CPT_S 540 Homework 8
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Question 1

$$P(Uniform = crimson) = 0.18 + 0.08 + 0.05 + 0.06 + 0.07 + 0.08 = 0.52$$
$$P(Uniform = gray) = 0.08 + 0.10 + 0.09 + 0.08 + 0.09 + 0.04 = 0.48$$
$$P(Weather = clear) = 0.18 + 0.08 + 0.06 + 0.08 = 0.40$$
$$P(Weather = cloudy) = 0.08 + 0.10 + 0.07 + 0.09 = 0.34$$
$$P(Weather = rainy) = 0.05 + 0.09 + 0.08 + 0.04 = 0.26$$

$$P(win|Uniform = crimson, Weather = clear)$$
$$= a < P(win = T|Uniform = crimson, Weather = clear), P(win = F|Uniform = crimson, Weather = clear) >$$
$$= a < P(win = T, Uniform = crimson, Weather = clear)/(P(Uniform = crimson, Weather = clear)), P(win = F, Uniform = crimson, Weather = clear)/(P(Uniform = crimson, Weather = clear)) >$$
$$= a < 0.18/(0.52 * 0.4), 0.06/(0.52 * 0.4) >= a < 0.865, 0.288 >=< 0.75, 0.25 >$$

$$P(win|Uniform = gray, Weather = clear)$$
$$= a < P(win = T|Uniform = gray, Weather = clear), P(win = F|Uniform = gray, Weather = clear) >$$
$$= a < P(win = T, Uniform = gary, Weather = clear)/(P(Uniform = gray, Weather = clear)), P(win = F, Uniform = gray, Weather = clear)/(P(Uniform = gray, Weather = clear)) >$$
$$= a < 0.08/(0.48 * 0.4), 0.08/(0.48 * 0.4) >= a < 0.42, 0.42 >=< 0.5, 0.5 >$$

$$P(win|Uniform = crimson, Weather = cloudy)$$
$$= a < P(win = T|Uniform = crimson, Weather = cloudy), P(win = F|Uniform = crimson, Weather = cloudy) >$$
$$= a < P(win = T, Uniform = crimson, Weather = cloudy)/(P(Uniform = crimson, Weather = cloudy)), P(win = F, Uniform = crimson, Weather = cloudy)/(P(Uniform = crimson, Weather = cloudy)) >$$
$$= a < 0.08/(0.52 * 0.34), 0.07/(0.52 * 0.34) >= a < 0.45, 0.4 >=< 0.53, 0.47 >$$

$$P(win|Uniform = gray, Weather = cloudy)$$
$$= a < P(win = T|Uniform = gray, Weather = cloudy), P(win = F|Uniform = gray, Weather = cloudy) >$$
$$= a < P(win = T, Uniform = gray, Weather = cloudy)/(P(Uniform = gray, Weather = cloudy)), P(win = F, Uniform = gray, Weather = cloudy)/(P(Uniform = gray, Weather = cloudy)) >$$
$$= a < 0.1/(0.48 * 0.34), 0.09/(0.48 * 0.34) >= a < 0.61, 0.55 >=< 0.53, 0.47 >$$

$$P(win|Uniform = crimson, Weather = rainy)$$

$$= a < P(win = T|Uniform = crimson, Weather = rainy), P(win = F|Uniform = crimson, Weather = rainy) >$$

$$= a < P(win = T, Uniform = crimson, Weather = rainy)/(P(Uniform = crimson, Weather = rainy)), P(win = F, Uniform = crimson, Weather = rainy)/(P(Uniform = crimson, Weather = rainy)) >$$

$$= a < 0.05/(0.52 * 0.26), 0.08/(0.52 * 0.26) >= a < 0.37, 0.59 >=< 0.39, 0.61 >$$

$$P(win|Uniform = gray, Weather = rainy)$$

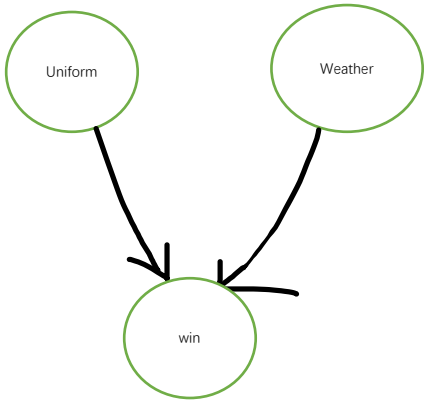
$$= a < P(win = T|Uniform = gray, Weather = rainy), P(win = F|Uniform = gray, Weather = rainy) >$$

$$= a < P(win = T, Uniform = gray, Weather = rainy)/(P(Uniform = gray, Weather = rainy)), P(win = F, Uniform = gray, Weather = rainy)/(P(Uniform = gray, Weather = rainy)) >$$

$$= a < 0.09/(0.48 * 0.26), 0.04/(0.48 * 0.26) >= a < 0.72, 0.0.32 >=< 0.69, 0.31 >$$

P(Uniform)	
Crimson	gray
0.52	0.48

P(Weather)		
clear	cloudy	Rainy
0.40	0.34	0.26



Uniform	Weather	P(Win| Uniform, Weather)	
		true	False
crimson	clear	0.75	0.25
gray	clear	0.50	0.50
crimson	cloudy	0.53	0.47
gray	cloudy	0.53	0.47
crimson	rainy	0.39	0.61
gray	rainy	0.69	0.31

Question 2

(a)

$$P(Uniform = Crimson, Weather = clear, win = T, CallFriend = T, Buyjersey = T)$$
$$= P(Uniform = Crimson) * P(Weather = clear) * P(Win = T|Uniform = Crimson, Weather = clear) * P(CallFriend = T|Win = T) * P(Buyjersey = T|Win = T)$$
$$= 0.6 * 0.3 * 0.9 * 0.7 * 0.6 = 0.06804$$

(b) CF=CallFriend
U=Uniform
BJ=BuyJersey
W=Weather
T/F=True/False

$$P(CF = T|U = gray, W = cloudy) = \alpha P(U = gray, W = cloudy, CF = T)$$

$$= \alpha \sum_{win} P(U = gray, W = cloudy, CF = T, win) = \alpha P(U$$

$$= gray) * P(W = cloudy) *< P(win = T|U = gray, W = cloudy)$$

$$* P(CF = T|win = T) +$$

$$(P(win = F|U = gray, W = cloudy) * P(CF = T|win = F),$$

$$(P(win = T|U = gray, W = cloudy) * P(CF = F|win = T) +$$

$$(P(win = F|U = gray, W = cloudy) * P(CF = F|win = F) >$$

$$= \alpha * 0.4 * 0.4 * [0.7 * 0.4 + 0.6 * 0.2, 0.3 * 0.4 + 0.6 * 0.8] = 0.16 * \alpha < 0.4,0.6 >$$

$$=< 0.4,0.6 >$$

(c)

$$P(U = cri|CF = T, BJ = T) = \alpha P(U = cri, CF = T, BJ = T)$$

$$= \alpha \sum_{weather} \sum_{win} P(CF = T, BJ = T, W, Win) = \alpha * P(Uniform$$

$$= crimson) \sum_{win} [P(win, CF = T, BJ = T) * P(weather = clear) +$$

$$\sum_{win} P(win, CF = T, BJ = T) * P(weather = cloudy) +$$

$$\sum_{win} P(win, CF = T, BJ = T) * P(weather = rainy)]$$

$$= \alpha * P(Uniform = crimson)$$
$$< [P(win = T|\ Uniform = crimson, weathe = clear) * P(CF = T|win = T)$$
$$* P(BJ = T|win = T) * P(weather = clear)$$
$$+ P(win = T|\ Uniform = crimson, weather = cloudy)$$
$$* P(CF = T|win = T) * P(BJ = T|win = T) * P(weather = cloudy)$$
$$+ P(win = T|\ Uniform = crimson, weathe = rainy) * P(CF = T|win = T)$$
$$* P(BJ = T|win = T) * P(weather = rainy)$$
$$+ P(win = F|\ Uniform = crimson, weathe = clear) * P(CF = T|win = F)$$
$$* P(BJ = T|win = F) * P(weather = clear)$$
$$+ P(win = F|\ Uniform = crimson, weathe = cloudy)$$
$$* P(CF = T|win = F) * P(BJ = T|win = F) * P(weather = cloudy)$$
$$+ P(win = F|\ Uniform = crimson, weathe = rainy) * P(CF = T|win = F)$$
$$* P(BJ = T|win = F)$$
$$* P(weather = rainy)], [P(win = T|\ Uniform = crimson, weathe = clear)$$
$$* P(CF = T|win = T) * P(BJ = T|win = T) * P(weather = clear)$$
$$+ P(win = T|\ Uniform = crimson, weather = cloudy)$$
$$* P(CF = T|win = T) * P(BJ = T|win = T) * P(weather = cloudy)$$
$$+ P(win = T|\ Uniform = crimson, weathe = rainy) * P(CF = T|win = T)$$
$$* P(BJ = T|win = T) * P(weather = rainy)$$
$$+ P(win = F|\ Uniform = crimson, weathe = clear) * P(CF = T|win = F)$$
$$* P(BJ = T|win = F) * P(weather = clear)$$
$$+ P(win = F|\ Uniform = crimson, weathe = cloudy)$$
$$* P(CF = T|win = F) * P(BJ = T|win = F) * P(weather = cloudy)$$
$$+ P(win = F|\ Uniform = crimson, weathe = rainy) * P(CF = T|win = F)$$
$$* P(BJ = T|win = F) * P(weather = rainy)] >$$

$$= \alpha *$$
$$< 0.6 * 0.3 * 0.9 * 0.7 * 0.6 +$$
$$0.6 * 0.4 * 0.6 * 0.7 * 0.6 +$$
$$0.6 * 0.3 * 0.4 * 0.6 * 0.7 +$$
$$0.6 * 0.3 * 0.1 * 0.2 * 0.3 +$$
$$0.6 * 0.4 * 0.4 * 0.2 * 0.3 +$$
$$0.6 * 0.3 * 0.6 * 0.2 * 0.3,$$

$$0.4 * 0.3 * 0.2 * 0.6 * 0.7 +$$
$$0.4 * 0.4 * 0.4 * 0.6 * 0.7 +$$
$$0.4 * 0.3 * 0.7 * 0.6 * 0.7 +$$
$$0.4 * 0.3 * 0.8 * 0.2 * 0.3 +$$
$$0.4 * 0.4 * 0.6 * 0.2 * 0.3 +$$
$$0.4 * 0.3 * 0.3 * 0.2 * 0.3 >$$
$$= a *< 0.17208, 0.08592 >$$
$$=< 0.67, 0.33 >$$

Question 3

$P(U = crimson) > P(U = gray)$, so choose $P(U = crimson) = 0.6$

$P(W = cloudy) > P(W = clear) > P(W = rainy)$ so choose $P(W = cloudy) = 0.4$

Based on these two, choose, we choose win, so $P(win = T | U = crimson, W = cloudy) = 0.6$

Given $win = T$, We choose corresponding max probability: $P(CF = T)\ and\ P(BJ = T)$

So we will have:

$$P(U = crimson, W = cloudy, win = T, CF = T, BJ = T) = 0.6 * 0.4 * 0.6 * 0.7 * 0.6$$
$$= 0.06048$$

Question 4

If they are independent, we will have $P(U) * P(W) = P(U\ and\ W)$

We pick one example:

$$P(U\ and\ W) = P(win = T | U = crimson, W = cloudy) + P(win = F | U = crimson, W = cloudy)$$
$$= 0.08 + 0.07 = 0.15$$
$$P(U = crimson, W = cloudy) = 0.52 * 0.34 = 0.1768$$

They are different, so they are not independent and information is not consistency.