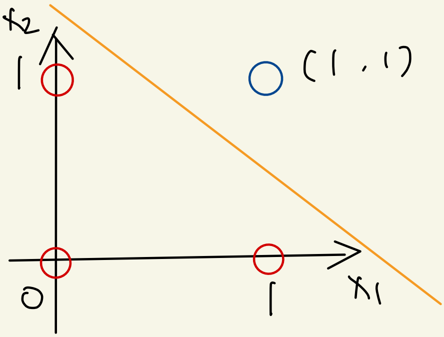
Problem 1. Perceptron Algorithm.

(1) We can plot NAND function on the 2D plane.



true.

false

$\therefore$ For NAND, negative class & positive class are linearly seperable.

(2). Since the initial line: $x_1 + x_2 - \frac{1}{2} = 0$

$\therefore$ we have $W = \begin{bmatrix} 1 \\ 1 \\ -\frac{1}{2} \end{bmatrix}$

Step 1. For. $P_1 \cdot (0, 1)$.

$$-1 \cdot W^T \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = -1 \cdot \begin{bmatrix} 1 & 1 & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = -\frac{1}{2} < 0$$

$\therefore$ P_1 is misclassified. on negative mistake

$$\therefore w' = w - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1-0 \\ 1-1 \\ -\frac{1}{2}-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -\frac{3}{2} \end{bmatrix}$$

$$\therefore w = w' = \begin{bmatrix} 1 \\ 0 \\ -\frac{3}{2} \end{bmatrix}$$

Step2. For $P_2(1,1)$

$$1 \cdot \begin{bmatrix} 1 & 0 & -\frac{3}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = -\frac{1}{2} < 0$$

$\therefore$ It is a positive mistake.

$$w' = w + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -\frac{1}{2} \end{bmatrix}$$

$$\therefore w = w'$$

Step 3. $P_3(1, 0)$

$$-1 \cdot \begin{bmatrix} 2 & 1 & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = -\frac{3}{2} < 0$$

$\therefore$ P_3 is misclassified on negative mistake.

$$\because w' = w - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2-1 \\ 1-0 \\ -\frac{1}{2}-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -\frac{3}{2} \end{bmatrix}$$

$\therefore$ $w = w'$

Step 4. $P_4(0, 0)$

$$-1 \cdot \begin{bmatrix} 1 & 1 & -\frac{3}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{3}{2} > 0$$

$\therefore$ P_4 is correctly classified.

Step 5. for P_1. $(0, 1)$

$$-1 \cdot \begin{bmatrix} 1 & 1 & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{2} > 0$$

$\therefore$ P_1 is correctly classified.

Step 6. for P_2 $(1, 1)$

$$1 \cdot \begin{bmatrix} 1 & 1 & -\frac{3}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{2} > 0$$

$\therefore$ P_2 is correctly classified.

Step 7. $P_3 : (1, 0)$.

$$-1 \cdot \begin{bmatrix} 1 & 1 & -\frac{3}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2} > 0$$

P_3 is correctly classified.

$\therefore$ P_1 P_2 P_3 P_4 are all correctly classified by the decision boundary $x_1 + x_2 - \frac{3}{2} = 0$. for boolean NAND function.