Chenrui Xu. HW1.

1. i. $f'(x) = (\sin(6x-1))' = 6\cos(6x-1)$.

ii. $f'(x) = 8x^7 + (-\frac{4}{x^5})$

iii. $f'(x) = \cancel{e^{x^{-1}} \cdot (-1) + (-2x^{-3}) = -e^{x^{-1}} - 2x^{-3}}$ $\exp(\frac{1}{x} + \frac{1}{x^2}) \cdot (-x^{-2} - 2x^{-3})$

iv. $f'(x) = 2\sin(6x-1) \cdot 6 \cdot \cos(6x-1) = 12\cos(6x-1)\sin(6x-1) = 6\sin(12x-2)$

2. $f'(x) = 6x^2 + 48x - 54 = 6(x^2+8x-9) \overset{set}{=} 0$ $(x-1)(x+9)=0$ $x=1 / x=-9$ have extreme value

$x=0$ plug in $f'(0) = -54$ $f'(2) = 6\times4 + 48\times2 - 54 = 66$

so we know that $f(1)$ is the local min value.

same way $f'(10) > 0$ $f(-9)$ is the local max value.

so when $x \in [-9, 1]$ $f(x)$ is decreasing.

$f''(x) = 12x + 48 = 0$ $x = -4$. $f''(0) = 48 > 0$ $f''(-5) = -12 < 0$ $\begin{cases} f''(-9) = \\ f''(1) > 0 \end{cases}$ concave up

we know if $f''(x) \geq 0$ it is concave up so when $x \geq -4$ $f(x)$ is concave up

3. ① critical point. $x = 1 / x = -9$. $f(1) = -28$ $f(-9) = 972$ so $(1, -28)$ and $(-9, 972)$

② inflection point. $x = -4$. $f(-4) = 472$ so $(-4, 472)$

③ $(1, -28)$ is local min. $(-9, 972)$ is local max

④ $x \in [-3, 3]$ global max $f(-3) = 324$ global min $f(1) = -28$.

$x \in (-\infty, 0]$ no global min global max $f(-9) = 972$

4. i. $\begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$.

ii. $(1,2)$: $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$. $(2,-1)$: $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $(0,0)$: $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

5. i. $f(x,y) = 2xy + x^2 + y$ $\frac{\partial f}{\partial x} = 2y + 2x$ $\frac{\partial f}{\partial y} = 2x + 1$ so $\begin{bmatrix} 2y+2x \\ 2x+1 \end{bmatrix}$.

ii. $(1,1)$: $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$. $(0,-1)$: $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $(0,0)$: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

iii. $\frac{\partial f}{\partial x_1} = 2x_1 + 2x_2 + x_2^2$ $\frac{\partial f}{\partial x_2} = 2x_1 + 2x_2 + 2x_1x_2$

6. i $y = 3x - 0.5$

ii. $y = ax + b$ $\begin{cases} 8 = 4a + b \\ 14 = 6a + b \end{cases}$ $2a = 6$ $a = 3$. $b = -4$ $y = 3x - 4$

iii $y = -\frac{1}{5}x + b$ $2 = -\frac{3}{5} + b$ $b = \frac{13}{5}$ $y = -\frac{1}{5}x + \frac{13}{5}$

iv $y = ax + 3$. $1 = 2a + 3$. $a = -1$. $y = -x + 3$.

V $\begin{cases} 4 = 6a+b \\ -1 = a+b \end{cases}$ $a=1$ $b=-2$. $y = x-2$.

7. i. $\begin{vmatrix} 2-\lambda & 0 \\ 0 & 5-\lambda \end{vmatrix} = (2-\lambda)(5-\lambda) = 0$ $\lambda=2 / \lambda=5$.

$\lambda=2$. $\begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow x_2 = 0$ x_1 can be any value so $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\lambda=5$ $\begin{pmatrix} -3 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow x_1 = 0$ x_2 can be any value so $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

ii. $\begin{vmatrix} 5-\lambda & 1 \\ 4 & 5-\lambda \end{vmatrix} = (5-\lambda)^2 - 4 = 0$ $(5-\lambda) = \pm 2$. $\lambda = 3 / 7$.

$\lambda=3$. $\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$ $\begin{matrix} 2x_1+x_2=0 \\ 4x_1+2x_2=0 \end{matrix}$ $x_1 = -\frac{1}{2}x_2 \Rightarrow \begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$

$\lambda=7$. $\begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$ $\begin{matrix} -2x_1+x_2=0 \\ 4x_1-2x_2=0 \end{matrix}$ $x_2 = 2x_1$ $\Rightarrow \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$

iii. $\begin{vmatrix} 3-\lambda & 5 \\ 3 & 1-\lambda \end{vmatrix} = (3-\lambda)(1-\lambda) - 15 = 0$ $3 - 3\lambda - \lambda + \lambda^2 - 15 = 0$ $\lambda^2 - 4\lambda - 12 = 0$ $\lambda = -2 / \lambda = 6$

$\lambda=-2$. $\begin{pmatrix} 5 & 5 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $x_1 + x_2 = 0$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\lambda=6$ $\begin{pmatrix} -3 & 5 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $\begin{matrix} -3x_1+5x_2=0 \\ x_2 = \frac{3}{5}x_1 \end{matrix}$ $\begin{pmatrix} \frac{5}{3} \\ 1 \end{pmatrix}$

8 i ① $f \in X$ $g \in X$ $f+g \in X$ ② $f+g = g+f$. ③ $(f+g)+h = f+(g+h)$ ④ $0 \in X$ $f+0 = f$ $\forall f \in X$

⑤ $(-f)$ is unique that $f+(-f) = 0$ ⑥ $af \in X$ ⑦ $1f = f \in X$ ⑧ $a(bf) = (a \cdot b)f$

⑨ $(a+b)f = af+bf$ ⑩ $a(f+g) = af+ag$

ii $A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \in X$ ① $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix} \in X$ ② $A+B = B+A$ ③ $(A+B)+C = A+(B+C)$

④ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in X$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + A = A$ $\forall A \in X$ ⑤ $(-A)$ is unique $(-A)+A = 0$ ⑥ $aA \in X$ ⑦ $1 \cdot A = A \in X$

⑧ $a(bA) = (a \cdot b) \cdot A$ ⑨ $(a+b)A = aA + bA$. ⑩ $a(A+B) = aA + aB$.

9. i. $a\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + b\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + c\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 0$ $\begin{cases} a+b+c=0 \\ 2a+2c=0 \\ 3a+b+c=0 \end{cases}$ $a=b=c=0$. $\Rightarrow$ linear independent dim $= 3$

ii. $a\sin(t) + b\cos(t) + c\cos(2t) = 0$ $\Rightarrow a\sin t + b\cos t + c - 2c\sin^2 t = 0$ the only solution is all parameters are 0. So it is linear independent. dim $= 3$

$a\begin{pmatrix} \sin 0 \\ \sin\frac{\pi}{2} \\ \sin \pi \end{pmatrix} + b\begin{pmatrix} \cos(0) \\ \cos(\frac{\pi}{2}) \\ \cos(\pi) \end{pmatrix} + c\begin{pmatrix} \cos(2 \cdot 0) \\ \cos(\pi) \\ \cos(2\pi) \end{pmatrix} = 0$ $a=b=c=0$ dim $= 3$

iii $a(1+t) + b(1-t) = 0$ $a+b+(a-b)t = 0$ $a+b=0$ $a-b=0$ $a=b=0$ linear independent dim $= 2$

iv $a\begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} + b\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + c\begin{pmatrix} 3 \\ 4 \\ 4 \\ 3 \end{pmatrix} = 0$ $\begin{matrix} a+b+3c=0 \\ 2a+4c=0 \end{matrix}$ $\begin{cases} c = a \\ b = -i \end{cases}$ $\Rightarrow c = 2a+b$ so not linear independent dim $= 2$

10. $a\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} + c\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \Rightarrow$ $\begin{matrix} -a+b+c=1 \\ a+b+c=2 \\ \rightarrow b=2 \end{matrix}$ $\Rightarrow \begin{cases} b=-1 \\ c=\frac{5}{2} \\ a=\frac{1}{2} \end{cases} \Rightarrow \begin{pmatrix} \frac{1}{2} \\ -1 \\ \frac{5}{2} \end{pmatrix}$