 0, \tag{B.26}$$

with strict inequality on a set of positive measure when $B' \geq B^*(y)$.[22]

Key evaluation at the baseline optimum. Evaluate g_θ at $B_1' \equiv B_1'(y,B)$. Using (B.25) and (B.26),

$$\begin{aligned} g_\theta(B_1') &= u'\big(c_\theta(B_1')\big)\Big[q_\theta(y,B_1') + \big(B_1' - (1-\delta)B\big) q_{\theta,B'}(y,B_1')\Big] + \beta \frac{\partial}{\partial B'} \mathbb{E}[V_\theta(y',B')]\Big|_{B'=B_1'} \\ &\leq u'\big(c_\theta(B_1')\big) q_\theta(y,B_1') + \beta \frac{\partial}{\partial B'} \mathbb{E}[V_1(y',B')]\Big|_{B'=B_1'}. \end{aligned} \tag{B.27}$$

By the baseline FOC (B.22) for $i = 1$,

$$0 \;=\; g_1(B_1') \;=\; u'\big(c_1(B_1')\big)\Big[q_1 + \big(B_1' - (1-\delta)B\big) q_{1,B'}\Big] + \beta \frac{\partial}{\partial B'} \mathbb{E}[V_1(y',B')]\Big|_{B'=B_1'},$$

[21] If net issuance is negative at the optimum, the conclusion below is even stronger because the self-impact term becomes favorable.

[22] This follows from monotonicity of the Bellman operator in the price schedule and Proposition 2. For $B' \geq B^*(y)$, pessimism implies lower prices $q_\theta < q_1$ and (via the probability ordering embedded in the pricing operator) higher default likelihood, both of which tighten the next-period budget more when B' increases. Hence the marginal effect of B' on the continuation value is weakly more adverse under θ.

and therefore, again using $q_{1,B'} \le 0$,

$$\beta \frac{\partial}{\partial B'} \mathbb{E}[V_1(y', B')]\bigg|_{B'=B_1'} = -u'\big(c_1(B_1')\big)\Big[q_1 + \big(B_1' - (1-\delta)B\big)q_{1,B'}\Big] \le -u'\big(c_1(B_1')\big)\, q_1(y, B_1'). \tag{B.28}$$

Substituting (B.28) into (B.27) yields

$$g_\theta(B_1') \le u'\big(c_\theta(B_1')\big)\, q_\theta(y, B_1') - u'\big(c_1(B_1')\big)\, q_1(y, B_1'). \tag{B.29}$$

Sign of the bound on the risky side. Under (B.24), (B.23) gives $q_\theta(y, B_1') < q_1(y, B_1')$. Moreover, $c_\theta(B_1') < c_1(B_1')$ because the only difference in the current-period budget is the price multiplying the same issuance $[B_1' - (1-\delta)B]$, hence by concavity $u'\big(c_\theta(B_1')\big) > u'\big(c_1(B_1')\big)$. Combining these two facts in (B.29) implies

$$g_\theta(B_1') < 0.$$

Since $B' \mapsto W(y, B, B'; q_\theta)$ is strictly concave, g_θ is strictly decreasing, so $g_\theta(B_1') < 0$ forces the θ-optimizer to lie strictly to the left of B_1':

$$B_\theta'(y, B) < B_1'(y, B).$$

From modes to distributions. Under the logit (Gumbel) regularization, the choice probabilities satisfy $\Pr_i(B' = b \mid y, B) \propto \exp\{W(y, B, b; q_i)/\eta\}$, which are unimodal and concentrate around the unique maximizer as $\eta \downarrow 0$. Because the mode shifts left from B_1' to B_θ', the θ-distribution first-order stochastically dominates (to the left) the baseline distribution, implying $\mathbb{E}_\theta[B' \mid y, B] < \mathbb{E}_1[B' \mid y, B]$.

Remark. If $B_1'(y, B) \le B^*(y)$ (safe side), the same argument yields the weak inequality $B_\theta'(y, B) \le B_1'(y, B)$; strictness may fail when both prices and continuation effects coincide on the relevant grid. □

B.10 Proof of Proposition 6

Proof. **Step 0: Bellman operators.** Let J_i be the Bellman operator for economy $i \in \{1, \theta\}$:

$$(J_i V_{in})(y, B) = \eta \ln\left(\exp\left(\frac{V^D(y; V_{in})}{\eta}\right) + \exp\left(\frac{V^R(y, B; q_i, V_{in})}{\eta}\right)\right),$$

where V^D and V^R are the default and repayment values respectively. The equilibrium value V_i is the unique fixed point of J_i on the space of bounded continuous functions on $\mathcal{Y} \times \mathcal{B}$ with the sup norm.

Step 1: Default value is identical across i. The default value

$$V^D(y;V_{in}) = u(h(y)) + \beta\left[\gamma\,\mathbb{E}_{y'|y}V_{in}(y',B_0) + (1-\gamma)\,\mathbb{E}_{y'|y}V^D(y';V_{in})\right]$$

is independent of the current price schedule q_i; hence

$$V^D(y;V_{in}) \quad \text{is the same for } i=1 \text{ and } i=\theta. \tag{B.30}$$

Step 2: Repayment value difference comes from prices. The repayment value is

$$V^R(y,B;q_i,V_{in}) = \rho\,\ln\left(\sum_{B'\in\mathscr{B}}\exp\left(\frac{W(y,B,B';q_i,V_{in})}{\rho}\right)\right),$$

with

$$W(y,B,B';q_i,V_{in}) = u\big(c_i(B')\big) + \beta\,\mathbb{E}_{y'|y}V_{in}(y',B'), \quad c_i(B') = y - \kappa B + [B' - (1-\delta)B]\,q_i(y,B').$$

The continuation term $\mathbb{E}_{y'|y}V_{in}(y',B')$ is identical across i; differences in W come solely from $c_i(B')$.

Step 3: Price ordering on the risky side. From Proposition 2, if $B' \geq B^*(y)$ then

$$q_\theta(y,B') < q_1(y,B'),$$

with strict inequality on a set of B' with positive measure. If the baseline optimal choice $B_1'(y,B) \geq B^*(y)$, the relevant B' in the repayment maximization fall in this region with positive probability.

Step 4: Consumption and utility ordering. For any such B', $c_\theta(B') < c_1(B')$, hence by strict concavity of u,

$$u(c_\theta(B')) < u(c_1(B')).$$

Because the continuation term in W is identical, we have

$$W(y,B,B';q_\theta,V_{in}) < W(y,B,B';q_1,V_{in}) \quad \text{for such } B'. \tag{B.31}$$

Step 5: Aggregating over choices. The log-sum-exp in V^R is strictly increasing in each W. From (B.31), and because the inequality is strict for some B' in the summation,

$$V^R(y,B;q_\theta,V_{in}) < V^R(y,B;q_1,V_{in})$$

whenever $B_1'(y,B) \geq B^*(y)$. On the safe side $B_1'(y,B) \leq B^*(y)$, the weak inequality holds.

Step 6: Bellman operator dominance and fixed points. By (B.30) and the above repayment ordering,

$$(J_\theta V_{in})(y,B) \le (J_1 V_{in})(y,B),$$

strictly when $B_1'(y,B) \ge B^*(y)$. The operators J_i are monotone and contractions (Blackwell conditions), hence the pointwise ordering is preserved at the unique fixed points:

$$V_\theta(y,B) \le V_1(y,B),$$

with strict inequality under the risky-side condition.

□

B.11 Proof of Proposition 7

Proof. **Step 1 (Intertemporal implementability).** Multiply (16) by β^t and sum expectations over t, and use $\beta(1+r)=1$ together with (17). Writing $\Delta_t \equiv B_{t+1}-(1-\delta)B_t$, feasibility under schedule q_i is equivalent to the *implementability constraint*

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t c_t\right] = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\big(y_t-\kappa B_t\big)\right] + \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\,\Delta_t\, q_i(y_t,B_{t+1})\right], \tag{B.32}$$

together with $c_t \ge 0$ and $B_{t+1}\in\mathscr{B}$. In particular, the transfer sequence $\{\tau_t\}$ only enforces period constraints and has zero present value, so it does not affect (B.32).

Step 2 (Set inclusion of feasible (c,B)). Fix any feasible pair (c,B) under q_θ; it satisfies (B.32) with $i=\theta$. Consider the same (c,B) under q_1. Subtracting the two versions of (B.32) gives the slack generated by switching from q_θ to q_1:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\,\Delta_t\big(q_1(y_t,B_{t+1})-q_\theta(y_t,B_{t+1})\big)\right] = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t c_t\right]_{q_1} - \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t c_t\right]_{q_\theta}. \tag{B.33}$$

By (A2), each summand in $\mathscr{S}(c,B)$ is nonnegative and strictly positive with positive probability whenever $|\Delta_t|>0$. Hence

$$\mathscr{S}(c,B)\ \ge\ 0 \quad\text{and}\quad \mathscr{S}(c,B)\ >\ 0\ \text{ if }\Pr\big(|\Delta_t|>0 \text{ and } q_1\neq q_\theta\big)>0.$$

Therefore, every (c,B) feasible under q_θ is also feasible under q_1, and generically with strictly more present-value resources available for consumption under q_1.

Step 3 (Value comparison). Let (c^θ,B^θ) be Ramsey-optimal under q_θ. By Step 2, (c^θ,B^θ) is feasible under q_1 and produces a weakly larger right-hand side in (B.32); because u is strictly increasing, the planner can raise consumption at (at least) one date by an $\varepsilon>0$

while keeping feasibility under q_1, yielding strictly higher utility. Hence

$$W_1^R \geq \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t^\theta)\right], \quad \text{with strict inequality under the strict part of (A2).}$$

Taking the supremum over all feasible (c, B) under q_1 gives $W_1^R > W_\theta^R$.

Remark (Why transfers cannot undo the loss). Condition (17) fixes the present value of transfers at zero, so transfers only reshuffle consumption across dates without changing the right-hand side of (B.32). The loss stems from the lower proceeds from intertemporal trade encoded in q_θ whenever Δ_t and the price dominance have the same sign; no transfer sequence can increase the present value on the right-hand side of (B.32). □

B.12 Proof of Proposition 8

Proof. We establish each part through rigorous analysis of the belief updating dynamics.

Part (i): Persistent Pessimism. Define the conditional maximum likelihood estimators:

$$\hat{\theta}_D(\{d_s\}_{s=0}^t) = \arg\max_{\theta\in[\underline{\theta},\bar{\theta}]} \prod_{s:d_s=1} P_\theta(y_s, B_s) \prod_{s:d_s=0} (1 - P_\theta(y_s, B_s)) \tag{B.34}$$

$$\hat{\theta}_R(\{d_s\}_{s=0}^t) = \arg\max_{\theta\in[\underline{\theta},\bar{\theta}]} \prod_{s:d_s=0} (1 - P_\theta(y_s, B_s)) \prod_{s:d_s=1} P_\theta(y_s, B_s) \tag{B.35}$$

Under negativity bias, the belief updating mechanism assigns different weights $w_D > w_R$ to default versus repayment observations:[23]

$$\hat{\theta}(\{d_s\}_{s=0}^t) = \frac{w_D \sum_{s:d_s=1} \ln P_\theta(y_s, B_s) + w_R \sum_{s:d_s=0} \ln(1 - P_\theta(y_s, B_s))}{w_D N_D + w_R N_R} \tag{B.36}$$

where N_D and N_R are the numbers of defaults and repayments, respectively.

Taking expectations over the ergodic distribution $\mu(y, B)$ of state variables:

$$\mathbb{E}[\hat{\theta}] = \int \left[w_D P_1(y, B) \frac{\partial \ln P_\theta(y, B)}{\partial \theta} + w_R(1 - P_1(y, B)) \frac{\partial \ln(1 - P_\theta(y, B))}{\partial \theta} \right] d\mu(y, B) \tag{B.37}$$

Since $P_\theta(y, B) = L(\Delta V(y, B)/(\theta\eta))$ where L is the logistic CDF:[24]

$$\frac{\partial \ln P_\theta(y, B)}{\partial \theta} = -\frac{\Delta V(y, B)}{\theta^2 \eta}(1 - P_\theta(y, B)) < 0 \tag{B.38}$$

[23] The negativity bias reflects well-documented cognitive biases where agents overweight negative information relative to positive information, consistent with prospect theory and related behavioral findings.

[24] This follows from the discrete choice formulation in the main model where taste shocks follow Gumbel distributions.

Substituting (B.38) into (B.37) and using the negativity bias $w_D > w_R$:

$$\mathbb{E}[\hat{\theta}] > 1 + \frac{(w_D - w_R)}{w_D + w_R} \int P_1(y,B) \frac{|\Delta V(y,B)|}{\theta^2 \eta} (1 - P_\theta(y,B)) d\mu(y,B) > 1 \tag{B.39}$$

By the ergodic theorem and (18), $\lim_{t\to\infty} \mathbb{E}[\theta_t] = \mathbb{E}[\hat{\theta}] > 1$.

Part (ii): History Dependence. Let $\mathcal{H}_t = \{(y_s, B_s, d_s)\}_{s=0}^{t-1}$ denote the history up to time t. The belief updating process creates a Markov chain on the augmented state space $(y, B, \theta, \mathcal{H})$.[25]

For any two histories $\mathcal{H}^A$ and $\mathcal{H}^B$ with different default patterns but identical fundamentals, define the corresponding stationary distributions Θ^A and Θ^B. The key insight is that the transition operator:

$$T_{\mathcal{H}}(\theta, \theta') = \lambda \delta(\theta' - \theta) + (1 - \lambda)\delta(\theta' - \hat{\theta}(\mathcal{H})) \tag{B.40}$$

depends explicitly on the history $\mathcal{H}$.

Since $\hat{\theta}(\mathcal{H}^A) \neq \hat{\theta}(\mathcal{H}^B)$ for histories with different default intensities, we have $\Theta^A \neq \Theta^B$. Moreover, if $\mathcal{H}^A$ contains more surprising defaults than $\mathcal{H}^B$, then Θ^A first-order stochastically dominates Θ^B:

$$\int_{\underline{\theta}}^{\theta} d\Theta^A(\tilde{\theta}) \leq \int_{\underline{\theta}}^{\theta} d\Theta^B(\tilde{\theta}) \quad \forall \theta \in [\underline{\theta}, \bar{\theta}] \tag{B.41}$$

Part (iii): Slow Convergence. Let π_t denote the default probability in period t under the current belief θ_t. The frequency of belief updates follows a Poisson process with intensity π_t.[26]

Between updates, beliefs evolve according to:

$$\theta_{t+1} = \lambda \theta_t \quad \text{(no default)} \tag{B.42}$$

The time between defaults follows an exponential distribution with rate $\pi_t \approx \pi^*$ in steady state. The expected duration between updates is $1/\pi^*$, and during each interval of length τ, beliefs decay by factor λ^τ.

For the convergence rate, consider the deviation from the stationary mean: $\epsilon_t = \theta_t - \mathbb{E}[\Theta^*]$. The dynamics satisfy:

$$\mathbb{E}[\epsilon_{t+1}] = \lambda \mathbb{E}[\epsilon_t] + (1 - \lambda)(\mathbb{E}[\hat{\theta}] - \mathbb{E}[\Theta^*]) \tag{B.43}$$

[25] The history dependence arises because the MLE $\hat{\theta}(\mathcal{H}_t)$ depends on the entire sequence of past defaults, not just the current state.

[26] This approximation is valid when defaults are rare events, which is the empirically relevant case for most sovereigns.

In steady state, $\mathbb{E}[\hat{\theta}] = \mathbb{E}[\Theta^*]$, so:

$$\mathbb{E}[\epsilon_{t+1}] = \lambda \mathbb{E}[\epsilon_t] \tag{B.44}$$

Therefore, $\|\theta_t - \mathbb{E}[\Theta^*]\| = O(\lambda^t)$. Since empirical persistence in sovereign spreads requires $\lambda \approx 0.95 - 0.99$, convergence is indeed slow with half-life approximately $\ln(2)/\ln(1/\lambda) \approx 14 - 69$ periods. □

B.13 Proof of Proposition 9

Proof. We establish each part through detailed analysis of the sovereign's optimization problem under strategic communication.

Setup: The Communication Problem. Let $\mathcal{W}(\alpha;\theta)$ denote the sovereign's value function under transparency level α and baseline pessimism θ. This satisfies the Bellman equation:

$$\mathcal{W}(\alpha;\theta) = \max_{\{c_t, B_{t+1}\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t (u(c_t) - \phi(\alpha)) \right] \tag{B.45}$$

subject to:

$$c_t = y_t - \kappa B_t + [B_{t+1} - (1-\delta)B_t] q_{\theta_{\text{eff}}}(y_t, B_{t+1}) \tag{B.46}$$

$$\theta_{\text{eff}}(\alpha, \theta) = \alpha \cdot 1 + (1-\alpha) \cdot \theta \tag{B.47}$$

Part (i): Interior Solution. Define the value function net of communication costs: $V(\alpha) = \mathcal{W}(\alpha;\theta) - \mathbb{E}[\phi(\alpha)]$. The first-order condition for optimality is:

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\alpha^*} = \left.\frac{\partial \mathcal{W}(\alpha;\theta)}{\partial \alpha}\right|_{\alpha=\alpha^*} - \gamma \alpha^* = 0 \tag{B.48}$$

To characterize $\frac{\partial \mathcal{W}}{\partial \alpha}$, note that transparency affects welfare through the bond pricing channel. Using the envelope theorem:[27]

$$\frac{\partial \mathcal{W}}{\partial \alpha} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u'(c_t) \frac{\partial c_t}{\partial \alpha} \right] \tag{B.49}$$

From the budget constraint (B.46):

$$\frac{\partial c_t}{\partial \alpha} = [B_{t+1} - (1-\delta)B_t] \frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} \frac{\partial \theta_{\text{eff}}}{\partial \alpha} = [B_{t+1} - (1-\delta)B_t] \frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} (1-\theta) \tag{B.50}$$

Since $\frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} < 0$ (higher perceived volatility reduces bond prices) and $(1-\theta) < 0$ for $\theta > 1$,

[27] The envelope theorem applies because the sovereign optimally chooses consumption and debt policies for any given price schedule.

we have:

$$\frac{\partial \mathscr{W}}{\partial \alpha} = (\theta - 1)\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u'(c_t)[B_{t+1} - (1-\delta)B_t] \left| \frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} \right| \right] > 0 \tag{B.51}$$

For an interior solution, we need both concavity and appropriate boundary conditions. The second-order condition requires:

$$\left. \frac{\partial^2 V(\alpha)}{\partial \alpha^2} \right|_{\alpha=\alpha^*} = \left. \frac{\partial^2 \mathscr{W}}{\partial \alpha^2} \right|_{\alpha=\alpha^*} - \gamma < 0 \tag{B.52}$$

Since $\frac{\partial^2 \mathscr{W}}{\partial \alpha^2} < 0$ (diminishing returns to transparency) and $\gamma > 0$, condition (B.52) is satisfied.[28]

The boundary conditions require $\frac{\partial V}{\partial \alpha}|_{\alpha=0} > 0$ and $\frac{\partial V}{\partial \alpha}|_{\alpha=1} < 0$, which hold for intermediate values of γ satisfying:

$$\left. \frac{\partial \mathscr{W}}{\partial \alpha} \right|_{\alpha=0} > \gamma > \left. \frac{\partial \mathscr{W}}{\partial \alpha} \right|_{\alpha=1} \tag{B.53}$$

Part (ii): Pessimism Amplifies Transparency. Differentiating the first-order condition (B.48) with respect to θ using the implicit function theorem:

$$\left. \frac{\partial^2 \mathscr{W}}{\partial \alpha \partial \theta} \right|_{\alpha=\alpha^*} + \left. \frac{\partial^2 \mathscr{W}}{\partial \alpha^2} \right|_{\alpha=\alpha^*} \frac{\partial \alpha^*}{\partial \theta} - \gamma \frac{\partial \alpha^*}{\partial \theta} = 0 \tag{B.54}$$

Rearranging:

$$\frac{\partial \alpha^*}{\partial \theta} = \frac{\frac{\partial^2 \mathscr{W}}{\partial \alpha \partial \theta}}{\gamma - \frac{\partial^2 \mathscr{W}}{\partial \alpha^2}} \tag{B.55}$$

To sign the numerator, differentiate (B.51) with respect to θ:

$$\frac{\partial^2 \mathscr{W}}{\partial \alpha \partial \theta} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u'(c_t)[B_{t+1} - (1-\delta)B_t] \left| \frac{\partial q_{\theta_{\text{eff}}}}{\partial \theta_{\text{eff}}} \right| \right] + (\theta - 1)\frac{\partial}{\partial \theta}\mathbb{E}[\cdots] \tag{B.56}$$

The first term is positive (direct effect), and the second term captures the indirect effect through equilibrium adjustments. Under reasonable parameter values, the direct effect dominates, ensuring $\frac{\partial^2 \mathscr{W}}{\partial \alpha \partial \theta} > 0$.[29]

Since the denominator in (B.55) is positive by the second-order condition (B.52), we conclude:

$$\frac{\partial \alpha^*}{\partial \theta} > 0 \tag{B.57}$$

Part (iii): Welfare Dominance. Define the net welfare gain from communication as:

$$\Delta W(\theta) = \mathscr{W}(\alpha^*(\theta);\theta) - \mathscr{W}(0;\theta) - \mathbb{E}[\phi(\alpha^*(\theta))] \tag{B.58}$$

[28] The diminishing returns arise because transparency has the largest impact when moving from very opaque to moderately transparent, with smaller gains from further increases.

[29] The direct effect dominates because the immediate benefit of transparency increases linearly with the degree of baseline pessimism, while the indirect effects through equilibrium adjustments are second-order.

For small $\theta - 1$, using a second-order Taylor expansion around $\theta = 1$:

$$\Delta W(\theta) \approx \frac{1}{2} \frac{\partial^2 \mathscr{W}}{\partial \alpha^2}\bigg|_{\alpha=\alpha^*(1)} [\alpha^*(\theta)]^2 - \frac{\gamma[\alpha^*(\theta)]^2}{2} \tag{B.59}$$

Using the first-order condition and part (ii), $\alpha^*(\theta) \approx C(\theta - 1)$ for some constant $C > 0$. Substituting:

$$\Delta W(\theta) \approx \frac{C^2(\theta-1)^2}{2} \left[\frac{\partial^2 \mathscr{W}}{\partial \alpha^2} - \gamma\right] \tag{B.60}$$

Since the second-order condition requires $\frac{\partial^2 \mathscr{W}}{\partial \alpha^2} - \gamma < 0$, we have $\Delta W(\theta) < 0$ for θ close to 1. However, as θ increases, the benefits of transparency grow faster than the costs.[30]

The critical threshold θ_c is defined by $\Delta W(\theta_c) = 0$. For $\theta > \theta_c$, the welfare gains from reduced borrowing costs exceed the communication costs, establishing welfare dominance.

□

[30]This occurs because the marginal benefit of transparency is proportional to $(\theta - 1)$, while the marginal cost is constant in θ.

C Computational Algorithm

I solve the model numerically using value function iteration on a discretized state space, following the computational strategies developed by (Mihalache, 2024). The algorithm is implemented in Fortran and parallelized using OpenMP to accelerate computation. The process is as follows:

C.1 Discretization and Initialization

The state space is discretized first.

1. I discretize the exogenous endowment process, an AR(1) in logarithms defined in equation (1), into a $N_y = 201$ state Markov chain using the (Tauchen, 1986) method, as implemented in my `discretizeAR1` subroutine. This yields a grid for income levels $\{y_i\}_{i=1}^{N_y}$ and a transition matrix $\Pi(y_i, y_j)$.

2. I construct the space for bond holdings, $\mathscr{B}$, as a grid of $N_B = 600$ points, uniformly spaced between $B_{\min} = 0$ and $B_{\max} = 0.75$ of mean output, created using a `linspace` function.

3. I initialize the algorithm with guesses for the value functions and the bond price schedule. I set the price of bonds, $q_0(y, B')$, initially to 1 for all states. I initialize the value function, $V_0(y, B)$, to the utility of consuming the endowment net of coupon payments, and the value of default, $Vd_0(y)$, to the utility of consuming the post-default output $h(y)$.

C.2 Value Function and Price Iteration

The core of my solution method is iterating on the value functions (V, V^D) and the bond price function q until they converge to a joint fixed point. This process solves the system of equations defined in Section 3. My main `DO WHILE` loop in the Fortran code continues until the maximum absolute difference between successive iterates for both the value function and the bond prices falls below a specified tolerance ($\epsilon_V = 10^{-6}, \epsilon_q = 10^{-6}$). A single iteration, from step $k-1$ to k, is a carefully ordered sequence of updates, parallelized with OpenMP where possible.

1. **Update the Value of Default.** I begin each iteration by updating the value of default, $V_k^D(y)$. This calculation uses the total value function V_{k-1} and default value V_{k-1}^D from the *previous* iteration. For each income state y_i, I follow the discrete version of equation (7):

$$V_k^D(y_i) = u(h(y_i)) + \beta \sum_{j=1}^{N_y} \Pi(y_i, y_j) \left[\gamma V_{k-1}(y_j, B_1) + (1-\gamma) V_{k-1}^D(y_j) \right],$$

where B_1 is the grid point corresponding to zero debt.

2. **Update the Value of Repayment and Sovereign Policies.** This is a multi-step process that I compute inside a large parallel loop over the state space (y_i, B_j).

   (a) First, for each state (y_i, B_j), I calculate the **choice-specific value of borrowing**, $W(y_i, B_j, B'_l)$, for *every* possible next-period debt level B'_l. This uses the bond price q_{k-1} and value function V_{k-1} from the prior iteration:

   $$W(y_i, B_j, B'_l) = u(y_i - \kappa B_j + [B'_l - (1-\delta)B_j] q_{k-1}(y_i, B'_l)) + \beta \sum_{m=1}^{N_y} \Pi(y_i, y_m) V_{k-1}(y_m, B'_l).$$

   (b) With the full vector of W values, I compute the **value of repayment**, $V_k^R(y_i, B_j)$, using the "log-sum-exp" formulation following (Mihalache, 2024). This numerically stable technique, which uses the borrowing taste shock parameter ρ (or `rhoB` in my code), replaces a non-differentiable 'max' operation over the borrowing grid:

   $$V_k^R(y_i, B_j) = \rho \ln\left(\sum_{l=1}^{N_B} \exp \frac{W(y_i, B_j, B'_l)}{\rho}\right).$$

   (c) Simultaneously, I compute the state-contingent **borrowing policy**, $\Pr(B'_l | y_i, B_j)$, as the softmax of the choice-specific values:

   $$\Pr(B'_l | y_i, B_j) = \frac{\exp(W(y_i, B_j, B'_l)/\rho)}{\sum_{s=1}^{N_B} \exp(W(y_i, B_j, B'_s)/\rho)}.$$

   (d) Finally, using the *just-updated* values $V_k^R(y_i, B_j)$ and $V_k^D(y_i)$, I calculate the new **ex-ante value function** $V_k(y_i, B_j)$, again using the log-sum-exp formula but with the default taste shock parameter η (`rhoD` in my code):

   $$V_k(y_i, B_j) = \eta \ln\left(\exp \frac{V_k^D(y_i)}{\eta} + \exp \frac{V_k^R(y_i, B_j)}{\eta}\right).$$

   (e) I also determine the sovereign's true **default probability**, $\Pr\{d = 1 | y_i, B_j\}$, at this point from the logistic choice formula:

   $$\Pr\{d = 1 | y_i, B_j\} = \frac{\exp(V_k^D(y_i)/\eta)}{\exp(V_k^D(y_i)/\eta) + \exp(V_k^R(y_i, B_j)/\eta)}.$$

3. **Update Bond Prices.** I update the bond price schedule $q_k(y, B')$ based on the lenders' zero-profit condition. This is the crucial step where I introduce the behavioral pessimism wedge, θ (`thetaD`). For each state (y_i, B'_l):

(a) I calculate the **lender's perceived default probability**, $\tilde{P}(y_m, B'_l)$. It uses the most recently updated value functions, V^R_k and V^D_k, but is distorted by the inflated taste shock parameter $\tilde{\eta} = \theta\eta$:

$$\tilde{P}(y_m, B'_l) = \frac{\exp(V^D_k(y_m)/(\theta\eta))}{\exp(V^D_k(y_m)/(\theta\eta)) + \exp(V^R_k(y_m, B'_l)/(\theta\eta))}.$$

(b) The new price, $q_k(y_i, B'_l)$, is the expected discounted payoff from the lender's perspective. This expectation is taken over future income states y_m and future bond choices B''_s. It uses the lender's perceived default probability $\tilde{P}$, the sovereign's *true* borrowing policy $\Pr(B''_s|y_m, B'_l)$ (computed in step 2), and the bond prices from the *previous* iteration, q_{k-1}. The full pricing equation implemented in the code is:

$$q_k(y_i, B'_l) = \frac{1}{1+r}\sum_{m=1}^{N_y}\Pi(y_i, y_m)(1-\tilde{P}(y_m, B'_l))\left(\kappa + (1-\delta)\sum_{s=1}^{N_B}\Pr(B''_s|y_m, B'_l)\cdot q_{k-1}(y_m, B''_s)\right).$$

4. **Check for Convergence.** The iteration concludes by calculating the supremum norm of the change in the value functions and price functions: errV $= \max|V_k - V_{k-1}|$ and errQ $= \max|q_k - q_{k-1}|$. If the errors are within tolerance, the loop terminates. Otherwise, the algorithm sets $V_0 \leftarrow V_k$, $Vd_0 \leftarrow V^D_k$, and $q_0 \leftarrow q_k$, and proceeds to the next iteration.

C.3 Simulation

Once the functions have converged, the `simulate` subroutine is called to generate the business cycle moments. For reproducibility, the random number generator is initialized with a fixed seed. The simulation generates a long time series of 100,000 periods, with the first 299 periods discarded as a burn-in phase.

The simulation starts from a deterministic state: zero initial debt and the median level of the endowment grid. For each subsequent period, the simulation proceeds as follows:

1. The new endowment level is determined by drawing from the discretized Markov transition matrix, Π.

2. If the economy was in default in the previous period, a random draw determines if it regains market access (with probability γ). If not, it remains in default.

3. If the economy has market access, a uniform random draw is compared against the converged default probability function, $\Pr\{d = 1|y, B\}$, to determine if a default occurs.

4. If the economy does not default, a second uniform random draw is used to select the next period's debt level, B', from the converged borrowing probability distribution, $\Pr\{B'|y, B\}$.

5. Key economic variables (consumption, trade balance, GDP, credit spreads) for the current period are calculated based on the state and choices, and then stored.

The business cycle moments reported in Table 3 are calculated from this simulated data series. The code consistently uses the standard numerical stability technique of subtracting the maximum value within any `exp` calculation in the log-sum-exp formulas to prevent floating-point overflow.

C.4 Numerical Stability Techniques

The implementation of the log-sum-exp formulation for discrete choice models requires careful attention to numerical stability to prevent floating-point overflow and underflow errors, as emphasized by (Mihalache, 2024). This subsection details the specific techniques I employ in my algorithm to ensure robust computation, particularly in the evaluation of choice probabilities and expected values corresponding to the discrete choice formulations in the main text.

C.4.1 The Log-Sum-Exp Numerical Stability Problem

The naive implementation of the log-sum-exp function

$$\mathrm{LSE}(x_1, \ldots, x_n) = \ln\left(\sum_{i=1}^{n} \exp(x_i)\right) \tag{C.61}$$

is prone to numerical instability when the values x_i are large in magnitude. For large positive values, $\exp(x_i)$ may overflow, returning infinity. For large negative values, $\exp(x_i)$ may underflow to zero, causing a loss of precision in the sum. In my sovereign default model, these issues arise when the choice-specific values $W(y, B, B')$ span a wide range due to varying consumption levels across borrowing choices, when the value functions $V^R(y, B)$ and $V^D(y)$ differ substantially near the default boundary, and when the small scale parameters $\rho \approx 10^{-5}$ and $\eta \approx 10^{-4}$ amplify the ratios W/ρ or V/η to extreme magnitudes.

C.4.2 Stabilized Log-Sum-Exp Implementation

To address these issues, I employ the standard "max subtraction" technique following (Mihalache, 2024). For any set of values $\{x_1, \ldots, x_n\}$, I first compute their maximum:

$$x_{\max} = \max\{x_1, \ldots, x_n\} \tag{C.62}$$

Then I rewrite the log-sum-exp as:

$$\mathrm{LSE}(x_1,\ldots,x_n) = x_{\max} + \ln\left(\sum_{i=1}^{n}\exp(x_i - x_{\max})\right) \tag{C.63}$$

This transformation ensures that all terms satisfy $(x_i - x_{\max}) \leq 0$, yielding $\exp(x_i - x_{\max}) \in (0,1]$, where the largest exponential term equals exactly 1 to prevent overflow, while smaller terms decay exponentially but remain numerically representable to preserve precision.

C.4.3 Application to Borrowing Choice Probabilities

In my computation of borrowing choice probabilities corresponding to equation (5), I use the stabilized implementation:

$$\overline{W}(y,B) = \max_{B'} W(y,B,B') \tag{C.64}$$

$$\Pr(B' = B_i|y,B) = \frac{\exp\frac{W(y,B,B_i)-\overline{W}(y,B)}{\rho}}{\sum_{j\in\mathscr{B}}\exp\frac{W(y,B,B_j)-\overline{W}(y,B)}{\rho}} \tag{C.65}$$

Similarly, the expected value of repayment corresponding to equation (9) becomes:

$$V^R(y,B) = \overline{W}(y,B) + \rho\ln\left(\sum_{j\in\mathscr{B}}\exp\frac{W(y,B,B_j)-\overline{W}(y,B)}{\rho}\right) \tag{C.66}$$

C.4.4 Application to Default Choice Probabilities

For the default decision corresponding to equation (4), I apply the same stabilization technique:

$$\overline{V}(y,B) = \max\{V^D(y), V^R(y,B)\} \tag{C.67}$$

$$\Pr(d=1|y,B) = \frac{\exp\frac{V^D(y)-\overline{V}(y,B)}{\eta}}{\exp\frac{V^D(y)-\overline{V}(y,B)}{\eta} + \exp\frac{V^R(y,B)-\overline{V}(y,B)}{\eta}} \tag{C.68}$$

The corresponding expected value function is:

$$V(y,B) = \overline{V}(y,B) + \eta\ln\left[\exp\frac{V^D(y)-\overline{V}(y,B)}{\eta} + \exp\frac{V^R(y,B)-\overline{V}(y,B)}{\eta}\right] \tag{C.69}$$

C.4.5 Lender Pessimism and Numerical Stability

The behavioral extension introducing lender pessimism as in equation (11) requires computing perceived default probabilities using the distorted parameter $\tilde{\eta} = \theta\eta$. I compute the lender's perceived default probability using the same stabilization technique as in the default choice probabilities:

$$\tilde{V}(y,B) = \max\{V^D(y), V^R(y,B)\} \tag{C.70}$$

$$\tilde{P}(y,B) = \frac{\exp \frac{V^D(y)-\tilde{V}(y,B)}{\tilde{\eta}}}{\exp \frac{V^D(y)-\tilde{V}(y,B)}{\tilde{\eta}} + \exp \frac{V^R(y,B)-\tilde{V}(y,B)}{\tilde{\eta}}} \tag{C.71}$$

where $\tilde{V}(y,B)$ serves as the stabilizing maximum for the lender's computation. An additional safeguard prevents division by zero when both exponential terms become numerically zero, setting $\tilde{P}(y,B) = 0.5$ in such extreme cases.

C.4.6 Additional Numerical Safeguards

Beyond the log-sum-exp stabilization, my implementation includes several other numerical safeguards. For consumption bounds, I prevent evaluation of the utility function at non-positive consumption levels by setting $W(y,B,B') = -10^6$ whenever the implied consumption is non-positive, which effectively removes infeasible choices from consideration. I set the convergence tolerances to $\epsilon_V = \epsilon_q = 10^{-6}$, balancing computational accuracy with reasonable iteration counts. All my computations use double precision arithmetic to minimize accumulated rounding errors.

These numerical stability techniques, following the approach of (Mihalache, 2024), are essential for ensuring convergence of my value function iteration algorithm, particularly in the presence of the small taste shock parameters that characterize the discrete choice approach.

References

Abbas, S. M. Ali, Alex Pienkowski, and Kenneth S. Rogoff, eds, *Sovereign Debt: A Guide for Economists and Practitioners*, Oxford University Press, 2019.

Aguiar, M. and G. Gopinath, "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, 2007, *115* (1), 69–102.

Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent, and Juha Seppälä, "Optimal Taxation without State-Contingent Debt," *Journal of Political Economy*, 2002, *110* (6), 1220–1254.

Amador, M. and C. Phelan, "Reputation and Sovereign Default," *Econometrica*, 2021, *89* (4), 1979–2010.

__ **and** __ , "Reputation and Partial Default," *American Economic Review: Insights*, 2023, *5* (2), 158–172.

Arellano, C., "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 2008, *98* (3), 690–712.

_ **and A. Ramanarayanan**, "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 2012, *120* (2), 187–232.

Baker, M. and J. Wurgler, "Investor Sentiment and the Cross-Section of Stock Returns," *The Journal of Finance*, 2006, *61* (4), 1645–1680.

Barberis, N. and R. Thaler, "A survey of behavioral finance," in George M. Constantinides, Milton Harris, and René M. Stulz, eds., *Handbook of the Economics of Finance*, Vol. 1, Elsevier, 2003, pp. 1053–1128.

Baumeister, Roy F., Ellen Bratslavsky, Catrin Finkenauer, and Kathleen D. Vohs, "Bad Is Stronger Than Good," *Review of General Psychology*, 2001, *5* (4), 323–370.

Bocola, L. and A. Dovis, "Self-Fulfilling Debt Crises: A Quantitative Analysis," *American Economic Review*, 2019, *109* (12), 4343–4377.

Bordalo, P., N. Gennaioli, Rafael La Porta, and A. Shleifer, "Diagnostic Expectations and Credit Cycles," *Quarterly Journal of Economics*, 2023, *138* (2), 941–994.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer, "Diagnostic Expectations and Credit Cycles," *The Journal of Finance*, 2018, *73* (1), 199–227.

Brunnermeier, M. K. and S. Nagel, "Hedge Funds and the Technology Bubble," *The Journal of Finance*, 2004, *59* (5), 2013–2040.

Calvo, G. A., L. Leiderman, and C. M. Reinhart, "Inflows of Capital to Developing Countries in the 1990s," *Journal of Economic Perspectives*, 1996, *10* (2), 123–139.

Cavallo, A., "Online and Official Price Indexes: Measuring Argentina's Inflation," *Journal of Monetary Economics*, 2013, *60* (2), 152–165.

Chatterjee, S. and B. Eyigungor, "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 2012, *102* (6), 2674–2699.

Cole, H. L. and P. J. Kehoe, "Models of Sovereign Debt: Partial versus General Reputations," *International Economic Review*, 1998, *39* (1), 55–70.

_ **and T. J. Kehoe**, "Self-Fulfilling Debt Crises," *The Review of Economic Studies*, 2000, *67* (1), 91–116.

Cole, Harold L., James Dow, and William B. English, "Default, Settlement, and Signalling: Lending Resumption in a Reputational Model of Sovereign Debt," *International Economic Review*, 1995, *36* (2), 365–385.

Cruces, J. J. and C. Trebesch, "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 2013, *5* (3), 85–117.

D'Erasmo, P., "Government Reputation and Debt Repayment in Emerging Economies," *Working Paper*, 2011. Federal Reserve Bank of Philadelphia.

Dovis, A. and R. Kirpalani, "Fiscal Rules, Bailouts, and Reputation in Federal Governments," *American Economic Review*, 2020, *110* (3), 860–888.

— **and** — , "Reputation, Bailouts, and Interest Rate Spread Dynamics," *Working Paper*, 2021. University of Wisconsin-Madison.

Dvorkin, M., J. M. Sanchez, H. Sapriza, and E. Yurdagul, "Sovereign Debt Restructurings," *American Economic Journal: Macroeconomics*, 2021, *13* (2), 26–77.

Eaton, J. and M. Gersovitz, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 1981, *48* (2), 289–309.

Egorov, K. and M. Fabinger, "Reputational Effects in Sovereign Default," *Working Paper*, 2016. University of Chicago.

Fourakis, S., "Sovereign Default and Government Reputation," *Working Paper*, 2024.

Gabaix, X., "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance," *The Quarterly Journal of Economics*, 2012, *127* (2), 645–700.

Gennaioli, N., A. Martin, and S. Rossi, "Sovereign Default, Domestic Banks, and Financial Institutions," *The Journal of Finance*, 2014, *69* (2), 819–866.

— **and A. Shleifer**, *A Crisis of Beliefs: Investor Psychology and Financial Fragility*, Oxford University Press, 2018.

Gilboa, I. and D. Schmeidler, "Maxmin expected utility with non-unique prior," *Journal of Mathematical Economics*, 1989, *18* (2), 141–153.

Grajewski, Silvio and Elzbieta Raszewski, "Argentina Fines Private Economists for Inflation Reports," Reuters February 2013. Online news article; Accessed 20 June 2025.

Hansen, L. P. and T. J. Sargent, *Robustness*, Princeton University Press, 2008.

Hatchondo, J. C. and L. Martinez, "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 2009, *79* (1), 117–125.

International Monetary Fund, "IMF Executive Board Issues Declaration of Censure on Argentina," Press Release No. 13/33, February 1, 2013 2013. Accessed 20 June 2025.

Kahneman, D. and A. Tversky, "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 1979, *47* (2), 263–291.

Kindleberger, C. P., *Manias, Panics and Crashes: A History of Financial Crises*, New York: Basic Books, 1978.

Lizarazo, S. V., "Default risk and risk averse international investors," *Journal of International Economics*, 2013, *89* (2), 317–330.

Long, J. B. De, A. Shleifer, L. H. Summers, and R. J. Waldmann, "Noise Trader Risk in Financial Markets," *Journal of Political Economy*, 1990, *98* (4), 703–738.

Longstaff, F. A., J. Pan, L. H. Pedersen, and K. J. Singleton, "How Sovereign Is Sovereign Credit Risk?," *American Economic Journal: Macroeconomics*, 2011, *3* (2), 75–103.

Mendoza, E. G. and V. Z. Yue, "A General Equilibrium Model of Sovereign Default and Business Cycles," *The Quarterly Journal of Economics*, 2012, *127* (2), 889–946.

Meyer, J., C. M. Reinhart, and C. Trebesch, "Sovereign Bonds Since Waterloo," *The Quarterly Journal of Economics*, 2022, *137* (3), 1615–1680.

Mihalache, G., "Solving default models," *Oxford Research Encyclopedia of Economics and Finance*, 2024. Forthcoming.

Mitchener, K. J. and C. Trebesch, "Sovereign Debt in the Twenty-first Century," *Journal of Economic Literature*, 2023, *61* (2), 565–623.

Morelli, J. M. and M. Moretti, "Information Frictions, Reputation, and Sovereign Spreads," *Journal of Political Economy*, 2023, *131* (11), 3066–3102.

Neumeyer, P. A. and F. Perri, "Business cycles in emerging economies: the role of interest rates," *Journal of Monetary Economics*, 2005, *52* (2), 345–380.

Paluszynski, R., "Learning about Debt Crises," *American Economic Journal: Macroeconomics*, 2023, *15* (1), 106–134.

Phelan, C., "Public trust and government betrayal," *Journal of Economic Theory*, 2006, *130* (1), 27–43.

Pouzo, D. and I. Presno, "Sovereign Default under Model Uncertainty," *American Economic Review*, 2016, *106* (9), 2337–2371.

Roch, F. and F. Roldán, "Uncertainty, Sovereign Default Risk, and State-Contingent Debt," *Journal of Political Economy Macroeconomics*, 2023, *1* (2), 334–370.

Shefrin, Hersh and Meir Statman, "The Disposition to Sell Winners Too Early and Ride Losers Too Long: Theory and Evidence," *The Journal of Finance*, 1985, *40* (3), 777–790.

Stangebye, Z. R., "Beliefs and long-maturity sovereign debt," *Journal of International Economics*, 2020, *127*, 103381.

Tauchen, G., "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, 1986, *20* (2), 177–181.

The Economist, "Don't Lie to Me, Argentina," The Economist February 2012.

Tomz, Michael and Mark L. J. Wright, "Empirical Research on Sovereign Debt and Default," *Annual Review of Economics*, 2013, *5*, 247–272.

Tversky, A. and D. Kahneman, "Judgment under Uncertainty: Heuristics and Biases," *Science*, 1974, *185* (4157), 1124–1131.