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Environmental Finance and Investments

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1 Introduction

During the last decade, climate change has been claimed to be the most serious environmental problem facing mankind. This current situation cannot be neglected: rather it deserves urgent action to curb the greenhouse gas (GHG) emissions from which this problem arises.

Paradoxically, at the international level, policy makers have been unable to successfully implement an agreement leading to a reduction in emissions. Up until now, the enacted agreement (the Kyoto protocol) has been unable to reverse the trend of rising CO_2 equivalent emissions, with an increase between 1990 and 2010 of 45 % which is deeply concerning for future generations.

The current situation generates fundamental questions that this book aims to answer: under which conditions could a market-based approach contribute to a decrease in emissions and therefore to sustainable economic growth? What are the new investment strategies that the Kyoto protocol and a scheme such as the European Emission Trading Scheme (EU ETS) should generate or promote? In the context of the European Union Emission Trading Scheme (EU ETS), what is the trade-off between production, technological changes and pollution?

The book is designed to give students and practitioners the knowledge and the theoretical tools to answer these and other more general questions in the context of the so-called environmental finance theory. This is a new research strand that investigates the economic, financial and managerial impacts of market-based environmental policies.

In order to better address these questions, Chap. 2 starts with a brief introduction on the issues at stake, giving some details on the causes of climate change, the considered scenarios and their foreseen impacts and the two broad strategies of mitigation and adaptation.

Chapter 3 gives the historical and institutional background behind climate markets, giving details on the important climate institutions, the type of players and the markets they operate in and the type of products at their disposal.

Chapter 4 follows with the economic source of the climate change issue, the existence of externalities and then proceeds to detail the three key instruments to

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overcome them: taxes, subsidies and permits. The chapter also touches upon the complex relationship between economic growth and the environment.

Chapter 4 focuses on optimal investment decisions in terms of technology changes and trading volume of emission rights. The chapter provides a comprehensive introduction to real option, a unique economic and financial framework to deal with the uncertain and irreversible nature of climate change decisions. Through a set of examples, the chapter details in a robust and theoretical setting the decision processes of industries and traders faced with the new carbon markets' specificities.

Finally, the last chapter overviews the main econometric studies that investigate energy prices, weather events, and macroeconomic shocks as determinants of allowance prices. Given the importance of understanding the emission permit price formation, the second half of this chapter provides an overview of deterministic and stochastic equilibrium permit price models currently investigated in the literature. Several market design institutions are considered. Banking and borrowing limitations, strategic trading interactions, and asymmetric information in the permit market are the most relevant. This part has been written together with Georg Gruell. This last chapter is more quantitative in nature and provide theoretical foundations to better understand the CO_2 emission price dynamics. This last section is of particular interest for Masters and PhD students or traders involved in the modeling of trading strategies.

By the end of this book, we hope that the reader have a clear understanding of how economic incentives can be used to implement environmental programs, and manage investment options (i.e., changes in production technology or construction of new facilities) under uncertainty in an emissions-trading regime. This understanding should encompass a familiarity with most market-based products (i.e., emission permits) that are currently issued and traded on exchanges and over-the-counter.

The reader will also be granted many insights into the conceptual understanding of the effects of such new markets. In particular, besides the usual aspects, he or she should understand the dimensions inherent in the above-mentioned trade-off between production, technological changes, and pollution. Enlarging production implies increasing offending gas emissions unless the latter is corrected by means of technological-abatement measures.

In fact, technological investment (and disinvestment) decisions in those particular sectors covered by the emissions programs have the typical essential characteristics of other investment decisions. They are partially or totally irreversible and may be delayed. But a new source of uncertainty/constraint is present: environmental compliance. The real-option approach, presented in Chap. 5, is precisely a decision-making tool adapted to investment decisions under uncertainty in the presence of constraints. The book is intended to show how this tool can be implemented in the environmental setting in order to make optimal decisions concerning production technology changes and the trade volume of emission rights. This approach takes into account the flexibility inherent in decision-making processes and the dynamic aspects of project selection in this particular setting. In particular, it aims to explain the basic principle that regulates tradable permit markets focusing in particular on the EU ETS.

Some basic models and case studies are presented and explained. The presence of strategic interactions among companies requires the investigation of optimal strategies by means of simple game-theoretical methods. Examples of the real option approach in this multiplayer setting are then presented.

The objective is for the reader to familiarize himself or herself with the decision making process in emission permit markets of regulated or non-regulated companies such as industries or banks and NGOs and to understand the empirical effects of environmental policies on this process.

Writing this book, our goal was to address in equal proportion future or current students enrolled in environmental finance studies and practitioners working either in regulated industries or in trading companies on carbon desks. With this in mind, this book was designed to be either read linearly, chapter after chapter or as a reference book, with a selection of relevant chapters.

To that purpose, each chapter has been designed as a stand-alone unit. For readers new to environmental finance, Chap. 6 could however benefit from a combined reading with Chap. 5 on real options and optimal investment.

Considering the large variety of practitioners and the large range of their exposure to carbon markets, it is difficult to recommend a unique sequence through this book. Non-technical readers new to the carbon world would certainly benefit from the overview offered in Chap. 2 followed by Chap. 3 on the history and institutions of carbon finance. Sections 5.2 to 5.5 of Chap. 5 should also provide a great introduction to the decision-making process of these new instruments. Technical practitioners or practitioners familiar with the topic should focus on Chaps. 4, 5 and 6 which, after an introduction to economic models, provide critical new insights on how to understand and model carbon trading in a scientific framework. Chapters 5 and 6, which represent the contributive core of this book, sum up for the first time in a book the forefront of research in carbon finance decision-making.

The structure and contents of this manual are the result of a semester long Master course on Environmental Finance offered at the University of Zurich since 2007 and several years of research in this field. It has benefited greatly from interactions with our students and with numerous practitioners. We are confident that students willing to complement their curriculum should read the book in its entirety and linearly. We hope this book may also prepare students to pursue further study in related areas, and equip them with the knowledge they would need to begin working professionally in the environmental finance industry, banks, brokerages and in related governmental (or non-governmental) organizations. Our goal with this book is to contribute to the creation of a "carbon curriculum" which could respond to the upcoming demand for graduate and post-graduate level positions.

Last but not least. We wish to thank many people with whom we have been interacting over these years, either in carbon finance industry of in academia, and led us to write this book: Olivier Bahn, Regina Betz, Santiago Moreno-Bromberg, Federica Buricco, Dallas Burtraw, Denny Ellerman, Raphael Calel, Frank Convery, Paolo Falbo, Sam Fankhauser, Max Fehr, Harrison Fell, Carolyn Fischer, Marc Gronwald, Cameron Hepburn, Beat Hintermann, Juri Hinz, Ruediger Kiesel, Reto Knutti, Jérémy Laurent-Lucchetti, Chuck Mason, Juan Pablo Montero, Jonas

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2 The Issue of Climate Change

2.1 The Causes of Climate Change

The warnings about global warming have been extremely clear for a long time. We are facing a global climate crisis. It is deepening. We are entering a period of consequences.

Al Gore

2.1.1 The Carbon-Temperature Conundrum

According to the Intergovernmental Panel on Climate Change (IPCC) which aggregates international research efforts on climate change, "global atmospheric concentrations of CO_2, CH_4 and N_2O have increased markedly as a results of human activities since 1750 and in 2005 exceeded by far the natural range of the last 650,000 years" IPCC (2007), with an increase of 70 % of the global greenhouse gases (GHG)[1] emissions due to human activities between the two periods.

The parts-per-million metric (ppm), that describes the concentration of carbon dioxide in the atmosphere, went from 280 ppm at the early stage of the industrialized revolution (around 1850) to more than 391 ppm in 2012, a value 39 % higher than the maximum level that was observed in the last 800,000 years (as shown in Fig. 2.1).[2]

In the meanwhile, the average global temperature has followed a strikingly similar pattern of increasing and accelerating warming. Eleven of the last twelve years (2000–2012) were among the warmest years in the instrumental record of global surface temperature (since 1850) with almost permanent occurrences of positive

[1] The recognized GHG are carbon dioxide CO_2, methane CH_4, nitrous oxide N_2O, hydrofluorocarbons HFC, perfluorocarbons PFC and sulfur hexafluoride SF_6. Despite its influence on climate due to its ability to absorb infrared radiation, water vapor is not listed among the GHG gases.

[2] Source: NOAA.gov.

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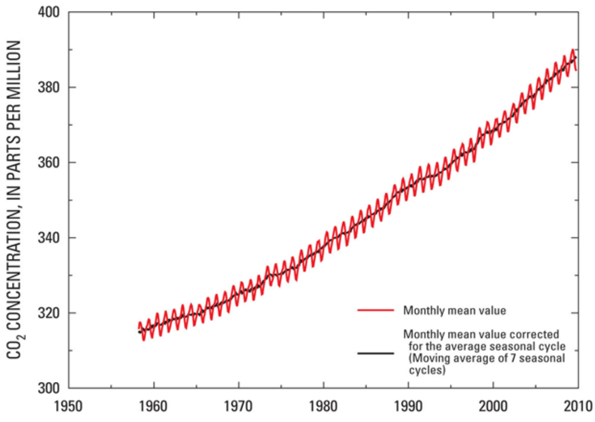

Fig. 2.1 Atmospheric carbon dioxide concentration measured at NOAA's Mauna Loa Observatory on Hawaii

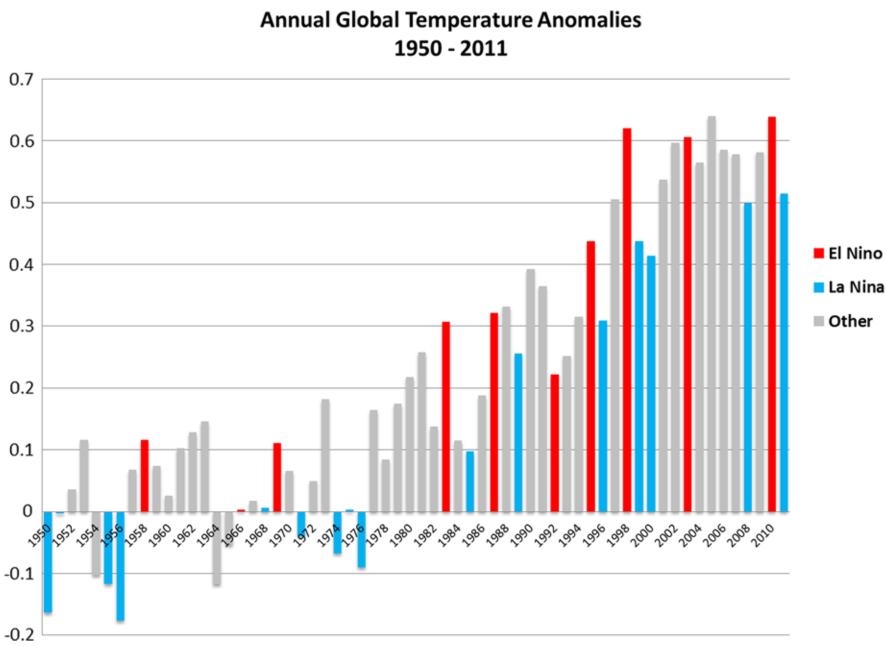

Fig. 2.2 Global temperature anomalies compared to long-term average (1950–2012). Source: NOAA.gov

temperature anomalies since 1980. Temperature anomalies from 1950 are presented in Fig. 2.2.

According to the fourth Assessment Report of the IPCC, the globally averaged surfaced temperature rose by approximately 0.7 °C between 1900 and 2009. For the last 50 years, global temperature rose at an average rate of about 0.13 °C per decade, almost twice as fast as the 0.07 °C per decade increase observed over previous periods.

The GHG Role in the Climate-Temperature Cycle

GHG are naturally present in the atmosphere and are not solely the results of anthropogenic activities. In the climate-temperature cycle, they play a fundamental role by absorbing and re-emitting solar radiation and causing the necessary warming of the earth's temperature. In the pre-industrial era, concentrations of GHG were stable but they rapidly increased afterwards (see table below: ppb means parts-per-billion).

Preindustrial levels of GHG concentration (source: IPCC)			
CO_2	CH_4	N_2O	CFC-12
280 ppm	700 ppb	270 ppb	0

Current levels of GHG concentration (2009)			
CO_2	CH_4	N_2O	CFC-12
387 ppm	1745 ppb	1045 ppb	533 ppm

As their concentrations in the atmosphere intensify, GHG act as a radiation trap that forces more energy to stay on surface and more heat to be produced, therefore causing global warming. In a general manner, each gas has a specific and complex cycle that involves interactions between the atmosphere, the terrestrial biosphere, the oceans, the sediments and the earth's crust. CO_2 for instance is produced, captured and dissolved through a short-to-medium-term carbon cycle involving carbon sources (fuel consumption, organic respiration, volcanic eruptions. . .) and sinks (forest uptake, sedimentation). Over the long term, CO_2 concentration in the atmosphere is subject to a decay rate permitted by the permanent sink role of oceans' sedimentation. Scientists tend to consider that it takes 55 years for emissions to be permanently removed from the atmosphere, with a half-life time of 38 years. Any attempt to reduce emissions has therefore to deal with the unavoidable inertia in the system and the existence of potential saturation limits of the natural sinks. Put differently, an efficient policy to reduce emissions, if it does not expect for slow practical results, could prove to be deceptive over the sort term.

Among the different GHG, CO_2 is the most important anthropogenic GHG responsible for global warming in terms of volume and absolute impact. According to the European EDGAR project on Global Emissions,[3] total global CO_2 emissions in 2011 had increased 3 % from 2010 to 34.0 billion tons and 45 % since 1990, the base year of the Kyoto Protocol. By comparison, global emissions in 1990 were

[3] http://edgar.jrc.ec.europa.eu.

Table 2.1 GWP values and lifetime, Source: IPCC AR4 report

Global Warming Potential (GWP)				
Gas	Lifetime	TH: 20 years	TH: 100 years	TH: 500 years
CO_2		1	1	1
CH_4	12	72	25	7.6
N_2O	114	289	298	153
HFC-23	270	12000	14800	12200
HFC-134a	14	3830	1430	435
SF_6	3200	16300	22800	32600

22.7 billion tons, an increase of 45 % on the 1970 level of 15.5 billion tons, as the consequence of increased use of fossil fuels and accelerated deforestations (while growth rates in CH_4 and N_2O emissions are mainly due to agriculture expansion).

In relative terms, gases have not the same effect on radiation retention: compared to CO_2, CH_4 and N_2O are present in less quantity in the atmosphere but have a greater capacity to create greenhouse effect. To compare their relative influence on global warming, scientists rely on a global warming potential (GWP) measurement instrument. The GWP is a relative scale that compares the greenhouse effect of a specific mass of gas to the same mass of CO_2 (having a normalized value of 1). The GWP measure accounts for the different decay rates of gases: a gas that generates relatively high greenhouse effect but that is dissolved rapidly has a high short-term GWP coefficient but a low long-term one. To account for this factor, GWP tables are given for specific time horizons (TH) (see Table 2.1).

While the relationship between carbon and temperature is no longer debated, it seems fair to acknowledge that the specific role of anthropogenic emissions in global warming is still the subject of specific scientific feuds, the most recent of them having occurred in November 2009.[4]

The so-called "climatoskeptics" are representing disparate groups of ideas and interests, gathered by their disbelief that climate change is an important issue to tackle, either because they consider that the scientific evidence remain flimsy and weakened by too much uncertainty or because they judge that other issues are much more important and effective and should be prioritized. Proponents of the first line of argumentations suggest that data are not entirely reliable, incapacitating any meaningful comparison of past patterns into current trends or that anthropogenic emissions are just a fraction of larger natural interactions not yet completely understood. However uncertain some results might be, it seems clear that a scientific consensus has now formed (embodied by the IPCC and other scientific institutions) and seems

[4]On November 19th 2009, the email server of the Climate Research Unit at the University of East Anglia (one of the most prominent research outlet on the issue of climate change) was hacked and email correspondences among its researchers were publicly disseminated. Dubbed "Climategate" by the press, the incident has revealed the bitter acrimony between climate change proponents and opponents and forced additional statements to reaffirm the existence of uncertainty in scientific evidence and results.

Table 2.2 Selection of modeling scenarios from the IPCC AR4 report (2007)

Illustration of the SRES storylines

Storyline	*Schematic structure of scenario (horizon 2090–2099)*	*Projected temp (°C)*	*Sea level rise (cm)*
A1	Future world of very rapid economic growth, global population that peaks in mi-century and decline thereafter and rapid introduction of new and more efficient technologies	1.4–6.4	20–59
A2	Very heterogeneous world with continuously increasing global population and regionally oriented economic growth that is more fragmented and slower than in other storylines	2.0–5.4	23–51
B1	Convergent world with the same global population as in the A1 storyline but with rapid changes in economic structures toward a service and information economy, with reductions in material intensity, and the introduction of clean and resource-efficient technologies	1.1–2.9	18–38
B2	World in which the emphasis is on local solutions to economic, social, and environmental sustainability, with continuously increasing population (lower than A2) and intermediate economic development	1.4–3.8	20–43

largely backed by the most recent measurements. Worryingly, those measurements tend to support a rather pessimistic prediction for climate change.

2.1.2 Global Warming Scenarios and Mitigation Strategies

For the purpose of policy decision-making and scientific discussions, the IPCC defined for its third assessment[5] (TAR, 2000) a set of scenarios exploring future development for GHG emissions. The IPCC improved on them for its fourth assessment,[6] despite some concern that the recent evolutions in emissions from the 2000–2007 period were not fully taken into account.

The scenarios, starting in 2000, differ by their storylines (A1, A2, B1, B2), which represent different demographic, social, economic, technological and environmental developments that diverge in increasingly irreversible ways. An overview is given in Table 2.2.

For each storyline, modeling teams of economists and scientists have computed sets of scenarios using integrated assessment models (IAM). Storylines A2, B1 and B2 have each one set of scenarios while the A1 storyline has three different sets that depend on alternative development of energy technologies: A1FI (fossil intensive), A1T (predominantly non-fossil) and A1B (balanced across energy sources).

[5] Special Report on Emissions Scenarios (SRES).

[6] by shifting the time horizon from 2100 to 2090–2099 and changing the method for the inclusion of uncertainties.

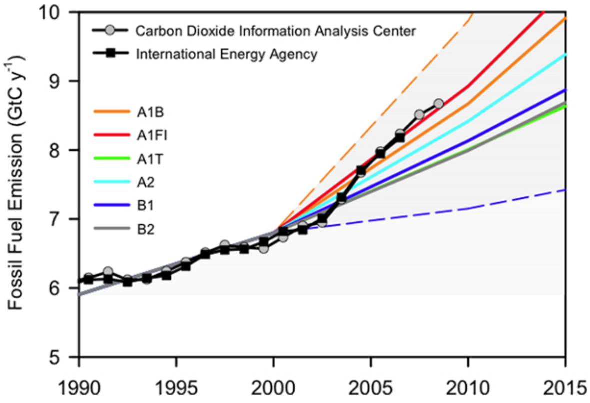

Fig. 2.3 Fossil fuel emissions: actual emissions compared to IPCC modeled projections. Source: Global Carbon Project (2010)

Scenarios are not ranked and are not attached to probability of occurrence. However, using the scenarios of the third assessment for the completion of the fourth assessment gave the advantage of backtesting the proposed models. By the end of 2009, it appears that the world is closely enough following the path of the A1FI model, with rapid economic growth and heavy reliance on fossil energies for countries such as China and India (Fig. 2.3).

The recent context of financial crises has had a modest disruptive impact on this trend, with an estimated −1.3 % reduction of fossil fuel emissions for the year 2009,[7] immediately absorbed by a rapid increase of 5.9 % per year in 2010 (due for a large part to a rise in energy inefficiency/carbon intensity).[8] This was expected to be the case, the reduction coming from a contraction of the economy but not from a change in the energy mix, see Fig. 2.4.

In order to assess mitigation costs, the IPCC has computed simulations[9] for stabilization scenarios around six specific CO_2-equivalent[10] concentration levels in the atmosphere, acknowledging that attempt to reduce concentration to 445–490 ppm CO_2-eq would require negative emissions for several decades (that is, higher uptakes than emissions). The different scenarios are presented in Table 2.3.

According to the sensitivity projections of the IPCC, any commitment to limit the global average temperature increase within a +2 °C limit would force to stabilize CO_2 concentration around 350–400 ppm. In December 2009, the latest concentration was estimated at 387 ppm, slowly increasing from the 375 ppm concentration recorded in 2005.

[7]Source: Global Carbon Project, Carbon Budget 2010.

[8]Carbon intensity is the amount of carbon (in terms of weight) emitted per unit of energy consumed.

[9]These simulations are to be amended for the fifth Assessment Report which will be issued in 2013 (for the Physical Science Basis) and 2014 (for the Impacts, Adaptation and Vulnerability and the Mitigation of Climate Change reports).

[10]Carbon dioxide equivalency is a quantity that describes, for a given mixture and amount of greenhouse gas, the amount of CO_2 that would have the same global warming potential. It is measured over a specified timescale, generally, 100 years.

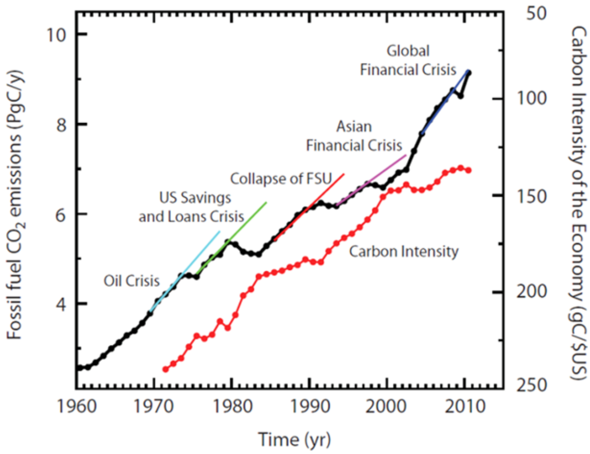

Fig. 2.4 Impact of recent financial crises on CO_2 emissions. Source: Global Carbon Project (2010)

Table 2.3 Concentration stabilization scenarios and impact on temperature increase and sea level rise. Source: IPCC

Scenario	CO_2 concentration at stabilization	CO_2-e concentration at stabilization	Change in global CO_2 emissions in 2050 (% of 2000 emissions)	Global average temperature increase (in °C)	Global average sea level rise (in m)
I	350–400	445–490	−85 to −50	**2.0–2.4**	0.4–1.4
II	400–440	490–535	−60 to −30	**2.4–2.8**	0.5–1.7
III	440–485	535–590	−30 to +5	**2.8–3.2**	0.6–1.9
IV	485–570	590–710	+10 to +60	**3.2–4.0**	0.6–2.4
V	570–660	710–855	+25 to +85	**4.0–4.9**	0.8–2.9
VI	660–790	855–1130	+90 to +140	**4.9–6.1**	1.0–3.7

A targeted concentration of 445–490 ppm CO_2-eq would represent a stabilized increase of temperature around +2–2.4 °C above the pre-industrial level. In the current context of increasing emissions, achieving a 350–400 ppm stabilization level will require a set of mitigation measure with different costs, areas of applicability and timing. IPCC has introduced in the stabilization scenarios a set of usable mitigation strategies, with increasing marginal cost: technology efficiency improvement, source of energy switching (ex: from coal to natural gas), development of renewable energies, demand reduction and carbon capture and storage. In all its storyline (A1, A2, B1, B2), the IPCC has included elements of mitigation (technology change and energy efficiency) that ensure emission reduction, up to 80 % compared to a “frozen 1990 technology” baseline.[11]

[11] Among other attempts to define mitigation strategies and abatement supply curves, the International Energy Agency (IEA) regularly publishes the Energy Technology Perspective Reports that include a detailed roadmap for energy technology, from both the supply and the demand sides.

2.1.3 The Environmental and Economic Impacts

Until damages are elicited and adaptation cost monetized, the urgency of taking measure against global warming remains for many elusive. In a traditional cost-benefit analysis, investing in mitigation makes sense only if it reduces damages and impacts up to the point where the local marginal costs of abatement and local marginal damages are equal. However, the issue of attaching costs to global warming and assessing impacts is a complex task that needs to overcome several hurdles: (i) regional and sectoral implications of a public good problem (externalities), (ii) presence of high degree of uncertainty, (iii) dynamic aspects and (iv) ethical issues.

Climate change damages are complex to precisely assess in a cost-benefit analysis because the main causes of the damages are not generated locally but are the results of collective externalities (in broad terms, the climate is a public good). Since some countries or regions will disproportionately suffer from the impacts in comparison with their emissions, it may prove difficult for them to define mitigation and adaptation strategies and adjust precisely to the severity of the damages, since they control only a limited share of the collective responsibility. This aspect leads to the second main difficulty, the importance of uncertainties in the impact valuation.

As Tol (2002) reminds us, *"the uncertainty about the impact of climate change is known to be large, because, climate change itself is rather uncertain in its magnitude and regional pattern, research on the impact of climate change needs substantial improvement, and the bulk of climate change will occur in a distant future"*.

Damages and costs are highly regional/sectoral and impose long-term, costly bottom-up studies with the extra difficulty to make them comparable (same set of baseline hypotheses, impacts defined on compatible storylines). This would require a concerted effort under the supervision of a centralized body, which has been the role played by the IPCC so far. However, its impact assessment does not provide clear monetized impacts, forcing further studies to rely on disconnected regional and sectoral assessments or to come up with ad hoc assumptions.

Damages and impacts are also dynamic in nature and susceptible of reinforcing loops or switching periods of positive and negative effects. To be able to compare across periods, a sound cost assessment requires the definition of a reliable and sensible discount rate: if the discount rate is low (down to zero), the model would almost imply an equal impact sensitivity across periods and generations, the view of the proponents of intergenerational equity. On the other hand, if the discount rate is high (or close to the market rates observed before the financial crisis), the damage assessment would mostly limit itself to the view of the current generation.

Most studies conduct impact analysis on a subset of the global regional/sectoral matrix, with important researches targeted towards agriculture, forestry and costal economic sectors. In a comprehensive effort, Tol (2005b) has summed up many

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Worth also noting but limited to the supply side, the consultancy McKinsey has computed an often-used abatement cost curve that includes all technologies available for less than the carbon permit's price limit of the first EU ETS phase. (A cost curve for greenhouse gas reduction, McKinsey Quarterly, 2007.)

Table 2.4 Estimates of the regional impacts of climate change as % of GDP (Source: Tol, 2005)

Estimates of the regional impacts of climate change (Horizon = 2100)				
	Studies			
	Pearce et al.	Mendelsohn et al.	Nordhaus and Boyer	Tol
Temperature increase (°C)	2.5 °C	2.5 °C	2.5 °C	1 °C
North America	−1.5 %			+3.4 %
USA	−1 % to −1.5 %	+0.3 %	−0.5 %	
OCDE Europe	−1.3 %			+3.7 %
EU	−1.4 %		−2.8 %	
OCDE Pacific	−1.4 % to −1.8 %			+1 %
Japan		−0.1 %	−0.5 %	
Eastern Europe/Former USSR	+0.3 %			+2 %
Eastern Europe			−0.7 %	
Middle East	−4.1 %		−2 %	+1.1 %
Latin America	−4.3 %			−0.1 %
Latin America	−4.3 %			−0.1 %
Brazil		−1.4 %		
South and Southeast Asia	−8.6 %			−1.7 %
India		−2 %	−4.9 %	
China	−4.7 % to −5.2 %	+1.8 %	−0.2 %	+2.1 %
Africa	−8.7 %		−3.9 %	−4.1 %

of those papers, both in their static and dynamic effects (see Table 2.4 where the estimates are expressed as per cent of Gross Domestic Product).[12]

From those estimates, it is apparent that the burden of climate change will not be borne equally across regions. One the hand, some countries should benefit from a temperature increase, which will positively mitigate the harsh conditions of their winters and increase economic outputs (for instance, Russia and Canada should experience positive GDP growth). Unfortunately, on the other hand, least developed countries (Africa, Southeast Asia) are predicted to be the ones suffering the most from global warming with an expected impact on GDP ranging from −3.9 % to −8.6 %.

Why Is the Developing World Especially Affected?

- The livelihood of the poor is known to be significantly dependent on natural resources.

[12] Pearce et al. (1996), Mendelsohn et al. (1998), Nordhaus and Boyer (2000), Tol (1999).

- When natural disasters destroy capital (be it machinery, cattle, or otherwise), the poor typically lack access to financial resources to restore the level of capital to its pre-disaster level.
- Areas of poverty are often located in places that are more susceptible to high variability in temperature and rainfall, such as hilly and steep slopes, and flood plains.
- Richer societies are more resilient societies as a result of the positive correlation between income and education, openness, financial development, and greater institutional capacity.
- In the words of the World Bank (Margulis and Narain 2009): *"developing countries face not only a deficit in adapting to current climate variation, let alone future climate change, but also deficits in providing education, housing, health, and other services. Thus, many countries face a more general "development deficit", of which the part related to climate events is termed the "adaptation deficit".*

The recognition of the partial ineluctability of global warming combined with the slow deployment of mitigation strategies have forced economists and policy makers to reconsider the importance of adaptation as a complementary measure to climate mitigation.

While adaptation is defined as the set of activities conducted to *offset* partially or in totality the averse impacts of damages due to global warming, mitigation covers the strategies to *reduce* the amount of GHG emissions. Adaptation can be divided between anticipative (*ex ante*) and reactive (*ex post*) strategies. For instance, the selection (and R&D) of drought-resistant crops prior to explicit climatic changes can be considered as a proactive measure, while emergency vaccinations in case of climate-related pandemics belong to reactive adaptation. In practice however, *"the distinction between anticipative and reactive adaptation is intuitively clear, but difficult to delineate with precision in a dynamic setting"* (Lecocq and Shalizi 2007).

Strengths and Weaknesses of Adaptation Measures

Strengths

1. Adaptation is by definition local/regional/sector-based: adaptation measures in effect privatize policies against climate changes by largely limiting the benefits of adaptation to those having invested in it.
2. Adaptation avoids the free-riding problem traditionally associated with mitigation and does not require concerted and simultaneous actions, fostering the advancement of regional or local projects.
3. Adaptation projects are often less costly and easier to set up.

4. Adaptation provides short-term protection against early damages.
5. For developing countries without mitigation issues, it represents the main set of strategies (ex: Africa).
6. Adaptation should be able to deal with extreme events.

Weaknesses

1. Larger uncertainties for anticipative projects.
2. Absence of a common performance indicator to compare the results of different adaptation projects.
3. Could lure countries with large emissions to give up on their mitigation projects, especially if they have short-term views (or equivalently, high discount rates).
4. Creating private goods and benefits, adaptation can foster or reinforce inequalities.
5. Projects are easily mixed with development targets already in place, impeding access to additional resources (ex: The Copenhagen Green Climate Fund).

Strengths and Weaknesses of Mitigation Measures

Strengths

1. Mitigation is the only long-term solution to reduce the anthropogenic part of climate change.
2. In general, mitigation strategies and efficiencies have been more studied and present less uncertainty about their benefits. As the IPCC notes, uncertainties are much larger at the local/sectoral level than at the global level.
3. Mitigation will have global benefits that are not excludable (equity value).
4. In negotiations, mitigation strategies could be different but they have the same performance metric which allows for comparisons and allocations.

Weaknesses

1. Mitigation is a public good: non-excludable and non-rivalrous. It creates agency problems, either through free-riders or barriers to collective action.
2. It involves international negotiations that are extremely difficult to manage with elusive search for consensus.
3. It is a long-term process that has no impact on short-term damages.
4. For numerous developing countries with little emissions but large exposures to impacts, it does not represent an effective policy.

Table 2.5 Estimates of adaptation costs in developing countries for 2010–2015 (Source: IIED, 2009)

Adaptation costs in developing countries for 2010–2015 (Source: IIED, 2009)	
Source	US$ billion per year
World Bank (2009)	75–100
UNFCCC (2007)	27–66
UNDP (2007)	86–109
Oxfam (2007)	more than 50
Stern (2006)	4–37

It is now clear that adaptation policies will have to be put in place, both as a way to cope with dramatic and extreme events and as a way to adapt to permanent changes of our environments. However, adaptation cost assessments are still lagging behind damage impacts studies and still lack a homogeneous corpus of evidence and measures.

Considering this limited amount of research conducted on adaptation strategies, it remains unclear how and to what extent adaptation and mitigation strategies interact with each other in a dynamic setting.

A classical example is Air-Conditioning: A/C systems are adaptation measures deployed in building to limit effect of global warming but at the same time, they increase energy consumption and the potential release of GHG. In this simple case, the correlation would be *negative* between the two measures. At the other end of the correlation spectrum, *positive* correlation may be found in deforestation finance (REDD), for which mitigation measures (decreased deforestation) provide adaptive instruments against floods and landslides.

While being cheaper in the short term than the range of available mitigation strategies, adaptation will have nonetheless important costs. Table 2.5 shows a range of estimates covering adaptation costs in developing countries.

In conclusion, it seems clear that an a optimal policy against climate changes and their impacts will have to combine both mitigation and adaptation. While adaptation are easier to implement, bear less uncertainties and can be privatized (partially avoiding free-riding effects), mitigation strategies are the only capable to reduce GHGs in the atmosphere in order to reestablish a viable long-term CO_2 concentration. Simply relying on adaptation measures could increase risk of reaching tipping points while being more and more costly to keep up with increased damages.

The Rise of the Emission Markets

3

3.1 The CO_2 Emission Market: History and Institutions

3.1.1 From the UNFCCC to the Kyoto Protocol

The UNFCCC First scientific evidence of human activity affecting the world's climate emerged during the World Climate Conference (WCC) held on February 1979 in Geneva. For the first time, a large group of politicians were concerned about human interferences with the climate and the environment. As a result of the global attention for climate change, the United Nations Environmental Program and the World Meteorological Organization established the International Panel on Climate Change (IPCC) in 1988.

The key task of the IPCC was to assemble and assess scientific information on the impact of the human carbon footprint. In 1990, the IPCC issued its First Assessment Report which reflected the views of 400 scientists on the threats posed by global warming. The report stated that global warming was a real problem caused by humans. Furthermore, the IPCC urged that the international community take measures to curb GHG emissions.

As a result, throughout the second meeting of the WCC held in Geneva later that year, the IPCC called for an international treaty to tackle climate change. For this purpose, the IPCC formed the Intergovernmental Negotiation Committee (INC). The INC met first in February 1991 when its representatives discussed and established the United Nations Framework Convention on Climate Change (UNFCCC).

The UNFCCC sets an overall framework for intergovernmental efforts to tackle the challenge posed by climate change and entered into force in March 1994. By that date, it was signed by 166 countries and only ten years later that number rose to 188 countries.[1] This almost worldwide membership makes the Convention one of the most universally supported international agreements on the environment.

[1] We refer to UNFCCC web site for an updated and detailed list. As of August 2011, UNFCCC has 194 parties.

M. Chesney et al., *Environmental Finance and Investments*,
Springer Texts in Business and Economics, DOI 10.1007/978-3-642-36623-9_3,

Table 3.1 List of Annex I Parties to the Convention (Source: UNFCCC)

Countries			
Australia	Austria	Belarus	Belgium
Bulgaria	Canada	Croatia	Czech Republic
Denmark	Estonia	European Union	Finland
France	Germany	Greece	Hungary
Iceland	Ireland	Italy	Japan
Latvia	Liechtenstein	Lithuania	Luxembourg
Monaco	Netherlands	New Zealand	Norway
Poland	Portugal	Romania	Russian Federation
Slovakia	Slovenia	Spain	Sweden
Switzerland	Turkey	Ukraine	UK and Northern Ireland
United States of America			

Under the agreement, concerned parties claimed that a substantial rise of GHG emissions had the ability to affect terrestrial and maritime ecosystems, resulting in an average raising of the temperature of the earth's surface and atmosphere. The ultimate objective of the Convention was therefore to stabilize GHG concentration in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system.[2] Furthermore, all members would have to promote sustainable technologies to grant an environmentally less harmful economic growth.

In practice, the Convention was only a proposal with no time constraint or mandatory emission cap for the parties. Article 4 of the Convention simply suggests that the parties should lower their emissions based on their levels of 1990. Additionally, the treaty holds provisions for updates and leaves to the Conference of Parties (COP) the task to set mandatory goals under the legal form of protocols. The Convention divides country members into three groups.

The first group is comprised of the Annex I parties. It includes industrialized countries which were members of the Organization for Economic Cooperation and Development (OECD) in 1992[3] and countries with Economies in Transition (EIT), that is, the Russian Federation and several other Central and Eastern European countries. Table 3.1 lists Annex I parties under the Convention.

These countries were asked to adopt climate change measures with the aim of reducing their GHG emissions to 1990 levels. However, no legally binding targets are set by the Convention. The EIT countries are granted some flexibility in implementing commitments as they are allowed to choose another year than 1990 as their base year.

[2] UNFCCC—Article 2.

[3] The European Union membership represented initially the 15 States who were EU members in 1997 (European Commission) when the Kyoto Protocol was adopted. It represents today the 27 country members of the European Union.

The second group is called Annex II parties. This group consists of the Annex I members without the EIT countries. Members of this group can help developing countries finance emission reduction activities. The scope of such an opportunity is twofold: fighting the adverse effects of climate change in other regions and enhance the transfer of environmental-friendly technology to EIT and developing countries.

The third and last group, called the Non-Annex I countries, consists of developing countries. These countries have no commitment to reduce emissions under the Convention and therefore have no part in the legally binding targets. The reason behind this exclusion is that it was considered at the time that climate change problems were essentially caused by the industrialization of developed countries and that the contributing share of developing countries was minimal (a fact which is no longer true, due to the high emissions profiles of countries like China and India).

To ensure an appropriate level of coordination, the convention set up an institutional body, the UNFCCC secretariat that has administrative responsibilities on behalf of the UNFCCC and its protocols (i.e., the Kyoto protocol). It is hosted in Bonn (Germany) since 1996. The secretariat is staffed by international civil servants and supports all institutions involved in the climate change process, particularly the COP (see next section), the subsidiary bodies and their bureaus.

The UNFCCC secretariat has several ongoing missions: apart from establishing the first national greenhouse gas inventories used to set 1990 levels, the secretariat has the recurring tasks of (i) compiling the annual GHG inventory data,[4] (ii) coordinating the in-depth reviews of Annex I party national communications and (iii) preparing the official documents for the COP and the subsidiary bodies.

The Conference of the Parties (COP) and the Subsidiary Bodies The supreme body of the Convention is the Conference of the Parties (COP), which meets at least once a year to assess efforts and improvements to tackle climate change. COP regularly reports the progresses towards the implementation of the Convention and makes public all policy instruments adopted under the Convention. Furthermore, the COP takes all necessary decisions to promote the effective implementation of the Convention.[5] Beside the COP, two subsidiary bodies have been introduced with the aim to steer preparatory work for the COP:

- The Subsidiary Body for Scientific and Technological Advice (SBSTA), defined under Article 9 of the Convention, provides the COP with scientific and technological matters. The SBSTA identifies innovative technologies and provides assessments of the state of scientific knowledge related to climate change. It promotes the transfer of environmentally-friendly technologies and also carries

[4]The UNFCCC Reporting Guidelines on Annual Inventories require Parties included in Annex I to the Convention (Annex I Parties), by 15 April each year, to provide annual national GHG inventories covering emissions and removals of direct GHGs (CO_2, CH_4, N_2O, HFCs, PFCs and SF_6) from six sectors (Energy, Industrial processes, Solvents, Agriculture, LULUCF, Waste), and for all years from the base year or period to the most recent year.

[5]We refer here to the UNFCCC Article 7, al. 2.

out methodological work in specific areas (LULUCF, REDD, HFC) as well as adaptation and vulnerability.

- The Subsidiary Body for Implementation (SBI), defined under Article 10 of the Convention, reports to the COP about the overall effectiveness of the implementation of the Convention. The SBI examines national communications and emission inventories submitted by parties. Furthermore, the SBI assists the COP during the preparation of its decisions with reviews of the environmental state of the art.

The Kyoto Protocol (KP) At the first conference of the UNFCCC, the negotiations for a Protocol with binding targets started. The Kyoto Protocol (KP) is the result of intensive negotiations at the third meeting held by the COP in 1997 in Kyoto, Japan. The KP commits Annex I countries to individual, legally binding targets to limit or reduce their GHG emissions.[6] The first phase of the KP entered into force in February 2005 due to Russia's ratification with a first commitment period that started in 2008 and ended in December 2012. As specified by Article 25 of the KP, the criteria to make the Protocol active is that at least 55 parties have the Protocol signed. Or, under a different measure, that at least 55 % of the total worldwide GHG emissions be covered. With Russian ratification, both criteria have been fulfilled. At the time of writing, 191 countries ratified the Protocol.

Under the KP, countries are separated into two different groups: those who are committed to binding targets, that is, developed countries, and those who do not face mitigation targets, referred to as non-Annex I countries. Out of 191 countries, only 40 countries plus the European Union (EU) are referred to as Annex I countries (see Table 3.2). These countries alone accounted for 61 % of GHG emissions in 2009. As of December 2009, 40 of the 41 Annex I countries have ratified the Protocol (the notable exception being the United States of America). However, Canada announced in December 2011 that he would not comply with its obligations under the first phase of the KP and withdraw from it, hereby limiting the scope and effectiveness of the KP.[7]

Based on the claim that developed countries are largely responsible for the past GHG emissions, the KP places logically a heavier burden on Annex I countries. More precisely, these countries are committed to control that their GHG emissions do not increase above a certain percentage of a specified base year by 2012[8] (see Table 3.2).

Positive targets were set for countries that had limited emissions in 1990 and were currently experiencing an expansion phase of their economies.

[6]The KP covers six main GHG: Carbon dioxide (CO_2), Methane (CH_4), Nitrous oxide (N_2O), Hydrofluorocarbons (HFC), Perfluorocarbons (PFC), and Sulphur hexafluoride (SF_6).

[7]The same year, Japan, Canada and Russia announced that they would not take on further Kyoto targets post-2012.

[8]The targets differ per country; see Kyoto Protocol Article 3, Sects. 5–8 for a more details.

Table 3.2 Quantified emission limitation as defined in Annex B of KP (Source: UNFCCC)

Annex I parties	Emission reduction or limitation (base year or period inscribed in Annex B)
Austria, Belgium, Bulgaria, Czech Republic, Denmark, Estonia, European Community,[a] Finland, France, Germany, Greece, Ireland, Italy, Latvia, Liechtenstein, Lithuania, Luxembourg, Monaco, Netherlands, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, UK and Northern Ireland	−8 %
United States of America	−7 %
Canada, Hungary, Japan, Poland	−6 %
Croatia	−5 %
New Zealand, Russian Federation, Ukraine	0 %
Norway	+1 %
Australia	+8 %
Iceland	+10 %

[a]Among the European Community, EU-15 has a common commitment to reduce emissions by 8 % between 2008 and 2012, while the EU-27 does not have a common Kyoto target

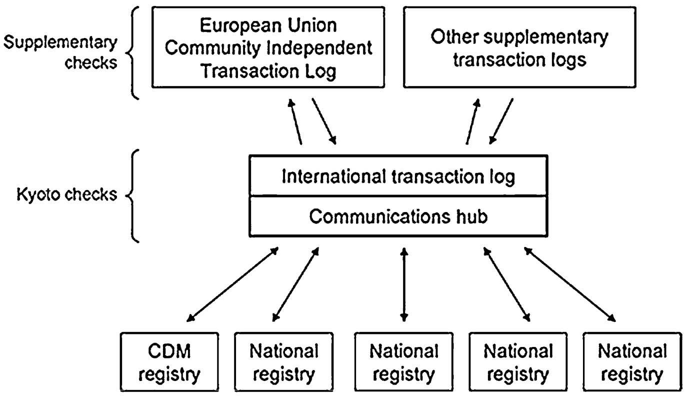

Fig. 3.1 Source: UNFCCC

To enforce their commitment, countries have to set up accounting registries and fulfill a review process in order to properly report their mitigation processes to the UNFCCC (see Fig. 3.1). Two types of registries are implemented:

- Each of the 38 Annex II countries[9] has a national registry that contains accounts used to register and exchange permits, either in the name of the government or

[9]Annex I countries without Belarus and Turkey.

in the name of the legal entities authorized to hold and trade units (i.e., cap-and-trade).

- Clean development mechanism units (see Sect. 3.1.3) are centralized in a CDM registry under the authority of the CDM executive board. It allows the distribution of CDM units to countries participating in CDM projects.

These registries serve to settle emissions trades through exchanges between accounts. Each registry will operate through a link established with the International Transaction Log (ITL) put in place and administered by the UNFCCC secretariat. The ITL verifies registry transactions in real time to ensure they are conducted in accordance with the set of rules agreed under the Kyoto Protocol.

In verifying registry transactions, the ITL provides an independent check that unit holdings are being recorded accurately in registries. After the Kyoto commitment period is finished, the number of units held in each country's registry is compared with the country's emissions over the commitment period in order to assess whether it has complied with its emission target under the Kyoto Protocol. Regional emissions trading schemes (i.e., EU ETS) that also comply trough the use of Kyoto units undertake their settlement through registry systems. EU allowances are specific Kyoto units that have been designated as being valid for trading under the scheme. Transactions in EU allowances are therefore recorded automatically as transactions under the Kyoto Protocol.

As EU trading has specific rules that differ from those of the Kyoto Protocol, a specific transaction log has been implemented by the European Commission and its members. The Community Independent Transaction Log (CITL) has been in place since the start of the EU ETS scheme in 2005 and EU registries are now operating with it. In 2008, EU registries had to switch their connections from the CITL to the ITL. The ITL has to conduct "Kyoto checks" on transactions proposed by both EU and non-EU registries. In the case of transactions involving EU registries, the ITL will forward information to the CITL so that it can conduct "supplementary checks" defined under the EU scheme.[10]

As part of the quantified emission limitations, every country has been assigned an amount of Assigned Amount Units (AAUs). These units are calculated in tons of CO_2 equivalent (CO_2-e) and are allocated at the beginning of each commitment period. To ease the accounting of the six different GHG, offending gases are weighted by their global warming potential (GWP, see Chap. 1).

Beside strict policy regulations, the KP establishes three so-called flexible mechanisms in order to give Annex I countries more flexibility to reduce emissions. The Kyoto Protocol demands that the use of the mechanisms is supplemental to domestic action and that domestic action should constitute a significant element of the effort made by each Party included in Annex I to meet its quantified emission limitation and reduction. The three flexible mechanisms of the Kyoto Protocol are:

[10] In 2012 the European Commission has activated the single EU registry for all users (EUTL). The single registry includes the accounts for stationary installations and personal accounts previously held in national registries.

- International Emission Trading (as defined in Article 17 of the Kyoto Protocol): Annex I Parties can acquire so-called Assigned Amount Units (AAUs) from other Annex I Parties and use them for compliance under the Kyoto Protocol.
- Joint Implementation (JI, as defined in Article 6 of the Kyoto Protocol): Annex I parties can contribute to their emission targets by investing in emission reduction projects in other Annex I countries. These investments eventually result in Emission Reduction Units (ERUs) that can be used for compliance under the Kyoto Protocol.
- Clean Development Mechanism (CDM, as defined in Article 12 of the Kyoto Protocol): Annex I Parties can undertake emission reduction projects in developing countries (non-Annex I), which lead to Certified Emission Reduction (CER) credits. These credits can be used for compliance in the industrialized countries. Contrary to AAUs and ERUs, CERs are coming from countries without emission reduction requirement, therefore augmenting the defined emission caps of the Annex I countries (since there is no cap attached to these countries).

These mechanisms should help all parties to achieve GHG emission reductions at the lowest cost possible. We refer the reader to Sect. 3.1.3 for a detailed overview of the CDM and JI flexible mechanisms.

3.1.2 The EU ETS

The European Union Emission Trading Scheme (EU ETS) is currently the largest cap-and-trade scheme in the world. It encompasses the 27 European countries (EU-27) and covers more than 11.500 installations of heavy emitters, representing 42 % of Europe's global GHG emissions in 2008 and close to half of its CO_2 emissions. The installations included in the scheme[11] are combustion plants, oil refineries, coke ovens, iron and steel plants and factories making cement, glass, lime brick, ceramic, pulp and paper. Currently, the scheme solely covers carbon dioxide emissions while planning to include other greenhouse gases in a later phase.

The EU ETS is legally completely independent of the UNFCCC and the Kyoto Protocol. It came into life in 2005 through the application of a 2003 Emission Trading directive (Directive 2003/87/EC) and was already operational when the KP came into force.

It completed its first trading phase between 2005 and 2007. This first period was intended to test and evaluate the performance of emission market. The EU ETS just ended its second trading phase (2008–2012), which is aligned with the KP's first commitment period. The third trading period (2013–2020) went through a major revision of the system's operational design. Relevant design differences are an EU-wide cap on allowances, as opposed to 27 individual Member State caps, decreasing by 1.74 % annually, up to and beyond 2020; auctioning as the default system of allocation in phase three; and stricter rules on the type of international credits that are allowed for use in the EU ETS. Since the European scheme is independent of

[11] Above certain capacity thresholds: for instance energy industries must have combustion installations with a rated thermal input exceeding 20 MW.

any binding protocol under the UNFCCC, the third trading phase will take place without consideration for a successor of the Kyoto Protocol (ensuring continuity of the trading activities between phase 2 and 3).

As a cap-and-trade scheme, in phase one and two caps were defined for each country through a National Allocation Plan (NAP) procedure that ensures that the total amount of allowances issued to installations was less than the amount that would have been emitted under a Business-As-Usual scenario (BAU). Prior to the start of a trading period, each member state had to prepare and submit a NAP that the European Commission reviews on the basis of 12 criteria (see Annex III of the Emission Trading directive).

Since the EU ETS does not covers all the global GHG sources that are accounted for in the Kyoto protocol, the Directive relates allowance limits to a "path" towards Kyoto compliance.[12] This restricted aspect about the coverage extent of emissions could be misleading since the EU ETS is by no mean designed to ensure absolute compliance. The Directive recognizes that the EU ETS should be complemented with other national measures (i.e., taxes, subsidies or regulations) to meet its Kyoto targets.

As a way to reduce the cost of compliance, the Commission allowed (in the second phase onward) states and companies to use flexible mechanisms (CDM and JI) up to a certain level defined in their respective NAP.

The Commission could reject NAPs if they are not compatible with the twelve criteria: for the first trading period, the Commission had rejected several plans on the basis of excessive allocations and ex-post adjustments.[13] Considering the poor performance of this first trading period (see Sect. 3.1.2), the Commission was especially worried about granting excessive allowances for the second phase (the so-called hot air) and asked for more stringent measures coming from former Eastern Block countries:[14] Bulgaria was asked to reduce by 37 % its requested allowances, Romania by 20 % and Hungary by 12 %.

Once a national allocation plan has been accepted, the member state has to take a final allocation decision and can make changes to the number of allowances for individual plants as a result of improved data. However, countries are not allowed to increase the overall number of allowances validated by the Commission.

For the 2008–2012 trading period and with some exceptions, the caps imposed to the states were below the official BAU emission projections defined in each NAP. It corresponds to a reduction of 7 % of the total emissions within the EU ETS (153 Mt/year). However, according to research,[15] the inclusion of the total amount

[12] It is therefore explicit that the EU ETS should be complemented by additional measures for the sectors that are not included in the scheme (transports, households and small businesses, agriculture).

[13] Ex-post adjustment means that a member state plans to intervene on the market once the allocation is done with the aim to redistribute allowances between companies.

[14] In 2007, the Commission set a EU-wide CO_2 cap of 2.08 billion tones for the second phase, 10 % less allowances than requested.

[15] Initial Assessment of national allocation plans for phase II of the EU emission trading scheme, ECOFYS, November 2006.

of JI/CDM credits that could be used by participating companies corresponds to 355 Mt/year, almost 17 % of the total emissions within the EU ETS. This inclusion of cheaper flexible units created de facto an increased cap and a potentially long position in permits, posing the real risk of postponing expensive abatement measures in favor of a less-than-stringent market.

Within each trading period under the scheme, companies exchanges units of European Union Allowance (EUA). For identification purposes, units are labeled by their year of surrender and are called vintage. The EU ETS allows firms to bank and borrow allowances between years within a trading phase: they can cover a short position by using previous unused units (banking) or by using permits allocated to following years (borrowing). The scheme allows companies to bank allowances between the current trading phase and the next one but forbids borrowing between trading phases.

EUA Characteristics

Transferable permits (also called allowances) can be considered as a *pseudo*-commodity whose price, as any standard commodity, is a function of permits demand and supply. In particular, the permit price reflects the expected supply–demand imbalance (Fig. 3.2).

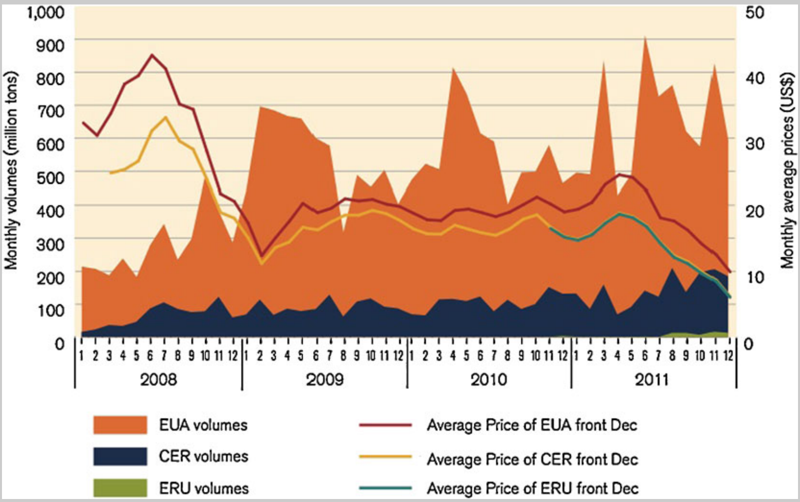

Fig. 3.2 Price and volume for EUAs, CERs and ERUs in the secondary market (2008–2011). Source: World Bank

The supply in the EU ETS is determined by three components. The first component is the initial allocation of permits, that is, the emission cap. Banking and borrowing provisions constitute the second component. Banking refers to the possibility for firms to save unused permits for future use, while borrowing represents the possibility to borrow permits from future allocations for use in the current period. Together with the initial permit allocation, banking and borrowing provisions are set by the regulatory authority. Offsets are the third component. Eligible offsets can be used for compliance purposes. For instance, Certified Emission Reduction (CERs), offsets generated under the CDM system, could be used for compliance in the EU ETS. Typically, restrictions are placed on the quality and quantity of offsets that can be used.

The demand side of permits depends on the evolution of the underlying regulated pollution. Under the EU ETS, CO_2 pollution emissions are regulated. The drivers of the underlying emissions include (long and short-term) pollution abatement options, economic cycles, energy-related prices and weather conditions.

Abatement Options Long and short-term abatement strategies arise whenever a firm faces a basic choice: adopting a new technology that lowers the marginal cost of pollution abatement and increases the abated volume of pollution (long term option) or relying on the trading of allowances to cover the amount of CO_2 emitted (short term option). We will not examine the dynamic properties of a cap-and-trade system here. We refer interested readers to Chao and Wilson (1993), Xepapadeas (2001), Zhao (2003), and Taschini (2011), among others.

Economic Cycles As discussed in Ellerman and Joskow (2008), in a cap-and-trade system, a change in the economic conditions is (and should be) reflected in the demand for permits. For instance, a growth in the economy would result in higher demand for permits and, consequently, it will increase the permit price. Such a price increase will ultimately encourage further adoption of new, low-polluting technologies. Conversely, a deterioration of the economy would result in lower demand for permits and, consequently, it will reduce the permit price. Unsurprisingly, the larger the amount of unused permits available in the market, the lower the permit price. This is what we observed in the EU ETS markets in 2006 and, again, during the economic slowdown in 2009.

Energy Prices Energy related prices can unsurprisingly exert a great influence on the demand for emission permits. The energy industry is in fact one of the largest CO_2 emitting sector. For instance, if gas prices are lower than coal prices, electricity will more likely be produced by burning gas. The use of gas in power generation emits less GHG than the use of coal, therefore a switch from coal to gas implies a lower demand of emission permits. Looking at fuel-switching, the studies of Convery and Redmond (2007), Alberola et al. (2008), and Creti et al. (2012), among others, highlight the importance of energy-commodities prices. Fuel-switching may be considered a short-term abatement option. Installations may lower their emissions by implementing long-term abatement measures in order to improve energy efficiency as well. Whether or not such measures are implemented depends on the cost of the technologies compared to the projected cost of offsetting emissions by purchasing (or not selling) allowances and eligible offsets.

Weather Factors that influence electricity generation are bound to affect the demand of allowances. For instance, a hot summer can lead to a higher demand for electricity because of air-conditioning. When electricity is produced by fuel-fired power plants, extremely hot summer or cold winter leads to higher CO_2 emissions, increasing the demand for permits. Using European weather data, Mansanet-Bataller et al. (2007) identify extreme weather events as CO_2 emission price drivers. Employing precipitations data, Houpert and de Dominicis (2006) argue that rain is a non-negligible price driver in the EU ETS. High precipitations make it possible to use more non-CO_2 emitting power sources and therefore reduces emissions in energy production. When the hydroelectric production is low, electricity has to be produced by other means generally quite CO_2 intensive, like coal or gas. In particular, most of the Scandinavian countries heavily rely on hydroelectric power production. When rainfall is scarce, a country like Norway for instance has to import electricity from neighboring countries, very likely Denmark. Danish electricity is in large part generated from coal-fired plants, implying higher emissions. Yet, in Scandinavian countries cold and dry weather can also lead to water shortage in the winter, because frozen water cannot be used for power production. Conversely, when hydroelectric production is high (because of an increase in rainfall, or because of melting ice in spring), less emissions are emitted compared to coal-fired power production. However, too much rainfall may also cause off-time for the hydroelectric installations. Other studies that investigate various combinations of macroeconomic factors such as energy-related prices and weather are Hintermann (2010), Gronwald et al. (2011), and Mansanet-Bataller et al. (2007).

The EU ETS has a "third party" monitoring and verification approach. Companies are asked to submit each year (latest March 31st) a report with their emissions for the preceding year. A private verifier independent of the company conducts the verification of the emission report. If a company does not surrender sufficient allowances by April 30th to cover the emissions of the preceding year, it has to pay an excess emission penalty of EUR 100 for each ton in excess.[16] Additionally, the company has to surrender permits covering the amount of excess emissions in relation to the following calendar year.

To resolve the over-allocation issue and ensure that by 2020, European Union countries will have reached a targeted emission reduction of 20 %, the Commission

[16] The excess emission penalty fee for the 2005–2007 trading period was EUR 40.

has proposed a revision of the EU ETS that was approved in December 2008. We give herein an overview of the main modification to the scheme that will be applied between 2013 and 2020, during the third trading phase. We shall remind the readers that at the time of writing, these modifications of the EU ETS are still subjects to amendments and can be outdated by future revisions:

- A first modification of the EU ETS is the technical inclusion of the aviation sector in 2013. The sector represents the second-largest emitting sector covered by the scheme, after the power sector.
- An enlargement of the scope of the scheme to include new sectors (petrochemical, ammonia and aluminum sectors) and two new gases (N_2O and PFCs). However, transport, shipping, agriculture and forestry will remain outside of the scope of the third trading period.[17]
- A EU-wide target will replace the current 27 national targets (and NAPs). To reach the global EU target of reducing emissions by 21 % below 2005 levels by 2020, allowances will be limited to a maximum of 1.72 billion units,[18] with total emission allowances cut by 1.74 % annually as of 2013 and until 2020.
- The preliminary cap for the year 2013 has been set at 2,039 million tons of CO_2 equivalent ($MtCO_2e$). The final cap will be adjusted to better represent the broadened scope of the scheme starting in 2013, the small operators that member states have chosen to exclude, the inclusion of the aviation sector, and the inclusion of emissions from Norway, Iceland, and Liechtenstein.
- A proposed auctioning of around 20 % of the total number of allowances in 2013. Furthermore, 100 % of allowances for the power sector should be auctioned. The target is a progressive phase-out of the grandfathering practice, to reach a global 70 % of allowances auctioned by 2020 and a 100 % auctioning by 2027. As of January 2013, auctioning will take place on a common EU-wide platform for most European member states (with the notable exceptions of Germany, Poland and the UK who decided to use national auction platforms). However, certain energy-intensive sectors that are at risk of leakage (offshoring) will continue to receive their allowances for free.
- A distribution method for the share of free allowances, based on benchmarks, will be developed by the end of 2010.
- The use of flexible mechanism units (CDM/JI) will be conditional to the passage of a global agreement pushing the required EU reduction to 30 % by 2020. Without an agreement, usage of flexible credits may be limited to 3 % of member states' total emissions in 2005.[19] Under Phase III, Kyoto credits will no longer be de facto compliant with the EU ETS. Their conformity with the European scheme will be conditioned and their fungibility into EUAs will be conditional. CERs representing emission reductions occurring before January 2013

[17] Shipping is considered for inclusion at a latter stage. For sectors not covered by the EU ETS, an average GHG reduction of 10 % should be achieved, proportionate to countries' GDP.

[18] To be compared to the current allocation of 2.08 billion tons for the second trading period.

[19] Provided that the additional quantity does not exceed 50 % of EU-wide reduction between 2008 and 2020.

will have to be swapped with EUA for full fungibility. CERs related to reduction occurring after December 2012 will not be swapped but will be considered fully equivalent with EUA of phase III. For new projects (registered after December 31, 2012), future CERs will be eligible if they come from a project in a Least Developed Country.

- An Energy Efficiency Directive (EED) was proposed by the European Commission on June 2011. This directive, which aim is to save energy and to reach the EC's self-imposed target of a 20 % cut in primary energy consumption by 2020, may create downward pressure on EUA prices buy providing a competing command-and-control measure to reduce the amount of emissions. Since the measure will not be accompanied by a more stringent cap, it could create a surplus of allowances that may lower EUA prices. It is expected to be implemented by January 2014.

The future of an efficient EU ETS lies also in its capacity to adjust for economic downturn and unexpected drop in yearly emissions. With the combined conjunction of the financial crisis at the international level and the multiple European crises, verified emissions in Europe declined 2.4 % year-over-year in 2011, after a significant and continuous decline of emissions in 2008 and 2009. The new Energy Efficiency Directive proposes to set aside a certain share of EUAs to account for the weak demand for permits. According to the World Bank[20] and Deutsche Bank,[21] "*the decline translates into an additional surplus of about 380 million EUAs in the scheme, now expected to be oversupplied by about one billion tons until 2020.*" As such, the idea of setting aside permits touches upon two important limitations of the EU scheme: (i) the capacity to maintain a constant incentive towards mitigation and (ii) the ability for the scheme to have dynamic and conditional rules instead of static ones. While a solution for the first problem has been proposed by the UK government with a carbon floor price set to be implemented as of April 2013,[22] the second issue is still very much unresolved.

3.1.3 The Kyoto's Flexible Mechanisms: Clean Development Mechanism and Joint Implementation

The Clean Development Mechanism (CDM) The Clean Development Mechanism (CDM) is one of the two project-based mechanisms defined as Kyoto flexible mechanisms (along with Joint Implementation presented in the next section). As such, the CDM has two main objectives: The first objective is to contribute to sus-

[20] State and Trends of the Carbon Market 2012.

[21] EU Emissions: 2011 VED Raises the Pressure, April 4, 2012.

[22] The floor price will be targeting fossil fuel power generators, taxing the difference between the price of EUAs and a UK's notional carbon floor price.

tainable investments in developing countries[23] through transfer of clean technology, foreign direct investment, and income streams from the sale of generated Certified Emission Reduction Units (CERs). The second objective is to realize reductions of GHG emissions in developing countries in order to help Annex I countries to meet their emission targets in a cost efficient way.

The CDM was established under Article 12 of the Kyoto Protocol with detailed rules and modalities subsequently agreed upon by Kyoto Protocol parties in 2001, as part of the so-called Marrakech Accords. The CDM Executive Board (CDM EB) was formed the same year and began building the structure and processes of the international CDM system. The first CDM projects were officially registered with the Executive Board in 2004.

To allow reduction in emissions in Annex I countries, projects have to generate additional "offsetting" permits by reducing project emissions from Business as Usual (BaU) emission scenarios in the host countries. Generated CERs are consequently registered in the CDM registry supervised by the UNFCCC and can be used by countries to comply with their national Kyoto target or by companies involved in a cap-and-trade scheme up to the allowed quotas (see the section on the EU ETS for the current and future eligibility rules).

Since CDM creates new emissions units above the caps set by Annex I countries, it was clear from the beginning that the UNFCCC and Annex I countries had to ensure that each CDM project was really reducing emissions. The Kyoto Protocol[24] hence introduced the set of project eligibility criteria necessary to guarantee that a CDM project meets the objectives of sustainable development and actual reduction of emissions.

To ensure the integrity of the CDM projects, the concept of additionality was adopted. This means that reductions in emissions must be additional to any that would occur in the absence of the certified project activity. Additionality is however a complex assessment procedure, relying on the existence and validation of a fair and justifiable BaU baseline scenario.[25] Among the different methods proposed by the CDM EB, the most commonly used methodologies to assess additionality in project rely on investment analysis, where the project developer demonstrates that the CDM revenue from selling CERs is required in order to put the return to the project above the investment threshold (the Internal Rate of Return, see Fig. 3.3).

The complexity of justifying and monitoring projects has forced the creation of a long and multi-stage validation process involving the project manager, the host

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[23]To participate in a CDM project, developing countries should have ratified the Kyoto Protocol ("a Party not included in Annex I may participate in a CDM project if it is a Party to the Kyoto Protocol", Art. 12).

[24]Paragraph 5, Article 12.

[25]The Marrakech Accord defines the baseline for a CDM project activity as the scenario that reasonably represents the anthropogenic emissions by sources of GHG that would occur in the absence of the proposed project activity.

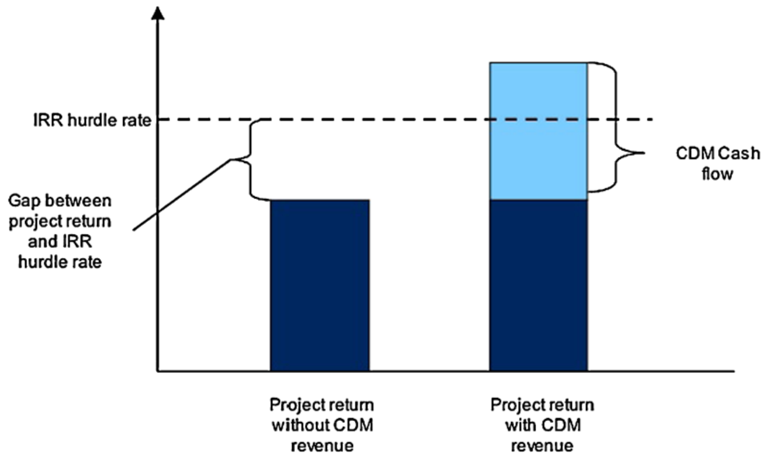

Fig. 3.3 Source: Guidebook to Financing CDM projects, CD4CDM, 2007

country, the UNFCCC and the supervision of independent auditors. Originating CERs takes 8 different stages,[26] from project design to the issuance of permits:

1. As a first step, the project manager issues a project design document (PDD) sketching the main aspects of the project to be implemented (methodologies, additionality, project boundary, crediting period, main impacts, monitoring method).
2. The PDD is then sent to the host country Designated National Authority (DNA) that approves or reject the project based on the information given. This first stage validation takes the form of a Letter of Approval (LoA) that serves as a proof of acceptance from the host country for the project manager and is a requirement for registering the project activity under the CDM.
3. For gaining compliance under the CDM, the project must be validated by a Designated Operational Entity (DOE) that verifies that the project meets the basic eligibility requirements, consults with stakeholders and finally provide a request for registration to the Executive Board in the form of validation report.
4. The validated project, submitted by the DOE to the CDM EB[27] is then registered (or rejected) on the basis of its eligibility requirements, methodologies, baselines and impacts. Registration is a key stage in the CDM project cycle, representing the point where a project activity is accepted as a CDM project,

[26]Readers interested in a more detailed view of the stages and rules of the CDM should refer to the CDM Rulebook website (www.cdmrulebook.org), a joint effort by the legal practice Baker & McKenzie and eight donor organizations.

[27]It is often assumed that the CDM EB is solely responsible for the registration of projects. The actual process is slightly more complex: after being processed by the UNFCCC secretariat, a validated project submission is reviewed by a Registration and Issuance Team (EB-RIT) appointed by the CDM EB. On the advice of this team, the CDM EB either approves or rejects the proposed project activity.

making it eligible to generate CERs. Usually, while waiting for the validation of their projects, project managers start consolidating the project financial structure.[28]

5. Once registered, the project starts its monitoring phase which provides the collection and analysis of all data relevant to the calculation of emission reductions from the project, on the basis of an approved monitoring methodology.
6. Once the project in place, a periodic verification is conducted by a different DOE to ensure the authenticity of the emission reductions from the project. This validation is done using the monitoring emission data.
7. When the validation process is completed, the DOE certifies that the emission reductions set out in the verification report where actually achieved. The certification report is considered to constitute a request for issuance of CERs.
8. Finally, when emission reduction units have been generated, verified and certified, they are officially issued by the CDM registry administrator on behalf of the CDM EB.

The entire CDM cycle is a long process. In their report about financing CDM projects,[29] the consultancy Ecosecurities and the United Nation Environment Program (UNEP) acknowledge a processing period of 6 to 12 months before the project validation, and between 1 and 2 additional months for the project registration by the CDM EB. Published in 2007, this report is however too optimistic about the actual delays experienced in the origination process of new permits. Attacked on its ability to closely assess and validate an increasing number of projects, the CDM EB has been accused in the past of granting ("rubber stamping") projects that lacked additionality, therefore generating hot air. The institutional body was then under pressure to approve new methodologies,[30] especially in difficult areas (i.e., afforestation/deforestation). The criticisms have forced to supervising body to increase the length of the validation-registration-issuance stages with the effect of creating bottlenecks in the CDM project pipeline.

According to the World Bank in its 2010 report on carbon markets, the average delay between a request of registration and a registration had nearly doubled between 2007 and 2009. Overall, the processing period from registration to issuance had more than tripled from around 200 days to more than 600 days. In 2005 it took an average of 100 days for a project to move from registration to first issuance. In 2010, this average had ballooned to more than 700 days.

The most recent data tend to attenuate however this bottleneck image of the Executive Board, which had to handle the great run for compliance and registration pushed by the new EU ETS policies (see Sect. 3.1.2). Recent efforts from the CDM EB in 2011 has permitted greater permit issuance and the cleaning up of the old

[28]The financing takes usually the form of Emissions Reduction Purchase Agreement (ERPA) that legally binds transfers of carbon credits from the project in exchange of scheduled payments.

[29]Guidebook to Financing CDM projects, CD4CDM, 2007.

[30]Through the Methodology Panel.

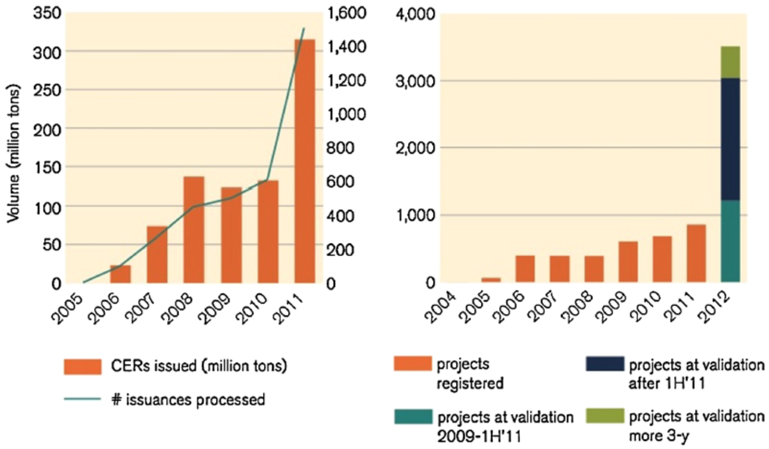

Fig. 3.4 CER issuance and CDM projects registered. Source: World Bank, 2012

backlogs. According to the World Bank, in 2001 alone, 315 million CERs were issued, representing a 140 % increase over 2010 and about 40 % of all issuances until that year (see Fig. 3.4). Validation time has been reduced to 525 days in average.[31]

The CDM market is now at the junction between a thriving but somehow experimental first phase and a more difficult and regulated future. Since the first CERs in 2004, the CDM primary market has increased extremely rapidly, benefiting from low cost projects in China (and to a lesser extent India and Brazil) involving leveraged[32] greenhouse gases (HFC destruction projects, methane avoidance, landfill gas) and large projects with marginal abatement costs (i.e. hydro projects in China). By the end of 2012, the UNFCCC have registered more than 4000 projects with a projected issuance of 1.1 billion CERs until the end of 2012, according to the UNEP Riso center.[33]

The “low hanging fruit” approach has nevertheless recently changed towards clean energy investments (renewable energy, energy efficiency and fuel switching projects), reflecting the EU ETS ban on high-GWP gases[34] in the new, post-2012 CDM framework (see Fig. 3.5).

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[31] The registration process remains however too slow to ensure the full registration of all projects before the end of 2012: according to the World Bank, 3500 projects were still in the validation by the end of 2011, with 1800 projects that are expected to registration in 2013 or later.

[32] Here, leveraged means that projects concentrate on reductions from gases with high global warming multiplicative potential (GWP).

[33] http://www.cdmpipeline.org/.

[34] The European Commission has formally adopted a ban on the use of industrial gas credits in the EU Emissions Trading System (EU ETS) as of May 2013. The ban applies to credits from projects which destroy HFC-23 and nitrous oxide (N_2O).

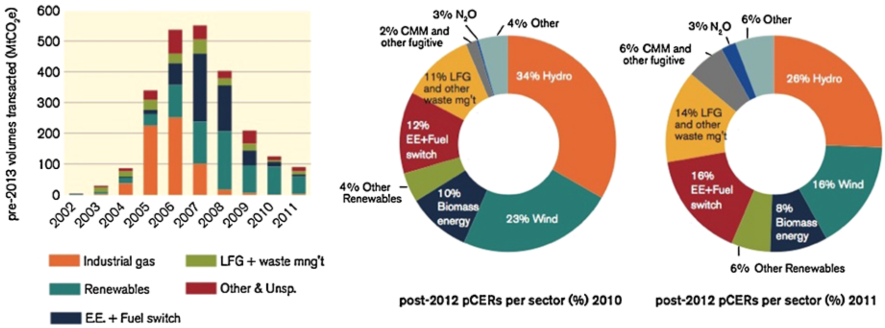


Fig. 3.5 CDM transacted per sector, pre-2013 and post-2012. Source: World Bank, 2012

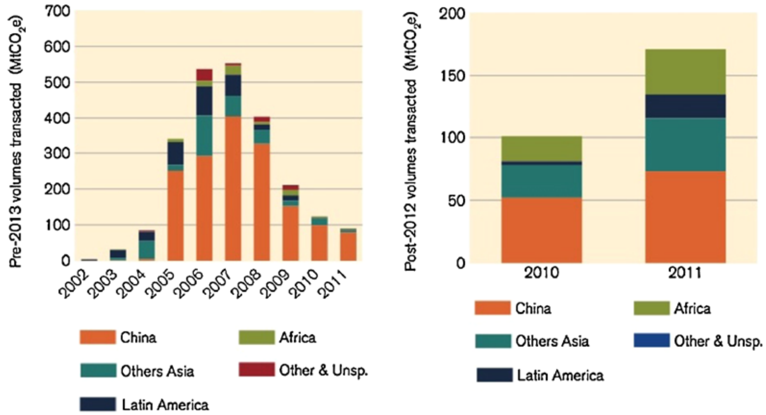


Fig. 3.6 CDM transacted per seller, pre-2013 and post-2012. Source: World Bank, 2012

In 2008, the World Bank estimated that 84 % of all ongoing projects were coming from China,[35] with 4 % respectively for India, 4 % for the rest of Asia, 3 % for Brazil and only 2 % for Africa. This profile is starting to change (see Fig. 3.6). In 2011, while post-2012 CERs were still largely from China (43 %), other Asian countries (India, Vietnam and Indonesia) started to divert significant shares of the supply (25 %), while Africa boomed to 36 million tons and 21 % of post-2012 CERs.

CER Characteristics

Certified Emission Reduction (CER) contracts are permits issued through the Clean Development Mechanism with the dual purpose of reducing cost of

[35]Over the period 2002–2008, China accounted for 66 % of all contracted CDM supply in the market.

abatement for companies subjected to emission caps and increasing capital and technological inflows in developing countries.

Contrary to EUA, CER certificates (also called permits) are not allocated to companies but are originated from "clean" projects in non-Annex I countries that must be verified and certified by independents auditors (Designated Operational Entities, DOEs) before being formally issued by the CDM registry administrator. The long issuance process and the numerous uncertainties surrounding developing countries help explain the relevance of project risk factors in the price valuation and has given rise to specific actors and contracts with the purpose of hedging those risks.

Multiple Project Risks As shown in Fig. 3.7, CER prices are *discounted* compared to EUA contracts with the same maturities. This holds for strip contracts (a basket of all vintages with an average price), spot contracts or primary CERs (contracts directly issued by the CDM project). In contrast to primary CER, pCER, secondary CER, sCER, are contracts sold by primary buyers that have often cleared the initial development risks. The discount factor (or "risk premium") is more or less pronounced depending on the number of risk factors associated to that specific contract. As such, pCER are the *riskiest* permits since they include the full array of project and country risks, from failure to generate permits to hosts' political instabilities and issuance failures. We refer to Labre and Atkinson (2010) for a comprehensive description and quantification of these risks.

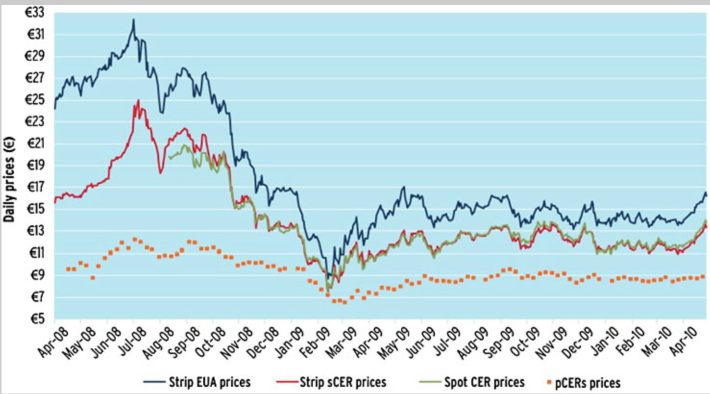


Fig. 3.7 Carbon prices from April 2008 to April 2010. Source: ECX, BlueNext, IDEAcarbon and World Bank

According to the World Bank, the main risks facing CDM projects are (i) country and political risks, (ii) rejected methodology leading to issuance failure, (iii) validation and registration failures, (iv) delays, (v) regulatory risks and (v) market price risks.

- Country and political risk is a classical problem faced by most project managers in developing countries. Conducting businesses in most of the relevant non-Annex I countries requires flexibility and a strong knowledge of the political and administrative institutions as well as the cultural and political contexts. Business environment, rules and regulations can change fast and can be plagued by political disruptions (coups, wars) and malpractices (tradition of bribery, state's unwanted influence). Moreover, as any CDM project requires the approval of the host country's DNA, projects are heavily dependent on the establishment of good relationships with the host's administrative bodies.
- The origination of CER permits is attached to the demonstration that the flexible mechanism allows for less emission-intense activities compared to a business-as-usual benchmark by using an approved methodology. While many projects are now using methodologies that have already been approved, new sectors are facing risks of rejections by the Executive Board, a problem currently faced by some afforestation and deforestation projects.
- Risks of validation and registration failures are important in the early stages of a project's development and are mainly due to either the inability to justify emission reductions (the principe of "additionality") or to discrepancies between the business plan proposed in the Project Design Document (PDD), the actual implementation and the eligibility requirements set out by the Executive Board. According to the CDM Rulebook, the requirements for validation by a DOE are the following: Project participants are voluntarily participating in the project activity, the host Party of the project has designated a national authority for the CDM, the non-Annex I country authorising participation in the project by the proponents is a Party to the Kyoto Protocol, the comments of local stakeholders have been received and taken into account, the likely environmental impacts of the project activity have been assessed, the project will result in greenhouse gas reductions that are additional and the baseline, monitoring, verification and reporting proposals (and all other aspects of the project) comply with the CDM requirements.

To conclude this section on CDM, few aspects that are likely to change in the post-2012 regulatory framework are worth mentioning:

- The biggest problem facing the CDM markets is a long-lasting oversupply problem: with the near end of the Kyoto Protocol in 2012 and the vast uncertainty surrounding the post-Kyoto agenda, CDM projects, which are medium to long-term in essence, face the increasing risk of being Phased out or limited with constraints on project types. Additional volume restrictions will apply in the EU ETS which will reduce the demand for CERs.

- The future CDM has to expand both its coverage of projects and its geographical boundaries: with new methodologies, projects could be developed in afforestation, reforestation and soil management while a greater emphasis will certainly be placed on development issues with a special focus on Africa. The recent trend of post-2012 CERs is encouraging to that respect, especially for the key countries of Democratic Republic of Congo, Burundi, and Nigeria.
- Finally, to reduce transaction costs and administrative burden, pooled projects under the umbrella of Programmes of Activities (PoA) will provide new ways to reduce marginal costs, spread risks and lessen the aforementioned bottleneck problem. At the time of writing, 269 PoAs have entered the CDM cycle, with a more diverse geographical distribution relative to standalone CDM projects, Africa accounting for 28 % of the total supply.

The Joint Implementation (JI) The Joint Implementation (JI) is the third flexible mechanism, defined under Article 6 of the Kyoto Protocol. Projects under the JI take place among Annex I countries with emission reduction or limitation commitment and most likely between economies in transition[36] (EIT parties) and developed countries. In such countries, due to lower efficiency levels, JI can stimulate emission abatements at lower costs while increasing the amount of foreign investments.

To be eligible, a JI project must provide a reduction in emissions by sources, or an enhancement of removals by sinks, that is additional to what would otherwise have occurred in a Business as Usual scenario. Projects must have approval of the host Party and participants have to be authorized to participate by a Party involved in the project. JI projects issue Emission Reduction Unit (ERUs) that could be used for a crediting period starting after the beginning of 2008 (ERUs were not allowed for the first trading phase of the EU ETS).

The JI project cycle is different and largely simpler than the CDM cycle. It comprises two procedures, called "Track 1" and "Track 2":

1. The "Track 1" procedure is the simplest one, acting as a fast tracking process for host countries meeting all the required eligibility requirements.[37] The host country is asked to verify without further supervision that emission reductions from a JI project are additional to the BaU scenario. If the verification is satisfactory, the host Party may issue the appropriate quantity of ERUs. This simpler approach is authorized because contrary to CDM projects that create additional permits, JI projects simply transfer permits between countries at a lesser cost (since host Parties are among Annex B countries), ensuring constant caps.
2. The "Track 2" procedure has to be used when the host Party does not fulfill the eligibility requirements. In such a situation, the host Party is not allowed

[36] Annex B of the Kyoto Protocol identifies 12 economies in transition: Bulgaria, Croatia, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, the Russian Federation, Slovakia, Slovenia, and the Ukraine.

[37] Two of the most important requirements are (i) the national system for the estimation of anthropogenic emissions and (ii) the national registry which have to be in place in accordance with Article 7 of the Kyoto Protocol. See paragraphs 21 and 23 of the JI guidelines for further details.

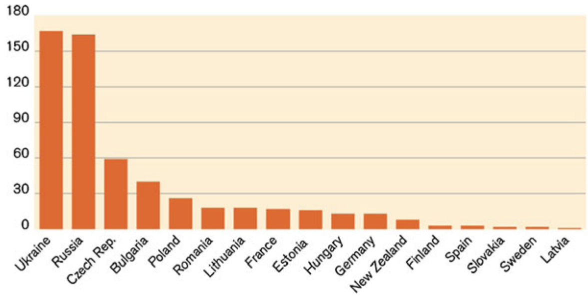


Fig. 3.8 Number of existing projects in the JI pipeline per country. Source: UNEP RISOE, 2011

to issue ERUs on its own and must require a verification procedure through the JI Supervisory Committee (JISC).[38] A project design document (PDD), which contains all information about the project, has to be submitted to the JISC. Following its submission, the JISC approves ("determines") or declines the project. As such, the Track 2 procedure is similar to the CDM Cycle: once such a project is running it has to be monitored by participants. An Accredited Independent Entity (AIE) has to review the reports and calculate the emission reductions generated by the project.

As with CDM, JI projects had to comply with new EU ETS rules: after 2012, ERUs can be traded if they were generated and issued before 2012 or if they were generated and issues from projects registered before 2013.

JI projects were distributed unequally between Track 1 and Track 2 procedures.[39] To date, most projects and issuances have taken place under Track 1. There were 686 existing projects at different stages of development in the JI pipeline. Almost 65 % of this pipeline was hosted in the Ukraine and the Russian Federation (256 and 189 projects, respectively), with other active countries including the Czech Republic and Bulgaria (59 and 39 projects, respectively). Quite surprisingly, France hosted the largest number of JI projects outside of Eastern Europe (17 projects).

Ukraine and the Russian Federation also monopolized ERU issuance. Out of the 253 million ERUs already issued until December 2012, 131 million tons (51 %) were generated in the Ukraine and 77 million (30 %), from the Russian Federation.

Bulgaria, Poland and the Czech Republic were competing for a leading share in registered projects: by the end of 2012, Poland had 25 projects in the JI pipeline, Bulgaria 39 projects and the Czech Republic 59 projects (Fig. 3.8). The Czech Republic was at the time of writing the largest issuer of this group (with 0.6 million ERUs issued).

The issued ERU were covering the main project types, with CH_4 reduction, Cement and Coal mine amounting for 47.5 % in 2012, energy efficiency projects for 28 % industrial gases for 16.2 % and renewables for 4.4 % (see Fig. 3.9). This distribution was somehow characteristic of the situation experienced by the CDM at its early stage. However, a current shift is clearly visible, with a progressive re-

[38] Paragraph 30–45 JI Guidelines Decision 9/CMP.1.

[39] According to UNEP RISO center that compiles JI project data (www.cdmpipeline.org).

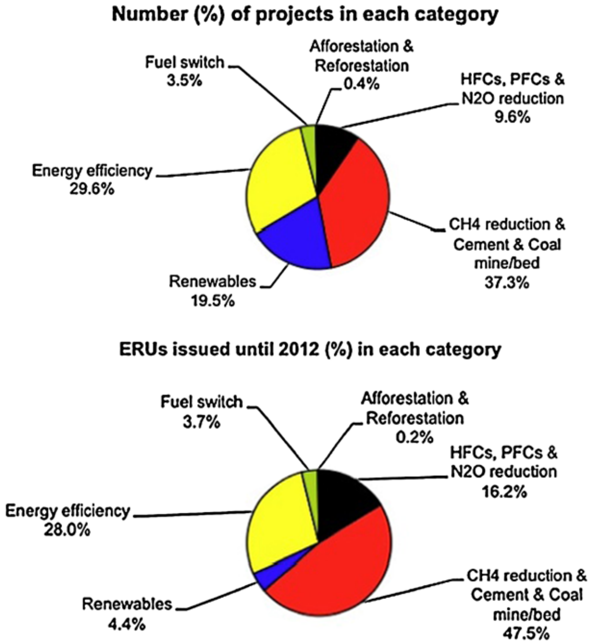


Fig. 3.9 Number of ERU issued and projects in the JI pipeline. Source: UNEP RISOE, World Bank 2012

placement of "low hanging fruit" strategies for "cleaner" ones: in terms of projects, renewable Energies (RE) have started to gain momentum (19.5 %), fuel switching projects were burgeoning (3 %), while industrial gases decreased comparatively (9.6 %).

To conclude, the withdrawal of Ukraine and Russia from the second phase of the Kyoto Protocol at the end of 2012 introduced great uncertainties for the JI scheme. At the time of writing, it was unclear how existing projects could be used and if the largest share of JI permits would become ineligible. The most plausible impact could be a drastic reduction in the overall supply, to the benefits of countries who agreed to pursue the second phase of the KP.

3.2 The Current State of the CO_2 Emission Markets

3.2.1 Carbon Exchanges and Market Players

Carbon Exchanges The buoyant environment that characterized the "pre-crisis" emission markets, with its multilayered and vastly diverse regulatory attempts (from regional enforcing schemes to local voluntary markets, see sections above), has set in motion an effort from exchanges to position themselves strategically for the next phase of maturity. By the end of 2011, it is still extremely difficult to clearly envision what will be the market landscape, both in terms of number of players, size of operation and types of products offered. However, it seems plausible that the exchanges that are already well implemented in the EU ETS will benefit first from

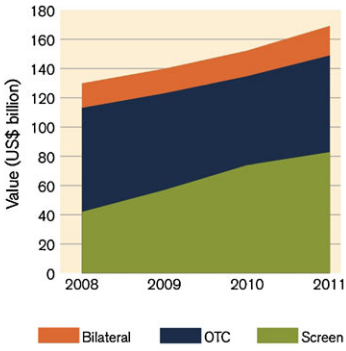


Fig. 3.10 Trading methods for EUAs, CERs and ERUs. Source: World Bank, 2011

a future market rebound and more stringent regulation at the national, regional or international levels. One of the biggest challenges for them will certainly be to incorporate both OTC, which represented in 2011 39 % of all EUAs, CERs and ERUs transactions (see Fig. 3.10), and bilateral trades (12 % of total).

In 2010, six exchanges are competing in the EUA and CER/JI trading segments:

1. By far the biggest in terms of volume and value, the European Climate Exchange (ECX) is part of the Climate Exchange plc. which also includes the Chicago Climate Exchange (see previous section). Launched in 2005 and based in London, ECX uses the electronic trading platform and clearance services of ICE (the former International Petroleum Exchange). In 2008, ECX represented 82 %[40] of exchanged EUA and 86 % of exchanged sCERs through a large palette of products: in addition to the traditional EUA and CER futures, the exchange offers spot prices (immediate settlement) for both types of contracts, option contracts (European call and put), spread trading (calendar spreads and CER-EUA spreads) and strip trading. To accommodate and lure OTC trading into the exchange, ECX also offers Exchange-for-Physical (EFP) facilities, as well as Exchange-for-Swap (EFS) and Block Trade Facility.
2. A distant second in volumes and values but currently the closest competitor of ECX, BlueNext, which is located in Paris, was founded in December 2007 by the French state-owned Caisse des Depots and NYSE Euronext. According to Thomson Reuters, the exchange served in 2008 10 % of all exchanged EUAs and less than 1 % of CER contracts. Like ECX, BlueNext provides a large array of contract types, with a strong focus on EUA and CER spot trading for which it is the main liquidity provider (the exchange was ahead of ECX with the creation of both spot contracts in 2008). It also offers CER-EUA spread and strip trading.
3. Nord Pool is one the largest power derivatives exchange, providing market places for trading in physical and financial contracts in the Nordic countries (it is based in Oslo and serves Finland, Sweden, Denmark and Norway). It was acquired by the NASDAQ OMX group in 2008 and now serves as a platform for

[40] Source: Thomson Reuters.

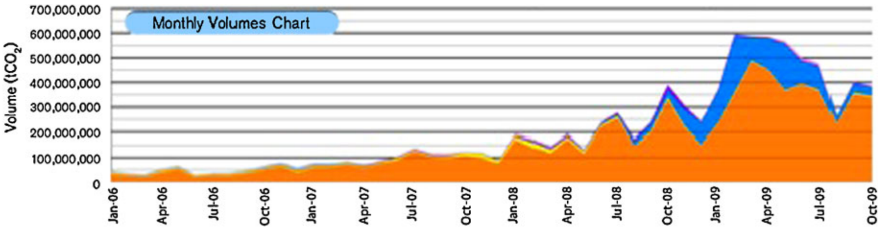


Fig. 3.11 Share of traded EUA by exchange. Source: Thomson Reuters, 2009

the carbon and energy offer of NASDAQ OMX. Reflecting its precursory role in emission trading, the exchange was the almost sole provider of exchanged CERs and the second platform for EUAs after ECX during the first years that followed the launch of the EU ETS. However, the emergence of BlueNext has since 2008 reduced gradually its market share: it represented in 2008 almost 4 % of EUAs and 9 % of CERs. Under the NASDAQ OMX name, it offers EUA and CER spot, futures and options.

4. The European Energy Exchange (EEX) was founded in 2002 as a result of the merger of the two German power exchanges Leipzig and Frankfurt. It trades spot and futures as well as coal and natural gas contracts. In an attempt to consolidate its position in continental Europe, EEX started in 2007 a partnership with the derivative exchange EUREX to provide for its clients emission trading capacities (using EUREX electronic platform). In 2008, the exchange represented 3.5 % of exchanged EUAs and 0.3 % of exchanged CERs.
5. Climex, which is based in the Netherlands, is the result of the 2008 merger of several carbon companies based in the UK, Hungary, the Netherlands and Spain. Contrary to the other exchanges, they strictly focus on spot trading for both EUA and CER. In 2008, 0.18 % of EUA (and a negligible share of CERs) were traded on Climex.
6. Finally, the NYME Green Exchange (based in New York) is worth mentioning for its strategic potential. Currently hosted by the NYME, the Green Exchange is venture owned by prominent finance US and international companies[41] with the aim to prepare for new emission regulations in the US. The exchange offers European allowances (EUA), UNFCCC permits (CERs) and specific US contracts for nitrogen oxide (NOX), Sulfur Dioxide (SO_2), California ETS contracts and RGGI markets. In real terms, the Green Exchange is still in infancy and currently represents tiny fraction of exchanged carbon permits.

Based on 2008 data, it seemed clear that ECX managed to gain the strongest momentum while consolidating its position as the premier exchange, both for EUAs and CERs (see Fig. 3.11).

The launch of BlueNext in 2008 diverted some EUA activities away from ECX in 2009 despite remaining uncertainties about its ability to fight on the same ground

[41] Constellation NewEnergy, Credit Suisse Energy, Evolution Markets, Goldman Sachs, ICAP Energy, J.P. Morgan Ventures Energy, Morgan Stanley Capital Group, RNK Capital, Spectron Energy, TFS Energy Tudor Investment, Vitol and CME Group.

as the London exchange (especially in the CER trading segment). ECX benefits from the high carbon activity level of London, with the world's largest numbers of carbon brokers, financiers, originators, dedicated industrial and financial trading desks. Climate Exchange plc. is also extremely well positioned to take advantage of any new regulation in the US, leveraging its European track record while competing for the dominance of the Chicago Climate Exchange and its derivative subsidiary, the Chicago Climate Futures Exchange.[42]

With few market leaders already in place under the EU ETS, the future of the carbon exchange industry certainly depends on the timing and geographical location of any new regulation involving cap-and-trade. We should expect a progressive consolidation following the same pattern of mergers observed in the past with more matured exchanges. It seems already clear that Europe will further consolidate around fewer exchanges, offering better liquidity and more complex products. What remains to be seen is whether European and North American existing exchanges will compete or consolidate in the future, a trend that will largely depend on the degree of similarity between the regulatory frameworks.

Market Players In general, we can distinguish four types of market players in the EU ETS:

- Governments;
- Industrial sectors;
- Energy sector;
- Financial institutions.

In their role as regulators, in phase one and two governments organized the allocation of emission allowances. At the highest level, the European Commission (EC) approved or rejected National Allocation Plans. Currently, the EC specifies guidelines for the flexibility mechanisms of the Kyoto Protocol. On a lower level, the EC General Directorate of the Environment and Climate controls and operates the Central EU Transaction Log and implements the design reform of the EU ETS.

A prominent player is the energy sector. Due to the ongoing liberalization process of the European electricity industry, most of the utilities have already well-developed trading desks. Therefore, the inclusion of emission permits in their trading portfolios was a relatively natural step. The rest of the industrial sectors covered by the EU ETS were instead less active on the market.

Because the access to the market of permits is generally not restricted to covered installations, financial institutions have been quite active player in phase I of EU ETS. Financial institutions include brokers, banks, insurers and private carbon funds. The rationale behind non restricting the market to compliance agents stems from the need for liquidity in the market. Financial institutions can play the role of intermediaries for a number of small emitters who are not familiar with markets and prefer delegating their allowance management to a third party.

[42] The Chicago Climate Futures Exchange is currently offering RGGI-regulated contracts as well as other US specific environmental contracts (NOX, SO_2 and California Climate Action Registry).

3.2.2 Carbon Products

In terms of type of contracts, carbon markets are comprised of spot, future and option contracts. While future contracts are the main tradable instrument, options on emission allowance were getting momentum at the early stage of the second phase of the EU ETS. According to Thomson Reuters, option trading represented in 2008 10 % of all EUA-related trades, and 6 % of CER-related trades, despite their recent availability.[43]

The offered options were and still are for the most part vanilla European call and put used either to hedge price and volume uncertainty or to speculate on price variation.

Apart from options, the second key innovation was due to the creation of immediate delivery spot contracts: the spot product segment experienced a large increase in activity, with spot transactions accounting for only 1 % of all transactions in the first half of 2008 but rising to 7 % in the third quarter of 2008 and 19 % in the fourth quarter.[44] Spot contracts are either used by institutional investors to seek arbitrage profits in the spot-future spread, to enter complicated structured operations requiring immediate settlements or to balance in short notice their allowance portfolios.

The recent market conditions of 2011, which favored pure speculative traders over regulated industries, has had the double effect of lifting the overall volume for carbon contracts and reenforcing the predominant role of future contracts, both for EUAs and Kyoto CERs and ERUs. According to the World Bank, options represented in 2011 10 % of EUA transaction value but were losing some relative ground to futures. The role of spot contracts, after the rising phase that lasted until 2009, is rapidly vanishing (2 % of total EUA trading value). As in the EUA market, the largest share of secondary CERs and ERUs were traded in the futures market. Secondary CER and ERU futures volumes represented 92 % of secondary offset volumes traded in 2011, with a very limited role for spot and option markets (see Fig. 3.12).

Since energy switching (from coal to natural gas) still represents the most efficient abatement option, emission exchanges are often attached to energy trading platforms. Nonetheless, the recent progresses and developments of clean technologies tend to put in place new portfolio combinations, with direct investments in renewable technologies and other form of abatement investments. In the case of CERs, the World Bank acknowledges a shifting pattern from pure ERPAs to hybrid and direct investments in clean companies, either trough "carbon loan"[45] or capital participations in carbon investment vehicles.[46]

[43] According to the World Bank, options volume on ECX grew five-fold to 240 $MtCO_2$-e between 2007 and 2008.

[44] Accounting for 36 % of all transactions in December 2008.

[45] A "carbon loan" is a contingent loan between two companies, where the service of the loan is linked to the supply of a certain volume of CERs.

[46] According to the World Bank, it represented 27 % of the total capitalization secured for 2008.

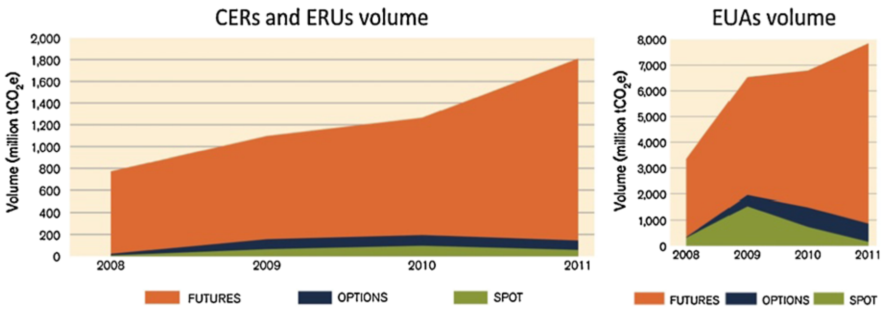


Fig. 3.12 Breakdown of product types for EUAs, CERs and ERUs. Source: World Bank, 2011

Food Price and Speculation

1. **The Context**
   Climate change has very diverse impact and one of them is the greater stress put on agricultural commodity supply. As a compounding factor, the role of investment players in the determination of food prices seemed to have partially contributed to some periods of famine in developing countries and especially in Africa. This is this interconnectedness between financial markets that is presented here.

   Recent years have seen a surge in commodity prices. The IMF's food price index increased by more than 80 % between the start of 2007 and the middle of 2008. According to the FAO, prices of maize, rice and wheat reached their highest levels in 30 years in 2008 (see Fig. 3.13).

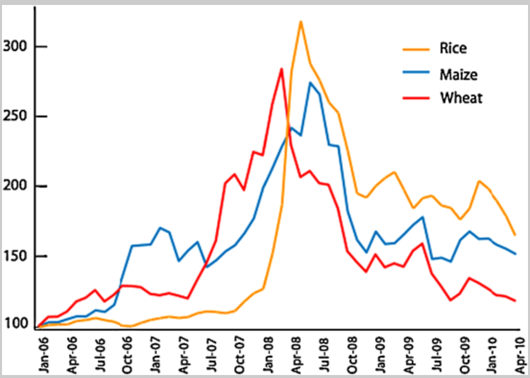


Fig. 3.13 Source: FAO 2010

   In terms of poverty, rising pricing of basic food commodities have aggravated an already dire situation. Many developing countries that promoted cash crops and exportation over productions for the home markets are net importers of staple foods and are immediately impacted by a rise in market prices.

For the poorest households in those countries, who spend between 50 % and 90 % of their income on food (Source: IMF), even a slight increase in the price of key food supplies can have tremendous impacts and trap those people into long-term poverty. According to the World Food Programme, high food prices led to the number of chronically malnourished people increasing by 75 million in 2007 and a further 40 million in 2008.

Increases in food price also have long-lasting rippling effects and reinforce poverty traps:

- Households are forced to reduce their intakes of fruits, vegetables, dairy and meat in order to afford staple foods, increasing risk of illnesses and reducing labor productivity.
- They may have to tap into their savings, sell assets or take out additional loans all of which increase their burdens and erode their safety nets.
- They often have to reduce spending on luxury items such as health care, education and family planning.

2. **The Supply-Demand Drivers**

Food price variations have multiple drivers. The most intuitive are derived from a combination of short-term shocks and long-term structural changes in the supply and demand for those products.

Supply drivers that could impact positively the price of staple commodities are drivers which contracts the outputs available on the global markets:

- Reduction in the annual agricultural yields or real crop failures due to (i) variations in weather's seasonalities, (ii) increased numbers of droughts or (iii) harsher winters.
- Reallocation of agricultural land to more rentable activities and away from staple food production. According to several research papers (C. Gilbert, *How to Understand High Food Price* (2008) and G. Rapsomanikis, *The 2007–2008 food price swing*. FAO Commodities and Trade Technical Paper 12 (2009)), the recent promotion of biofuels and their relatively high price (which is correlated to the price of oil) compared to classical alternatives have pushed farmers in the US and producers in developing countries to drastically reduce their productions of food commodities.
- Long-term climate change impacts will reduce production potentials in areas that greatly depend on the availability of food at an acceptable price. According to the UNDP (as reported by the

Special Rapporteur on the Right to Food, Olivier de Schutter (http://www.srfood.org)), the number of additional people at risk of hunger could reach 600 million as a direct result of climate change (UNDP Development Report 2008). In Southern Africa, yields from rainfed agriculture could be reduced by up to 50 % between 2000 and 2020 (IPCC).

- Several countries have enacted policies that limit the amount of agricultural productions to be exported as a response to rising food price. In the case of rice, key rice exporters such as India, Vietnam and Thailand introduced export bans to protect rice availability nationally, reducing the international supply.

On the demand side, a conjunction of factors have recently push up the consumption of food commodities:

- The first element is the compound effect of a increased global population with improved purchasing power of several developing countries, creating additional customers for food resources that put pressure on food prices.
- A second element is the alimentation mix of developing countries (along with developed countries) that include more and more meat (and dairy). Producing meat requires large amount of cereals (with relatively low transformation yield) that are diverted from traditional human consumptions as a final product to become an intermediate input in meat production.

3. **Can Speculation be Partially Responsible?**
With those short and long-term factors into consideration, the role of speculation is currently up to debate. Compared to traditional investments that are tied to the nature of the physical underlying, speculation is often defined as a purely financial position taken to generate profits from a specific market strategy (according to the FAO, only 2 % of future contracts end in the delivery of the physical commodity). Food commodities are in this case part of a new asset class.

As such, speculation is not constrained to the available volume of physical assets and can be easily leveraged: According to the United Nations Conference on Trade and Development's report (2009), between 2006 and 2008, it is estimated that speculators dominated long positions in food commodities, holding 65 % of long maize contracts, 68 % of soybeans and 80 % of wheat while commodity index trading strategies have risen from $13 billion at the end of 2003 to $260 billion as of March 2008 (Testimony of M. Masters before the Committee on Homeland Security and Government Affairs, US Senate, 2008 (see Fig. 3.14)).

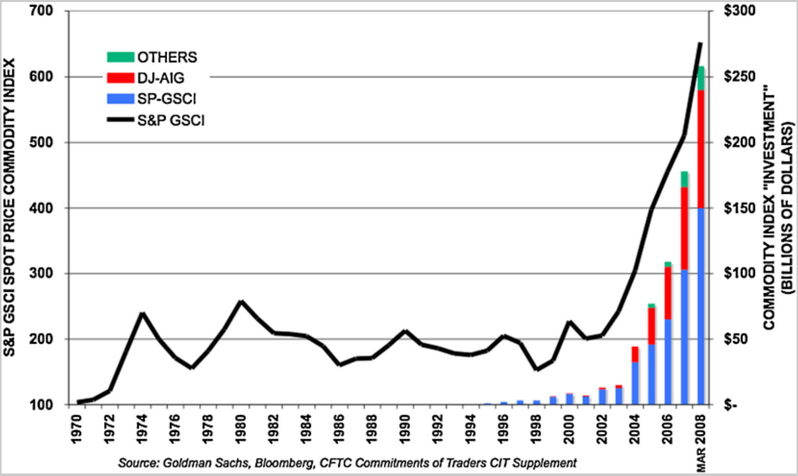


Fig. 3.14 Source: FAO 2010

Speculation has obvious benefits, from bringing liquidity and improving price revelation to offering hedging strategies to producers. However, two conditions have to be met for those benefits to actually materialize: (a) the participating speculators should not have market powers and (b) the commodity market should be free of trend-following investors.

While it is extremely difficult to test for the existence of market power in transactions that are largely OTC, recent events have been reported that seem to indicate that very large and potentially distorting players are present in the commodity markets. ARMAJARO, a large commodity hedge fund based in London with $2 billion under management, took delivery of 7 % of cocoa annual production in a single day of 2010, effectively drying up the market (http://www.guardian.co.uk/business/2010/jul/19/speculators-commodities-food-price-rises). During the same year, the price of coffee shot up 20 % in just three days as a direct result of hedge funds reversing their position from a initial bet that the price of coffee would be falling. In presence of deregulated and highly opaque hedge funds benefiting from low interest rates and high leverages, the risk of market distortion has become a real threat for food commodities.

According to testimonies before the Committee on Homeland Security and Government Affairs (2008), the presence of trend-following investors is even more troublesome as it could have the largest negative impacts on food prices. Traditional speculators generate most of their value from arbitrages of short-term market distortions and as such, stay close to market fundamentals and expected equilibrium prices. They *buy* *and sell* commodities and represent the representative speculator with the positive impacts depicted in most economic books.

However, a new class of speculators, the index speculators, has recently emerged. Those speculators are more investors in the sense that they allocate a portion of their portfolios to "investments" in the commodities futures market. They represent corporate and government pension funds, sovereign wealth funds, university endowments and other institutional investors.

Those investors are usually not concerned with the price per unit since the main motivation behind this investment class comes from being historically uncorrelated with traditional asset classes (equity and fixed income). These investors buy the required number of contracts until their allocation has been entirely used, mostly in popular commodity indices (Standard & Poors, Goldman Sachs Commodity Index and the Dow Jones—AIG Commodity Index) and *keep their positions*.

A sign of the trend-following behavior is shown in Fig. 3.15:

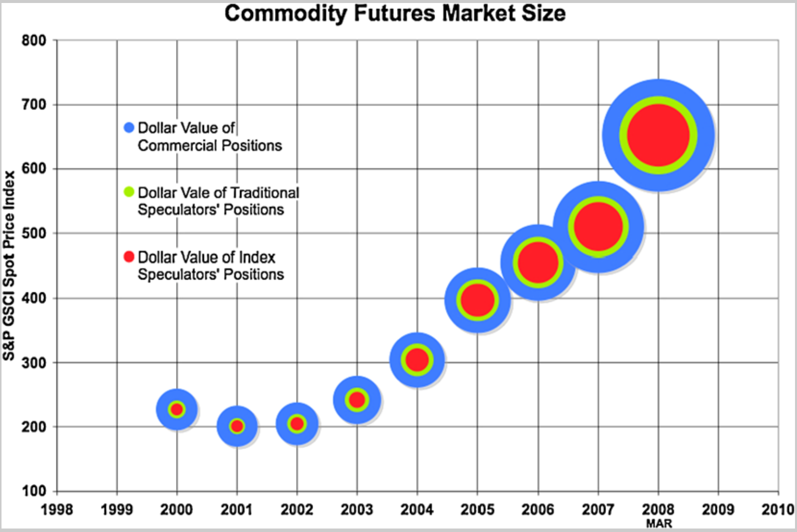


Fig. 3.15 Source: Bloomberg, CFTC Commitments of Traders CIT supplements

As money pours into the markets, two things happen at the same time: the markets expand and prices rise. Rising prices attract more index speculators whose tendency is to increase their allocations as prices rise (since it increases the current absence of correlation with other market classes).

When this type of investors, who value commodities as diversification instruments and not as real and vital items, is allowed to participate in commodity trading, price distortion is to be expected. With many of them relying on a portfolio allocation "consensus" (or trend), it could also generate massive swings if funds were to unload their positions at a latter stage (of if the uncorrelation potential of the asset class were to vanish).

3.3 The Future of the CO_2 Markets

3.3.1 At the International Level: The Limited Steps of COP 16, 17 and 18

Despite a disappointing COP 15 in Copenhagen in 2009, the 2010 COP 16 in Cancun has managed to relaunch the international discussions and renew optimism in the future of post-Kyoto climate regulations. At the Cancun conference, UNFCCC members agreed to keep average global temperature below 2 °C in comparison to pre-industrial levels, leaving open the possibility of moving to a 1.5 °C target.

However, according to the analysis of the UNEP emission gap report, the reduction pledges proposed during Cancun by the developed and developing countries fall short of reaching the target: under a best case scenario, only 60 % of the required emission reduction would be covered by the non-biding efforts of the members.

To compensate for the lack of global mitigation efforts, adaptation measures were promoted during Cancun as high-impact instruments that could be directly targeted towards the most pressing needs of the developing countries.[47] As such, the Cancun agreements formalized a commitment by developed countries to mobilize $100 billion a year by 2020 to address the mitigation and adaptation policies of non-Annex I countries.

Members also decided to establish a "Green Climate Fund" with the aim of enabling and supporting enhanced action on mitigation, including substantial finance to reduce emissions from deforestation and forest degradation (REDD-plus), adaptation, technology development and transfer and capacity-building.

The source of funding are not yet clear but it is expected that the Fund should receive a "*significant*" portion of the $100 billion pledge, which will amount to $US

[47] The Copenhagen Accord, in its final formulation stated the need for additional adaption finances as such: "*Adaptation to the adverse effects of climate change and the potential impacts of response measures is a challenge faced by all countries. Enhanced action and international cooperation on adaptation is urgently required to ensure the implementation of the Convention by enabling and supporting the implementation of adaptation actions aimed at reducing vulnerability and building resilience in developing countries, especially in those that are particularly vulnerable, especially least developed countries, small island developing States and Africa. We agree that developed countries shall provide adequate, predictable and sustainable financial resources, technology and capacity-building to support the implementation of adaptation action in developing countries.*"

30 billion a year for the period 2010–2012. In general, funding for adaptation will be prioritized for the most vulnerable developing countries, such as the least developed countries, small island developing states and Africa.

The main uncertainty however, unresolved at the time of writing, is whether or not this new pledge really constitutes additional capital inflows or a rebranding of existing bilateral and multilateral agreements.

Additionally, members agreed to facilitate transfers of capital and technologies to developing countries and least developed countries through (i) an improved CDM structure, (ii) the recognition of the developing countries' contributions to mitigation and (iii) a better representation of forestry-related activities.

- The main area of improvement for the CDM world will be the development of standardized baselines combined with monitoring methodologies in order to reduce the costs of development and the risks associated with bottleneck issues (see Sect. 3.1.3) and regulatory failures. At the same time, a demand shock that will be caused by the EU ETS restricting post-2012 projects to those located in least developed countries will reinstate low-growth economies at the center of the flexible mechanisms (see box "the future of the CDM market").
- The Cancun conference formally recognized developing countries' Nationally Appropriate Mitigation Actions (NAMAs) which are aimed at reducing emissions relative to business-as-usual emissions in 2020, contingent upon the provision of finance, technology and capacity building. At the time of writing, 45 countries have registered a wide range of mitigation actions with the UNFCCC.
- Finally, the Cancun conference recognized the need for a broader incorporation of forest-related activities in efforts to limit climate change. Forest-related and agriculture-based mitigation projects have the dual benefit of being relevant for most developing countries while providing rippling effects in productivity gains, expanded supply of staple foods and improved stewardship of natural resources. Specific recognition was given to the reduction from deforestation and degradation through REDD and REDD+ initiatives. This means that forests will be included in any future agreement with the possibility of generating international credits from these activities.[48]

The COP held in Durban in 2011 offered definitely mixed results. On the one hand, it seemed clear that many countries were in the process of disengaging themselves from the mitigation efforts required by the UNFCCC, a obvious consequence of the significant crisis that started in 2008. On the other hand, Durban set the stage for future negotiations on an international accord, while improving the effectiveness and efficiency of several Kyoto instruments.

COP 17 has managed to put forward three key "results":

- A provisional accord was agreed upon for a second commitment period of the Kyoto Protocol, by which the rules of the KP will be extended for a second effective period that will start in 2012 until either 2017 or 2020. The prolongation

[48] COP 16. 2010, Outcome of the work of the Ad Hoc Working Group on Long-term Cooperative Action under the Convention.

still requires eventual accession or ratification by the requisite number of parties to enter into force and will be represented by a "provisional application" in the interim period. The new commitment measures for the Annex I countries (referred to as quantified emission limitations or reductions objectives—QELROs) under the second period have to be determined at the end of 2012. It is however clear that the scope of the accord will be greatly reduced, with the defection of countries like Canada and possibly Japan and the Russian Federation. In effect, the second commitment period is expected to be limited to the 27 Parties forming the European Union (EU), as well as Norway, Switzerland, and Iceland.

- The conference also put in motion a new roadmap toward a global legal agreement on climate change by 2015, called the "Durban Platform". Under this platform, a "protocol, legal instrument, or an agreed outcome with legal force" should be defined by 2015, to be implemented by 2020. The platform will consider the new results of the fifth assessment report of the IPCC to set more stringent mitigation measures.
- Initiated during COP 16 in Cancun, the Green Climate Fund progressed towards operationalization. The results of the Transitional Committee were acted and a governing instrument formalized. The Fund will start operations with the World Bank as interim trustee and the UNFCCC and Global Environment Facility as interim secretariat.

COP 18 in Doha: An Unclear Path Towards a Post-Kyoto World

The conclusions of the Doha conference were slim and largely uncertain, as it was clear that developed and developing countries had still multiple issues unsolved on the topic of shared effort, redistribution and assistance. However, few things of limited impact were agreed under the Doha 'Climate Gateway':

- The agenda set during the Durban negotiations was maintained with a successor to the Protocol set to be developed by 2015 and implemented by 2020.
- The Kyoto Protocol was extended for a second phase until 2020. However, Japan, New Zealand, Canada, Russia, Belarus and Ukraine decided to opt out from this second phase, which will only cover developed nations that are responsible for 15 percent or less of global emissions (mainly the European Union).
- For the first time, the concept of "loss and damage" was included in the negotiation documents, stating that richer nations could be financially responsible to other nations for their failure to reduce carbon emissions.
- Wealthy nations put off for a year resolution of the dispute over the implementation of the Green Climate Fund to countries most heavily affected by climate change.

With the important exits of active users of flexible mechanisms and their permits (AAUs, CERs, ERUs), it remains to be seen how these markets will thrive in the second phase of the Kyoto Protocol.

3.3.2 At the Domestic Level: A Fragmented Landscape

With the future international space partially left empty, countries and regions are in the process of developing domestic solutions that differ in their scope and pace but almost always combine elements of cap-and-trade schemes, baseline and credit mechanism, carbon taxes, subsidies, emission standard and renewable energy and energy efficiency certificates.

Along with the European Union which will remain the main player in the carbon field, multiple competing solutions will emerge in the near future at the national or regional levels, creating a fragmented regulatory environment with specific deadlines and requisites.

The following section will provide a rapid overview of the most promising regulations in Annex I countries, starting with the EU ETS (for more details on the future of the EU ETS, please refer to Sect. 3.1.2). Some important non-Annex I countries are also covered.

The EU ETS From 2013, the EU ETS will tighten its centralized cap. It will be reduced by 1.74 % each year of the average annual level of the Phase II cap. To ensure that adequate mitigation efforts are undertaken within European borders, access to UNFCCC project offsets will be limited to no more than 50 % of the reduction required in the EU ETS.

The EU ETS will also incorporate three new directive designed to increase the perimeter of regulation and to improve the mitigation process within the borders of the European Union: (i) the partial auctioning of allowances, (ii) the inclusion of new industries and (iii) a practice of "effort sharing" for sectors not included in the EU ETS.

The United States Considering the large uncertainties surrounding any new legislations for emission reductions, the US efforts are for now essentially spearheaded by the new AB-32 legislation in California and its role in the Western Climate Initiative.

AB 32 requires California to cut greenhouse gas emissions to 1990 levels by 2020. It also identifies a cap-and- trade program as one of the strategies the state will employ to reduce GHG emissions. During the program's first compliance period (2013–2014), large stationary sources that emit at least 25,000 tCO_2-e per year in the industry and electricity sectors will be covered, including out-of-state generation (i.e., imports). As a cost-control measure, AB 32 allows entities covered by the scheme to purchase and use offsets for compliance purposes, but volumes are limited to 8 percent of annual emissions. Offsets will come from a domestic offsets program with the possibility of importing international forest offsets. California also has a strong renewable energy mandate and a requirement that the carbon content of the state's vehicle fuels be cut by 10 percent by 2020.

California is also the leading member of the Western Climate Initiative (WCI), an association of American and Canadian states which aims to reduce regional GHG emissions to 15 percent below 2005 levels by 2020. California and Québec (the other regional jurisdiction that passed a cap-and-trade regulation) are now working toward linking their systems from the start of their programs in January 2013.

Australia Australia decided in 2010 to prepare a carbon plan over a three-to-five year period that will transition into an emissions trading scheme. In November 2011, the country passed a Clean Energy Legislative Package with a goal of reducing net emissions by 5 % below 2000 levels by 2020. The package includes a Carbon Price Mechanism (CPM), which will start with a fixed price for the first three years. The price will be set at A$23/ton (€18.50) (indexed annually by 2.5 %) and will operate similar to a tax. From July 2015, the CPM price will float but will be constrained within a price floor and a ceiling until July 2018 when the price will float freely. The floor will be set at A$15 and the ceiling will be set at A$20 above the international price. Starting in 2015, the number of carbon permits (Carbon Units, CU) will be limited by a cap which will be set by mid-2014 for the first five years of the flexible price period of the CPM. Through a rollover window, each year an additional year's cap will be determined such that there will always be caps set five years in advance. The CPM is expected to cover approximately 500 businesses representing 60 % of Australian GHG emissions from electricity generation, industrial facilities, fugitive emissions, and some landfills sectors. In 2015, from July 2015, participants to the program will be able to supply 50 % of their emissions obligation with international units, without the constraints imposed by the EU ETS.

The country is also developing a domestic offsets scheme known as the Carbon Farming Initiative (CFI), which aims to provide new economic opportunities to farmers, forest growers, and landholders. The aim of the CFI is to reduce or avoid emissions in specific land-based projects (reducing emissions from Savannah burning, fertilizer use, livestock production and capture of methane emissions from landfills) and to sink carbon in bio-sequestration projects (soil management and tree afforestation). CFI projects will be able earn Kyoto ACCUs that will be fungible in both the CPM and in the Kyoto compliance market.

New Zealand In early 2011, a review of the NZ ETS commenced, as required by Climate Change Response Act 2002.

The scope of the review included the following elements: coverage of agriculture, allocation mechanisms for New Zealand Units, whether or not to keep the fixed-price cap of NZ$25 and the one-for-two obligation for emitters, and whether synthetic greenhouse gases should be included in the ETS.

The report defined also a framework of operation for the post-2012 period. It recommends that an absolute cap be placed on covered emissions, that the ceiling pricing be maintained beyond 2015 and that international offset credits be limited in the scheme. The ability to fully cover emissions with Kyoto offsets has so far depressed the price of the forestry national market, limiting the willingness of foresters to work with the scheme.

However, at COP 18 in Doha, the country announced that he was opting out from the second phase of the Kyoto Protocol.

Table 3.3 Most recent state climate change policies in Canada (Source: World Bank)

Policy	Implementation	Comment
Act to amend the environment quality act and other legislative provisions in relation to climate change	Quebec	The amendment enables Quebec to have a cap-and-trade emissions trading scheme, and to link to the emissions trading schemes being developed with other jurisdictions
Greenhouse gas reduction (cap and trade) act 2008	Ontario	The amendment enables Ontario to have a cap-and-trade emissions trading scheme, and to link to the emissions trading schemes being developed with other jurisdictions
Greenhouse gas reduction (cap and trade) act 2008	British Columbia	British Columbia is the first province in Canada to introduce an act allowing a cap-and-trade scheme. The proposed scheme enables British Columbia to link to the emissions trading schemes being developed with other jurisdictions
Climate change and emissions management act	Alberta	Covers facilities with GHG emissions greater than 100,000 tons. Requires emissions intensity reductions of 12 percent
Management and reduction of greenhouse gases act	Saskatchewan	Covers facilities with GHG emissions greater than 50,000 tons. Requires emission reductions from a baseline by 2 percent per year from 2010 to 2019

Canada Canada has aligned its international commitment with the United States and plans to reduce total greenhouse gas emissions by 17 percent from 2005 levels by 2020. The target is inscribed in the Copenhagen Accord. At the federal level however, Canada has announced in December 2011 its unilateral decision to quit the first commitment period of the Kyoto protocol, without achieving the targeted reduction in emissions. Canada restated in 2012 its choice of quitting the Protocol for its second phase, which implied that the country had no longer an emissions reduction to commit to.

With disappointing decisions at the federal level, the burden of emission reduction rests within the provinces (Table 3.3).

Japan As with Canada, Japan decided to exit the second phase of the Kyoto Protocol, which put the burden of emissions reduction on national or local initiatives. Until 2011, the government of Japan considered the ETS component an important policy measure for Japan to achieve its announced target of reducing GHG emissions by 25 percent by 2020 compared to 1990 levels. However, Japan has tied the development of an ETS to broad international agreement, which has the foreseeable result to defer its implementation. To compensate for the ETS deferral, other components (introducing a carbon tax and establishing a feed-in tariff for all renewable energy sources) may pass in 2011 (Table 3.4).

Brazil On December 29, 2009, the Brazilian Parliament adopted a new law that establishes the National Climate Change Policy (NCCP) of Brazil and sets a volun-

Table 3.4 Most recent state climate change policies in Japan (Source: World Bank)

Policy	Implementation	Comment
Emissions trading scheme (deferred)	Japan	On March 12, 2010, the government of Japan proposed the "Basic Act on Global Warming Countermeasures," an overall climate change policy framework that includes introducing an ETS
Feed-in tariffs	Japan	Feed-in tariff for all renewable energy sources with the goal of increasing domestic energy generation from renewable sources to 10 percent of total primary energy supply by 2020
Anti-global warming measure tax	Japan	Anti-global warming tax is proposed as add-on to existing taxes covering wide range of fuels, of which rates are proportional to CO_2 emission
Voluntary experimental integrated ETS	715 organizations	715 organizations had applied to participate, of which 521 supplied targets (as of July 2009). The trial scheme aims to bring together several existing initiatives, such as the Keidanren Voluntary Action Plan, plans for a domestic offsets scheme, and the Japan-Voluntary Emissions Trading Scheme (J-VETS), which targets smaller emitters
Tokyo emissions trading scheme (cap-and-trade)	Tokyo	The Tokyo metropolitan area launched its own mandatory cap-and-trade scheme on April 1, 2010, which targets office and commercial buildings (including universities) and factories. The scheme covers approximately 1,400 installations and 1 percent of the country's emissions
Saitama prefecture trading scheme (cap-and-trade)	Saitama Prefecture	Starting April 1, 2011, Saitama, the fifth largest prefecture in Japan, will become the second Japanese prefecture to implement a mandatory emissions trading scheme. Saitama and Tokyo signed a pact to link their cap-and-trade schemes in the future

tary national greenhouse gas reduction target of between 36.1 and 38.9 percent of projected emissions by 2020.

Among other instruments, the NCCP considers the creation of a Brazilian Emission Reductions Market (BERM) to achieve the voluntary emission reduction target (will be operationalized by Brazilian stock exchanges and the Securities Commission) as well as several new commitments for LULUCF, energy efficiency measures and sectoral measures in the steel industry.

In 2010, a new law passed to establish the System of Incentives for Environmental Services (SISA) to preserve and foster a forest-based, low-carbon economy, in coordination with a new comprehensive REDD+ policy.

At the local level, Rio de Janeiro is the main driver of innovation in the environmental finance sector. An emissions trading scheme (ETS) for the State of Rio de Janeiro will have its first legally binding period for private companies starting in 2013. The initial targets will primarily cover the oil and gas, steel, chemical, petrochemical, and cement sectors. Additionally, a partnership between the city of Rio and the private sector for the creation of a Environmental Asset Exchange, a platform that will facilitate the trading of allowances, offsets, and other carbon-linked financial products between private companies.

China China released in March 2011 its 12th Five-Year Plan of National Economic and Social Development. It sets a carbon-intensity reduction target of 17 % and aims to cut energy intensity by 16 percent by 2015. These targets are consistent with the 40 to 45 percent reduction in carbon intensity from 2005 levels that was first announced at the Copenhagen Conference and reaffirmed at the Cancun Conference.

As part of the 12th Five-Year Plan China will increase forest cover by 12.5 million hectares by 2015, improve GHG emissions and energy monitoring systems, promote energy efficiency in industrial plants and buildings, support the expansion of public rail transport infrastructure, and continue the development of nonfossil fuel energy sources.

China is introducing pilot emission trading schemes in two provinces (Guangdong and Hubei) and five cities (Beijing, Tianjin, Shanghai, Chongqing, and Shenzhen) that may be expanded to a national scheme by 2015.

India In 2008, India launched the National Action Plan on Climate Change (NAPCC), which involves the establishment of eight programs on solar technology, energy efficiency, sustainable habitat, water, Himalayan ecosystem, green India, agriculture, and strategic knowledge. Two market-based mechanisms were launched: the Renewable Energy Certificate (REC) schemes and the Perform Achieve and Trade (PAT).

Under the NAPCC, a Renewable Purchase Obligations (RPOs) requires that 5 % of the nationwide share of electricity be sourced from renewable energy in 2010, (with an annual increase of 1 % for ten years). To achieve this goal, companies can purchase RECs and are traded on the India Energy Exchange (IEX) and Power Exchange India Ltd (PXIL).

The PAT is part of the energy efficiency mission, approved in 2010, that is expected to achieve total avoided capacity addition of 19,598 MW, representing fuel savings of around 23 million tons per year and greenhouse gas emission reductions of 98.55 million tons per year, over the next four years. To this goal, the PAM is a market-based mechanism set to enhance cost effectiveness of meeting energy efficiency improvement targets in energy-intensive large industries and facilities. The scheme mandates reduction targets to Designated Consumers (DCs) that collectively account for 45 % of the nation's commercial energy use. The DCs that performed above their benchmarks will receive Energy Efficiency Certificates (ESCerts) to be traded bilaterally or through the two national power exchanges.

Finally, a solar mission, approved in 2009, is expected to enable setting up of 200 MW of off-grid solar power and cover 7 million square meters with solar collectors in its first phase from 2010–2013. It has set a voluntary target of 20,000 MW by 2022.

In 2011, India submitted its voluntary emission reduction objective under the Copenhagen Accord, a voluntary target of reducing emissions intensity of its GDP by 20–25 percent by 2020 in comparison to the 2005 level.

It is however expected that India's per capita CO_2 emissions grow from 1.1 ton in 2001 to 3–5 tons in 2030.

4 The Economics of Mitigation Strategies

"Climate change, like other environmental problems, involves an externality: the emission of greenhouse gases damages others at no cost to the agent responsible for the emissions."
Sir Nicolas Stern

4.1 The Cause of GHG Pollution: The Negative Externalities

Environmental externality is the materialization of a simple market failure: when agents conduct economic activities that engage imperfectly priced environmental assets, either as inputs (excessive consumption) or outputs (pollution), they engage in socially excessive levels of harmful activities.

Consider an cement producer that emits a polluting level of GHG by burning combustibles in order to produce cement. The primary goal of the company is to maximize its private profits, such that:

$$\max_{q}\big(p.q - c(q)\big) \tag{4.1}$$

with q represents the units of cement, p the unit selling price and $c(q)$ the cost function associated with producing q units of cement. It is usual (and realistic) to consider $c(q)$ as function monotonically increasing in q (producing more cement cost more, not less), which means, at the very least, that $c'(q) > 0$ (in general, marginal costs also increase: $c''(q) > 0$).

The optimum (and highest possible profit) for the company is obtained when the private marginal cost is equal to its private marginal revenue:

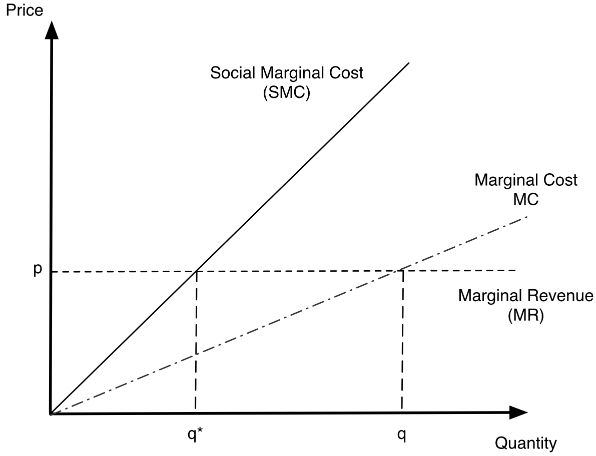
$$\text{Marginal revenue} \quad \leftarrow \quad p = c'(q) \quad \rightarrow \quad \text{Marginal cost}$$

In a competitive market, firms with free access to environmental resources will continue to engage in polluting activities until the marginal return (the difference between the marginal revenue and the marginal cost) of their production is zero.

However, at the social level, the production of cement incurs a social cost which is the release of GHG emissions in the atmosphere (causing global warming at the global level but potentially health and respiratory problems locally around the plant).

M. Chesney et al., *Environmental Finance and Investments*,
Springer Texts in Business and Economics, DOI 10.1007/978-3-642-36623-9_4,

Fig. 4.1 Optimum quantity produced in presence of externality



For simplicity, we model this desutility as a quadratic function of the amount of cement produced, q^2, which implies that the marginal social cost of the GHG emissions equals $2.q$.

If, through an internalizing mechanism (that we do not define for the moment), the producer were aware of and responsible for this additional social cost, its new profit function would be modified such that:

$$\max_q \left(p.q - c(q) - q^2\right)$$

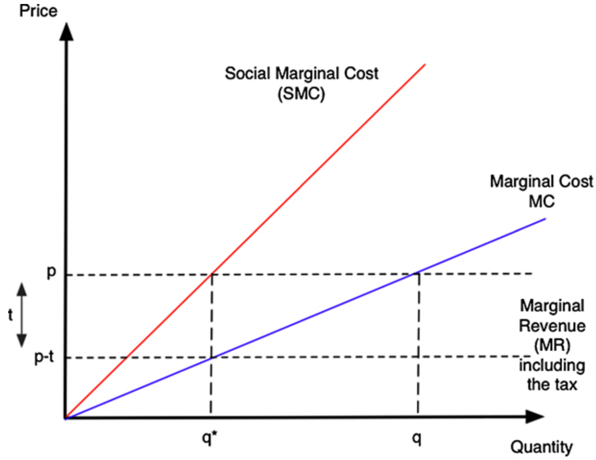
resulting in the following optimum:

$$p = c'(q) + 2.q$$

When the cement producer takes into account the total cost function (private and social), the new social optimum results in a lower production of cement. In presence of externalities, the polluting company produces too much, to the point of being harmful for the society, as depicted in Fig. 4.1 (when the producer accounts for the social desutility of its pollution, the quantity of cement produced is no longer q but q^*, a lower level).

The policy implication of this result is clear. The costs (at this stage, social or private) of polluting or depleting activities need to be "internalized" and agents benefiting from those externalities need to be confronted with a price equal to the marginal external cost of their polluting activities to induce a social optimum.

Fig. 4.2 Optimum quantity produced in presence of tax



4.2 Using Price Constraint as a Centralized Solution: Taxes and Subsidies

To ensure the right level of pollution (or the right amount of resource depletion), a price distortion can be enforced to internalize in the producer's maximization the social cost resulting from the polluting activity. The idea, promoted by Pigou in a seminal article (Pigou 1918), is to use a tax that plays the role of a levy on the polluting agent. To work, this tax must be set in a way that ensures that the new optimum (where the marginal cost equals the levied marginal revenue) ends up at the optimal quantity q^*. Consider now the maximization of the producer in presence of a unit tax per unit of produced good:

$$\max_q \left(p.q - c(q) - tq\right)$$

If the marginal cost function $c'(q)$ and the social cost function (in our previous example q^2) are known, setting up the tax level t is not difficult:

$$t = p - c'\left(q^*\right)$$

Figure 4.2 shows graphically the effect of using a tax system to reach the socially optimal level of production.

The imposition of a unit tax on the production produces a downward shift of the marginal revenue line by the amount of the tax, from p to $(p - t)$. Faced with this revenue decrease, a company has an incentive to reduce the output from the initial q to q^*, where q^* is the social optimum.

The solution proposed by Pigou is apparently simple and efficient. It relies however on the very strong hypothesis that both the private and social marginal cost functions are perfectly observable, which is rarely if ever the case. Moreover, while it can be assumed that the social marginal cost is unique across the population (already a strong assumption), the private marginal costs are unique to each polluting company. The optimal t would imply a perfect knowledge of every marginal cost curve, which is cumbersome and quite unrealistic even for the most autocratic government.

The theory also recognizes that a subsidy per unit of emission reduction could establish the same incentive for abatement activity as a tax of the same magnitude per unit of pollution emitted. Instead of charging the polluter for its emissions and forces him to reduce its production, a subsidy incentivizes costly abatement strategies. Consider the example of the cement company presented in Sect. 4.1. Instead of paying a tax for each unit of cement produced, the company is now offered a subsidy for each unit of pollution abated, such that:

$$\max_q \big(p.q - c(q) + v\big[q^* - q\big]\big)$$

where v is the subsidy per unit of pollution abated and q^* is the production benchmark used to calculate the subsidy. For simplicity, we consider here that the marginal abatement cost is negligible for all levels of pollution (or that the only abatement solution is a reduction of the production of cement q).

In the case where $v = t$, it is clear that the subsidy has exactly the same effect and result as the tax instrument. This is the case because the subsidy can be divided between a lump-sum (vx^*) which has no influence on the producer's decision and a embedded tax of the form $v \cdot x$ which has the same role as the simple tax shown earlier.

In practice, there are however important asymmetries between these two policy instruments. In particular, taxing and subsiding have quite different implications for production profitability in a polluting industry: subsidies increase profits, while taxes decrease them. At the sector level, a tax results in the contraction of both profitability and emission because tax decreases both companies' marginal and average benefits. Conversely, the subsidy induces the entry of new companies attracted by increased profits and produces an increase in output supply that translates into an increase in pollution.[1]

In the dynamic context of climate change, subsidies can be useful in early stage of emission reduction to promote technology changes and carbon-constrained activities. In this case, they should not be targeted towards abatement efforts by polluting companies but towards new carbon-constrained technologies, renewable energies and R&D (such as carbon capture and storage). According to Aghion et al. (2009), subsidies directed to "radical emission-free backstop technologies" are required to both jump-start and ease the economic transition towards a "green growth" economy.

Subsidies by themselves are nonetheless not enough. They should be used in conjunction with a carbon price high enough to foster long-term substitutions towards clean activities: the substitutions ensure that the investments in the not-immediately

[1] Readers interested in a more detailed analysis should refer to Baumol and Oates (1988).

productive technologies do no halt economic growth while a sufficiently high price on carbon incentivizes the use of new technologies over older, cheaper ones. Said differently, using a combination of subsidies and taxes allows to deal with the two problems behind intensive emissions: the environmental one and the innovation one. The use of a carbon price deals with the environmental externality while the subsidy ensures that enough R&D is dedicated to clean energy solutions. Along these lines, it is clear that reverting to a single solution is detrimental to its effectiveness: as shown in Acemoglu et al. (2009) and under certain specifications, using a carbon tax alone would have to be 15 times higher during the first five years of transition and 12 times higher over the following five years to reach the same level of substitution towards clean technology as a joint policy using subsidies and carbon price Aghion et al. (2009). The authors computed that it would represent a cumulative loss of 1.33 % of total consumption over the next 100 years. It also bears the risk of an excessive drop in consumption in the short run. On the other hand, relying exclusively on subsidies would have to be on average 115 % higher during the first 10 years compared to their level when used in combination with a carbon price.

4.3 Using a Decentralized Solution: Tradable Permits

As seen in Sect. 4.2, one of the main issues with taxes (and subsidies) is its reliance on a perfect knowledge by the social planner of both the private and social marginal cost curves to optimally infer the correct tax rate t (or the subsidy v). If one or both of them are misjudged by the social planner, it could result in a sub-optimal level of production q^{**}, with the remaining cost either borne by the firm (if the tax is too high) or the society (if the tax is too low).

In a fundamental contribution, Coase (1960), while recognizing the need for a price on externality, rebuked the "Pigouvian" tax as inefficient for the reasons mentioned above. His argument is that if ownership rights on the pollution were in place and enforced, the polluting company and the people affected by the pollution could negotiate and eventually find an equilibrium level that would correspond to the social optimum (see Fig. 4.3).

Consider the situation in Fig. 4.3 for which the current pollution level q would be on the right of the optimal pollution level q^*, indicating excessive pollution. If pollution units are associated with tradable (i.e., valuable) ownership rights, the social group affected by the pollution would be able to offer the polluting company a compensation for abating its emissions, as long as the social marginal damage is higher than the marginal abatement cost. By bargaining its way to a reduced level of pollution, the "society" will decrease the remaining pollution towards q^*. On the contrary, if q were on the left of q^*, representing an excessive effort to abate, the firm would be able to pay for more pollution, since the society would be ready to accept a compensation higher than the experienced damage. In this case, q would increase until it reaches q^*.

In a world of perfect foresight, the q^* resulting for the bargaining would be exactly the quantity obtained using a tax instrument. The permit solution has however one great advantage, which is that it does not require a perfect knowledge of the

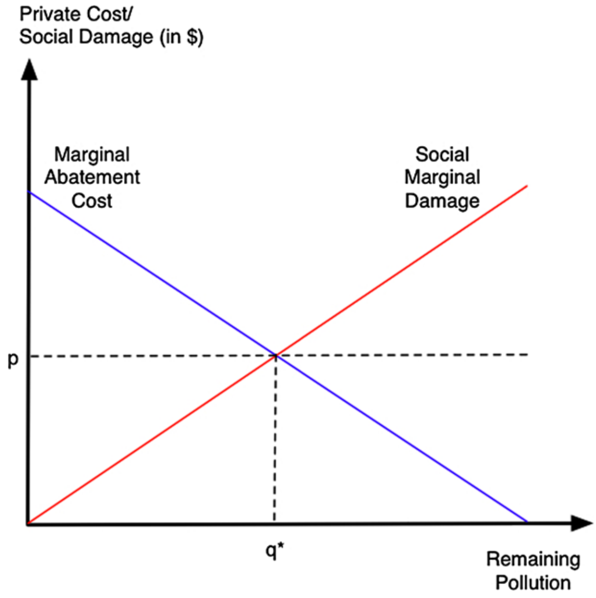


Fig. 4.3 Optimum quantity produced in presence of tax

marginal curves: the marginal cost functions are being elicited during the negotiation, ensuring that the equilibrium level of pollution is the correct one. Moreover, this solution is completely independent of who owns the initial rights. If it happens that the polluting firm owns all the rights to pollute, the society will have to buy abatement permits. If the society owns the environment and its quality, it will be able to sell pollution rights. This neutral—almost amoral—approach has indeed been criticized for not favoring a "*polluter pays*" principle.

In a situation with multiple polluting firms with different marginal cost of abatement, tradable permits also have the advantage of ensuring the least costly approach to reduce pollution. For each company, the equilibrium permit price is equal to its marginal cost, which is also the marginal cost of the aggregated abatement curve across the firms, such that:

$$S_t = MC_t \quad \forall t \in [0, T] \tag{4.2}$$

with S_t the permit price at period t and MC_t the marginal cost of abatement (see Montgomery 1972a and Chap. 5 for further details).[2]

However, permits—even in a deterministic setting—are not without their limitations. They are motivated by the assumption that ownership rights exist and that the members of the negotiations (on both sides) can be clearly identified. However,

[2]The relationship between permit price and marginal abatement cost is based on strong assumptions. As presented in Chap. 5, in presence of uncertainties and asymmetries of information about the pollution levels, permit prices may differ from the marginal abatement cost.

whenever it is difficult to link a point of emissions to its damages, the existence of negotiation is either in jeopardy or can lead to sub-optimal results.

In a similar way, a negotiation to reach the optimum requires no transaction costs. While it can be assessed that transaction costs can be inexistent or rather low in a local context with clear responsibilities, they become much more significant when the decisions are international and the responsibilities multiple and difficult to delineate. In those situations, it is also difficult to prevent free-riding, whereby some pollution firms decide to skip the negotiations because their responsibilities have not been clearly established and because no institution is in position to force them to participate (see the long-lasting American and Canadian positions about climate change and reduced GHG emissions).

Relying on tradable permits has also an ethical (or "value-based") bias that can very much be a strong weakness in the eyes of some: it implies that all the environmental damages can be associated with a monetary value. This aspect can become problematic to value morbidity/mortality damages, cultural degradation, existence value and some forms of irreversibility.

Finally, to be able to negotiate and reach an optimum, the variations in the appreciation and valuation of the damages have to be smooth. If the damage function takes either an infinite value in presence of pollution or 0 in absence of it, there is no bargaining possible as the polluting agents are not in a position to offer a compensation (or the society to pay for clean-up efforts). In those circumstances, the most common approach is to use command-and-control measures by which pollution legislations are enforced without monetization. Legal measures are often use for health-related pollutions (when it is not ethically feasible to price mortality and morbidity) or when irreversibility, combined with existence value, would imply an almost infinite price tag (for instance, the convention on international trade in endangered species, CITES, that protects rare species with plummeting populations).

4.4 The Quest for the Best Solution and the Influence of Uncertainty

4.4.1 The Quest for the Best Solution Using a Quantity Instrument: Cap-and-Trade

Due to the lack of defined ownership rights on common goods, the pure market approach promoted by Coase is practically impossible to implement in the climate change context. How would each individual household negotiate individually for the damages caused by global warming when it implies infinite peculiars assessments, strict delimitations of damages and endless negotiations, a more than cumbersome perspective. Without an imposed scarcity on pollutions (or natural resources), problems involving public goods (such as global warming) would remain outside of market's reach due to a lack of global coordination, extremely high transactions costs and free riding.

Private Goods or Public Goods?

In environmental and resources economics, some characteristics of the natural assets are essential to design effective regulations that would use the most suitable instruments. Environmental goods are often categorized among two groups: (i) whether they are rivalrous or not and (ii) whether they are excludable or not.

A Rivalrous Good is a good whose consumption by someone reduces simultaneous consumption by others. Any finite, scarce, depletable good can be considered rivalrous: fish stocks, livestocks, land and forests are all members of this group.

An Excludable Good as the name implies, can be "privatized", which means that one can find ways to prevent outsiders to access it. In theory, everything can be made excludable through the appropriate level of control and restrictions. In practice, high transaction and monitoring costs, international or global jurisdictions, inadequate legal system and lack of ownership titles favor the existence of non-excludable goods, like air or international waters.

- *Private goods* Private goods are both rivalrous and excludable goods. Agricultural capital (livestocks, land) and productive capital (machines, financial capital) are all private goods. Private goods are at the core of the Coase's theorem on market equilibrium since they can be singled out, monetized and bargained for.
- *Club goods* Club goods are goods that are excludable but non-rivalrous: their access can be limited but the consumption of each member has no effect on the consumption of the others. Members of this category are service and network assets, like R&D, satellite imageries, internet or radio broadcasting. In certain cases (like internet), the non-rivalrous characteristic is heavily dependent on the number of people using it and their intensity of usage. When pressure on the service is too high, the good reverts to either a private or a common type, depending on the ability to individualize the service.
- *Common goods* Common goods are both rivalrous and non-excludable, resulting in what is often called "the tragedy of the commons": an over-exploitation of resources that ends up with the resources' total degradation. Situations with common goods are frequent in environmental and resources economics, mostly because it is difficult and costly to monitor consumption of scarce natural resources that are not attached to property rights. Forest and land degradation or depletion of fish stocks are famous example of the problem.
- *Public goods* Public goods are non-excludable and non-rivalrous, which means there are particularly prone to free-riding and lack of effective coordination. A prime example of public goods is climate change.

To overcome the empirical limitation of the "pure market" theory, quotas and cap-and-trade markets have hence appeared as the combination of centralized agents, governments or dedicated agencies that represent the society and decentralized polluters. In a cap-and-trade mechanism (but quotas are similar), a quantity of pollution is fixed a priori by the responsible authorities, after a complete assessment of the maximum bearable level of damages. This quantity, called the cap, is set (with great hopes) to replicate the optimal level of pollution q^* as the optimum between the social desutility of pollution and the cost of abatement for the polluting firms. In a way, the agencies that determine the cap can be seen as representatives of a society whose marginal social cost is perfectly elastic around q^*: the damages are infinitely sensitive to the smallest variation of pollution.

Once the cap is set, tradable units of pollution are created and allocated to the polluter either for free (*grandfathering*) or through a compensation scheme. The differences between free and costly allocations are detailed in Sect. 4.4.2. Polluting firms are then allowed to trade those permits between them as in the pure market devised by Coase: as long as the permit unit price is higher than the cost of abatement for one unit of pollution, a firm will abate and sell the corresponding permit (it if owns it) for a profit to a firm with higher abatement costs. This will progressively lower the permit price. Conversely, a firm whose costs of abatement are higher than the permit price will buy permits on the market, pushing the price higher.

At equilibrium (when all firms are satisfied), using a market ensures that the compliance to the capped pollution is achieved at the lowest possible cost, since the permit price is equivalent to the lowest marginal cost of abatement possible. Think about it: if price were above the lowest marginal cost of abatement, the firm with the lowest cost would be selling permits at a profit, forcing down the price. Symmetrically, if the price were below the lowest marginal cost, the same firm would reduce its abatement effort and cover it by purchasing permits, therefore pushing the price up. Moreover, by offering the opportunity to sell permits and generate profits, markets incentivize technology changes and competitiveness towards clean activities (more on that in Sect. 4.5).

As shown in Sect. 4.2, in a deterministic setting with perfect knowledge, taxes are a fully equivalent alternative to a cap-and-trade scheme with tradable emission permits. An environmental authority can set a price (i.e., a tax) and adjust it such that emissions are sufficiently reduced to prescribe environmental standard and reach the optimal pollution quantity. Alternatively, it can issue the requisite number of permits directly and allow polluters to determine the market-clearing price through permit trading. The regulator can, in short, set either "price" (tax) or "quantity" (emission cap) and achieve the desired result.

However, this basic equivalence obscures some crucial practical differences between the two approaches.

Under the price approach, the regulator set a tax based on a sound and comprehensive evaluation of private and social marginal cost functions (which are, in fact, rarely known precisely). If the designed tax turns out to be too low, the pollution will exceed permissible levels and cause a sub-optimal level of damage for the society. If the tax is too high, it will harm firms and end up with sub-optimal production and growth levels. The regulatory agency might therefore have to enact periodic (and unpopular) increases in taxes to readjust its estimates.

In contrast and in theory, a system of marketable permits automatically accommodates itself to growth and inflation. Since there can be no change in the aggregate quantity of emissions without some explicit action on the part of the agency (i.e., the total quantity of permits is fixed), increased demand will simply translate itself into a higher market-clearing price for permits, with no effects on levels of pollution. On the other hand, polluters are likely to prefer the permit approach because it ensures the lowest levels of compliance costs (since the marginal cost is obtained with certainty when the proper market conditions are present).

Cap-and-trade schemes are however not impervious to a set of factors that affect their performances. They could be the objects of (i) market manipulations, (ii) transaction costs, (iii) enforcement degrees, (iv) free riding and (v) regulatory picking to which we can add the strong issue of uncertainty, presented in the next Sect. 4.4.2.

(i) Market manipulation[3] covers a varied range of socially harmful malpractices, none of them exclusive to "cap-and-trade" schemes. For instance, exclusionary manipulation, where a market monopolist forces competition out of activity by refusing to sell permits, can aggravate the inefficiencies that occur in both the market for permits and the product market. Similarly, the efficiency of marketable permit systems depends on the competitiveness of the output markets in which polluting firms compete. The introduction of marketable permits increases aggregate "welfare" if output markets are competitive (Malueg 1990). In contrast, in the presence of non-competitive output markets, a system of emission permits may reduce social "welfare" even if the market for the emission permits is perfectly competitive.

    Manipulation could also happen in the distribution of permits and could extort non-optimal permit price and social equilibrium (whenever a monopolist with high abatement cost manages to retain a higher share of initial permits than optimally allocated).

(ii) Markets are also vulnerable to transaction costs and enforcement degrees, where the costs of compliance and trading are a non-negligible component of the permit price. It was demonstrated by Keeler (1991) that under plausible penalty functions and when permit enforcement is incomplete, marketable permits could allow more pollution for non-compliant firms than under a system of uniform standard (command-and-control). This would be permitted by the trade-offs from the benefits of permit trading and the expected cost of being detected and punished. Moreover, as a result of operational and regulatory costs, trades are less frequent (reduced volume of permits traded) and equilibrium prices are diverging conditionally on the initial assignment of property rights (see Stavins 1995).

[3]For more detailed references on the topic of market manipulation, see Hahn (1984), Misiolek and Elder (1989) and Malueg (1990).

(iii) In a world without asymmetry, free riding should not be a problem observed in market. However, free riding has been recognized as a major problem for optimality by delaying adoption of new abatement technologies and limiting trading activities. Companies have an incentive to "wait and see", wishing that competitors, initially hammered by high prices, invest and adopt new technologies before them, therefore reducing subsequent compliance costs. The issue (a combination of free riding and option timing) also ensues in case of regulatory uncertainty, pushing companies to wait longer than socially optimal. With the approaching end of the Kyoto Protocol, uncertainties about the future of a globally binding protocol has been pinpointed by companies as one the main factors pushing them to postpone clean investments and trading strategies.

(iv) The strength and effectiveness of the incentives created by a cap-and-trade scheme will depend in large part on the rules that regulators apply to marketable permits schemes and on the existence of a concerted approach between the different regulators. If there is no unified front, as it is still the case for GHG mitigation, firms are incentivized to move in jurisdictions with low or no regulation in place. It creates a legal competitive advantage for them with lower costs. This displacement of polluting activities for legal purposes is called *leakage*.

4.4.2 The Influence of Uncertainty

Besides practical differences, the symmetry between price and quantity approaches is critically dependent upon the assumption of perfect knowledge and on the presence (or absence) of uncertainty. In a setting mixing imperfect (i.e., asymmetric) information concerning social and private cost functions and uncertainties about the reality of the pollution damages, the outcomes under the two approaches can differ in important ways.

In a seminal paper, Weitzman (1974) investigated the asymmetry between price and quantity instruments and produced a theorem with extremely important policy implications. In short, the theorem states that in the presence of uncertainty concerning the costs of pollution control, the preferred policy instrument depends on the relative steepness of the marginal social and private cost curves.[4]

Consider, for example, the case where the marginal social cost curve is quite steep and increasing but marginal abatement costs are fairly constant over the relevant range, as shown in Fig. 4.4. This could reflect some kind of compounding, non-linear effect where the pollutant concentration build-up creates increasing envi-

[4]Many economists and policy makers have long favored the use of a price instrument to control greenhouse gases because they are a stock pollutant and as such the marginal benefit of abatement is relatively flat. While the early literature on the problem is consistent with this view, the later literature is less categorical. It showed that the choice between a price or quantity control depends, in part, upon the assumption on the dynamic structure of cost uncertainty. For a discussion on the role of stocks and shocks concepts in the debate over price versus quantity we refer to Parsons and Taschini (2013) and references therein.

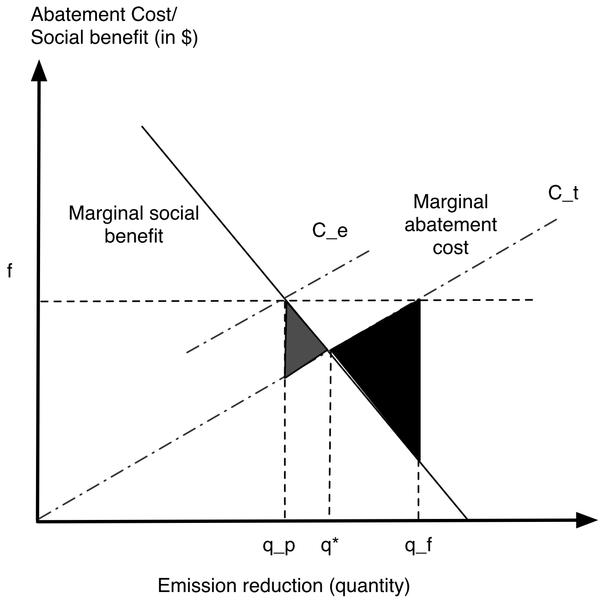


Fig. 4.4 Comparison of social deadweight loss under price and quantity instruments for

ronmental damages. In such a setting, it is clearly important that the environmental authority has a close control over the quantity of emissions. Unfortunately, the authority does not know the real marginal abatement cost function C_t and can only rely on the expected one, C_e. The marginal social benefit (black line) is perfectly known in this example.

If a price instrument f were employed (instead of a quantity instrument) and the authority were to underestimate the true costs of pollution control, emissions might exceed the critical range and reach the level q_f (where the fee f crosses the true marginal abatement cost function C_t), triggering a resulting environmental disaster. In such a case, the Weitzman theorem tells us that the regulator should choose the quantity instrument because the marginal benefit curve has a steeper absolute slope than the marginal cost curve.

In Fig. 4.4, it is clear that while the authority has a limited to guess the right marginal abatement cost, the cost for the collectivity will be lower under a quantity instrument (which would impose the quantity q_p) than under the price instrument f. The difference is directly observable by comparing the two deadweight loss areas: the area on the left represents the loss from the permit instrument, the one on the right represents the loss under the tax system.

Suppose now the opposite situation. The marginal abatement cost curve is now steep and that the marginal benefits from pollution control are relatively constant over time, as shown in Fig. 4.5. The danger here is that because of imperfect information, the regulatory agency might select an overly stringent standard, thereby imposing excessive costs on polluters and society. Under these circumstances, the expected welfare gain is larger under the price instrument. Polluters will not get

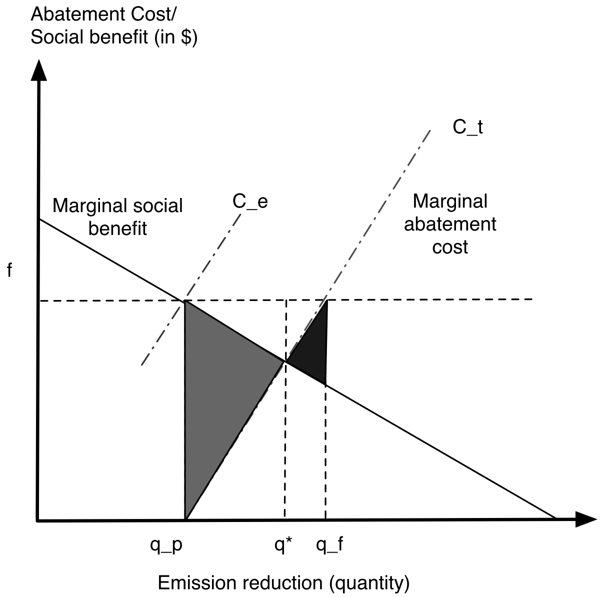


Fig. 4.5 Optimum quantity produced in presence of externality

stuck with inordinately high control costs, since they always have the option of paying the unit tax on emissions rather than reducing their pollution further.

In practice, one of the biggest problems in the design of a cap-and-trade scheme is the assessment of the private marginal abatement functions. While the damages of pollution are typically estimated by the regulatory agencies, these agencies are only provided with partial, incomplete information about the costs of abatement, similar to a principal-agent setting: polluting firms, playing the roles of the agents, know their cost curves but benefit either (i) from inflating them in order to get additional permits (in the case of grandfathering) or (ii) from deflating them to get a lower permit price (when the marginal damage curve is steep and/or when the firm has to pay for her permits). Since the agency, the principal, cannot be aware of the true cost curves, a first option is to believe the polluters, resulting in sub-optimal allocations and excessive emissions.

Figure 4.6 describes the distortion and excessive pollution resulting from the grandfathering approach.

By announcing a higher marginal abatement cost, the company can transfer some of its private abatement cost to the society (grey area). It no longer has to abate until q^* but simply q', a clear decrease in its abatement efforts. Simultaneously, the strategy comes with some sub-optimality in the form of a deadweight social loss (black area) which represents the absence of an optimal bargain from both the society and the firm. At the level q', it is clear that (i) total cost is not minimized and (ii) that the society would have been willing to pay the company to reduce its emissions to q^* (since the emitting company would have accepted some permit prices from

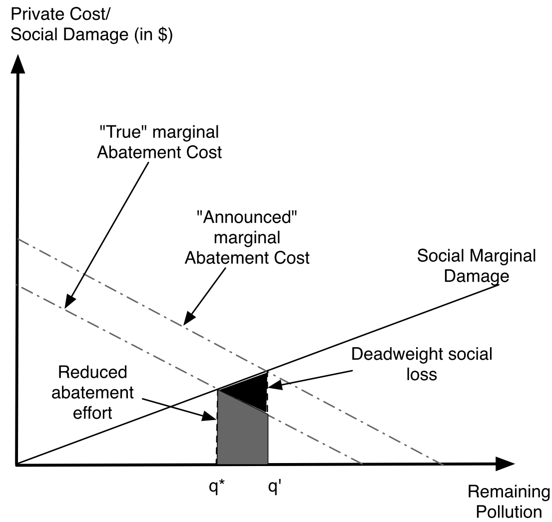


Fig. 4.6 Sub-optimal pollution levels when the firm over-reports

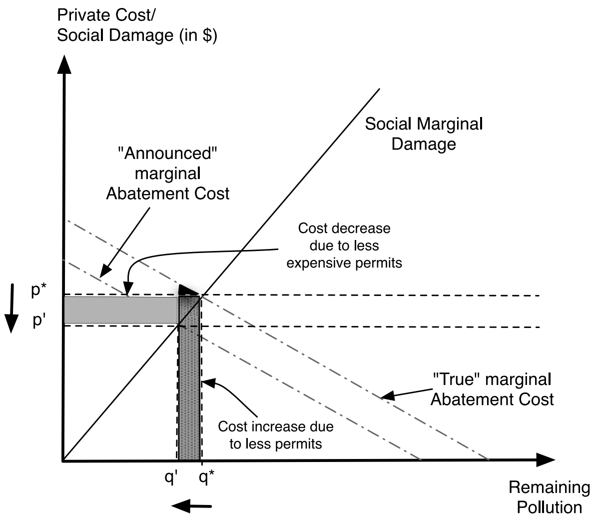
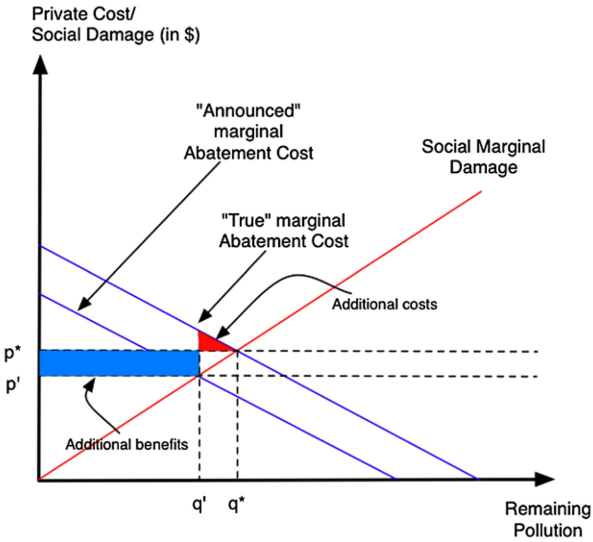


Fig. 4.7 Trade-off for the firm when under-reporting

q' to q^* above its marginal abatement cost but below the social damages). Hence, "over-announcement" from the polluting company is a clear sub-optimal in terms of total welfare (sub-optimal which is even more marked if the marginal social damage function is steep while the marginal abatement cost function is flatter).

Figure 4.8 depicts a situation where that firm is faced with an initial auction of emission permits and has an incentive to announce a lower marginal abatement to reduce the permit unit price. The under-reporting depends on the trade-off between the gains of a lower permit cost $(p'q' - p^*q^*)$ and the increase in marginal abatement costs (the total area formed by the dotted and the black surfaces), as shown in Fig. 4.7.

Fig. 4.8 Sub-optimal pollution levels when the firm under-reports



From Figs. 4.7 and 4.8, the net gains for the company can be decomposed as follows: first, the company benefits from paying less permits, in effect gaining from the lower permit cost $(p'q' - p^*q^*)$. It has however to abate more (from q^* to q'), which cost him the dotted area in Fig. 4.7 plus the black area which represents the additional costs from abatement. In net terms, the firm should assess whether the gray area of Fig. 4.8 (the gains from a lower permit price) is larger than the black area (the additional abatement costs).

To solve this problem, a possible and very effective solution would be to use a special auction that forces polluting firms to reveal their true marginal abatement cost functions. The model presented is a simplified version of Montero (2008) with only one polluting firm.[5]

Consider a polluting firm under a cap regulation, such that the firm should cover its emissions by abating them or purchasing permits. The inverse demand function for permits is defined as $P(x)$ with x representing the pollution level of the firm. This function, which is equivalent in absolute terms to the marginal abatement cost function $C'(x)$, is only known by the firm (private knowledge). On the opposite end of the pollution bargain, the marginal social cost is defined by the function $D'(x)$ such that $D'(x) \geq 0$. $D(x)$ is the primitive of the marginal social cost, i.e. the accumulated social cost function.

In absence of regulation, as shown in Sect. 4.1 (Eq. (4.1)), the firm will emit the unregulated amount x_0, such that $P(x_0) = 0$.

[5]Readers should refer to Montero (2008) for the version of the model with multiple firms.

In a regulated context, the goal of the environmental agency is to minimize the social damage and private cost (abatement):

$$\min_l C(l) + D(l)$$

Doing so allows for a social optimum such that at the optimum, $C'(l^*) = D'(l^*)$, with $C(l)$ representing the true abatement cost function for the firm.

In a perfect information setting whereby the regulator knows exactly the demand function of the firm ($P(x) \equiv -C'(x)$), as shown in Sect. 4.3 (Eq. (4.2)), the agency sells the permits at price p^*, which ensures the optimal level of pollution l^* since:

$$p^* = P\big(l^*\big) = D'\big(l^*\big)$$

However, in presence of uncertainty about the inverse demand function P, the regulator can implement an auction mechanism that will force the revelation of the polluter's true inverse demand function. In this auction, the polluter bids his full inverse demand function $P()$ (i.e., announcing what it will be willing to pay for each level of x) and the regulator determines the number of permits l to be sold during the auction and the corresponding price p.

The key revelation mechanism is a payback of the form $\alpha(l)pl$ that the firm obtains shortly after the auction (and about which the firm is informed). It is interesting to note that the two extreme for the payback policy, the grandfathering ($\alpha(l) = 1$) and the traditional costly auction without payback ($\alpha(l) = 0$) give the expected sub-optimal results: grandfathering gives incentive to over-report by as much as to postpone any abatement effort while no payback gives incentives to under-report to some optimal extent.

Since the structure of the payback (i.e., that it depends on the amount of supplied permits) is of common knowledge, the firm will try to minimize its compliance cost:

$$\min_l C(l) + pl - \alpha(l)pl$$

From the auction clearing condition, $p = D'(l)$, which implies that the first order condition for the firm is as follow:

$$C'(l) + D'(l) + D''(l)l - \alpha'(l)D'(l)l - \alpha(l)\big(D''(l)l + D'(l)\big) = 0$$

The regulator, knowing the minimization procedure of the firm, chooses $\alpha(l)$ such that $C'(l^*) + D'(l^*) = 0$. From the FOC, it is clear that the optimum is achieved if:

$$\alpha'(l)D'(l)l + \alpha(l)\big(D''(l)l + D'(l)\big) = D''(l)l$$

which can be simplified:

$$\alpha'(l) + \alpha(l)\left(\frac{(D''(l)l + D'(l))}{D'(l)l}\right) = \frac{D''(l)}{D'(l)}$$

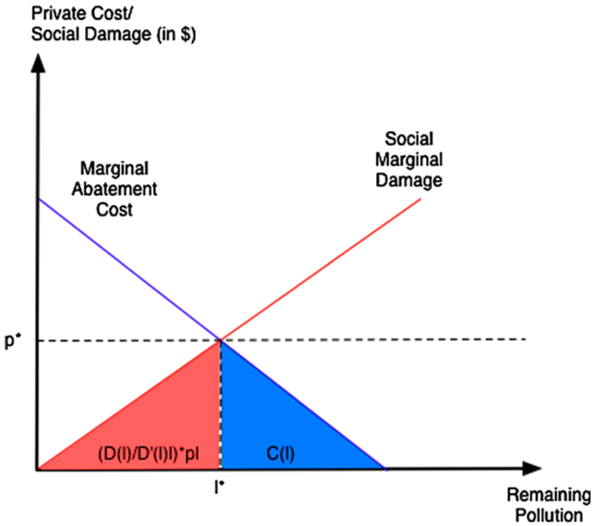


Fig. 4.9 Sub-optimal pollution levels when the firm under-reports

Solving this differential equation gives the payback function ensuring the optimality:

$$\alpha(l) = 1 - \frac{D(l)}{D'(l)l}$$

In economic terms, the optimal payback policy $\alpha(l)$ ensures that the firm internalizes the social cost within the private cost minimization, solving in a decentralized manner the regulator's problem. The private cost for the firm becomes:

$$\min_l \left(C(l) + \left(\frac{D(l)}{D'(l)l} \right) pl \right)$$

which can be simplified (since $p = D'(l)$) as:

$$\min_l \big(C(l) + D(l) \big)$$

which the initial problem posed to the environmental agency. Figure 4.9 presents the solution graphically.

4.5 Growth and the Environment: Is It Possible to Have Both?

Irrespective of the instrument used to abate pollution levels, a lingering question that stalled numerous regulatory efforts is the impact that pollution reduction will have on growth and its corollaries, jobs and investments. It is often said (especially in the US) that the deployment of stringent measures to curb pollution will have the fundamental flaw of preventing innovation (due to increased costs) and contracting investments.

This line of arguments does not account for the important role environment plays in the growth engine, whether as an vital input (in the case of resources) or as a negative factor (pollution). Using simplified versions of the growth models presented in Aghion and Howitt (2009), it is clear that abating pollution has complex interactions on economic development. Similar to the debate on mitigation instruments, the importance of environment as a driver of growth depends on strong assumptions about our ability to replace it and our capabilities to adapt from its decline.

4.5.1 Growth and the Environment: A Curse?

The interactions between the environment and economic growth can be divided between problems of resources and issues of pollution. The climate change angle of this book tends to overemphasize the pollution problem but it remains essential to understand the importance of natural resources as necessary production inputs and the consequences of their depletion. Even in a closed climate change framework, lack of resources will become a decisive problem: water shortage, access to labor in case of higher mortality or forced migration, forest degradation and genetic pool degradation have already become new problems for producing firms. This is without accounting for the numerous and free services ecosystems provide, from water purification, nutrient cycling to flood and disease regulations.

Consider for instance a simple global production model requiring some natural inputs to produce a global, undifferentiated, good Y:

$$Y = AKR^{\phi}$$

with K the current aggregated capital stock, R the current flow of natural resources, A an indicator of productivity and ϕ indicates the level of marginal productivity of the natural resources. Since natural resources are by nature finite, each inflow of natural input R used depletes the existing natural stock S such that:

$$\Delta S(t) = \frac{dS(t)}{dt} = \dot{S} = -R$$

where $\dot{S}$ is the variation of the natural stock after one period.

At the same time, the stock capital K evolves according to the following dynamics:

$$\Delta K(t) = \dot{K} = sY - \delta K$$

where s is the saving rate and δ is the rate of capital depreciation. Since the stock of natural resources is finite, it will eventually be fully depleted (since S cannot be negative), meaning that the flow R will reach 0 at some point in time t. From that point on, and due to the form of production function, no Y will be produced, resulting in the progressive depletion of the capital K, since:

$$\Delta K(t) = \dot{K} = -\delta K \quad \text{and} \quad \lim_{t \to \infty} K(t) = 0$$

Without any capital remaining, the economy will rapidly collapse and with it the population well-being.

To prevent such a gloomy outcome, economists have thought of different approaches to counteract the depletion of natural resources, evolving around the ideas of substitution and technological progress. The next section gives some insights about the potential and inherent limitations of the substitution approach.

4.5.2 A Possible Solution: Substituting Nature

The main idea behind the substitution approach is that to overcome the depletion curse, the production engine should be able to substitute natural capital with another form of abundant capital (human capital) in order to sustain the flow of output. There is two key ideas here: the first idea is that there is no uniqueness about natural capital, in the sense that technologies and innovations will provide ways to substitute extractive resources with synthetic, human produced ones. If innovations continue on desalinization of water, it should be possible in the foreseeable future to replace fresh water by seawater, transforming a scarce resource into an abundant one.

The second ideas is that it will be possible to increase productivity at a rate that will compensate for the decrease inflow of capital resources. Take the car of yesterday and compare it to the car of today. Despite providing the same (or maybe even more) amount of utility, the average car of today consumes much less gasoline by relying on a more efficient combustion engine.

However, to spur R&D in the first place, the depletion of environmental resources is initially required. In economic terms, the resource depletion is often justified to promote the technologies that will substitute natural capital at a later stage. Conceptually, the idea is reinforced by (i) its reliance on the largely available human capital, which goes in the same direction as the population's growth and (ii) by a track record of past innovations that invites optimism.

In a technology-based model[6] (a.k.a. Schumpeterian world with innovations), the main aspect that differentiates it from the previous, simpler model of Sect. 4.5.1, growth can be internalized within the production engine through R&D and innovations.

Lets consider such a production function:

$$Y = L^{1-\alpha} A^{1-\alpha} x^{\alpha} R^{\phi}$$

[6] Readers interested by the full fledged model should refer to Aghion and Howitt (2009), p. 379.

where the production function incorporates an additional labor sector L in addition to the productivity parameter A, the produced capital K which takes the form of an intermediate input x and the natural input R. For simplicity, we assume few things: (i) the price of the final output is the numeraire which is a common practice ($p_y = 1$); (ii) the labor market is composed of manufacturing labor L and research labor n. Total labor is assumed to be 1, therefore labor-market clearing condition imposes that: $L + n = 1$; (iii) it takes 1 unit of Y to produce one unit of intermediate good x; (iv) the producer of the intermediate good is a monopoly while the final good Y is produced in a competitive market.

The intermediate producer uses the final good (with one to one relationship) to produce the intermediate input x. Since the intermediate producer is a monopolist, she can sell x units of the intermediate good to the final sector at price:

$$p(x) = \frac{\partial Y}{\partial x} = \alpha L^{1-\alpha} A^{1-\alpha} x^{1-\alpha} R^{\phi}$$

She chooses x such that her profits are maximized:

$$\pi = \max_x \{p(x) \cdot x - x\}$$

since she needs one Y for each x at numeraire price (therefore the $-x$), which yields after substituting the optimal x^* in the Y production function:

$$Y = \alpha^{\frac{2\alpha}{1-\alpha}} LAR^{\frac{\phi}{1-\alpha}}$$

Note that the intermediate good is now a combination of the three primary inputs R, A and L.

The growth rate of Y, $\hat{Y}$ can be assessed by taking the derivative of the logarithm of Y over t, which gives:

$$\hat{Y} = \frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\phi}{1-\alpha}\frac{\dot{R}}{R}$$

Why $\hat{Y} \approx \frac{d\ln(Y)}{dt}$?

For the variable Y, the growth rate $\hat{Y}$ could be written down:

$$\hat{Y} = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

such that:

$$\hat{Y} + 1 = \frac{Y_t}{Y_{t-1}}$$

By taking the log on both sides of the expression, we have:

$$\hat{Y} \approx \log(\hat{Y}+1) = \log Y_t - \log Y_{t-1} = \frac{d\ln(Y)}{dt}$$

which is the case if $\hat{Y}$ remains small (from a Taylor Series approximation).

Assume now that government can impose restriction on the extraction rate, such that S remains always positive: $\dot{R} = -qR$ with $q > 0$. The growth rate of the final production becomes:

$$\hat{Y} = g_A - \phi q$$

where g_A is the rate of productivity growth.

In this case, the economy will sustain growth as long as $g_A > \phi q$. In other terms, as long as the productivity growth outpaces the productive rate of natural extraction, the economy will expand. This is what is commonly called a weak sustainability. To get a better intuition about productivity growth, lets assume that productivity and its growth are the results of innovation and ultimately of labor put in R&D. g_A can be expressed as

$$g_A = (\gamma - 1)\lambda n$$

where n is the labor put in R&D, γ is the size of innovations (i.e., the effectiveness of new discoveries) and λ is an R&D productivity parameter. It is clear that there will be production growth for Y only if the growth rate of productivity is higher than the modified rate of extraction, such that:

$$\frac{\phi q}{(\gamma - 1)\lambda}$$

since n is at best equal to 1 (if all labor is allocated to R&D). As stated in Aghion and Howitt (2009), "*the conditions will be satisfied if the productivity of R&D λ or the size of innovations γ is sufficiently large, or if the depletion rate q is sufficiently small.*"

This situation has however two main drawbacks.

The first drawback is the strong assumption that R&D and innovation can always grow faster than resource depletion. It would mean that research is a continuous process of improvement, which is rarely the case. More often than not, innovations are the results of periods of great discoveries and expansions followed by either periods of stagnation or periods "*creative destruction*" (following Schumpeter). The next technological revolution is therefore never a certainty and could mean long periods of natural deficits not being compensated by improvements in productivity.

The second drawback is the important ethical dilemma posed by a strict replacement (and eventual disappearance) of nature by man-made products. Even if we are capable of truly living without natural inputs (processed food, artificial forests to capture carbon…), it can be argued that the opportunity to enjoy the ecosystems and environments that are at our disposal should remain an option available for the future generations. This is the main idea behind the principle of strong sustainability.

Sustainability: Some Definitions

- *Weak sustainability*: A weak sustainability is concerned about the total stock of capital available and not about the form in which the stocks of capital (man-made, natural and human) are passed to future generations. As long as future generations obtain stocks of capital not less than those of the present generation, the conditions of sustainable development are satisfied even if this is done at the expense of drawing down the stocks of natural capital (Gowdy, 1999).
- *Strong sustainability*: A strong sustainability approach is not only concerned about keeping the aggregate stocks of capital constant, but it also requires that the stocks of natural capital (ecological assets) should not decrease over-time (Pearce et al., 1994). This is mainly because natural capital is associated with ecological assets, which are non-substitutable, and very essential for the welfare and survival of human beings. These assets are often referred to as "critical natural capital". Examples of "critical natural capital" include biological diversity, ozone layer, and carbon cycle (Pearce et al., 1994). This suggests that environmental problems such as global warming, ozone layer depletion, and land degradation will tend to deplete "critical natural capital".
- *Brundtland commission's definition of sustainability*: The World Commission on Environment and Development (1987) defines sustainable development as development that "meets the needs of the present without compromising the ability of future generations to meet their own needs". In other words sustainable development means that the well being of the people should at least be increasing over-time.
- *Environmental sustainability*: Environmental sustainability is defined as "a process of maintaining or improving the integrity of the life support system of the earth". The maintenance and improvement of the life support system of the earth is a necessary condition for welfare of the present and future generations.
- *Economic sustainability*: Economic sustainability is the maintenance of the stocks of capital or assets in order to produce a non-declining set of benefits. this is a situation whereby an investment produces non-declining or constant benefits over-time or leads to constant stream of consumption over-time.

- *Social sustainability*: Social sustainability as the name suggests is concerned about the conservation of social and cultural diversity. The issues of equity, gender equality, and democracy are central to the notion of social sustainability.

4.5.3 A Possible Solution: Targeting the Clean Economy

The previous section deals with the depletion of natural resources but does not mention pollution and their negative effects on production. It is however clear that pollution is not only harmful at the individual level but is also damaging for production and growth. In the case of climate change, potential damages may cover drops in productivity due to higher temperature. destruction of productive assets from climatic catastrophes, increased pressure on labor supply from health problems and land migrations. Combined with the resource depletion problem presented in Sects. 4.5.1 and 4.5.2, it appears essential to limit pollution to its minimum.

To model the influence of pollution on global production, we consider now a model where Y could be either produced by a clean technology or by a dirty technology:

$$Y_t = \begin{cases} Y_t^d + Y_t^c & \text{if } S_t > 0 \\ 0 & \text{otherwise} \end{cases}$$

with: $\dot{S} = -\phi Y_t^d + \omega S$ ($\phi > 0$) and where:

$$Y_d = L^{1-\alpha} A_d^{1-\alpha} x_d^{\alpha} \tag{4.3}$$

$$Y_c = L^{1-\alpha} A_c^{1-\alpha} x_c^{\alpha} \tag{4.4}$$

It is important to note that in this model, there is no need for a natural input R.

The natural stock S is depleted by the dirty production process with a rate ϕ but also organically regenerates at rate ω. Note that here the natural resource is no longer a fixed stock to be depleted but a renewable resource providing a potentially recurring flow of input (as long as the depletion rate $\hat{Y}_t^d$ remains lower than ω, which is the supremum of $\hat{S}$).[7]

As defined by the model, for production to be strictly positive, S should remain positive. It is therefore essential to know the limit on the growth of Y_t^d that ensures positive production.

[7] The supremum of S or $sup(S)$ is defined to be the smallest real number that is greater than or equal to every number in S.

Using the logic of the model presented in Sect. 4.5.2 with intermediate monopoly and with intermediate good prices defined as the value of their marginal product in the Y production, we obtain:

$$p_d(x_d) = \alpha L^{1-\alpha} A_d^{1-\alpha} x_d^{\alpha-1}$$
$$p_c(x_c) = \alpha L^{1-\alpha} A_c^{1-\alpha} x_c^{\alpha-1}$$

Using this price function, the intermediate producers of intermediate dirty and clean goods propose the optimum quantities:

$$x_c^* = \alpha^{\frac{2}{1-\alpha}} A_c L_c$$
$$x_d^* = \alpha^{\frac{2}{1-\alpha}} A_d L_d$$

at the optimum monopolistic profits:

$$\pi_c = \frac{1-\alpha}{\alpha} x_c$$
$$\pi_d = \frac{1-\alpha}{\alpha} x_d$$

By replacing the optimal quantities of intermediates goods x_c^* and x_d^*, we obtain following expressions for Y_t^d and Y_t^c:

$$Y_c = \alpha^{\frac{2\alpha}{1-\alpha}} L_c A_c \quad (4.5)$$
$$Y_d = \alpha^{\frac{2\alpha}{1-\alpha}} L_d A_d \quad (4.6)$$

The principal input being labor, his allocation between the two production processes is central for the existence and intensity of pollution. From introductory microeconomics, we now that labor is allocated where wages are highest, which depends on the comparative marginal productivity of labor between the two sectors $\{\frac{dY}{dL_c}, \frac{dY}{dL_d}\}$. Using Eq. (4.3), the two marginal products are:

$$\frac{\partial Y}{\partial L_c} = (1-\alpha)\frac{Y_c}{L_c}$$
$$\frac{\partial Y}{\partial L_d} = (1-\alpha)\frac{Y_c}{L_c}$$

which can be reformulated, using Eq. (4.5), as:

$$\frac{\partial Y}{\partial L_c} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_c$$
$$\frac{\partial Y}{\partial L_d} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_d$$

Since the two production functions only differ by their respective productivity component A_i, it is clear that if $A_c \neq A_d$, labor will be fully allocated to the highest productivity.

When $A_c > A_d$, all labor is allocated to the clean production and conversely when $A_d > A_c$, all labor is allocated to the dirty production. In this case, if $\hat{Y_d} > \omega$ (which largely depends on the population dynamics behind $\hat{L_d}$), the natural stock will deplete and annihilate production over the long run.

To prevent an economic collapse, it is therefore fundamental to ensure that the productivity of the dirty production remains below the productivity of the clean production. This is what justifies the introduction of a targeted tax, as a price mechanism that can help to produce incentives toward the socially beneficial clean technology.

By imposing a tax τ only on dirty technologies, one modifies the optimal production of intermediate goods in the dirty sector (since the price of the intermediate good is lowered by the tax), such that:

$$p_d(x_d) = (1-\tau)\alpha L^{1-\alpha} A_d^{1-\alpha} x_d^{\alpha-1}$$
$$x_d^* = (1-\tau)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} A_d L_d$$

In this case, while the production function for the clean final good Y_c remains unchanged, we obtain a new production function for the dirty good Y_d:

$$Y_c = \alpha^{\frac{2\alpha}{1-\alpha}} L_c A_c$$
$$Y_d = (1-\tau)^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_d A_d$$

where the profit on Y is diminished by the imposed tax.

The modified marginal productivity of labor are now as follow:

$$(1-\tau)\frac{\partial Y}{\partial L_d} = (1-\tau)(1-\alpha)\frac{Y_d}{dL_d} = (1-\tau)^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_d$$
$$\frac{\partial Y}{\partial L_c} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_c$$

If $A_c > (1-\tau)^{\frac{1}{1-\alpha}} A_d$, all labor is shifted to clean production. To ensure it is the case, the minimum tax rate should be equal to:

$$\tau^{\min} = 1 - \left(\frac{A_c}{A_d}\right)^{1-\alpha}$$

The solution of abandoning polluting technologies for greener ones is a promising idea, one of the main justification behind the promotion of renewable energies and green productions. And to achieve such a goal, putting a price (through a price mechanism like a tax or a quantity mechanism like a marketable permit) is key.

However, even renewable energies consume depletive resources and emit some forms of pollution, only at lower rate (therefore a lower ϕ). Wind farms for instance are associated with decimation of migratory birds and land appropriation (in). Nuclear energy is famous for its proliferation and waste management risks. Electric cars need batteries which requires rare earth elements such as lanthanum, neodymium, terbium and dysprosium to operate. Since there is no *fully* clean technologies, ensuring the permanence of growth will require a combination of innovations in targeted sectors with limited environmental footprints.

It is nevertheless quite likely that our environment, in the transitory periods leading to sustainability, will be damaged. To maintain our current levels of wellbeing while accommodating the future expansion of the developing countries, a straightforward solution would be to adapt our economy to its changing environmental reality. Wouldn't it be easier to adapt our economies to a warmer world than trying—with large uncertainties—to mitigate the causes of climate change? This is the subject of the next section.

4.5.4 A Possible Solution: Mixing Adaption and Mitigation Strategies

One of the most difficult aspect of the climate change problem is the identification of a successful strategy amongst the set of available options. Is it better or even efficient to reduce the emission at the source through mitigation efforts or should we accept that the climate will change irreversibly and adapt accordingly (if it is less costly)? Should we specialize in only one of these two solutions or should we combine them and if so, how? If limiting our emissions proves to be too costly in terms of reduced growth, wouldn't it be easier to try to engineer to climate by designing tools that can break the warming loops? (see box on Geoengineering the Climate).

Answering this vast and complex question depends on several key factors. The first required element is a clear understanding of the strengths and limitations of each approach and how those approaches interact with each other dynamically. The second is the ability to construct models that give a summarized view of the system as a whole (economics-carbon-climate) as well as a clear appreciation for the trade-offs and preferences attached to each possible solution.

This section gives a short overview of the main economic tool used by economists to try to answer the question, the integrated assessment model (IAM), which combines the long term environmental-carbon system with the perturbations generated by the economic activities.

The section starts with basic definition of mitigation, adaptation (and geoengineering).

According to the IPCC, mitigation represents the technological change and substitution that reduce resource inputs and emissions per unit of output. Mitigations measures are closely linked to the traditional policies (see Sects. 4.2 and 4.3) that attempt to curb emissions at the source by imposing additional costs on the polluters (or additional benefits from the abatement).

Mitigation has been so far the core strategy of the Kyoto protocol and the main subject of the different negotiation rounds, for reasons that are summarized in Bosello (2010):

- the policy makers are more familiar with the economic instruments used for mitigation than with those required to promote adaptation measures (i.e., technology transfers, R&D);
- the adaptation strategies are still new and often lack proper testings. In a situation where the consequences of climate change remain uncertain, it is difficult to predict if the chosen methods will be able to cope with deviations from their designed targets;
- mitigation strategies have a satisfying moral message, by reducing the levels of damages instead of simply shielding the population from them.

However and despite the urgency of the situation, global GHG emissions are still increasing, in particular because there is not yet an overall agreement to curb world emissions. In this context, and since future climate changes appear now unavoidable to some extent, adaptation measures gained in the past years a new political momentum as an important component of climate polices.

Contrary to mitigation options, adaptation measures do not reduce emission levels, but provide strategies to deal effectively with climate change effects by reducing their impacts (Tol 2005a; Adger et al. 2007; Klein et al. 2007b). Adaptation strategies cover a large array of sectors and options, from new agricultural crops, modified urban planning (dikes, sewerage systems), medical preventions against pandemic to controlled migrations of population and activity changes.

Depending on the degree of anticipation (and requirement for it), adaptation measures can be reactive or preventive: vaccination campaigns can be made mandatory without any materialized threat (as precautionary principle) or could be offered only in case of pandemic urgency, for instance.

Compared to mitigation strategies, adaptation measures have two main strengths. First, their benefits are often immediate or very short-term, which reduces their exposure to uncertainty and discounting preferences. This immediacy is also beneficial for populations already vulnerable to certain impacts of climate change (Parry et al. 2009). Second, adaptation measures in effect privatize policies against climate changes by largely limiting the benefits of adaptation to those having invested in it. Adaptation avoids the free-riding problem traditionally associated with mitigation[8] and does not require concerted and simultaneous actions, fostering the advancement of regional or local projects. As pointed by Olson (1965), "*only a separate and 'selective' incentive will stimulate a rational individual in a latent group to act in a group-oriented way*" and to that goal, adaptation is effective.

Both international institutions and governments have recognized these strengths and have now started to conceive and finance portfolios of adaptation projects. For instance, the World Bank has initiated a US$500 million Pilot Program for Climate

[8] A country say may hesitate to pay for emission reductions that will also impact favorably those who did not participate in any mitigating efforts, thus unbalancing its competitiveness (Olson 1965; Baumol and Oates 1988).

Resilience and prepared in 2009 a new study to assess adaptation costs, areas and applicability in developing countries (Margulis and Narain 2009). Under the United Nations Framework Convention on Climate Change (UNFCCC), a new adaptation fund has also been launched, financed with 2 % of the shares of proceeds coming from the issuances of certified emission reduction units (CERs) under the clean development mechanism (CDM). During the recent Copenhagen conference (COP15), it was also decided to create the Copenhagen Green Climate Fund (CGCF), with a first budget of US$30 billion in the 2010–2012 period to invest in mitigation and adaptation projects. This fund should eventually reach US$100 billion by 2020.[9] In addition to those dedicated projects, adaptation strategies are now more and more blended into more traditional development projects and official development assistances (ODA) (Klein et al. 2007a). They are also pushed forward in developed countries albeit without the kind of targeted recognition used for developing countries.

Considering the simultaneous promotion of adaptation strategies and the relative weaknesses of mitigation policies so far, the question of their respective role has to be evaluated, both for policy and investment purposes.

As already mentioned, it could be that adaptation strategies become inexpensive alternatives to mitigation approaches, at least as long as no clear international agreement forces the world's economies to transition into an more efficient economy (in terms of GHG emissions).

Finding the optimal solution to this problem requires that economists not only account for the economic trade-offs and systems but also recognize the impacts economic pollutions have on the global environment (and its temperature) and the set of feedbacks loops—through damages and natural capital depletion—that constraint economic growth. To do so, they have developed integrated assessment models (IAM) which serve the purpose of describe the complex interactions between the environment and the economy in ways that are rich enough to ensure a proper description but simple enough for their findings to be meaningful. Examples of IAMs are DICE (Nordhaus 1994, 2007; de Bruin et al. 2009), MERGE (Manne et al. 1995; Manne and Richels 2005), RICE (Nordhaus and Yang 1996) and TIAM (Loulou and Labriet 2008; Loulou 2008).

From the most recent research (Bosello et al. 2010; Bahn et al. 2012), it appears that the optimal portfolio of mitigation and adaptation strategies heavily depends on few key assumptions and on the respective attributes of each strategy.

- The first key assumption is the preference for the present (which is represented by the discount rate): if the discount rate is high with a low valuation of the future damages from climate change, adaptation policies are favored since they are "short-term" effective. The opposite is true in the case of a low discount rate.

[9]Copenhagen Accord, Conference of the Parties (COP-15), December 2009, articles 8 and 11 (http://unfccc.int/resource/docs/2009/cop15/eng/l07.pdf).

- In almost all cases of "acceptable" discount rates, adaptation and mitigation have to be deployed together: they are more complements than substitutes due their interconnectedness. This depends however on the model used to account for the cost of adaptation. If adaptation cost is considered to be independent of the level of emissions and damages (as in de Bruin et al. 2009), the positive feedback of mitigation on adaptation efficiency is almost inexistent. However, when adaptation costs are correlated to the level of damages, which seems more appropriate in a large array of situation without leveraging power (i.e., vaccines), the positive feedbacks pushes for preemptive mitigation strategies followed by adaptation efforts.
- Finally, as shown in Felgenhauer and de Bruin (2009) the presence of uncertainty has the effect of delaying both mitigation and adaptation compared to a certain scenario about the damages caused by climate change. This delay could however prove dangerous in case of unforeseen "nonlinearities" in the climate loops. If damages spike abruptly and without notice, the optimal solution would be to reduce the waiting period and act faster, even in presence of remaining uncertainties.

The optimal allocation of efforts between mitigation and adaptation is still a work in progress, with choices that will be revealed and evolve along the improved appreciation of the systems and possible impacts linked to climate change. Our personal view is that acting now is eventually less costly than waiting to apply solutions when the situation becomes difficult to manage. It makes the assumptions (that we consider realistic) that progress and development are long processes full of errors and inertia. Acting now by limiting emissions while working on adaptation measures will ensure enough time to test and validate the instruments we will be counting on to limit the damages.

This view is however no shared by everyone as shown in the following box.

The "Geoengineering Solution"

Geoengineering is the concept according to which one can control and tweak the earth's system using engineering techniques applied at the planetary level. While it has been considered a fringe field of science since the 1960, it has regained popularity as a possible way to limit the impacts of climate change.

Geoengineering has been promoted for multiple reasons, the first one being its supposedly low cost of implementation compared with other mitigation and adaptation strategies. It has also the benefit of being the "worst-case" scenario final solution, in a specific case where mitigation and traditional adaptation would have failed to limit the negative impacts of climate change. The Copenhagen consensus, a think-tank that gather several prominent Nobel prize economists under the idea that mitigation is not the most efficient strategy, has listed three climate engineering projects among the 15 best responses to climate change:

- *Marine cloud whitening* is part of a set of solutions called solar radiation management. The main idea is to create large-scale surface that are capable of reflecting the solar radiation and limit the greenhouse effect. Marine cloud whitening consists of specifically-designed boats that could spray seawater droplets into marine clouds to make them reflect more sunlight. It should be noted that the long lasting effects of the droplets are still poorly understood by scientists and are expected to cause side-effects and perturbations in the ecosystems in which they will be introduced. Another possible solution and less harmful solution would be to use pale-coloured roofing or use space-based mirrors to obstruct solar radiation.
- *Aerosol insertion* is also part of the solar radiation management options. The idea is to launch fine material into the stratosphere to scatter and absorb sunlights. As with marine cloud whitening, the consequence of the release of particles in the atmosphere remain to be studied. Scientists however agree that this method may cause important disruptions in the water circulation of the lower and upper atmospheres.

  As the unique large-scale natural field test, the Pinatubo's eruption of 1991, spewing the equivalent of 10 million tons of sulfur has been assumed to have cooled the planet by half a degree Celsius the following year.
- *GHG removal* solutions cover all the attempts to remove GHG from the atmosphere, either through chemical reactions that transform the gases into something neutral for the climate's cycle or by capturing it and storing it. Among the main possible strategies are iron fertilization of the oceans (which increases the production of phytoplankton that feed on carbon dioxide), the production of biochar and carbon capture and storage (CCS) techniques at plants that emit the gases.

Despite being attractive for their low expected costs and ability to deal with extreme scenarios, geoengineering solutions pose numerous problems that limit their attractiveness. The biggest flaw of most of these options is that they do nothing to stop the main cause of climate change, the production of GHG. They just try to ensure that its consequences are not felt. The downside is that (i) the engineering solutions have to be put in place forever and possibly continuously expended and that (ii) they cannot prevent tipping-points and dramatic climatic variations.

A second crucial problem is the absence of reliable data regarding the effectiveness and potential side effects of these methods. While the current climate system is still far from being understood, it would imply acting on some of its levers not knowing the full scope of their effects and consequences.

Finally, it promotes a "wait-as-we-progress" attitude, weakening policies and efforts to reduce climate change through mitigation.

4.6 Investing in a Uncertain Environment: The Importance of Quasi-option as a Decision Tool

So far in this economic section, most of the decisions (whether they are polluting or abating existing sources of pollution) have been modeled under certainty. Private and public costs were known and decisions were taken accordingly.

However, dealing with the environment is usually not that simple, for two basic reasons.

The first one is that environmental systems are tremendously complex and are still the subjects of ongoing research. While some of the relationships between pollution and its consequences on the environment are known, the full extent of their implications are rarely completely understood, especially over the long term. Chemical components that were assumed to be safe are reevaluated differently (i.e. the family of Bisphenol compounds), safety measures are tested to new and extreme extents (the BP's golf spill in the US, the Fukushima nuclear plant in Japan) and environmental systems remain mysterious (the impact of clouds on global warming, the oil absorption capacity of the gulf's marsh). In practice, this implies that decisions regarding the environment have to account for a fair amount of *uncertainty*.

The second reason that makes environmental decisions difficult is the long lasting impact pollution (or depletion) has on natural assets. While resource and environmental economics separates natural assets that are renewable (capable of endogenous regeneration) from those that are solely depletable, in most situations slow recoveries implies that decisions have to be considered as *irreversible*. For instance, the decision to cut a old-growth forest does not imply that forest cannot grow back on the land but it means that the primal nature of the forest won't ever be recovered. Moreover, during the recovery process, the forest won't be able to provide the environmental services that were provided before, a net loss that could last decades.

To account for the uncertain and irreversible nature of most environmental decisions, the concept of quasi-option[10] was first used by Arrow and Fisher (1974) to prove that in a situation where the benefits of exploiting a pristine natural area are uncertain, it could be optimal to not entirely develop the area and to keep some part of it as an hedge against possible increase of value from preservation in the future.

The main intuition to this result is that, considering the uncertainty surrounding the preservation value, exploiting the natural asset in its entirety could imply renouncing to the benefits of preservation for perpetuity. The optimal decision would account for this risk (which is independent of any risk aversion) and insure against it by keeping a share of the pristine area untouched.

This idea is very strong and applicable to a large set of environmental problems. Its rationale is at the core of the concept of real options defined in the next Chap. 5 and critical for the optimality of investment decisions in environmental contexts. Such a filiation justifies a closer look at the Arrow and Fisher's model of quasi-option, for the numerous insights it provides to enlighten the concept of option value.

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[10] The concept was formally defined by Weisbrod (1964).

Their initial model is the simple investment choice under uncertainty: exploiting a pristine area or preserving it. The investment decisions occurs over two periods (t_1, t_2), with the decisions in t_2 affecting the investment decision in t_2.

The relevant variables and parameters of the model are defined as follow:

$$\begin{cases} d & \text{a unit of land} \\ d_1 & \text{the amount of land developed in the first period} \\ d_2 & \text{the amount of land developed in the second period} \\ b_p & \text{the benefits from preservation of } d \text{ in the first period} \\ b_d & \text{the benefits from development of } d \text{ in the first period} \\ \beta_p & \text{the expected benefits from preservation of } d \text{ in the second period} \\ & \text{conditional on } b_p \text{ and } b_d \\ \beta_d & \text{the expected benefits from development of } d \text{ in the second period} \\ & \text{conditional on } b_p \text{ and } b_d \\ c_1 & \text{the investment costs in the first period} \\ c_2 & \text{the investment costs in the second period} \end{cases}$$

There is no discounting between the two periods. Developing the area incurs development costs while protecting does not. The solution is obtained by backward induction, starting from the optimal decision in the second period. In the second period t_2, the net benefits of development either exceed their costs or not:

$$\begin{cases} (\beta_d - \beta_p) > c_2 \rightarrow d_2 = (1 - d_1) \\ (\beta_d - \beta_p) < c_2 \rightarrow d_2 = 0 \end{cases}$$

For ease of reading, the authors define $z = (\beta_d - \beta_p)$ and $w = b_d - b_p - c_1$ which are the benefits in second and first periods of developing the area. Consider now the special situation A such that the future benefits of development at time t_2 are higher than the development costs for the period $(z > c_2)$. It will justify a full development of the natural resource.

If A happens, the net benefits in t_2 are:

$$V = b_p(1 - d_1) + b_d d_1 - c_1 d_1 + \beta_d - c_2(1 - d_1) \Rightarrow w d_1 + c_2 d_1 + b_p + \beta_d - c_2$$

However, if A does not materialize:

$$V = b_p(1 - d_1) + b_d d_1 - c_1 d_1 + \beta_p(1 - d_1) + \beta_d d_1 \Rightarrow w d_1 + z d_1 + b_p + \beta_p$$

Knowing this, the expected gains of starting development in t_1 $(d_1 > 0)$ are:

$$\textbf{(1)} \quad \Rightarrow \quad E\big[\big(w + \min(c_2, z)\big)d_1 + b_p + \max(\beta_d - c_2, \beta_p)\big]$$

In the situation where $d_1 = 0$ and A occur, $V = b_p + \beta_d - c_2$.

However, in a situation where $d_1 = 0$ and A do not occur, $V = b_p + \beta_p$. The expected gains of protecting the area in the first period $(d_1 = 0)$ are therefore:

$$\textbf{(2)} \quad \Rightarrow \quad E\big[b_p + \max(\beta_d - c_2, \beta_p)\big]$$

The economic value under uncertainty between immediately starting the development in t_2 and waiting for the resolution of the uncertainty in t_2 is equal to the difference between **(1)** and **(2)**:

$$(\mathbf{3}) \quad \Rightarrow \quad E\left[w + \min(c_2, z)d_1\right]$$

If this last equation is strictly positive ((**3**) > 0), it is preferable to start immediately the development, since the added value of a development in t_1' s higher than the "information value" gained from waiting one period.

While Arrow and Fisher did not compute the optimal values of d_1 and d_2, they are interested in the role played by uncertainty in the value of waiting. They hence compare variations of equation (**3**) between a scenario of certainty and a scenario of uncertainty. If (**3**) is lower under uncertainty than under certainty, it would mean that there is greater value in waiting when information is not yet fully available.

Under certainty:

$$E\left[w + \min(c_2, z)\right] \quad \text{becomes } E[w] + \min\left(c_2, E[z]\right)$$

If A occurs, the choice criterion becomes:

$$E[w] + c_2$$

However, $\min(c_2, z) \leq c_2$ with $p(\min(c_2, z) \leq c_2) > 0$ (p being the probability of realization of $\min(c_2, z) \leq c_2$)

We then get:

$$E\left[\min(c_2, z)\right] \leq c_2$$

and finally:

$$E\left[w + \min(c_2, z)\right] < E[w] + c_2$$

What the authors are able to show using this simple model is that the expected development benefit is lower under uncertainty than under certainty, a sign that not developing immediately (or waiting) has some specific values under uncertainty. As they put it, "*There exists a range of values for z and ω for which development should not, then, take place under uncertainty but should under certainty*."

In their seminal approach, Arrow and Fisher suggest that a decision under uncertainty about the payoff to investment presenting some irreversibilities will lead to optimally "under-invest" rather than "over-invest". This is the case because in presence of uncertainties, "*we should err on the side of underinvestment, rather than overinvestment, since development is irreversible. Given an ability to learn from experience, underinvestment can be remedied before the second period, whereas mistaken overinvestment cannot, the consequences persisting in effect for all time.*"

They call it a "feel" of risk aversion produced by the restriction on reversibility, despite the fact that not risk aversion is present in this problem. Therefore, all benefits coming from an irreversible decision (but one that can be postponed) should also account for the waiting value (or quasi-option) of reducing the uncertainty that a the decision to act destroys.

In the next chapter, this powerful principle is extended and formalized within the real option framework as a decision tool for investment.

5 The Finance of Environmental Investments

5.1 Introduction

A proper selection and valuation of investment (or disinvestment) projects is crucial for financial managers. This is particularly difficult in the context of risks, globalization of economy, technology changes, strong competition, and in the presence of information asymmetry and environmental constraints. In the last context, it is essential for governments to decide which policy to adopt in response to a threat to the environment and when to implement it. For firms, it is also important to adapt to environmental constraints. The logic of the current EU regulation (EU ETS) with respect to the regulated companies consists either in incentivizing the adoption of cleaner technologies in order to reduce their emissions or in imposing the purchase of new CO_2 allowances. For these companies, it obviously implies taking strategic investment decisions in order to meet the requirements. In such cases, the decision making tool usually recommended by the corporate finance theory is the so called NPV (Net Present Value) approach.

This conventional approach, generated by the neo-classical theory, does not fulfill the needs as it ignores the flexibility inherent to the decision making process and the dynamic aspects of project selection. These limitations could be detrimental in each of the three steps required by an investment analysis approach: valuation of the possible investment projects, selection and timing of the chosen projects. As shown in this chapter, the real options approach provides a new and powerful decision making tool that overcomes the limitations of the traditional DCF (Discount Cash Flow) method, and permits a more appropriate valuation of investment projects related to the environment in particular.

In this chapter, we rely on the real options approach for valuing and selecting investment projects in less CO_2 intensive production technologies and identifying the best timing within which to undertake such projects. This chapter is one of the first attempts to connect real options with investment decisions in the setting of an emission allowance market. There are many books that focus on real options. The reader interested in in-depth knowledge on this topic should consult in particular Dixit and Pindyck (1994).

M. Chesney et al., *Environmental Finance and Investments*,
Springer Texts in Business and Economics, DOI 10.1007/978-3-642-36623-9_5,

5.2 Characteristics of Investment Projects

Investments projects are mainly characterized by six features:

- Price uncertainties
  Future input and output price dynamics are often unknown and should therefore be modeled as stochastic processes and not simply as deterministic or possibly constant processes.
- (Ir)-reversibility of the investment decision
  If the investment project is irreversible, then the investment decision will be taken with more caution, that is, could be delayed. On the contrary, a possible ability to switch or to abandon the project once started, will have an impact on the timing of the decision, that is, will generate more incentives to invest sooner.
- Time dimension
  An investment is not only a now-or-never decision. In most cases, the possibility to delay the investment decision exists and is worth considering. More information can be obtained in order to reduce the downside risk inherent in the project.
- Implementation delay (time-to-build)
  Most of the projects require some time before being implemented. The decision to invest is not instantaneously implemented. This feature obviously has an impact on the decision making process of investment projects.
- Sequential decision
  Confronted with an investment decision, a firm could choose to proceed sequentially, that is, step by step and not as a one shot decision. This ability to consecutively plan the investment is susceptible to limit the risk inherent in the investment decision. Indeed, if at a given step, the downside risk is perceived as being too high, the company will stop the process and only lose what has been spent until this date.
- The presence or absence of competition
  In the presence of competition, the possible first mover advantage could influence the decision making process in a determinant way. The danger of losing the market will generate incentives to invest sooner in order to try to preempt the competition. On the contrary, the absence of competition will give the company more flexibility in the timing of the decision making process, that is, a monopolistic situation will allow the company to slow down this process in order to take the time to gather enough relevant information.

5.3 The Neoclassical Approach: The Net Present Value (NPV)

According to the neoclassical approach, the Net Present Value (NPV) criteria is the relevant tool to use whenever taking investment decisions. The NPV is computed as the sum of discounted difference between expected earnings and costs:

$$\mathrm{NPV} = -F_0 + \frac{F_1}{(1+r)} + \frac{F_2}{(1+r)^2} + \cdots + \frac{F_n}{(1+r)^n}$$

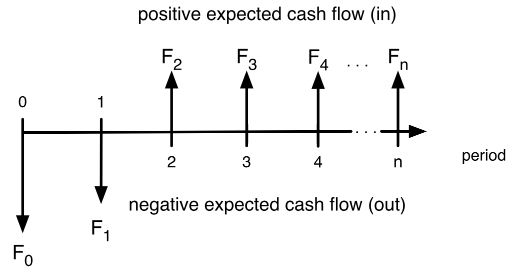


Fig. 5.1 Example of a sequence of expected cash flows

where n is the lifetime of the investment, r the discount rate (constant in our setting) and F_i, $i = 1, \dots, n$ the negative or positive expected cash flows. Negative expected cash flows correspond to costs. Figure 5.1 illustrates graphically the context.

NPV Criterion In this context, the decision rule of the NPV criterion is very simple. Indeed, according to the NPV criterion, the investment should be undertaken only if the NPV is positive.

5.3.1 Limitations of the NPV Approach

The following assumptions are intrinsic to the NPV approach.

- A reliable estimation of earnings and costs expectations.
- An explicit derivation of the discount rates (and therefore of the risk premia).
- A static investment rule: In other words, the investment can be undertaken either at the current date or never.

However, in reality investments are undertaken in a random, risky environment. Therefore, a reliable estimation of earnings and costs expectation could be highly questionable. As shown later in this chapter, the incertitude inherent to future flows is inappropriately considered by assuming that only the discount rate will take into account this incertitude. Furthermore, the static investment rule is suboptimal. In most cases, indeed, it is possible and recommended to delay the investment decision in order to determine whether or not the investment makes sense. Sequential characteristics of investments allow for the partial delay of the decision to invest, in order to gather relevant information before possibly confirming the investment. A major component of the so-called managerial flexibility is therefore neglected by the NPV criteria. The decision making process should be encompassed within a dynamic setting where, at each step in time, possibilities to invest and to wait before investing are evaluated and compared, and where interactions between strategic behaviors of the firm and its possible competitors are present. This setting should obviously be stochastic in order to take into account the above-mentioned sources of uncertainty inherent in the project. Only in such a dynamic and stochastic setting, will the risk premia required for the computation of the discount rate be appropriately derived.

Furthermore, the NPV criteria neglects the opportunity costs associated with the decision to invest. Indeed, as soon as this decision is taken, the opportunity to invest at a later, more suitable stage vanishes. This idea will be developed in the first example presented in this chapter and underpins the real options approach.

5.3.2 Relationship to Option Pricing Theory

The limitations of the NPV criteria demonstrate that investment opportunities should be comprehended in a richer setting. The real options approach goes beyond these limitations by perceiving these opportunities as options. In this new framework, the two essential questions, when should the investment be undertaken and what is its value, can be answered in an efficient and coherent way.

At this level it is worthwhile to briefly explain or recall what financial options are. A Call (or respectively a Put) Option is a contract giving its owner:

- the right to buy (sell) a security asset (stock, bond…) or a commodity (oil, gas…)
- at a given price (the so-called strike price)
- at a given date (for European type options) or during a given period of time (American type).

The option pricing theory initially developed by Black and Scholes (1973) and also by Merton (1973), under specific simplifying assumptions, has been, since then, significantly extended in order to value different types of financial contingent claims.

An investment opportunity can be precisely considered as a real option because it corresponds to the right, and not to the obligation, of undertaking an investment for a given cost, which corresponds to the strike price of the financial option, and, during a given period of time, corresponds to the exercise period. Therefore, an investment opportunity can in most cases be understood to be an American option.

5.4 Investment Opportunities as Options

In order to use option pricing models for the valuation of investment opportunities, the similarities between real and financial options should be highlighted. Table 5.1 emphasizes the main aspects of this analogy.

The real options approach allows one to properly take into account options such as:

- deciding when the investment project should be undertaken (option to delay),
- investing in new or additional capacity (option to expand),
- closing or abandoning an existing project (option to abandon),
- undertaking sequential investments (multi-stage options),
- undertaking horizontal or geographical diversification for multinational companies (option to grow),

Table 5.1 Analogy between financial and real options

Financial options	Real options
Stock price	Expected present value of future cash flows (CF)
Exercise price	Expected present value of investment costs (IC)
Time to expiration	Lifetime of the investment opportunity
Volatility	Project value return's volatility
	(Volatility of the ratio $\frac{CF}{IC}$ if costs are stochastic.)
Dividend	Cost of keeping the investment opportunity alive
Risk free interest rate	Risk (adjusted) interest rate (WACC)

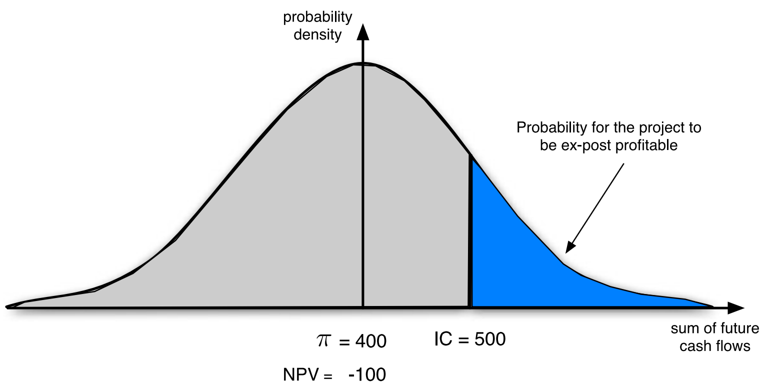


Fig. 5.2 Probability density function of the sum of future cash flows

- switching technologies (option to change technologies). This last opportunity is key in the environmental setting.

It also allows one to take into account the costs associated with the possible reversibility of the project.

Using the NPV approach corresponds to the exercise of an American option as soon as it is "in the money", that is, as soon as the underlying value is higher than the strike price. As shown in the next example, this might be sub-optimal.

5.4.1 An Intuitive Example

A utility company plans the construction of a new facility to generate electricity by burning gas. The initial investment cost IC is €500 million. Let us first assume that the sum of future expected cash flow π is €400 million and for simplicity that the interest rate is equal to 0.

As shown in Fig. 5.2, the corresponding NPV is $400 - 500 = -€100$ million, therefore the decision is to not undertake the project.

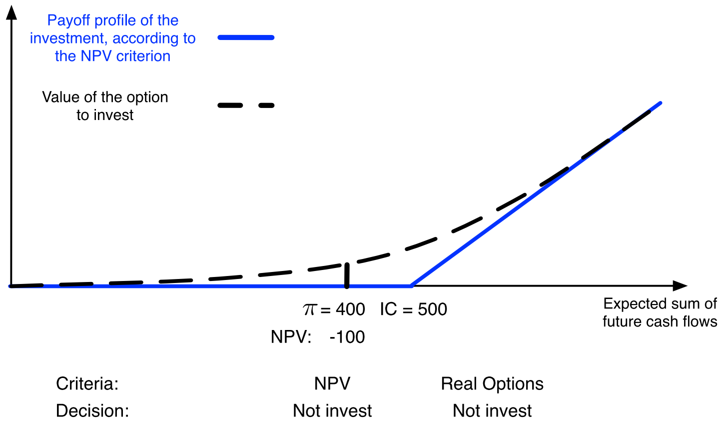


Fig. 5.3 Value of the option to invest and payoff profile of the investment

However, it does not mean that the value of the project is zero. Indeed, the realized sum of future cash flows could be higher than €500 million and, in this case, the project will be financially profitable ex post. Therefore, the project value is precisely the value of a call option with strike price €500 million. This idea is illustrated by the blue area in Fig. 5.2 and the vertical black line above 400 in Fig. 5.3.

If, at a future date, the situation improves, that is, if, the expected discounted cash flow increases and reaches €600 million for example, the NPV will become positive, being equal to €100 million. According to the NPV criteria, the investment should be undertaken and should generate an expected net profit of €100 million. In reality, the decision to invest deactivates definitively the investment opportunity. As soon as the decision to invest materializes, the opportunity to invest obviously disappears. Therefore, in order to take an optimal decision, the expected discounted profits generated by the investment, that is, the NPV, should be compared with the opportunity costs associated with the disappearance of the option to invest. The NPV should therefore be compared with the value of the "real" option to invest. The decision to invest should be delayed as long as the NPV, that is, the net benefits, are strictly smaller than the value of the real option. In other words, the real costs associated with the investment should not only comprehend the investment costs, but also the opportunity cost linked to the possibly irreversible loss of the investment opportunity, that is, the value of the real option. This is precisely what is done in the real options setting. In this example, as illustrated in Fig. 5.4, if the value of the real option to invest is €120 million, then in spite of a positive NPV, the investment should not be undertaken. The project has more value as such, that is, as an opportunity, than when it materializes.

If, at a later date, the situation improves further, that is, if the sum of future cash flow increases and now reaches €700 million for example, then, as shown in Fig. 5.5, the NPV will be equal to €200 million. If the value of the real option is also €200 million, there are no longer incentives to wait and the project should be undertaken.

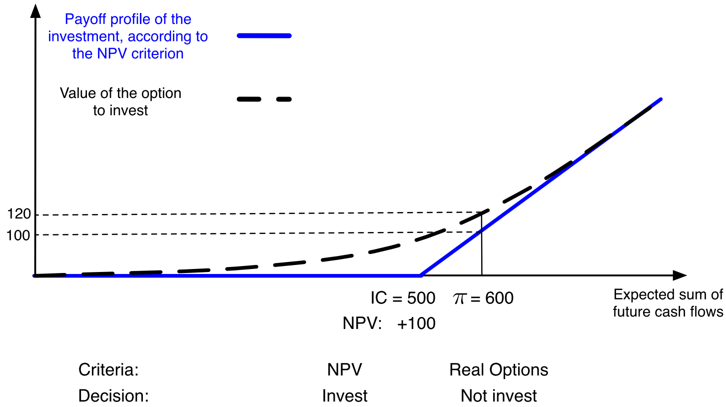


Fig. 5.4 Value of the option to invest and payoff profile of the investment

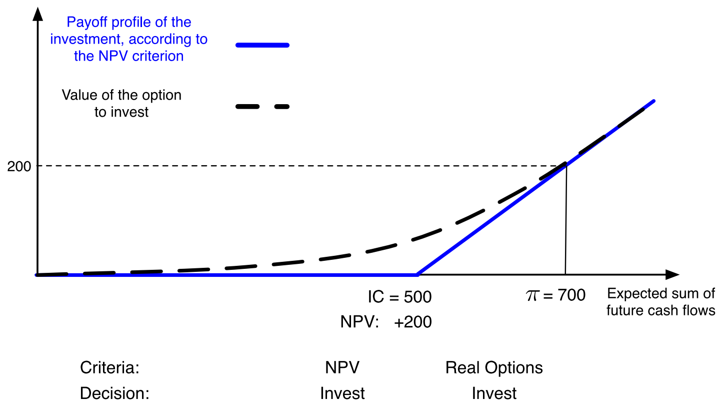


Fig. 5.5 Value of the option to invest and payoff profile of the investment

This following simple example further investigates the time dimension of investment opportunities.

5.4.2 From NPV to Real Options: A Second Example

A company can irreversibly invest €4 million in a carbon capture and storage project (see Sect. 3.1.3 for more details). The annual "reduction" corresponds to 20,000 tons of CO_2, and the corresponding perpetual cash flow is constant. The current price P_0 of an emission certificate (certificates from CDM projects[1]) is €25/ton but by assumption will increase the following year by 50 % (with prob-

[1] Please refer to Sect. 3.1.3 for more details.

$$W_0 = 0.5 \begin{cases} p & 0.75 \text{ --- } 0.75 \text{ --- } 0.75 \\ 1-p & 0.25 \text{ --- } 0.25 \text{ --- } 0.25 \end{cases}$$

Fig. 5.6 Project value in million €

$$\text{at } t_0\text{: } V_0 = 5.5 \begin{cases} p = 0.6 & \text{at } t_1\text{: } V_1^u = \sum_{t=0}^{\infty} \frac{0.75}{(1.1)^t} = 8.25 \\ 1-p = 0.4 & \text{at } t_1\text{: } V_1^d = \sum_{t=0}^{\infty} \frac{0.25}{(1.1)^t} = 2.75 \end{cases}$$

Fig. 5.7 Discounted sum of cash flows after one year

ability $p = 0.6$) or decrease by 50 % (with probability $1 - p = 0.4$) and then remain constant every year thereafter. Then, in order to simplify the setting, it is assumed that the price will perpetually remain at the level reached: €12.5 or €37.5.

Figure 5.6 illustrates the context, where W_0 is the value of the project (quantity times price). The company can either invest at the current date or wait until next year and decide depending on the price level of the emission certificate. Assuming a discount rate of 10 %, the NPV in millions corresponds to:

$$\text{NPV} = -4 + 0.5 + \sum_{t=1}^{\infty} \frac{0.6 \cdot 0.75 + 0.4 \cdot 0.25}{(1.1)^t} = -4 + 6 = 2 > 0$$

According to the NPV criteria, the project should be undertaken without delay.

However, the NPV approach ignores the possibility to postpone the investment in order to avoid the bad scenario (the price falls to €12.50 and the investment generates losses each year) and to invest only in good circumstances (the price goes up to €37.50). In order to analyse this possibility, let us calculate an adjusted NPV if the firm waits and invests only when the price is €37.50. This adjusted NPV, which takes into account the option to wait, is, therefore, in this simple example the value of the real option to invest.

$$\text{NPV}^* = (0.4 \cdot 0) + 0.6 \cdot \left[-\frac{4}{1.1} + \sum_{t=1}^{\infty} \frac{0.75}{(1.1)^t} \right] \approx 2.3$$

Since the adjusted NPV is higher than the standard one, postponing the CCS investment is rewarding, in this example.

The binomial description of the discount cash flows (DCF) is given by Fig. 5.7, where V_1^u (resp. V_1^d) represents at year one the discounted sum of cash flows if the price of an emission certificate goes up (down, respectively). V_0 is this expected discounted value at the initial date (if the project is not undertaken at this date, but after one year).

Since undertaking the CCS investment is like holding a call option, the investment payoff follows an option payoff (Fig. 5.8).

$$C_0 = \begin{cases} C_1^u = \max[0; (V_1^u - I_0)] = \max[0; 8.25 - 4] = 4.25 \\ C_1^d = \max[0; (V_1^d - I_0)] = \max[0; 2.75 - 4] = 0 \end{cases}$$

Fig. 5.8 Payoff construction

If the price of emission certificates moves to €37.50 after one year, the option will be in-the-money and, therefore, be exercised, that is, the investment costs will be paid and the investment will be realized. The payoff, €4.25 million, will be the difference between the discounted cash flows, €8.25 million and the costs €4 million. But if the price drops to €12.50, the option will be worthless and not be exercised.

In this example, the value of the option to invest is therefore €2.3 million, while the NPV, which is the value of the investment if undertaken at initial time, is 2 million, and the difference, 0.3 million is the time value of the option, that is, the opportunity cost of exercising the option at initial time. As explained in the beginning of this chapter, if the option to invest is exercised too early, the decision might be not optimal and the owner of the option incurs an opportunity cost. The option to invest should be exercised only when the opportunity cost is equal to zero, that is, if the price of the emission certificate increases in this example after one year.

Assuming that the market is complete and the project payoffs are fully replicable, the value of the investment opportunity at initial time C_0 can also be derived by constructing a risk-free portfolio. Such a portfolio consists of a long position in a CCS investment and an unknown number n of short positions in emission certificates that can be originated by a similar project.

- The current value of this portfolio is:

$$W_0 = -C_0 + n \cdot (\text{annual reduction}) \cdot P_0 = -C_0 + n \cdot (20000) \cdot 25$$

  where the number n has to be determined, the current price of the emission certificate is known and the quantity of permits is fixed.
- After one year, the portfolio will be worth:

$$W_1 = -C_1 + n \cdot (20000) \cdot P_1$$

  where C_1 and P_1 are random variables.

Graphically, we obtain Fig. 5.9, where W_1 is given in millions €. The number n is determined in such a way that the portfolio is riskless (its value is independent of the price evolution). Therefore after one year, the two payoffs should be equal:

$$-4.25 + 0.75 \cdot n = 0.25 \cdot n$$

Fig. 5.9 Value of portfolio after one year

$$W_0 \begin{cases} W_1 = 0 + 0.25 \cdot n & \text{if} \quad P_1 = 12.5 \\ W_1 = -4.25 + 0.75 \cdot n & \text{if} \quad P_1 = 37.5 \end{cases}$$

The solution to this equation is $n = 8.5$ and therefore $W_1 = 2.125$. The absence of arbitrage implies a riskless return, thus:

$$W_0 \cdot (1 + r) = W_1$$

and

$$(-C_0 + n \cdot 20000 \cdot P_0) \cdot 1.1 = 2.125 \cdot 10^6$$

Therefore, by relying on the portfolio approach, the initial value of the option to invest in the CCS investment is re-derived: $C_0 \approx 2.3$ millions of Euros.

5.4.3 Real Options and Incentives to Invest: A Third Example

The last two simple examples could lead one to think that the introduction of the real options approach always generates incentives to wait longer before investing than with the NPV criteria. However, this conclusion is limited to a specific simplified static framework, without interactions between the decisions of the different companies (for example, in a monopolistic setting) where the profit generated by the potential investment is independent of the investment decisions of the other players. In a dynamic and competitive setting where decisions of other companies are taken into account, the real options criteria might generate incentives to invest sooner than with the static NPV approach. The following example illustrates this idea.

Within the EU ETS, regulated companies are confronted with the following choice in order to comply with current regulation. They should either invest in a new technology with the intention to reduce their CO_2 emissions, or buy emission rights. These two possible solutions should allow them to avoid penalties at maturity.

If the price of the emission right is small, the NPV approach would recommend buying allowances, instead of investing in the clean technology. Indeed, if the market price of the emission right is smaller than the marginal cost of changing the technology, then it is cheaper to buy rights than to adopt a clean technology.

However, by focusing on one company, the static NPV approach neglects the other regulated firms and the impact of their decisions to invest or not to invest on the price of the emission right. Such a low price will also push the other regulated companies to buy rights, instead of reducing their emissions. The submission of their buy orders could induce a rapid increase in the market price which would in turn generate a growing interest for a technology switch. This switch might become the

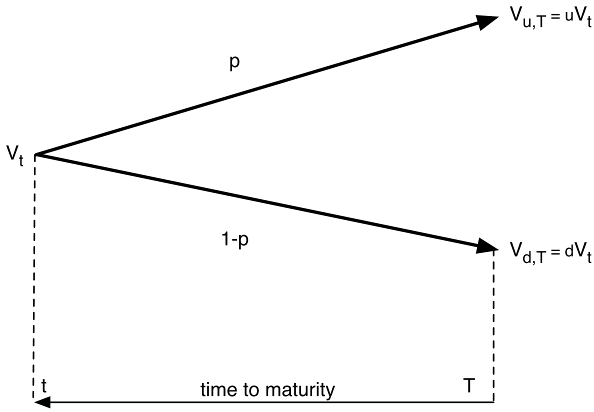


Fig. 5.10 The one-step binomial model

cheapest solution for companies who were not fast enough to buy emission rights. At the end of the day, in a dynamic setting, one will easily understand that it could make more sense to invest in a new technology and sell the rights in surplus than to try to purchase allowances at an increasing price. This phenomenon is embedded in the more realistic real options approach.

In order to value environmental investment opportunities in a real option setting, a general option pricing methodology is required. This is the goal of the next section.

5.5 Option Pricing with the Binomial Model

5.5.1 The One-Step Binomial Model

In this section, the fundamental characteristics of the binomial model will be recalled (Fig. 5.10).

- Time indicators t and T are the current time and the maturity respectively. The difference $T - t$, which is the step length, is denoted Δt. In the one-step binomial model: $t = 0$ and $T = 1$.
- V_t is the underlying value at time t. It moves either up to $u \cdot V_t$, with $u > 1$ or down to $d \cdot V_t$, with $d = \frac{1}{u}$.
- $\Pr(V_T = u \cdot V_t) = p$ is the probability of an upward movement.
- By assumption the market is arbitrage free and therefore:

$$V_t \cdot (1 + r) \cdot \Delta t = p \cdot u \cdot V_t + (1 - p) \cdot d \cdot V_t$$

hence

$$p = \frac{e^{r\Delta t} - d}{u - d} \quad \text{where } 0 \leq p \leq 1$$

5.5.2 Multi-step Binomial Model

In a multi-period binomial model, the same logic applies with a number of steps higher than one.

The continuous stochastic dynamics of the underlying asset can be discretized by a binomial model where Δt is infinitesimally small and hence the final outcome is normally distributed.

The following example describes the evolution of a random investment of 1€, when the number n of steps is 4. Between the initial step and the final one, there are 16 possible paths. One allows to reach u^4 and another one d^4, four lead to $d \cdot u^3$ and another group of four to $d^3 \cdot u$ and finally six to d^2u^2.

$$
\begin{array}{cccccccc}
 & & & & u^4 & 1 & \\
 & & & u^3 & & & \\
 & & u^2 & & du^3 & 4 & \\
 & u & & du^2 & & & \text{Total} \\
€1 & & du & & d^2u^2 & 6 & = 16 \\
 & d & & ud^2 & & & \\
 & & d^2 & & d^3u & 4 & \\
 & & & d^3 & & & \\
 & & & & d^4 & 1 & \\
t=0 & t=1 & t=2 & t=3 & t=4 & &
\end{array}
$$

For n-period binomial trees with general risk neutral probabilities p and $(1-p)$, the probability of reaching a certain node is given by:

$$P\left(V_{n\Delta t} = u^j d^{n-j} V_0\right) = \left(\frac{n!}{j!(n-j)!}\right) p^j (1-p)^{n-j}$$

with $(\frac{n!}{j!(n-j)!})$ the number of possible paths to reach a given node $u^j d^{n-j} V_0$ and $p^j (1-p)^{n-j}$ the probability of reaching the node $u^j d^{n-j} V_0$ within a given path. For example, for $p = 0.6$,

$$P\left(V_{n\Delta t} = u^2 d^2 V_0\right) = \left(\frac{4!}{2!2!}\right) \cdot 0.6^2 \cdot 0.4^2 = 6 \cdot 0.6^2 \cdot 0.4^2 = 0.35$$

where

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

In other words, in this example there are six paths which, at maturity, lead to the node d^2u^2 and the probability corresponding to each of these paths is $0.6^2 \cdot 0.4^2$, as for any of these six possibilities, the stock price will rise twice and also decrease twice.

The probability of reaching at least a minimal level at maturity is also very useful, in particular in terms of option pricing. The following formula gives the requested result in the n-period binomial tree setting:

$$P\left(V_{n\Delta t} \geq u^j d^{n-j} V_0\right) = \Phi[a; n, q] \equiv \sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!}\right) p^j (1-p)^{n-j}$$

It gives precisely the probability of reaching at least the node $u^j d^{n-j} V_0$ at maturity. The parameter a is defined as the minimum number of upward moves which will allow the underlying process to reach at least this node at time $n\Delta t$.

In the following example, the initial price V_0 is €50, the number of steps n, is equal to four, the probability p of an upward movement is 0.6, the factor u characterising the upward movement is 1.03 and the probability of reaching at least the level $u^3 d V_0$ (i.e., 53) at maturity, is computed.

$$\begin{array}{ccccccccccc}
 & & & & & & & \nearrow & u^4 \cdot 50 & & = 56.18 \\
 & & & & & & u^3 \cdot 50 & & & & \\
 & & & \nearrow & u^2 \cdot 50 & \nearrow\searrow & & \searrow\nearrow & du^3 \cdot 50 & & = 53 \\
 & \nearrow & u \cdot 50 & & & & du^2 \cdot 50 & & & & \\
€50 & & & \searrow\nearrow & du \cdot 50 & \searrow\nearrow & & \searrow\nearrow & d^2u^2 \cdot 50 & & = 50 \\
 & \searrow & d \cdot 50 & & & & ud^2 \cdot 50 & & & & \\
 & & & \searrow & d^2 \cdot 50 & \nearrow\searrow & & \searrow\nearrow & d^3u \cdot 50 & & = 47.17 \\
 & & & & & & d^3 \cdot 50 & & & & \\
 & & & & & & & \searrow & d^4 \cdot 50 & & = 44.5
\end{array}$$

$$\begin{aligned}
P(V_{n\Delta t} \geq 53) &= \sum_{j=3}^{n} \left(\frac{n!}{j!(n-j)!}\right) p^j (1-p)^{n-j} \\
&= \left(\frac{4!}{4!0!}\right) \cdot 0.6^4 \cdot 0.4 + \left(\frac{4!}{3!1!}\right) \cdot 0.6^3 \cdot 0.4 = 0.48
\end{aligned}$$

In this case, parameter a is equal to three, because at least three upward moves are required in order to reach at least euros 53 at maturity.

5.5.3 Multi-period Binomial Model and Option Pricing

Assuming a multiplicative binomial process, with discrete intervals for a stock price, allows for the valuation of different types of options. Let us start with European options.

With a one-step binomial tree, the following dynamics are obtained:

$$
\begin{tikzcd}[column sep=huge]
 & uV \quad \text{with probability} \quad p \\
V \arrow[ur, dash] \arrow[dr, dash] & \\
 & dV \quad \text{with probability} \quad (1-p)
\end{tikzcd}
$$

where V is the initial stock price. In this framework, the call value at maturity takes two possible values:

$$
\begin{tikzcd}[column sep=huge]
 & c_u = \max[0; uV - K] \quad \text{with probability} \quad p \\
c \arrow[ur, dash] \arrow[dr, dash] & \\
 & c_d = \max[0; dV - K] \quad \text{with probability} \quad (1-p)
\end{tikzcd}
$$

At initial time, in an arbitrage free setting, the option value c is the expected discounted value of the call price at maturity:

$$c = \left[p \cdot c_u + (1-p) \cdot c_d\right] e^{-r\Delta t}$$

where r is the risk-free interest rate.

Proceeding along the same lines and assuming that the stock price dynamics correspond to a two-period binomial tree, the call price at initial time can be derived. The approach is illustrated in the following figure.

$$
\begin{tikzcd}
 & & u^2V & & & & c_{uu} = \max[0; u^2V - K] \\
 & uV \arrow[ur] \arrow[dr] & & & & c_u \arrow[ur] \arrow[dr] & \\
V \arrow[ur] \arrow[dr] & & duV & \Rightarrow & c \arrow[ur] \arrow[dr] & & c_{ud} = \max[0; udV - K] \\
 & dV \arrow[ur] \arrow[dr] & & & & c_d \arrow[ur] \arrow[dr] & \\
 & & d^2V & & & & c_{dd} = \max[0; d^2V - K]
\end{tikzcd}
$$

where

$$
\begin{aligned}
c_u &= \left[p \cdot c_{uu} + (1-p) \cdot c_{du}\right] e^{-r\Delta t} \\
c_d &= \left[p \cdot c_{ud} + (1-p) \cdot c_{dd}\right] e^{-r\Delta t}
\end{aligned}
$$

The backward induction procedure allows to find the option price c at initial time.

$$
\begin{aligned}
c &= \left[p^2 \cdot c_{uu} + (1-p)^2 \cdot c_{du} + 2p(1-p) \cdot c_{du}\right] e^{-2r\Delta t} \\
c &= \Big[p^2 \cdot \max\big[0; u^2V - K\big] + (1-p)^2 \cdot \max\big[0; d^2V - K\big] \\
&\qquad + 2p(1-p) \cdot \max[0; udV - K]\Big] e^{-2r\Delta t}
\end{aligned}
$$

The same logic can be applied in order to derive the value of a call within a n-period binomial tree.

The value of a call for an n-step binomial tree is:

$$c = \left\{ \sum_{j=0}^{n} \left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^{n-j} \cdot \max\left[0; u^j d^{n-j} S_0 - K\right] \right\} e^{-nr\Delta t}$$

This equation can be rewritten as follows:

$$c = \left\{ \sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^{n-j} \cdot \left(u^j d^{n-j} S_0 - K\right) \right\} e^{-nr\Delta t}$$

where a is the minimal number of upward moves, such that the option is in the money at maturity ($(u^j d^{n-j} S_0 - K) \geq 0$).

By splitting the right-hand side of the latter equation into two terms, the next expression is obtained:

$$\begin{aligned} c = S_0 &\left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^{n-j} \cdot \left(u^j d^{n-j}\right) e^{-nr\Delta t} \right] \\ &- K e^{-nr\Delta t} \left[\left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^{n-j} \right] \end{aligned}$$

i.e.:

$$\begin{aligned} c = S_0 &\left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!} \right) \left(\frac{pu}{e^{r\Delta t}} \right)^j \left(\frac{(1-p)d}{e^{r\Delta t}} \right)^{n-j} \right] \\ &- K e^{-nr\Delta t} \left[\left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^{n-j} \right] \end{aligned}$$

Finally, the call price is obtained:

$$\begin{aligned} c = S_0 &\left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!} \right) \hat{p}^j \cdot (1-\hat{p})^{n-j} \right] \\ &- K e^{-nr\Delta t} \left[\left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^{n-j} \right] \end{aligned}$$

with:

$$\begin{aligned} \hat{p} &= \frac{pu}{e^{r\Delta t}} \\ 1 - \hat{p} &= \frac{(1-p)d}{e^{r\Delta t}} \end{aligned}$$

The first (the second respectively) expression in brackets corresponds to the complementary binomial distribution function $\Phi[a; n, \hat{p}]$ (resp. $\Phi[a; n, p]$). Therefore, the expression above can be simplified to:

$$c = S_0 \Phi[a; n, \hat{p}] - K e^{-nr\Delta t} \Phi[a; n, p]$$

and a being the minimal number of upward moves, such that the option is in the money at maturity. At the limit, when n tends to infinity, the Black and Scholes option pricing formula is obtained.

5.6 The Black–Scholes Formula

5.6.1 Pricing European Options

The Black–Scholes formula for a European call c and put option p on a stock without dividend payments is given by:

$$c = S_0 N(d_1) - K e^{-rT} N(d_2) \tag{5.1}$$

$$p = K e^{-rT} N(-d_2) - S_0 N(-d_1) \tag{5.2}$$

with

$$d_1 = \frac{\ln(\frac{S_0}{K}) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(\frac{S_0}{K}) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

and where the following notations are used: S_0 is the stock price at initial time t_0, K is the exercise price, c is the European call price, C is the American call price, p is the European put price, P is the American put price, r is the riskless interest rate, T is time to maturity, σ is the annualized volatility of the stock price returns and $N(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution. Note that the expected stock return plays no role in this formula.

The CDF of the standard normal distribution is the integral of its density:

$$N(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2}\, dx$$

Since there is no analytical solution for this integral, the distribution function of a standard Normal distribution can be approximated by relying on numerically computed tables that can be found in any statistics textbook (or on the web).

The assumptions of the Black and Scholes model are:

- There are no transaction costs or taxes.

- There are no riskless arbitrage opportunities.
- The risk-free rate of interest, r, is constant and the same for all maturities.
- Securities trading is continuous.
- The short selling of securities with full use of proceeds is permitted.
- All securities are perfectly divisible.
- Stock returns are normally distributed.
- The volatility is constant.
- There are no dividends until option maturity.

The following example allows one to compare results generated by the Black–Scholes and by the Binomial models. Let us consider the following values for the five inputs in the Black and Scholes formula:

$$\begin{aligned} S_0 &= €50 \qquad K = €51 \quad T = 0.5 \\ r &= 0.05 \qquad \sigma = 0.12 \end{aligned}$$

Given these inputs, the parameters d_1 and d_2 required in the Black and Scholes formula, can be derived. By relying on the aforementioned tables, the relevant values taken by the cumulative distribution function $N(\cdot)$ are then approximated:

d_1	$\frac{\ln(\frac{50}{51})+(r+\frac{1}{2}0.12^2)0.5}{0.12\sqrt{0.5}} \approx 0.104$
d_2	$d_1 - 0.12\sqrt{0.5} \approx 0.019$
$N(d_1)$	0.541
$N(-d_1)$	$1 - N(d_1) \sim 0.459$
$N(d_2)$	0.508
$N(-d_2)$	$1 - N(d_2) \sim 0.492$

Thus, the value of the European call and put options can be obtained:

$$\begin{aligned} &c = S_0 N(d_1) - Ke^{-rT} N(d_2) \\ &\quad \Rightarrow \quad c \sim 50 \cdot 0.5398 - 51 \cdot e^{-0.05 \cdot 0.5} \cdot 0.508 \sim €1.82 \\ &p = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \\ &\quad \Rightarrow \quad p \sim 51 \cdot e^{-0.05 \cdot 0.5} \cdot 0.492 - 50 \cdot 0.4602 \sim €1.56 \end{aligned}$$

The relationship between the put and call values, the so-called put-call parity, holds:

$$c - p = S_0 - Ke^{-rT} \quad \Rightarrow \quad 1.82 - 1.56 = 50 - 51 \cdot e^{-0.05 \cdot 0.5} = €0.26$$

The option price can also be calculated in a two-period binomial setting, defined as follows. The current stock price of €50 will either increase by 6 % or decrease by almost 6 % in the next three months. Indeed, the volatility, 12 %, corresponds to the subsequent parameters in this binomial setting.

$$u = 1.06 \qquad d = \frac{1}{u} \approx 0.94 \quad \text{and} \quad \Delta t = 0.25$$

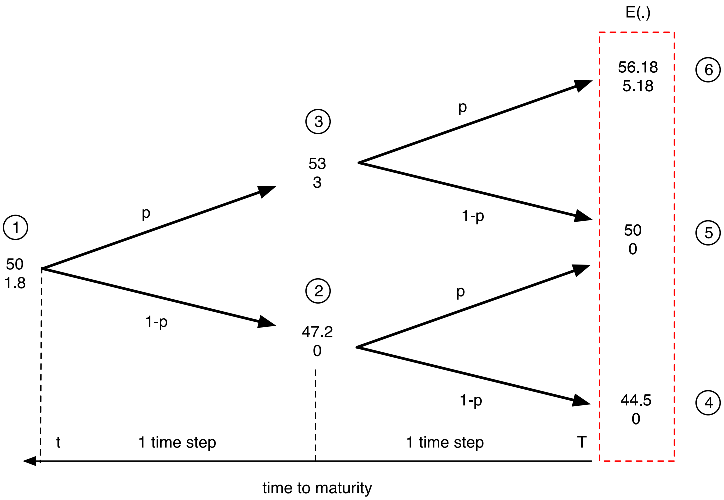


Fig. 5.11 Two-period binomial call pricing

Since $u = e^{\sigma\sqrt{\Delta t}}$, then $\sigma = \frac{\ln(u)}{\sqrt{\Delta t}} = \frac{\ln(1.06)}{\sqrt{0.25}} \approx 0.12$. The risk neutral probability of an upward movement is given by:

$$p = \frac{e^{\frac{r(T-t)}{2}} - d}{u - d} = \frac{e^{0.05\cdot 0.25} - \frac{1}{1.06}}{1.06 - \frac{1}{1.06}} \approx 0.6$$

The continuous annual risk free interest rate is 5 %. The factor 0.25 is used in the exponential because with a time to maturity of six months and a two period binomial approach, one period corresponds to three months, that is, to 0.25 year. Figure 5.11 illustrates the approach where 1.78 is approximated by 1.8.

By relying on a backward induction procedure, the possible values of the stock and of the option at maturity are first computed. The option prices are equal to their intrinsic value, that is, either to the difference between the stock and the strike prices if this difference is positive, or zero otherwise. If the stock price increases twice, the final price €56.18 is reached and the option price is equal to its intrinsic value, €5.18. Otherwise, if the stock price increases only once or decreases twice, its value will be smaller than the strike price, the option will be out-of-the-money at maturity with a value equal to zero.

The European call value is the risk neutral expected discounted value of the payoffs at maturity:

$$c = e^{-0.05\cdot 0.5} \cdot \left[0.6^2 \cdot 5.18 + 2 \cdot 0.6 \cdot (1 - 0.6) \cdot 0 + (1 - 0.6)^2 \cdot 0\right] = €1.78$$

At initial time (node one), the probability that the stock price will reach €56.18 (node 6) at maturity is 0.36 ($= 0.6^2$), and the probability of hitting the node €50 (node five) is $2 \cdot 0.6 \cdot (1 - 0.6)$, because two paths will lead to €50 at maturity and the probability of each of these paths is $0.6 \cdot 0.4$ (the stock price must increase and decrease once). Finally, the probability of reaching €44.5 (node four) is 0.16 ($= 0.4^2$), because in this case, the stock price decreases twice.

Obviously, the same value is obtained when the two possible values of the option after three months are discounted. For the nodes 3, corresponding to an upward movement, and 2, corresponding to a downward movement, the option values, c_u and c_d, respectively, are obtained.

$$c_u = e^{-0.05 \cdot 0.25} \cdot \left[0.6 \cdot 5.18 + (1 - 0.6) \cdot 0\right] = €3.00$$
$$c_d = 0$$

The value of the option at initial time (node one) is the risk neutral expected discounted value of the two possible payoffs after one period.

$$c = e^{-0.05 \cdot 0.25} \cdot \left[0.6 \cdot 3 + (1 - 0.6) \cdot 0\right] = €1.78$$

The value of the call option generated by the two-period binomial approach (€1.78), nearly corresponds to that of the Black and Scholes model (€1.82). The difference is mainly due to the limited number of steps, only two for a six month period.

5.6.2 Pricing American Options

The value of American Options can also be derived within a multi-period binomial model. For non-dividend paying stocks, American and European call options have the same price. However, this is not true for put options. Put options are also useful in the real option setting as they correspond to disinvestment decisions.

American put options can be exercised before maturity and, in some situations, it is indeed optimal to do so instead of keeping the option alive. These situations play a key role in the valuation of American put options. They can easily be identified within a multi-period binomial tree. At each node of the tree, the risk neutral expected discounted value of the two next possible payoffs must be compared with the intrinsic value of the option. The idea is to compare the payoffs corresponding to future and instantaneous exercises. If the difference between the former and the latter, i.e. the so-called time value of the option, is strictly positive, then the optimal decision consists in waiting. Intuitively, if time has value, the owner of the option should wait.

Conversely, if the payoffs generated by future and instantaneous exercises are equal, that is, if the time value of the option is zero, then the option should be exercised. Intuitively, if time has no value, the owner of the option should not wait. At each node of the tree, the value of the American put option is either the risk neutral expected discounted value of the next two possible payoffs if the time value is strictly positive, or the intrinsic value, if the time value is equal to zero. Therefore, at

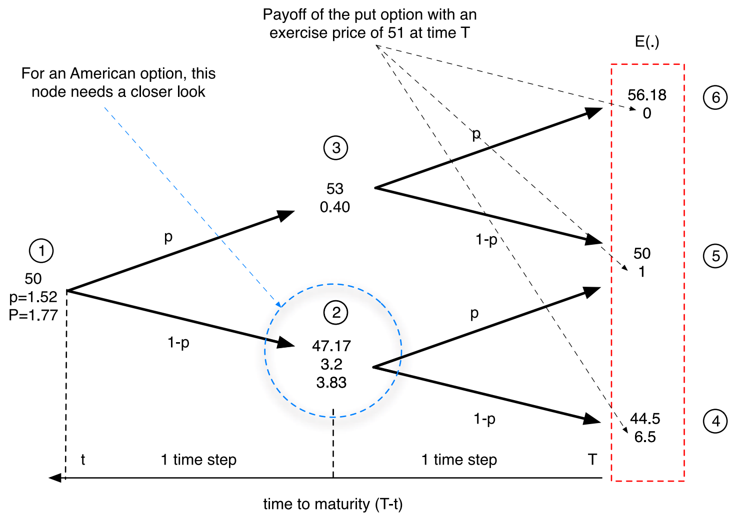


Fig. 5.12 The value of an American put option within a two-period binomial model

each node, this put price is the maximum of these two possible values. The following figure illustrates these ideas within the two-period binomial tree already used for American call options (Fig. 5.12).

By proceeding along the same lines as for the American call option, the backward induction procedure is used. The option prices at maturity are first computed. They are equal to their intrinsic value, that is, either to the difference between strike and stock prices if this difference is positive, or zero otherwise. At maturity, European and American option prices are equal. If the stock price increases twice, the final stock price €56.18 will be reached and the put option price will be equal to its intrinsic value which is zero. Otherwise, if the stock price increases and decreases once, its value will be equal to €50 and the option price will be equal to €1. Finally, if the stock price decreases twice and reaches €44.5, then the put price will be €6.5.

One period before maturity, the stock price is either €53 (node three) or €47.17 (node two). In the first case, the intrinsic value is equal to zero and the American put price is equal to the risk neutral expected discounted value of the two next possible payoffs: €0.4. This is also the price of the European put.

In the second case, the risk neutral expected discounted value of the two next possible payoffs is €3.2. This is not the value of the American put option, because its intrinsic value, €3.83, is higher. At this node, instead of keeping the option alive, it is optimal to exercise it. €3.2 represents the value of the European put option at this node. At initial time, the risk neutral expected discounted value of the two next possible payoffs value is given by:

$$P = e^{-0.05 \cdot 0.25} \cdot \left[0.6 \cdot 0.40 + (1 - 0.6) \cdot 3.83\right] = €1.77$$

This is the American put price. The intrinsic value of this option at this node is €1. Its time value is €0.77. The European put price is given by the following expression:

$$p = e^{-0.05 \cdot 0.25} \cdot \left[0.6 \cdot 0.40 + (1 - 0.6) \cdot 3.16\right] = €1.52$$

and is obviously smaller than the American one.

5.6.3 How Can the Volatility Be Estimated?

One of the five inputs required in order to use the Black and Scholes model is volatility. Unfortunately, this parameter is not observable. There are basically two methods that could be employed for its estimation.

The first one consists in computing the so-called historical volatility which corresponds to the variance of historical returns. The trivial example, which comes next, illustrates this approach.

The last four daily prices are considered: €100 at time t_0, €101 at t_1, €100 at t_2 and finally €101 at current time, t_3. Based on these four values, three stock returns can be computed: $+1\ \%$ from t_0 to t_1, approx. $-1\ \%$ from t_1 to t_2 and $+1\ \%$ during the last period. The average return is then obtained:

$$E(r) = \frac{0.01 - 0.01 + 0.01}{3} = 0.0033$$

Finally, the variance of the returns can be derived:

$$E(r) = \frac{(0.01 - 0.0033)^2 + (-0.01 - 0.0033)^2 + (0.01 - 0.0033)^2}{3} = 0.000089$$

The historical volatility is the annualized standard deviation (the number of trading days in one year is approximated by 270):

$$\sigma = \sqrt{0.000089} \cdot \sqrt{270} = 0.155 = 15.5\ \%$$

The second approach consists in computing the so-called implied volatility which is the volatility that, when used with a specific option pricing model, the Black and Scholes model for example, yields a theoretical value for the option equal to the current market price of that option. The following simple example sheds light on this approach.

Let us use the following parameters: The stock price at initial time, S_0, is equal to €99. The exercise price K is €100. The riskless interest rate, r, is equal to $1\ \%$, The time to maturity, T, is one year and the initial market price of the call option, c_m, is €9.9.

According to Eq. (5.1), the volatility implied by the market price of the option based on the Black and Scholes model satisfies the following equation:

$$c = S_0 N(d_1) - K e^{-rT} N(d_2)$$

with

$$d_1 = \frac{\ln(\frac{99}{100}) + 0.01 + \frac{\sigma^2}{2}}{\sigma} \approx \frac{\sigma}{2}$$
$$d_2 \approx -\frac{\sigma}{2}$$

Indeed, $\ln(\frac{99}{100}) + 0.01 \approx 0$.

Then $N(d_1) = N(\frac{\sigma}{2})$ and $N(d_2) = N(-\frac{\sigma}{2}) = 1 - N(\frac{\sigma}{2})$ Hence, the implied volatility satisfies:

$$99 \cdot N\left(\frac{\sigma}{2}\right) - 100 \cdot e^{-0.01} \cdot \left[1 - N\left(\frac{\sigma}{2}\right)\right] = 9.9$$

which means that

$$N\left(\frac{\sigma}{2}\right) \approx \frac{(\frac{9.9}{99} + 1)}{2} = 0.55$$

and by relying on a table for the normal distribution: $\frac{\sigma}{2} \approx 0.13$ In this example, the implied volatility is therefore 26 %.

Is the Volatility Constant? According to the Black and Scholes model, the volatility should remain constant. It is worthwhile to understand that it is a strong assumption and not only for stocks but also for the environmental setting. As shown for example in Fig. 3.2 of Chap. 3 on the price evolution of clean development mechanism (CDM) permits (CERs), this assumption has certain limitations (i.e. the presence of clusters of volatility instead of a constant one).

Intuitively, it seems quite clear that the volatility was not constant during this period of time. In general, most of the empirical results confirm this observation. The assumption of normality of the log-stock return distribution should be mainly comprehended as a useful approximation of reality, while bearing in mind its own limitations.

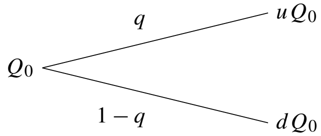
5.7 The Real Options Approach as a Decision Making Tool for Compliance with Environmental Regulation

In the remaining part of this chapter, uncomplicated examples of companies facing carbon regulations will be considered. More comprehensive models, in continuous time, will be presented in the last part of Chap. 6. These examples could be easily extended to broader and more complex settings. These firms will have to comply with environmental regulation within a trading scheme which will be a simplified version of the EU ETS. They will have to make strategic investment decisions concerning possible reduction of their emissions and the trade of emission allowances.

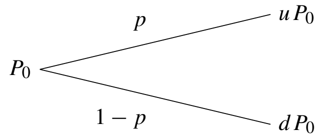
The use of the real options approach, instead of the standard static NPV criteria will appear to be more judicious.[2] Within a multi-period binomial model, the choice of this dynamic approach will indeed allow for optimal investment decisions.

5.7.1 A First Example: One-Period Binomial Model for the Emissions and Price Processes. What Is the Optimal Decision in Terms of Emission Rights Trading?

This basic case describes a one-period model where both the instantaneous emissions level of a regulated company and the emission permits price are exogenous and independent processes. The length of the period is one year. By assumption these two processes evolve according to a binomial model. The uncertainty of the market is introduced in the model through the existence of two possible states, at final time, for the emissions process and the allowance permit price dynamics. The following figures illustrates the context. The first one depicts the emissions process:



while the second one represents the price dynamics:



where $0 < d < 1 < u = \frac{1}{d}$ and where Q_0 and P_0 are the initial emissions level and permit price, respectively. The parameters q and p are the probabilities of an upward movement for pollution and prices, respectively. The final emission level and price, $\widetilde{Q}_1$ and $\widetilde{P}_1$, are perceived at initial time as random variables. Across the different examples that we present here, the tilde is used to indicate that the variable is random.

In this simple one-period model, trading opportunity exists only at initial time and the maturity of this trading scheme is one year. This first example does not distinguish between the real options approach and the NPV criteria. Indeed, for such a purpose, the possibility to delay the trade would be required. This will be the case in the next example. In spite of that, the option vocabulary is introduced in the current example.

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[2]Readers interested in the use of real options models to environmental problems should refer to Baranzini et al. (2003), Barrieu and Chesney (2003) and Loubergé et al. (2002) for examples of full-fledged applications.

X_0 denotes the requested initially unknown optimal quantity of permits that the company should buy (if X_0 is positive) or sell (if negative) at initial time, and N the permits endowment at that time. By the end of the period, at time 1, the company must possess enough permits. If the firm fails to meet compliance, it will pay a penalty equal to P plus the price of the permits, that is, P_1 for each ton of uncovered pollution.

Concerning the emission allowances owned by the company at the end of the period, either a shortage or a surplus situation is expected. The company runs the risk of either paying the price for being in shortage, that is, being uncovered, or being in excess, that is, holding worthless emission allowances. In the former case, the costs are generated by the possession of too few emission rights at maturity. In the latter case, the incurred costs are due to the purchase at initial time of too many emission allowances. The reader should bear in mind that at maturity, the emission rights have a redemption value equal to zero. Therefore, a regulated company should be particularly cautious in its decision to buy emission rights.

Let $g(\cdot)$ be defined as the company's net final position in terms of emission permits at t_0:

$$g(\widetilde{Q}_1) = Q_0 + \widetilde{Q}_1 - X_0 - N$$

with $\widetilde{Q}_1$ the random emission level that can take either the value uQ_0 or dQ_0 at t_1.

A shortage situation for the regulated company corresponds to a positive net final position g. Conversely, an excess situation will be characterized by a negative g.

Given the initial endowment of permits and the expected future permits net position, at time zero the firm minimizes its expected discounted costs. Therefore, the total cost is simply the sum of the costs incurred at time zero and the potential costs (i.e., the penalty plus the price of the permits, per unit of uncovered pollution) at time one. The initial cost corresponds to the purchase of emission allowances. A negative cost (i.e., a profit) would instead result from a sale. The resulting minimization problem is:

$$\min_{X_0}\left\{P_0 \cdot X_0 + (1+\eta)^{-1} E\left[g(\widetilde{Q}_1)^+ \cdot (\widetilde{P}_1 + P)\right]\right\} \tag{5.3}$$

where $g()^+$ is defined as the positive value of the company's net final position:

$$g(\widetilde{Q}_1)^+ = \begin{cases} Q_0 + \widetilde{Q}_1 - X_0 - N & \text{if } Q_0 + \widetilde{Q}_1 - X_0 - N \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

In this setting, the objective of the regulated firm is to trade an optimal number X_0 of emission rights. Optimal means that the company will determine this number in such a way that its net final position, $Q_0 + \widetilde{Q}_1 - X_0 - N$, will be as close to zero as possible. The objective of the company is, as much as possible, to avoid both excess and shortage situations which generate costs at initial time in the former case and at maturity of the trading scheme, in the latter.

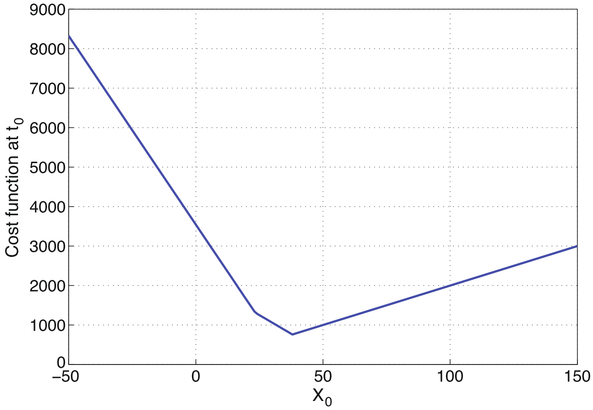


Fig. 5.13 Cost function at t_0

Interestingly, $g()^+$ corresponds to the payoff (in volume) at maturity of a call option with strike price $X_0 + N$ and a final underlying value equal to $Q_0 + Q_1$. Therefore through this option pricing prism, the objective of the company is at initial time to choose the exercise price, thanks to the choice of the trading quantity X_0, in such a way that the call option will be possibly at-the-money at maturity. Obviously, in the current stochastic context, an optimal choice of exercise price does not mean that the option will indeed be at-the-money with certainty. Indeed, at initial time, $\widetilde{Q}_1$ is a random variable.

Usually, in a minimization problem setting, a partial derivative of the function to minimize, with respect to the relevant variable, is computed. Unfortunately, to proceed along these lines is not possible because the cost function considered in expression (5.3) is not differentiable everywhere. Indeed, the positive part of function g, which is a key factor in the cost function, is not differentiable when the above mentioned option is at-the-money, i.e. precisely when the expected discounted cost are minimized. A numerical approach is therefore employed. The solution is graphically identified (see Fig. 5.13) with the following set of parameter values:

- The initial endowment of emission permits, N, is equal to 50;
- The discount rate, η, is equal to 6 %;
- The penalty, P, is equal to €100;
- The initial permit price, P_0, is assumed to be €20;
- The upward probability for the price process, p, is 0.8;
- The factors u and d characterizing the upward and the downward movement of prices and emissions are 1.2 and $\frac{1}{1.2}$ ($\approx$0.83), respectively;
- The initial CO_2 emission, Q_0, is assumed to be 40 tons;
- The upward probability, q, for the emission process is 0.5;

The cost function has been plotted for different possible values of X_0. This function is minimized when the regulated company purchases a number of emission allowances equal to 38.

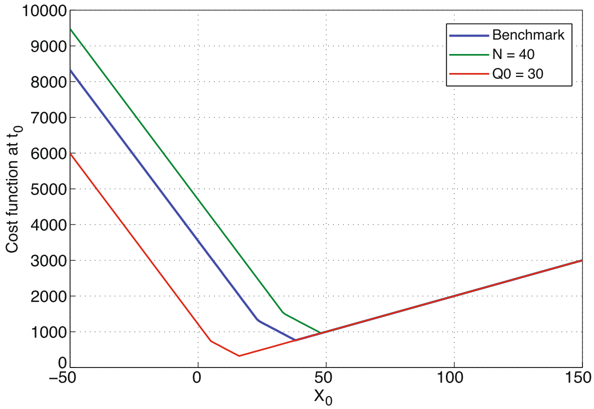


Fig. 5.14 Sensitivity of the cost function to change in N and Q_0

This is the quantity of permits to be bought in order to perfectly hedge the emissions exposure in the worst case scenario where the emission increases between the initial and the final date. In this case indeed, the cumulated emission level of the firm will be 88 tons $(40 + 40 \cdot 1.2)$, and given the initial endowment corresponding to 50 tons of emissions, allowances matching 38 tons are required in order to avoid penalties.

Intuitively, the initial permit price of €20 is small compared to a penalty of €100. Therefore, it is cheaper for the firm to buy the allowances corresponding to this scenario and to possibly own too many rights than to buy the number of permits related to the most favorable scenario (when emissions decrease) and to take the risk of paying penalties and buying emission allowances at maturity. As seen later, this result holds true as long as the probability q of the worst case scenario remains high enough.

Obviously, the lower N, the larger the quantity of permits the company has to buy at initial time. Similarly, the lower Q_0, the smaller the quantity of permits the company should buy at this time. These results are emphasized in Fig. 5.14.

Along the same lines, if the upward probability q of the emission process is small enough, the quantity of permits the company has to buy at initial time in order to hedge the pollution exposure, decreases.

In the first case where q is equal to only 10 %, the company should buy 22 emission permits, but no more. This number corresponds to the hedge against the favorable scenario only, that is, against the case where the emission process decreases. In this case, the firm will indeed require $Q_0 + d \cdot Q_0 - N$ that is, 22 permits precisely, because the weight given to the worst case scenario is quite small.

In the second and opposite case where q is equal to 90 %, X_0 stays at 38. Indeed, the weight given to the worst case scenario is in this case predominant. Figure 5.15 illustrates the results of these comparative statics.

Finally, as shown in Fig. 5.16, if the initial price P_0, is high enough, the number of emission permits X_0 purchased by the regulated firm decreases. For example, if P_0 is equal to 160, the company will hedge only against the favorable scenario and hence limit its purchase to 22 emission allowances.

Fig. 5.15 Sensitivity of the cost function to change in the probability q

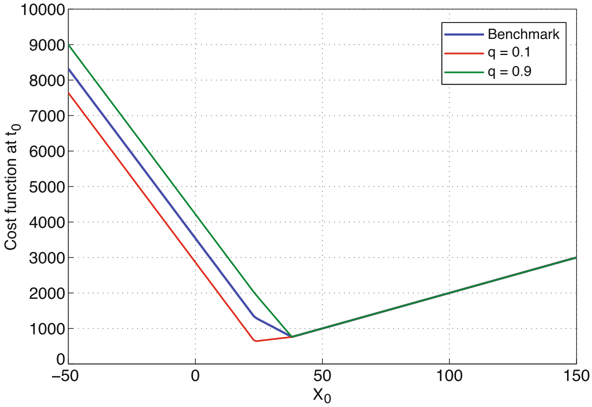
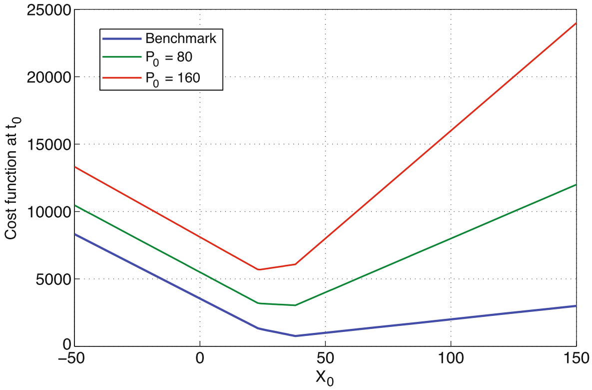


Fig. 5.16 Sensitivity of the cost function to change in the price P_0



5.7.2 A Second Example: Two-Period Binomial Model for the Emissions and Price Processes. What Are the Optimal Decisions in Terms of Emission Rights?

This case is an extension of the last example. It corresponds to a two-period model where both the instantaneous emissions of a regulated company and the emission permits price are exogenous and independent processes. The length of each period is one year. Except for this extension, the context of the last example holds true and the previous notations are used.

By assumption these two processes evolve according to a binomial model. The uncertainty of the market is introduced in the model through the existence of three possible states, at final time, for pollution, and the emission allowance permit price. Figure 5.17 illustrates the context with $0 < d < 1 < u = \frac{1}{d}$ and where Q_0 and P_0 are the initial emissions level and initial permit price respectively. Parameters q and p are the probabilities of an upward movement for emissions and prices, respectively. The time one and final emission level and price, $\widetilde{Q}_1$, $\widetilde{Q}_2$ and $\widetilde{P}_1$, $\widetilde{P}_2$, are perceived as random variables at initial time.

Fig. 5.17 Emissions and price dynamics

In this two-period model, trading opportunities exist at initial time and after one year. This second example, by introducing the possibility to partially delay the potential trade, will allow, in contrast to the first one, to distinguish between the real options approach and the NPV criteria.

In this new setting, there are now two unknowns that need to be determined: X_0 and X_1 are the optimal quantity of permits that the company should buy or sell, respectively, at initial time and after one year. As in the first example, N denotes the permits endowment at initial time.

At maturity, at the end of the second period, the company must have enough permits. If the firm fails to achieve compliance, it must pay a penalty equal to P plus the price of the permits, that is, P_2 for each ton of uncovered pollution.

By proceeding along the same lines as in the last example, at initial time the company minimizes its cost function $f()$:

$$\min_{X_0} f(X_0) \tag{5.4}$$

where

$$f(X_0) = \left\{P_0X_0 + E\left[(1+\eta)^{-1} \cdot \widetilde{P}_1 \cdot X_1 + (1+\eta)^{-2} h(\widetilde{Q}_1, \widetilde{Q}_2)^+ \cdot (\widetilde{P}_2 + P)\right]\right\} \tag{5.5}$$

and where function $h()$ represents the company's net final position in terms of emission permits:

$$h(\widetilde{Q}_1, \widetilde{Q}_2) = Q_0 + \widetilde{Q}_1 + \widetilde{Q}_2 - X_0 - X_1 - N$$

and depends on two factors in this second example. In line with the last example, $h(\cdot)^+$ represents the positive value of function $h(\cdot)$. With this approach, the real option to trade emissions rights at time 1 is considered.

The cost function is the sum of three components: the costs incurred at initial time and corresponding to the trade of allowances, the expected discounted costs resulting from the trade of permits at the end of the first period, and finally the expected discounted costs at maturity (i.e. the penalty plus the price of the permits, per unit of uncovered pollution). A negative cost, that is, a profit, at time zero or at time one would result from the decision to sale.

The number of traded emission permits after one year, X_1, depends on the current cumulated emissions, $Q_0 + Q_1$, the realized permit price, P_1, and the number of allowances owned by the company $N + X_0$, at that time:

$$X_1 = X_1(Q_0 + Q_1, P_1, N + X_0)$$

Indeed, after one year, the regulated company will be confronted with a second choice, the choice of the number of allowances that it should trade, X_1. The context of this second choice corresponds to the one-period binomial model considered in the first example:

$$\min_{X_1}\left\{P_1 \cdot X_1 + (1+\eta)^{-1} E\left[g(\widetilde{Q}_2)^+ \cdot (\widetilde{P}_2 + P)\right]\right\}$$

where the company's net final position in terms of emission permits, $g()$, is defined by the following one argument function:

$$g(\widetilde{Q}_2) = Q_0 + Q_1 + \widetilde{Q}_2 - X_0 - X_1 - N$$

Therefore, in order to solve this problem, a backward induction procedure can be applied. The idea consists in starting first at the end of the first period, and in finding the optimal volume of traded emission X_1, at that date as a function of the possible initial traded quantity X_0. Then, the problem will be resolved at initial time and the initial optimal traded quantity will be derived.

As previously mentioned, after one period, the context of the minimization costs problem is a one-period binomial model and therefore corresponds to the first example. According to this example and given the set of parameters used in this case, the objective of the company is always to hedge against the worst case scenario.

The intuition remains the same. The initial permit price, €20, is small compared to a penalty of €100. Therefore, it is cheaper for the firm to buy the allowances corresponding to this scenario and to possibly own too many rights than to buy the number of permits related to the most favorable scenario (a decrease in the emissions) and to take the risk of paying penalties and buying emission allowances at maturity. After one year, the permit price will be either €24 or €16.67. A permit price of €24 is not high enough to change this hedging strategy.

A fortiori for a price of €16.67 which is smaller than €20. According to the one-period model, the optimal volume of traded emissions at date one is therefore:

$$X_1^* = \left(1 + u + u^2\right) Q_0 - X_0 - N$$

if the emissions after one year, Q_1, increase and are equal to uQ_0 and

$$X_1^* = (1 + d + du) Q_0 - X_0 - N$$

if the emissions after one year, Q_1, decrease and are equal to dQ_0.

Now that the problem has been solved at year one, the initial minimization problem at initial time, given by Eqs. (5.4) and (5.5), can be considered:

$$\min_{X_0}\left\{P_0 X_0 + (1+\eta)^{-1} E\left[\widetilde{P}_1 \cdot \widetilde{X}_1^*\right] + (1+\eta)^{-2} E\left[h(\widetilde{Q}_1, \widetilde{Q}_2)^+\right] \cdot E\left[(\widetilde{P}_2 + P)\right]\right\}$$

where:

$$
\begin{aligned}
E\big[\widetilde{P}_1 \cdot \widetilde{X}_1^*\big] = pq\big(uP_0 \cdot \big(\big(1+u+u^2\big)Q_0 - N - X_0\big)\big) \\
+ p(1-q)\big(uP_0 \cdot \big((2+d)Q_0 - N - X_0\big)\big) \\
+ (1-p)q\big(dP_0 \cdot \big(\big(1+u+u^2\big)Q_0 - N - X_0\big)\big) \\
+ (1-p)(1-q)\big(dP_0 \cdot \big((2+d)Q_0 - N - X_0\big)\big)
\end{aligned}
$$

and where function $h(\cdot)$, that is, the company's net final position in terms of emission permits, is given:

$$h(\widetilde{Q}_1, \widetilde{Q}_2) = Q_0 + \widetilde{Q}_1 + \widetilde{Q}_2 - X_0 - \widetilde{X}_1^* - N$$

if the emissions after one year, Q_1, increase and are equal to uQ_0 and

$$h(uQ_0, \widetilde{Q}_2) = Q_0 + uQ_0 + \widetilde{Q}_2 - X_0 - \big(\big(1+u+u^2\big)Q_0 - X_0 - N\big) - N$$

if the emissions after one year, Q_1, decrease and are equal to dQ_0.

$$h(dQ_0, \widetilde{Q}_2) = Q_0 + dQ_0 + \widetilde{Q}_2 - X_0 - \big((2+d)Q_0 - X_0 - N\big) - N$$

In both cases, it is straightforward to check that this function is negative. The intuition being that the company will avoid penalties at maturity, because after one year, it will always hedge against the worst case scenario.

Therefore:

$$h(\widetilde{Q}_1, \widetilde{Q}_2)^+ = 0$$

The minimization problem is therefore trivial as the cost function is a linear function of X_0:

$$
\begin{aligned}
f(X_0) = P_0X_0 + (1+\eta)^{-1} pq\big(uP_0 \cdot \big(\big(1+u+u^2\big)Q_0 - N - X_0\big)\big) \\
+ p(1-q)\big(uP_0 \cdot \big((2+d)Q_0 - N - X_0\big)\big) \\
+ (1-p)q\big(dP_0 \cdot \big(\big(1+u+u^2\big)Q_0 - N - X_0\big)\big) \\
+ (1-p)(1-q)\big(dP_0 \cdot \big((2+d)Q_0 - N - X_0\big)\big)
\end{aligned}
$$

i.e.:

$$
\begin{aligned}
f(X_0) = P_0X_0\left(1 - \frac{pu + (1-p)d}{1+\eta}\right) \\
+ (1+\eta)^{-1}\big(pq\big(uP_0 \cdot \big(1+u+u^2\big)Q_0 - N\big) \\
+ p(1-q)\big(uP_0 \cdot (2+d)Q_0 - N\big) \\
+ (1-p)q\big(dP_0 \cdot \big(1+u+u^2\big)Q_0 - N\big) \\
+ (1-p)(1-q)\big(dP_0 \cdot (2+d)Q_0 - N\big)\big)
\end{aligned}
$$

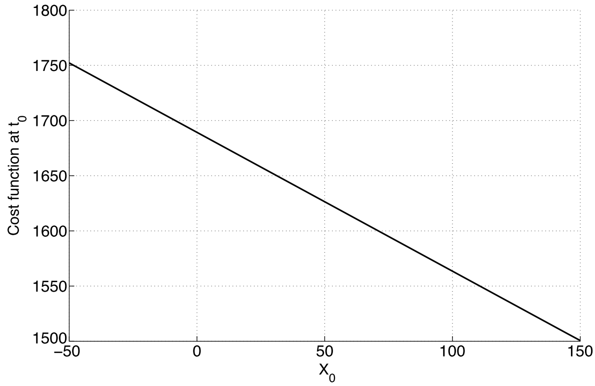


Fig. 5.18 Cost function at time t_0

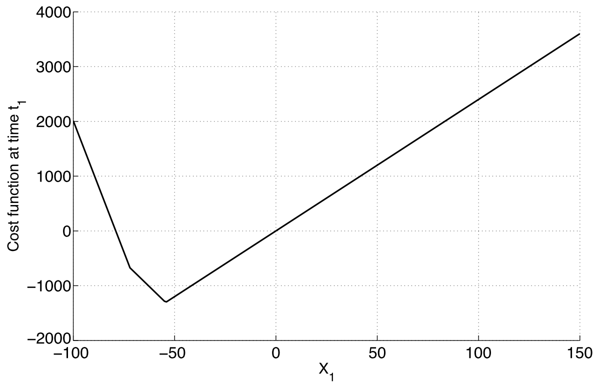


Fig. 5.19 Cost function at time t_1 if emissions increased at t_1

With the selected set of parameters, the factor of X_0 is negative and therefore the optimal choice is to buy as many emission permits as possible. In our framework, X_0 is constrained in the interval $(-N, 3N)$. Hence, the company should buy 150 emission allowances at initial time and should therefore possess a total of 200 permits at initial time. Figure 5.18 sheds light on this strategy.

If the emissions increase after one year, the regulated company will sell 54 permits in order to keep 146 allowances that will correspond to the hedge in the worst case scenario. The cumulated emissions are indeed almost equal to 146 tons (i.e., $40(1 + u + u^2) = 145.6$), if the emissions level increases twice. Figure 5.19 illustrates the minimization problem.

If the emissions decrease after one year, the company should sell 87 permits in order to hold 113 allowances which will allow the firm to be hedged if emissions increase during the second period. The cumulated emissions are indeed approximatively equal to 113 tons (i.e., $40(1 + d + 1) = 113.3$), if the pollution level increases and then decreases during the second period (Fig. 5.20).

It is important to mention that those results largely depend on the price dynamics for the permits. Under our initial assumptions, the discounted expected future price $\frac{E[P_1]}{(1+\eta)}$ is higher than its initial value P_0, which justifies to buy as many permits as

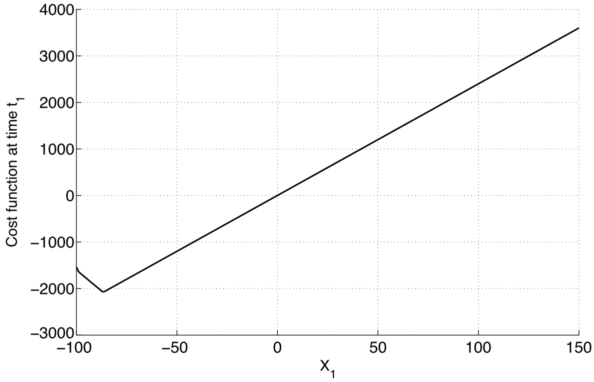


Fig. 5.20 Cost function at time t_1 if emissions decreased at t_1

possible in the initial period to benefit from the increase in value between the two periods.

If we consider instead the discounted price dinamics as a martingale, such that $\frac{(pu+(1-p)d)}{(1+\eta)} = 1$, it is evident from the minimization problem at period 0 that X_0 does no longer enter the minimization problem. Intuitively, the company is no longer able to benefit from a positive evolution of the price (in expectation), which eliminates the buying strategies at period 0. There would be no difference in expectation between a purchase at $t = 0$ and a purchase at $t = 1$. In period 1 however, the hedging strategies are obviously maintained.

Under our initial assumptions where the discounted future price is higher than P_0, it is worthwhile to compare the NPV criterion with the real options setting. According to the NPV procedure, the first decision that the company should take is to decide whether or not it is preferable to wait one period before trading emissions permits or not. If the company trades immediately, the second decision will be the ability to buy or sell again at t_1, knowing that the decision at t_1 does not enter in the minimization problem at t_0. Indeed, the NPV decision is based on a sequence of static decisions: If the company prefers to wait, the number of emissions permits X_0 are set to 0 and the second decision will be how many permits X_1 to trade to avoid paying the penalty at t_2.

We first need to assess which NPV strategy ensures the best cost minimization: waiting one period (NPV_1) or starting trading immediately (NPV_0).

Under NPV_0, the company will first decide either to purchase (or sell) emissions permits X_0 at t_0. The optimal choice of X_0 will maximize the following NPV:

$$\text{NPV}_0(t_0) = -\left\{P_0X_0 + E\left[(1+\eta)^{-2} \cdot h(\widetilde{Q}_1, \widetilde{Q}_2)^+(\widetilde{P}_2 + P)\right]\right\}$$

Only then at t_1 the company will decide on additional trading of permits X_1. The static NPV approach would recommend purchasing 96 emission allowances instead of 150 at the initial date.

In this simple setting, the company is confronted with a choice between different negative NPV projects, that is, between different choices of X_0. The minimized costs under the NPV are bigger than under the real option setting (2000 at Fig. 5.21

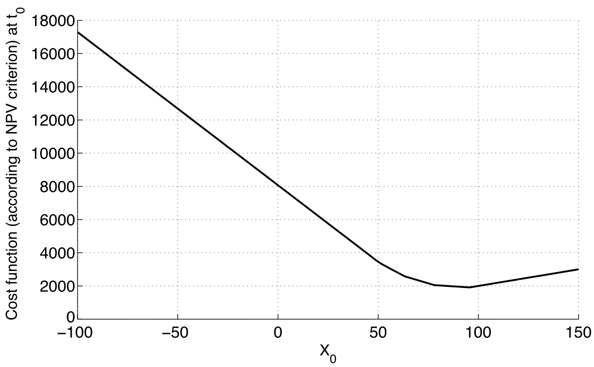


Fig. 5.21 Cost function (according to the NPV criterion) at time t_0

is higher than 1500 at Fig. 5.18) because the dynamic strategy linked to the existence of the second date is not taken into account. The objective being to minimize the cost function, that is, to maximize the (negative) expected discounted profit.

The standard static NPV approach considers only the current investment decision and recommend trading volume X_0 of permits which maximizes the NPV. Given the set of parameters, as previously mentioned, this criteria would lead to the purchase of 96 permits which represents a hedge in the worst case scenario, if the emission level increases twice.

Figure 5.21 shows the cost function at time t_0 under the NPV criterion NPV_0.

At initial time, the $\mathrm{NPV}_0(t_0)$ is negative since the purchase of 96 permits represents a cost. According to this approach, the acquisition of 150 permits, resulting from the real option approach, is no longer recommended because the possibility to disinvest at a later time, that is, to sell extra rights after one year, is not appropriately taken into account, that is to say, dynamically. This possibility will be only considered only once it appears, that is, after one year. At this date, the NPV of the project will be:

$$\mathrm{NPV}_0(t_1) = -\left\{P_1 X_1 + (1+\eta)^{-1} \cdot E\left[g(\widetilde{Q}_2)^+(\widetilde{P}_2 + P)\right]\right\}$$

where the company's net final position in terms of emission permits, $g()$, is defined by the following one argument function:

$$g(\widetilde{Q}_2) = Q_0 + Q_1 + \widetilde{Q}_2 - X_0 - X_1 - N$$

Given the set of parameters employed in this example, if, during the first year emissions decrease, the NPV criteria will recommend to sell 33 permits. In this case indeed the regulated company will own 146 less 33 permits, that is to say 113 allowances, and will be therefore hedged against the worst case scenario in which the emission level increases during the second year.

If emissions increase during the first year, according to the NPV criteria, the company should not trade permits. Its total of 146 permits generates a hedge if emissions increase further during the second year.

The other NPV denoted $\mathrm{NPV}_1(t_1)$ would be to wait until t_1 before trading emission permits. In this case, $X_0 = 0$ and the optimal solution depends on whether emissions increased or not. If emissions decrease, the company should purchase 63 permits to fully hedge its worst case scenario (113). If emissions increase, the company need to purchase 96 permits to avoid having to pay penalties in the worst case (146). It is clear that the strategy of waiting one period is not interesting for the firm. The company has still to detain the same amount of permits at t_1 but will have to pay in expectation more to get them. Therefore, the NPV_0 is the best NPV strategy. As shown in this example, in the NPV setting, a sequence of static decisions are implemented, instead of the dynamic decision making process of the real options framework. When a decision is taken at initial time, it neglects the possibility of taking a further decision later, after one year, and therefore neglects the possibility to buy them at t_0 and then to sell them at a higher price.

The analysis of the net value of the project, under the NPV criterion and the real options setting, sheds light on this difference. Under the NPV, the sequence of static decisions results in a cost of 1920 euros at t_0 (to purchase the permits, e.g., $96 \cdot 20$). In expectation, the company will be able to either sell 33 permits (if emissions decrease) or do nothing (if emissions increase), resulting in the expected profits from the t_1 strategy:

$$\frac{0.5 \cdot (33 \cdot E[P_1]) + 0.5 \cdot 0}{1.06} = \frac{0.5 \cdot (33 \cdot (0.8 \cdot 20 \cdot 1.2 + 0.2 \cdot \frac{20}{1.2}))}{1.06} \approx 351$$

The net cost for the company at t_0 under the NPV_0 is therefore $1920 - 351 = 1569$ euros.

In this simple setting, the full hedge, that is to say the purchase of 96 permits, is recommended, while in the real options framework, the over hedge, i.e. the purchase of 150 permits, is suggested. The latter strategy encompasses the full hedge strategy of the 96 permits acquired, but goes beyond this with 54 extra emission rights. These extra permits generate the option to disinvest after one year, that is to say to sell these rights at a higher price. Therefore, from the real options perspective, the optimal choice consists not only in exercising 96 call options at initial time, but 54 more. The difference in value, for which the real options approach yields lower costs, comes from the time value created by the purchase of the put options. This is the fact of detaining and waiting one year that allows the company to resell the emissions permits at a profit in expectation. This time value is non-existent in the NPV, because the value to detain the put options and to disinvest after one period is never taken into account.

These 54 extra rights will add a new dimension to the investment/disinvestement strategy. They can be sold after one year and they therefore represent the creation of 54 put options. These put options should be exercised after one year (when their time value will be equal to zero), in the absence of future exercise dates. They will add value to the strategy. Indeed, the emission price increases with a probability

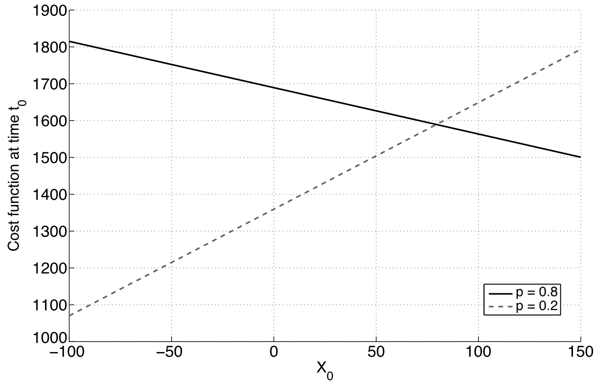


Fig. 5.22 Sensitivity of the cost function to change in p

p equal to 80 %. Therefore, the expected discounted profit corresponding to these extra rights is:

$$54 \cdot \left(\frac{0.8 \cdot 24 + 0.2 \cdot 16.7}{1.06} - 20 \right) \approx 68$$

The positive sign of this amount corroborates the idea that the real options approach dominates the NPV criteria. Under the real options setting, the net cost at t_0 is therefore $1920 - 351 - 68 = 1501$ euros, an improvement of 68 euros when compared to the sequence of static decisions of the NPV. This difference may seem small but it is due to the constraint we put on possible purchase of permits X_0 at t_0 ($\leq 3N$).

Comparative Statics Obviously, with other parameter values different results would be obtained. For example, if the upward probability for the price process, p, is equal to 20 % instead of 80 %, then the permit price is expected to fall and it makes more sense to sell as many allowances as possible—in this case 50—with the objective of buying at a lower price the required permits after one year in order to comply with regulation at maturity. Indeed, the possibility to trade permits again after one year allows for such a strategy. The latter is emphasized in Fig. 5.22.

The use of the dynamic decision making process allows the company at date zero, to take advantage of the expected permit price by adjusting the strategy based only on the full hedge of cumulated emissions. To adjust the latter strategy means to over- or under-hedge at the initial date, depending on the expected trend of the permit price process until time one, in order to profit from expected price movements. This distorted hedge will be transformed into a perfect one at date one.

Obviously, if the assumption that the volume of traded permits has no impact on the permit price evolution, is relaxed, it could enhance the performance of these strategies based on real options.

5.7.3 A Third Example: One-Period Binomial Model for the Emission and Price Processes. What Is the Optimal Decision in Terms of Emission Rights Trading and Technology Changes?

As in the first example, a one-period model is considered where both the instantaneous pollution and the emission permits price are exogenous and independent processes. As mentioned previously, a one-period model does not allow for the distinction between the real options approach and the NPV criteria. Indeed, for such a purpose, the possibility to take the investment decision at a later date would be required. This will be the case in the next example.

In this third example, there is no trading opportunity. The focus is instead on technology changes in order to meet compliance. It is assumed that at initial time the company can either switch the technology of its production process ($\theta = 1$) for a lump-cost C or do nothing ($\theta = 0$). The new technology will reduce the emissions by Γ tons of offending gas.

According to the first example, given the initial endowment of permits, the potential reduction from the change of technology and the expected future permits net position, the firm minimizes its cost function $f(\cdot)$ at initial time.

The cost function is simply the sum of the costs incurred at initial time if the firm changes its technology and the potential costs at final time. The latter consists of the penalty plus the price, times the number of missing emission rights in the case of shortage. The resulting minimization problem is:

$$\min_{\theta \in (0,1)} f(\Gamma, \theta) \quad \text{with}$$

$$f(\Gamma, \theta) = \theta \cdot C + (1+\eta)^{-1} E\big[g(\widetilde{Q}_1, \theta)^{+} \cdot (\widetilde{P}_1 + P)\big]$$

where $g(\widetilde{Q}_1, \theta) = Q_0 + \widetilde{Q}_1 - N - \theta \cdot \Gamma$.

In line with the first example, the costs at maturity correspond to the payoff of a call option with strike price $N + \theta \cdot \Gamma$ and a final underlying value equal to $Q_0 + Q_1$. Therefore, through this option pricing prism, the objective of the company is, at initial time, to choose the exercise price, thanks to the choice of technology, i.e. Γ, in such a way that the call option will possibly be at-the-money at maturity. Obviously, in the current stochastic context, an optimal choice of exercise price does not mean that the option will be at-the-money with certainty. Indeed, at initial time, Q_1 is a random variable.

In order to take an optimal decision, the comparison of the two values taken by the cost function f for the two possible values of the control variable, i.e. $\theta \in \{0, 1\}$, is required.

$$f(\Gamma, 0) = (1+\eta)^{-1} E\big[g(\widetilde{Q}_1, 0)^{+} \cdot (\widetilde{P}_1 + P)\big]$$

and

$$f(\Gamma, 1) = (1+\eta)^{-1} E\big[g(\widetilde{Q}_1, 1)^{+} \cdot (\widetilde{P}_1 + P)\big] + C$$

Table 5.2 Parameters used in the numerical example

Parameter	Value
N	50
η	0.06
P (penalty)	100
P_0	20
p	0.8
u	1.2
d	$\frac{1}{1.2} \approx 0.8$
Q_0	40
q	0.5
Γ	30 (tons)
C	€1000

By relying on the definition of the function g and by separating the two values into a product of independent expectations, the following equivalence is obtained:

$$
\begin{aligned}
&f(\Gamma,0) > f(\Gamma,1) \\
&\quad\Leftrightarrow\quad E\big[(Q_0+\widetilde{Q}_1-N)\cdot \mathbf{1}_{Q_1\geq N-Q_0}\big]\cdot\big(E[\widetilde{P}_1]+P\big) \\
&\qquad\qquad > E\big[(Q_0+\widetilde{Q}_1-N-\Gamma)\cdot \mathbf{1}_{Q_1\geq N+\Gamma-Q_0}\big]\cdot\big(E[\widetilde{P}_1]+P\big)+(1+\eta)C
\end{aligned}
$$

And moving the expectations to the left-hand side:

$$
\begin{aligned}
&f(\Gamma,0) > f(\Gamma,1) \\
&\quad\Leftrightarrow\quad \big\{(Q_0-N)\cdot \Pr\big(Q_1\in[N-Q_0,N+\Gamma-Q_0]\big) \\
&\qquad\qquad -\Gamma\cdot\Pr(Q_1>N+\Gamma-Q_0) \\
&\qquad\qquad + E[Q_1\cdot \mathbf{1}_{Q_1(\omega)\in[N-Q_0,N+\Gamma-Q_0]}]\big\}\cdot\big(E[\widetilde{P}_1]+P\big) > C(1+\eta)
\end{aligned}
$$

For the numerical approach, the set of parameter values employed in the first example is kept, with two extra parameters Γ and C (Table 5.2).

Figure 5.23 represents the relationship between the cost function (for $\theta=0$ and $\theta=1$) and the reduction of emissions: Γ in tons of offending gas for $\Gamma\in[0,50]$ and $C=€1000$.

Obviously $f(0)$ does not depend on Γ since $\theta=0$ corresponds to the case where there is no technology change. In this example, $f(0)=3545$. For a given cost C of adopting the cleaner technology, $f(1)$ is a decreasing function of the realized reduction in emissions.

For Γ smaller than 9 tons of offending gas, it is not worthwhile to adopt the new technology. Switching costs are too high compared to a limited reduction in offending gas emissions. On the contrary, if Γ is higher than 9 tons, a technology change is optimal in order to avoid incurring the penalty. In this case, the cost function is minimized for $\theta=1$.

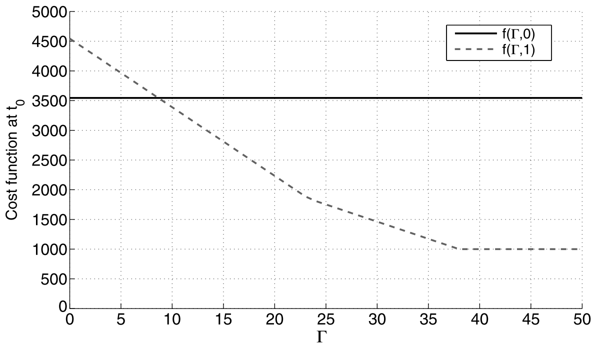


Fig. 5.23 Cost function for different value of $\Gamma \in [0, 50]$

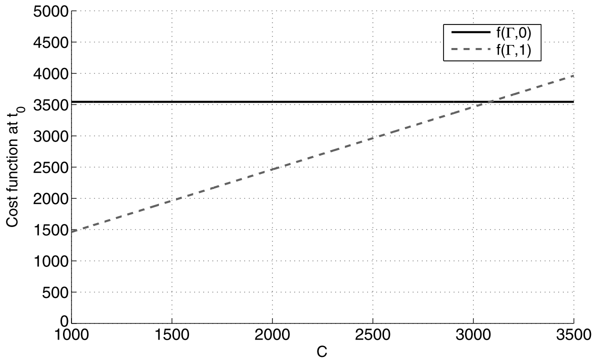


Fig. 5.24 Cost function for different values of the switching cost $C \in [1000, 3500]$

Figure 5.24 represents the relationship between the cost function (for $\theta = 0$ and $\theta = 1$) and the switching cost C, for $C \in [1000, 3500]$ and $\Gamma = 30$ tons.

Obviously, the lower the switching cost C, the higher the profitability of the new production process. In particular, if C is smaller than €3084, then it is optimal for the company to change its technology. Otherwise, the cost function f is minimized with $\theta = 0$, that is, when the initial polluting technology is kept.

A relationship between the switching cost C and the reduction in emissions Γ generated by the new technological process is now introduced. A standard quadratic cost function is employed:

$$C(\Gamma) = \left(\frac{\alpha}{2}\right)\Gamma^2$$

In such a situation, the objective of the company is to determine the optimal quantity of tons of GHG to abate in order to minimize its cost function.

The resulting minimization problem is:

$$\min_{\Gamma} F(\Gamma)$$
$$F(\Gamma) = C(\Gamma) + (1+\eta)^{-1} E\big[G(\widetilde{Q}_1)^{+} \cdot (\widetilde{P}_1 + P)\big]$$

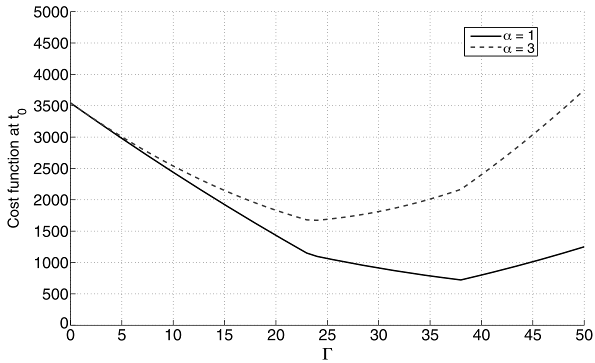


Fig. 5.25 Cost function for different parametrisations of the switching cost function C and different values of Γ

where: $G(\widetilde{Q}_1) = Q_0 + \widetilde{Q}_1 - N - \Gamma$.

An analytical approach is unfortunately not possible, since, as in the first example, the cost function F is not differential everywhere. A numerical approach is therefore employed. The solution is graphically identified.

Figure 5.25 represents the relationship between the cost function and the possible abatement Γ, in tons of offending gas, for $\Gamma \in [0, 50]$, and different values of the coefficient α.

For $\alpha = 1$ the optimal amount of abatement corresponds to $Q_0 + uQ_0 - N = 38$, i.e. to the worst case scenario. Indeed, in this case where emissions increase and reach $uQ_0 = 48$ tons at final time, the company is fully hedged. Total emissions will be 88 tons and the purchase of 38 emission rights will allow to avoid the penalty, given the initial endowment of 50 rights.

For $\alpha = 3$, abatement costs are more expensive and therefore the optimal amount of reduction corresponds only to a partial hedge, where emissions will decrease in the next period and reach approx. 32 tons. In this case, total emissions amount to 72 tons and the purchase of 22 ($= Q_0 + dQ_0 - N$) emission rights will circumvent the penalty. The non-complying risk at maturity, i.e. the risk of incurring a penalty at this date if emissions increase, is more than compensated by the risk of abating too much, that is, to incur too many abatement costs at initial time.

5.7.4 A Fourth Example: A Two-Period Binomial Model for the Emission and Price Processes—What Are the Optimal Decisions in Terms of Emission Rights Trading and Technology Changes?

This case combines the last two examples. It corresponds to a two-period model where both the instantaneous pollution of a regulated company and the emission permits price are exogenous and independent processes. The length of each period is one year.

In this new and more realistic setting, the company can take two decisions in order to comply with regulations: trade emission rights and change its technology. The sequence of decisions is the following. At initial time, the company is confronted

Table 5.3 Parameters used in the numerical example

Parameter	Value
N	50
η	0.06
P (penalty)	100
P_0	20
p	0.8
u	1.2
d	$\frac{1}{1.2} \approx 0.8$
Q_0	40
q	0.5
Γ	40 (tons)
C	€3000

with trading opportunities only. One period later, at intermediate time, it is assumed that the company can either switch the technology of its production process ($\theta = 1$) for a lump-cost of C or do nothing ($\theta = 0$). The new technology reduces the emissions by Γ tons of offending gas.

Except for this extension, the context of example two holds true and the previous notations are used.

By introducing the possibility to partially delay and transform the investment decision, this new example allows us, in contrast to the first and third one, to distinguish between the real options approach and the NPV criteria.

By assumption these two processes evolve according to a binomial model. The uncertainty of the market is introduced in the model through the existence of three possible states: at final time, for pollution, and for the emission allowance permit price. The following figure illustrates the context.

with: $0 < d < 1 < u = \frac{1}{d}$ and where Q_0 and P_0 are the initial pollution level and initial permit price, respectively. Parameters q and p are the probabilities of an upward movement for pollution and prices, respectively. Time one and final emission level and price, Q_1, Q_2 and P_1, P_2, are perceived as random variables at initial time.

Based on the past examples, let us consider the following parameter values (Table 5.3).

In this new setting, there are now two unknowns that need to be determined: X_0, the optimal quantity of permits that the company should buy or sell at initial time, and θ, which determines whether or not the company will change its technology after one year. As in the first example, N denotes the permits endowment at initial time. At maturity, at the end of the second period, the company must have enough permits. If the firm fails to achieve compliance, it must pay a penalty equal to P plus the price of the permits, that is, P_2 for each ton of uncovered pollution.

By proceeding along the same lines as in the last examples, at initial time the company minimizes its cost function:

$$\min_{X_0} f(X_0) \tag{5.6}$$

with

$$f(X_0) = P_0 \cdot X_0 + E\left[\frac{\theta C}{1+\eta} + (1+\eta)^{-2} \cdot h(\widetilde{Q}_1, \widetilde{Q}_2)^{+} \cdot (\widetilde{P}_2 + P)\right] \tag{5.7}$$

and where function $h(\cdot)$ represents the company's net final position in terms of emission permits:

$$h(\widetilde{Q}_1, \widetilde{Q}_2) = Q_0 + \widetilde{Q}_1 + \widetilde{Q}_2 - N - X_0 - \theta \cdot \Gamma$$

and depends on two factors in this example. In line with the last examples, $h(\cdot)^{+}$ represents the positive value of function h.

The cost function is the sum of three components: costs incurred at initial time and corresponding to the trade of allowances, discounted costs resulting from a possible technology change at the end of the first period, and finally, expected discounted costs at maturity (i.e., the penalty plus the price of the permit per unit of uncovered pollution). A negative cost, that is, a profit, at time zero would result from the decision to sell.

At intermediate time, the control variable θ, depends on the cumulated emissions, $Q_0 + Q_1$, the realized permit price, P_1, and the number of allowances owned by the company $N + X_0$, at that time:

$$\theta = \theta(N + X_0, Q_0 + Q_1, P_1)$$

Indeed, after one year, the regulated company will be confronted with a second choice: either a technology switch ($\theta = 1$) or the status quo in terms of technology ($\theta = 0$). The context of this second choice corresponds to the one-period binomial model considered in the third example:

$$\min_{\theta \in \{0,1\}} = F(X_0, \theta)$$

with

$$F(X_0, \theta) = \theta \cdot C + (1+\eta)^{-1} E\big[G(\widetilde{Q}_2, \theta)^{+} \cdot (\widetilde{P}_2 + P)\big]$$

where:

$$G(\widetilde{Q}_2, \theta) = Q_0 + Q_1 + \widetilde{Q}_2 - N - X_0 - \theta \cdot \Gamma$$

Therefore, in order to solve this problem, a backward induction procedure will be applied. This involves starting at the end of the initial period, and deriving θ, which corresponds to the optimal decision in terms of technologies as a function of the possible initial traded quantity as well as the observed emissions and price at that time. Then, the problem will be resolved at initial time and the initial optimal traded quantity will be derived.

Let us therefore rewrite θ as a function of X_0, the realized emissions Q_1 and price at intermediate time, P_1:

$$\theta = \theta(X_0, Q_1, P_1)$$

As previously mentioned, after one period, the context of the minimization costs problem is a one-period binomial model and therefore corresponds to the third example. Let us consider the four possible cases for the emissions and the price during the first period.

Case 1: $Q_1 = dQ_0$ and $P_1 = dP_0$ If the level of emissions and the allowance price decrease during the first period, the comparison of the two values taken by the cost function $F(\cdot)$ for the two possible values of the control variable, that is, $\theta \in \{0, 1\}$, is required in order to take an optimal decision in terms of a possible technology change.

$$F(X_0, 0) = (1+\eta)^{-1} E\big[G(\widetilde{Q}_2, 0)^+ \cdot (\widetilde{P}_2 + P)\big]$$

and

$$F(X_0, 1) = C + (1+\eta)^{-1} E\big[G(\widetilde{Q}_2, 1)^+ \cdot (\widetilde{P}_2 + P)\big]$$

By relying on the definition of function $G(\cdot)$ and by separating the two values into a product of independent expectations, the following equivalence is obtained:

$$\begin{aligned} & F(X_0, 0) > F(X_0, 1) \\ & \quad \Leftrightarrow \quad E\big[(Q_0 + dQ_0 + \widetilde{Q}_2 - N - X_0)^+\big] \cdot \big(E[\widetilde{P}_2] + P\big) \\ & \qquad\qquad > E\big[(Q_0 + dQ_0 + \widetilde{Q}_2 - N - X_0 - \Gamma)^+\big] \cdot \big(E[\widetilde{P}_2] + P\big) + (1+\eta)C \end{aligned}$$

i.e.

$$\begin{aligned} & F(X_0, 0) > F(X_0, 1) \\ & \quad \Leftrightarrow \quad \big((1-q)\big(\big(1+d+d^2\big)Q_0 - N - X_0\big)^+ \\ & \qquad\qquad + q\big((1+d+1)Q_0 - N - X_0\big)^+\big)\big(pudP_0 + (1-p)d^2P_0 + P\big) \\ & \qquad\quad > (1+\eta)C + (1-q)\big(\big(\big(1+d+d^2\big)Q_0 - N - X_0 - \Gamma\big)^+ \\ & \qquad\qquad + q\big((1+d+1)Q_0 - N - X_0 - \Gamma\big)^+\big)\big(pudP_0 + (1-p)d^2P_0 + P\big) \end{aligned}$$

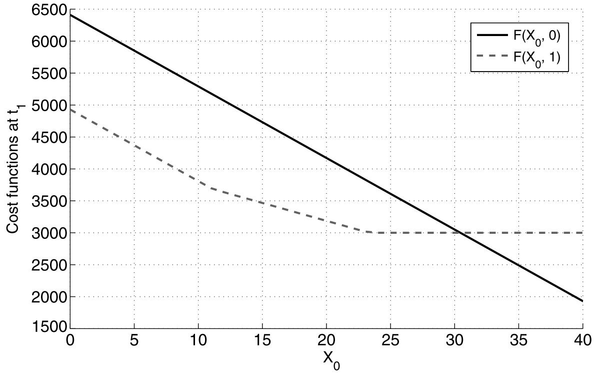


Fig. 5.26 Cost functions at t_1 for different values of X_0, $Q_1 = dQ_0$, $P_1 = dP_0$

And moving the expectations to the left-hand side:

$$\begin{aligned}
&F(X_0, 0) > F(X_0, 1) \\
&\quad \Leftrightarrow \quad (1+\eta)C < \Big((1-q)\Big(\Big(\big(1+d+d^2\big)Q_0 - N - X_0\Big)^+ \\
&\qquad - \Big(\big(1+d+d^2\big)Q_0 - N - X_0 - \Gamma\Big)^+\Big) \\
&\qquad + q\Big(\big((1+d+1)Q_0 - N - X_0\big)^+ \\
&\qquad - \big((1+d+1)Q_0 - N - X_0 - \Gamma\big)^+\Big)\Big)\big(pudP_0 + (1-p)d^2P_0 + P\big)
\end{aligned}$$

The company's optimal choice is to change its technology only if the initial purchase of emission allowances it too limited. Given the set of parameters used in this case, the following result is obtained:

$$F(X_0, 0) > F(X_0, 1) \quad \Leftrightarrow \quad X_0 < 30.5$$

that is, $\theta(X_0, dQ_0, dP_0) = 1_{X_0 < 30.5}$.

That is to say, if emissions and the allowance price decrease after one period, a technology switch is optimal only if the initial number of emission rights purchased at initial time is smaller than 31 (see Fig. 5.26).

If for example, this initial number is 30, then the firm will own 80 emission rights after one period and will decide to switch to the new technology. The status quo in terms of technologies would be inefficient. It would correspond to an under-hedge, that is, owning 80 rights (initial endowment of 50 plus and a purchase of 30 rights) at maturity would be too little compared with potential needs: 101 ($\approx 40 + \frac{40}{1.2} + \frac{40}{1.2^2}$) or 113 ($\approx 40 + \frac{40}{1.2} + 40$) with 50 % probability each.

Conversely, if the initial number is 40, then the firm will own 90 emission rights after one period and will decide to keep the old technology. A technology switch would be too expensive. It would correspond to an over-hedge, that is, owning 90 rights (initial endowment of 50 plus a purchase of 40 permits) at maturity would be too much compared with potential needs: 73 ($= 113 - 40$) or 61 ($= 101 - 40$) with 50 % probability each, permitted by the technology change abatement.

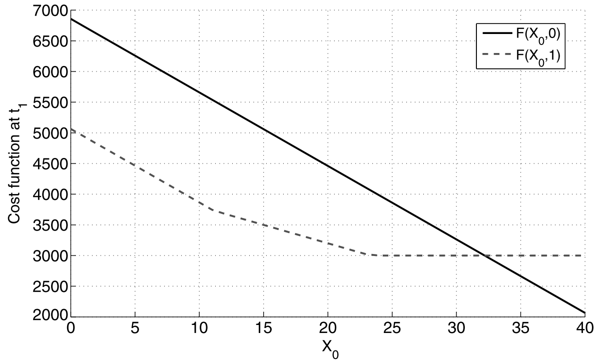


Fig. 5.27 Cost function at t_1 for different values of X_0, $Q_1 = dQ_0$, $P_1 = uP_0$

Continuing along the same lines, the results concerning the three other cases are obtained.

Case 2: $Q_1 = dQ_0$ and $P_1 = uP_0$

$$\begin{aligned}
&F(X_0,0) > F(X_0,1)\\
&\quad\Leftrightarrow\quad (1+\eta)C < \Big((1-q)\Big(\big((1+d+d^2)Q_0 - N - X_0\big)^+\\
&\qquad - \big((1+d+d^2)Q_0 - N - X_0 - \Gamma\big)^+\Big)\\
&\qquad + q\Big(\big((1+d+1)Q_0 - N - X_0\big)^+\\
&\qquad - \big((1+d+1)Q_0 - N - X_0 - \Gamma\big)^+\Big)\Big)\big(u\big(pu + (1-p)d\big)P_0 + P\big)
\end{aligned}$$

In this case:

$$F(X_0,0) > F(X_0,1) \quad\Leftrightarrow\quad X_0 < 32.5$$

and

$$\theta(X_0, dQ_0, dP_0) = 1_{X_0<32.5}$$

An increase in the price during the first period, which could generate higher costs at maturity in case of shortage, enhances the attractiveness of the technology switch (Fig. 5.27).

Case 3: $Q_1 = uQ_0$ and $P_1 = dP_0$

$$\begin{aligned}
&F(X_0,0) > F(X_0,1)\\
&\quad\Leftrightarrow\quad (1+\eta)C < \Big((1-q)\Big(\big((1+u+d^2)Q_0 - N - X_0\big)^+\\
&\qquad - \big((1+u+d^2)Q_0 - N - X_0 - \Gamma\big)^+\Big)\\
&\qquad + q\Big(\big((1+u+u^2)Q_0 - N - X_0\big)^+\\
&\qquad - \big((1+u+u^2)Q_0 - N - X_0 - \Gamma\big)^+\Big)\Big)\big(d\big(pu + (1-p)d\big)P_0 + P\big)
\end{aligned}$$

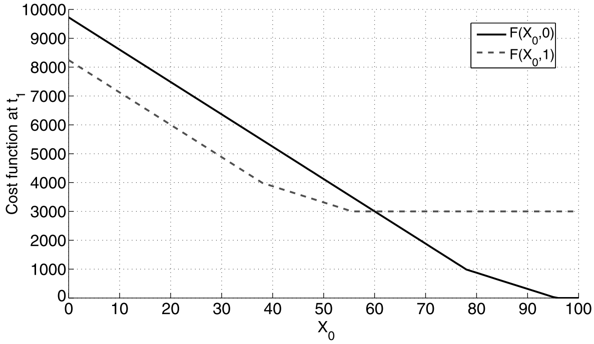


Fig. 5.28 Cost function at t_1 for different values of X_0, $Q_1 = uQ_0$, $P_1 = dP_0$

In this case,

$$F(X_0, 0) > F(X_0, 1) \quad \Leftrightarrow \quad X_0 < 60$$

and

$$\theta(X_0, dQ_0, dP_0) = 1_{X_0 < 60}$$

An increase in the emissions process during the first period, which could generate higher costs at maturity in case of shortage, enhances the attractiveness of the technology switch (Fig. 5.28).

Case 4: $Q_1 = uQ_0$ and $P_1 = uP_0$

$$\begin{aligned} &F(X_0, 0) > F(X_0, 1) \\ &\quad \Leftrightarrow \quad (1+\eta)C < \big((1-q)\big(\big(\big(1+u+d^2\big)Q_0 - N - X_0\big)^+ \\ &\qquad\qquad - \big(\big(1+u+d^2\big)Q_0 - N - X_0 - \Gamma\big)^+\big) \\ &\qquad\qquad + q\big(\big(\big(1+u+u^2\big)Q_0 - N - X_0\big)^+ \\ &\qquad\qquad - \big(\big(1+u+u^2\big)Q_0 - N - X_0 - \Gamma\big)^+\big)\big)\big(u\big(pu + (1-p)d\big)P_0 + P\big) \end{aligned}$$

In this case:

$$F(X_0, 0) > F(X_0, 1) \quad \Leftrightarrow \quad X_0 < 61.5$$

and

$$\theta(X_0, dQ_0, dP_0) = 1_{X_0 < 61.5}$$

When emissions and price increase, the advantage of the technology switch is even clearer (see Fig. 5.29).

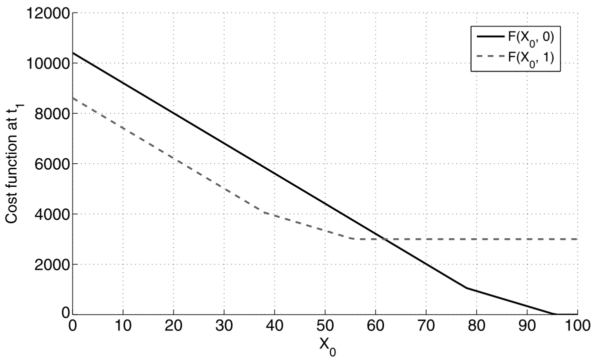


Fig. 5.29 Cost function at t_1 for different values of X_0, $Q_1 = uQ_0$, $P_1 = uP_0$

Now that θ as a function of X_0, Q_1 and P_1 has been derived, the minimization problem at initial time, given by Eq. (5.6), can be rewritten as follows:

$$\min_{X_0} f(X_0)$$

with

$$f(X_0) = P_0 \cdot X_0 + E\left[\frac{\theta(X_0, Q_1, P_1)C}{1+\eta} + (1+\eta)^{-2} \cdot h(Q_1, Q_2)^+ \cdot (P_2 + P)\right]$$

and where function $h(\cdot)$ represents the company's net final position in terms of emission permits:

$$h(Q_1, Q_2) = Q_0 + Q_1 + Q_2 - N - X_0 - \theta\Gamma$$

Hence, according to the values taken by the function θ, all the 16 different combinations of pairs of emissions and price paths are taken into account:

$$\begin{aligned}
f(X_0, \Gamma) = P_0 X_0 + p^2 \sum_{i=0}^{1}\sum_{j=0}^{1} q^i (1-q)^{2-i-j} q^j \Bigg[&\frac{\theta(X_0, u^i d^{1-i} Q_0, uP_0)C}{1+\eta} \\
&+ (1+\eta)^{-2} \cdot h\big(u^i d^{1-i} Q_0, u^{i+j} d^{2-i-j} Q_0\big)^+ \cdot \big(u^2 P_2 + P\big)\Bigg] \\
+ p(1-p) \sum_{i=0}^{1}\sum_{j=0}^{1} q^i (1-q)^{2-i-j} q^j \Bigg[&\frac{\theta(X_0, u^i d^{1-i} Q_0, uP_0)C}{1+\eta} \\
&+ (1+\eta)^{-2} \cdot h\big(u^i d^{1-i} Q_0, u^{i+j} d^{2-i-j} Q_0\big)^+ \cdot (P_2 + P)\Bigg] \\
+ (1-p)p \sum_{i=0}^{1}\sum_{j=0}^{1} q^i (1-q)^{2-i-j} q^j \Bigg[&\frac{\theta(X_0, u^i d^{1-i} Q_0, dP_0)C}{1+\eta}
\end{aligned}$$

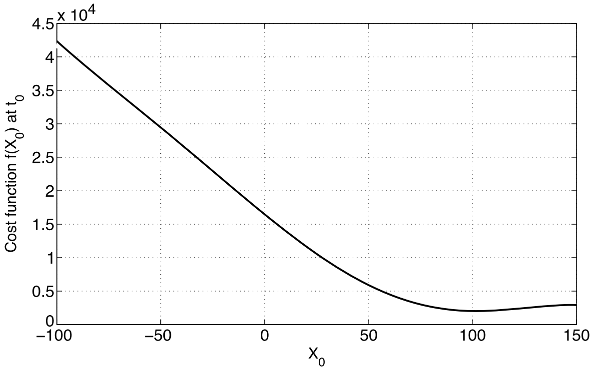


Fig. 5.30 Cost function at t_0 for different values of X_0

$$+ (1+\eta)^{-2} \cdot h\big(u^i d^{1-i} Q_0, u^{i+j} d^{2-i-j} Q_0\big)^+ \cdot (P_2 + P)\Bigg]$$

$$+ (1-p)^2 \sum_{i=0}^{1} \sum_{j=0}^{1} q^i (1-q)^{2-i-j} q^j \left[\frac{\theta(X_0, u^i d^{1-i} Q_0, d P_0) C}{1+\eta} \right.$$

$$\left. + (1+\eta)^{-2} \cdot h\big(u^i d^{1-i} Q_0, u^{i+j} d^{2-i-j} Q_0\big)^+ \cdot \big(d^2 P_2 + P\big) \right]$$

A numerical approach is required in order to solve this problem. With the set of parameters used in this case, Fig. 5.30 is obtained.

The optimal solution for the company involves the purchase of 96 emission rights at initial time and maintaining the status quo in terms of technology ($\theta = 0$, that is, keeping the initial technology) at the end of the first period. This purchase of rights represents a hedge in the worst case scenario, that is, when the volume of emissions increases twice (see Example 2). Obviously, this strategy incorporates drawbacks. Indeed, if this level does not increase twice, this strategy will correspond to an over-hedge and thus will not be efficient, because the redemption price of the emission allowances is zero at maturity. However, the disadvantage is more than compensated by the advantage of buying cheap emission allowances.

The intuition is the following. The initial permit price, €20, is small compared to a technology switching cost of €75 (€3000/40) or a penalty of €100. Therefore, it is cheaper for the firm to buy the allowances corresponding to this scenario and to possibly own too many rights than to buy the number of permits related to the most favorable scenario (a decrease in the emissions) and to take the risk of incurring the higher cost of the technology switch, or paying penalties and buying emission allowances at maturity.

While we are in a dynamic setting, the comparison between the NPV criterion and real options is more relevant in Example 2 (Sect. 5.7.2) because decisions at times t_0 and t_1 are on the same variable (the number X_0 of emissions rights traded). This is what allows a possible under- or over-hedge, which is not possible in this example. In spite of that, a comparison between the two settings sheds light on the benefits of a real options approach in a dynamic setting.

In the NPV setting, a sequence of static decisions is considered with first the purchase of emission permits without considering the technology change (t_0) followed by a second decision, which considers this switch (t_1). This represents the NPV_0 criterion. Another static strategy (NPV_1, see Exercise 2) would correspond to a unique choice at t_1. In the real options framework, a dynamic decision making process is implemented. As shown in the second example (Sect. 5.7.2), the static NPV approach neglects this adjustment. Under the NPV_0 criterion and with the current parameter values, the company will perfectly hedge for the worst-case scenario at t_2 (146 tons) and will purchase at t_0 96 permits ($X_0 = 96$). It can be easily shown that the strategy corresponding to the NPV_1 is dominated by its counterpart strategy NPV_0: indeed, a unique technology switch at t_1 corresponds to emissions from 61 (best case) to 106 (worst case), that is to say, a higher cost than for NPV_0.

The result of the NPV_0 criterion is similar to the real options strategy simply because in the dynamic setting, the sub-optimality of technology switch has already been derived at t_0. Indeed the cost associated with the technology change (€75 per ton) is too high compared with the price level of the emission right (€20 per ton). As soon as the strategy recommends the technology change (as in the comparative statics case below), the real options strategy benefits from under-hedging at initial time, which gives more value to the subsequent option to change the technology. Under the NPV, the best-strategy would still be a perfect hedge in a emission path without technology chance, a clear loss of value.

Comparative Statics Obviously, with other parameter values different results would be obtained. For instance, if the switching cost C was smaller: $C = 500$, and if the reduction of emissions associated with the change of technology was higher: $\Gamma = 60$ tons, then an alternative strategy would be optimal. It consists of an under hedge at initial time ($X_0 \approx 63$) followed by a possible change in technology at intermediary time, if the emission level increases. The number of emissions rights owned by the company at time t_0 is $50 + 63 = 113$ and corresponds precisely to the permits required at maturity if the emissions decrease and increase once. The cost associated with the technology change (€8.33 per ton) is now relatively cheap compared with the price level of the emission right (€20 per ton) and it makes sense to favor a delay in the decision making process in order to take advantage of this cheaper opportunity to meet compliance targets. In other words, if at time t_1, the emissions increase, it will result in a technology change. Otherwise, the company will maintain the status quo.

Figure 5.31 is obtained.

5.7.5 A Last Example: One-Period Binomial Model for Emission: What Are the Optimal Decisions in Terms of Emission Rights Trading when Price Dynamics Are Endogenously Derived?

In the previous examples, price dynamics of the emission allowance were given and are therefore independent of demand and supply. In this last example, we go one step further, by deriving endogenously these dynamics. This more realistic setting

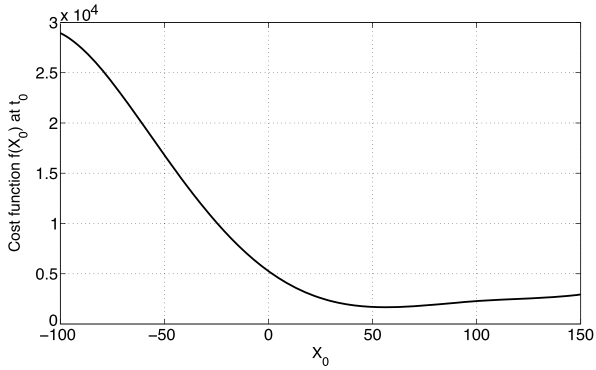


Fig. 5.31 Cost function at t_0 for different values of X_0

allows us to shed light on interesting features, in particular the impact of strategic behavior of companies on the price process.

In this new situation, a second company is introduced. The strategic interaction between the two firms is the key factor explaining the price formation. Along the same lines as examples one and three, the emission process evolves according to a one-period binomial model. The instantaneous pollution process remains exogenous but now the emission permits price is the equilibrium price resulting from the supply/demand balance.

$X_{i,0}$ and $N_{i,0}$ denote respectively the quantity of permits that the ith company, $i \in \{1, 2\}$, buys or sells at initial time t_0 and the permits endowment at that time. In a cap and trade, such as the EU ETS, the gas reduction target is settled at the inception of each phase. Hence, the supply side of pollution permits is indeed fixed and given by:

$$N = N_{1,0} + N_{2,0}$$

Let us denote the net amount of permits that the ith company possesses at time t by:

$$\delta_{i,0} = N_{i,0} + X_{i,0}$$

Given that the total number of permits is fixed, the market clearing condition is:

$$\delta_{1,0} + \delta_{2,0} = N \quad \text{or, in another form} \quad X_{1,0} = -X_{2,0}$$

This condition implies that in equilibrium the permit positions are in zero net supply. Hence, it satisfies the competitive equilibrium condition that requires equality between supply and demand for emission permits in the market.

At the end of the period we expect either a surplus or a shortage situation between the issued emission allowances and the realized pollution level. Indeed, given the random setting inherent to the emission process, in most cases, the company

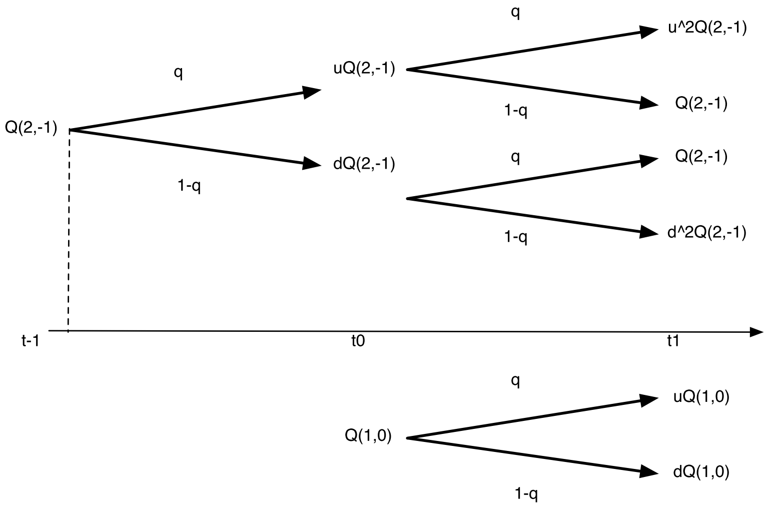


Fig. 5.32 Example of possible emissions of firm 2 from firm 1's viewpoint

will either be holding worthless emission allowances or paying the price for being uncovered. The following expression:

$$Q_{i,0} + Q_{i,1} - X_{i,0} - N_{i,0}$$

represents the company's i final position in emission permits in a one-period model where trading opportunities exist only at t_0.

Let us consider the first company ($i = 1$). At time t_0 this company has full knowledge of its own permit endowment, its starting pollution value, $N_{1,0}$ and $Q_{1,0}$, the permit endowment of the second company $N_{2,0}$ and only partial knowledge of the level of emissions of the second company, $Q_{2,0}$. More precisely, it is assumed that from the point of view of the first company, $Q_{2,0}$ may take two values: $u.Q_{2,-1}$ and $d.Q_{2,-1}$, respectively, with probabilities q_2 and $1 - q_2$.

$Q_{2,-1}$ is a parameter which allows to build a one period binomial tree before initial time and therefore permits to introduce uncertainty in the emission process of the second company, from the standpoint of the first company. However, in this simple setting, this parameter is not a component of the cumulated emissions of the second companies which is at final time: $Q_{2,0} + Q_{2,1}$.

The presence of asymmetric information imposes a lag-effect on the dynamics of the pollution process of the other company. Indeed, $Q_{2,0}$ will be revealed to the first company only at the end of the first period. From the point of view of the first company, the situation can be illustrated in Fig. 5.32, where the previous notations are used.

For the second company, the problem is symmetric and therefore from its point of view, Fig. 5.33 is obtained:

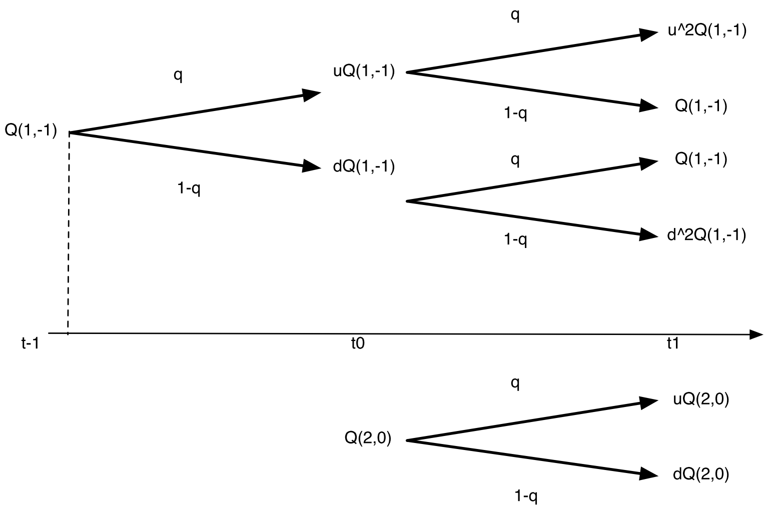


Fig. 5.33 Example of possible emissions of firm 1 from firm 2's viewpoint

At final time t_1, each company must have enough permits. In order to simplify the setting, it is assumed that if a firm fails to achieve compliance, it will pay only the penalty P for each ton of uncovered pollution, but without having to buy permits.

In order to avoid the penalty, the firm can also purchase permits at final time. By assumption, if at least one of the two firms wants to buy allowances at maturity, the final price will the penalty P. Therefore, the final cost for the first company will be $X_{1,1} \cdot P$, where:

$$X_{1,1} = (Q_{1,0} + Q_{1,1} - X_{1,0} - N_{1,0})^{+} - \Gamma_1$$

with

$$\Gamma_1 = \min\left\{(X_{1,0} + N_{1,0} - Q_{1,0} - Q_{1,1})^{+}, (Q_{2,0} + Q_{2,1} - X_{2,0} - N_{2,0})^{+}\right\}$$

Indeed, if this company is in shortage, it will either incur a penalty P or purchase allowances for each ton of uncovered pollution. These two solutions are financially equivalent because if the firm wants to buy permits, their price is equal to the penalty. Contrarily, if this firm is in excess of permits at final time, its sales will be constrained by the number of allowances that the other company wants to buy. More precisely, this company will sell a number of allowances equal to the minimum of what it wants to sell and what the other company wants to purchase.

It is worth noting that in this two-company setting where it is possible to sell permits at maturity, the final cost can also be negative, that is, can correspond to a profit.

Given the initial endowment of permits and the dynamics of the emission process, the first company minimizes its cost function at t_0. The total expected cost is simply the sum of the costs at t_0 and the potential penalty at t_1. Therefore, the resulting minimization problem is:

$$\min_{X_{1,0}} f_1(X_{1,0})$$

where

$$f_1(X_{1,0}) = P_0 \cdot X_{1,0} + E\big[(1+\eta)^{-1} \cdot \widetilde{X}_{1,1} \cdot P\big]$$

Since the penalty level is constant, it can be taken out of the expectation operator:

$$f_1(X_{1,0}) = P_0 \cdot X_{1,0} + (1+\eta)^{-1} P \cdot E[\widetilde{X}_{1,1}]$$

Similarly, company two minimizes its cost function at initial time. Its total expected costs are simply the sum of the costs at t_0 and the potential penalty at t_1.

$$\min_{X_{2,0}} f_2(X_{2,0})$$

where

$$f_2(X_{2,0}) = P_0 \cdot X_{2,0} + (1+\eta)^{-1} P \cdot E[\widetilde{X}_{2,1}]$$

and

$$X_{2,1} = (Q_{2,0} + Q_{2,1} - X_{2,0} - N_{2,0})^+ - \Gamma_2$$

with

$$\Gamma_2 = \min\big\{(X_{2,0} + N_{2,0} - Q_{2,0} - Q_{2,1})^+, (Q_{1,0} + Q_{1,1} - X_{1,0} - N_{1,0})^+\big\}$$

For the sake of simplicity, the same discounting factor is used for both companies. The generalization of taking two different discounting factors is straightforward.

To find the equilibrium solutions for P_0, $X_{1,0}$ and $X_{2,0}$, we use a numerical grid search method that seek a fixed point between companies 1 and 2, such that their optimal individual decisions (knowing the best response of the other company) ensures the maximum volume traded.

Since the problem is solved numerically, we select specific parameters and boundaries for our variables.

The initial endowments of emission permits for the two companies are $N_{1,0} = 50$ and $N_{2,0} = 60$, respectively;

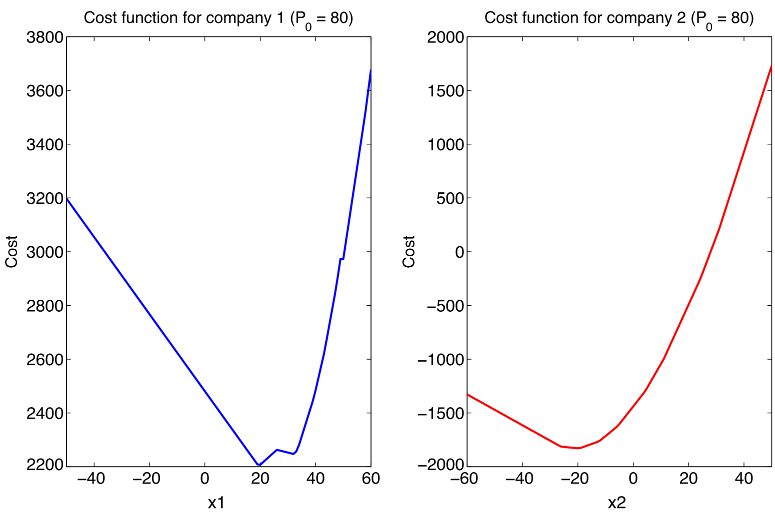
The discount rate, η, is equal to 6 %;

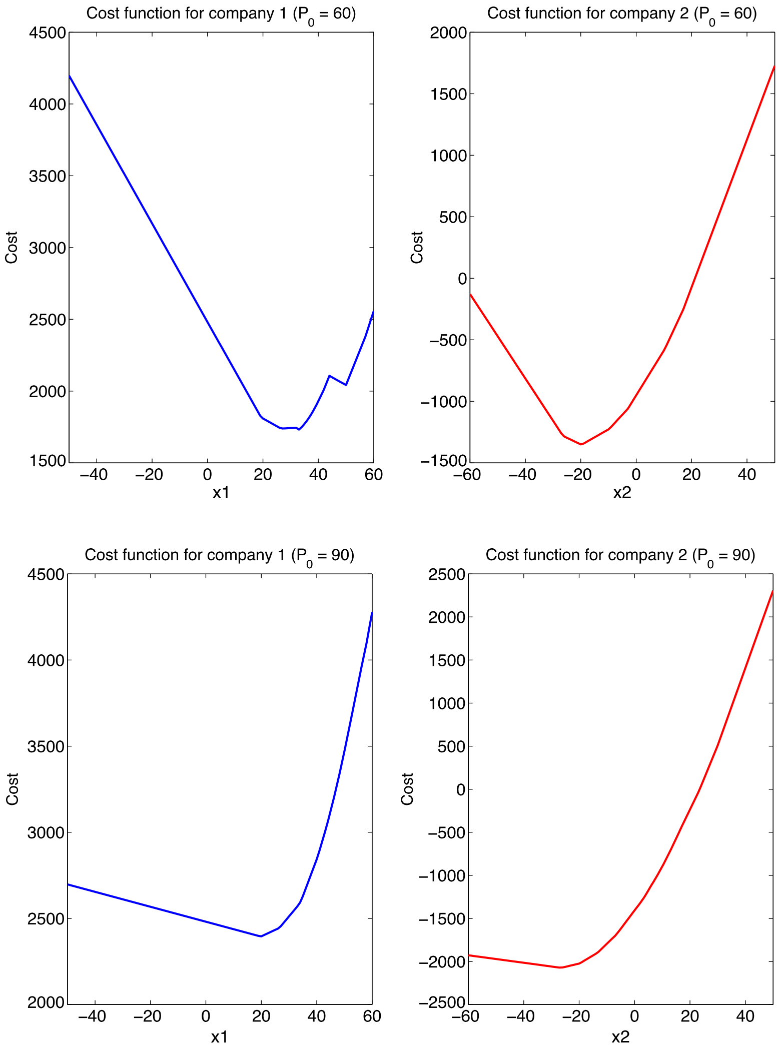
The penalty, P, is equal to €100; The initial CO_2 emissions are: $Q_{1,0} = 37.8$ and $Q_{2,-1} = 21.9$ tons, respectively, from the stand point of the first company, as well as $Q_{2,0} = 18.2$ and $Q_{1,-1} = 45.4$ tons, from the point of view of the second company. It is worth noting that the actual emissions of each company are indeed one of the possible values expected by the other ($18.2 = 21.9 \cdot d$ and $37.8 = 45.4 \cdot d$). The upward probabilities, q_1, and q_2 characterizing the emission processes of the two companies are 0.5 for both;

The factor u and d characterizing the upward and downward movement of the emission processes are the same for the two firms: 1.2 and $\frac{1}{1.2}$ (approximately 0.8), respectively.

In this example, the number of permits is such that there is an equilibrium price at $P_0 = 80$ such that company 1 asks optimally for 20 permits ($X_{1,0} = +20$) to buy while company 2 decides optimally to sell the same number of permits ($X_{2,0} = -20$). At this price, demand and supply intersect. The following graphs represent the cost functions under the selected parameters for the two companies: Under the selected parameters, company 1 is in shortage of permits at initial time while company 2 has permits in excess. The purchase of the 20 additional permits represents a full cover of the excess emissions of company 1 in the best case scenario, where emissions from company 1 decrease in period 1 ($37.8 + \frac{37.8}{1.2} - 50 - 20 \approx 0$). For company 2, the 20 permits sold represent a full cover under the worst case scenario, where emissions for company 2 increase in period 1 ($18.2 + 18.2 \cdot 1.2 - 60 + 20 \approx 0$). It is worth noting that company 2 received many allowances in comparison with her expected emissions, creating a situation of excess rights (generating an incentive to sell) while company 1 is in the opposite situation with important emission levels and limited number of emission rights at initial time (creating an incentive to buy).

Lets now consider the trading strategies of the two companies for an imposed lower permit price $P_0 = 60$. At this price, the optimal strategy for company 1 is to buy 33 permits ($X_{1,0} = +33$, to cover her worst case scenario). However, company 2





is not willing to sell this amount of permits (due to the reduced price) and optimally maintains its offer of 20 permits ($X_{2,0} = -20$). In that case, company 2 would aim to cover her best case scenario (where her emissions decrease), which would provide an excess of 20 permits available to sell.

This shows that a price P_0 does not permit the market clearing. On the contrary, the price of $P_0 = 80$ minimizes the cost functions of the two companies.

If the initial permit price were higher (90), we would obtain a different result: company 1 in this case would want to buy 20 permits (to cover her best case scenario) but at this high price, company 2 would prefer selling 27 permits (covering her best case), which does not clear the market.

6 The Emission Price Dynamics

6.1 Econometric Analysis of the EUA Price

Recently, in an effort to bridge the gap between theory and observed market-price behavior, an increasing number of empirical studies have investigated the historical time series of the price of emission allowances.

The first task of such literature consisted in the identification of the properties of the time-series, particularly with regard to its data-generation process. The early literature investigated the first EU ETS Phase (2005–2007) and established that univariate time series modelling based on ARMA-GARCH processes provide the best fit to data, see Paolella and Taschini (2008) and Benz and Trück (2008), among others. These models specify the evolution of the variance based on past innovations and distinguish between conditional and unconditional components of the volatility. In particular, Paolella and Taschini (2008) investigated the asymmetric responses to different shocks in the allowances market and proposed a parametric model which places more weight both on more recent returns and on negative returns.

The second task of this literature consisted in explicitly modelling the jumps present in the data. When the time-series exhibit several jumps it seems natural to incorporate this phenomena in the modelling exercise. This can be done, for instance, by using a geometric Brownian motion augmented by jumps, see Daskalakis et al. (2009).

More recently, a burgeoning academic literature has emerged proposing more sophisticated classes of processes.[1] A detailed description of this later literature is out of the scope of this chapter and we refer to the book of Chevallier (2011) for a comprehensive discussion of such econometric studies. Instead, we proceed by reporting the major statistic of the EUAs price and propose a simple modelling exercise that could be easily used for exercise and replication purposes.

[1] While the following list of papers is certainly not complete, it contains most of the processes that have been suggested in the literature: Alberola and Chevallier (2009), Bunn and Fezzi (2009), Hintermann (2010), Gronwald et al. (2011), and Creti et al. (2012).

M. Chesney et al., *Environmental Finance and Investments*,
Springer Texts in Business and Economics, DOI 10.1007/978-3-642-36623-9_6,

Table 6.1 Summary statistics of the futures and spot prices of the CO_2 emission allowances. EUA10 is the futures contracts with maturity December 2010; EUA12 is the futures contracts with maturity December 2012; Spot I is the spot price in Phase I; and Spot II is the spot price in Phase II. Source: ECX and Bluenext

	EUA10	EUA12	Spot I	Spot II
Mean	16.79	17.99	10.36	15.92
Standard deviation	5.13	5.17	10.32	4.25
Skewness	1.30	1.49	0.33	1.37
Kurtosis	0.37	0.97	−1.51	0.95

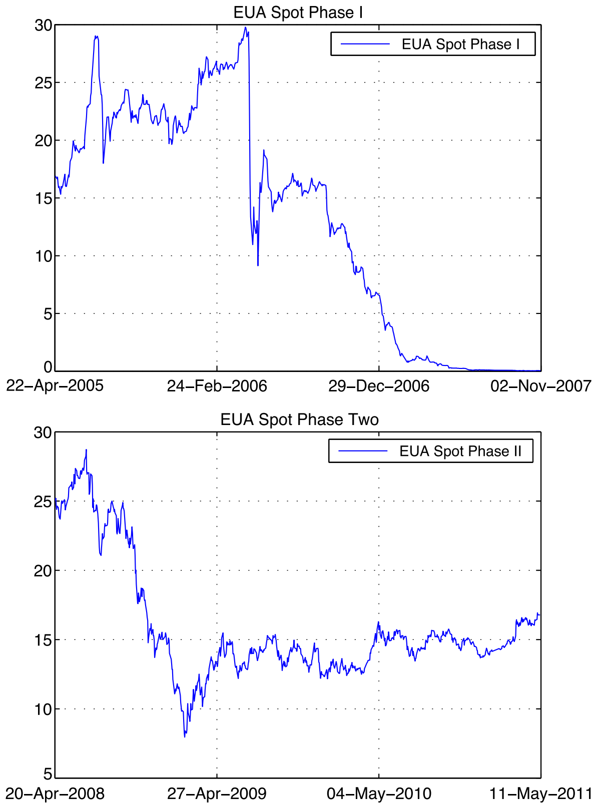


Fig. 6.1 EUA Spot Phase I is the spot price of the emission allowances in Phase I from June 2005 until November 2007 (*upper diagram*). EUA Spot Phase II is the spot price of the emission allowances in Phase II from April 2008 until May 2011 (*lower diagram*). Source: Bluenext

6.1.1 Key Statistics for the EUA Price

Table 6.1 reports the major statistics for the futures with maturity December 2010 and December 2012, respectively, and for the spot price in Phase I and Phase II, respectively, of the CO_2 emission allowances. Similarly, Table 6.2 considers log-returns. Figure 6.1 represents visually the price evolution registered on Bluenext of the spot prices in Phase I from June, 24th 2005 until December, 30th 2007 and the spot prices in Phase II from April, 21st 2008 until May, 9th 2011. Similarly,

Table 6.2 Summary statistics of the futures and spot log-returns of the CO_2 emission allowances. EUA10 LR is the log-returns of futures contracts with maturity December 2010; EUA12 LR is the log-returns of futures contracts with maturity December 2012; Spot I LR is the log-returns of spot price in Phase I; and Spot II LR is the log-returns of spot price in Phase II. Source: ECX and Bluenext

	EUA10 LR	EUA12 LR	Spot I LR	Spot II LR
Mean	0	0	−0.01	0
Standard deviation	0.02	0.02	0.1	0.02
Skewness	−0.13	−0.15	−0.67	−0.19
Kurtosis	1.91	2.53	15.82	2.31

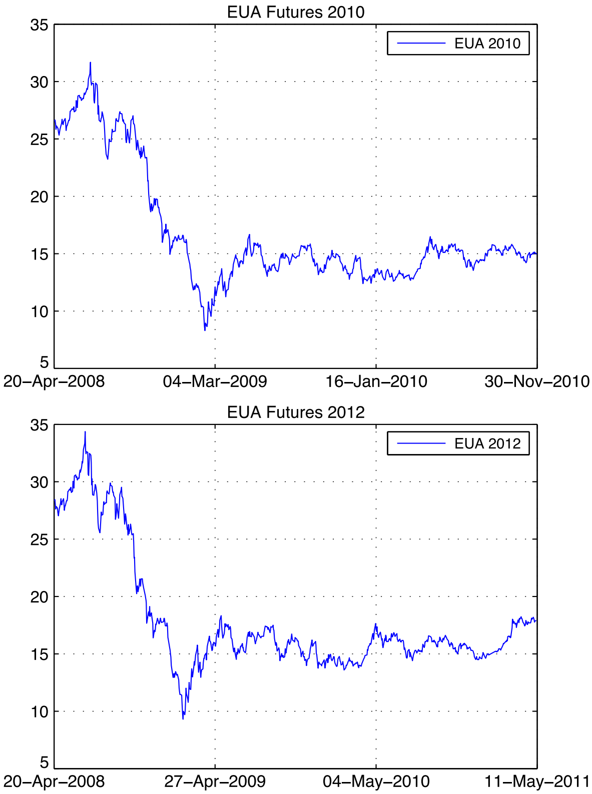


Fig. 6.2 EUA 2010 is the futures price of the emission allowances with maturity December 2010 (*upper diagram*); EUA 2012 is the futures price of the emission allowances with maturity December 2012 (*lower diagram*). Time spans from April 2008 until November 2010 for futures with maturity December 2010; and from April 2008 until May 2011 for futures with maturity December 2012. Source: European Climate Exchange

Fig. 6.2 depicts the price evolution registered on the European Climate Exchange of the futures with maturity December 2010 from April, 22nd 2008 until November, 29th 2010 and futures with maturity December 2012 from April, 22nd 2008 until May, 9th 2011. All prices are quoted in €/ton of CO_2. The futures and spot Phase I plots show an abrupt discontinuous shift in May 2006. At that time, after it became clear that the total amount of expected pollution was over-estimated, a significant market-price correction occurred. Then, due to banking restrictions the spot price in phase I decreased toward zero. Whether in Phase I or II, spot and futures prices have been often characterized by a relatively high degree of volatility. However, neither spot nor futures markets were characterized by a good level of liquidity.

6.1.2 Fuel Switch

Since Montgomery (1972b), research on the theoretical price dynamics of emission allowances has concentrated on the substitution principle between emission allowances and abatement technology. After Montgomery (1972b), such authors as Tietenberg (1985), Cronshaw and Kruse (1996) and Rubin (1996) also show that, in a deterministic setup, the allowance price corresponds to the (cheapest) marginal cost of abatement.[2]

In the context of the EU ETS, fuel-switching is the cheapest abatement technology that can also be implemented easily in the short run. Fuel-switching corresponds to the option to switch from coal- to gas-fired power generation. Gas-fired power production emits less emissions per MWh of electricity than coal-fired power generation, therefore a fuel switch from coal to gas lowers emissions. Because coal and gas have different relative carbon intensities, rising CO_2 allowance prices can make gas-fired power generation more competitive than coal-fired generation. A fuel switch from coal to gas yields a reduction of CO_2 emissions per MWh of produced electricity, which implies that less emissions have to be covered with EUAs. The price of gas compared to the price of coal affects the operating choices for the power generation industry. If the coal prices are low compared to the price of gas, more coal will be used. Since coal emits more emissions than gas to produce one MWh of electricity, this leads to higher emissions, therefore raising the demand of EUAs. If the gas prices are low compared to coal, then installations may switch to gas. Since gas has a lower carbon emission output than coal, the demand for EUAs would fall.

Let us define e_c and e_g be the CO_2 emission factors of coal and gas, respectively. Also, let h_c and h_g be the heating rates of coal and gas, respectively. Heating rates measure how much fuel is consumed for the production of one MWh of electricity. Using these parameters and the coal C_t and gas G_t time-series, we can derive the historical path of the coal-to-gas switch price E_t:

$$E_t = \frac{h_g G_t - h_c C_t}{e_c - e_g} \quad t = 0, \ldots, T \tag{6.1}$$

Aligning to conventional results in environmental economics and assuming E_t to be the medium- and long-term CO_2 price benchmark, then one can model it as being a (rough) proxy of the marginal cost to fuel switch. Figure 6.3 represents the log of E_t and the log-price of the emission allowances in Phase I. The difference between

[2]A more detailed description of some of these models is provided in the next section.

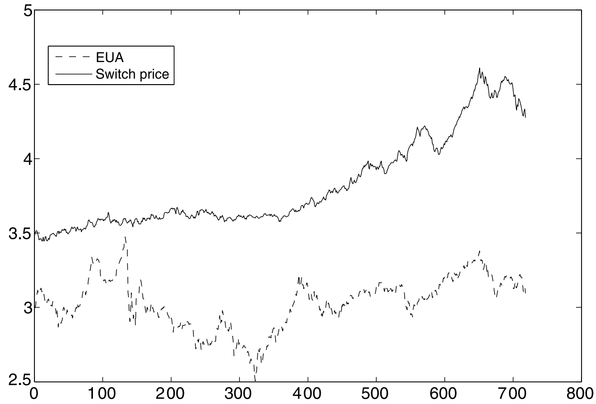


Fig. 6.3 Log-price of the fuel-switch and log-price of spot emission allowances in the period from September 29, 2005 to October 6, 2008

the (theoretical) fuel-switching price, E_t, and the observed price of the emission allowances can be explained by the fact that power plants are not all equally efficient. In other words, power plants have different energy efficiency and, consequently, face firm-specific fuel switching levels. Yet, E_t can be considered a good proxy of the (representative) marginal cost to fuel switch.

For the calculation of the fuel switch price we consider National Balancing Point (NBP) gas futures and API#2 coal prices with maturity December 2008.[3] The NBP gas prices are quoted pence/Therm and are converted to €/Therm.[4] The API#2 coal contracts are quoted in \$/ton and converted to €/ton. The currency conversions are performed using exchange rates from the European Central Bank. The total length of the series is 717 observations and the time period is from September 29, 2005 to October 6, 2008.

As mentioned in Sect. 6.1.2, fuel switch is the most important short-run abatement measure for fuel-fired power plants in the context of the EU ETS. The continuous-time fuel switch price process can be modeled as a combination of two processes:

$$\varepsilon_t = P_t + X_t \tag{6.2}$$

where P_t is a deterministic process and X_t is a stochastic process. The deterministic part models the trend and seasonal fluctuations. Gas prices have a marked winter-summer seasonality. Winter prices for gas are generally higher than summer prices. This is due to an increase in gas heating demand. Furthermore, cold weather renders the conditions for production and supply of gas difficult. Therefore, in general

[3] The National Balancing Point is a virtual trading location for the sale, purchase and exchange of UK natural gas. It is the pricing and delivery point for the Intercontinental Exchange natural gas futures contract. Currently, it is the most liquid gas trading point in Europe. The Argus-McCloskey Coal Price Index service is the source of the API prices, which are the key indexes used for international physical and derivatives coal business. The API#2 index is the industry standard reference price used to trade coal imported into northwest Europe. The API 2 index is an average of the Argus CIF Rotterdam assessment and McCloskey's northwest European steam coal marker.

[4] Therm stands for 100,000 British thermal units (Btu).

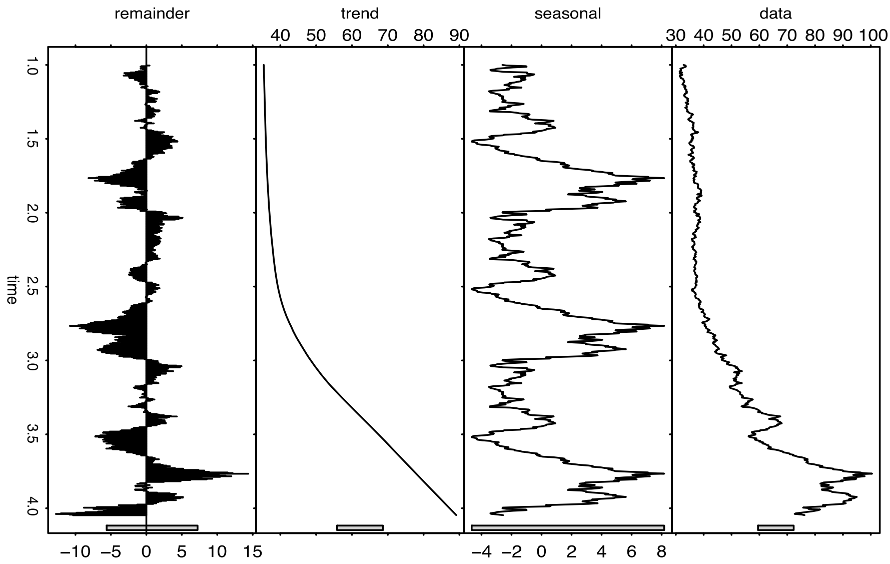


Fig. 6.4 Plot of the time series, the seasonality, trend and remainder

Table 6.3 STL parameters using the *stl* function in *R*

Parameters for the STL algorithm	Value
Span (in lags) of loess window for seasonal extraction	7171
Span (in lags) of loess window for trend extraction	353
Span (in lags) of loess window for the loess window of the low-pass filter	235

the switching point is higher during the winter months, and switching from coal to gas is less likely to occur. The fuel switch price is de-seasonalized using the Seasonal Trend with Local Regression Smoothing or Loess (STL) procedure of Cleveland et al. (1990), which provides trend and seasonal components that are robust to outliers. In Fig. 6.4 the decomposition of the series into seasonality, trend and remainder with the STL algorithm is shown. Table 6.3 shows parameters for the STL algorithm.

Once the trend and the seasonal components are removed from the time series, we are left with the residual X_t:

$$X_t = \varepsilon_t - P_t, \quad t = 0, \dots, T. \tag{6.3}$$

Table 6.4 reports the summary statistics of X_t. As expected, the Jarque-Bera and Shapiro-Wilk tests indicate the non-normality of the time series under investigation.

A geometric Brownian motion (GBM) process of the form

$$dX_t = \mu_1 X_t\, dt + \sigma_1 X_t\, dW_t, \tag{6.4}$$

Table 6.4 Summary statistics of the deseasonalized log returns X_t

	Log prices	Log returns
Number of observations	717	716
Mean	−0.01	0
Stdev	0.05	0.01
Skewness	−0.15	−0.16
Kurtosis	3.4	3.82
Jarque-Bera (p-val)	0.02	0
Shapiro Test (p-val)	0	0

Table 6.5 Parameters for the geometric Brownian motion (GBM) and Ornstein–Uhlenbeck (OU) model

Parameters	GBM process	OU process
λ		2.2520
μ	−0.0002	−0.0030
σ	0.0114	0.0243

and an Ornstein–Uhlenbeck (OU) process of the form

$$dX_t = \lambda(\mu_2 - X_t)\,dt + \sigma_2\,dW_t, \tag{6.5}$$

are calibrated on the time series X_t, where W_t is a standard Brownian motion; μ_1 is the drift term; σ_1 and σ_2 are the volatility terms; λ measures the speed of adjustment to the long run mean μ_2.[5] Table 6.5 reports the calibration results.

Industry routinely switches fuels in response to relative prices or allowance shortages. Fuel-switching is a medium-term abatement option and, therefore, could be used as a proxy for the allowance price over such an horizon. As such, the fuel-switching process can be used for hedging and pricing purposes. For a review on pricing and hedging in the power industry, see Fusai and Roncoroni (2008). For an example on how this modelling could be used to evaluate the value and the activation frequencies of a generation system consisting of coal-fired and a gas-fired power plants, see Taschini and Urech (2010).

6.2 Deterministic and Stochastic Equilibrium Models

This section provides an overview of deterministic and stochastic equilibrium allowance price models existing in the literature. The aim of this section is to highlight the relationship between these models and discuss their general findings. For the sake of comparison, the notation has been modified. A summary of the numerous variables used in this section can be found in Tables 6.6 and 6.7 at the end of the chapter.[6]

[5] A detailed description of the calibration procedure can be found in Brigo et al. (2009). We refer to Daskalakis et al. (2009) and Chevallier (2011) for a discussion of the model extensions.

[6] This section has been written together with Georg Grüll.

6.2.1 Deterministic Equilibrium Models

The following deterministic models assume that regulated firms are profit maximizers or cost minimizers. Regulated firms are mandated to strictly comply with the regulation by choosing an optimal production strategy and an optimal allowance trading strategy.

6.2.2 Model of Montgomery 1972

One of the first references to marketable allowances can be found in the seminal works of Crocker (1966) and Dales (1968). In these papers, pollution-abatement problems are viewed within an economic, cost-benefit framework in conjunction with the concept of property rights. Based on such an idea, Montgomery (1972b) provides a rigorous theoretical justification of how transferable emission allowances lead to an efficient allocation of abatement costs across various "sources of pollution". Necessary and sufficient conditions for market equilibrium and efficiency are derived, given the setting of multiple profit-maximizing firms who attempt to minimize total compliance costs.

We now present a slightly modified and simplified version of Montgomery's model. The original model is simplified by neglecting the spatial dimension of the pollution problem under investigation in the original paper. Such simplification is possible because we are interested in the problem of regulating greenhouse gas emissions which belong to the class of global pollutants. As discussed in Chap. 2, the pollution level of global pollutants is not influenced by the location of the polluting activity. Only in the case of local pollutants the location matters. In particular, the original model computes the location-dependent concentration of pollution by multiplying the dispersion factor with the corresponding emission quantities. By reducing the analysis to one location and assuming that the dispersion factor is the same across all locations—this is the case for global pollutants such as CO_2—the pollution concentration is the product of the constant dispersion factor and the emission quantity. The constant dispersion factor allows us to formulate the optimization problem in terms of the emission quantity as opposed to the pollution concentration.

Given a discrete and finite horizon setting, Montgomery solves a deterministic problem where n profit maximizing firms minimize their costs to comply with regulation (hereafter compliance costs).

Let us denote the price of good r by G^r and assume that firm i produces $y^i = (y^{i,1}, \ldots, y^{i,R})$ of the goods $1, \ldots, R$. When choosing the emission level Q^i, production costs are equal to the twice differentiable and convex function $C^i_{\text{good}}(y^i, Q^i)$. Given y^i and Q^i, firm i's profit is equal to:

$$\pi^i\big(y^i, Q^i\big) = R^i_{\text{good}}\big(y^i\big) - C^i_{\text{good}}\big(y^i, Q^i\big) = \sum_{r=1}^{R} G^r y^{i,r} - C^i_{\text{good}}\big(y^i, Q^i\big),$$

where Q^i denotes the emission quantity and $R^i_{\text{good}}(y^i)$ is the total return. Under an emission constrained scenario, the firm must choose an emission level Q^i and adjust its output in order to obtain maximum profit for the fixed level of emission.

Montgomery quantifies the cost of meeting regulation of the ith firm, $C^i(Q^i)$, by solving the maximization problem under different production constraints. This cost corresponds to the difference between the unconstrained maximum of profit and firm ith maximum of profit when emissions are constrained. Analytically, we can express $C^i(Q^i)$ as:

$$C^i\left(Q^i\right) = \max_{y^i}\left\{\pi^i\left(y^i, Q^i\right)\right\} - \max_{y^i, Q^i}\left\{\pi^i\left(y^i, Q^i\right)\right\}.$$

Montgomery (1972b) has shown that $C^i(Q^i)$ is a twice differentiable, decreasing and convex function.

Let N^i be the number of emission allowances; θ^i be the number of allowances (positive and negative values correspond to buying and selling positions, respectively); and let $\bar{S}$ denote the allowance price. First, Montgomery defines a market equilibrium relative to an initial allocation of allowances and derives necessary and sufficient conditions for its existence. Second, a subsidiary construction, called a *joint cost minimum*, is defined and shown to exist. Montgomery generates a second set of necessary and sufficient conditions. It is shown that the emission vector and shadow prices which satisfy the conditions of a *joint cost minimum* for given totals of allowances also satisfy the conditions of competitive equilibrium relative to any initial allocation of allowances in which the given totals are completely distributed among firms. To prove that a competitive equilibrium achieves the joint cost minimum defined below, Montgomery shows that when allowances totals equal desired air qualities any emission vector and price vector which satisfy the equilibrium conditions also satisfy the conditions for efficiency. Let us introduce the definition of market equilibrium first.

Definition 6.1 (Market equilibrium) Let $\bar{Q} = (\bar{Q}^1, \ldots, \bar{Q}^n) \geq 0$, $\bar{\theta} = (\bar{\theta}^1, \ldots, \bar{\theta}^n)$ and $\bar{S} \geq 0$. A market equilibrium is the $(2n+1)$-tuple $(\bar{Q}, \bar{\theta}, \bar{S})$ such that $\bar{Q}$ and $\bar{\theta}$ are the solution of the following n maximization problems:[7]

$$\max_{Q^i, \theta^i}\left\{-C^i\left(Q^i\right) - \bar{S}\theta^i\right\},$$

$$\text{subject to} \quad N^i + \theta^i - Q^i \geq 0$$

which also satisfy the market clearing conditions

$$\sum_{i=1}^{n} \bar{\theta}^i \leq 0 \quad \text{and} \quad \bar{S}\sum_{i=1}^{n} \bar{\theta}^i = 0.$$

And the definition of joint cost minimum is the following.

[7] The formulation as a maximization problem is chosen to simplify the comparison exercise with the other models under investigation.

Definition 6.2 (Joint cost minimum) A joint cost minimum is the n-tuple $\widetilde{Q} = (\widetilde{Q}^1, \ldots, \widetilde{Q}^n) \geq 0$ that is the solution of the following maximization problem:

$$\begin{aligned} &\max_{Q^1,\ldots,Q^n} \left\{ -\sum_{t=1}^{n} C^i(Q^i) \right\}, \\ &\text{subject to} \quad \sum_{t=1}^{n} (N^i - Q^i) \geq 0. \end{aligned} \tag{6.6}$$

The following theorem shows the existence of market equilibrium.

Theorem 6.3 (Existence of market equilibrium) *A market equilibrium* $(\bar{Q}, \bar{\theta}, \bar{S})$ *exists if and only if there exist* $\bar{u} = (\bar{u}_1, \ldots, \bar{u}_n) \geq 0$ *and* $\bar{S} \geq 0$ *such that* $(\bar{u}, \bar{S})$ *satisfy the Karush–Kuhn–Tucker conditions of the n maximization problems*

$$-\frac{\partial C^i}{\partial Q^i}(\bar{Q}^i) - \bar{u}_i \geq 0, \qquad \bar{Q}^i \left[\frac{\partial C^i}{\partial Q^i}(\bar{Q}^i) + \bar{u}_i \right] = 0, \quad \bar{Q}^i \geq 0, \tag{6.7}$$

$$-\bar{S} + \bar{u}_i = 0, \tag{6.8}$$

$$N^i + \bar{\theta}^i - \bar{Q}^i \geq 0, \qquad \bar{u}_i \left[N^i + \bar{\theta}^i - \bar{Q}^i \right] = 0, \quad \bar{u}_i \geq 0, \tag{6.9}$$

and the market clearing conditions

$$\sum_{i=1}^{n} \bar{\theta}^i \leq 0, \qquad \bar{S} \sum_{i=1}^{n} \bar{\theta}^i = 0. \tag{6.10}$$

Proof 1 $C^i(Q^i)$ is a twice differentiable and convex function in Q^i. $\bar{S}\theta^i$ and $N^i + \theta^i - Q^i$ are linear functions. It follows that both $-C^i(Q^i) - \bar{S}\theta^i$ and $N^i + \theta^i - Q^i$ are twice differentiable and concave functions. Theorem 6.21 in the Appendix completes the proof. □

The following theorem shows the existence of the joint cost minimum.

Theorem 6.4 (Existence of joint cost minimum) *A joint cost minimum* $\widetilde{Q}$ *exists if and only if there exists* $\tilde{u} \geq 0$ *satisfying the Karush–Kuhn–Tucker conditions of the maximization problem given in Eq.* (6.6):

$$-\frac{\partial C^i}{\partial Q^i}(\widetilde{Q}^i) - \tilde{u} \geq 0, \qquad \sum_{i=1}^{n} \widetilde{Q}^i \left[\frac{\partial C^i}{\partial Q^i}(\widetilde{Q}^i) + \tilde{u} \right] = 0, \quad \bar{Q}^i \geq 0, \tag{6.11}$$

$$\sum_{i=1}^{m} (N^i - \widetilde{Q}^i) \geq 0, \qquad \tilde{u} \left[\sum_{i=1}^{n} (N^i - \widetilde{Q}^i) \right] = 0, \quad \tilde{u} \geq 0. \tag{6.12}$$

Proof 2 Constraints are linear and $C^i(Q^i)$ is a twice differentiable and convex function in Q^i, this implies that $-\sum_{i=1}^n C^i(Q^i)$ is twice differentiable and concave. Theorem 6.21 in the Appendix completes the proof. □

Montgomery shows that if we choose the license totals correctly, any market equilibrium is a joint cost minimum. The relation between the optimality conditions is explained in detail in the Appendix at the end of this chapter.

Results When pollution emissions are positive, ($\bar{Q}^i > 0$), Montgomery (1972b) shows that the equilibrium allowance price, $\bar{S}$, is equal to the marginal abatement costs, $-\frac{\partial C^i}{\partial Q^i}(\bar{Q}^i)$. This is a remarkable and important result in the literature. Second, Montgomery (1972b) proves that the market equilibrium is equivalent to the joint cost minimum. Therefore, transferable allowance systems are cost-optimal: an essential property of a transferable allowance system.

6.2.3 Model of Rubin (1996)

Rubin (1996) extends the work of Montgomery (1972b) by proving cost-optimality of transferable allowance systems in a continuous-time, finite horizon setting. By solving a deterministic problem where n profit maximizing firms minimize their compliance costs, Rubin analyses the allowance price evolution over time. Moreover, Rubin investigates the effect of banking and borrowing on the allowance price. Yet, in what follows we concentrate on the proof of cost-optimality of transferable allowance systems.

Let N^i be the number of emission allowances; θ^i be the number of allowances (positive and negative values correspond to buying and selling positions, respectively); and let $\bar{S}$ denote the allowance price. Further, Q_t^i denotes the emission quantity at time t, $C^i(Q_t^i)$ denotes the costs that the ith firm faces when choosing the emission level Q_t^i. The twice differentiable, decreasing and convex function $C^i(\cdot)$ is defined like in Montgomery's model. Finally, let B_t^i model the number of allowances that are available at time t. A positive (negative) value corresponds to banking (borrowing) allowances at time t. Similarly to Montgomery, Rubin defines first the market equilibrium and the joint cost minimum. Than he shows that they are equivalent. As done in the previous section, let us introduce the definition of market equilibrium first.

Definition 6.5 (Market equilibrium) A market equilibrium consists of the vectors $\bar{Q}_t = (\bar{Q}_t^1, \dots, \bar{Q}_t^n) \geq 0$, $\bar{\theta}_t = (\bar{\theta}_t^1, \dots, \bar{\theta}_t^n)$ and $\bar{S}_t \geq 0$ such that $\bar{Q}_t$ and $\bar{\theta}_t$ maximize the following optimization problem of each firm given $\bar{S}_t$:

$$\max_{Q_t^i, \theta_t^i} \left\{ \int_0^T e^{-rt} \left[-C^i(Q_t^i) - \bar{S}_t \theta_t^i \right] dt \right\},$$

subject to the constraints of each firm for each $t \in [0, T]$

$$\dot{B}_t^i := \frac{\partial B_t^i}{\partial t} = N_t^i + \theta_t^i - Q_t^i,$$
$$B_0^i = 0, \quad B_t^i \geq 0, \quad Q_t^i \geq 0,$$

which also satisfy the market clearing condition on allowances and the terminal stock condition

$$\sum_{i=1}^{n} \bar{\theta}_t^i = 0, \quad \forall t,$$
$$\bar{S}_T \sum_{i=1}^{n} \bar{B}_T^i = 0.$$

The definition of joint cost minimum is the following.

Definition 6.6 (Joint cost minimum) A joint cost minimum $\widetilde{Q}_t = (\widetilde{Q}_t^1, \ldots, \widetilde{Q}_t^n) \geq 0$ is the solution of the following maximization problem:

$$\max_{Q_t^1, \ldots, Q_t^n} \left\{ \int_0^T e^{-rt} \left[-\sum_{i=1}^{n} C^i \left(Q_t^i \right) \right] dt \right\},$$

subject to the constraints of each firm for each $t \in [0, T]$

$$\dot{B}_t := \frac{\partial B_t}{\partial t} = \sum_{i=1}^{n} \left(N_t^i - Q_t^i \right),$$
$$B_0 = 0, \quad B_t \geq 0, \quad Q_t^i \geq 0 \quad \text{for all } i = 1, \ldots, n.$$

Rubin shows the existence of the market equilibrium and the joint cost minimum. He generates sets of necessary and sufficient conditions and shows that the emission vector and shadow prices which satisfy the conditions of a joint cost minimum for given totals of allowances also satisfy the conditions of competitive equilibrium relative to any initial allocation of allowances in which the given totals are completely distributed among firms. The following two theorems show the existence of the market equilibrium and the joint cost minimum. A brief interpretation follows each theorem.

Theorem 6.7 (Existence of market equilibrium) *Under the assumption that regulated companies are not buying or selling at the minimum or maximum rate, a market equilibrium* $(\bar{Q}_t, \bar{\theta}_t, \bar{S}_t)$ *exists if and only if there exist for all* $t \in [0, T]$ *non-negative multipliers* $\bar{u}_t = (\bar{u}_t^1, \ldots, \bar{u}_t^n)$, $\bar{\beta}_t = (\bar{\beta}_t^1, \ldots, \bar{\beta}_t^n)$ *and an optimal allowance* *price* $\bar{S} \geq 0$ *such that the following conditions hold for all* $i = 1, \ldots, n$:

$$e^{-rt}\frac{\partial C^i}{\partial Q_t^i}(\bar{Q}_t^i) - \bar{u}_t^i \geq 0, \qquad \frac{\partial \bar{u}_t^i}{\partial t} = \bar{\beta}_t, \qquad e^{-rt}\bar{S}_t + \bar{u}_t^i = 0, \tag{6.13}$$

$$\bar{Q}_t^i\left[e^{-rt}\frac{\partial C^i}{\partial Q_t^i}(\bar{Q}_t^i) - \bar{u}_t^i\right] = 0, \qquad \bar{B}_t^i\bar{\beta}_t^i = 0, \qquad \bar{B}_T^i\bar{u}_T^i = 0, \tag{6.14}$$

$$\bar{Q}_t^i \geq 0, \qquad \bar{B}_t^i \geq 0, \qquad \frac{\partial B_t^i}{\partial t} = N_t^i + \bar{\theta}_t^i - \bar{Q}_t^i, \tag{6.15}$$

and the market clearing condition and the terminal stock condition

$$\sum_{i=1}^n \bar{\theta}_t^i = 0, \qquad \bar{S}_T\sum_{i=1}^n \bar{B}_T^i = 0. \tag{6.16}$$

Proof 3 (Idea) Forming the Lagrangian of the corresponding minimization problem

$$L^i = e^{-rt}\big(C^i\big(Q_t^i\big) + \bar{S}_t\theta_t^i\big) + u_t^i\big(N_t^i + \theta_t^i - Q_t^i\big) - \beta_t^i B_t^i$$

and using both the conditions given in Eqs. (A.13)–(A.16) in Theorem 6.22 (Appendix) and the Karush–Kuhn–Tucker conditions completes the proof. The conditions can then be retrieved from

$$\frac{\partial L^i}{\partial Q_t^i} \geq 0, \qquad \frac{\partial \bar{u}_t^i}{\partial t} = -\frac{\partial L^i}{\partial B_t^i}, \qquad \frac{\partial L^i}{\partial \theta_t^i} = 0,$$

$$\bar{Q}_t^i\frac{\partial L^i}{\partial Q_t^i} = 0, \qquad \bar{B}_t^i\frac{\partial L^i}{\partial B_t^i} = 0, \qquad \bar{B}_t^i\bar{u}_T^i = 0,$$

$$\bar{Q}_t^i \geq 0, \quad \bar{B}_t^i \geq 0, \quad \frac{\partial L^i}{\partial u_t^i} = \frac{\partial B_t^i}{\partial B}.$$

□

Interpretation

(a) If $\bar{Q}_t^i > 0$, then

$$\bar{u}_t^i = e^{-rt}\frac{\partial C^i}{\partial Q_t^i}(\bar{Q}_t^i), \qquad \bar{S}_t = -\frac{\partial C^i}{\partial Q_t^i}(\bar{Q}_t^i).$$

This means that the allowance price is equal to the marginal abatement costs. Moreover, the marginal value of a banked emission allowance (the shadow price) is equal to the discounted marginal abatement costs.

(b) Rubin (1996) shows that the allowance price is growing at the risk-free interest rate if there are no restrictions on banking and borrowing. However, the allowance price will have a lower growth rate in case borrowing is forbidden, but regulated companies wish to do so.

(c) The transversality condition, $\bar{B}^i_T \bar{u}^i_T = 0$, implies that banked allowances are worthless at time T.

Theorem 6.8 (Existence of joint cost minimum) *A joint cost minimum exists if and only if there exist for all $t \in [0, T]$ non-negative multipliers $\bar{u}_t, \bar{\beta}_t$ such that the following conditions hold for $i = 1, \ldots, n$:*

$$e^{-rt} \frac{\partial C^i}{\partial Q^i_t}\big(\widetilde{Q}^i_t\big) - \tilde{u}_t \geq 0, \qquad \frac{\partial \tilde{u}_t}{\partial t} = \tilde{\beta}_t, \qquad \frac{\partial B_t}{\partial t} = \sum_{i=1}^{n} \big(N^i_t - \widetilde{Q}^i_t\big), \tag{6.17}$$

$$\widetilde{Q}^i_t \left[e^{-rt} \frac{\partial C^i}{\partial Q^i_t}\big(\widetilde{Q}^i_t\big) - \tilde{u}_t \right] = 0, \qquad \widetilde{B}_t \tilde{\beta}_t = 0, \qquad \widetilde{B}_T \tilde{u}_T = 0, \tag{6.18}$$

$$\widetilde{Q}^i_t \geq 0, \qquad \widetilde{B}_t \geq 0. \tag{6.19}$$

Proof 4 Similar to Theorem 6.7. The conditions can be retrieved from

$$\frac{\partial L}{\partial Q^i_t} \geq 0, \qquad \frac{\partial \tilde{u}_t}{\partial t} = -\frac{\partial L}{\partial B_t}, \qquad \frac{\partial L}{\partial u_t} = \frac{\partial B_t}{\partial t},$$

$$\widetilde{Q}^i_t \frac{\partial L}{\partial Q^i_t} = 0, \qquad \widetilde{B}_t \frac{\partial L}{\partial B_t} = 0, \qquad \widetilde{B}_T \tilde{u}_T = 0,$$

$$\widetilde{Q}^i_t \geq 0, \qquad \widetilde{B}_t \geq 0,$$

where the Lagrangian is given by

$$L = e^{-rt} \sum_{i=1}^{n} C^i\big(Q^i_t\big) + u_t \sum_{i=1}^{n} \big(N^i_t - Q^i_t\big) - \beta_t B_t.$$

□

Interpretation If $\widetilde{Q}^i_t > 0$, then

$$-\tilde{u}_t = -e^{-rt} \frac{\partial C^1}{\partial C^1_t} = \cdots = -e^{-rt} \frac{\partial C^n}{\partial Q^n_t}\big(\widetilde{Q}^n_t\big). \tag{6.20}$$

This means that the marginal cost of banking allowances is equal to the discounted marginal abatement costs. In particular, marginal abatement costs are the same for all the regulated companies that are polluting.

Results Similarly to Montgomery (1972b), Rubin (1996) shows that also in continuous-time the equilibrium allowance price, $\bar{S}$, is equal to the marginal abatement costs, $-\frac{\partial C^i}{\partial Q^i}(\bar{Q}^i)$. Moreover, Rubin proves that the market equilibrium is equivalent to the joint cost minimum.

6.2.4 Model of Kling and Rubin (1997)

Kling and Rubin (1997) extend the work of Rubin (1996) and consider both cost- and social-optimality. Whereas Montgomery (1972b) and Rubin (1996) show that a system of transferable allowances leads to the least-cost solution, Kling and Rubin (1997) analyze the socially optimal solution by incorporating the damage function associated to pollution emissions. In what follows, we present the major contributing results of Kling and Rubin (1997). The mathematical derivations are not discussed here.

Let the convex function $D(Q,t)$ describe the damages (costs) imposed on the society and associated to the quantity of pollution Q. Also, let us assume that the firm i produces goods y_t^i for a unit price G_t. Revenues from the production of the goods at time t are given by

$$R_{\text{good}}^i\big(y_t^i\big) = G_t y_t^i.$$

Furthermore, the cost function $C_{\text{good}}^i(Q,y)$ describes the total costs of producing the output y when choosing the emission level Q. It is assumed that $C_{\text{good}}^i(Q,y)$ is strictly convex in (Q,y), that is,

$$\frac{\partial C_{\text{good}}^i}{\partial y} > 0, \qquad \frac{\partial C_{\text{good}}^i}{\partial Q} < 0, \qquad \frac{\partial^2 C_{\text{good}}^i}{\partial y \partial Q} < 0,$$
$$\frac{\partial^2 C_{\text{good}}^i}{\partial y^2} > 0, \qquad \frac{\partial^2 C_{\text{good}}^t}{\partial Q^2} > 0.$$

Kling and Rubin first solve the profit maximization problem of a regulated firm and than they consider the central planner's optimization problem.[8] Let us first introduce the firm's maximization problem.

Definition 6.9 (Firm i's profit maximization problem) Given the allowance price $\bar{S}_t$, firm i chooses an optimal emission level $\bar{Q}_t^i \geq 0$, buys and sells an optimal number of allowances θ_t^i and produces an optimal quantity of output $\bar{y}_t^i$:

$$\max_{Q_t^i, \theta_t^i, y_t^i} \left\{ \int_0^T e^{-rt} \big[R_{\text{good}}^i\big(y_t^i\big) - C_{\text{good}}^i\big(Q_t^i, y_t^i\big) - \bar{S}_t \theta_t^i \big] \, dt \right\},$$

subject to the following constraints for each $t \in [0,T]$ and each firm

$$\dot{B}_t^i := \frac{\partial B_t^i}{\partial t} = N_t^i + \theta_t^i - Q_t^i,$$
$$B_0^i = 0, \quad B_T^i \geq 0, \quad Q_t^i \geq 0.$$

[8] It is interesting to observe that the firm's target function is similar to the one used in the stochastic models where a fine for non-compliance is explicitly modelled in the objective function.

And the Central planner's control problem is defined as follows.

Definition 6.10 (Central planner's optimization problem) The central planner chooses optimal emission levels $\bar{Q}_t = (\bar{Q}_t^1, \dots, \bar{Q}_t^n) \geq 0$ and output quantities $\bar{y}_t = (\bar{y}_t^1, \dots, \bar{y}_t^n)$:

$$\max_{Q_t^1,\dots,Q_t^n,y_t^1,\dots,y_t^n} \left\{ \int_0^T e^{-rt} \left[\sum_{i=1}^n R_{\text{good}}^i(y_t^i) - \sum_{i=1}^n C_{\text{good}}^i(Q_t^i, y_t^i) - D\left(\sum_{i=1}^n Q_t^i, t \right) \right] \right\},$$

subject to the following constraints for each $t \in [0, T]$ and each firm

$$\dot{B}_t := \frac{\partial B_t}{\partial t} = \sum_{i=1}^n (N_t^i - Q_t^i),$$

$$B_0 = 0, \quad B_T \geq 0, \quad Q_t^i \geq 0 \quad \text{for all } i = 1, \dots, n.$$

Results Kling and Rubin show that the equilibrium allowance price is equal to the marginal abatement costs

$$\bar{S}_t = -\frac{\partial C^1}{\partial Q_t^1}(\bar{Q}_t^1, \bar{y}_t^1) = \dots = -\frac{\partial C^n}{\partial Q_t^n}(\bar{Q}_t^n, \bar{y}_t^n).$$

By investigating firms' incentives to bank or borrow emission allowances, Kling and Rubin show that under unconstrained provisions firms will suboptimally choose excessive damage and output levels in early periods and correspondingly too few in later periods. Under the assumption that social damages are linear, Kling and Rubin show that a social optimum could be achieved by introducing a modified banking rules that penalize borrowing by discounting borrowed allowances. In Rubin (1996), a firm has to hand in one allowance per unit of emission independent of when emissions take place. Such a banking regulations is described by the constraint:

$$\dot{B}_t := \frac{\partial B_t}{\partial t} = \sum_{i=1}^n (N_t^i - Q_t^i). \tag{6.21}$$

The proposed modified banking rule of Kling and Rubin (1997) requires firms that borrow one unit of emission for a time period of length t to hand in $e^{rt}(>1)$ allowances at time t. Analytically, this corresponds to the new constraint:

$$\dot{B}_t := \frac{\partial B_t}{\partial t} = e^{-rt} \sum_{i=1}^n (N_t^i - Q_t^i). \tag{6.22}$$

We now consider the stochastic equilibrium models proposed by Seifert et al. (2008), Carmona et al. (2009), and Chesney and Taschini (2012). A common feature of the previous literature is the assumption (or constraint) that the prescribed quota is strictly met. The models we consider below relax such a constrain by explicitly modelling noncompliant event, that is, under-compliance is a possible outcome. When regulated entities are noncompliant, they pay a per unit penalty for excess emissions.

6.2.5 Model of Seifert et al. (2008)

Seifert et al. (2008) solve the representative agent's cost minimization problem. The equivalence of the market equilibrium and the joint cost optimum justify the representative agent framework that significantly simplifies the problem at hand.

Below we introduce some notation and definitions used in Seifert et al. (2008). Let the stochastic process β_t^i represent firm ith's emission rate before abatement activities. For a constant β_0^i and σ_i^2, the pollution emission rate β_t^i is assumed to follows:

- a White–Noise process, that is, $\beta_t \sim N(\beta_0, \sigma^2)$, or
- α_t^i an arithmetic Brownian motion of the form $\beta_t = \beta_0 + \sigma W_t$.

Let the stochastic process α_t^i represent the abatement rate; and let the stochastic process θ_t^i represent the (positive or negative) number of allowances bought and sold, respectively. The compliance strategy depends on the expected pollution emitted in the interval $[0, T]$, the abatement volume and on the number of purchased and sold allowances:[9]

$$q_t^i = \mathbb{E}\left[\int_0^t \beta_s^i \, ds \mid \mathcal{F}\right] - \int_0^t \alpha_s^i \, ds - \int_0^t \theta_s^i \, ds.$$

Finally, N^i corresponds to the number of allocated allowances to firm i, and P is the per unit penalty for excess emissions.

Seifert et al. (2008) model marginal abatement costs as a deterministic, increasing function. In particular, the instantaneous abatement costs are

$$C^i\left(\alpha_t^i\right) = \frac{1}{2} c_i \left(\alpha_t^i\right)^2,$$

where c_i is a positive constant. Regulated firms can buy and sell allowances. The per-period costs and profits associated to the trading strategy are equal to $S_t \theta_t^i$, where S_t is the allowance price at time t. The potential, final fine for non compliance corresponds to $P(q_T^i - N^i)^+$.

[9] There are modelling choice that make Seifert et al. (2008) different from Chesney and Taschini (2012) and Carmona et al. (2009). In particular, pollution emissions are broken down into emissions before abatement activities and after abatement activities.

Definition 6.11 (Firm i's optimization problem) Given the allowance price S_t, firm i minimizes its expected costs by choosing an optimal abatement strategy and buying and selling an optimal number of allowances, that is,

$$\max_{\alpha_t^i,\theta_t^i} \mathbb{E}\Big[-\int_0^T e^{-rt} C^i\big(\alpha_t^i\big)\, dt - \int_0^T e^{-rt} S_t \theta_t^i - e^{-rT} P\big(q_T^i - N^i\big)^+\Big]$$
$$= \max_{\alpha_t^i,\theta_t^i}\Big[-\int_0^T e^{-rt}\Big(\frac{1}{2}c_i\big(\alpha_t^i\big)^2\Big)\, dt - \int_0^T e^{-rt} S_t \theta_t^i - e^{-rT} P\big(q_T^i - N^i\big)^+\Big]. \tag{6.23}$$

The following lemma describes the dynamics of the firm's cumulative emissions.

Lemma 6.12 (SDE for firm i's cumulative "emissions" q_t^i) *Assume that the emission rate before abatement activities, β_t^i, follows* (i) *a White–Noise process or* (ii) *an arithmetic Brownian motion.*

Then the SDE of firm i's cumulative emissions are given by

$$dq_t^i = -\big(\alpha_t^i + \theta_t^i\big)\, dt + H_t^i\, dW_t,$$

where H_t^i is (i) $H_t^i = \sigma_i$ *and* (ii) $H_t^i = \sigma_i(T-t)$.

Proof 5 See online appendix of Seifert et al. (2008). □

The dynamics of the equilibrium permit price, S_t, is described in the following Lemma.

Lemma 6.13 (First order conditions of firm i's optimization problem) *Let $V^i(t, q_t^i)$ be the expected value of an optimal policy for firm i's cost minimization problem (cf. Definition 6.11) between time t and T. Denote its partial derivatives by V_t^i, V_q^i, V_{qq}^i. Then the first order conditions of this optimization problem are given by*

$$\alpha_t^i = -\frac{1}{c} e^{rt} V_q^i$$
$$S_t = -e^{-rt} V_q^i = c_i \alpha_t^i = -\frac{\partial C^i(\alpha_t^i)}{\partial \alpha_t^i}.$$

Proof 6 (Idea) By the principle of optimality

$$V^i\big(t, q_t^i\big) = \max_{\alpha_t^i,\theta_t^i} \mathbb{E}\big[-e^{-rt} C^i\big(\alpha_t^i\big)\, dt - e^{-rt} S_t \theta_t^i\, dt + V^i\big(t + dt, q_t^i + dq_t^i\big)\big]. \tag{6.24}$$

Applying Itô's lemma to $V^i(t, q_t^i)$, we have

$$
\begin{aligned}
V^i&\left(t+dt,q_t^i+dq_t^i\right)-V^i\left(t,q_t^i\right)\\
&=V_t^i\,dt+V_q^i\,dq_t^i+\frac{1}{2}V_{qq}^i\,dq_t^i\,dq_t^i\\
&=V_t^i\,dt-V_q^i\left(\alpha_t^i+\theta_t^i\right)dt+H_t^iV_q^i\,dW_t+\frac{1}{2}\left(H_t^i\right)^2V_{qq}^i\,dt\\
&=\left(V_t^i-V_q^i\left(\alpha_t^i+\theta_t^i\right)+\frac{1}{2}\left(H_t^i\right)^2V_{qq}^i\right)dt+H_t^iV_q^i\,dW_t.
\end{aligned}
$$

This implies

$$
\mathbb{E}\left[V^i\left(t+dt,q_t^i+dq_t^i\right)-V^i\left(t,q_t^i\right)\mid\mathcal{F}_t\right]=\left(V_t^i-V_q^i\left(\alpha_t^i+\theta_t^i\right)+\frac{1}{2}\left(H_t^i\right)^2V_{qq}^i\right)dt.
$$

Subtracting $V^i(t,q_t^i)$ on both sides of Eq. (6.24) yields

$$
0=\max_{\alpha_t^i,\theta_t^i}\left[-e^{-rt}C^i\left(\alpha_t^i\right)-e^{-rt}S_t\theta_t^i+V_t^i-V_q^i\left(\alpha_t^i+\theta_t^i\right)+\frac{1}{2}\left(H_t^i\right)^2V_{qq}^t\right], \quad (6.25)
$$

with boundary condition

$$
V^i\left(T,q_T^i\right)=-e^{-rT}P\left(q_T^i-N^i\right)^+.
$$

Maximizing the right-hand side of Eq. (6.25) by taking the partial derivatives with respect to α_t^i and θ_t^i and setting it to zero yields

$$
\begin{aligned}
\alpha_t^i&=-\frac{1}{c_i}e^{rt}V_q^i\\
S_t&=-e^{rt}V_q^i=c_i\alpha_t^i=\frac{\partial C^i(t,\alpha_t^i)}{\partial\alpha_t^i}.
\end{aligned}
$$

□

In the Appendix, we report the optimization problem of the representative agent in Seifert et al. (2008) model

Results When the emission rate follows an arithmetic Brownian motion, it is not possible to derive a closed-form solution. However, Seifert et al. (2008) provide an illustration of the allowance price in this case. It is interesting to observe that the shape of this graph is identical to the one of the payoff of a binary call option written on the underlying process q_t, with a strike price N and expiry date T. Because the value of a binary option can be interpreted as the probability that the underlying process is greater than the strike value at time T, the allowance price can be seen as the per-unit penalty P multiplied by the probability of a shortage event, i.e. $q_T > N$. We refer to Chesney and Taschini (2012) for a more detailed discussion on this similarity.

6.2.6 Model of Carmona et al. (2009)

The stochastic equilibrium model of Carmona et al. (2009) captures several characteristics of an ordinary allowance scheme. The model considers an economy with n firms where profits and costs are expressed in time-T currency. Thus, no discount factors appear in the formulae. Firms are profit maximizer and choose an optimal production strategy and an optimal allowance trading strategy. Firm ith's profits from producing goods over the time period $[0, T]$ are:

$$R^t_{\text{good}}(y^i) - C^i_{\text{good}}(y^i) = \sum_{t=0}^{T-1}\left(\sum_{j,k} G^k_t y^{i,j,k}_t\right) - \sum_{t=0}^{T-1}\left(\sum_{j,k} \kappa^{i,j,k}_t y^{i,j,k}_t\right),$$

where the stochastic process $y^{i,j,k}_t$ represents the output quantity of goods k at time t using technology j; the stochastic process G^k_t represents the unit price of goods k at time t; the stochastic process $\kappa^{i,j,k}_t$ represents the marginal costs of production of goods k using technology j. Marginal costs are exogenous.

Gains and losses from allowances trading are represented as:

$$T^i(\Theta^i) = \sum_{t=0}^{T-1} \Theta^i_t (A_{t+1} - A_t) - \Theta^i_T A_T,$$

where the stochastic process $\Theta^{i,j,k}_t$ represents the total number of allowances bought and sold. A positive (negative) value indicates that firm ith is a net buyer (seller); the stochastic process A_t represents the price of the futures contracts with maturity T time-t.

The final, potential fine for non compliance is given by:

$$P \cdot \left(q^i(y^i) + \Delta^i - N^i - \Theta^i_T\right)^+$$

and it depends on the firm ith's cumulative emissions in the period $[0, T]$

$$q^i(y^i) + \Delta^i = \sum_{t=0}^{T-1} Q^i_t + \Delta^i = \sum_{t=0}^{T-1}\left(\sum_{j,k} e^{i,j,k} y^{i,j,k}_t\right) + \Delta^i,$$

where $e^{i,j,k}$ is the constant emission factor measuring pollution emitted per unit of goods k with technology j and the random variable Δ^i models uncontrolled emissions. It also depends on the policy parameters: compliance and allocation levels. The constant per unit penalty, P, for excess emissions and the total amount of allocated allowances, $N^i = \sum_{t=0}^{T-1} N^i_t$, i.e. the cap.

In Carmona et al. (2009), model the following constraints must hold:

1. Production cannot exceed capacity:

$$0 \le y^{i,j,k}_t \le K^{i,j,k}.$$

2. Demand is always smaller than the total production capacity:

$$0 \le D_t^k \le \sum_i \sum_j K^{i,j,k},$$

where D_t^k models the stochastic demand for good k and $K^{i,j,k}$ is the constant firm's capacity to produce goods k with technology j.

It is interesting to notice that the total fine is expressed in terms of Θ_t^i, the number of emission allowances hold by the firm. It is simple to express Θ_t^i in terms of θ_t^i:

$$\theta_0^i = \Theta_0^i,$$
$$\theta_t^i = \Theta_t^i - \Theta_{t-1}^i.$$

Therefore, gains and losses are

$$\begin{aligned}
T^i\left(\Theta^i\right) &= -\sum_{t=0}^{T} \theta_t^i A_t \\
&= -\Theta_0^i A_0 - \sum_{t=1}^{T}\left(\Theta_t^i - \Theta_{t-1}^i\right) A_t \\
&= -\Theta_0^i A_0 + \Theta_0^i A_1 - \Theta_1^i A_1 + \Theta_1^i A_2 - \Theta_2^i A_2 + \cdots + \Theta_{T-2}^i A_{T-1} \\
&\quad - \Theta_{T-1}^i A_{T-1} + \Theta_{T-2}^i A_T - \Theta_T^i A_T \\
&= \Theta_0^i (A_1 - A_0) + \Theta_1^i (A_2 - A_1) + \cdots + \Theta_{T-1}^i (A_T - A_{T-1}) - \Theta_T^i A_T \\
&= \sum_{t=0}^{T-1} \Theta_t^i (A_{t+1} - A_t) - \Theta_T^i A_T.
\end{aligned}$$

Finally, the uncontrollable emissions must satisfy a technical condition: conditional on the information available at time $T-1$, the sum of all uncontrollable emissions, $\sum_i \Delta^i$, must have a continuous distribution. This technical assumption is introduced in order to avoid pathological situations concerning the equilibrium prices.

We first introduce the firm's optimization problem.

Definition 6.14 (Firm i's optimization problem) Given the forward allowance price A and the prices of the produced goods G, firm i maximizes its expected terminal wealth by buying or selling an optimal number of allowances and producing an optimal quantity of goods, i.e.

$$\sup_{\Theta^i, y^i} \mathbb{E}\left[L^i\left(\Theta^i, y^i \mid A, G\right)\right] \tag{6.26}$$

where the terminal wealth is given by

$$
\begin{aligned}
&L^i\big(\Theta^i, y^i \mid A, G\big) \\
&\quad = \big[R^i_{\text{good}}\big(y^i \mid G\big) - C^i_{\text{good}}\big(y^i\big) + T^i\big(\Theta^i \mid A\big) - P \cdot \big(q^i\big(y^i\big) + \Delta^i - N^i - \Theta^i_T\big)^+\big].
\end{aligned} \tag{6.27}
$$

The global optimization problem is defined as follows.

Definition 6.15 (Global optimization problem) A fictitious central planner minimizes expected total costs by producing an optimal quantity of goods, that is, it faces the following optimization problem

$$
\inf_y \mathbb{E}\big[C_{\text{good}}(y) + P\big(q(y) + \Delta - N\big)^+\big], \tag{6.28}
$$

where

$$
\begin{aligned}
C_{\text{good}}(y) &= \sum_i C^i_{\text{good}}\big(y^i\big), \\
q(y) &= \sum_i q^i\big(y^i\big), \\
\Delta &= \sum_i \Delta^i, \\
N &= \sum_i N^i.
\end{aligned}
$$

The following definition and lemmas help understanding the relationship between market equilibrium and global optimum.

Definition 6.16 (Market equilibrium) $(\bar{A}, \bar{G})$ is a market equilibrium in emission allowances with associated strategies $\bar{\Theta}$ and $\bar{y}$ if for given

- $\bar{A}$ (one-dimensional stochastic process for forward price on allowances)
- $\bar{G}$ (multi-dimensional stochastic process for the prices of the products)

the associated optimal strategies

- $\bar{\Theta}$ (multi-dimensional stochastic process of optimal trading strategies)
- $\bar{y}$ (multi-dimensional stochastic process of optimal production strategies)

lead to a situation where all the firms ("maximize" their profits) are satisfied by their strategy in the sense that for all i

$$
\mathbb{E}\big[L^i\big(\bar{\Theta}^i, \bar{y}^i \mid \bar{A}, \bar{G}\big)\big] \geq \mathbb{E}\big[L^i\big(\Theta^i, y^i \mid \bar{A}, \bar{G}\big)\big] \quad \text{for all } \big(\Theta^i, y^i\big)
$$

and the following two conditions hold

- Market clearing condition on allowances

$$\sum_i \bar{\Theta}_t^i = 0.$$

- Supply meets demand for each good

$$\sum_{i,j} \bar{y}_t^{i,j,k} = D_t^k.$$

We notice that in the business as usual scenario (i.e. $P = 0$), the equilibrium price of the goods is given by

$$\bar{G}_t = \max_{i,j} \left\{ \kappa_t^{i,j,k} \mathbf{1}_{\{y_t^{i,j,k} > 0\}} \right\}.$$

Therefore, the equilibrium prices correspond to a merit-order type of equilibrium where all the production means of the economy are ranked by increasing production costs, $\kappa_t^{i,j,k}$ and demand is met by producing with the cheapest production means. The resulting equilibrium price of goods k is equal to the marginal cost of production using the most expensive production means to meet demand D_t^k.

Conversely, in the presence of a penalty $P > 0$, the equilibrium prices of the goods are given by

$$\bar{G}_t = \max_{i,j} \left\{ \left(\kappa_t^{i,j,k} + e^{i,j,k} \bar{A}_t \right) \mathbf{1}_{\{y_t^{i,j,k} > 0\}} \right\}.$$

Therefore, the equilibrium prices correspond to a merit-order type of equilibrium where costs are adjusted for the emissions associated to the production, $\kappa_t^{i,j,k} + e^{i,j,k} \bar{A}_t$.

Results Carmona et al. (2009) show that the market equilibrium is equivalent to the joint cost optimum. Prices of the produced goods correspond to a merit-order type equilibrium with adjusted costs. Consequently, products are becoming more expensive in the presence of an emissions trading scheme. The price increase is equal to the value of the allowances that are needed for the production of the good. Moreover, Carmona et al. (2009) show that the futures allowance price is equal to the penalty multiplied by the probability of allowance shortage at the end of the compliance period. The event of allowance shortage materialises when the net cumulative emissions (after abatement activities) of all regulated companies exceed the total number of allocated allowances. The equilibrium allowance price may be different when the allowance positions are not considered in aggregate terms. By considering firm specific allowance positions, Chesney and Taschini (2012) address this aspect.

6.2.7 Model of Chesney and Taschini (2012)

The stochastic models described above provide a fairly comprehensive treatment of the inter-temporal evolution of the allowance price. However, such studies have been framed in a setting where fundamental aspects of trading allowances and the presence of asymmetric information in the market for allowances were not taken into consideration. Kijima et al. (2010) confirms that the market for allowances does more than simply transfer allowances from regulated firms with a surplus in allowances to firms with a deficit. In an attempt to study supply and demand imbalance, Chesney and Taschini (2012) develop an equilibrium model for the price dynamics of emission allowances in the short term that accounts for the presence of asymmetric information. In particular, the equilibrium allowance price reflects the perceived scarcity or excess of allowances in the market.

We first recall the resulting equilibrium allowance prices when firms' emissions are aggregated. We rewrite the equilibrium allowance price derived in Carmona et al. (2009) and than introduce the equilibrium allowance price proposed in Chesney and Taschini (2012). Let $q_{[0,T]}$ be the random variable that denotes the aggregated cumulative emissions of all regulated companies at time T; N is the total number of allocated allowances, i.e. the cap; and P is the constant per unit penalty for excess emissions.

Allowing for stochastic production costs, revenues from selling produced goods and total pollution emission quantities, Carmona et al. (2009) show that the price of the futures contract with maturity T at time t is:

$$F(t,T) = P \cdot \mathbb{P}(q_{[0,T]} > N | \mathcal{F}_t), \tag{6.29}$$

in other words the allowance price equals the expected non-compliance cost, that is, the penalty multiplied by the probability of allowances shortage.

Assuming that interest rates r are deterministic and that there is no convenience yield as shown by Uhrig-Homburg and Wagner (2007), the theoretical allowance price at time t is given by:

$$\begin{aligned} S_t &= Pe^{-r(T-t)} \cdot \mathbb{P}(q_{[0,T]} > N | \mathcal{F}_t) \\ &= \begin{cases} Pe^{-r(T-t)} & \text{if } q_{[0,t]} \geq N \\ Pe^{-r(T-t)} \cdot \mathbb{P}(q_{[t,T]} > N - q_{[0,t]} | \mathcal{F}_t) & \text{if } q_{[0,t]} < N. \end{cases} \end{aligned} \tag{6.30}$$

The model of Chesney and Taschini (2012) specifies the process for the cumulative emissions in the framework of Carmona et al. (2009) by assuming that the firms' emission rate, Q_t, follows a geometric Brownian motion:

$$Q_t = Q_0 \exp\left\{ \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right\}. \tag{6.31}$$

Therefore, the cumulative emissions in $[0, t]$ are given by

$$q_{[0,t]} = \int_0^t Q_s \, ds. \tag{6.32}$$

This means that the cumulative emissions are described by the integral over a geometric Brownian motion for which no closed-form density is available. The model of Chesney and Taschini (2012) approximates the cumulative emissions in the time interval $[t_1, t_2] \subseteq [0, T]$ by the following linear approximation:

$$q_{[t_1,t_2]} \approx \tilde{q}^{\mathrm{Lin}}_{[t_1,t_2]} = Q_{t_2}(t_2 - t_1). \tag{6.33}$$

The following lemma introduces the cumulative emissions in the 1-firm model of Chesney and Taschini.

Lemma 6.17 (Cumulative emissions in the model of Chesney and Taschini) *Let μ and σ be the parameters of the geometric Brownian motion modelling the emission rate. Let $t \in [0, T]$, $\tau = T - t$ and $Z \sim N(0, 1)$.*

Then the cumulative emissions in the interval $[t, T]$ are given by

$$\tilde{q}^{\mathrm{Lin}}_{[t,T]} = Q_t \exp\left\{\ln(\tau) + \left(\mu - \frac{\sigma^2}{2}\right)\tau + \sigma\sqrt{\tau}Z\right\}. \tag{6.34}$$

Proof 7

$$\begin{aligned}\tilde{q}^{\mathrm{Lin}}_{[t,T]} &= (T - t) \cdot Q_T = \tau \cdot Q_T \\ &= \tau \cdot Q_t \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\tau + \sigma W_\tau\right\} \\ &= Q_t \exp\left\{\ln(\tau) + \left(\mu - \frac{\sigma^2}{2}\right)\tau + \sigma\sqrt{\tau}Z\right\}.\end{aligned}$$

□

The following lemma describes the allowance price dynamics in the 1-firm model of Chesney and Taschini.

Lemma 6.18 (Permit price—Linear approximation) *The allowance price at time $t < T$ is given by*

$$S^{\mathrm{Lin}}_t = \begin{cases} Pe^{-r\tau} & \text{if } q_{[0,t]} \geq N \\ Pe^{-r\tau} \cdot \Phi\left(\frac{-\ln\left(\frac{1}{\tau}\left[\frac{N - q_{[0,t]}}{Q_t}\right]\right) + \left(\mu - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}\right) & \text{if } q_{[0,t]} < N \end{cases} \tag{6.35}$$

where $\tau = T - t$ is the time to compliance.

When firms' emissions are aggregated, the equilibrium allowance price at time T can be rewritten as:

$$S^{\mathrm{Lin}}_T = P \cdot \mathbb{E}[\mathbf{1}_{\{q_{[0,T]} \geq N\}}].$$

Proof 8

$$S_t^{\text{Lin}} = \begin{cases} Pe^{-r\tau} & \text{if } q_{[0,t]} \geq N \\ Pe^{-r\tau} \cdot \mathbb{P}(\tau \cdot Q_T > N - q_{[0,t]} | \mathcal{F}_t) & \text{if } q_{[0,t]} < N. \end{cases}$$

Let $Z \sim N(0, 1)$. Then,

$$\begin{aligned}
&\mathbb{P}(\tau \cdot Q_T > N - q_{[0,t]} | \mathcal{F}_t) \\
&= \mathbb{P}\left(\tau \cdot Q_t \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\tau + \sigma\sqrt{\tau} Z\right\} > N - q_{[0,t]} \middle| \mathcal{F}_t\right) \\
&\overset{Q_t > 0}{=} \mathbb{P}\left(\exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\tau + \sigma\sqrt{\tau} Z\right\} > \frac{1}{\tau}\left[\frac{N - q_{[0,t]}}{Q_t}\right] \middle| \mathcal{F}_t\right) \\
&\overset{N > q_{[0,t]}}{=} 1 - \Phi\left(\frac{\ln(\frac{1}{\tau}[\frac{N - q_{[0,t]}}{Q_t}]) - (\mu - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}\right) \\
&= \Phi\left(\frac{-\ln(\frac{1}{\tau}[\frac{N - q_{[0,t]}}{Q_t}]) + (\mu - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}\right)
\end{aligned}$$

completes the proof. □

Chesney and Taschini (2012) extend the case where firm's emissions are aggregated (single-firm case) and propose a multi-firm case with $\mathcal{I}$ profit maximizer firms. The resulting allowance price dynamics is described below. As Hintermann (2010) has tested empirically, in the short run firms comply with the regulation by adjusting their allowance portfolios ($\delta_{i,t} = N_i + \sum_{s=0}^{t} x_{i,s}$). Permits portfolios are adjusted by choosing the optimal amount of allowances to purchase ($x_{i,t} > 0$) and to sell ($x_{i,t} < 0$). Chesney and Taschini show that in a multi-firm framework, the equilibrium allowance price S_t reflects at each instant t the net accumulated pollution of company ith ($\int_t^T Q_{i,s}\, ds - \delta_{i,T-1}$) and the expected net accumulated pollution of the other I^- companies, where $I^- := \mathcal{I} - i$

$$S_t = P \cdot \mathbb{E}_t[\mathbf{1}_{\{q^i_{[0,T]} \geq N^i\}}] \cdot \mathbb{E}_t[\mathbf{1}_{\{q^{I^-}_{[0,T]} \geq N^i\}}].$$

The equilibrium allowance price in Chesney and Taschini (2012) is similar to the equilibrium allowance price computed in Carmona et al. (2009), although in this framework pollution abatement is not explicitly modelled, see Hintermann (2010) for data-driven evidence. Asymmetric information is introduced by imposing a one-time *lag-effect*. So, each firm ith has complete knowledge about its own net accumulated pollution at time t:

$$\int_0^t Q_{i,s}\, ds - \delta_{i,t-1}.$$

Table 6.6 Survey on the variables of the different equilibrium models (Part 1)

Variable	Description
α	Abatement rate in the model of Seifert et al. (2008)
A_t	Time-t futures allowance price (maturity at time T) in the model of Carmona et al. (2009)
B	Number of allowances in the "bank" corresponding to the number of allocated allowances plus the purchased allowances minus the emissions. This variable is used in the models of Rubin (1996), Kling and Rubin (1997)
β	Emission rate before abatement activities in the model of Seifert et al. (2008)
$C(\cdot)$	Abatement costs in the models of Montgomery (1972b) and Rubin (1996)
$C_{\text{good}}(\cdot)$	Production costs in the models of Montgomery (1972b), Rubin (1996), Kling and Rubin (1997), and Carmona et al. (2009)
D	Demand for the goods in the model of Carmona et al. (2009)
Δ	Aggregated uncontrollable emissions in $[0, T]$ in the model of Carmona et al. (2009)
e	Emission factor in the model of Carmona et al. (2009)
G	Prices of the produced goods in the model of Montgomery (1972b), Rubin (1996), Kling and Rubin (1997) and Carmona et al. (2009)
K	Production capacity in the model of Carmona et al. (2009)
κ	Marginal production costs in the model of Carmona et al. (2009)
μ	Parameter of the process modelling the emission rate in the models of Chesney and Taschini (2012) and Grüll and Kiesel (2009)
N	Number of allocated emission allowances in the models of Montgomery (1972b), Rubin (1996), Kling and Rubin (1997), Seifert et al. (2008), Carmona et al. (2009), and Chesney and Taschini (2012)
P	Penalty per unit of emission that is not covered by an allowance at compliance time. Variable is used in the models of Seifert et al. (2008), Carmona et al. (2009), and Chesney and Taschini (2012)

However, it does not have complete knowledge about the net accumulated pollution of others firms:

$$\int_0^{t-1} Q_{I^-,s}\, ds - \delta_{I^-,t-1}.$$

In practice, asymmetric information generates a different information with respect to which the expected net emissions are computed.

Though an explicit form of the equilibrium allowance price is not provided, its numeric evaluation has proven to be unproblematic. In their numerical section, Chesney and Taschini (2012) show that the equilibrium allowance price is sensitive to the different characterizations of the pollution processes ($\mu, \sigma \in R^{\mathcal{I}}$). Not surprisingly, the higher the expected pollution growths, the higher the probability of each firm being in shortage by the end of the trading period. Consequently, the allowance price will also be higher. Similarly, the higher the uncertainty about each single net allowance position before the compliance date, the higher the uncertainty

Table 6.7 Survey on the variables of the different equilibrium models (Part 2)

Variable	Description
$\pi(\cdot)$	Profit/Loss from the production of goods in the models of Montgomery (1972b) and Rubin (1996)
Q	Emission rate (including abatement activities) in the models of Montgomery (1972b), Rubin (1996), Kling and Rubin (1997), and Chesney and Taschini (2012)
q	Expected total cumulative emissions in $[0, T]$ in the model of Seifert et al. (2008)
$q(y)$	Cumulative emissions in $[0, T]$ (excluding uncontrollable emissions) in the model Carmona et al. (2009)
$q_{[t_1,t_2]}$	Total cumulative emissions in $[t_1, t_2]$ in the models of Chesney and Taschini (2012)
$R_{\text{good}}(\cdot)$	Revenues from the production of goods in the models of Montgomery (1972b), Rubin (1996), Kling and Rubin (1997), and Carmona et al. (2009)
S	Permit price in the models of Montgomery (1972b), Rubin (1996), Kling and Rubin (1997), Seifert et al. (2008), and Chesney and Taschini (2012)
σ	Parameter of the process modelling the emission rate in the models of Chesney and Taschini (2012)
T	End of the (compliance) period in the models of Rubin (1996), Kling and Rubin (1997), Seifert et al. (2008), Carmona et al. (2009), Chesney and Taschini (2012)
θ	Number of allowances bought and sold at time t in the models of Montgomery (1972b), Rubin (1996), Kling and Rubin (1997), and Seifert et al. (2008)
Θ	Number of allowances bought and sold until time t in the model of Carmona et al. (2009)
y	Output quantity of the produced good in the models of Montgomery (1972b), Rubin (1996), Kling and Rubin (1997), and Carmona et al. (2009)

about each probability of shortage. Consequently, the allowance price will also be higher, again.

Appendix

This Appendix provides an overview of the techniques used to solve static and dynamic linear optimization problems and some details about the models introduced before.

Solving Static Optimization Problems

Definition 6.19 (Lagrangian) Let $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ and let $f(x), g_1(x), \ldots, g_m(x)$ be functions.

Then the Lagrangian of the following static nonlinear optimization problem

$$\min_{x_1,\ldots,x_n} \{f(x)\} \quad \text{subject to } g_j(x) \leq 0 \quad \text{for } j = 1, \ldots, m$$

is given by

$$L(x,u) = f(x) + \sum_{j=1}^{m} u_j g_j(x).$$

Definition 6.20 (Static convex optimization with non-negative control variables) Let $x = (x_1, \dots, x_{n'}, x_{n'+1}, \dots, x_n)$. Assume that $f(x), g_1(x), \dots, g_m(x)$ are convex functions that are continuously differentiable. Furthermore, assume that there exists $\tilde{x} \in \mathbb{R}^n$ such that $g_j(\tilde{x}) < 0$ holds for all non-linear constraints. We consider the following optimization problem:

$$\begin{array}{lll} \min\limits_{x_1,\dots,x_n} \left\{ f(x) \right\} & & \\ \text{subject to} & g_j(x) \le 0 & \text{for } j = 1, \dots, m \text{ and} \\ & x_i \ge 0 & \text{for } i = 1, \dots, n' \ (n' \le n). \end{array}$$

Theorem 6.21 (Karush–Kuhn–Tucker conditions) $x = (\tilde{x}_1, \dots, \tilde{x}_n)$ *is the optimal solution of the optimization problem of Definition* 6.20 *if and only if there exists* $\tilde{u} \in \mathbb{R}^m$ *such that all the Karush–Kuhn–Tucker conditions are satisfied*:

For $i = 1, \dots, n'$

$$\frac{\partial L}{\partial x_i}(\bar{x}, \bar{u}) = \frac{\partial f}{\partial x_i}(\bar{x}) + \sum_{j=1}^{m} \bar{u}_j \frac{\partial g_j}{\partial x_i}(\bar{x}) \ge 0, \tag{A.1}$$

$$\bar{x}_i \frac{\partial L}{\partial x_i}(\bar{x}, \bar{u}) = \left[\frac{\partial f}{\partial x_i}(\bar{x}) + \sum_{j=1}^{m} \bar{u}_j \frac{\partial g_j}{\partial x_i}(\bar{u}) \right] = 0, \tag{A.2}$$

$$\bar{x}_i \ge 0, \tag{A.3}$$

and for $i = n' + 1, \dots, n$

$$\frac{\partial L}{\partial x_i}(\bar{x}, \bar{u}) = \frac{\partial f}{\partial x_i}(\bar{x}) + \sum_{j=1}^{m} \bar{u}_j \frac{\partial g_j}{\partial x_i}(\bar{x}) = 0, \tag{A.4}$$

and for $j = 1, \dots, m$

$$\frac{\partial L}{\partial u_j}(\bar{x}, \bar{u}) = g_j(\bar{x}) \le 0, \tag{A.5}$$

$$\bar{u}_j \frac{\partial L}{\partial u_j}(\bar{x}, \bar{u}) = \bar{u}_j g_j(\bar{u}) = 0, \tag{A.6}$$

$$\bar{u}_j \ge 0. \tag{A.7}$$

Solving Dynamic Optimization Problems

Theorem 6.22 (Dynamic optimization problem) *Let $T < \infty$ and let f and g be twice differentiable concave functions. Consider the following dynamic, deterministic optimization problem*

$$\max_{x(t)} \left\{ \int_0^T f\big(s(t), x(t), t\big)\, dt \right\} \tag{A.8}$$

$$\textit{subject to} \quad \dot{s}(t) := \frac{\partial s}{\partial t}(t) = g\big(s(t), x(t), t\big) \quad \textit{for } t \in [0, T] \tag{A.9}$$

$$s(0) = 0 \tag{A.10}$$

$$s(T) \geq 0. \tag{A.11}$$

Then the Hamiltonian is defined as

$$H := H\big(s(t), x(t), t\big) = f\big(s(t), x(t), t\big) + u(t) \cdot g\big(s(t), x(t), t\big) \tag{A.12}$$

and the solution of the optimization problem must satisfy

$$\frac{\partial H}{\partial x} = 0, \tag{A.13}$$

$$-\frac{\partial H}{\partial s} = \frac{\partial u}{\partial t} =: \dot{u}, \tag{A.14}$$

$$-\frac{\partial H}{\partial u} = \frac{\partial s}{\partial t} =: \dot{s}, \tag{A.15}$$

$$u(T)s(T) = 0. \tag{A.16}$$

Remark

(a) $x(t)$ is called control variable. The state variable $s(t)$ is influenced by the choice of the control variable.
(b) Equation (A.13) is similar to the condition in a static non-linear optimization problem.
(c) Equation (A.15) restates the condition on the state variable (cf. Eq. (A.9)).
(d) Equation (A.16) is called transversality condition.

Proof 9 (Idea) A rigorous proof can be found in Pontryagin et al. (1962). The following proof is along the lines of Barro and Sala-i Martin (1995).

First, we rewrite the constraint as an integral and set up the Lagrangian function with the continuum of multipliers $u(t)$ for the dynamic constraint and the multiplier v for the terminal condition of the state variable:

$$L = \int_0^T f\big(s(t), x(t), t\big)\, dt + \int_0^T u(t) \cdot \big[g\big(s(t), x(t), t\big) - \dot{s}(t)\big]\, dt + vs(T).$$

Second, integration by parts of

$$\int_0^T u(t)\dot{s}(t)\,dt = \big[u(t)s(t)\big]_0^T - \int_0^T \dot{u}(t)s(t)\,dt$$
$$= u(T)s(T) - u(0)s(0) - \int_0^T \dot{u}(t)s(t)\,dt$$
$$= u(T)s(T) - \int_0^T \dot{u}(t)s(t)\,dt$$

and using the definition of the Hamiltonian yields

$$L = \int_0^T \big[H\big(s(t), x(t), t\big) + \dot{u}(t)s(t)\big]\,dt + \big(v - u(T)\big)\cdot s(T). \tag{A.17}$$

Third, solve the problem using perturbation analysis:

Let $\bar{x}(t)$ be the optimal path for the control variable. The constraint $\dot{s}(t) = g(s(t), x(t), t)$ yields an optimal path for the state variable that we denote by $\bar{s}(t)$. Define the perturbations of the optimal paths by

$$x := x(t) = \bar{x}(t) + \varepsilon p^{(x)}(t), \qquad s := s(t) = \bar{s}(t) + \varepsilon p^{(s)}(t),$$
$$s(T) = \bar{s}(T) + \varepsilon dS(T),$$

where ε is a scalar and $p^{(x)} := p^{(x)}(t)$ and $p^{(s)} := p^{(s)}(t)$ are called perturbation functions. The perturbation analysis is completed by using that near the optimum small perturbations do not affect the maximum value of our optimization problem, that is,

$$\frac{\partial L}{\partial \varepsilon}\big(\bar{s}(t), \bar{x}(t), t\big) = 0. \tag{A.18}$$

Applying the chain rule to Eq. (A.17) yields

$$\frac{\partial L}{\partial \varepsilon} = \int_0^T \left[\frac{\partial H}{\partial s}\cdot\frac{\partial s}{\partial \varepsilon} + \frac{\partial H}{\partial x}\cdot\frac{\partial x}{\partial \varepsilon} + \dot{u}\cdot\frac{\partial s}{\partial \varepsilon}\right]dt + \big(v - u(T)\big)\cdot\frac{\partial s(T)}{\partial \varepsilon}$$
$$= \int_0^T \left[\frac{\partial H}{\partial s}\cdot p^{(s)} + \frac{\partial H}{\partial x}\cdot p^{(x)} + \dot{u}\cdot p^{(s)}\right]dt + \big(v - v(T)\big)\cdot dS(T)$$
$$= \int_0^T \left[\frac{\partial H}{\partial x}\cdot p^{(x)} + \left(\frac{\partial H}{\partial s} + \dot{u}\right)\cdot p^{(s)}\right]dt + \big(v - v(T)\big)\cdot dS(T).$$

Since $\frac{\partial L}{\partial \varepsilon}(s(t), \bar{x}(t), t) = 0$ must hold for any choice of perturbation functions, we obtain

$$\frac{\partial H}{\partial x} = 0, \qquad \frac{\partial H}{\partial s} + \dot{u} = 0, \qquad v = u(T).$$

Combining $v = u(T)$ and $v \cdot s(T) = 0$, the complementary slackness condition from the terminal constraint, yields the so-called transversality condition

$$u(T)s(T) = 0. \qquad \square$$

Relationship Between Optimality Conditions

Lemma 6.23 *A solution of the joint cost minimization problem satisfies the conditions of a market equilibrium.*

Proof 10 Using the conditions given in (6.11) and (6.12) we show that

$$\bar{Q}^i = \bar{Q}^i, \qquad N^t + \bar{\theta}^i - \bar{Q}^i = 0, \qquad \bar{u}_1 = \bar{u} = \bar{S}$$

satisfy the conditions given in (6.7)–(6.10).

Conditions (6.11) and (6.12) imply Eq. (6.7):

Since $\frac{\partial C^i}{\partial Q^i}(\widetilde{Q}^i) + \tilde{u} \leq 0$ for and $\widetilde{Q}^i \geq 0$ all $i = 1, \ldots, n$, it follows from $\sum_{i=1}^n \widetilde{Q}^i [\frac{\partial C^i}{\partial Q^i}(\widetilde{Q}^i) + \tilde{u}] = 0$ that $\widetilde{Q}^i [\frac{\partial C^i}{\partial Q^i}(\widetilde{Q}^i) + \tilde{u}] = 0$ holds for all $i = 1, \ldots, n$. Therefore, $\widetilde{Q}^i$ and $\tilde{u}$ satisfy Eq. (6.7) for all $i = 1, \ldots, n$.

Conditions (6.11) and (6.12) imply Eq. (6.8):

If $\bar{u}_i = \tilde{u} = \bar{S}$, $\bar{S} - \bar{u}_i = 0$ satisfied for all $i = 1, \ldots, n$ by any $\bar{\theta}^i$.

Conditions (6.11) and (6.12) imply Eq. (6.9):

By $\bar{Q}^i = \widetilde{Q}^i$ and $N^i + \bar{\theta}^i - \widetilde{Q}^i = 0$, Eq. (6.9) is satisfied for any $\bar{u}_i$.

Conditions (6.11) and (6.12) imply Eq. (6.10):

$$0 \leq \sum_{i=1}^n \left(N^i - \widetilde{Q}^i\right) \stackrel{N^i + \bar{\theta}^i - \widetilde{Q}^i = 0}{=} -\sum_{i=1}^n \bar{\theta}^i,$$

$$0 = \tilde{u}\left[\sum_{i=1}^n \left(N^i - \widetilde{Q}^i\right)\right] \stackrel{\bar{S} = \tilde{u}}{=} \bar{S}\left[\sum_{i=1}^n \left(N^i - \widetilde{Q}^i\right)\right] \stackrel{N^i + \bar{\theta}^i - \widetilde{Q} = 0}{=} -\bar{S}\sum_{i=1}^n \bar{\theta}^i. \qquad \square$$

Lemma 6.24 *Any emission vector that satisfies the conditions of a market equilibrium is a solution of the joint cost minimization problem.*

Proof 11 Using the conditions given in (6.7)–(6.10) we show that

$$\widetilde{Q}^i = \bar{Q}^i, \qquad \tilde{u} = \bar{S}$$

satisfy the conditions given in (6.11) and (6.12).

Conditions (6.7)–(6.10) imply Eq. (6.11):

By Eq. (6.8), $\bar{u}_i = \bar{S}$. Therefore,

$$-\frac{\partial C^i}{\partial Q^i}(\bar{Q}^i) - \bar{S} \geq 0, \quad \bar{Q}^i \left[\frac{\partial C^i}{\partial Q^i}(\bar{Q}^i) + \bar{S}\right] = 0,$$

which implies $\sum_{i=1}^{n} \bar{Q}^i [\frac{\partial C^i}{\partial Q^i}(\bar{Q}^i) + \bar{S}] = 0$. Therefore, $\bar{Q}^i$ and $\tilde{u} = \bar{S}$ satisfy Eq. (6.11).

Conditions (6.7)–(6.10) imply Eq. (6.12):

By Eq. (6.9) and (6.10), $\sum_{i=1}^{n}(N - \bar{Q}^i) \geq -\sum_{i=1}^{n} \bar{\theta}^t \geq 0$.

By Eq. (6.8), Eq. (6.9) becomes $\bar{S}[N^t + \bar{\theta}^i - \bar{Q}^i] = 0$. By Eq. (6.10),

$$0 = \sum_{i=1}^{n} \bar{S}\left[N^i + \bar{\theta}^i - \bar{Q}^i\right] = \bar{S} \sum_{i=1}^{n} \left(N^i - \bar{Q}^i\right) + \bar{S} \sum_{i=1}^{n} \bar{\theta}^i = \bar{S} \sum_{i=1}^{n} \left(N^i - \bar{Q}^i\right).$$

Therefore, $\bar{Q}^i$ and $\tilde{u} = \bar{S}$ satisfy Eq. (6.12). □

The Solution for a Representative Agent in Seifert et al. (2008)

After showing the equivalence between the market equilibrium and the joint cost problem, Seifert et al. (2008) solves the cost optimization problem of a representative agent as follows.

Let us define q_t, the total cumulative emissions minus the number of allowances bought and sold, as:

Definition 6.25 (Market equilibrium) A market equilibrium with associated optimal strategies, consisting of $\bar{S}_t$, $(\bar{\alpha}_t^1, \ldots, \alpha_t^n)$ and $(\bar{\theta}_t^1, \ldots, \theta_t^n)$, solves the firms' individual cost optimization problems as given in Definition 6.11 and satisfies the market clearing condition

$$\sum_{i=1}^{n} \bar{\theta}_t^i = 0.$$

Definition 6.26 (Global optimization problem) The central planner minimizes joint costs of the firms by choosing optimal abatement strategies:

$$\begin{aligned} &\max_{\alpha_t^1,\ldots,\alpha_t^n} \mathbb{E}\left[-\int_0^T e^{-rt} \sum_{i=1}^{n} \left(\frac{1}{2} c_i \left(\alpha_t^i\right)^2\right) dt - e^{-rT} P \sum_{i=1}^{n} \left(q_T^i - N^i\right)^+\right], \\ &q_t = \mathbb{E}\left[\int_0^t \beta_s \, ds \mid \mathcal{F}_t\right] - \int_0^t \alpha_s \, ds. \end{aligned} \tag{A.19}$$

The dynamics of the total cumulative emissions are given in Lemma 6.27. A characteristic partial differential equation (PDE) and an analytical expression for the allowance price can be found in Theorem 6.29.

Lemma 6.27 (SDE for emissions of the representative agent q_t) *Assume that the emission rate before abatement activities, β_t, follows*

(i) *the White–Noise process* $\beta_t \sim N(\beta_0, \sigma^2)$ *or*
(ii) *the arithmetic Brownian motion* $\beta_t = \beta_0 + \sigma W_t$.

Then the SDE for the cumulative emissions of the representative agent are given by

$$dq_t = -\alpha_t\, dt + H_t\, dW_t,$$

where H_t *is* (*i*) $H_t = \sigma$ *and* (*ii*) $H_t = \sigma(T - t)$.

Proof 12 See online appendix of Seifert et al. (2008). □

Definition 6.28 (Optimization problem of the representative agent) Given the allowance price S, the representative agent minimizes its expected costs by choosing an optimal abatement strategy:

$$\begin{aligned}&\max_{\alpha_t} \mathbb{E}\left[-\int_0^T e^{-rt} C(\alpha_t)\, dt - e^{-rt} P(q_T - N)^+\right]\\ &\quad = \max_{\alpha_t} \mathbb{E}\left[-\int_0^T e^{-rt}\left(\frac{1}{2} c(\alpha_t)^2\right) dt - e^{-rT} P(q_T - N)^+\right].\end{aligned} \tag{A.20}$$

Theorem 6.29 (Permit price dynamics) *Let* $V(t, q_t)$ *be the expected value of an optimal policy for the optimization problem in Definition 6.28 between time* t *and* T.

Denote its partial derivatives by V_t, V_q, V_{qq}.

(a) *Assume that the emission rate before abatement activities is given by the arithmetic Brownian motion in Lemma 6.27.*

Then the characteristic PDE of the allowance price is given by

$$V_t + \frac{1}{2}\sigma^2 (T - t)^2 V_{qq} + \frac{1}{2c} e^{rt} (V_q)^2 = 0,$$

with boundary condition

$$V(T, q_T) = e^{rt} P(q_T - N)^+,$$

and the allowance price is given by

$$S_t = -e^{rt} V_q.$$

(b) *Assuming that the emission rate before abatement activities is given by the White–Noise process in Lemma 6.27. Then the characteristic PDE of the allowance price is given by*

$$V_t + \frac{1}{2}\sigma^2 V_{qq} + \frac{1}{2c} e^{rt} (V_q)^2 = 0,$$

and there is an analytical formula for the allowance price:

$$S(t,q_t) = P \cdot \frac{1}{1 - \frac{\exp\{\frac{-P[P(T-t)+2c(N-q_t)]}{2c^2\sigma^2}\}(-2+\operatorname{erf}c(\frac{N-q_t}{\sigma\sqrt{2(T-t)}}))}{\operatorname{erf}c(\frac{P(T-t)+c(N-q_t)}{\sigma\sqrt{2(T-t)}})}}$$

where $\operatorname{erf}c(x) = 1 - \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt$ *is the complementary error function.*

Proof 13 (Idea) Similar to the proof of Lemma 6.13, Itô's lemma and Lemma 6.27 imply

$$\begin{aligned}
&V(t+dt, q_t+dq_t) - V(t,q_t) \\
&\quad = \left(V_t - \alpha_t V_q + \frac{1}{2}(H_t)^2 V_{qq}\right) dt + H_t V_q\, dW_t, \\
&\mathbb{E}\big[V(t+dt, q_t+dq_t) - V(t,q_t) \mid \mathcal{F}\big] \\
&\quad = V_t dt - \alpha_t V_q\, dt + \frac{1}{2}(H_t)^2 V_{qq}\, dt.
\end{aligned}$$

By the principle of optimality

$$V(t,q_t) = \max_{\alpha_t} \mathbb{E}\big[-e^{-rt} C(\alpha_t)\, dt + V(t+dt, q_t+dq_t)\big]. \tag{A.21}$$

Subtracting $V(t,q_t)$ on both sides of Eq. (A.21) yields

$$0 = \max_{\alpha_t}\left\{-e^{-rt} C(\alpha_t) + V_t - \alpha_t V_q + \frac{1}{2}(H_t)^2 V_{qq}\right\}. \tag{A.22}$$

Maximizing the expression within the curly brackets by deriving it with respect to α_t and setting it to zero yields

$$\alpha_t = -\frac{1}{c} e^{rt} V_q, \quad \text{or} \quad c\alpha_t = -e^{rt} V_q. \tag{A.23}$$

The characteristic PDE is obtained by setting the expression within the curly brackets in Eq. (A.22) to zero and by inserting the formula for α_t into this equation:

$$\begin{aligned}
&-e^{-rt} C(\alpha_t)\, dt + V_t - \alpha_t V_q + \frac{1}{2}(H_t)^2 V_{qq} = 0 \\
&\Leftrightarrow \quad -e^{-rt}\frac{1}{2} c(\alpha_t)^2 + V_t - \alpha_t V_q + \frac{1}{2}(H_t)^2 V_{qq} = 0 \\
&\Leftrightarrow \quad -\frac{1}{2c} e^{rt}(V_q)^2 + V_t + \frac{1}{c} e^{rt}(V_q)^2 + \frac{1}{2}(H_t)^2 V_{qq} = 0 \\
&\Leftrightarrow \quad V_t + \frac{1}{2c} e^{-rt}(V_q)^2 + \frac{1}{2}(H_t)^2 V_{qq} = 0.
\end{aligned} \tag{A.24}$$

The spot price equals marginal abatement costs as shown in Lemma 6.13, that is,

$$S_t = \frac{\partial C(\alpha_t)}{\partial \alpha_t} = c\alpha_t \overset{(3.45)}{=} -e^{-rt} V_q .$$

(a) The proof is completed by using (ii) of Lemma 6.27.
(b) The characteristic PDE follows directly from Eq. (A.24) by using part (i) of Lemma 6.27. A solution of this PDE is derived by Seifert et al. (2008). □

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