IEOR E4004: Optimization Models and Methods

Take-home Mini-Midterm2 Examination

Posted: 6pm (EST), Monday, November 14, 2022
Due: 6pm (EST), Monday, November 21, 2022

November 14-21, 2022 **Professor D. Goldfarb**

YOU MAY CONSULT CLASS LECTURES, THE LITERATURE OR THE INTERNET FREELY, BUT YOU MAY **NOT** OBTAIN HELP FROM ANY OTHER PERSON. If you download any codes from the internet or use codes copied from the internet or the literature, you MUST provide citations to your sources. Your answers to the questions must be well documented and clearly presented. For example, any networks corresponding to a max-flow formulation must be neatly drawn with flows and arc capacities clearly labeled for you to obtain full credit.

This exam concerns the use of **Max-Flow/Min-Cut** problem formulations, and algorithms to solve such problems, to answer some questions about the which soccer team(s) in a **fictitious** league of 6 teams from the 6 Ivy League schools with the smallest undergrad enrollments ($4K - 8K$), Brown (B), Columbia (C), Dartmouth (D), Harvard (H), Princeton (P) and Yale (Y), can end up as the unique champion or tied for the championship given their standing about two-thirds of the way through the season and their remaining schedule of matches.

In a full season, each team plays the other team plays two matches with every other team in the league, one at and one away from home. Hence, each team plays 10 matches. Each match is identified by the first letters of the two teams, e.g., $\begin{pmatrix} B \\ C \end{pmatrix}$. If a match ends in regulation time with a winner, the winner and loser are awarded 3 points and 0 points, respectively. If a match ends in regulation time with a tie, there is a shootout, and the winner and loser are awarded 2 points and 1 point, respectively. *(This point system is not what is currently used in most national and international soccer leagues, but has been used in the past.)* The mini-Ivy-League champion(s) is (are) the team(s) that has (have) been awarded the most points after the completion of the 30 scheduled matches that season. Suppose that after 19 matches have been played the following results about these matches is known.

Table 1.

Matches		Regu-	lation	Shoot	-out		Matches
Teams	Played	Won	Lost	Won	Lost	Points	Left to Play
Y	7	5	2	0	0	15	3
P	6	4	1	1	0	14	4
H	7	3	2	0	2	11	3
D	6	2	2	1	1	9	4
C	6	1	4	1	0	5	4
B	6	1	5	0	0	3	4

Table 2.

Matches	Y P	Y H	Y D	Y C	Y B	P H	P D	P C	P B	H D	H C	H B	D C	D B	C B
Played	1	2	1	1	2	2	1	1	1	1	1	1	2	1	1
Remaining	1	0	1	1	0	0	1	1	1	1	1	1	0	1	1

All of the following questions refer to the data given in Tables 1 and 2. Where a formulation is asked for, you **must** give a **clearly** drawn and labeled diagram of the max-flow network.

Formulate a Maximum Flow Problem that can be used to either verify or contradict each of the following statements:

1. (*20 points*)

   it is possible for B to win or tie for the championship;

2. (*20 points*) **it is possible for C to win the championship**;

3. (*20 points*) **it is possible for Y, P, H, and D to end the season tied for the championship, each having earned 20 points, with C and B tied for last place, each with 5 points.**

   For both Parts 1, 2, and 3 above, you must state what must be true about the solution to your formulation that provides a verification that the statement in bold type is *true* **or a** *false***.** For Parts 1 and 2, recall the application of max-flow to **the baseball elimination problem** in the course slides, and for Part 3, recall the application of max-flow to **network flow feasibility verification problems**.

4. (*15 points*) Write a **code** for solving maximum flow problems that implements the **Edmonds-Karp shortest augmenting path variant of the Ford-Fulkerson (FF) Augmenting Path Method**. (Shortest here means augmenting paths that have the fewest number of arcs, which are identified by searching for augmenting paths from the source node s using breadth-first search.)

5. (*15 points*) Use your code to solve the three max-flow formulations in Parts 1, 2, and 3.

6. (*15 points*) Now that you have verified whether the statements in Parts 1, 2, and 3 are true or false, explain your conclusion in simple terms, referring only to the networks in your Maximum Flow Formulations in Parts 1, 2 and 3, **without solving** for the maximum flow solution. **Note:** Even if you can argue in answer to this Part, why the statements in Parts 1, 2, and 3 are true or false, without formulating a maximum flow problem, you **MUST** provide such a formulation to receive credit for Parts 1, 2, and 3.