Geometry of transfer matrix

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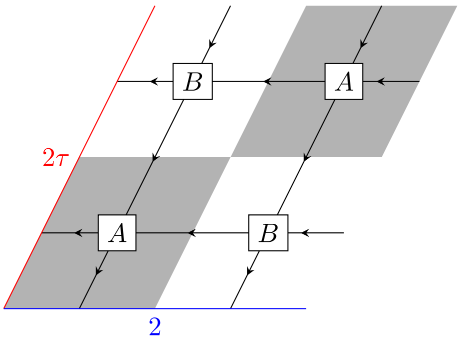
Based on the intuition that a fixed-point tensor represents a partition function on a block of system:



we expect that

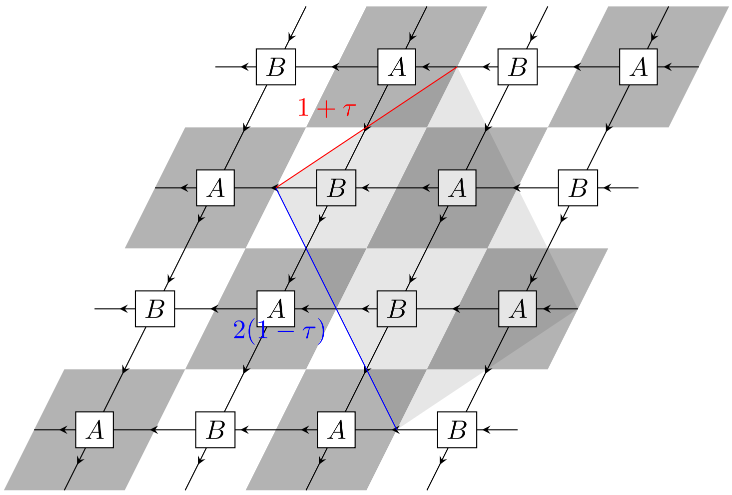
$$\lambda_{ij} = \exp\left[2\pi \mathrm{i}\tau \left(h_i - \frac{c}{24}\right) - 2\pi \mathrm{i}\bar{\tau} \left(h_j - \frac{c}{24}\right) + \epsilon A\right].$$

Here A is the area of the cell.

Thus the shape factor corresponds to the case when $i = j = 0$:

$$\lambda_{00} = \exp\left[2\pi \mathrm{i}(\bar{\tau} - \tau)\frac{c}{24} + \epsilon A\right].$$

Now we choose another cell that has the same area as the first one



The new modular parameter of this cell is

$$\tau' = \frac{1+\tau}{2(1-\tau)}.$$

And the area of the new cell is the same as the previous one. Let λ'_{ij} be the eigenvalue of the new transfer matrix, then

$$\frac{\lambda'_{ij}}{\lambda_{00}} = \exp\left[2\pi \mathrm{i}\tau' \left(h_i - \frac{c}{24}\right) - 2\pi \mathrm{i}\bar{\tau}' \left(h_j - \frac{c}{24}\right) - 2\pi \mathrm{i}(\bar{\tau} - \tau)\frac{c}{24}\right].$$

In particular, the central charge can be obtained from

$$\frac{\lambda'_{00}}{\lambda_{00}} = \exp\left[2\pi\mathrm{i}\left(\tau - \tau' - \bar{\tau} + \bar{\tau}'\right)\frac{c}{24}\right] = \exp\left[-\pi\Im(\tau - \tau')\frac{c}{6}\right].$$

And the conformal dimensions can be obtained from

$$\frac{\lambda'_{ij}}{\lambda'_{00}} = \exp\left[-2\pi\Im\tau'(h_i + h_j) + 2\pi\mathrm{i}\Re\tau'(h_i - h_j)\right].$$

When $\tau = \mathrm{i}$, we have $\tau' = \frac{\mathrm{i}}{2}$. Then the conformal data are calculated via

$$\frac{\lambda'_{00}}{\lambda_{00}} = \mathrm{e}^{-\frac{\pi c}{12}}, \quad \frac{\lambda'_{ij}}{\lambda'_{00}} = \mathrm{e}^{-\pi(h_i + h_j)}.$$