

$$\begin{array}{r} \times 46 \\ \hline 57 \end{array}$$

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2/27/2011

CSE 331 Homework 4

1. Prove that an n -element heap has height $h = \log_2 n$. You need to prove this for any given n . Showing $h = 3$ for $n = 8$ is not a proof. (5 points)

Theorem: an n -element heap has height $h = \log_2 n$

$$\begin{array}{r} 12 \\ \hline 5 \end{array}$$

h is the height of an n -element heap

There are 2^i elements at depth $i = 0, \dots, h-2$ and at least one element at depth $h-1$

So: $n \geq 1 + 2 + 4 + 8 + \dots + 2^{h-2} + 1$

$n \geq \frac{2^h - 1}{2 - 1} \therefore h \leq \log_2 n + 1$

This is true, but does not imply $h \leq \log_2 n + 1$. logs deal with exponents

2. Apply ExtractMIN to the heap $H[] = \{2, 3, 7, 4, 9, 8, 15, 13, 10\}$ using the pseudocode in lect10 (<http://www.cse.msu.edu/cse331/Section02/lec10.pdf>).

Show the contents of the array H whenever there is a change. (3 points)

2, 3, 7, 4, 9, 8, 15, 13, 10

10, 3, 7, 4, 9, 8, 15, 13

3, 10, 7, 4, 9, 8, 15, 13

3, 7, 10, 4, 9, 8, 15, 13

3, 7, 4, 10, 9, 8, 15, 13

3, 4, 7, 10, 9, 8, 15, 13

*not $\times 2/3$
10 should be swapped w/ 4 here. Heap sort swaps w/ smallest child.*

3. Insert key 1 to the following heap H using the pseudocode in lect10 (<http://www.cse.msu.edu/cse331/Section02/lec10.pdf>). Show the contents of the array H whenever there is a change. $H[] = \{2, 3, 7, 4, 9, 8, 15, 13, 10\}$. (4 points)

2, 3, 7, 4, 9, 8, 15, 13, 10

2, 3, 7, 4, 9, 8, 15, 13, 10, 1

2, 3, 7, 4, 1, 8, 15, 13, 10, 9

2, 1, 7, 4, 3, 8, 15, 13, 10, 9

1, 2, 7, 4, 3, 8, 15, 13, 10, 9

4

4. This problem consists of two parts. First, give an $O(n \log_2 k)$ -time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists. (Hint: Use a min-heap for k -way merging.) Note, if you need to use any defined operation on a heap, just treat it as a function and don't need to worry about the details. For example, if you want to build a heap on an array A , just

say buildHeap(A). However, you need to describe on what elements you build this heap. For your designed algorithm, analyze its time complexity.

In the second part of this problem, show the contents and changes of the heap you built in order to merge three sorted lists [0, 1, 3], [4], and [2, 5]. (15 pts)

a)

```
merge_heaps(){
    create new_heap;
    for i = 0 through k heaps {
        for j = 0 through n elements{
            insert heap[k] in to new_heap;
        }
    }
}
```

1
k
nk
n log₂ k

$\times 3$
10

How is this $n \log_2 k$?

b)

```
<empty>
start to merge [0,1,3]
0
0, 1
0, 1, 3
start to merge [4]
0, 1, 3, 4
start to merge [2, 5]
0, 1, 3, 4, 2
0, 1, 3, 4, 2, 5
done!
```

$\times 5$
5

5. This problem is a programming problem. Implement a program which

1
builds a heap for any input array using the algorithm described in our class (see Chapter 7 or the notes). A test program is provided at <http://www.cse.msu.edu/~cse331/Section02/examples/heap-shell.cpp>. We will compile your program on arctic.cse.msu.edu. Please make sure it can compile and run correctly on arctic. Otherwise you cannot get full credit. (30 pts)

$\times 30$

Nice job!