Washington State University
School of Electrical Engineering and Computer Science
Fall 2021

CptS 440/540 Artificial Intelligence
Homework 8 - Solution
Due: October 28, 2021 (11:59pm pacific time)

General Instructions: Put your answers to the following problems into a PDF document and upload the document as your submission for Homework 8 for the course CptS 440 Pullman (all sections of CptS 440 and 540 are merged under the CptS 440 Pullman section) on the Canvas system by the above deadline. Note that you may submit multiple times, but we will only grade the most recent entry submitted before the deadline.

1. Recall the full joint probability distribution from HW7 (reproduced below). Suppose we are told that Uniform and Weather are independent of each other, and that Win depends on both Uniform and Weather. Show a Bayesian network consistent with this information. Be sure to show all nodes, links, and conditional probability tables (CPTs). Use the full joint probability distribution below to compute the CPT entries.

Win		true		false	
Uniform		crimson	gray	crimson	gray
Weather	clear	0.18	0.08	0.06	0.08
	cloudy	0.08	0.10	0.07	0.09
	rainy	0.05	0.09	0.08	0.04

Solution:

First, some sample CPT calculations:

P(Uniform=crimson) = 0.18+0.08+0.05+0.06+0.07+0.08 = 0.52
P(Uniform=gray) = 1 – P(Uniform=crimson) = 0.48
P(Weather=clear) = 0.18+0.08+0.06+0.08 = 0.40
P(Weather=cloudy) = 0.08+0.10+0.07+0.09 = 0.34
P(Weather=rainy) = 1 – P(Weather=clear) – P(Weather=cloudy) = 0.26
P(Win=true | Uniform=crimson, Weather=clear)
= P(Win=true | Uniform=crimson, Weather=clear) / P(Uniform=crimson, Weather=clear)

At this point, we can compute P(Uniform=crimson, Weather=clear) in two ways:

(1) We can use the information that Uniform is independent of Weather to write:
P(Uniform=crimson, Weather=clear) = P(Uniform=crimson) * P(Weather=clear)
P(Win=true | Uniform=crimson, Weather=clear)

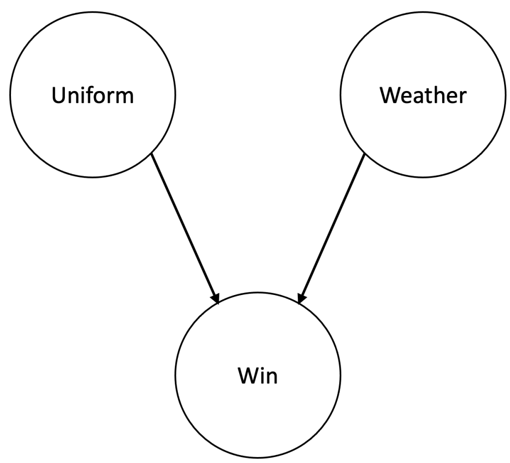
= P(Win=true, Uniform=crimson, Weather=clear) / [P(Uniform=crimson)*P(Weather=clear)]
= (0.18) / [(0.52)*(0.40)] = 0.87
P(Win=false | Uniform=crimson, Weather=clear)
= 1 – P(Win=true | Uniform=crimson, Weather=clear) = 0.13

(2) We can compute P(Uniform=crimson, Weather=clear) from the FJPD table:
P(Uniform=crimson, Weather=clear) = 0.18 + 0.06
P(Win=true | Uniform=crimson, Weather=clear)
= P(Win=true, Uniform=crimson, Weather=clear) / P(Uniform=crimson,Weather=clear)
= (0.18) / (0.18 + 0.06) = 0.75
P(Win=false | Uniform=crimson, Weather=clear)
= 1 – P(Win=true | Uniform=crimson, Weather=clear) = 0.25

Either way is okay, and the network below includes both possibilities. The reason for the discrepancy is that according to the FJPD table, Uniform and Weather aren't really independent (which is addressed in question #4). If they really were independent, then the FJPD table values would be different, and the two methods for computing **P**(Uniform,Weather) would yield the same result.

The remainder of the probabilities in the CPTs are computed as above, but only the remaining final values are shown in the network below.

P(Uniform)	
crimson	gray
0.52	0.48



P(Weather)		
clear	cloudy	rainy
0.40	0.34	0.26

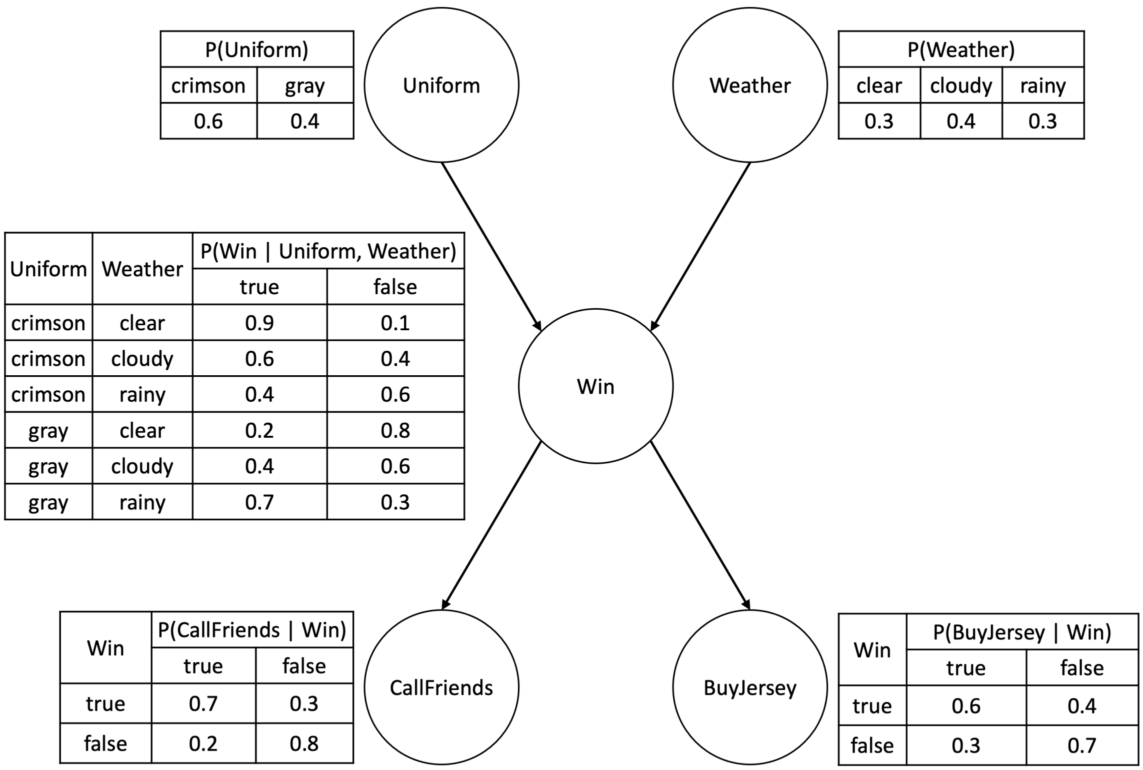
Uniform	Weather	P(Win | Uniform, Weather)	
		true	false
crimson	clear	0.87	0.13
crimson	cloudy	0.45	0.55
crimson	rainy	0.37	0.63
gray	clear	0.42	0.58
gray	cloudy	0.61	0.39
gray	rainy	0.72	0.28

(1) Assume **P**(Uniform, Weather) = **P**(Uniform)***P**(Weather)

Uniform	Weather	P(Win | Uniform, Weather)	
		true	false
crimson	clear	0.75	0.25
crimson	cloudy	0.53	0.47
crimson	rainy	0.38	0.62
gray	clear	0.50	0.50
gray	cloudy	0.53	0.47
gray	rainy	0.69	0.31

(2) **P**(Uniform, Weather) taken from table.

2. Using the Bayesian network below, compute the following probabilities. Show your work.
   a. P(Uniform=crimson, Weather=clear, Win=true, CallFriends=true, BuyJersey=true)
   b. P(CallFriends=true | Uniform=gray, Weather=cloudy)
   c. P(Uniform=crimson | CallFriends=true, BuyJersey=true)



P(Uniform)	
crimson	gray
0.6	0.4

P(Weather)		
clear	cloudy	rainy
0.3	0.4	0.3

Uniform	Weather	P(Win | Uniform, Weather)	
		true	false
crimson	clear	0.9	0.1
crimson	cloudy	0.6	0.4
crimson	rainy	0.4	0.6
gray	clear	0.2	0.8
gray	cloudy	0.4	0.6
gray	rainy	0.7	0.3

Win	P(CallFriends | Win)	
	true	false
true	0.7	0.3
false	0.2	0.8

Win	P(BuyJersey | Win)	
	true	false
true	0.6	0.4
false	0.3	0.7

Solution:

a. P(Uniform=crimson, Weather=clear, Win=true, CallFriends=true, BuyJersey=true)
   = P(Uniform=crimson) * P(Weather=clear) * P(Win=true | Uniform=crimson, Weather=clear) * P(CallFriends=true | Win=true) * P(BuyJersey=true | Win=true)
   = (0.6)*(0.3)*(0.9)*(0.7)*(0.6)
   = 0.068

b. P(CallFriends=true | Uniform=gray, Weather=cloudy)
   Note: BuyJersey ignored because not an ancestor of query or evidence variables.
   = α Σ_{Win} P(Uniform=gray) * P(Weather=cloudy) *
   P(Win | Uniform=gray, Weather=cloudy) * P(CallFriends=true | Win)
   = α P(Uniform=gray) * P(Weather=cloudy) *
   Σ_{Win} P(Win | Uniform=gray, Weather=cloudy) * P(CallFriends=true | Win)
   = α (0.4)*(0.4) *
   [P(Win=true | Uniform=gray, Weather=cloudy) * P(CallFriends=true | Win=true) +
   P(Win=false | Uniform=gray, Weather=cloudy) * P(CallFriends=true | Win=false)]

$= \alpha$ (0.4)*(0.4) * [(0.4)*(0.7) + (0.6)*(0.2)]

$= \alpha$ (0.064)

Solving in the same way, but for CallFriends=False:

P(CallFriends=false | Uniform=gray, Weather=cloudy)

$= \alpha$ (0.4)*(0.4) *

[P(Win=true | Uniform=gray, Weather=cloudy) * P(CallFriends=false | Win=true) +
P(Win=false | Uniform=gray, Weather=cloudy) * P(CallFriends=false | Win=false)]

$= \alpha$ (0.4)*(0.4) * [(0.4)*(0.3) + (0.6)*(0.8)]

$= \alpha$ (0.096)

Normalizing with α = 1 / (0.064 + 0.096) = 6.25

P(CallFriends=true | Uniform=gray, Weather=cloudy) = 6.25*(0.064) = 0.4

c. P(Uniform=crimson | CallFriends=true, BuyJersey=true)

$= \alpha$ * P(Uniform=crimson, CallFriends=true, BuyJersey=true)

$= \alpha\ \Sigma_{\text{Weather}}\ \Sigma_{\text{Win}}$ P(Uniform=crimson) * P(Weather) * P(Win | Uniform=crimson, Weather) *
P(CallFriends=true | Win) * P(BuyJersey=true | Win)

$= \alpha$ P(Uniform=crimson) Σ_{Weather} P(Weather) Σ_{Win} P(Win | Uniform=crimson, Weather) *
P(CallFriends=true | Win) * P(BuyJersey=true | Win)

$= \alpha$ (0.6) *

{ P(Weather=clear) *
[P(Win=true | Uniform=crimson, Weather=clear) * P(CallFriends=true | Win=true) *
P(BuyJersey=true | Win=true) +
P(Win=false | Uniform=crimson, Weather=clear) * P(CallFriends=true | Win=false) *
P(BuyJersey=true | Win=false)] +
P(Weather=cloudy) *
[P(Win=true | Uniform=crimson, Weather=cloudy) * P(CallFriends=true | Win=true) *
P(BuyJersey=true | Win=true) +
P(Win=false | Uniform=crimson, Weather=cloudy) * P(CallFriends=true | Win=false) *
P(BuyJersey=true | Win=false)] +
P(Weather=rainy) *
[P(Win=true | Uniform=crimson, Weather=rainy) * P(CallFriends=true | Win=true) *
P(BuyJersey=true | Win=true) +
P(Win=false | Uniform=crimson, Weather=rainy) * P(CallFriends=true | Win=false) *
P(BuyJersey=true | Win=false)] }

$= \alpha$ (0.6) * { (0.3) [(0.9)(0.7)(0.6) + (0.1)(0.2)(0.3)] +
(0.4) [(0.6)(0.7)(0.6) + (0.4)(0.2)(0.3)] +
(0.3) * [(0.4)(0.7)(0.6) + (0.6)(0.2)(0.3)] }

$= \alpha$ (0.172)

Solving in the same way for Uniform=gray:

P(Uniform=gray | CallFriends=true, BuyJersey=true)

= α (0.4) *

{ P(Weather=clear) *

[P(Win=true | Uniform=gray, Weather=clear) * P(CallFriends=true | Win=true) * P(BuyJersey=true | Win=true) +

P(Win=false | Uniform=gray, Weather=clear) * P(CallFriends=true | Win=false) * P(BuyJersey=true | Win=false)] +

P(Weather=cloudy) *

[P(Win=true | Uniform=gray, Weather=cloudy) * P(CallFriends=true | Win=true) * P(BuyJersey=true | Win=true) +

P(Win=false | Uniform=gray, Weather=cloudy) * P(CallFriends=true | Win=false) * P(BuyJersey=true | Win=false)] +

P(Weather=rainy) *

[P(Win=true | Uniform=gray, Weather=rainy) * P(CallFriends=true | Win=true) * P(BuyJersey=true | Win=true) +

P(Win=false | Uniform=gray, Weather=rainy) * P(CallFriends=true | Win=false) * P(BuyJersey=true | Win=false)] }

= α (0.4) * { (0.3) [(0.2)(0.7)(0.6) + (0.8)(0.2)(0.3)] +
(0.4) [(0.4)(0.7)(0.6) + (0.6)(0.2)(0.3)] +
(0.3) * [(0.7)(0.7)(0.6) + (0.3)(0.2)(0.3)] }

= α (0.08592)

Normalizing with α = 1 / (0.172 + 0.08592) = 3.877

P(Uniform=crimson | CallFriends=true, BuyJersey=true) = 3.877*(0.172) = 0.67

3. What would be the most likely sample from applying direct sampling to the Bayesian network in Problem 2? What is this sample's probability?

Solution:

There are two interpretations of "most likely sample". One is that you choose the highest probability value at each node in the network given the parents' values. In this case, the most likely sample is:

(Uniform=crimson, Weather=cloudy, Win=true, CallFriends=true, BuyJersey=true)

Probability = (0.6)(0.4)(0.6)(0.7)(0.6) = 0.06048

Another interpretation is to choose the sample with the highest probability. Given a sample's frequency is F out of N samples, then the probability of the sample will be F/N for large N. So, the highest-probability sample will be the sample most often generated. In this case, the most likely sample is:

(Uniform=crimson, Weather=clear, Win=true, CallFriends=true, BuyJersey=true)

Probability = (0.6)(0.3)(0.9)(0.7)(0.6) = 0.06804

4. *CptS 540 Students Only*. In Problem 1 above, we are told that Uniform and Weather are independent of each other. Is that information consistent with the full joint probability distribution? Justify your answer.

Solution:

No. If Uniform and Weather are independent, then **P**(Uniform, Weather) = **P**(Uniform) * **P**(Weather). But take for example the case where Uniform=crimson and Weather=clear.

Summing the consistent entries from the table:

P(Uniform=crimson, Weather=clear) = 0.18 + 0.06 = 0.24

Multiplying the two independent probabilities, again summed up from the table:

P(Uniform=crimson) = 0.18+0.08+0.05+0.06+0.07+0.08 = 0.52
P(Weather=clear) = 0.18+0.08+0.06+0.08 = 0.40
P(Uniform=crimson) * P(Weather=clear) = 0.52 * 0.40 = 0.208

So, **P**(Uniform, Weather) ≠ **P**(Uniform) * **P**(Weather), and therefore Uniform is not independent of Weather.