HW7. Chenrui Xu.

1. (a). $P(Win=T, Uniform=crimson, Weather=clear) = 0.18$

(b). $P(Weather=clear) = 0.18+0.08+0.06+0.08 = 0.4$

(c). $P(Uniform=crimson) = 0.18+0.08+0.05+0.06+0.07+0.08 = 0.52$

(d). $P(Win=T \mid Weather=clear) = P(Win=true \wedge Weather=clear) / P(Weather=clear)$

$= (0.18+0.08)/0.38 = 0.65$

(e). $P(Win=T \mid Weather=cloudy \vee Weather=rainy)$ #

$= P(Win=T \wedge (Weather=cloudy \vee Weather=rainy)) / P(Weather=cloudy \vee Weather=rainy)$

$= (0.08+0.1+0.05+0.09)/(0.08+0.1+0.09+0.07+0.05+0.09+0.08+0.04)$

$= 0.32/0.6 = 0.53$.

2. $P(Win \mid Practise=T \cup Healthy=T) =$

$P(Win \wedge (Practise=T \vee Healthy)) / P(Practise=T \vee Healthy)$

$= P(Practise=T \vee Healthy \mid Win) \cdot P(Win) / P(Practise=T \vee Healthy)$.

$= \alpha \cdot P(Practise=T \vee Healthy \mid Win) \cdot P(Win)$.

$= \alpha \langle P(Practise=T \vee Healthy \mid Win=T) \cdot P(Win=T),$

$Practise=T \vee Healthy \mid Win=F) \cdot P(Win=F) \rangle$

$= \alpha \langle 0.8 \times 0.7, 0.3 \times 0.4 \rangle = \alpha \langle 0.56, 0.12 \rangle = \langle 0.82, 0.18 \rangle$

Breeze $\{\neg B_{1,1}, B_{2,1}, B_{1,2}, B_{3,2}\}$

Known $\{\neg P_{1,1}, \neg P_{2,1}, \neg P_{1,2}, P_{3,1}\}$

Frontier $\{P_{1,3}, P_{3,2}\}$

Other $\{P_{2,3}, P_{3,3}\}$

Question3a

Question3b

$$
\begin{aligned}
&P(P_{22} \mid B, k) \\
&= \alpha P(P_{22}, B, k) \\
&= \alpha \sum_{Uk} P(P_{22}, B, k, Uk) \\
&= \alpha \sum_{f} \sum_{0} P(P_{22}, B, k, f, 0) \\
&= \alpha \sum_{f} \sum_{0} P(B \mid P_{22}, k, f, 0) \cdot P(P_{22}, k, f, 0) \\
&= \alpha \sum_{f} P(B \mid P_{22}, k, f) \cdot \sum_{0} P(P_{22}) \cdot P(k) \cdot P(f) \cdot P(0) \\
&= \alpha P(P_{22}) \cdot P(k) \sum_{f} P(f) \cdot P(B \mid P_{22}, k, f) \\
&= \alpha' P(P_{22}) \sum_{f} P(f) \cdot P(B \mid P_{22}, k, f) \\
&= \alpha' \langle P(P_{22}) \sum_{f} P(f) \cdot P(B \mid P_{22}, k, f), P(\neg P_{22}) \sum_{f} P(f) \cdot P(B \mid \neg P_{22}, k, f) \rangle \\
&= \alpha' \langle P(P_{22}) (P(B \mid P_{22}, k, P_{13}, P_{32}) \cdot P(P_{13}, P_{32}) , \\
&\qquad P(P_{22}) (P(B \mid P_{22}, k, P_{13}, \neg P_{32}) \cdot P(P_{13}, \neg P_{32}) , \\
&\qquad P(P_{22}) (P(B \mid P_{22}, k, \neg P_{13}, P_{32}) \cdot P(\neg P_{13}, P_{32}) , \\
&\qquad P(P_{22}) (P(B \mid P_{22}, k, \neg P_{13}, \neg P_{32}) \cdot P(\neg P_{13}, \neg P_{32}) ,
\end{aligned}
$$

$P(\neg P_{22})(P(B|P_2, k, P_{13}, P_{32}) \cdot P(P_{13}, P_{32}),$

$P(\neg P_{22})(P(B|\neg P_{22}, k, P_{13}, \neg P_{32}) \cdot P(P_{13}, \neg P_{32}),$

$P(\neg P_{22})(P(B|\neg P_{22}, k, \neg P_{13}, P_{32}) \cdot P(\neg P_{13}, P_{32}),$

$P(\neg P_{22})(P(B|\neg P_{22}, k, \neg P_{13}, \neg P_{32}) \cdot P(\neg P_{13}, \neg P_{32}) >$

Given $P_{x,y} = \langle 0.2, 0.8 \rangle$

$= \alpha' \langle 0.2 \cdot (1 \times 0.2 \times 0.2 + 1 \times 0.2 \times 0.8 + 1 \times 0.8 \times 0.2 + 1 \times 0.8 \times 0.8),$

$0.8 (1 \times 0.2 \times 0.2 + 1 \times 0.2 \times 0.8 + 0 \times 0.8 \times 0.2 + 0 \times 0.8 \times 0.8)$

$= \alpha' \langle 0.2, 0.16 \rangle = \langle 0.56, 0.44 \rangle$

No, it won't change $P(P_{22})$

So the sets will be

Breeze $\{\neg B_{1,1}, B_{3,1}, B_{1,2}, B_{3,2}, B_{33}\}$ (No change for others)

$P(P_{2,2} \mid B, k) = \alpha' P(P_{22}) \sum_f P(B \mid P_{22}, k, f) P(f)$

$= \alpha' \langle P(P_{22}) [\sum_f P(B \mid P_{22}, k, f) P(f)], P(\neg P_{22}) [\sum_f P(B \mid \neg P_{22}, k, f) \cdot P(f)] \rangle$

As Set frontier $\{P_{13}, P_{32}\}$ not change.

$P(P_{2,2} \mid B, k)$ will not change then.

So the result will still be $\langle 0.56, 0.44 \rangle$

Question4