1 Problem

Given a directed graph $G = (V, E)$ and one sequence of constraints $P = (L \times L)^k$ of length k, where L is a set of labels and $|L| = |V|$, we want to find a sequence of sequences of adjacent mappings $S = \langle F^1, F^2, \ldots, F^k \rangle$ minimizing the total cost, such that all constraints $p_i \in P$ are satisfied by the mapping $F^i_{m_i}$ ($m_i = |F^i|$).

2 Adjacent Mapping

First of all, a mapping is a bijective function $f : V \to L$ that maps the vertices of G into the labels L.

From the definition above, given two mappings f and f', we say that they are adjacent iff there is exactly one edge $(u, v) \in E$ such that $f(u) = f'(v)$, $f(v) = f'(u)$ and $\forall w \in V \setminus \{u, v\}(f(w) = f'(w))$.

3 Constraints

A constraint is defined by an ordered pair (a, b) such that $a, b \in L$. We say that a mapping f satisfies a constraint (a, b) iff there is an edge $(u, v) \in E$ such that $f(u) = a$ and $f(v) = b$.

4 Output

We seek a sequence of sequences of adjacent mappings $S = \langle F^1, F^2, \ldots, F^k \rangle$, such that its total cost is minimized.

Consider that the operation of transition from one mapping to its adjacent mapping is called swap operation. Each $F^i \in S$ is a sequence of adjacent mappings $\langle f^i_0, f^i_1, \ldots, f^i_{m_i} \rangle$ from an initial mapping (f^i_0) to a final mapping ($f^i_{m_i}$), that should satisfy the i-th constraint. In other words, $m_i - 1$ represents the number of swap operations required in order to satisfy constraint p_i from mapping $f^i_0 = f^{i-1}_{m_{i-1}}$ (note that $f^1_0 = f^0$ – an initial mapping).

Also, given the i-th constraint of P $p_i = (a, b)$, if $f^i_{m_i}$ does not satisfy p_i, then it should satisfy $p'_i = (b, a)$, using a reverse operation.

The total cost of such sequence is given by the sum of all swap and reverse operations. Given that the swap and reverse operation costs C_{swap} and C_{rev}, respectively, and the function h (Equation 1), the total cost is (Equation 2):

$$h(f, i) = \begin{cases} 0, & \text{if } f \text{ satisfies } p_i \\ C_{rev} \end{cases} \tag{1}$$

$$C_{total} = \sum_{i=1}^{k} |F^i| \cdot C_{swap} + h(f^i_{m_i}, i) \tag{2}$$