CO 351 course note

Chenxuan Wei

Sep 2022

Contents

1 Week 1-1

1. Graph
   A graph G is a pair(V, E) where V is finite set and E is a set of unordered pairs of elements of V

2. Walk, path, cycle
   A walk Q is a non-empty sequence of vertices v_i
   if all v_i distinct, it is a path
   if $n \geq 3$, $v_1 = v_n$ is is a cycle

3. Cut
   Let $G = (V, E), S \subseteq V$
   A S cut is $\delta(S) = \{uv \in E : u \in S, v \notin S\}$
   A v, t cut of G is $\delta(S)$ where $v \in S, t \notin S$

4. Theorem 1.1
   let s, t be vetices of gaph $G = (V, E)$
   there exists an s,t-path $\iff$ there is not st cut $\delta(S)$ where $\delta(S) = \emptyset$

5. Connected graph
   if $\forall u, v \in V$, there is a uv path, then it is connected
   Corollary1.2: a graph is connected $\iff$ there is no $S \subset V, S \neq \emptyset, \delta(S) = \emptyset$

6. subgraph
   G' is subgraph if $V(G') \subseteq V(G)$ and $E(G') \subseteq E(G)$
   if is spanning subgraph if $V(G) = V(G')$

7. Digraph
   a digraph $D = (N, A)$ where N is finte set and A is a set or ordered pair of element of N
   elements of N are called nodes and elements of A are arcs
   $d_{out}(u)$ and $d_{in}(u)$ are the number of arcs that out and in of node u

8. Cut in digraph
   Now cut have in and out too
   $\delta_{out}(S) = \{(u, v) \in A : u \in S, v \notin S\}$
   $\delta_{in}(S) = \{(u, v) \in A : u \notin S, v \in S\}$
   v, t-cut of D is the a set $\delta_{out}(S)$ where $v \in S, t \notin S$ (have order)

9. Theorem 1.3
   Let s, t be nodes of a digraph $D = (N, A)$, there exists an s,t-dipath in D $\iff$ there is no st-cut $\delta_{out}(S)$ that $= \emptyset$

10. Proposition 1.4
    Let Q be an s,t-diwalkt, if $s = t$ then Q can be decomposed into collection of dicycles
    if not if can be decomposed into a s,t-dipath and dicycles

11. Corollary 1.5
    If there exists a u,v-dipath and a vw-dipath, then there exists an u,w-dipath

12. Proposition 1.6
    If evey node of D has in degree, then there is a directed cycle

13. Incidence matrix
    a matrix that represent a digraph, where

    - Row = nodes
    - columns = arcs
    - tails-1, head +1
    - 0 everywhere else

14. Theorem 1.7
    Let $T = (V, E)$ be a tree

    - T+e has exactly one cycle C
    - Let e' be any edge of C then $T + e - e'$ is a tree

15. Lemma 1.8
    if there exists a uv path and uw path, there is uwpath

16. Lemma 1.9
    Let G be a connected with a cycle C, remove a edge will result a connected graph

17. Lemma 1.10
    In a tree T, any two vertices are connected by a unqiue path

2 Week 2-1

(a) Week duality
If x is a feasible solution of the primal and y is a feasbile solution of the dual, then $c^\intercal x \leq b^\intercal y$

(b) Duality theorem
if the primal has an optimal solution x and dual has an optimal solution y, then $c^\intercal x = b^\intercal y$

(c) Basic solution
Asssume A is $q * n$ and has full row rank, A basis B of A is a subset of q column of A such that $A_B = [Aj : j \in B]$ is non singular
Given a basis B, the unique solution x of Ax = b x_B is a basic colution and feasible if $x_B \geq 0$

(d) Theorem 1.15
if P is in SEF and has full row rank
if P has feasible solution, then it has a basic feasible solution
if it has optimal, then it has a optimal basic solution

(e) Complementary slackness
Let x y be feasible to P, D
$c^\intercal x = \sum_j c_j x_j \leq \sum_j (\sum_i a_{ij} y_i) x_j = \sum_i (\sum_j a_{ij} x_j) y_i \leq \sum_i b_i y_i = b^\intercal y$
if they are optimal

- $x_j = 0$ or $\sum_i a_{ij} y_i = c_j$
- $y_i = 0$ or $\sum_j a_{ij} x_j = b_i$

(f) Theorem
Suppose x and y are optimal solution of (P1) and (D1)
then they satisifies the complementary slackness condition