Problem Chosen	2022	Team Control Number
E	**MCM/ICM Summary Sheet**	**# 2224142**

Forestry for Carbon Sequestration

Forests play a crucial role in resistance to the adverse effects of climate change via absorbing greenhouse gases to fix carbon. At the same time, carbon sequestrations are the basis for forest growth. Based on carbon sinks, natural hazards, and human interventions, the forest growth models indicate the growth patterns of the forests. Usually, when the forest biomass reaches half of its maximum value, the growth rate of the forest will gradually slow down. Forest development is facilitated by cutting down some of the mature/over-mature trees.

Being committed to calculating carbon sinks and exploring forest growth patterns to **determine forest management strategies**, humans have determined many models to **find the appropriate rotation period**, and therefore to accelerate forest development and improve the benefits that humans can receive. Calculating forest carbon sequestrations requires consideration of many influences and mathematical modeling. In the present paper, we have constructed a forest carbon sequestration model and a forest growth model.

On the one hand, aiming at achieving an accurate estimate of carbon fixation, we improve the traditional CASA model and build **a new carbon sequestration model**. In this model, we take factors such as solar radiation, monthly mean temperature, precipitation, and vegetation type into account. Based on all the impact factors, we considered two active driving variables of NPP, i.e., light, effective radiation, and light energy utilization. We simplified the effects of some influencing factors to make it simpler and more practical, ensuring the error of the results are within a reasonable range at the same time.

On the other hand, we also **modeled the forest growth dynamics**. Up to now, human intervention has been performed on most of the existing forests, which greatly affects the usability of multiple forest growth models. We select the rotation period of trees from idealized single tree growth but consider the effects of pests and wood price fluctuations meanwhile. All these works make our forest growth dynamics model more realistic.

We **analyze the sensitivity of the model, taking precipitation**, solar radiation, and temperature as examples to demonstrate the results of changing the degree of emphasis on carbon sequestrations and growth. What is more, we evaluate the model and give several suggestions for future improvement. Eventually, we also show the carbon sequestrations over 100 years and the best management plan of the forests in Shaanxi.

Keywords: CASA, carbon sequestration model, forest growth dynamics model

Contents

1 Introduction

1.1 Problem Background

Carbon emission is the major cause of climate change, which greatly threatens lives. To mitigate the negative effect, human intervention to better manage forests and thus increase carbon sequestration is the most effective way up to date. It is known that harvesting can increase the output of woods, therefore contributing to economic growth. However, overharvesting limits carbon sequestration, devastating the ecosystem. As a consequence, balancing the value of forest products derived from harvesting and the value of allowing the forest to continue growing and carbon sequestering carbon as living trees should be taken into consideration by forest managers.

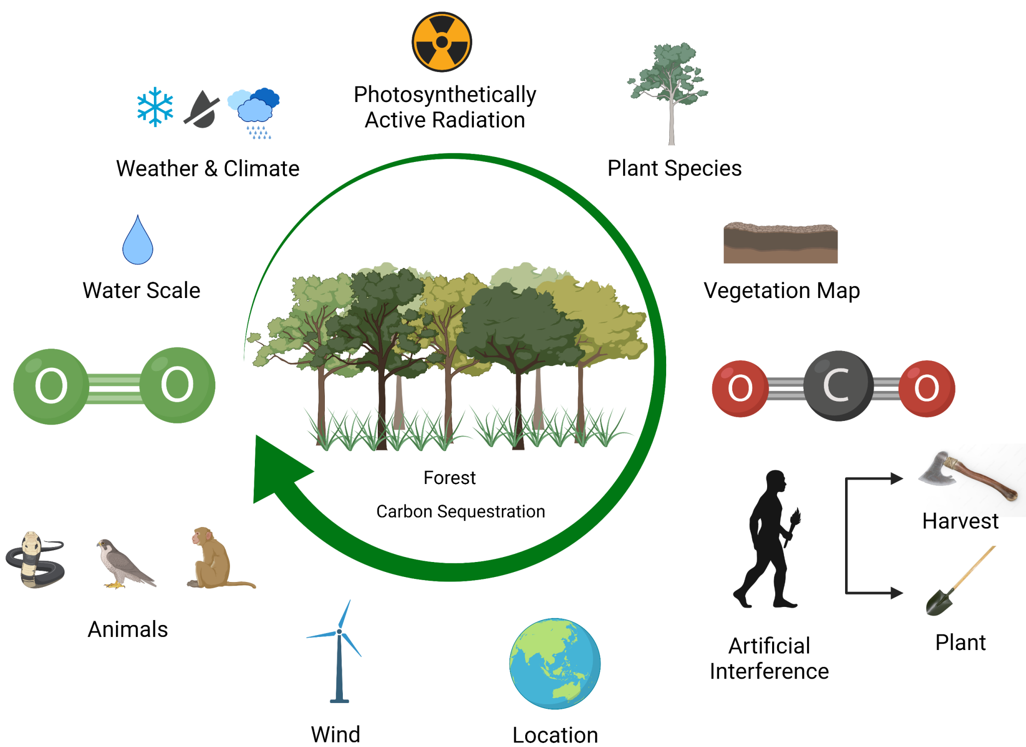

Figure 1: Climate Change and Carbon Emission

1.2 Problem Restatement

Our main goals are to help managers maximize forest benefits by developing a carbon sequestration model integrating multiple aspects. The transition points would also be found out for applying the model to different forests. To solve these problems, several small parts are as follows:

• Develop a carbon sequestration model to determine the quantity of carbon sequestration. Additionally, using this model on various forests with multiple characteristics and thus find out the most effective forest management plan at sequestering carbon dioxide.

• The plan best for carbon absorption is not the plan best for society. Based on the carbon sequestration model, we need to suggest a spectrum of management plans to balance the various ways that forests are valued, including carbon sequestration, and explain why they are the best approaches.

• After computing our models on various forests, we should identify a forest that our decision model would suggest the inclusion of harvesting in its management plan. Suppose that the best management plan includes a time between harvests that is 10 years longer than current practices in the forest, we also need to discuss a strategy for transitioning from the existing timeline to the new timeline in a way that is sensitive to the needs of forest managers and all who use the forest.

• Finally, we will make clear why our analysis identified including harvesting in the management of this forest rather than it being left untouched and convince the local community that this is the best decision for their forest.

1.3 Previous Research

The measurement of forest biomass is the key to calculating the amount of carbon sequestered. At present, there are mainly actual measurements and model estimates. Among them, the actual measurement method is more accurate but time-consuming and laborious. Model estimation is limited in accuracy but saves time and effort. In addition, it can clearly illustrate regular conclusions.

The measurement of forest biomass is the key to calculating the amount of carbon sequestered. At present, there are mainly actual measurements and model estimates. Among them, the actual measurement method is more accurate but time-consuming and laborious. Model estimation is limited in accuracy but saves time and effort. It can clearly illustrate regular conclusions. Three main types of models have been developed, including models with biological processes as the driving mechanism, models with forest survey and stand growth as the driving mechanism, and hybrid models with biological processes and forest survey as the driving mechanism. Several studies have been reported on the amount of carbon sequestered in Chinese forests.

With the development of statistical methods and the deepening of human understanding of forest growth mechanisms, forest growth models have gradually developed from simple empirical models to complex process models. The amount of carbon sequestered by forest vegetation in China has been successively estimated by domestic and foreign scholars. Dixon, Winjum, and others were the first to estimate the amount of carbon sequestered by forest vegetation in China as 6.078 PgC and 17 PgC. Later, the OBM model BIOME3 was applied to the carbon storage of terrestrial ecosystems in

China yielding 57.9 PgC Liu used four times of Chinese forest inventory data to measure its carbon sequestration, which yielded from 3.75 PgC at the beginning to 4.20 PgC in the fourth time. In addition, statistical models were used to calculate the amount of carbon sequestered in China's forests as early as 1987. Due to the different models used by different scholars to estimate forest biomass, the estimation results of forest vegetation carbon sequestration vary widely.

1.4 Our Approach

We first established a forest carbon sequestration model based on the CASA model, and considered the weather, geographical location, and soil environment. At the same time, we also simplified some factors to reduce the data requirements of the constructed forest carbon sink model under the condition of ensuring the accuracy. Based on the constructed model, we calculated the carbon sink of Shaanxi forests from the collected data and made corresponding pictures. On the other hand, we also developed an idealized forest growth model and specified the cutting period and the biomass cut. To make the model more realistic, we considered the effects of insect infestation and timber prices on the biomass and yield of cutting.

2 General Assumptions and Model Overview

In fact, according to the data of previous years, we agreed that the average humidity RH in Shaanxi Province is 62.9% The biomass of the forest growth model begins from zero.

1. Canopy surface area and leaf biomass are anisotropically correlated.
2. The relationship between leaf biomass and fine heel biomass is maintained as a constant.
3. According to Pipe theory, the ratio of leaf biomass to sapwood area at the crown base is constant.
4. Canopy radius and crown length are proportional.
5. Root length of the tree transmission and tree height are proportional.

3 Model Preparation

3.1 The Data

3.1.1 Data Collection

The data we used mainly include precipitation data, solar radiation data, temperature data and plant types data. The data sources are summarized in Table.1.

Table 1: Data Source Collection

Database Names	Database	Data Type
National Meteorological Science Data Center	http://data.cma.cn/	Geography
EarthExplorer	https://earthexplorer.usgs.gov/	Geography
Google Scholar	https://scholar.google.com/	Academic Paper
CNKI	https://www.cnki.net/	Academic Paper

3.1.2 Data Cleaning

The data in Table.1 are from National Meteorological Science Data Center. We think it the most accurate data that we can get, so we just omit the steps for data cleaning.

4 Model I: Forestry Carbon Sequestration Model

4.1 Notations

Table 2: Notations

Symbols	Description	Unit
$Tree(x, y)$	forest type locally	/
$Sl(x, y, t)$	monthly solar radiation locally in month t	$(gC \cdot m^{-2} \cdot month^{-1})$
$Te(x, y, t)$	monthly average temperature locally in month t	$°C$
$Pr(x, y, t)$	monthly average precipitation locally in month t	mm
$NDVI(x, y, t)$	NDVI locally in month t	/
$NDVI\_max(i)$	maximum NDVI value of the NDVI of forest type i	/
$NDVI\_min(i)$	minimum NDVI value of the NDVI of forest type i	/
$\eta(i)$	maximum light energy utilization of forest type i	/

4.2 Methods for Calculating Carbon Sequestration

To understand how forests store carbon in the atmosphere, we need to understand the carbon cycle in nature first. In the carbon cycle, green plants absorb carbon dioxide from the atmosphere and synthesize organic matter through photosynthesis. At the same time, they also break down organic matter through respiration, releasing carbon dioxide into the atmosphere. When the intensity of photosynthesis is

Table 3: Continued

Symbols	Description	Unit
$Tm(x,y)$	optimum growth temperature of the forest located in (x,y)	$^{\circ}C$
$Ta(x,y,t)$	temperature stress coefficient of enzyme for photosynthesis locally in month t	/
$Tb(x,y,t)$	temperature stress coefficient of enzyme for respiration locally in month t	/
$W(x,y,t)$	water stress coefficient locally in month t	/
$Ep(x,y,t)$	potential evaporation locally in month t	mm
$E(x,y,t)$	actual evaporation locally in month t	mm
$Ltt(x,y)$	latitude locally	/
$Ra(x,y,t)$	extraterrestrial radiation locally in month t	$(gC \cdot m^{-2} \cdot month^{-1})$
$\mu(x,y,t)$	ratio of photosynthetically efficient radiation	/
$ER(x,y,t)$	active radiation of absorbed light energy locally in month t	$(gC \cdot m^{-2} \cdot month^{-1})$
$\gamma(x,y,t)$	utilization rate of light energy of a tree locally in month t	$(gC \cdot MJ^{-1})$
$NPP(x,y,t)$	net primary productivity locally in month t	$(gC \cdot m^{-2} \cdot month^{-1})$
$NPP\_sum(x,y)$	annual net primary productivity locally	$(gC \cdot m^{-2})$

greater than the intensity of respiration throughout the forest, it will accumulate organic matter. To accommodate human needs, factories turn some of the trees into products that preserve carbon in the products.

We focus on how much stronger photosynthesis is than respiration. We choose NPP as the indicator to measure the carbon content of the forest, because it has many direct relationships with photosynthesis, respiration and carbon sequestration. And it can represent the difference of the strength of photosynthesis and respiration directly [1]. We need to know which factors are directly related to photosynthesis. The first is light and water, because under the action of light, chloroplasts can photolyze water to generate energy for synthesis with carbon. The sun's radiation can directly provide light energy to the earth. Second, temperature is also essential. The enzymes of photosynthesis and respiration are only active at suitable temperature, which promotes the occurrence of photosynthesis and respiration and ensures the normal operation of carbon cycle. Third, the efficiency of light energy utilization by different vegetation is also different [2]. Therefore, the type of vegetation in a certain area should also be considered.

4.3 CASA Carbon Sequestration Model Construction

Our work is based on the Carnegie-Ames-Stanford approach (CASA) model [3], combined with (the) simplification of the model for obtaining regional potential evapotranspiration [4], to achieve a

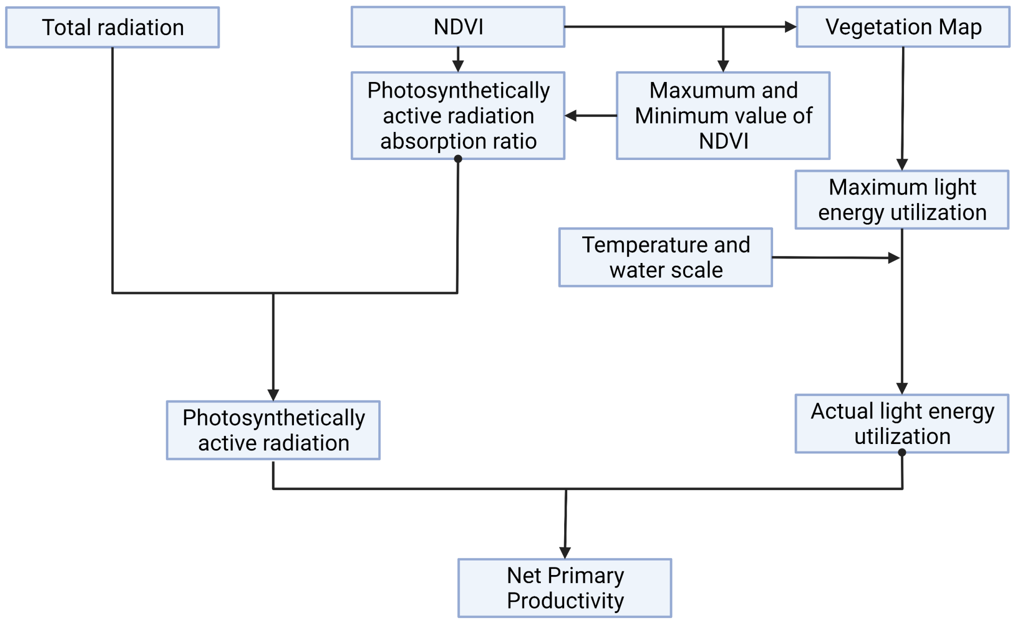

Figure 2: CASA Model Flowchart

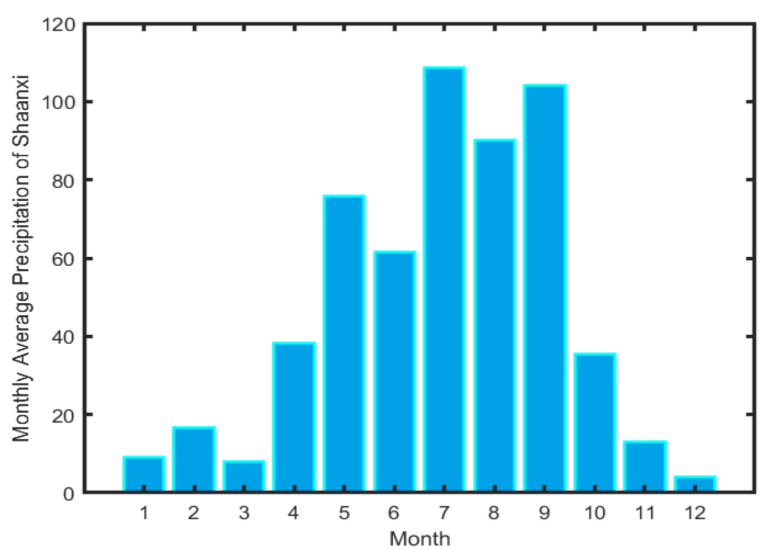

Figure 3: Monthly Average Precipitation of Shaanxi

simple import of vegetation, precipitation , solar radiation and temperature data to estimate the NPP method.

In fact, according to the data of previous years, we assume that the average humidity RH in Shaanxi Province is 62.9% [5]. At the same time, National Meteorological Science Data Center of China also provided us with the precipitation, total solar radiation and average temperature of each month in Shaanxi Province in 2006, and organized the information in units of 8km*8km pixels. Also, we derived monthly of Normalized Difference Vegetation Index(NDVI) at each pixel by MODIS. We found the satellite photos of Shaanxi Province in 2000 in the data provided by EarthExplorer, and obtained the vegetation distribution of each pixel through projection transformation, as shown in Fig.4

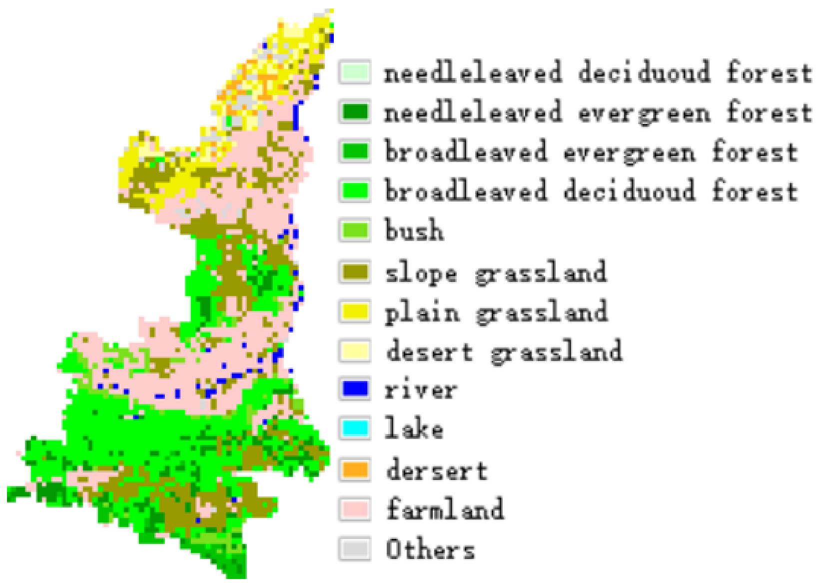

Figure 4: Plant Type of Each Pixel in Shaanxi

4.4 Calculation of Variables Related to Light Energy

The variables related to light energy are ER and μ. ER is the portion of the spectrum of solar radiation absorbed by green plants during photosynthesis that puts chlorophyll molecules in an excited state. It is determined by the solar radiation (SL) that can be absorbed by the vegetation and the proportion of incident photosynthetically effective radiation (μ) absorbed by the vegetation, which is estimated by the formula: [3]

$$ER(x,t) = SL(x,t) \cdot \mu(x,t) \cdot 0.5 \tag{1}$$

The constant 0.5 represents the proportion of solar effective radiation available to vegetation to total solar radiation. (Photosynthetically active radiation on average accounts for about 50% of total solar radiation.) There is a linear relationship between μ and the normalized vegetation index (NDVI)

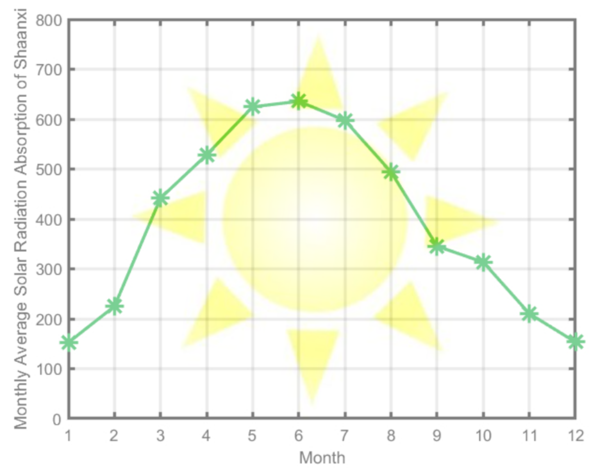

Figure 5: Monthly Average Solar Radiation of Shaanxi

within a certain range [6]. Furthermore, this relationship can be determined based on the maximum and minimum values of NDVI for a particular vegetation type and the corresponding maximum and minimum values of ER. NDVI is to detect the vegetation growth status, vegetation cover and to eliminate some of the radiation errors etc. NDVI can reflect the background effects of the plant canopy. The linear relationship is estimated by the formula [6]:

$$\mu(x,t) = 0.949 * \frac{NDVI(x,t) - NDVI_{i,min}}{NDVI_{i,max} - NDVI_{i,min}} + 0.001 \tag{2}$$

4.5 Calculation of Stress Factors

γ refers to the ratio of the chemical potential contained in the dry matter produced by a unit area of plants in a certain period of time to the ER to which it is irradiated. γ is regulated by various environmental factors, such as air temperature, soil moisture status, and atmospheric water vapor pressure difference. Enzymes in photosynthesis and respiration both have optimum temperatures, and if the temperature is too low or too high, the activity of the enzymes will be affected, thereby affecting photosynthesis. [7]

$$\gamma(x,y,t) = T_a(x,y,t) \cdot T_b(x,y,t) \cdot W_\gamma(x,y,t) \cdot \eta \tag{3}$$

In the formula above, Ta describes how temperature affects the activity of enzymes in photosynthesis, which greatly reduces the efficiency of photosynthesis, and Tb represents how temperature widens the gap between the intensity of respiration and the intensity of photosynthesis. η is only related to vegetation type.

$$T_a(x,y,t) = 0.8 + 0.002 * T_m(x) - 0.005 * [T_m(x)]^2 \tag{4}$$

$$T_b(x,y,t) = \frac{1.184}{1 + e^{0.2*T_m(x)-10-T(x,y,t)}} \cdot \frac{1}{1 + e^{0.3*(-T_m(x))-10-T(x,y,t)}} \tag{5}$$

Tm(x) is the most suitable temperature for plants to grow, which equals to the average monthly temperature when NDVI reaches to the top in this area. In the Fig.6 above, different kinds of forests

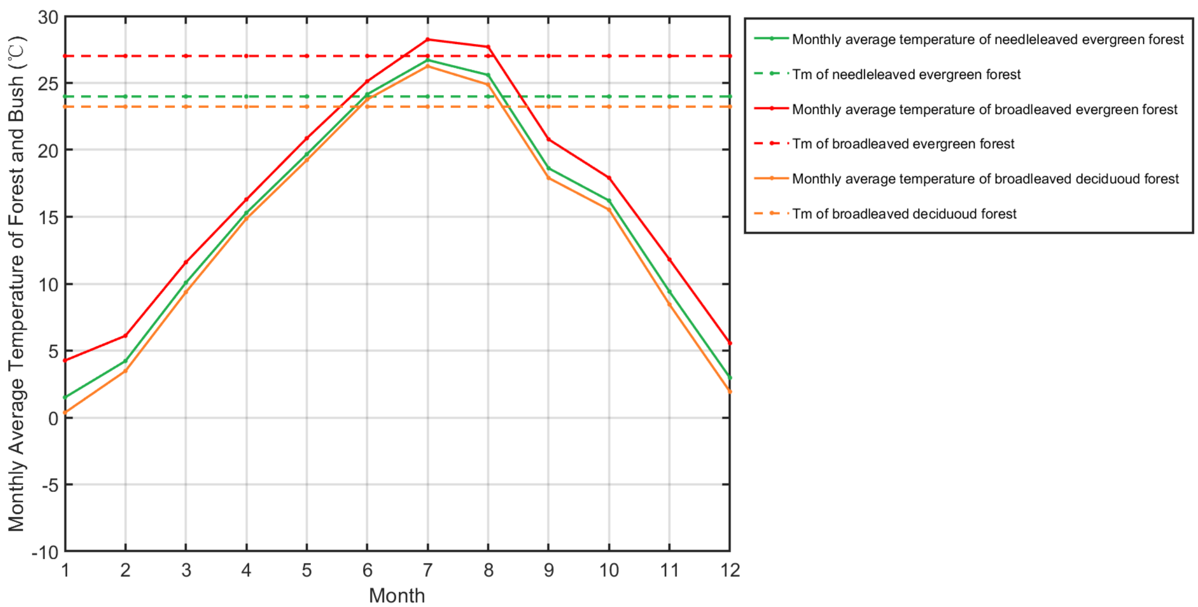

Figure 6: Monthly Average Temperature and T_m of Different Forest Types in Shaanxi

have different kinds of most suitable temperature. And it is June, July and August that have the smallest difference between the average monthly temperature and the most suitable temperature in three types of forest. W reflects the effect of the available water conditions available to plants on γ. If an area is drier, then photosynthesis is more affected by water stress.[8] We calculate W(x,y) by the following formula:

$$W(x,t) = \frac{0.5 + 0.5 * E(x,t)}{Ep(x,t)} \tag{6}$$

4.6 The Simplification of the Calculation Process of Ep and E

To calculate the quantities E and Ep, we used Valiantzas's model. Through this model, we can calculate the stress factor coefficient without knowing the surface atmospheric pressure and wind speed.

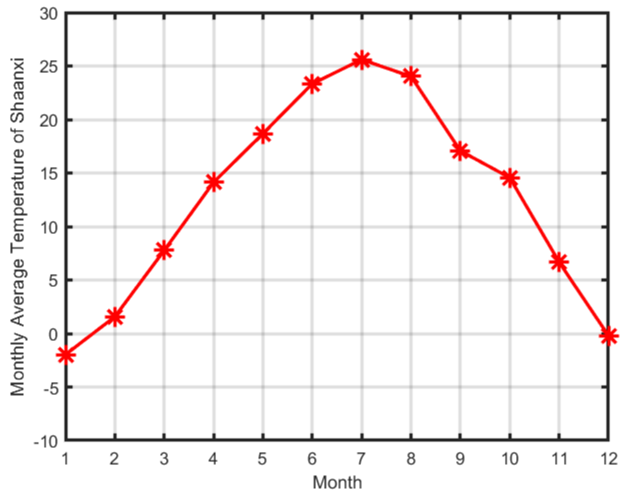

Figure 7: Monthly Average Temperature of Shaanxi

That is:

$$Ep = 0.047Rs \cdot sqrt(T + 9.5) - 2.4 \cdot (\frac{Rs}{Ra})^2 + 0.09(T + 20)(1 - 0.01 \cdot RH) \tag{7}$$

$$Rn = \sqrt{Ep(x, y, t) \cdot Pr(x, y, t)} \cdot \left(0.369 + 0.598 \cdot \sqrt{\frac{Ep(x, y, t)}{Pr(x, y, t)}}\right) \tag{8}$$

$$E = Pr(x, y, t) \cdot Rn \cdot \frac{Pr(x, y, t)^2 + Rn^2 + Pr(x, y, t) \cdot Rn}{(Pr(x, y, t) + Rn) \cdot (Pr(x, y, t)^2 + Rn^2)} \tag{9}$$

The unit of T is ℃, while items of the formula (7) from left to right are calculated by simplifying the calculation of incoming net short wave radiation, outgoing net longwave radiation and aerodynamic component, by this method, the water stress factor can be estimated by substituting other readily available data for surface atmospheric pressure and wind speed. [4]

4.7 NPP Calculation

The carbon that a forest can generate through photosynthesis can be represented by two factors: light and effective radiation (ER) of plants and actual light energy utilization γ, which is estimated by the following equation: [3]

$$NPP(x, t) = ER(x, y, t) \cdot \gamma(x, y, t) \tag{10}$$

As shown in Fig.8, we calculated the total sum of NPP of Shaanxi province in 2006. Futhermore, as shown in Table.4, we derived NPP of several forest types. For example, the needleleaved evergreen forest in the south of Shaanxi produced NPP of $858.12775 gC/m^2$ per year.

By this model, we can have a further solution for the mass of carbon sequestered by this forest in

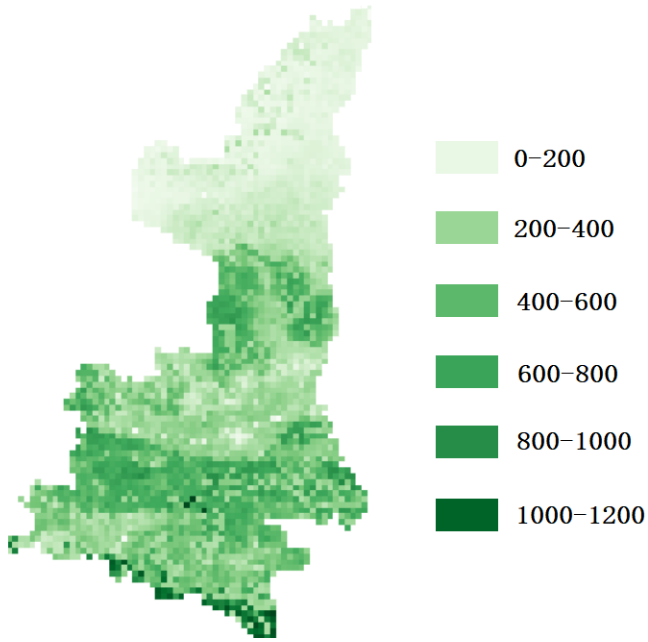

Figure 8: NPP of Each Pixel in Shaanxi

Table 4: Relationship between Forest Type and NPP

Forest Type	NPP(Unit: $gC/(m^2 \cdot year)$)
needleleaved evergreen forest	858.12775
broadleaved evergreen forest	597.23737
broadleaved deciduoud forest	367.71793
bush	384.04153
slope grassland	344.87766
plain grassland	189.22005
desert grassland	167.46722

100 years.

5 Model II: Forest Growth Dynamics Model

5.1 Notations

Table 5: Notations

Symbols	Description
y	Time of forest growth model
$X(y)$	Forest biomass at y
V	Forest growth rate
a	Response parameters of original forest biomass and forest nutrients to form new biomass
b	Response parameters of new forest biomass and forest nutrients to form another batch of new biomass
c	Response parameters related to the biomass of forest organisms that die or otherwise return to the forest
XM	The maximum biomass that a forest can support
$\bar{V}$	Average Forest growth rate
$P(y)$	Price of wood at y
$L(y)$	Loss rate of the forest caused by insect damage at y
d	Discount factor for wood damaged by insects
e	Discount Rate

5.2 Construct the Growth Model

The structure of a forest is not static.[9] As mentioned earlier, carbon sinks eventually create new biomass, a process also known as forest growth. Forest growth can be modeled to assess and predict the development trends. Modeling also facilitates selective human intervention in forest development. For example, when a forest grows slowly due to an excess of trees, some trees can be artificially cut down to reduce competition for sunshine and other nutrients to accelerate forest growth. At the same time, keeping the high rate of the forest growth is also helpful to increase the value of the forest for human.[10] As for the factors affecting forest growth, besides carbon sequestration, plant species and their photosynthesis, human intervention and pests, and even fire and drought are all factors. Similar to the carbon sequestration model, the forest growth depends on nutrients from the outside world, including water, sunshine, and others, and the forest partially returns to the soil, such as dead leaves

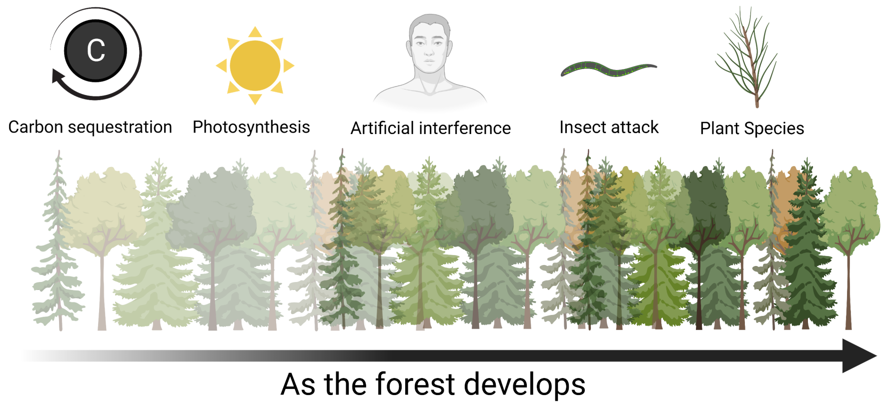

Figure 9

and dead trees. In addition, as a constant process, the newly grown forest also consumes nutrients. So, determine the following formulas:

$$\frac{dx}{dt} = rX\left(1 - \frac{X}{X_M}\right) \tag{11}$$

$$U = aX - cX^2 - bX \tag{12}$$

5.3 Determination of Periodical Forest Harvest

When forest biomass reaches its maximum, growth is essentially close to zero. Thus it must be cut before then. To maximize the use of land resources, we chose to determine the rotation period of the forest when the average growth of the whole growth cycle is maximum. The ecological benefits in the years before and after the rotation period of mature forests are very small and difficult to measure, so they are generally not considered. So, determine the following formulas: [9]

$$\frac{d\bar{x}}{dt} = \frac{1}{t}\int_0^t \left(\frac{dx}{dt}\right) dt \tag{13}$$

$$\frac{(a-b)\,X - cX^2}{a-b} ln\left[\frac{(a-b-cX)}{X}\right] - \left[\frac{1}{2}(a-b)\,X^2 - \frac{1}{3}cX^3\right] = 0 \tag{14}$$

When

$$\left(\frac{d\bar{x}}{dt}\right)' = 0 \tag{15}$$

in forest production, the cut is generally chosen when the real growth rate V is equal to the average growth rate $\bar{V}$, which ensures that the maximum biomass is obtained over the entire growth cycle.

$$\frac{d\bar{x}}{dt} = \frac{1}{t}\int_0^t \left(\frac{dx}{dt}\right) dt \tag{16}$$

When it equals to 0, then

$$\frac{(a-b)\,X - cX^2}{a-b} ln\left[\frac{(a-b-cX)}{X}\right] - \left[\frac{1}{2}\,(a-b)\,X^2 - \frac{1}{3}cX^3\right] = 0 \tag{17}$$

The biomass of felling during the rotation period is:

$$\bar{x} = \int_0^t \left(\frac{dx}{dt}\right) dt = \int_0^x \left(\frac{dx}{dt}\right) dx = \frac{1}{2}\,(a-b)\,X^2 - \frac{1}{3}cX^3 \tag{18}$$

5.4 Deforestation Strategy Model Optimization

To maximize benefits and to consider the effects of natural disasters, the model was further optimized. It is assumed that, in case of local insect infestation, the measure is to cut down and sell the forest trees that received insect infestation at a discount. It is assumed that insect damage does not affect the growth rate of trees. The discounted return is maximized by determining the cutting period, considering the price fluctuations and the risk of insect infestation.[11] So, the formulas are as follows:

$$\underset{Y}{max}(1+e)^{-(Y-y_0)}S(Y)P(Y) + \sum_{y=y_0}^{Y-1} L(t)S(y)P(y)\cdot d(1+e)^{-(y-y_0)} \tag{19}$$

$$s.t. \qquad S(y+1) = S(y)(g(y) - L(y)) \tag{20}$$

Since the occurrence of insect pests depends on stochastic factors such as climate with the pre-infestation of the pests. The forest damage caused by insect pests is discretized into m classes:

Assume that the probability of a transfer from $l1$ to lm is $Ph[lj|li] = aij$, then the matrix $A = a[i,j]m * m$ is the state transfer matrix of the pest occurrence process, and obviously, by the probability condition we can get:

$$\sum_{j=1}^{m} a_{ij} = 1, \qquad i = 1, 2, \cdots, m$$

Similarly, the wood price fluctuation P can be discretized into n steps as follows:

$$P = \{p_1, p_2, \cdots, p_n\}$$

Then the state transfer matrix of timber price change is:

$$\sum_{i=1}^{n} b_{kl} = 1, \qquad k = 1, 2, \cdots, n$$

Assuming that the price of lumber is not affected by whether or not an infestation occurs, then the state of loss and lumber price can be represented by the binary relation (li, pk) between the set L and P. Let this combination be F. Then F is the Cartesian product of the set L and P, i.e., $F = L \cdot P$, $|X| = |L| \cdot |P| = m * n \triangleq N$, where $|F|$ denotes the number of elements in the set. From probability theory, the probability of a combinatorial process moving from (li, pk) to (lj, pl) in one step is:

$$P_r[(l_j, p_l)|(l_i, p_k)] = P_r[l_j|l_i] \cdot P_r[p_l|p_k]$$

So, the transfer matrix of the combined procedure C is:

$$C = A \otimes B$$

$\otimes$ represents the Kronecker matrix product. After describing the value of woods and SPB loss, this problem can be solved by random dynamic programming: [12]

$$I = (i-1)n + k \tag{21}$$

$$\pi_J(y+1) = \sum_{I=1}^{N} \pi_I(y) C_{IJ} \tag{22}$$

$$S_J(y+1) = \sum_{I=1}^{N} S(y)[g(y) - L_i(y)] C_{IJ} \tag{23}$$

$$V_J(y+1) = \sum_{I=1}^{N} C_{IJ}[V_I(y) + (1+e)^{-(y-y_0)} \alpha P_k(y) S_I(y) L_i(y)] \tag{24}$$

$$F_J(y+1) = [V_J(y+1) + (1+e)^{-(y-y_0+1)} P_k(y+1) S_J(y+1)]/\pi_J(y+1) \tag{25}$$

$$F_j(y+1) = \max_{h_J(i)}(1 - h_J(y) F_j(y) + h_J(y) F_J(y+1)), \qquad J = 1, 2, \cdots, N \tag{26}$$

while $h_J(y) = 1$ represents preserving the forest and $h_J(y) = 0$ represents cutting down the forest.

6 Test the Model

6.1 Sensitivity Analysis

We consider three main factors related to meteorology: temperature, precipitation, and solar radiation. Observe the average change in NPP by changing one of the variables slightly. We calculate the average variation rate of NPP by changing those variable with the multiplier of 0.9 to 1.1, and 0.01 for an interval. As shown in Fig.10, solar radiation influence NPP most at the rate of 0.766, and the absolute value of the influence rate precipitation and temperature are 0.223 and 0.448.

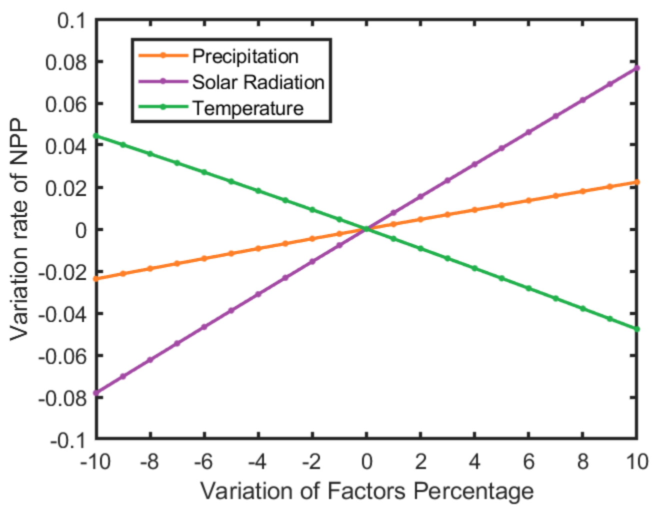

Figure 10: Average Variation Rate of NPP by Fine-tuning Variables

7 Conclusion

7.1 Summary of Results

7.1.1 Result of Problem 1

Based on the constructed carbon sequestration model, we calculate the carbon sequestrations over 100 years of the forests in Shaanxi. The result are approximately the NPP sum of the selected pixels multiplies $6.4 * 109$ grams.

7.1.2 Result of Problem 2

Based on the constructed forest growth model, we decide the best management plan of the forests in Shaanxi. The plan is to harvest mature/over-mature trees when the biomass is between $\frac{X_M}{2}$ and $\bar{x}$, which is $\frac{1}{2}(a-b)X^2 - \frac{1}{3}cX^3$. There is no condition that all the trees will not be cut for a long time.

7.2 Strength and Weakness

7.2.1 Strength

We consider a combination of different weather and location factors.

In the forest carbon sequestration model, we take into account the changes in carbon sequestration caused by weather and geographical location, such as precipitation and temperature. Since some data are difficult to measure and collect, we selectively ignore some influencing factors while ensuring the accuracy of the model. This work makes the model much more applicable and better understood.

In the forest growth model, we consider the growth of a single tree from an ideal situation and decide on its cutting period. When such a model is applied to the entire forest, it is only necessary to estimate the age and growth of each tree. To cope with the various factors and maximize profitability, we consider the impact of pests and timber costs on the final profitability. Pests and other hazards can be interchanged in the model, as their damage effects are similar in principle in the model. The simplicity and feasibility of the integrated model are high.

7.2.2 Weakness

The constructed model remains idealized, such as the environmental biomass in the growth model will not be constant. In the data statistics, the nutrients that forest growth receives from the environment in each time cycle are difficult to measure, although the carbon sequestration can be used to approximate the equivalence, but also to consider factors such as nitrogen and water. In addition, most of the available data are after human intervention. Substituting these data into the calculation is subject to significant errors.

8 Nontechnical Newspaper Article

Cut Down a Tree? Or Not?

The forest management strategies all point out moderate tree cutting. Will cutting trees be beneficial to the forest? Isn't it said that everyone has the responsibility to care for trees?

Forests are one of the most important parts of the ecological environment. They not only regulate the climate and purify the air but are also the basis of human survival. Every country takes the protection of forests very seriously so that everyone knows "no deforestation". The traditional perception is that cutting down trees destroys the forest ecosystem. But is this the case? Deforestation means excessive cutting of trees, which means the destruction of the original stable ecosystem and the deviation from the original development trajectory. With the development of knowledge and technology, people have realized that a stable forest structure does not grow anymore, which means carbon storage and emissions are in balance. The stability is naturally beneficial, but wouldn't it be better to properly manage the forest to keep it in a state of growth? A mature forest arises from a small forest that develops over a long period. During this process, the forest structure grows, and carbon storage is greater than carbon emissions. The process continues until a stable forest is formed. Although the growth rate of the forest varies during this process, it follows a certain pattern. Therefore, a good management strategy including moderate deforestation not only accelerates forest growth but also provides wood for humans to improve the quality of life.

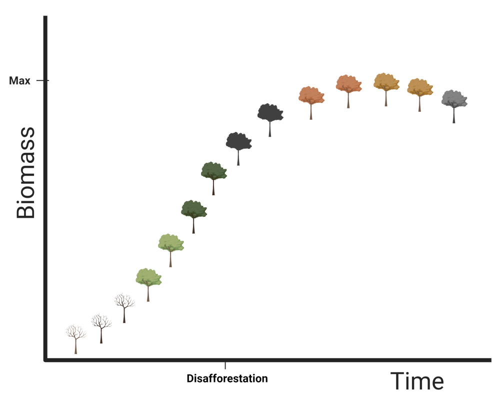

Figure 11: The Timecourse of Biomass

A stable forest structure is not the optimal solution. Why does the biomass of ancient forests that are not disturbed by humans not expand with each passing day? This is because, under the regulation only by the natural environment, the forest would tend to develop in an equilibrium. For example, small trees that do not receive sunlight do not grow well under large trees, and too close planting does not produce good yields. All of this happens during the development of a forest. The structure of the forest is slowly selected and adjusted as it develops, and eventually reaches a stable structure. When the biomass reaches the maximum capacity of the forest, the forest will develop at a rate close to 0. This is when the forest is low yielding for humans. This is because the forest biomass change converges to 0, which means that there is no change in the amount of carbon sequestered (carbon balance), i.e., no excess oxygen is released. It is worth noting that forest biomass develops rapidly when it is within a suitable range, as shown by the large increase in carbon sequestration. In addition, the development of forest biomass can be sustained with a high yield by constantly adjusting it back to the appropriate range. Therefore, we need to regulate the biomass within this appropriate range. Deforestation would be the most direct and effective way to do this.

Proper management of the forest structure will maximize the benefits. Deforestation requires clear planning and strategy. For example, new trees are growing, and their benefits are higher compared to mature trees. In addition, priority should be given to cutting down trees with high plant density in the area. Most forests are now subject to human intervention, even a significant number of planted forests. A sound management strategy is then the key to maximizing the use of forests for human benefit. What is a reasonable management strategy? Achieving a steady high rate of growth in carbon sinks coupled with moderate annual tree cutting to generate income. In research, a rich diversity of species is preferred to a single species. In addition, appropriate forest density, anthropogenic water, fertilizer supplementation, etc. all contribute to the development of forest structure.

Having said so much, there won't be anyone else who insists that not cutting down trees is the best option, right?

9 References

[1] Field Christopher B., Behrenfeld Michael J., Randerson James T., Falkowski Paul (1998). Primary production of the biosphere:integrating terrestrial and oceanic components, Science , 281 , 237-240.

[2] W. Zhu, Y. Pan, H. He, D. Yu and H. Hu. Simulation of maximum light use efficiency for some typical vegetation types in China, Chinese Science Bulletin, 2006, 51(4): 457-463.

[3] W. Zhu, Y. Pan, J. Zhang. "Remote sensing estimation of net primary productivity of terrestrial vegetation in China," Chinese Journal of Plant Ecology, 2007, 31(3): 413-424.

[4] John D. Valiantzas. "Simplified versions for the Penman evaporation equation using routine weather data," Journal of Hydrology (2006) 331, 690– 702.

[5] China National Meteorological Administration, Data Set of China's Surface Climate Standard Value Monthly(1981-2010), 2012.

[6] Ruimy A., Saugier B. (1994). Methodology for the estimation of terrestrial net primary production from remotely sensed data. Journal of Geophysical Research , 97, 18515-18521 .

[7]Potter CS, Randerson JT, Field CB , Matson PA, Vitousek PM , Mooney HA, Klooster SA(1993).Terrestrial ecosy stem production -a process model based on global satellite and surface data. Global Biogeochemical Cycles, 7 , 811 -841.

[8] Piao S., Fang J., Guo Q.(2001). Application of CASA Model to the estimation of Chinese terrestrial net primary productivity .Acta Phytoecologica Sinica, 25, 603 -608.

[9] Ouyang Hui. Analysis of non-linear, non-equilibrium dynamics model of forest ecosystem and its application[J]. Agricultural Systems Science and Integrated Research,1993(04): 261-264.

[10] Jiang X., Wang X. Optimal Harvest Decision Model for the Forest: A New Analysis Framework to Chinese Forestry Economic Policy, Scientia Silvae Sinicae, 2013(09).

[11] Wang D. W., Gu J. Z., T. P. Holmes. A stochastic dynamic programming model for optimizing forest harvesting strategies[J]. Information and Control, 1992(03): 180-183.

[12] Dreyfus S., Law A. The Art and Theory of Dynamic Programming, New York: Academic Press, 1977.

[13] Chen Guonan. A preliminary attempt to measure biological production in China using the Miami model [J]. Journal of Natural Resources, 1987, 2(3): 270 278.

[14] China National Meteorological Administration, Data Set of China's Surface Climate Standard Value Monthly(1981-2010), 2012.

[15] Dong Wenfu, Guan Dongsheng. The role of forest ecosystems in the carbon cycle. Chongqing Environmental Science, 2001, 24(3):25 27.

[16] Field C., Randerson J., Malmstrom C. (1995). Global net primary production: combining ecology and remote sensing.Remote Sensing of Environment, 51, 74-88.

[17] IPCC.Intergovermental Panel on Climate Change Scientific Assessment of Climate Changes [M]. UNEP, UN, New York. 1992

[18] Los SO, Justice CO, Tucker CJ (1994). A global 1 degree by 1 degree NDVI dataset for climate studies derived from the GIMMS continental NDVI data.International Journal of Remote Sensing, 15, 3493-3518.

[19] Winjum J K, Dixon, R K, Paule S. Forest management and carbon storge:an analysis of key forest nations [J]. Water, Air, and Soil Pollution, 1993(70): 239 257.

[20] Tietenberg T. 2003. Environmental and Natural Resource Economics, 5th Edition, published by Beijing Economic Science Press, 271.

[21] Bartels. Barker, Zhang XD, Peng ZH, et al. A modeling approach to the study of carbon cycle in forest ecosystems [J]. World Forestry Research, 2008, 21(1): 9 13.

10 Appendix

Here is the code for the process of calculating NPP.

```
#Main Code of Forestry Carbon Sequestration Model
#Calculate ndvi_min, ndvi_max
for i in range(0,Row):
    for j in range(0,Column):
        if(bound[i][j]):
            for t in range(0,12):
                ndvi_min[tree[i][j]]=Min(ndvi_min[tree[i][j]],ndvi[i][j][t])
                ndvi_max[tree[i][j]]=Max(ndvi_max[tree[i][j]],ndvi[i][j][t])
#Calculate Tm, Ta, Tb
for i in range(0,Row):
    for j in range(0,Column):
        if(bound[i][j]):# If this pixel is in the surveyed province
            ndvi_maxi=-1
            ndvi_mini=100
            for t in range(0,12):
                ndvi_maxi=Max(ndvi_maxi,ndvi[i][j][t])
                ndvi_mini=Min(ndvi_mini,ndvi[i][j][t])
            for t in range(0,12):
                if(abs(ndvi[i][j][t]-ndvi_maxi)<1e-6):
                    Tm[i][j]=Te[i][j][t]
            if(ndvi_mini<=-10):
                continue
            for t in range(0,12):
                Ta[i][j][t]=0.8+0.02*Tm[i][j]-0.0005*(Tm[i][j]**2);
                Tb[i][j][t]=1.184/(1+exp(0.2*(Tm[i][j]-10-Te[i][j][t])))*1/(1+exp(0.3*(-Tm[i][j]-10+Te[i][j][t])))
#Calculate NPP(x,y,t)
for i in range(0,Row):
    for j in range(0,Column):
        if(bound[i][j]):
            for t in range(0,12):
                mu[i][j][t]=(ndvi[i][j][t]-ndvi_min[tree[i][j]])/(ndvi_max[tree[i][j]]-ndvi_min[tree[i][j]])*0.949+0.001
                ER[i][j][t]=mu[i][j][t]*Sl[i][j][t]*0.5
                ltt=get_latitude(i,Row,39.5,31.67)*pi/180
                N=4*ltt*sin(0.53*(t+1)-1.65)+12
                Ra=3*N*sin(0.131*N-0.95*ltt)*month_day[t]
                Ep[i][j][t]=0.047*Sl[i][j][t]*sqrt(Te[i][j][t]+9.5)-2.4*((Sl[i][j][t]/Ra)**2)+0.09*(Te[i][j][t]+20)*(1-HMDT/100)
                r=Pr[i][j][t]
                if(r==0):
                    W[i][j][t]=0.5
                else:
                    Rn=((Ep[i][j][t]*r)**0.5)*(0.369+0.598*((Ep[i][j][t]/r)**0.5))
                    E[i][j][t]=r*Rn*(r**2+Rn**2+r*Rn)/((r+Rn)*(r**2+Rn**2))
                    W[i][j][t]=0.5+0.5*E[i][j][t]/Ep[i][j][t]
                gamma[i][j][t]=eta[tree[i][j]]*W[i][j][t]*Ta[i][j][t]*Tb[i][j][t]
                NPP[i][j][t]=ER[i][j][t]*gamma[i][j][t]
                NPP_sum[i][j]+=NPP[i][j][t]
```