Geometry of transfer matrix

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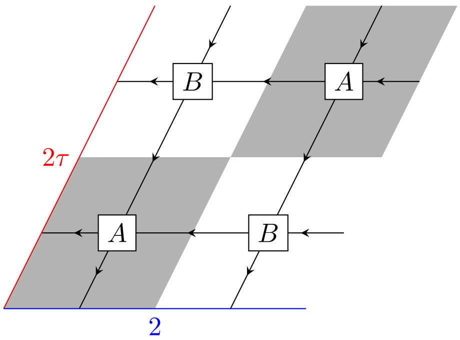
Based on the intuition that a fixed-point tensor represents a partition function on a block of system:



we expect that

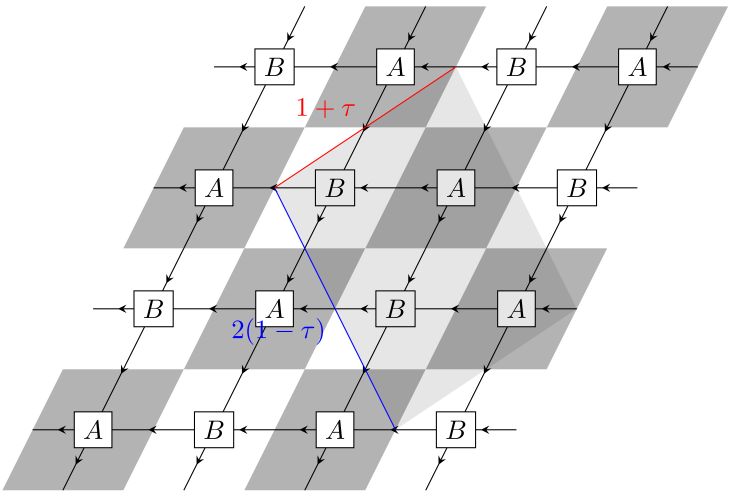
$$\lambda_{ij} = \exp\left[2\pi \mathrm{i}\tau\left(h_i - \frac{c}{24}\right) - 2\pi \mathrm{i}\bar{\tau}\left(h_j - \frac{c}{24}\right) + \epsilon A\right].$$

Here A is the area of the cell.

Thus the shape factor corresponds to the case when $i = j = 0$:

$$\lambda_{00} = \exp\left[2\pi \mathrm{i}(\bar{\tau} - \tau)\frac{c}{24} + \epsilon A\right].$$

Now we choose another cell that has the same area as the first one



The new modular parameter of this cell is

$$\tau' = \frac{1+\tau}{2(1-\tau)}.$$

And the area of the new cell is the same as the previous one. Let λ'_{ij} be the eigenvalue of the new transfer matrix, then

$$\frac{\lambda'_{ij}}{\lambda_{00}} = \exp\left[2\pi \mathrm{i}\tau'\left(h_i - \frac{c}{24}\right) - 2\pi \mathrm{i}\bar{\tau}'\left(h_j - \frac{c}{24}\right) - 2\pi \mathrm{i}(\bar{\tau} - \tau)\frac{c}{24}\right].$$

In particular, the central charge can be obtained from

$$\frac{\lambda'_{00}}{\lambda_{00}} = \exp\left[2\pi\mathrm{i}\,(\tau - \tau' - \bar{\tau} + \bar{\tau}')\,\frac{c}{24}\right].$$