

## CSE331 Homework 8

Due by the start of class on April 24

1. (10 pts) We used dynamic programming to solve the longest common subsequence (LCS) problem in class. This algorithm needs to use a 2 dimensional array with size  $(|x|+1) * (|y|+1)$  given that  $x$  and  $y$  are two input sequences. Now, modify the dynamic programming algorithm so that we only need to use a linear table with size either  $|x|$  or  $|y|$ . Describe your idea (or modification) and then write your pseudocode.

*Note: you don't need to find the longest common subsequence between  $x$  and  $y$ . You only need to find the size of an LCS.*

2. (10 pts) Consider the problem of computing the optimal order of pairwise matrix multiplications to compute a matrix chain multiplication, where we seek the matrix chain multiplication result of  $A_1 A_2 A_3 A_4 A_5 A_6$ , where the dimensions of the matrices are:  $A_1: 10 \times 20$ ,  $A_2: 20 \times 1$ ,  $A_3: 1 \times 40$ ,  $A_4: 40 \times 5$ ,  $A_5: 5 \times 30$ ,  $A_6: 30 \times 15$ . Please use the following pseudo-code to fill in the matrix  $M$  when  $s = 1, 2$ , and  $3$ .

```
//Compute minimum cost to multiply  $M_L, M_{L+1}, \dots, M_R$ 
For  $s = 1$  to  $R-L+1$  //s is the number of input matrices
    for  $i = L$  to  $R+1-s$ 
         $j = i+s-1$ 
        if ( $i == j$ )  $M[i, j] = 0$ 
        else
             $M[i, j] = \min_{i \leq k \leq j-1} \{ M[i, k] + M[k+1, j] + C_{i-1} C_k C_j \}$ 
```

3. (5 pts) Show that “compute the cheapest multiplication” for chained matrix multiplication does not work. In order to show that this greedy choice does not work, provide a counter-example.

4. [10 pts] The binomial coefficients  $C(N, k)$  can be defined recursively as follows. For  $0 \leq i \leq N$ ,  $C(i, 0) = C(i, i) = 1$ . For  $k > N$ ,  $C(N, k) = 0$ . For  $0 < k < N$ ,  $C(N, k) = C(N-1, k) + C(N-1, k-1)$ . Write a function and give an analysis of the running time to compute the binomial coefficients as follows: a. recursively; b. using dynamic programming.

5. (25 pts) This is a programming problem. Implement the dynamic programming algorithm to find the longest common subsequence between two input sequences. You need to fill in the table and then do the trace-back in order to output the LCS. Below you can find the specific requirements:

- 1) The program should be named as LCS and take two files as inputs.

LCS inputfile1 inputfile2

- 2) Both files contain a single-line string of numbers only, such as 10 22 33 etc.
- 3) Your file should output the LCS between the two input strings.
- 4) Two example input files can be on the course website as files inputfile1 and inputfile2.
- 5) We will test your program using two different files.
- 6) If needed, you can use the driver program hw8-driver.cpp on the course website.

Bonus problem (5 pts)

How many different orders are there for chain multiplication of  $n$  matrices?

For example, there are two different orders for the multiplication of three matrices:  $(A_1 A_2) A_3$  and  $A_1 (A_2 A_3)$ .