Insertion Loss Calibration for A Phase Shifter Network

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Contents

Motivations

- A phase shifter network (PSN) can significantly reduce the number of RF chains in MIMO systems and mitigate strong interference.
- Phased array calibration methods such as rotating element electric field vector (REV)[1], multi-element phase-toggle (MEP) method[2] , etc. are performed in the lab environment (almost noise free).
- For research field, over-the-air (OTA) phased array calibration approaches focus on phase deviations, but they do not consider the insertion loss of the PSN[3].

[1] S. Mano and T. Katagi, "A method for measuring amplitude and phase of each radiating element of a phased array antenna," *Electronics and Communications in Japan*, vol. 65, no. 5, pp. 58–64, Jan. 1982.

[2] G. A. Hampson and A. B. Smolders, "A fast and accurate scheme for calibration of active phased-array antennas," *IEEE Antennas and Propagation Society International Symposium*, vol. 2, Jul. 1999, pp. 1040–1043.

[3] X. Wei, *et al.*, "Calibration of phase shifter network for hybrid beamforming in mmwave massive mimo systems," *IEEE Transactions on Signal Processing*, vol. 68, pp. 2302–2315, 2020.

Contributions

- Introduce a novel PSN insertion loss calibration method under imperfect CSI, which adopts multiple beamformers generated by changing phases of b-bit PSN.
- Derive the Cramer-Rao Bound (CRB) of insertion loss deviation estimations as a benchmark and analyze the lower bound of measurement times.
- Evaluate the estimation performance by showing that the RMSEs of PSN insertion loss estimates can closely approach CRBs.

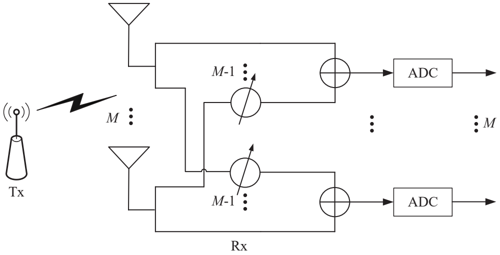
Contents

Signal Model

- In SIMO system, an $M \times M$ PSN is applied in between antennas and analog-to-digital converters (ADCs) at the receiver.
- The received signal at the output of ADCs can be expressed as

$$\mathbf{Y} = \mathbf{E}(\boldsymbol{\alpha})\mathbf{h}\mathbf{x} + \mathbf{Z},$$

where $\mathbf{Y} \in \mathbb{C}^{M\times 1}$ is the received signal; $\mathbf{x} \in \mathbb{C}^{1\times L}$ is the pilot with $|\mathbf{x}(l)| = 1, l = 1, 2 \ldots L$; $\mathbf{E}(\boldsymbol{\alpha}) \in \mathbb{C}^{M\times M}$ is a PSN matrix with $\boldsymbol{\alpha}$ being the insertion loss; $\mathbf{h} \in \mathbb{C}^{M\times 1}$ denotes channel response; $\mathbf{Z} \in \mathbb{C}^{M\times L}$ is i.i.d. Gaussian noise.



Problem Formulation

The effective channel can be written as

$$\breve{\mathbf{h}} = \frac{1}{N}\mathbf{Y}\mathbf{x}^H = \mathbf{E}(\boldsymbol{\alpha})\mathbf{h} + \breve{\mathbf{z}} \in \mathbb{C}^{M\times 1}.$$

Consider a uniform linear array (ULA) of M antenna elements with uniform element spacing $\lambda/2$. The channel response is $\mathbf{h} = \mathbf{a}(\theta)\gamma$, where

$$\mathbf{a}(\theta) = \left[1, e^{-j\pi\sin(\theta)}, \cdots, e^{-j\pi(M-1)\sin(\theta)}\right]^T.$$

Note that the parameters $\boldsymbol{\alpha}, \theta$ and γ are unknown and the PSN matrix

$$\mathbf{E}(\boldsymbol{\alpha}) = \begin{bmatrix} \beta & \alpha_{1,2}e^{j\phi_{1,2}} & \dots & \alpha_{1,M}e^{j\phi_{1,M}} \\ \alpha_{2,1}e^{j\phi_{2,1}} & \beta & \dots & \alpha_{2,M}e^{j\phi_{2,M}} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{M,1}e^{j\phi_{M,1}} & \alpha_{M,2}e^{j\phi_{M,2}} & \dots & \beta \end{bmatrix},$$

β: direct link loss, $\alpha_{p,q}$: insertion loss, $\phi_{p,q}$: phases

Problem Formulation

$$\mathbf{E}(\boldsymbol{\alpha}) = \beta\mathbf{I}_M + \underbrace{\begin{bmatrix} 0 & \alpha_{1,2} & \dots & \alpha_{1,M} \\ \alpha_{2,1} & 0 & \dots & \alpha_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{M,1} & \alpha_{M,2} & \dots & 0 \end{bmatrix}}_{\boldsymbol{\alpha}} \odot \underbrace{\begin{bmatrix} 0 & e^{j\phi_{1,2}} & \dots & e^{j\phi_{1,M}} \\ e^{j\phi_{2,1}} & 0 & \dots & e^{j\phi_{2,M}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\phi_{M,1}} & e^{j\phi_{M,2}} & \dots & 0 \end{bmatrix}}_{\boldsymbol{\Phi}}.$$

- During insertion loss calibration, the transmitter is required to send the pilot to pass through PSN repeatedly, and the receiver applies the training beamformer $\mathbf{E}(\boldsymbol{\alpha})$ generated by randomly changing PSN's phase multiple times for measurements.
- Denote nth beamformer is $\mathbf{E}_n(\boldsymbol{\alpha}) = \beta\mathbf{I}_M + \boldsymbol{\alpha} \odot \boldsymbol{\Phi}_n$.
- The nth effective channel

$$\breve{\mathbf{h}}_n = \mathbf{E}_n(\boldsymbol{\alpha})\mathbf{a}(\theta)\gamma + \breve{\mathbf{z}}_n, n = 1, \dots, N.$$

Problem Formulation

Given $\breve{\mathbf{h}}_n = \mathbf{E}_n(\boldsymbol{\alpha})\mathbf{a}(\theta)\gamma + \breve{\mathbf{z}}_n, \quad \mathbf{E}_n(\boldsymbol{\alpha}) = \beta\mathbf{I}_M + \boldsymbol{\alpha} \odot \boldsymbol{\Phi}_n.$

Stacking $\boldsymbol{\Phi}_n$ and $\breve{\mathbf{z}}_n$ into

$$\hat{\boldsymbol{\Phi}} \triangleq \left[\boldsymbol{\Phi}_1^T, \boldsymbol{\Phi}_2^T, \cdots, \boldsymbol{\Phi}_N^T\right]^T \in \mathbb{C}^{NM\times M}, \quad \hat{\mathbf{z}} \triangleq \left[\breve{\mathbf{z}}_1^T, \breve{\mathbf{z}}_2^T, \cdots, \breve{\mathbf{z}}_N^T\right]^T,$$

$$\hat{\mathbf{h}} \triangleq \left[\breve{\mathbf{h}}_1^T, \breve{\mathbf{h}}_2^T, \cdots, \breve{\mathbf{h}}_N^T\right]^T, \quad \hat{\mathbf{E}} = \mathbf{1}_N \otimes \beta\mathbf{I}_M + (\mathbf{1}_N \otimes \boldsymbol{\alpha}) \odot \hat{\boldsymbol{\Phi}} \in \mathbb{C}^{NM\times M}.$$

The stacked effective channel $\hat{\mathbf{h}} = \underbrace{\hat{\mathbf{E}}\mathbf{a}(\theta)\gamma}_{\tilde{\mathbf{h}}} + \hat{\mathbf{z}} \in \mathbb{C}^{NM\times 1}.$

$$\min_{\boldsymbol{\alpha},\theta,\gamma} \left\|\hat{\mathbf{h}} - \tilde{\mathbf{h}}(\boldsymbol{\alpha},\theta,\gamma)\right\|_2^2 \xrightarrow[\text{Optimization}]{\text{Alternative}} \boldsymbol{\alpha}, \{\theta, \gamma\}$$

Contents

Channel Estimation

- Fix $\boldsymbol{\alpha}$ and optimize γ and θ, where $\tilde{\mathbf{h}} = \hat{\mathbf{E}}\mathbf{a}(\theta)\gamma$

$$\min_{\theta,\gamma} P_0(\theta,\gamma) \triangleq \left\|\hat{\mathbf{h}} - \tilde{\mathbf{h}}(\theta,\gamma)\right\|_2^2 .$$

- Take the first derivative of γ^* and θ,

$$\frac{\partial P_0}{\partial \gamma^*} = \left[\hat{\mathbf{E}}(\boldsymbol{\alpha})\mathbf{a}(\theta)\right]^H \left(\tilde{\mathbf{h}} - \hat{\mathbf{h}}\right),$$

$$\frac{\partial P_0}{\partial \theta} = -2\pi \mathsf{Im}\left\{\left[\hat{\mathbf{E}}(\boldsymbol{\alpha})(\mathbf{a}(\theta) \odot \mathbf{b}(\theta))\gamma\right]^H \left(\tilde{\mathbf{h}} - \hat{\mathbf{h}}\right)\right\}.$$

where $\mathbf{b}(\theta) \triangleq [0, \cos(\theta), 2\cos(\theta), \cdots, (M-1)\cos(\theta)]^T$.

- Letting $\frac{\partial P_0}{\partial \gamma^*} \triangleq 0$ yields

$$\hat{\gamma} = \frac{\left[\hat{\mathbf{E}}(\boldsymbol{\alpha})\mathbf{a}(\theta)\right]^H \hat{\mathbf{h}}}{\left[\hat{\mathbf{E}}(\boldsymbol{\alpha})\mathbf{a}(\theta)\right]^H \left[\hat{\mathbf{E}}(\boldsymbol{\alpha})\mathbf{a}(\theta)\right]}$$

Insertion Loss Estimation

- Fix α and θ, optimize $\boldsymbol{\alpha}$, where $\breve{\mathbf{h}}_n = \mathbf{E}_n(\boldsymbol{\alpha})\mathbf{a}(\theta)\gamma + \breve{\mathbf{z}}_n$,

$$\min_{\boldsymbol{\alpha}} P_1(\boldsymbol{\alpha}) \triangleq \sum_{n=1}^{N} \left\| \breve{\mathbf{h}}_n - \mathbf{E}_n(\boldsymbol{\alpha})\mathbf{a}(\theta)\gamma \right\|_2^2 .$$

- $\mathbf{h}_{\mathsf{eff}_n} \triangleq \breve{\mathbf{h}}_n - \beta\mathbf{h}$, rewrite

$$\mathbf{h}_{\mathsf{eff}_n} = \mathbf{D}_n \boldsymbol{\alpha}_{\mathsf{vec}} + \mathbf{z}_n,$$

where $\mathbf{D}_n = (\mathbf{h}^T \otimes \mathbf{I}_M)\mathsf{diag}(\boldsymbol{\Phi}_{\mathsf{vec}_n}) \in \mathbb{C}^{M\times(M\times M)}$,
$\boldsymbol{\Phi}_{\mathsf{vec}_n} = \mathsf{vec}(\boldsymbol{\Phi}_n), \boldsymbol{\alpha}_{\mathsf{vec}} = \mathsf{vec}(\boldsymbol{\alpha})$,

- Remove the zero columns of $\mathbf{D}_n$ and the zero rows of $\boldsymbol{\alpha}_{\mathsf{vec}}$, and thus $\mathbf{D}_n$ and $\boldsymbol{\alpha}_{\mathsf{vec}}$ become $\bar{\mathbf{D}}_n \in \mathbb{C}^{M\times(M\times M-M)}$, $\bar{\boldsymbol{\alpha}}_{\mathsf{vec}} \in \mathbb{R}^{(M\times M-M)\times 1}$.

$$\min_{\boldsymbol{\alpha}} P_1(\boldsymbol{\alpha}) \triangleq \sum_{n=1}^{N} \left\| \breve{\mathbf{h}}_n - \mathbf{E}_n(\boldsymbol{\alpha})\mathbf{a}(\theta)\gamma \right\|_2^2 \rightarrow \min_{\bar{\boldsymbol{\alpha}}_{\mathsf{vec}}} P_2(\boldsymbol{\alpha}_{\mathsf{vec}}) \triangleq \sum_{n=1}^{N} \left\| \underbrace{\mathbf{h}_{\mathsf{eff}_n} - \bar{\mathbf{D}}_n \bar{\boldsymbol{\alpha}}_{\mathsf{vec}}}_{p_n} \right\|_2^2$$

Insertion Loss Estimation

- Take the derivative

$$\frac{\partial p_n}{\partial \bar{\boldsymbol{\alpha}}_{\text{vec}}} = \left[\bar{\mathbf{D}}_n^H \bar{\mathbf{D}}_n + \left(\bar{\mathbf{D}}_n^H \bar{\mathbf{D}}_n\right)^T\right] \bar{\boldsymbol{\alpha}}_{\text{vec}} - \left[\left(\mathbf{h}_{\text{eff}_n}^H \bar{\mathbf{D}}_n\right)^T + \bar{\mathbf{D}}_n^H \mathbf{h}_{\text{eff}_n}\right].$$

  Note that $\left[\bar{\mathbf{D}}_n^H \bar{\mathbf{D}}_n + \left(\bar{\mathbf{D}}_n^H \bar{\mathbf{D}}_n\right)^T\right]$ is singular.

- Stacking $\bar{\mathbf{D}}_n$ and $\mathbf{h}_{\text{eff}_n}$ into

$$\hat{\mathbf{D}} \triangleq \left[\bar{\mathbf{D}}_1^T, \bar{\mathbf{D}}_2^T, \cdots, \bar{\mathbf{D}}_N^T\right]^T, \ \hat{\mathbf{h}}_{\text{eff}} = \left[\mathbf{h}_{\text{eff}_1}^T, \mathbf{h}_{\text{eff}_2}^T, \cdots, \mathbf{h}_{\text{eff}_N}^T\right]^T$$

$$\hat{\mathbf{h}}_{\text{eff}} = \hat{\mathbf{D}} \bar{\boldsymbol{\alpha}}_{\text{vec}} + \hat{\mathbf{z}},$$

$$\min_{\bar{\boldsymbol{\alpha}}_{\text{vec}}} P_2(\boldsymbol{\alpha}_{\text{vec}}) \triangleq \sum_{n=1}^{N} \left\|\mathbf{h}_{\text{eff}_n} - \bar{\mathbf{D}}_n \bar{\boldsymbol{\alpha}}_{\text{vec}}\right\|_2^2 \to \min_{\bar{\boldsymbol{\alpha}}_{\text{vec}}} \left\|\hat{\mathbf{h}}_{\text{eff}} - \hat{\mathbf{D}} \bar{\boldsymbol{\alpha}}_{\text{vec}}\right\|_2^2.$$

- Hence, the estimated value of insertion loss can be determined by

$$\boxed{\hat{\boldsymbol{\alpha}}_{\text{vec}} = \left[\hat{\mathbf{D}}^H \hat{\mathbf{D}} + \left(\hat{\mathbf{D}}^H \hat{\mathbf{D}}\right)^T\right]^{-1} \left[\left(\mathbf{h}_{\text{eff}}^H \hat{\mathbf{D}}\right)^T + \hat{\mathbf{D}}^H \mathbf{h}_{\text{eff}}\right].}$$

Cramer-Rao Bound

Fisher Information Matrix (FIM)

$$\boldsymbol{F} = \frac{2}{\sigma_z^2} \sum_{n=1}^{N} \mathrm{Re}\left[\frac{\partial \boldsymbol{\mu}_n^H(\boldsymbol{\eta})}{\boldsymbol{\eta}} \frac{\partial \boldsymbol{\mu}_n(\boldsymbol{\eta})}{\boldsymbol{\eta}} \right],$$

$$\frac{\partial \boldsymbol{\mu}_n(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} = \left[\frac{\partial \boldsymbol{\mu}_n(\boldsymbol{\eta})}{\partial \boldsymbol{\mathcal{A}}}, \frac{\partial \boldsymbol{\mu}_n(\boldsymbol{\eta})}{\partial \theta}, \frac{\partial \boldsymbol{\mu}_n(\boldsymbol{\eta})}{\partial \mathrm{Re}\{\gamma\}}, \frac{\partial \boldsymbol{\mu}_n(\boldsymbol{\eta})}{\partial \mathrm{Im}\{\gamma\}} \right].$$

$$\boldsymbol{\eta} = \left[\boldsymbol{\mathcal{A}}^T, \theta, \mathrm{Re}\{\gamma\}, \mathrm{Im}\{\gamma\} \right]^T,$$

$$\boldsymbol{\mathcal{A}} = [\alpha_{2,1}, \cdots, \alpha_{M,1}, \alpha_{1,2}, \cdots, \alpha_{M-1,M}] \in \mathbb{R}^{(M\times M-M)},$$

$$\breve{\mathbf{h}}_n \sim \mathcal{CN}\left(\boldsymbol{\mu}_n, \sigma_z^2 \mathbf{I}_M\right), \boldsymbol{\mu}_n = \mathbf{E}_n \mathbf{a}(\theta)\gamma = (\boldsymbol{\alpha} \odot \boldsymbol{\Phi}_n)\, \mathbf{a}(\theta)\gamma \in \mathbb{C}^{M\times 1}.$$

The first $M \times M - M$ diagonal elements in $\boldsymbol{F}^{-1}$ are the CRBs of $\boldsymbol{\mathcal{A}}$.

Minimum Number of Measurements

- Unknown parameters to be estimated

$$\boldsymbol{\eta} = \left[\boldsymbol{\mathcal{A}}^T, \theta, \mathrm{Re}\{\gamma\}, \mathrm{Im}\{\gamma\}\right]^T,$$

- Received signal

$$\breve{\mathbf{h}}_n = \mathbf{E}_n(\boldsymbol{\alpha})\mathbf{a}(\theta)\gamma + \breve{\mathbf{z}}_n \in \mathbb{C}^{M\times 1}, n = 1, \ldots, N.$$

- The number of unknowns in $\boldsymbol{\eta}$ is $M^2 - M + 3$. Since each measurement of $\breve{\mathbf{h}}_n$ can provide $2M$ equations. To ensure the feasibility of insertion loss estimation, the number of required measurements N needs to satisfy

$$2MN \geq M^2 - M + 3,$$

which yields

$$N \geq \left\lceil \frac{M}{2} + \frac{3}{2M} - \frac{1}{2} \right\rceil.$$

Contents

Convergence

Parameter	Value
Degree of arrival (DoA)	$\theta \sim U(-90^\circ, 90^\circ)$
Direct link loss	$\beta = -2$ dB
Insertion loss	$\boldsymbol{\alpha} \sim \mathcal{N}(-11, 9)$ dB
Complex gain γ	Rician factor=2
Pilot length	100
Monte-Carlo simulations	1000
RMSE$_{\boldsymbol{\alpha}}$	$\frac{\sqrt{\mathbb{E}\left[\|\hat{\boldsymbol{\alpha}}-\bar{\boldsymbol{\alpha}}_{\text{vec}}\|_2^2\right]}}{M^2-M}$

Table 1: Simulation settings

Estimation error:$=\left\|\hat{\mathbf{h}} - \tilde{\mathbf{h}}(\boldsymbol{\alpha}, \theta, \gamma)\right\|_2^2$

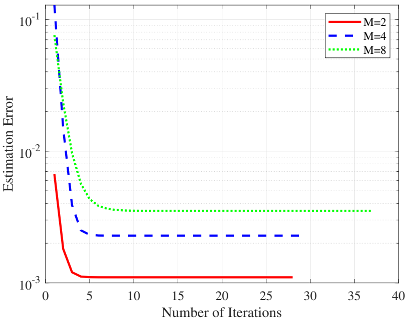


Figure 1: The convergence of proposed algorithm under a different number of antennas when SNR = 20dB, N = 8.

RMSE vs SNR/Number of measurements

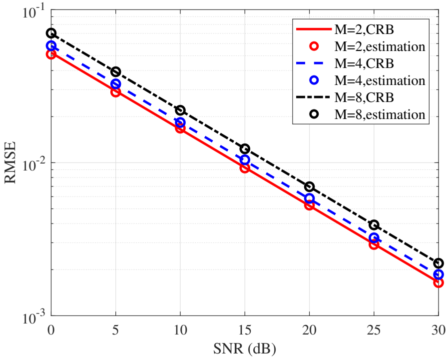


Figure 2: RMSE vs SNR, N=8

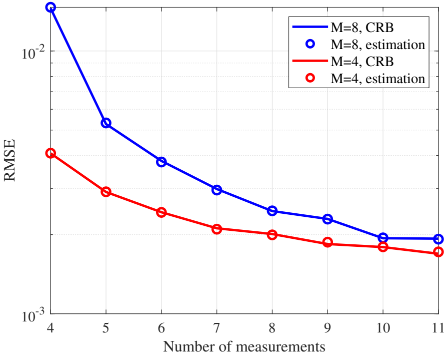


Figure 3: RMSE vs Number of measurements, SNR=20 dB

Hybrid interference mitigation

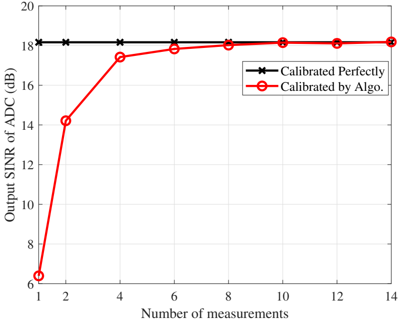


Figure 4: 4×4 6-bit PSN, SNR=20dB, ($N = 1$ represents uncalibrated PSN).

- A hybrid interference mitigation method[1] was proposed to cancel strong interferences before they reach ADCs by using a PSN for analog prewhitening.
- Two interferences where the noise power of each interference is 40dB stronger than the signal of interest.

[1] W. Zhang *et al.*, "Hybrid Interference Mitigation Using Analog Prewhitening," *IEEE Transactions on Wireless Communications*, vol. 20, no. 10, pp. 6595-6605, Oct. 2021.

Q & A

Thanks for your attention!

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