# CS 320: Examples Operational Semantics

December 2, 2023

**Problem 1. Booleans and Integers**

*Consider the following language with booleans, and integers $L_0$:*

$$\begin{array}{rl} \textit{constants} & \langle const \rangle ::= boolean \mid int \mid \texttt{error} \\ \textit{expressions} & \langle expr \rangle ::= \langle const \rangle \mid \textbf{add}\ (\langle expr \rangle, \langle expr \rangle) \mid \textbf{eq}\ (\langle expr \rangle, \langle expr \rangle) \end{array}$$

*Consider the following rules defining the operational semantics of $L_0$. In this operational semantics a configuration is just an expression, which we will denote using the meta-variables $x, y, \ldots$. We use the notation $x \Rightarrow y$ to say that from the configuration/expression $x$ we can get in one step to the configuration/expression $y$. Similarly, we use the notation $x \Rightarrow^n y$ to say that from the configuration/expression $x$ we can get, in $n$ steps, to the configuration/expression $y$. Here $\mathbb{Z}$ is the set of all integers, and $\mathbb{B}$ is the set of all booleans. $+$, and $=$ are the usual mathematical notion of integer addition, and equality.*

$$\frac{}{x \Rightarrow^0 x}\ \textsc{Multi-Base} \qquad \frac{x \Rightarrow^n y \qquad y \Rightarrow z}{x \Rightarrow^{n+1} z}\ \textsc{Multi-Ind} \qquad \frac{x \Rightarrow x'}{\textbf{add}\ (x, y) \Rightarrow \textbf{add}\ (x', y)}\ \textsc{Add-Left}$$

$$\frac{x \in \mathbb{Z} \qquad y \Rightarrow y'}{\textbf{add}\ (x, y) \Rightarrow \textbf{add}\ (x, y')}\ \textsc{Add-Right} \qquad \frac{x \in \mathbb{Z} \qquad y \in \mathbb{Z}}{\textbf{add}\ (x, y) \Rightarrow (x + y)}\ \textsc{Add-Success}$$

$$\frac{x \in \mathbb{B} \cup \{\texttt{error}\}}{\textbf{add}\ (x, y) \Rightarrow \texttt{error}}\ \textsc{Add-Left-Error} \qquad \frac{x \in \mathbb{Z} \qquad y \in \mathbb{B} \cup \{\texttt{error}\}}{\textbf{add}\ (x, y) \Rightarrow \texttt{error}}\ \textsc{Add-Right-Error}$$

$$\frac{x \Rightarrow x'}{\textbf{eq}\ (x, y) \Rightarrow \textbf{eq}\ (x', y)}\ \textsc{Eq-Left} \qquad \frac{x \in \mathbb{Z} \qquad y \Rightarrow y'}{\textbf{eq}\ (x, y) \Rightarrow \textbf{eq}\ (x, y')}\ \textsc{Eq-Right}$$

$$\frac{x \in \mathbb{Z}}{\textbf{eq}\ (x, x) \Rightarrow \textbf{true}}\ \textsc{Eq-True} \qquad \frac{x, y \in \mathbb{Z} \qquad x \neq y}{\textbf{eq}\ (x, y) \Rightarrow \textbf{false}}\ \textsc{Eq-False}$$

$$\frac{x \in \mathbb{B} \cup \{\texttt{error}\}}{\textbf{eq}\ (x, y) \Rightarrow \texttt{error}}\ \textsc{Eq-Left-Error} \qquad \frac{x \in \mathbb{Z} \qquad y \in \mathbb{B} \cup \{\texttt{error}\}}{\textbf{eq}\ (x, y) \Rightarrow \texttt{error}}\ \textsc{Eq-Right-Error}$$

*Prove the following judgements by drawing their derivation trees.*

- $\mathbf{add}\,(1,\ \mathbf{add}\,(2,3)) \Rightarrow^2 6$
- $\mathbf{eq}\,(\ \mathbf{add}\,(1,2)),\ \mathbf{add}\,(2,1)) \Rightarrow^3\ \mathbf{true}$
- $\mathbf{add}\,(\ \mathbf{add}\,(1,2),\ \mathbf{eq}\,(1,2)) \Rightarrow^3 \texttt{error}$