CS 320 : Formal Semantics

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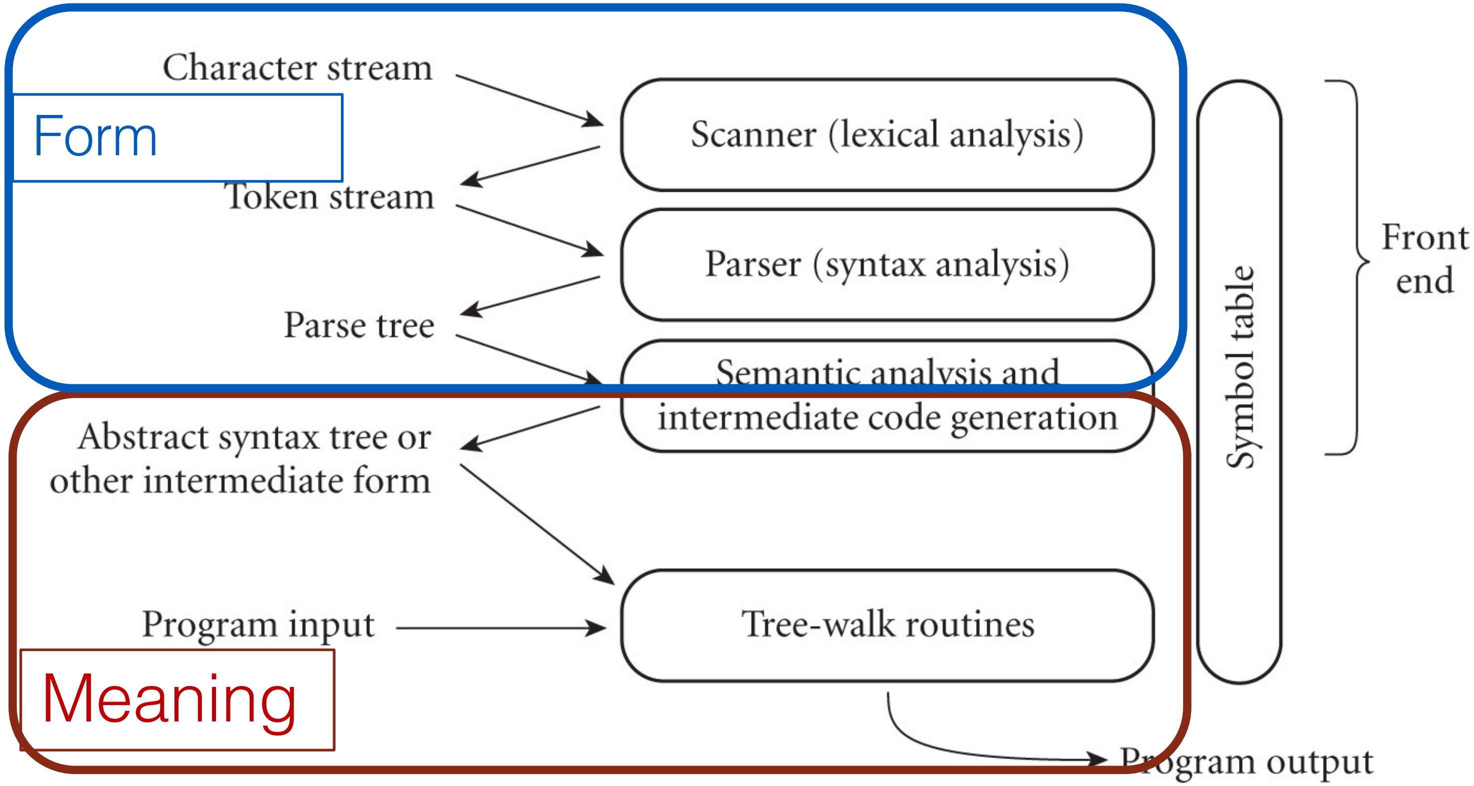
Form and Meaning



Syntax vs Semantics

- Syntax is about “form” and semantics about “meaning”.

```
<expr> ::= <expr> <addop> <term>
         | <term>
<term> ::= <term> <mulop> <term>
         | <factor>
<factor> ::= <const> | ( <expr> )
<const> ::= 1|2|3|4|5|6|7|8|9|0
<addop>   ::= + | -
<mulop>   ::= * | /
```

- How do we give meaning to sentences from this grammar?

We have two kinds of semantics: static and dynamic

Semantics: Static vs Dynamic

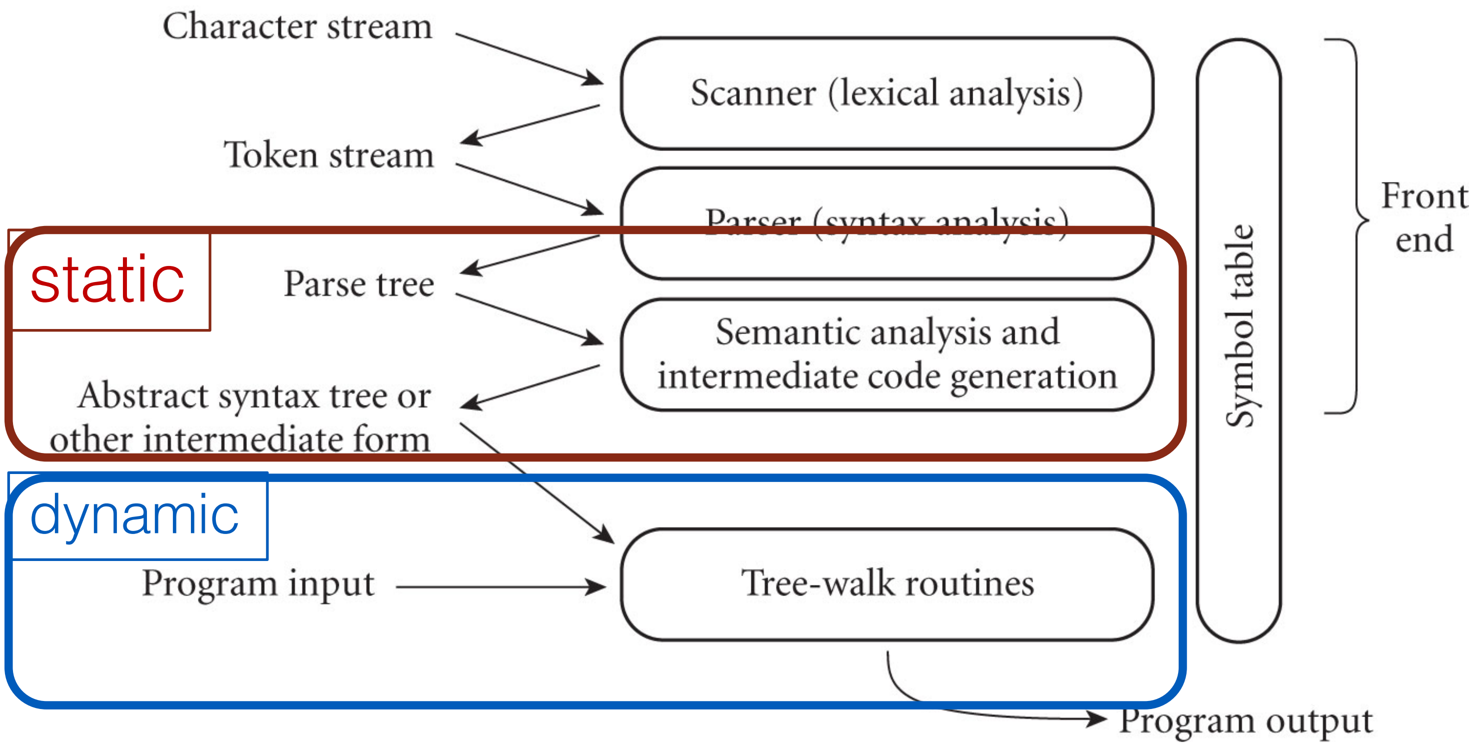
Static semantics:

- Set of rules attaching some high level meaning to the syntactic structure.
  Examples: typing information, well-formedness of commands, etc.
- It is usually enforced statically (before execution).

Dynamic semantics:

- Set of rules describing how the syntactic objects need to be executed.
  Examples: expression evaluation, commands execution, etc.
- It described the way the program must be executed at runtime.

Parsing and semantic analysis



Dynamic Semantics

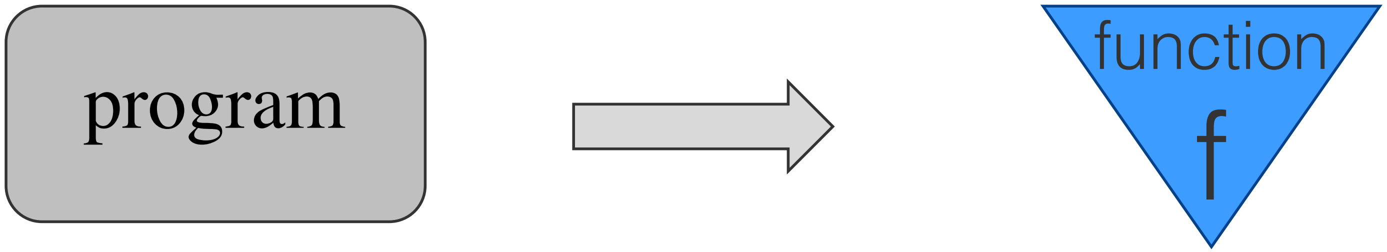
Why do we need to specify the semantics?

- Programmers need to know what statements and command mean
- Compiler writers must know exactly what language constructs do
- Correctness proofs with respect to the specifications,
- Designers need to detect ambiguities and inconsistencies
- …

How would you specify the semantics of a program?

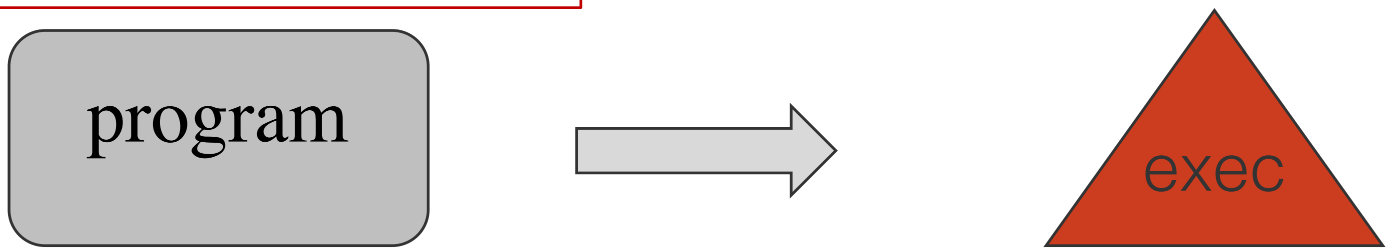
Formal Semantics

Denotational Semantics



A program is described by a mathematical function specifying the input-output relation that the program implements.

Operational Semantics



A program is described by the sequence of transformations that the program implements on the input to produce the output.

Operational Semantics

- Gives the meaning of a program by describing how the statements are executed.
- This can be provided through formal rules or through a description of a machine to execute the programs.
- The change in the state of the machine (memory, registers, etc.) defines the meaning of the statement.

Some examples

- https://docs.oracle.com/javase/specs/jvms/se7/html/
- http://www.open-std.org/jtc1/sc22/wg14/www/docs/n1570.pdf
- http://sml-family.org/sml97-defn.pdf
- Our interpreter.

Operational Semantics

- It is usually provided at a level of abstraction that is independent from the machine.
- The detailed characteristics of the particular computer would make actions difficult to describe/understand.
- Different formalism has been developed to describe the operational semantics in a machine-independent way.

We will look into formal rules and derivations.

Language for the Interpreter (simplified)

The language for the interpreter can be described by the following grammar:

```
<const> ::= int | name
<prog>  ::= <com>;<prog> | quit
<com>   ::= push <const> | pop | add | sub | mul | div
```

- A program is a sequence of commands followed by `quit`.
- A command is one the keywords above - in the case of push this is followed by a constant.
- A (simplified) constant is either an int or a string.
- We will denote arbitrary programs with `p,p’,…`

Operational semantics for the interpreter

$$\texttt{(p/S)} \rightarrow \texttt{(p'/S')}$$

Here `(p/S)` is a configuration where p is a program and S is a stack. We call these pairs configurations because we think in terms of an "abstract machine".

We can think about the stack as a list of values (denoted with `v`):

$$\texttt{v}_\texttt{n}\texttt{::}\ldots\texttt{::v}_\texttt{2}\texttt{::v}_\texttt{1}\texttt{::[]}$$

We say that from the configuration (p/S) we can step (or reduce) to the configuration (p'/S') in one step.

An example: `push const`

Informal description: Pushing `const` to the stack.

Formal description:

We use the variable `const` in both sides to represent the same const.

We use p and S because we want to consider all the possible situations.

`(push const;p/S)` $\rightarrow$ `(p/const::S)`

This says that the program we are processing has the form: `push const;p`

This says that the stack we are producing has the form: `const::S`

Another example: `pop`

Informal description: Remove the top value from the stack.

Formal description:

$$(\texttt{pop;p/v::S}) \rightarrow (\texttt{p/S})$$

We use p and S because we want to consider all the possible situations.

This says that the program we are processing has the form: `pop;p`

This says that the stack we are producing has the form: `S`

Does this rule capture all the possible situations?

Another example: `pop` from an empty stack

Informal description: If the stack is empty we stop and output Error.

Formal description:

$$(\texttt{pop;p}/\texttt{[]}) \longrightarrow \texttt{Error}$$

Another example: `add`

Informal description: Add consumes the top two values in the stack, and pushes their sum to the stack. If there are fewer then 2 values on the stack, terminate and report error. If two top values in the stack are not integers, terminate and report error.

Formal description:

What can we do for this case?

$$(\texttt{add};\texttt{p}/\texttt{v}_2\texttt{::v}_1\texttt{::S}) \longrightarrow (\texttt{p}/\texttt{v}_2\texttt{+v}_1\texttt{::S})$$

$$(\texttt{add};\texttt{p}/\texttt{v}_1\texttt{::[]}) \longrightarrow \texttt{Error}$$

$$(\texttt{add};\texttt{p}/\texttt{[]}) \longrightarrow \texttt{Error}$$

This + represents the addition of the two values

Revisiting the stack

First try: We can think about the stack as a list of values (denoted with `v`):

$$v_n :: \ldots :: v_2 :: v_1 :: []$$

What is the problem with this?

We don't distinguish values of different types.

Second try: We can think about the stack as a list of typed values (denoted with `type(v)`):

$$\mathtt{int}(v_n) :: \ldots :: \mathtt{name}\ (v_2) :: \mathtt{int}(v_1) :: []$$

Revisiting `add`

Informal description: Add consumes the top two values in the stack, and pushes their sum to the stack. If there are fewer then 2 values on the stack, terminate and report error. If two top values in the stack are not integers, terminate and report error.

Formal description:

$(\texttt{add;p/int}(v_2)\texttt{::int}(v_1)\texttt{::S}) \longrightarrow (\texttt{p/int}(v_2+v_1)\texttt{::S})$

$(\texttt{add;p/}v_1\texttt{::[]}) \longrightarrow \texttt{Error}$

$(\texttt{add;p/[]}) \longrightarrow \texttt{Error}$

$(\texttt{add;p/name}(v)\texttt{::S}) \longrightarrow \texttt{Error}$

$(\texttt{add;p/int}(v_1)\texttt{::name}(v_2)\texttt{::S}) \longrightarrow \texttt{Error}$

Are we done?