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Sift finds keypoint locations at particular scales and assign orientations to them. By ~~using~~ giving each keypoint one or more orientations based on local gradient directions, sift achieves invariance

P2

$$\frac{\partial L}{\partial y_i} = \frac{\partial L}{\partial p_i} \cdot \frac{\partial p_i}{\partial y_i}$$

So we have:

$$L = -\log(p_{gt})$$

$$p_{gt} = \frac{e^{y_{gt}}}{\sum_{i=1}^{n} e^{y_i}}$$

if $i = gt$

$$p_{gt} = p_i$$

$$\frac{\partial L}{\partial y_i} = -\frac{1}{p_{gt}} \cdot \frac{\partial p_i}{\partial y_i}$$

$$\frac{\partial p_i}{\partial y_i} = \frac{\partial \left(\frac{e^{y_i}}{\sum_{i=1}^{n} e^{y_i}} \right)}{\partial y_i}$$

$$= \frac{\Sigma \frac{\partial e^{y_i}}{\partial y_i} - e^{y_i} \frac{\partial \Sigma}{\partial y_i}}{\Sigma^2}$$

$$= \frac{e^{y_i}(\Sigma - e^{y_i})}{\Sigma^2} = \frac{e^{y_i}}{\Sigma} \cdot \frac{(\Sigma - e^{y_i})}{\Sigma}$$

$$= p_{gt} \cdot (1 - p_{gt})$$

$$\frac{\partial p_i}{\partial y_i} \cdot \frac{\partial L}{\partial p_i} = \frac{e^{y_i}(\Sigma - e^{y_i})}{\Sigma^2} \cdot -\frac{1}{p_{gt}}$$

$$= \frac{e^{y_i}(\Sigma - e^{y_i})}{\Sigma^2} \cdot -\frac{1}{p_{gt}}$$

$$= p_{gt} \cdot (1 - p_{gt}) \cdot -\frac{1}{p_{gt}}$$

$$= p_{gt} - 1$$

$$= p_i - 1$$

if $i \neq gt$.

$$\frac{\partial p_{gt}}{\partial y_i} = \frac{\partial \left(\frac{e^{y_{gt}}}{\sum e^{y_i}} \right)}{\partial y_i}$$

$$= \frac{\Sigma \frac{\partial e^{y_{gt}}}{\partial y_i} - e^{y_{gt}} \frac{\partial \Sigma}{\partial y_i}}{\Sigma^2}$$

$$= 0 - \frac{e^{y_{gt}} \cdot e^{y_i}}{\Sigma^2}$$

$$= -\frac{e^{y_{gt}}}{\Sigma} \cdot \frac{e^{y_i}}{\Sigma}$$

$$= -p_{gt} \cdot p_i$$

$$\therefore \frac{\partial p_i}{\partial y_i} \cdot \frac{\partial L}{\partial p_i} = p_i$$

P3. BP of FC

$x \sim R^{L\times 1}$ $\quad y \sim R^{O\times 1}$ $\quad \frac{\partial L}{\partial y} \sim R^{O\times 1}$

$$\underset{R^{O\times L}}{\frac{\partial L}{\partial W}} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial W} = \frac{\partial L}{\partial y} \cdot \frac{\partial (Wx+b)}{\partial W} = \underset{(O\times 1)}{\frac{\partial L}{\partial y}} \cdot \underset{(1\times L)}{X^T} \qquad W \sim O\times L$$

here $\frac{\partial L}{\partial y} \times X^T$ $\cancel{m\times n \quad n\times k}$

$$\underset{R^{O\times 1}}{\frac{\partial L}{\partial b}} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial b} = \frac{\partial L}{\partial y} \cdot \frac{\partial (Wx+b)}{\partial b} = \frac{\partial L}{\partial y} \cdot 1$$

$(O\times 1) \cdot 1$

$$\underset{R^{L\times 1}}{\frac{\partial L}{\partial x}} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial x} = \frac{\partial L}{\partial y} \cdot \frac{\partial (Wx+b)}{\partial x} = \underset{(O\times 1)}{\frac{\partial L}{\partial y}} \cdot \underset{(O\times L)}{W}$$

here $\frac{\partial L}{\partial y}$, adjust to $W^T \cdot \frac{\partial L}{\partial y}$

here $W^T \cdot \frac{\partial L}{\partial y}$. $\quad (L\times O) \quad (O\times 1)$

P4.

$$\frac{\partial L}{\partial w_k(t,m,n)} = \frac{\partial L}{\partial y} \cdot \frac{\partial \sum_{t=0}^{T-1} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(t, i\times s+m, j\times s+n)\, w_k(t,m,n) + b_k}{\partial w_k(t,m,n)}$$

$$\frac{\partial L}{\partial w_k(t,m,n)} = \sum_{i=0}^{H-M} \sum_{j=0}^{W-N} \frac{\partial L}{\partial y(k,i,j)} \cdot x(t, i\times s+m, j\times s+n)$$

$$\frac{\partial L}{\partial b(k)} = \sum_{m=0}^{K} \sum_{i=0}^{H-M} \sum_{j=0}^{W-N} \frac{\partial L}{\partial y(k,i,j)}$$

$$\frac{\partial L}{\partial x(t,m,n)} = \sum_{m=0}^{K} \sum_{i=0}^{H-M} \sum_{j=0}^{W-N} \frac{\partial L}{\partial y} \cdot w_k(t, m-i\times s, n-j\times s)$$

P5. Max2D

$$\frac{\partial L}{\partial X(t,m,n)} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$= \begin{cases} \frac{\partial L}{\partial y}. & \text{where } x(t,m,n) = x(c', i \times stm', j \times stn') \\ 0 & \text{otherwise.} \end{cases}$$