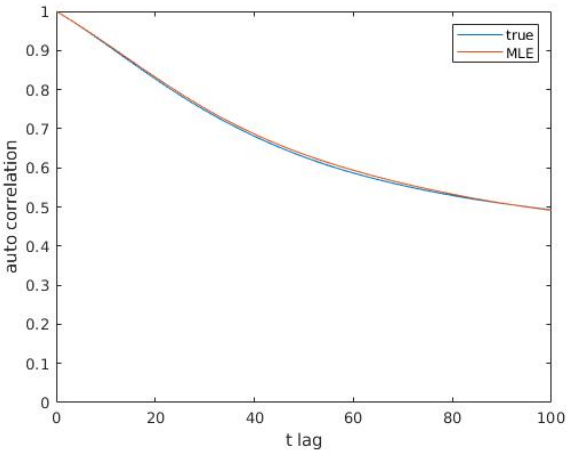
Assume that we have following reduced stochastic model:

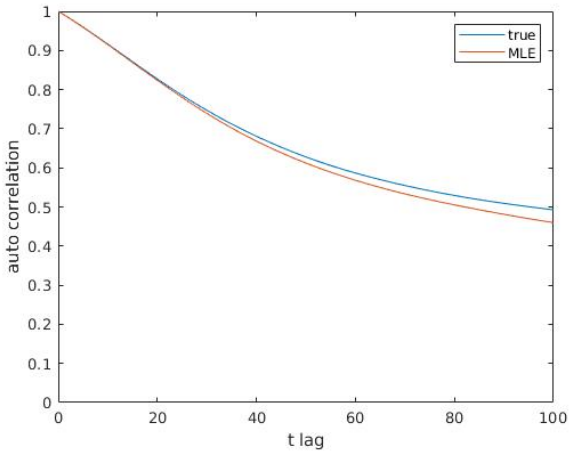
$$dx_i = \sum_{j=i-1}^{i+1} A_j x_j dt + \sum_{j,k=i-1}^{i+1} B_{j,k} x_j x_k dt + \sum_{j,k,l=i-1}^{i+1} C_{j,k,l} x_j x_k x_l dt + \sigma_1 dW_1 + \sigma_2 x_i dW_2 \tag{1}$$

(1) Try different initial set for the MLE, to set if it converges to same optimal result.

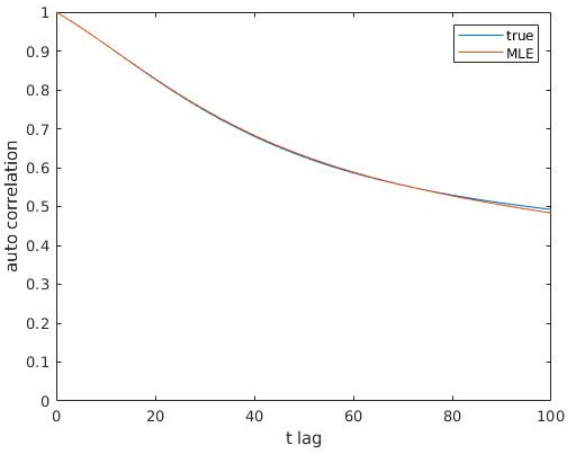
- user-defined guess Θ_1
- $\Theta_2 = 0.9 * [A; B; C; \sigma_1; \sigma_2]$
- $\Theta_3 = [0.25A; 3B; 3C; 3\sigma_1; 3\sigma_2]$
- $\Theta_4 = [3A; 0.25B; 0.25C; 3\sigma_1; 3\sigma_2]$
- $\Theta_5 \rightarrow N(0, 1)$



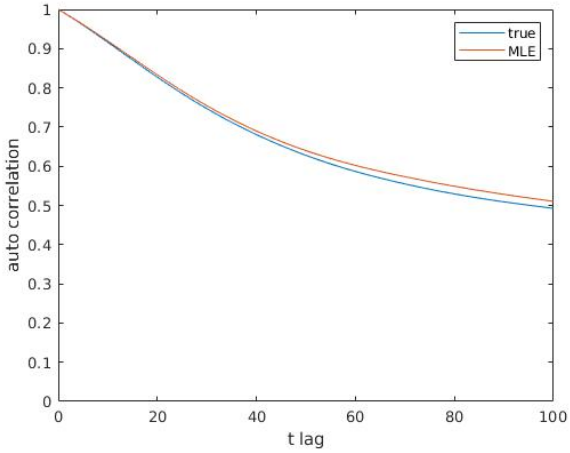
(a) user-defined guess Θ_1



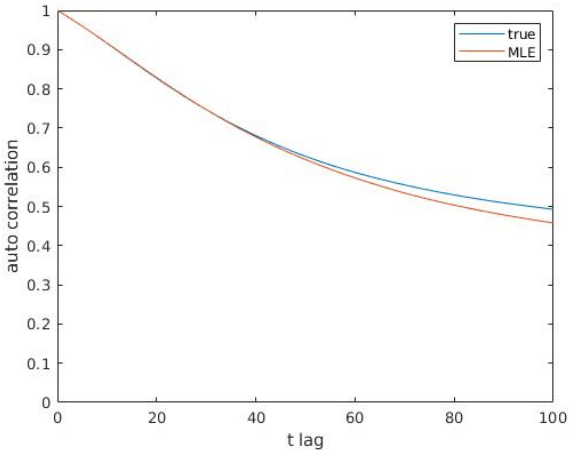
(b) $\Theta_2 = 0.9 * [A; B; C; \sigma_1; \sigma_2]$



(c) $\Theta_3 = [0.25A; 3B; 3C; 3\sigma_1; 3\sigma_2]$



(d) $\Theta_4 = [3A; 0.25B; 0.25C; 3\sigma_1; 3\sigma_2]$

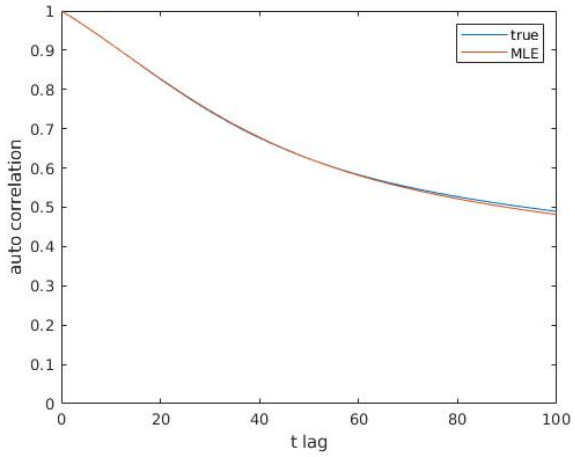


(e) $\Theta_5 \rightarrow N(0, 1)$

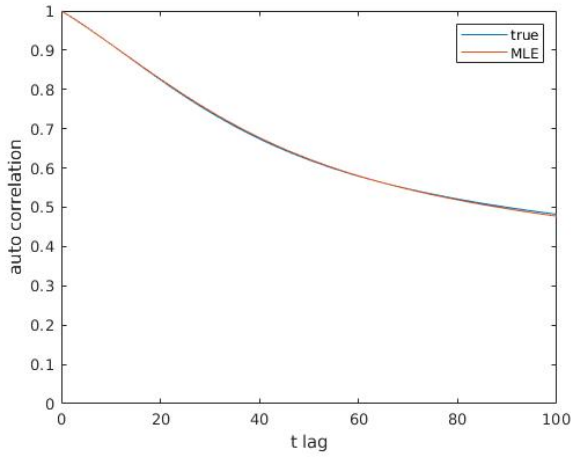
Figure 1: Different initiation for Estimation

(2) Does the estimation will be improved by increasing data size. Fix $\delta t = 0.01$, increase the trend.

- Trend=1000; Data size=10^5
- Trend=10000; Data size=10^6
- Trend=50000; Data size=5×10^6



(a) Trend=10000; Data size=10^6

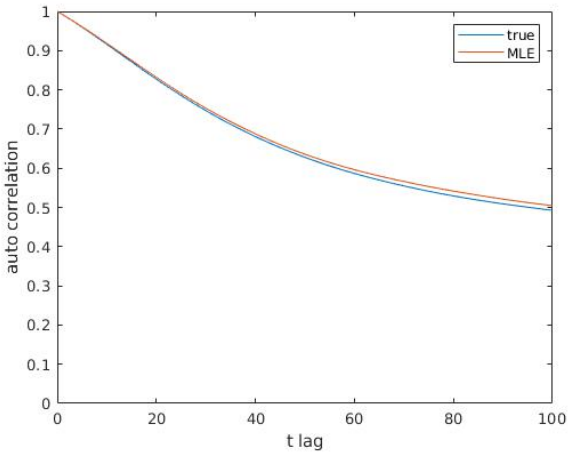


(b) Trend=50000; Data size=5×10^6

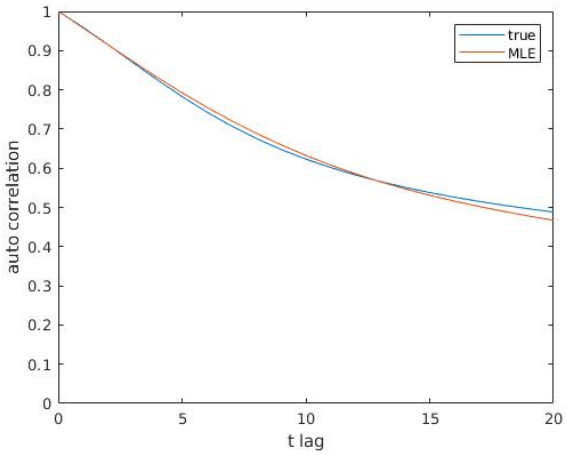
Figure 2: Different initiation for Estimation

(3) if we keep data size fixed $N = 10^5$, increase dt. How large dt could be, but still have good estimation.

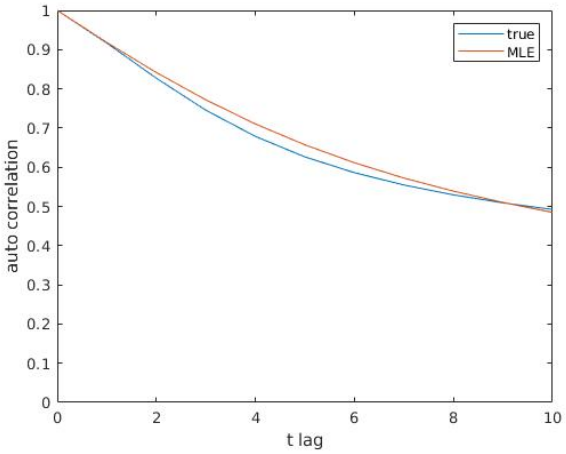
- dt=0.01
- dt=0.05
- dt=0.1
- dt=0.2



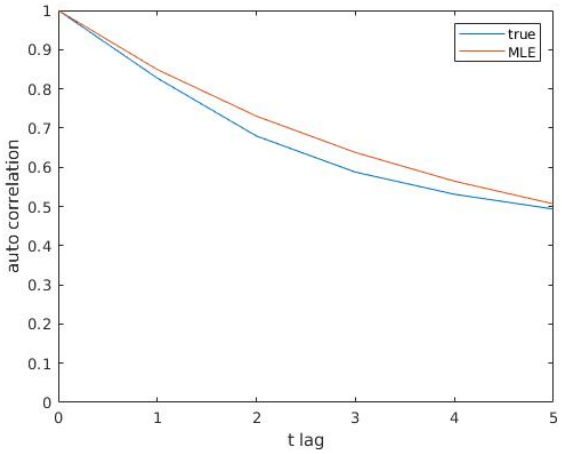
(a) Data size=10^5, dt=0.01



(b) Data size=10^5, dt=0.05



(c) Data size=10^5, dt=0.1



(d) Data size=10^5, dt=0.2

Figure 3: implement of dt