ECSE 362 - Fundamentals of Power Engineering

Lab 1: Three Phase Circuits and Power

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Lab section: 003

Introduction

The experiment is divided into two distinct segments. The initial segment is dedicated to an introductory exploration of three-phase circuits. This encompasses understanding the relationship between voltages and currents in these circuits, familiarization with Y and Delta configurations, and mastering the computation of power in three-phase systems. The subsequent segment delves deeper into the intricacies of three-phase power and its measurement techniques. This includes determining apparent, active, and reactive power in three-phase circuits, as well as the computation of the power factor.

Analysis

PART I:

1. Line voltage

d) The line voltages are measured and shown in the figure below using metering window of the data acquisition software.

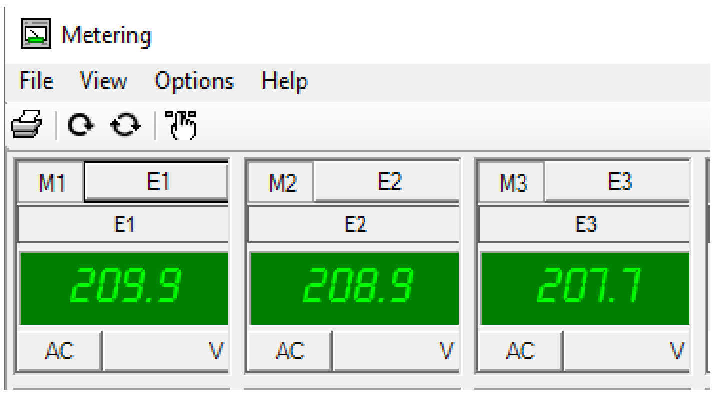


Figure 1: Recorded Line Voltage

$E_{45}^{rms} =$ 209.9 V

$E_{56}^{rms} =$ 208.9 V

$E_{64}^{rms} =$ 207.7 V

h) Line voltages are displayed using the oscilloscope window of the data acquisition module software. The peak amplitude and the phase shift between the three voltages are shown below with E_{45} as the reference signal. Horizontal cursors were used to measure the peak-to-peak value while the vertical cursors were used to measure the time difference of the two curves. The calculation of phase shifts ($\Delta\theta$) is shown below using the equation, knowing the frequency f and time difference between two voltages peaks Δt :

$$\Delta\theta = 360 \times \Delta t \times f$$

Peak voltages are derived from half of the value of the peak-to-peak voltages measured using horizontal cursor.

$PS = 360° \cdot td \cdot f$

$\Rightarrow \angle(E_{45}, E_{56})$
$= 360° \cdot (5.01 \times 10^{-3}) \cdot 60.02$
$= 121.2°$

$\Rightarrow \angle(E_{45}, E_{64})$
$= 360 \cdot (-5.46 \times 10^{-3})(60.02)$
$= -118.0°$

$\Rightarrow \angle(E_{56}, E_{64})$
$= 360(5.72 \times 10^{-3})(60.02)$
$= 123.6°$

Figure 2: Peak Amplitude and Phase Shift between the three Voltages

$\angle(E_{45}, E_{56}) =$ 121.2° $E_{45}^{peak} =$ 306.53 V

$\angle(E_{45}, E_{64}) =$ -118.0° $E_{56}^{peak} =$ 306.53 V

$\angle(E_{56}, E_{64}) =$ 123.6° $E_{64}^{peak} =$ 306.53 V

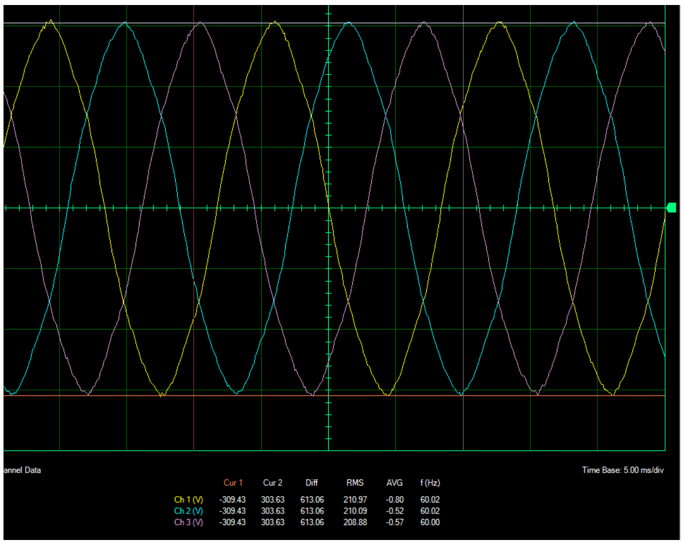


Figure 3: Peak-to-Peak Voltages

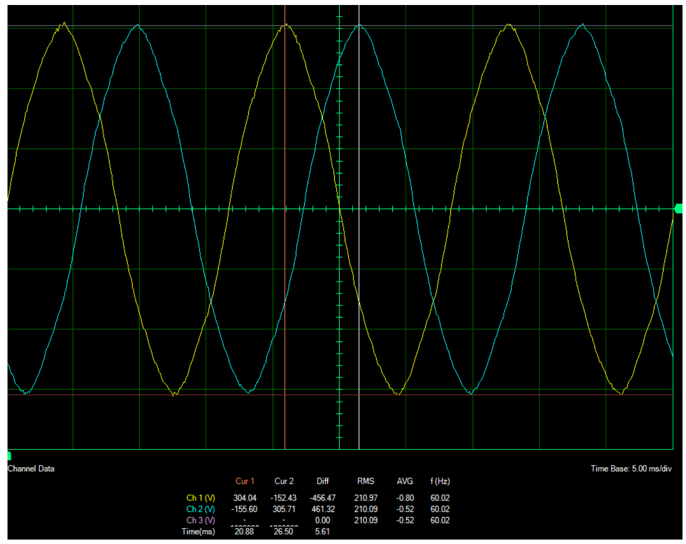


Figure 4: Phase Shift E45, E56

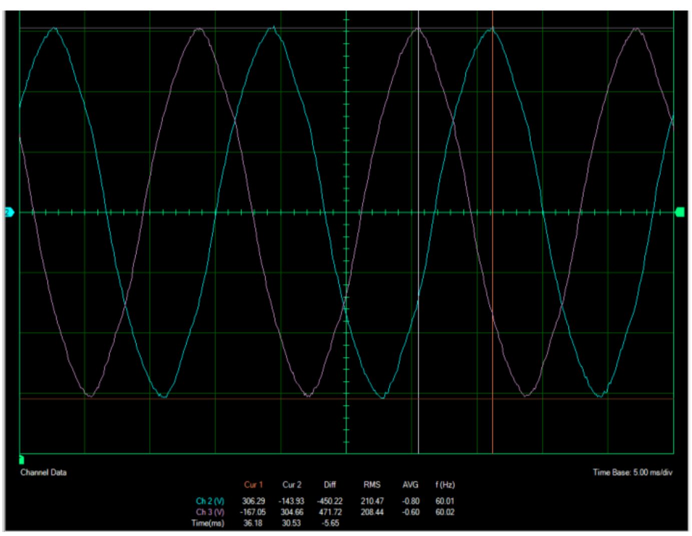


Figure 5: Phase Shift E45 E64

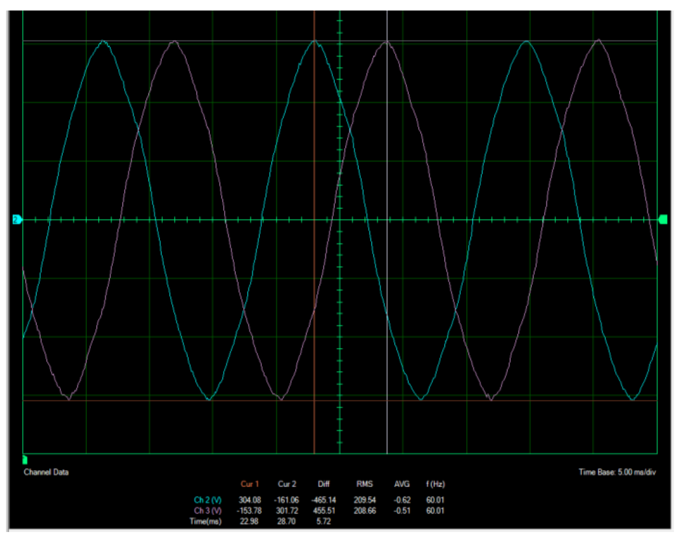


Figure 6: Phase Shift E56 E64

l) The voltage magnitudes and phase shifts between the three voltages are shown below with E_{45} as the reference phasor. It is measured by the phasor analyzer window using the data acquisition module software.

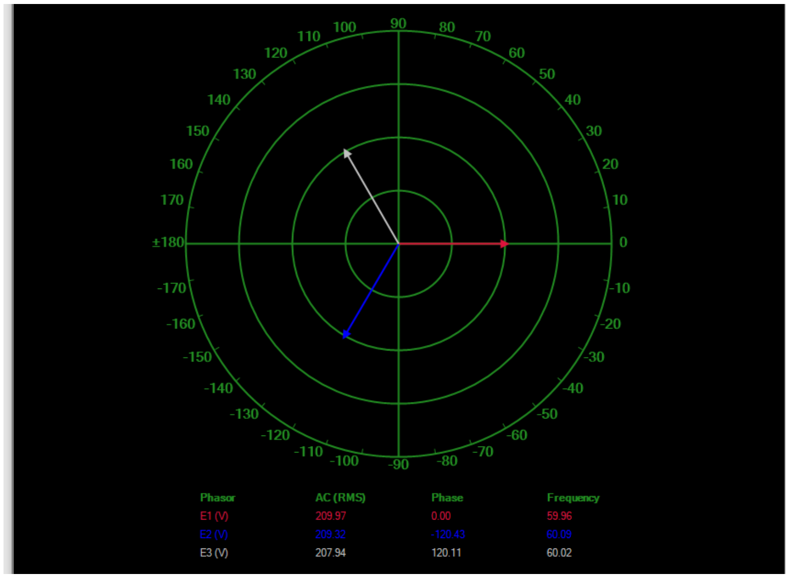


Figure 7: Voltages rms magnitude, phase, and frequency using the phase analyzer.

m) Three-line voltages are balanced three phase quantities. As shown in the figure above, the three generated voltages have the same amplitude and are phase shifted 120 degrees from each other. They also have the same frequency of ~60Hz.

2. Phase voltage

c) The rms phase voltages are measured and shown below:

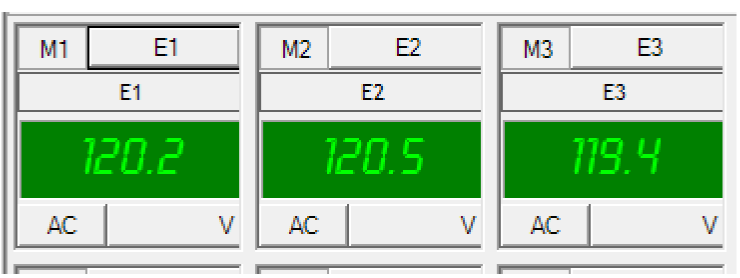


Figure 8: rms phase voltages

$E_{4N}^{rms} =$ 120.2 V

$E_{5N}^{rms} =$ 120.5 V

$E_{6N}^{rms} =$ 119.4 V

g) The peak-to-peak voltages were measured using the horizontal cursors. The peak voltages are derived from half of the peak-to-peak voltages. The time difference was measured using the vertical cursors. The calculations of phase shift are shown in Figure 9 below. It was derived using the equation:

$$\Delta\theta = 360 \times \Delta t \times f$$

where $\Delta\theta$ is the phase shift, frequency $f = 60Hz$, and time difference between two voltages peaks is Δt.

$\angle(E_{4N}, E_{5N})$
$= 360(5.51\times10^{-3})(59.94)$
$= 119.0^\circ$

$\angle(E_{4N}, E_{6N})$
$= 360(-5.71\times10^{-3})(59.97)$
$= -123.27^\circ$

$\angle(E_{5N}, E_{6N})$
$= 360(5.51\times10^{-3})(59.95)$
$= 119.0^\circ$

$\angle(E_{4N}, E_{5N}) =$ 119.0° $E_{4N}^{peak} =$ 166.04 V

$\angle(E_{4N}, E_{6N}) =$ -123.27° $E_{5N}^{peak} =$ 166.04 V

$\angle(E_{5N}, E_{6N}) =$ 119.0° $E_{6N}^{peak} =$ 166.04 V

Figure 9: Peak Amplitude and Phase Shift between the three Voltages

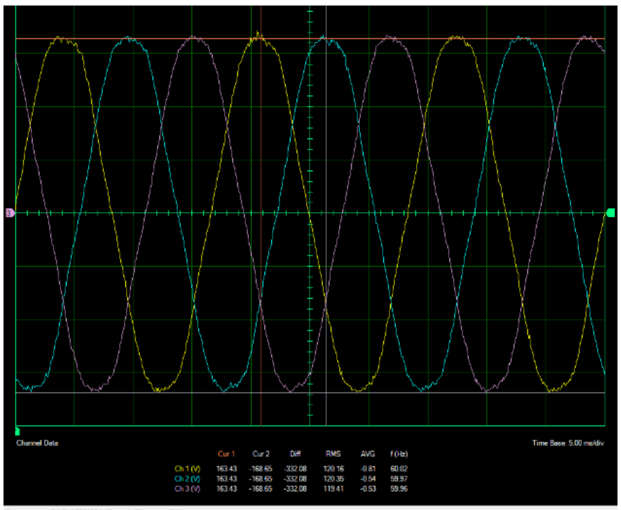


Figure 10: Peak-to-peak voltage amplitude

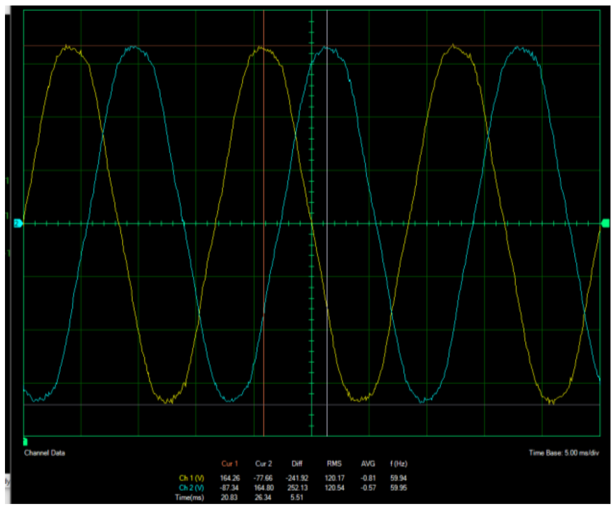


Figure 11: Phase Shift E4N E5N

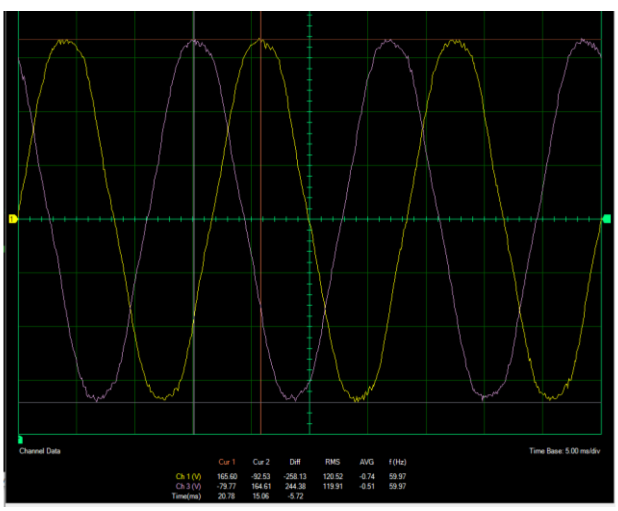


Figure 12: Phase Shift E4N, E6N

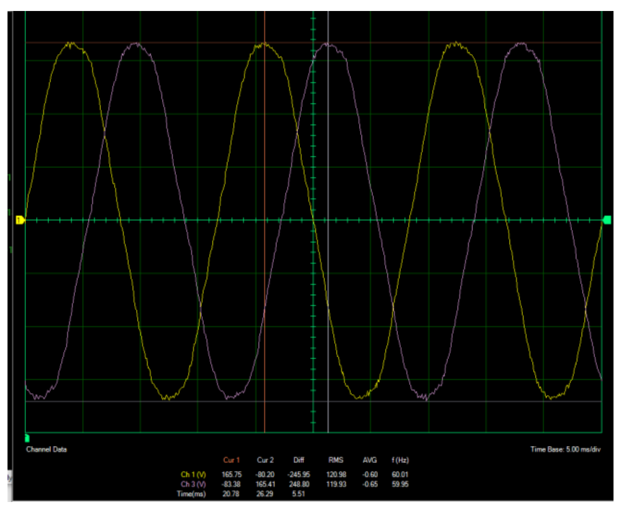


Figure 13: Phase Shift E5N, E6N

k) The magnitude and phase shifts of three voltages are shown in the figure below by using the phasor analyzer window of the data acquisition module software.

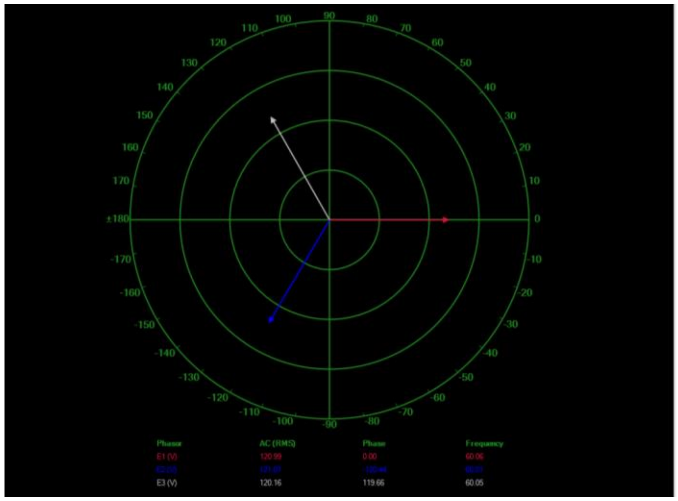


Figure 14: Voltage magnitude and phase shifts between three voltages

l) They are balanced three phase quantities. The phase difference between reference angle and any of the two angles is 120 degrees. The RMS values are equal.

m) Yes, the ratio is approximately $\sqrt{3}$.

$$\frac{E_{avg}^{l}}{E_{avg}^{p}} = \frac{\frac{209.9 + 208.9 + 207.7}{3}}{\frac{120.2 + 120.5 + 119.4}{3}} = \frac{208.8}{120.0} = 1.74 \approx \sqrt{3}$$

p) The RMS amplitude of the voltages and the phase shifts between them are measured using oscilloscope window of the data acquisition module. The calculation of phase shifts uses the following equation:
Phase Shift = 360 * Time Difference * Frequency.

$E_{4N}^{rms} =$ 121.39V $\quad E_{45}^{rms} =$ 210.87V

360(−1.31×10⁻³)·59.97

= −28.3

$\angle(E_{4N}, E_{45}) =$ −28.3°

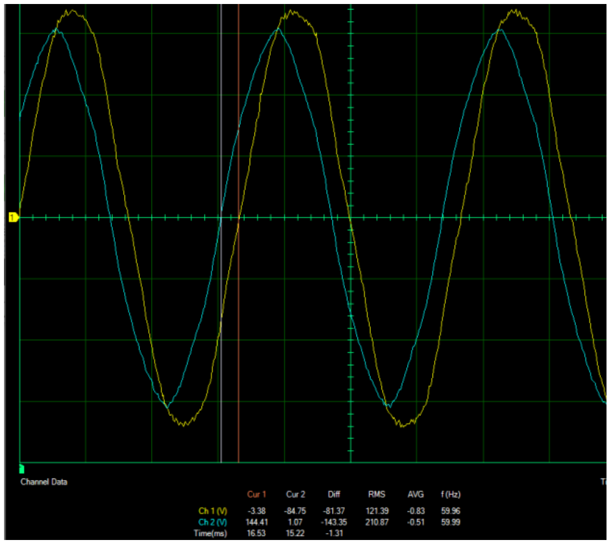


Figure 15 Amplitude of the Voltages and Phase Shifts between E4N and E5N

t) Using the phasor analyzer window, the voltage magnitude and phase shifts are shown in the figure below. The phase voltage lags the line voltage by 29.72 degrees. The line voltage is bigger than the phase voltage by factor of $\sqrt{3}$.

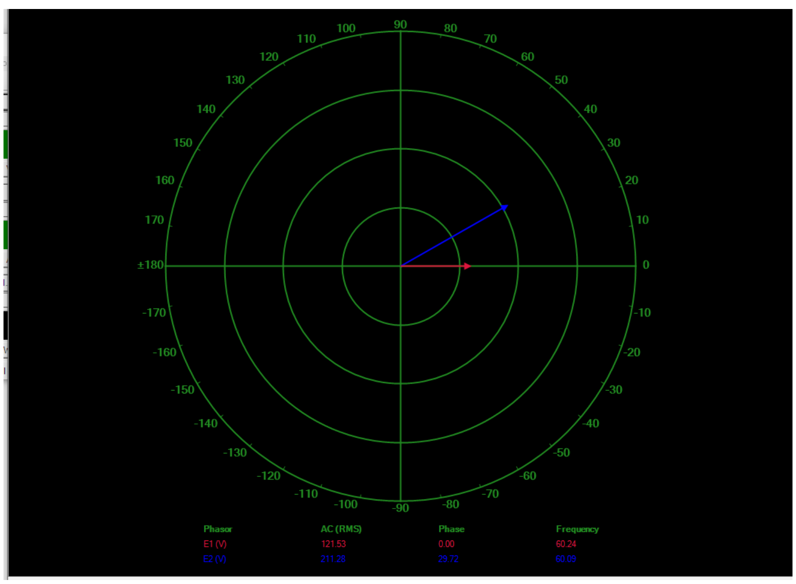
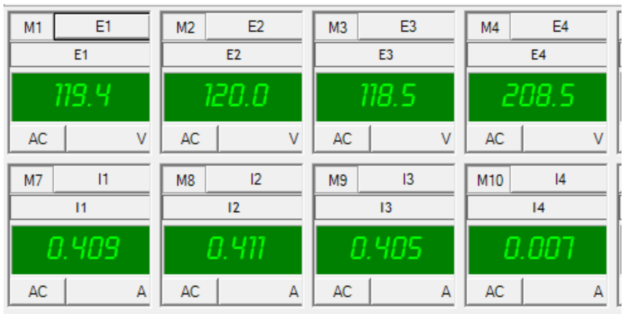


Figure 16: Voltage magnitude and Phase Shift of E4N and E45

u) Yes, the phase voltage is lagging by approximately 30 degrees. The measured value is 29.72 degrees.

3. Y connection

d)



$E_1 =$ 119.4 V $I_1 =$ 0.409 A

$E_2 =$ 120.0 V $I_2 =$ 0.411 A

$E_3 =$ 118.5 V $I_3 =$ 0.405 A

Figure 17: RMS Voltages and Currents across R1, R2, R3

e) The magnitudes and phase shifts between three voltages are measured using the phasor analyzer window as shown in the figure below. Three voltages have the same magnitude with minor discrepancies possibly due to age of wiring. The voltage has a difference of 120 degrees with reference voltages. It displays characteristics of balanced quantities.

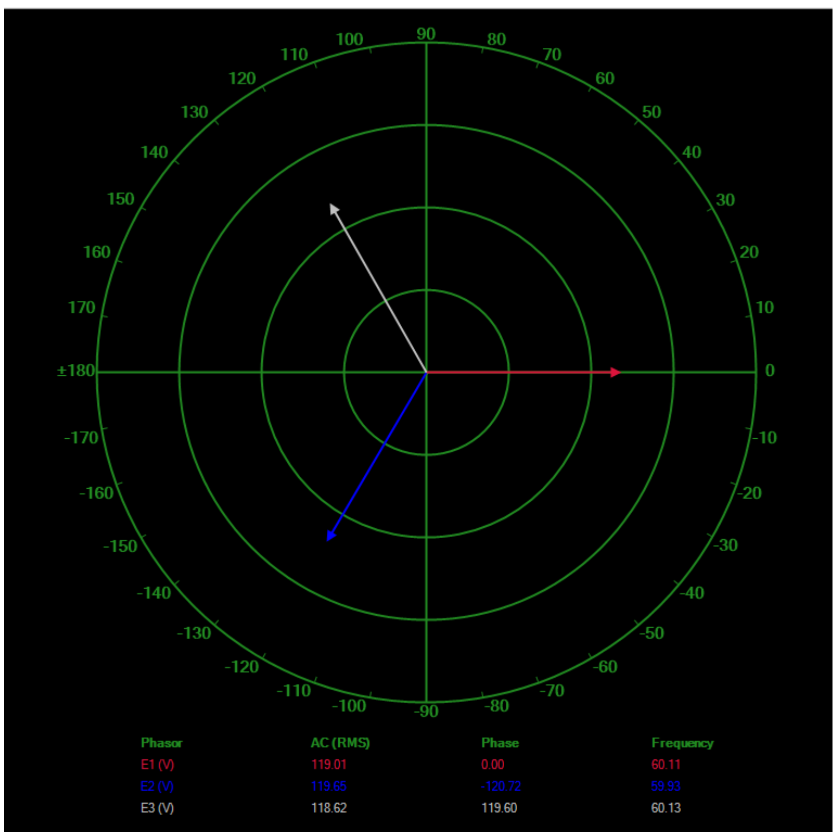


Figure 18: Magnitude and Phase Shifts between Three Voltages

f) The magnitudes and phase shifts between three currents are shown in the figure below. The three currents present same magnitudes with minor discrepancies possibly due to age of wires. The current has a difference of 120 degrees with the reference current. It displays balanced quantity characteristics.

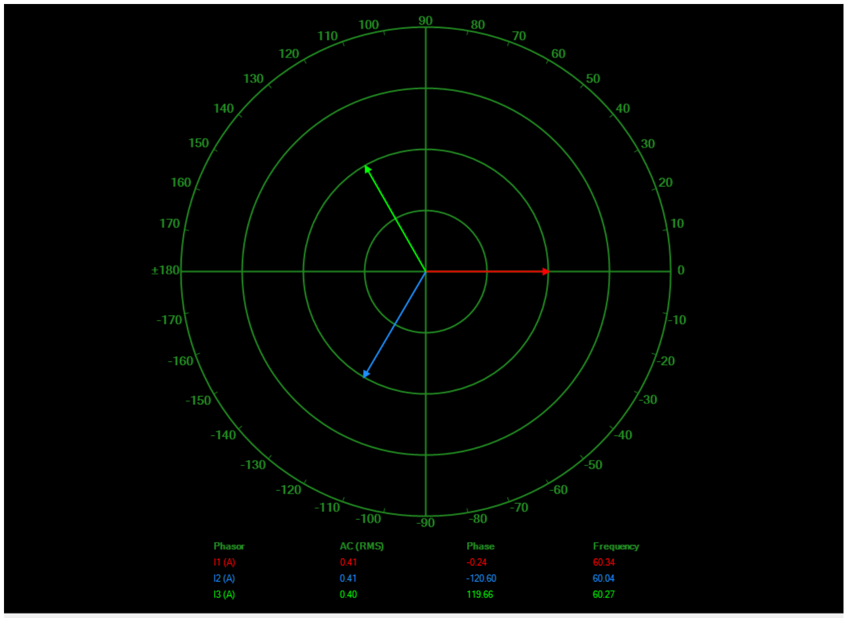


Figure 19: Magnitude and Phase Shifts between Three Currents

h) Yes, they are well balanced. Both current and voltages shows equal magnitude and evenly spaced angles.

i)

$$P_1 = 119.4 \times 0.409 = 48.835 \text{ W}$$

$$P_2 = 120.0 \times 0.411 = 49.320 \text{ W}$$

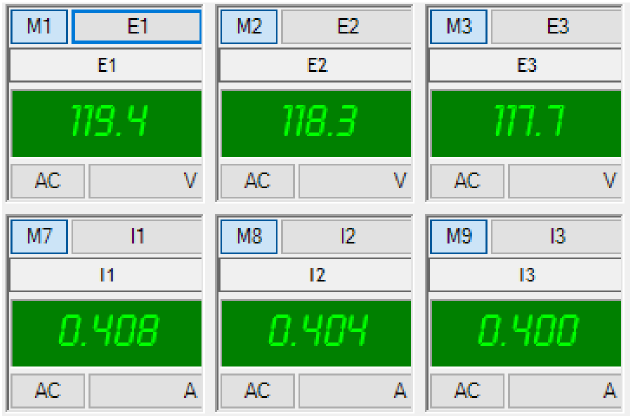
$$P_3 = 118.5 \times 0.405 = 47.993 \text{ W}$$

j)

$$P_T = P_1 + P_2 + P_3 = 146.148 \text{ W}$$

4. Δ connection

d) The rms voltage across and the rms current through the load resistance R1, R2, and R3 of Δ circuits are shown below using the metering window of the data acquisition software.



$E_1 =$ 119.4 V $I_1 =$ 0.408 A

$E_2 =$ 118.3V $I_2 =$ 0.404 A

$E_3 =$ 117.7V $I_3 =$ 0.400 A

Figure 20: Δ-connected RMS Voltage and Current of Load Resistance

e) Voltages E_1, E_2, and E_3 are shown in the figure below using the phasor analyzer window of data acquisition module software with E_1 as the reference phasor. Their magnitudes are approximately the same. E_2 and E_3 both exhibits a phase difference of roughly 120 degrees with the reference.

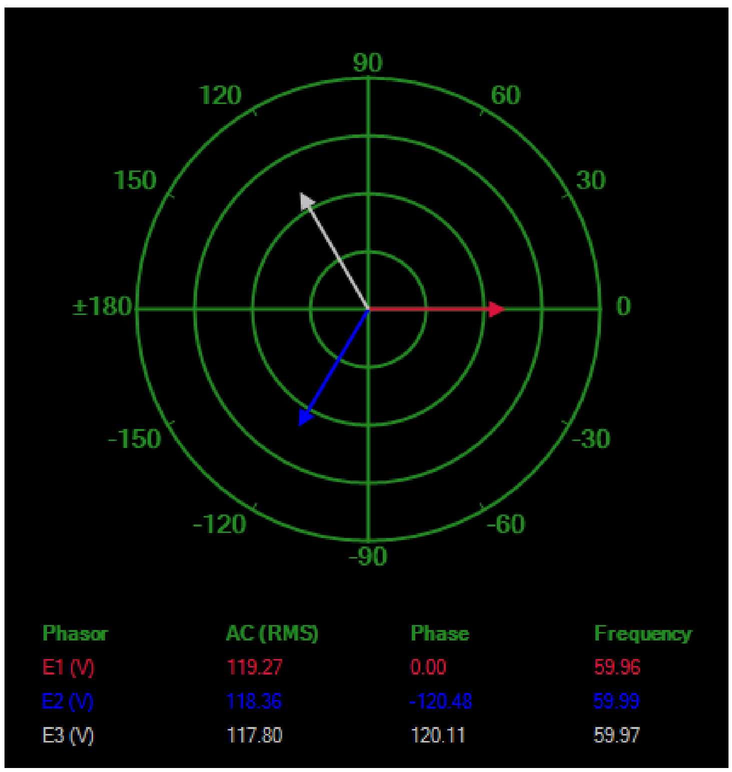


Figure 21: Voltage E1, E2, E3 using Phasor Analyzer Window

f) Current I_1, I_2, and I_3 are measured using the phasor analyzer window with I_1 as the reference. They exhibit approximately the same magnitude around 0.4. In addition, phase of I_2 and I_3 differ with reference by 120 degrees.

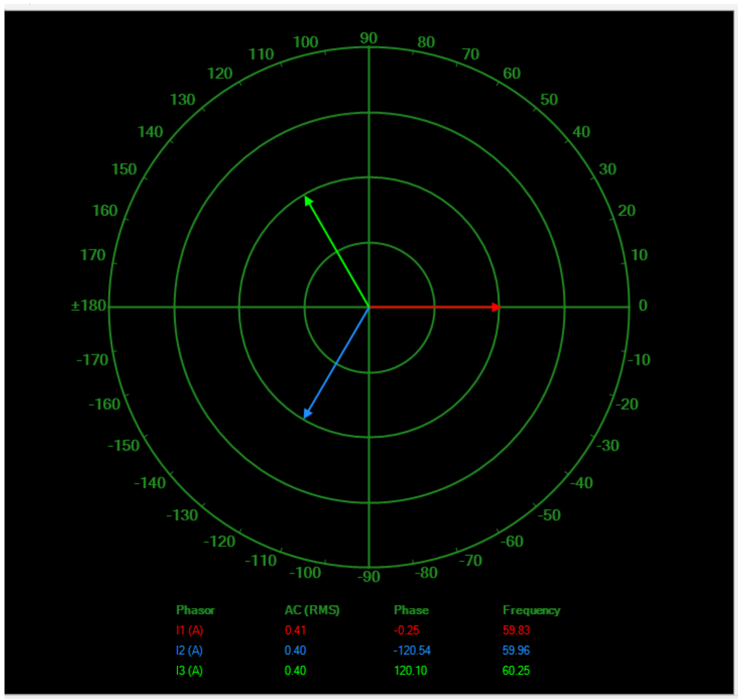


Figure 22: Current I1, I2, I3 using Phasor analyzer window

h) Yes. The current and voltages are well balanced. As mentioned before, they both exhibit balanced quality characteristics. The quantities have approximately the same RMS value and the phase has a difference of 120 degrees with the reference value.

k) Line currents are measured using the phase analyser (Figure 23 below):

$I_4 =$ 0.70 A

$I_5 =$ 0.70 A

$I_6 =$ 0.69 A

l) With I_4 as the reference phasor, currents, I_4, I_5, and I_6 are obtained using phasor analyzer window as shown below. I_4 is displayed as I_1, I_5 is displayed as I_2, and I_6 is displayed as I_3.

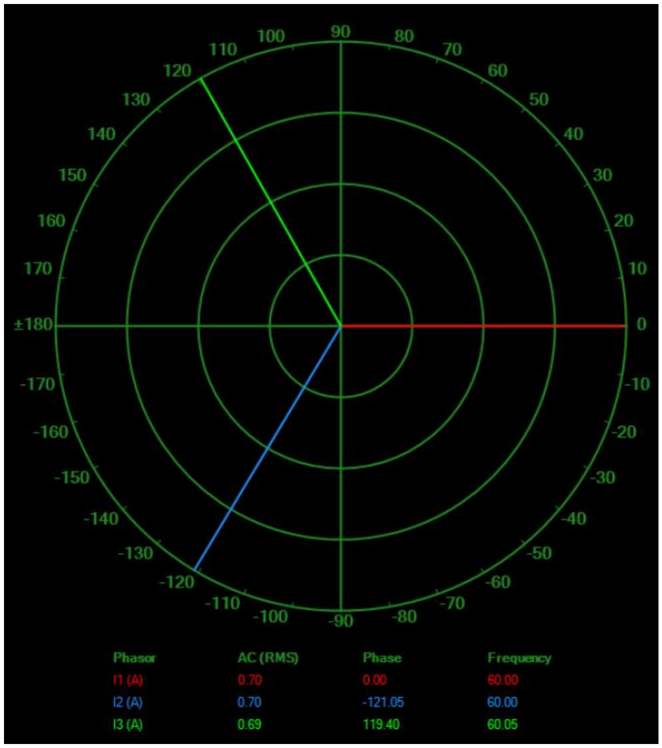


Figure 23: Currents I4, I5, and I6 using Phasor Analyzer Window

o) Ammeters measuring currents I_5 and I_6 are replaced with short circuits. The new current measurement I_7 (I_2 on the figure below) and reference current I_4 are measured using the phasor analyzer window. The current has a phase shift of 30 degrees with reference and the magnitude is smaller by a factor of $\sqrt{3}$.

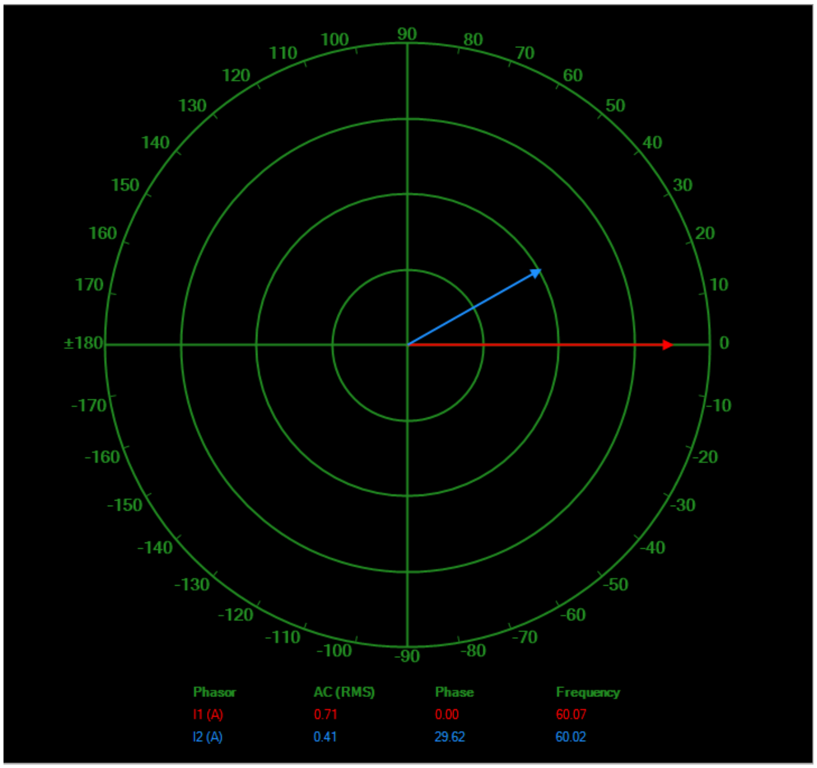


Figure 24 I4 and I7 Measurement Using Phasor Analyzer

p)

$$\frac{I_{avg}^{l}}{I_{avg}^{p}} = \frac{(0.70 + 0.70 + 0.69)/3}{(0.408 + 0.404 + 0.400)/3} = 1.724 \approx \sqrt{3}$$

q) Yes, the ratio is approximately $\sqrt{3}$

r) Relation between line and phase current in a Δ-connection is: $I_l \approx I_p\sqrt{3}\angle - 30°$

s)

$P = |I_p|^2 R$

$P_1 =$ $0.41^2 \cdot 300 = 50.43$ W

$P_2 =$ $0.40^2 \cdot 300 = 48.00$ W

$P_3 =$ $0.40^2 \cdot 300 = 48.00$ W

t)

$P_T =$ $P_1 + P_2 + P_3 = 146.43$ W

5. Questions

a) In a Y connected circuit, if the line voltage is 346 V, what is the phase voltage?

$$I_p = \frac{I_L}{\sqrt{3}}\angle 30° = \frac{346}{\sqrt{3}}\angle 30° = \mathbf{199.8\angle 30°\ (A)}$$

b) In a Δ connected circuit, the current is 20A in each resistance load. What is the line current?

$$I_L = \sqrt{3}\angle -30° I_p = \mathbf{34.64\angle -30°\ (A)}$$

c) In a Y connected circuit, the current is 10A in each resistance load. What is the line current?

<u>Answer:</u> The line current remains the same to be 10A in each resistance load.

d)

$P_{total} = 3kW \quad P_{single} = \frac{3}{3} = 1kW$

$P = \frac{V^2}{Z} \Rightarrow V_p = \sqrt{PZ} = \sqrt{1000 \cdot 10} = 100\ V \quad V_L = \sqrt{3} \cdot 100 = 173.2V$

e)

i. What is the line current?

$I_p = \frac{V}{Z} = 440/11 = 40\ A$

$I_L = 40 \cdot \sqrt{3} = 69.28\ A$

ii. What is the total three-phase power?

$P = \frac{3V^2}{R} = 3 \cdot \frac{440^2}{11} = 52.8\ KW$

PART II:

1. Reactive Power Measurement

d) The rms voltages and currents through the three inductive loads L_1, L_2, and L_3 are shown below.

$E_1 =$ 119.1 V $\quad I_1 =$ 0.298 A

$E_2 =$ 119.6 V $\quad I_2 =$ 0.298 A

$E_3 =$ 117.8 V $\quad I_3 =$ 0.296 A

e) Phasor analyzer window is used to display the voltages E_1, E_2, E_3, and currents I_1, I_2, I_3.

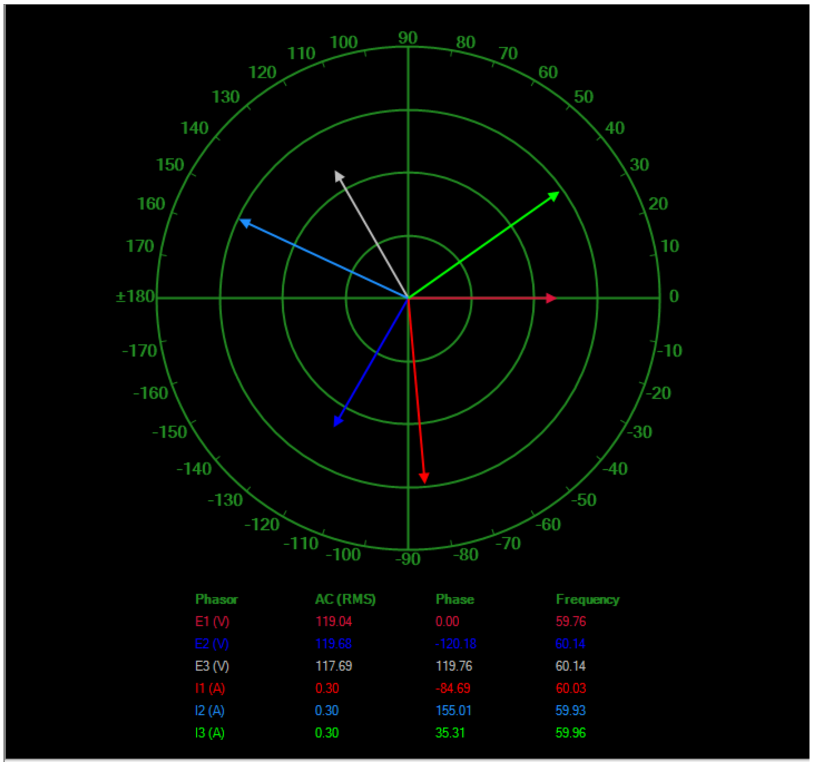


Figure 25: Voltages and Currents of inductors displayed by Phasor Analyzer Window

(g) What is the average phase shift between the phase voltages and phase currents?

$$\angle(E, I) = \underline{\quad 35.3° \quad}$$

(h) What is the average value of the phase current?

$$I^p_{avg} = \underline{\quad 0.297 \text{ A} \quad}$$

(i) What is the value of the phase voltage?

$$E^p_{avg} = \underline{\quad 118.83 \text{ V} \quad}$$

(j) Calculate the reactive power for each of the inductive loads.

$Q = E \times I$

$$Q_1 = \underline{119.1 \times 0.298 = 35.49 \text{ var}}$$

$$Q_2 = \underline{119.6 \times 0.298 = 35.64 \text{ var}}$$

$$Q_3 = \underline{117.8 \times 0.296 = 34.86 \text{ var}}$$

(k) Calculate the three-phase reactive power.

$$Q_T = \underline{Q_1 + Q_2 + Q_3 = 105.99 \text{ var}}$$

2. Active and Inductive reactive Power Measurement

d) The rms voltage across the three inductive loads L1, L2, and L3 are shown below.

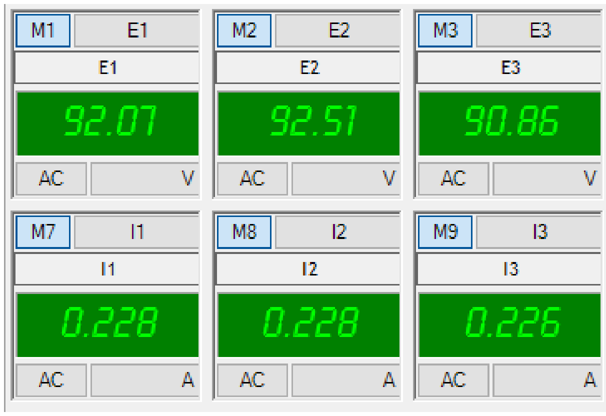


Figure 26: rms voltage and current across the three inductive loads

g) Phase voltage across the three resistive loads R1, R2, and R3 are shown below.

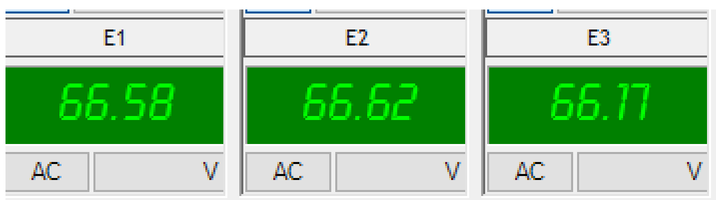


Figure 27: rms voltages across the three resistive loads

i) Phasor analyzer window is used to display the voltage and current across three loads. Across the same load, voltage and current have the same phase. Both voltage and current are three phase balanced quantities and are at a 120-degree phase difference with the reference.

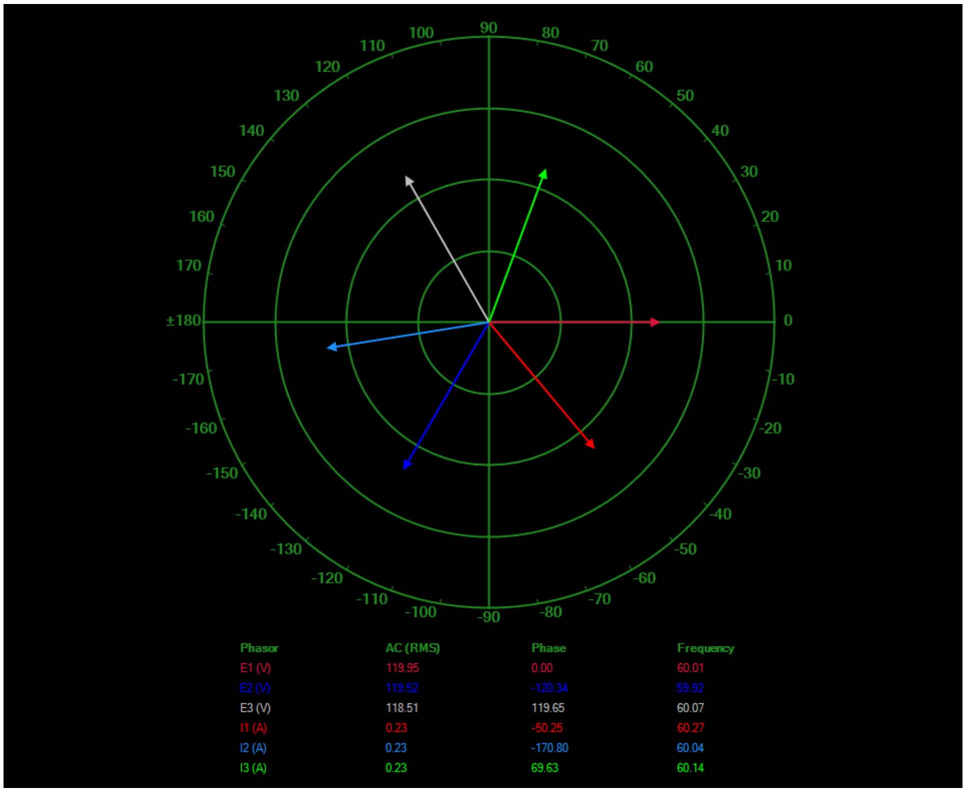


Figure 28: Phasor analysis of voltages and currents across the three loads

j) The phase shift between the line voltages and line currents are:

$$\angle(E_{45}, I_1) = -50.25°$$

$$\angle(E_{56}, I_2) = -50.46\ °$$

$$\angle(E_{64}, I_3) = -50.02°$$

Hence, we get:

$$\angle(E, I) = \frac{\sum_{n=1}^{3} \angle(E_n, I_n)}{3} = -50.24°$$

k) From (d) and (g), we have:

Calculate the active power dissipated in the three resistors using the results of (d) and (g).

E_4 66.58 $\times$ I_1 0.228 $=$ 15.18 W

E_5 66.62 $\times$ I_2 0.228 $=$ 15.19 W

E_6 66.17 $\times$ I_3 0.226 $=$ 14.95 W

The total active power is $P_T =$ 15.18 + 15.19 + 14.95 = 45.32 W l)

E_1	92.07	$\times$	I_1	0.228	$=$	20.99 var
E_2	92.51	$\times$	I_2	0.228	$=$	21.09 var
E_3	90.86	$\times$	I_3	0.226	$=$	20.53 var

The total reactive power is $Q_T =$ 20.99 + 21.09 + 20.53 = 62.61 var.

m) From (j), we have $\angle(E, I) = -50.24°$. Hence, we can derive power factor as

$$pf = \cos(-50.24°) = 0.64\ lagging$$

n) From (k) and (l), we get:

$$S_T = \sqrt{45.32^2 + 62.61^2} = 77.29\ (VA)$$

o) Using the line current and line voltage: $V_{line} = 208\ V_{ac}$ and $I_{line} = I_{ammeter}$

$$S_T = \sqrt{3} V_{line} I_{line} = \sqrt{3} \times 208 \times 0.228 = 82.14\ (VA)$$

p) The apparent power in (n) is slightly off from the total apparent power in (o).

q) Using the values calculated above:

$$pf = \frac{P}{S} = \frac{45.32}{77.29} = 0.59\ lagging$$

Compared to (m) the values very close.

r)

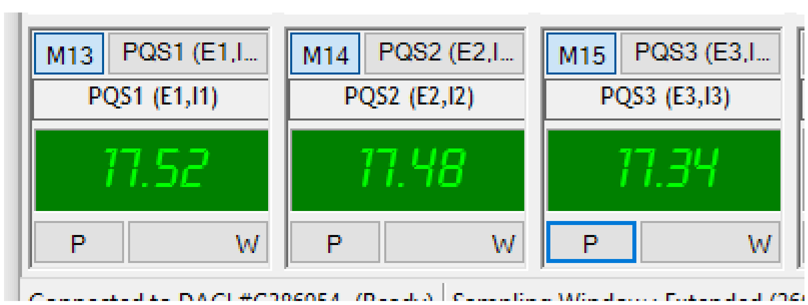


Figure 29: Active power generated

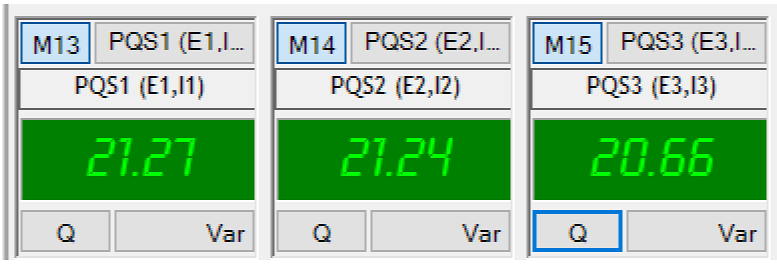


Figure 30: Reactive power generated

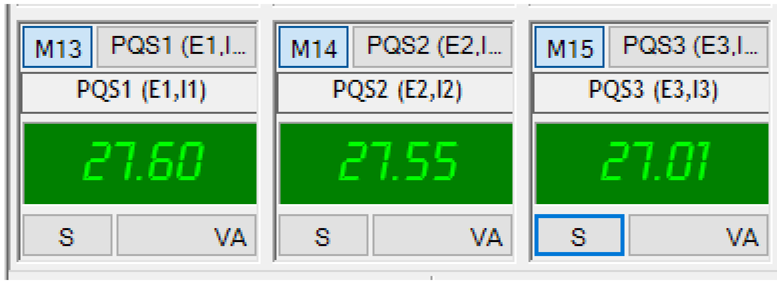


Figure 31: Total apparent power generated

Based on the above pictures, we summed up the active power P to be 52.33 W, the reactive power Q to be 63.17 Var, and the apparent power to be 82.16 VA. The results are very close.

3. Active and Capacitive reactive Power measurement

d)

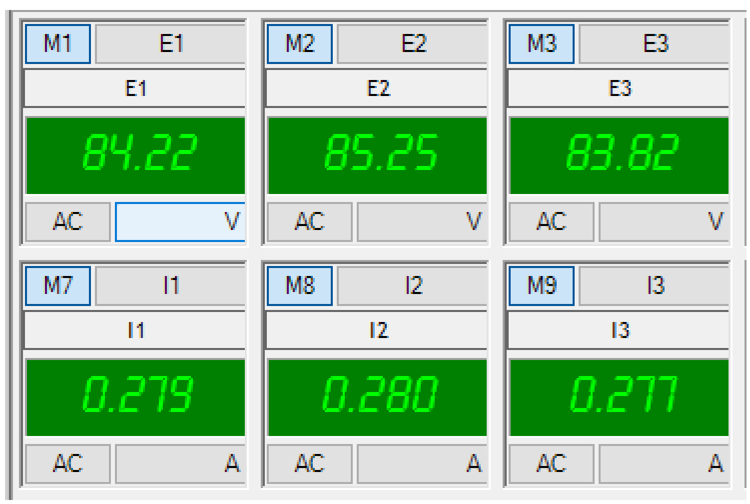


Figure 32: Voltages across and currents through the three capacitive loads

g)

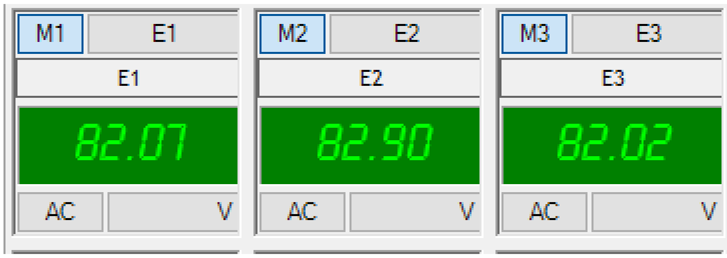


Figure 33 Voltages across the three resistive loads R1, R2, and R3

i)

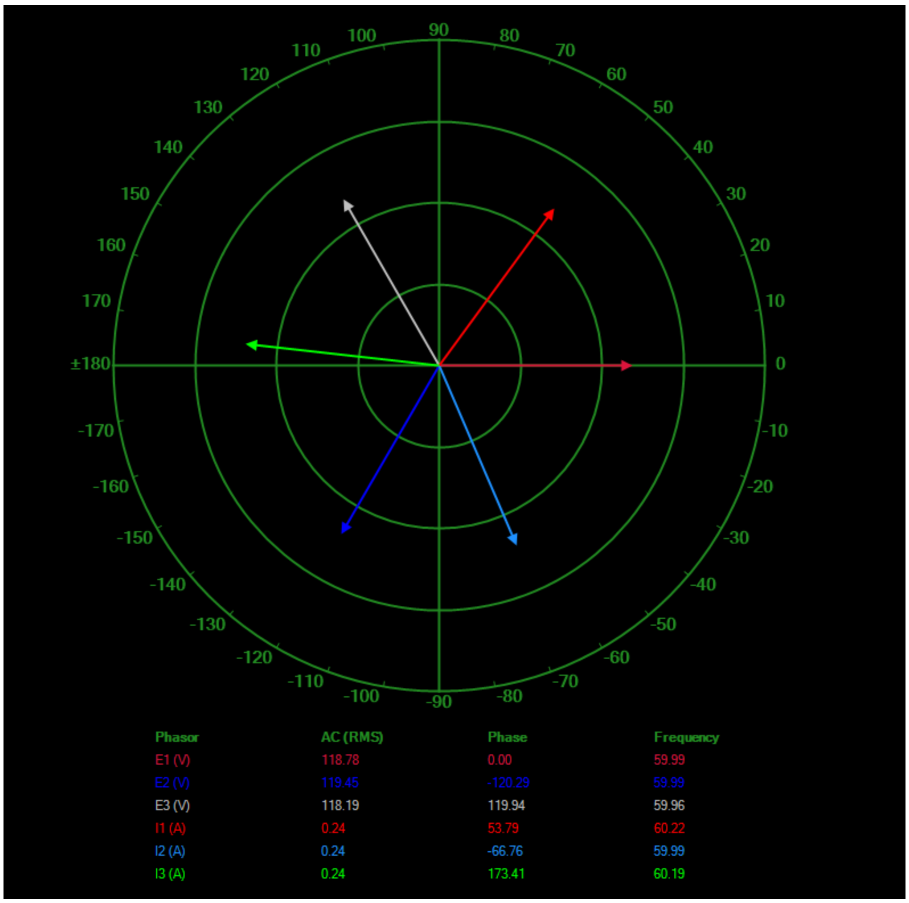


Figure 34: Voltages across the three loads (resistors and capacitors) and the line currents

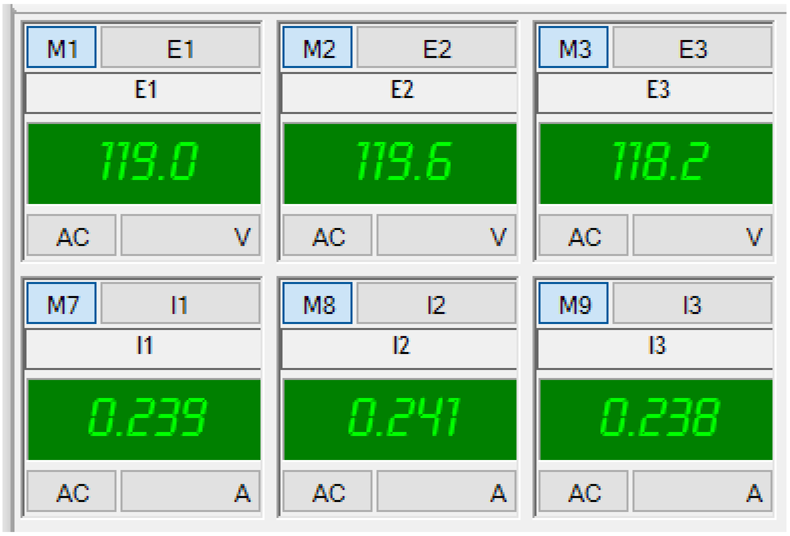


Figure 35 RMS Voltages and Magnitudes

j) The phase shifts between the line voltages and line currents are:

$$\angle(E_{45}, I_1) = 53.79°$$

$$\angle(E_{56}, I_2) = 53.53\ °$$

$$\angle(E_{64}, I_3) = 53.47°$$

Take the average, we get:

$$\angle(E, I) = 53.60°$$

k)

E_4 82.07 $\times$ I_1 0.279 = 22.90W

E_5 82.90 $\times$ I_2 0.280 = 23.21W

E_6 82.02 $\times$ I_3 0.277 = 22.72W

The total active power is P_T = 22.90+23.21+22.72 = 68.83 W.

l)

E_1	84.22	$\times$ I_1	0.279	=	-23.50 var
E_2	85.25	$\times$ I_2	0.280	=	-23.87 var
E_3	83.82	$\times$ I_3	0.277	=	-23.22 var

The total reactive power is $Q_T =$ -23.5 +(-23.87)+(-23.22) = -70.59 var

m) From (j), we have $\angle(E, I) = 53.60°$. Hence, we can derive power factor as

$$pf = \cos(53.60°) = 0.59 \ \ leading$$

n) From (k) and (l), we get:

$$S_T = 98.59\, VA$$

o) Using the line current and line voltage values:

$$S_T = \sqrt{3} * 208 * 0.280 = 100.87\ (VA)$$

p) The apparent power in (n) is … to total apparent power in (o).

q)

$$pf = \frac{P}{S} = \frac{68.83}{98.59} = 0.698\ leading$$

Compared to m the value is very close.

r)

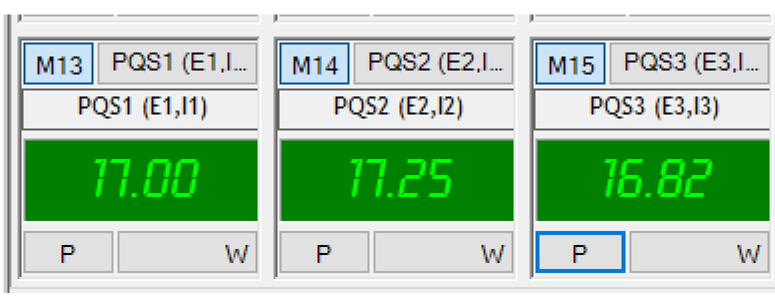


Figure 36 Active Power

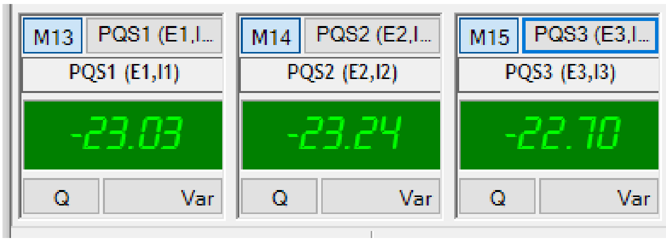


Figure 37 Reactive Power

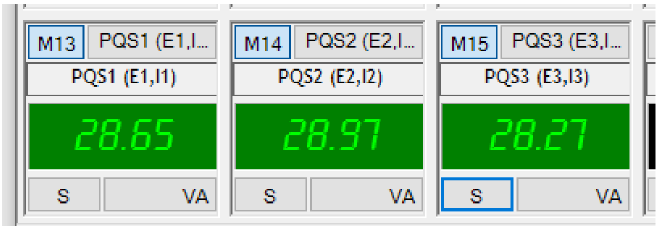


Figure 38 Apparent Power

Based on the above pictures, we summed up the active power P to be 51.07 W, the reactive power Q to be -69.13 Var, and the apparent power to be 85.89 VA. The results are very close.

s) Questions

(s-i) Question 1:

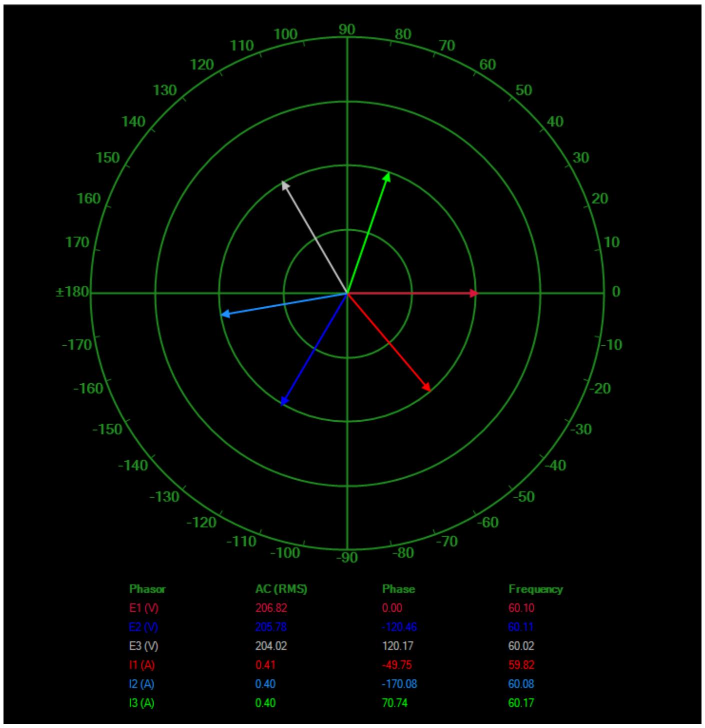


Figure 39: Phase analyser of a Δ-connected RL loads

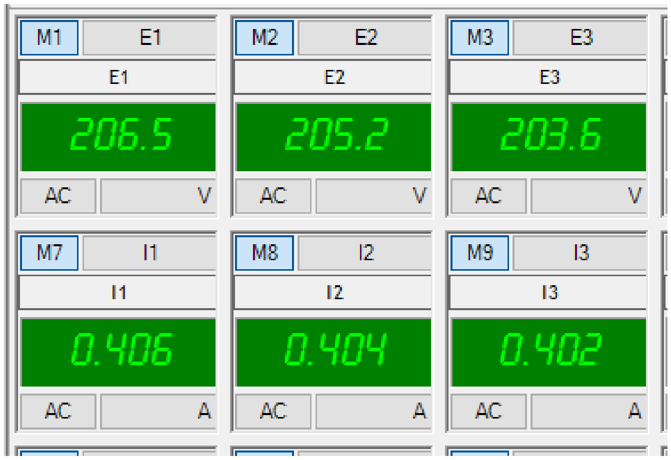


Figure 40: RMS Voltages and Currents

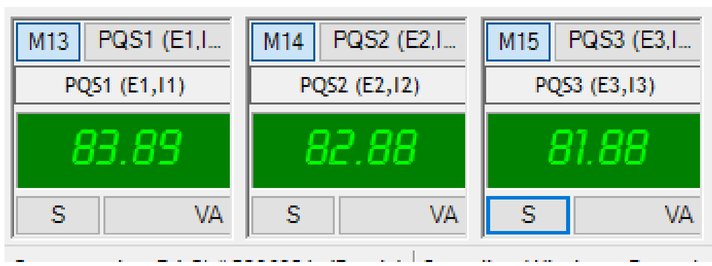


Figure 41: Apparent Power

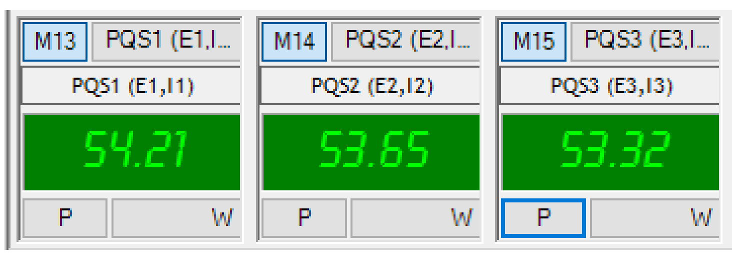


Figure 42: Active Power

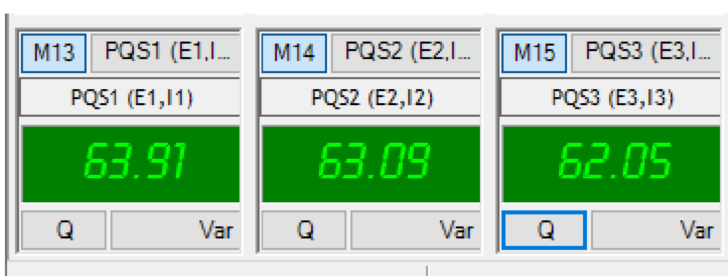


Figure 43: Reactive power generated by the source

By delta connecting the loads, the line voltage and phase voltage would be equal to 208 V. It is $\sqrt{3}$ times bigger than before (roughly 120V in Y connection). Power is proportional to the square of voltage. Therefore, the power in delta connection would be 3 times bigger than Y connection. As shown from the pictures above, the voltage is indeed very close to 208 V and the active, reactive, and apparent power fed by the source are three times bigger than the previous section.

(s-ii) Question 2:

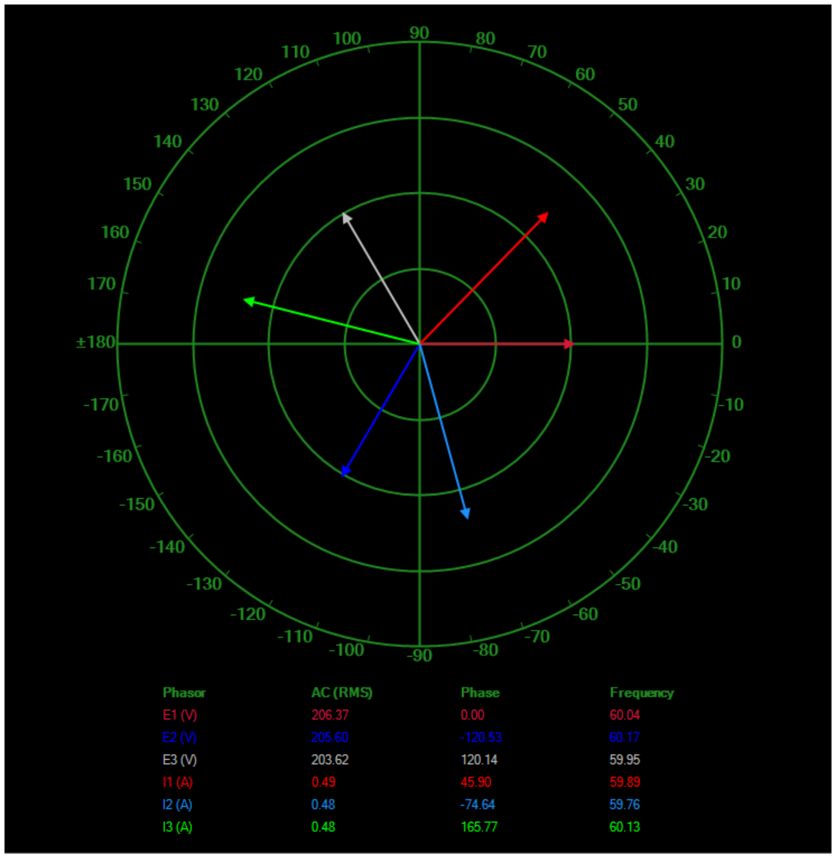


Figure 44: Phase analyser of a Δ-connected RC loads

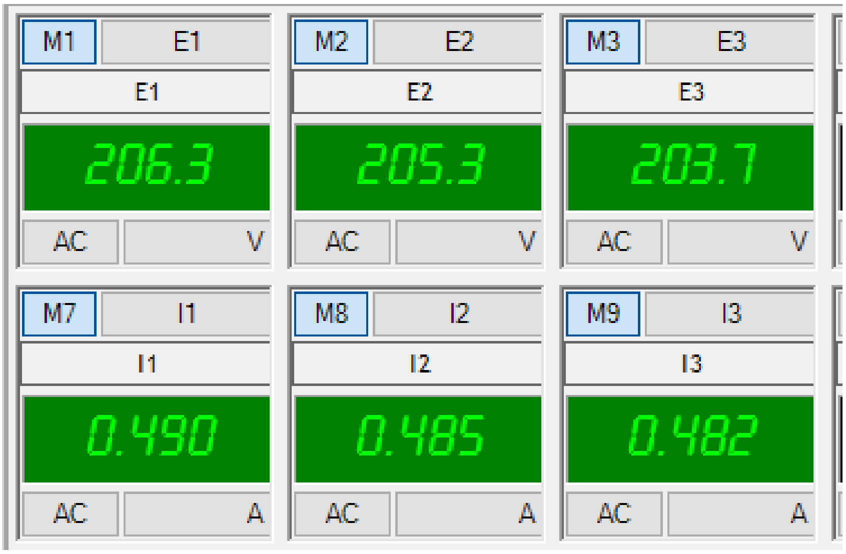


Figure 45 RMS Voltages and Currents

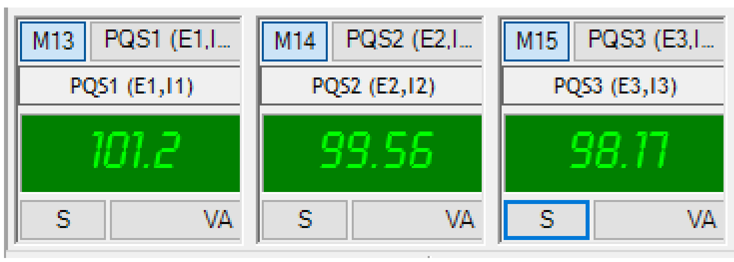


Figure 46 Apparent Power of the source

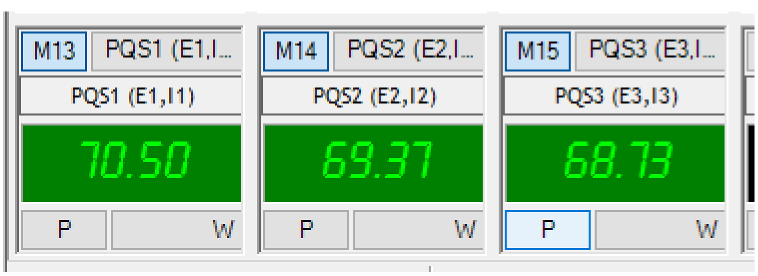


Figure 47 Active Power generated by the source

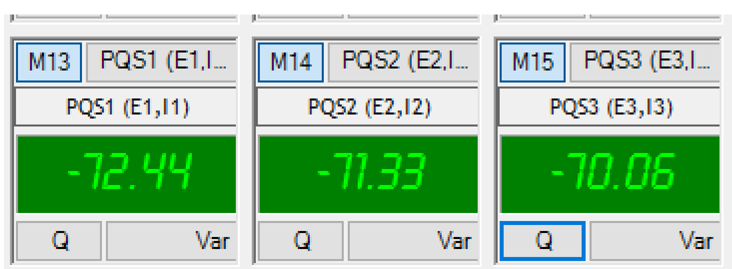


Figure 48 Reactive Power absorbed by the source (source convention)

The line and phase voltage would be equal to 208 V compared with the former section where the phase voltage equal to approximately 120 V (208/$\sqrt{3}$ V). It is now $\sqrt{3}$ times bigger. Power is proportional to the square of voltage. Thus, the power in this section should be 3 times bigger than Y connection. As shown in the pictures above, the voltage is indeed very close to 208 V and the active, reactive, and apparent power fed by the source are three times bigger than the previous section.