513HW1

Student name: Chenxu Wang

CWID: 10457625

Homework 1.1

Jerry and Susan have a joint bank account.

Jerry goes to the bank 20% of the days.

Susan goes there 30% of the days.

Together they are at the bank 8% of the days.

a. **Susan was at the bank last Monday. What' s the probability that Jerry was there too?**

   Assume that: A= { Jerry goes to the bank}=20%.

   B= { Susan goes to the bank} =30%, so P(AB)=8%,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.3} = 26.67\%$$

b. **Last Friday, Susan wasn' t at the bank. What' s the probability that Jerry was there?**

$$P\left(A|\overline{B}\right) = \frac{P\left(A \cap \overline{B}\right)}{P(\overline{B})} = \frac{P(A) - P(A \cap B)}{P(\overline{B})} = \frac{0.2 - 0.08}{0.7} = 17.14\%$$

c. **Last Wednesday at least one of them was at the bank. What is the probability that both of them were there?**

$$\frac{P(A\cap B)}{P(A\cup B)}=\frac{P(A\cap B)}{P(A)+P(B)-P(A\cap B)}=\frac{0.08}{0.2+0.3=0.08}$$

$$=19.05\%$$

Homework 1.2

Harold and Sharon are studying for a test.

Harold' s chances of getting a "B" are 80%. Sharon' s chances of getting a "B" are 90%.

The probability of at least one of them getting a "B" is 91%.

a. **What is the probability that only Harold gets a "B" ?**

Assume that: A= { Harold gets a "B" }= 80%,

B= { Sharon gets a "B" }=90%, so$P(A\cup B)=91\%$.

$$P(A\cap B)=P(A)+P(B)-P(A\cup B)=0.8+0.9-0.91=0.79$$

$$P\left(A\cap\overline{B}\right)=P(A)-P(A\cap B)=0.8-0.79=1\%$$

b. **What is the probability that only Sharon gets a "B" ?**

$$P\left(\overline{A}\cap B\right)=\ P(B)-P(A\cap B)=0.9-0.79=11\%$$

c. **What is the probability that both won' t get a "B" ?**

$$1-P(A\cup B)=1-91\%=9\%$$

Homework 1.3

Jerry and Susan have a joint bank account.

Jerry goes to the bank 20% of the days.

Susan goes there 30% of the days.

Together they are at the bank 8% of the days.

Are the events "Jerry is at the bank" and "Susan is at the bank" independent?

NO. According to HW 1.1: $P(A|B) = 26.67\%$, $P(A) = 20\%$.

then $P(A|B) \neq P(A)$.

Similarly, $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.08}{0.2} = 40\%$, $P(B) = 30\%$

$P(B|A) \neq P(B)$. So, the events "Jerry is at the bank" and "Susan is at the bank" are not independent.

Homework 1.4

You roll 2 dice.

a. **Are the events "the sum is 6" and "the second die shows 5" independent?**

Assume that A= { The sum is 6}, B= { The second die shows 5}.

		Die 2					
		1	2	3	4	5	6
Die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$P(A) = \frac{5}{6\times 6} = \frac{5}{36}$, $P(B) = \frac{1}{6}$,

$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$, so $P(A|B) \neq P(A)$.

Similarly, $P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{36}}{\frac{5}{36}} = \frac{1}{5}$, then $P(B|A) \neq P(B)$

So, the events "the sum is 6" and "the second die shows 5" are not independent.

b. **Are the events "the sum is 7" and "the first die shows 5" independent?**

Assume that A={ The sum is 7. }, B={ The first die shows 5.}. Then according to the figure above:

$P(A) = \frac{6}{36} = \frac{1}{6}$, $P(B) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{36}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{6}$, $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{6}$

then $P(A|B) = P(A)\ and\ P(B|A) = P(B)$, so the events "the sum is 7" and "the first die shows 5" are independent.

Homework 1.5

An oil company is considering drilling in either TX, AK and NJ. The company may operate in only one state. There is 60% chance the company will choose TX and 10% chance – NJ. There is 30% chance of finding oil in TX, 20% - in AK, and 10% - in NJ.

1. **What' s the probability of finding oil?**
2. **The company decided to drill and found oil. What is the probability that they drilled in TX?**

Let A_1, A_1, A_3 *respectively be that the Oil company decided to drill in {TX}, {AK}, {NJ}, and let* B_1, B_2, B_3, *be that the company found oil in {TX}, {AK}, {NJ}.*

Then $P(A_1) = 60\%, P(A_1) = 30\%, P(A_3) = 10\%$,

$P(B_1) = 30\%, P(B_2) = 20\%, P(B_3) = 10\%$,

1. The probability of finding oil is

$\sum_{i=1}^{3} P(A_i) * P(B_i) = P(A_1) * P(B_1) + P(A_1) * P(B_2) + P(A_3) * P(B_3) = 0.6 * 0.3 + 0.3 * 0.2 + 0.1 * 0.1 = 25\%$

2. The probability that they drilled in TX is

$$\frac{P(A_1) * P(B_1)}{\sum_{i=1}^{3} P(A_i) * P(B_i)} = \frac{0.6 * 0.3}{0.25} = 72\%$$

Homework 1.6

The following slide shows the survival status of individual passengers on the Titanic. Use this information to answer the following questions

- **What is the probability that a passenger did not survive?**

Let A be the probability that a passenger did not survive.

$$P(A) = \frac{P(passenger_{not\ survived}\ )}{P(passenger)} = \frac{1490}{2201} = 67.70\%$$

- **What is the probability that a passenger was staying in the first class?**

Let B be the probability that a passenger was staying in the first class.

$$P(B) = \frac{P\left(passenger_{first\ class}\ \right)}{P(passenger)} = \frac{325}{2201} = 14.77\%$$

- **<u>Given that a passenger survived</u>, what is the probability that the passenger was staying in the first class?**

Let C be the probability that the passenger was staying in the first class, given that a passenger survived.

$$P(passenger\ was\ in\ the\ first\ class | passenger\ survived)$$

$$= \frac{203}{711} = 28.55\%$$

- **Are survival and staying in the first class independent?**

The following probabilities can get from the figure,

$$P(survival) = \frac{711}{2201} = 32.30\%$$

$$P(people\ in\ the\ first\ class) = \frac{325}{2201} = 14.77\%$$

$$P(people\ in\ the\ first\ class|survival) = \frac{203}{711} = 28.55\%$$

$$P(survival|people\ in\ the\ first\ class) = \frac{203}{325} = 62.46\%$$

Then, $P(people\ in\ the\ first\ class|survival) \neq P(people\ in\ the\ first\ class)$

$$P(survival|people\ in\ the\ first\ class) \neq P(survival)$$

So, survival and staying the first class are not independent.

- **Given that a passenger survived, what is the probability that the passenger was staying in the first class and the passenger was a child?**

$$P(firstclass_{child}|passenger\ survived) = \frac{6}{711} = 0.844\%$$

- **Given that a passenger survived, what is the probability that the passenger was an adult?**

$$P(passenger\ was\ an\ adult|passenger\ survived) = \frac{654-57}{711}$$

$$= 83.97\%$$

- **<u>Given that a passenger survived</u>, are age and staying in the first class independent?**

Let P(C) be the adult, given a passenger survived, P(D) be people in the first class, given a passenger survived.

$$P(C) = P(adult|passenger\ survived) = \frac{654-57}{711}$$

$$= 83.97\%$$

$P(D) = P(\,first\ class|passenger\ survived) = \frac{203}{711} = 28.55\%$*, then* $P(C) \cdot P(D) = 23.97\%$*,*

$P(C \cap D) = \frac{197}{711} = 27.71\%$*.*

However, $P(C \cap D) \neq P(C) \cdot P(D)$*, so given that a passenger survived, age and staying in the first class independent are not independent.*

Survived

Age	Cabin: 1st	2nd	3rd	Crew	Sub Total
Adult	197	94	151	212	654
Child	6	24	27	-	57
Sub Total	**203**	**118**	**178**	**212**	**711**

Not Survived

Age	Cabin: 1st	2nd	3rd	Crew	Sub Total
Adult	122	167	476	673	1,438
Child			52		52
Sub Total	**122**	**167**	**528**	**673**	**1,490**

Total

Age	Cabin: 1st	2nd	3rd	Crew	Grand Total
Adult	319	261	627	885	2,092
Child	6	24	79		109
Grand Total	**325**	**285**	**706**	**885**	**2,201**