Sovereign Default with Bounded Rationality

[INCOMPLETE AND COMMENTS WELCOME]

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Outline

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Motivation

Motivation

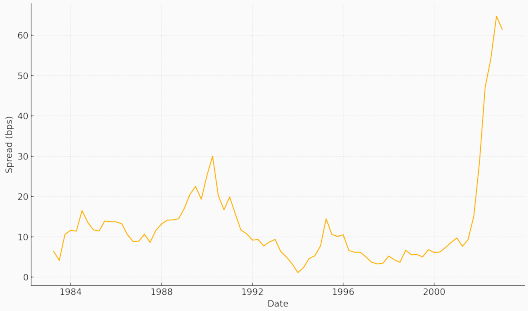
The Stylized Facts:

- Emerging market sovereign spreads $\Longrightarrow$ **high** and extremely **volatile**
- Crises $\Longrightarrow$ sudden **spikes in spreads**
- Standard models generated spreads $\Longrightarrow$ too low and too smooth

The Blank:

- Most of the papers in this literature assume full rationality

Figure 1: Argentina's Spread (1983-2003)



Data Source: (Arellano, 2008)

A Model with Bounded Rationality

Previous Solution: Long term debt, debt dilution, etc.

This paper: Relax the full rationality assumption $\Longrightarrow$ heterogeneous Lenders

- Boundedly rational agents (Information & Cognitive Costs $\Longrightarrow$ Heuristics)

Key Results:

Theoretical: Endogenous price discontinuity

- A new mechanism for sudden crises

Quantitative: Match high average spreads AND

- Extreme spread volatility
- Higher financial fragility (more debt & defaults)

Model

Model Setup

A small open economy with AR(1) potential output stream $\{y_t\}_{t=0}^{\infty}$:

$$\ln(y_{t+1}) = \rho \ln(y_t) + \varepsilon_{t+1}, \quad \text{where } \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$$

described by a kernel $p(y', y)$

Identical households maximize:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

The government smooth consumption by borrowing from (and lending to) foreign creditors

Government & Financial Market

The Sovereign Government

Decision: Default or Repay?

- If **Default**:
  - Output cost: $y \to h(y) \leq y$
  - Excluded from credit markets
  - Re-entry with probability θ each period
- If **Repay**:
  - Honors current debt B
  - Chooses next period's assets B'

Budget Constraint (when repaying):

$$c = y + B - q(B', y)B'$$

The Financial Market

- **Lenders:** Competitive & Risk-Neutral.
- **Asset:** One-period, non-contingent bond.
- **The Bond Pricing Equation:**

$$q(B', y) = \frac{1 - \delta(B', y)}{1 + r} \qquad (1)$$

$\delta(B', y)$: probability of default on the new debt B'

Recursive Government's Problem

The Value of Default, $V^D(y)$

$$V^D(y) = u(h(y)) + \beta \mathbb{E}_{y'} \left[\theta V(0, y') + (1 - \theta) V^D(y') | y \right] \tag{2}$$

The Value of Repayment, $V^R(B, y)$

$$V^R(B, y) = \max_{B' \geq -Z} \left\{ u \left(y + B - q(B', y) B' \right) + \beta \mathbb{E}_{y'} [V(B', y') | y] \right\} \tag{3}$$

The Optimal Decision & Overall Value Function, $V(B, y)$
Defaults if and only if $V^D(y) > V^R(B, y)$

$$V(B, y) = \max \left\{ V^R(B, y), V^D(y) \right\} \tag{4}$$

Default Probability with λ-Rationality

Assuming the market consists of two types of lenders:

Fraction λ: Rational Lenders

$$\delta_r(B', y) = \mathbb{E}_{y'}\left[\mathbb{I}_{\{V^D(y') > V^R(B', y')\}} | y\right] = \int \mathbb{I}_{\{\dots\}} p(y, y') dy' \tag{5}$$

Fraction $(1-\lambda)$: Boundedly Rational Lenders

$$\delta_{ir}(B', y) = \mathbb{I}_{\{V^D(\mathbb{E}[y'|y]) > V^R(B', \mathbb{E}[y'|y])\}} \tag{6}$$

Aggregate Market Belief

The weighted average of the two groups' beliefs

$$\delta(B', y; \lambda) = \lambda \delta_r(B', y) + (1-\lambda)\delta_{ir}(B', y) \tag{7}$$

Theoretical Results

The Discontinuous Price Schedule

Bounded rationality $\implies$ "price cliff"

Theorem

With a fraction $(1-\lambda) > 0$ of boundedly rational lenders, the equilibrium bond price schedule $q(B', y; \lambda)$ has a unique discontinuity at a critical debt threshold $\tilde{B}'(y)$. Specifically:

- *The price drop at the threshold*

$$\lim_{B' \to \tilde{B}'(y)^+} q(B', y; \lambda) - \lim_{B' \to \tilde{B}'(y)^-} q(B', y; \lambda) = \frac{1-\lambda}{1+r}$$

- *For "safe" debt levels ($B' > \tilde{B}'(y)$), $q(B', y; \lambda) \geq q(B', y; 1)$*
- *For "risky" debt levels ($B' < \tilde{B}'(y)$), $q(B', y; \lambda) \leq q(B', y; 1)$*

Intuition: Shift in the beliefs of boundedly rational lenders $\implies$ sudden drop in the market price

Implication 1: Endogenous Interest Rate Spikes

Corollary
For adverse states (B, y) where $B' < \tilde{B}'(y)$, the resulting interest rate

$$r^c(B, y) = \frac{1}{q(B'(B, y), y; \lambda)} - 1$$

*is **strictly higher** than in the fully rational benchmark*

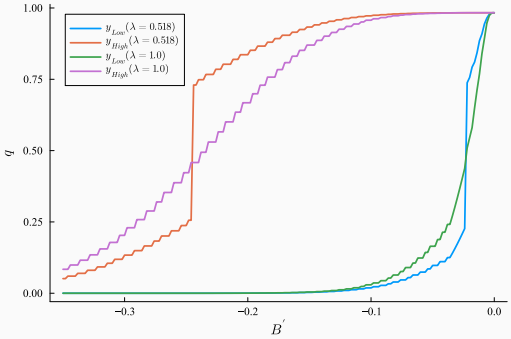
Implication 2: State-Dependent Financial Fragility

Proposition
*The critical debt threshold $\tilde{B}'(y)$ is **decreasing** in y, i.e., $\frac{d\tilde{B}'(y)}{dy} < 0$*

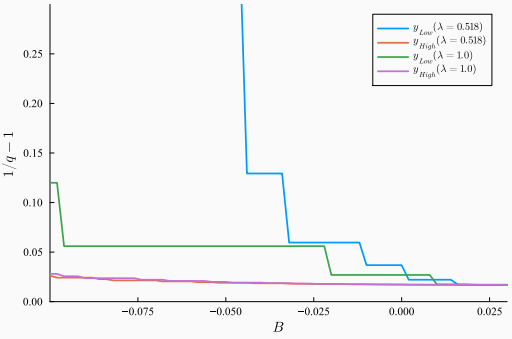
Intuition: In good times $\implies$ lenders are more optimistic $\implies$ the government sustain more debt before a crisis is triggered

Quantative Results

Price Drop & Interest Rate Spike



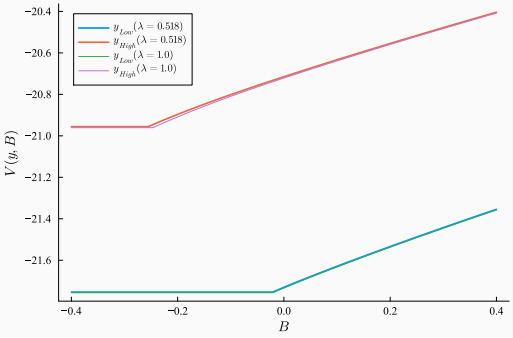
(a) Bond Price Schedule $q(B', y)$



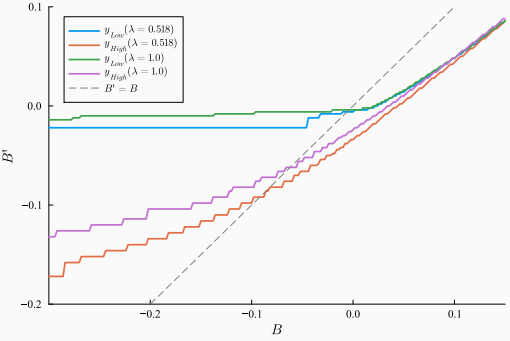
(b) Equilibrium Interest Rate $r^c(B, y)$

In adverse states (low y, high initial debt B) $\Longrightarrow$ "forced" over the cliff $\Longrightarrow$ **high** equilibrium interest rates

Equilibrium Policy & Financial Fragility



(a) Value Function $V(B, y)$



(b) Savings Function $B'(B, y)$

Negligible "value effect" + the "price effect" dominates $\Longrightarrow$ more aggressive borrowing $\Longrightarrow$ endogenously more **fragile**

Recap

Summary & Key Takeaways

A New Mechanism for Sovereign Default

- Heterogeneous lender beliefs $\Longrightarrow$ both rational and boundedly rational agents

Endogenous ”Price Cliff”

- Endogenously generates a discontinuous bond price schedule

Resolving the Spread Puzzles

- Can match the high average spreads observed in emerging markets

Endogenous Financial Fragility

- “Cheap” credit offered by optimistic lenders $\Longrightarrow$ induced to borrow more aggressively $\Longrightarrow$ endogenously more fragile