Fixed Income Quantitative Trading HW1
Name: Chenyu Wang
NetID: cw2945

Q1: The account has been created.

Q2: The data is downloaded and saved as "Treasury Yild.xlsx"

a. Explain how these CMT rates are constructed

Explain:

Firstly, constructed the yield curve:

The curve is based on the closing market bid yields on actively traded Treasury securities in the OTC market. These market yields are calculated from composites of indicative bid-side market quotations(not actual transactions) obtained by the Federal Reserve Bank of NY on near 3:30p.m. each trading day.

Then, read the CMT yield values from the curve:

Read the CMT yield values using interpolation method from the yield curve of fixed maturities, currently 1,2,3 and 6 months and 1,2,3,5,7,10,20 and 30 years.

Note that the yields are interpolated by the curve. This approach provides a yield for a exact maturity, for example 10 years, even if there is no outstanding securities has exactly 10 years remaining to maturity.

Q3: The solution is in the Notebook.

Q4:

a. Explain reasoning behind a choice of a risk-free rate

The major difference between the choice of risk-free rate in the paper and the traditional method is whether we consider collateralization. In traditional method, we just ignore this part, so the formula is quite simple. If we consider credit risk in the model, we find that the risk-free rate is actually default-risky rate. After collateralization, we need a default-free rate which is our new risk-free rate. Normally, secured borrowing can attract a better rate than unsecured borrowing. The unsecured borrowing rate is based on bank credit rating and is perceived probabilities of default. So, it's default-risky rate. While secured borrowing rate is default-free rate.

b. Prove that collateralized LIBOR forward contracts are different from Eurodollar futures, despite the conclusion reached by Johannes & Sundaresan(2007).

For collateralized LIBOR forward contracts, we have:

$$F_{t,T}^{CSA} = E_t^T[S(T)]$$

And when we calculate the daily difference of the forward contract:

$$V(t') - V(t) = E_{t'}[e^{-\int_{t'}^{T} r_C(u)du}](F_{CSA}(t',T) - F_{CSA}(t,T))$$

However, when we calculate the daily difference of the futures:

$$V(t') - V(t) = F_{t',T}^{Fut} - F_{t,T}^{Fut}$$

There is no discount factor here, so they are different.

c. What was the reason for the mistaken conclusion in the Johannes & Sundaresan(2007) paper? Explain your reasoning.

The conclusion reached in Johannes & Sundaresan (2007) is that swaps are priced close to portfolios of futures rather than portfolios of forwards discounted at the instantaneous LIBOR rates.

The paper reached this conclusion through empirical analysis. The logic of this part is the following:

$$\begin{cases} If\ \left|S_{0,T}^{Fut} - S_{0,T}^{Mar}\right| < \varepsilon, Swap\ is\ priced\ close\ to\ portfolios\ of\ futures\ contracts \\ If\left|S_{0,T}^{For} - S_{0,T}^{Mar}\right| < \varepsilon, Swap\ is\ priced\ close\ to\ portfolios\ of\ forward\ contracts \end{cases}$$

The empirical result in the paper is:

$$\left|S_{0,T}^{Fut} - S_{0,T}^{Mar}\right| < \varepsilon$$

So, Johannes & Sundaresan reached a conclusion that swaps are priced close to portfolios of futures.

However, according to Funding beyond discounting: collateral agreements and derivatives pricing (2010), the type of convexity effects in futures are different from what we see in CSA versus no-CSA forward contracts. For a futures contract, the difference will not be discounted!

This effect is similar to the one defined as “NPV effect” in the paper “Central Clearing of Interest rate Swaps: a Comparison of Offerings”

Through this point of view, we could easily find the mistakes in Johannes & Sundaresan(2007)’s logic, the details are displayed as below:

In order to make the expression more easily to understand, let me correspond the notations in two papers and make my notations clear.
In Johannes & Sundaresan(2007), if we assume that the principal is 1, then all the notations can correspond with the notations in Funding beyond discounting: collateral agreements and derivatives pricing(2010).

$$S_{0,T}^{Fut} = F_{0,T}^{Fut}, S_{0,T}^{Mar} = F_{0,T}^{CSA}, S_{0,T}^{For} = F_{0,T}^{NoCSA}$$

Here, there may be a confusion that why $S_{0,T}^{Mar} = F_{0,T}^{CSA}$? The reason is that we find the calculations in both papers. The calculation results of these two notations are exactly the same. In reality, one-period swap with collateral just looks like a forward contract with collateral. So, this could be easily understood.

Now, let’s consider the condition that suggest that swap is priced close to portfolios of futures. For futures contract, the price is adjusted everyday by marked to market. However, the swap rate is fixed at the first day and then remain the same until maturity. That brings a new challenge that we should not only make sure that at the beginning we have:

$$\left|F_{0,T}^{Fut} - F_{0,T}^{CSA}\right| < \varepsilon$$

We should also make sure that

$$\left|V_{t'}^{Fut} - V_{t'}^{CSA}\right| < \varepsilon \quad (4)$$

Note: at this part $V_{t'}^{Fut}$ means at time t', the value of the margin account of the future. According to Funding beyond discounting: collateral agreements and derivatives pricing (2010), we know that:

$$V_{t'}^{Fut} - V_{t}^{Fut} = F_{t',T}^{Fut} - F_{t,T}^{Fut} \quad (1)$$

It's easily to understand that the change of your margin account is the change of the future price. D^{CSA} is the discount factor.

$$V_{t'}^{CSA} - V_{t}^{CSA} = D^{CSA}\left[F_{t',T}^{CSA} - F_{t,T}^{CSA}\right] \quad (2)$$

$$V_{t'}^{NoCSA} - V_{t}^{NoCSA} = D^{NoCSA}\left[F_{t',T}^{NoCSA} - F_{t,T}^{NoCSA}\right] \quad (3)$$

In order to examine (4), let (1) – (2):

$$\left(V_{t'}^{Fut} - V_{t'}^{CSA}\right) - \left(V_{t}^{Fut} - V_{t}^{CSA}\right) = \left(F_{t',T}^{Fut} - D^{CSA}F_{t',T}^{CSA}\right) - \left(F_{t,T}^{Fut} - D^{CSA}F_{t,T}^{CSA}\right)$$

So, what the paper examine in the empirical part: $\left|F_{0,T}^{Fut} - F_{0,T}^{CSA}\right| < \varepsilon$ is not sufficient to prove that $\left|V_{t'}^{Fut} - V_{t'}^{CSA}\right| < \varepsilon$.

On the contrary, let (3) – (2):

$$\begin{aligned}\left(V_{t'}^{NoCSA} - V_{t'}^{CSA}\right) - \left(V_{t}^{NoCSA} - V_{t}^{CSA}\right)\\ = \left(D^{NoCSA}F_{t',T}^{NoCSA} - D^{CSA}F_{t',T}^{CSA}\right) - \left(D^{NoCSA}F_{t,T}^{NoCSA} - D^{CSA}F_{t,T}^{CSA}\right)\end{aligned}$$

So, what the paper examine in the empirical part: $\left|F_{0,T}^{Fut} - F_{0,T}^{CSA}\right| < \varepsilon$ is also not sufficient to prove that $\left|V_{t'}^{NoCSA} - V_{t'}^{CSA}\right| < \varepsilon$ is wrong.

In a nutshell, the logic in Johannes & Sundaresan is wrong. What really matter is that for collateralized swap contract, the change of the collateral is based on PV(which looks like a forward contract rather than futures). However, in future contract, the change of the margin account is based on FV. Johannes & Sundaresan ignored this difference (discount factor) and thus got the wrong conclusion.