

BU CS320 Assignment 5: Context Free Grammars

October 30, 2023

1. Given the following grammar where $\langle expr \rangle$ is the starting symbol

$\langle digit \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$ $\langle nat \rangle ::= \langle digit \rangle \mid \langle digit \rangle \langle nat \rangle$ $\langle int \rangle ::= \langle nat \rangle \mid -\langle nat \rangle$ $\langle expr \rangle ::= \langle int \rangle$ $\quad \mid (\langle expr \rangle)$ $\quad \mid \langle expr \rangle + \langle expr \rangle$ $\quad \mid \langle expr \rangle * \langle expr \rangle$
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Derive the sentence using *rightmost derivation*.

12 + 2 * -07

$\langle expr \rangle ::= \langle expr \rangle * \langle expr \rangle$
 $\Rightarrow \langle expr \rangle * \langle int \rangle$
 $\Rightarrow \langle expr \rangle * -\langle nat \rangle$
 $\Rightarrow \langle expr \rangle * -\langle digit \rangle \langle nat \rangle$
 $\Rightarrow \langle expr \rangle * -\langle digit \rangle \langle digit \rangle$
 $\Rightarrow \langle expr \rangle * -\langle digit \rangle 7$
 $\Rightarrow \langle expr \rangle * -07$
 $\Rightarrow \langle expr \rangle + \langle expr \rangle * -07$
 $\Rightarrow \langle expr \rangle + \langle int \rangle * -07$
 $\Rightarrow \langle expr \rangle + \langle nat \rangle * -07$
 $\Rightarrow \langle expr \rangle + \langle digit \rangle * -07$

$\Rightarrow \langle expr \rangle + 2 * -07$
 $\Rightarrow \langle int \rangle + 2 * -07$
 $\Rightarrow \langle nat \rangle + 2 * -07$
 $\Rightarrow \langle digit \rangle \langle nat \rangle + 2 * -07$
 $\Rightarrow \langle digit \rangle \langle digit \rangle + 2 * -07$
 $\Rightarrow \langle digit \rangle 2 + 2 * -07$
 $\Rightarrow 12 + 2 * -07$

2. Given the following grammar where $\langle stmt \rangle$ is the starting symbol.

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 $\langle digit \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$ 
 $\langle letter \rangle ::= a \mid b \mid c \mid \dots \mid z$ 
 $\langle nat \rangle ::= \langle digit \rangle \mid \langle digit \rangle \langle nat \rangle$ 
 $\langle int \rangle ::= \langle nat \rangle \mid - \langle nat \rangle$ 
 $\langle expr \rangle ::= \langle int \rangle$ 
                $\mid ( \langle expr \rangle )$ 
                $\mid \langle expr \rangle + \langle expr \rangle$ 
                $\mid \langle expr \rangle * \langle expr \rangle$ 
 $\langle id \rangle ::= \langle letter \rangle \mid \langle letter \rangle \langle id \rangle$ 
 $\langle stmt \rangle ::= \langle id \rangle = \langle expr \rangle$ 
                $\mid \text{for } \langle id \rangle = \langle expr \rangle \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$ 
                $\mid \{ \langle stmts \rangle \}$ 
                $\mid \text{pass}$ 
 $\langle stmts \rangle ::= \langle stmt \rangle \mid \langle stmt \rangle ; \langle stmts \rangle$ 

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Derive the sentence using *leftmost derivation*.

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for x = -12 to 10 do { y = 0; pass }
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$\langle stmt \rangle ::= \text{for } \langle id \rangle = \langle expr \rangle \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$

$\Rightarrow \text{for } \langle letter \rangle = \langle expr \rangle \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$

$\Rightarrow \text{for } x = \langle expr \rangle \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$

$\Rightarrow \text{for } x = \langle int \rangle \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$

$\Rightarrow \text{for } x = - \langle nat \rangle \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$

$\Rightarrow \text{for } x = - \langle digit \rangle \langle nat \rangle \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$

$\Rightarrow \text{for } x = - 1 \langle nat \rangle \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$

$\Rightarrow \text{for } x = - 1 \langle digit \rangle \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$

$\Rightarrow \text{for } x = - 12 \text{ to } \langle expr \rangle \text{ do } \langle stmt \rangle$

\Rightarrow for $x = -12$ to $\langle \text{int} \rangle$ do $\langle \text{stmt} \rangle$

\Rightarrow for $x = -12$ to $\langle \text{nat} \rangle$ do $\langle \text{stmt} \rangle$

\Rightarrow for $x = -12$ to $\langle \text{digit} \rangle \langle \text{nat} \rangle$ do $\langle \text{stmt} \rangle$

\Rightarrow for $x = -12$ to $1 \langle \text{nat} \rangle$ do $\langle \text{stmt} \rangle$

\Rightarrow for $x = -12$ to $1 \langle \text{digit} \rangle$ do $\langle \text{stmt} \rangle$

\Rightarrow for $x = -12$ to 10 do $\langle \text{stmt} \rangle$

\Rightarrow for $x = -12$ to 10 do $\{ \langle \text{stmts} \rangle \}$

\Rightarrow for $x = -12$ to 10 do $\{ \langle \text{stmt} \rangle; \langle \text{stmts} \rangle \}$

\Rightarrow for $x = -12$ to 10 do $\{ \langle \text{id} \rangle = \langle \text{expr} \rangle; \langle \text{stmts} \rangle \}$

\Rightarrow for $x = -12$ to 10 do $\{ y = \langle \text{expr} \rangle; \langle \text{stmts} \rangle \}$

\Rightarrow for $x = -12$ to 10 do $\{ y = \langle \text{int} \rangle; \langle \text{stmts} \rangle \}$

\Rightarrow for $x = -12$ to 10 do $\{ y = \langle \text{nat} \rangle; \langle \text{stmts} \rangle \}$

\Rightarrow for $x = -12$ to 10 do $\{ y = \langle \text{digit} \rangle; \langle \text{stmts} \rangle \}$

\Rightarrow for $x = -12$ to 10 do $\{ y = 0; \langle \text{stmts} \rangle \}$

\Rightarrow for $x = -12$ to 10 do { $y = 0$; $\langle \text{stmt} \rangle$ }

\Rightarrow for $x = -12$ to 10 do { $y = 0$; pass }