

## 问题二

### (1) 附录 2.1

计算单级杆泵功图的 Matlab 程序:

```
function [U,F]=jianmo12sudu
x=792.5;
Uo=[0.0000 0.0040 0.0120 0.0230 0.0370 0.0540 0.0740 0.0970 0.1230
0.1520 0.1830 0.2160 0.2510 0.2890 0.3280 0.3690 0.4120 0.4560
0.5010 0.5480 0.5960 0.6450 0.6940 0.7450 0.7970 0.8490 0.9010
0.9540 1.0080 1.0620 1.1160 1.1700 1.2250 1.2790 1.3340 1.3880
1.4420 1.4960 1.5500 1.6030 1.6550 1.7070 1.7580 1.8080 1.8580
1.9060 1.9530 1.9990 2.0430 2.0860 2.1270 2.1670 2.2050 2.2410
2.2750 2.3070 2.3370 2.3650 2.3910 2.4140 2.4350 2.4540 2.4710
2.4850 2.4970 2.5070 2.5150 2.5200 2.5240 2.5250 2.5240 2.5220
2.5180 2.5120 2.5040 2.4950 2.4840 2.4720 2.4590 2.4440 2.4270
2.4100 2.3910 2.3720 2.3510 2.3290 2.3050 2.2810 2.2560 2.2290
2.2010 2.1720 2.1420 2.1110 2.0790 2.0460 2.0110 1.9750 1.9380
1.9000 1.8600 1.8200 1.7780 1.7350 1.6900 1.6450 1.5980 1.5500
1.5010 1.4510 1.3990 1.3470 1.2940 1.2390 1.1840 1.1290 1.0720
1.0150 0.9580 0.9010 0.8440 0.7860 0.7290 0.6730 0.6180 0.5630
0.5100 0.4590 0.4090 0.3610 0.3150 0.2720 0.2310 0.1930 0.1580
0.1270 0.0980 0.0740 0.0520 0.0340 0.0200 0.0100 0.0030 0.0000];
D=[28.5460 31.1600 33.2510 35.4120 37.9570 40.7800 46.1480 50.4000 53.9900
55.9420 56.8830 57.0920 57.5450 58.1030 58.1380 58.2420 58.3470 58.0680
58.0680 58.2420 58.6610 58.9050 59.4620 60.0200 60.8910 61.2050 61.3440
60.8910 61.2750 61.0660 60.7520 59.8110 59.4970 58.8700 58.0330 57.6150
57.2320 56.4650 55.5240 55.0360 54.4080 53.9550 53.5020 53.5370 53.0490
53.2230 52.7000 52.2470 52.1780 52.2470 51.4810 50.7830 50.3300 49.8070
49.0410 48.2740 47.2980 46.7400 45.8340 45.5900 45.2070 44.9630 44.6840
44.4050 44.1960 43.9170 43.6030 43.1150 42.7320 41.9650 40.8500 40.2570
39.7000 39.6650 39.6650 38.1310 31.8220 30.1840 28.3720 27.0470 26.3850
23.8060 19.1700 16.7650 14.9880 13.2450 11.8850 14.2210 17.1140 19.6930
20.8080 22.4460 23.4220 21.0870 18.9960 16.4510 15.5450 13.6980 13.0010
14.4650 16.2770 18.1590 17.1830 17.3230 17.2880 16.5210 15.3710 14.7090
15.0570 14.4300 14.2560 14.3600 14.5340 14.7090 14.8830 14.7780 14.8130
14.7090 14.9530 15.2660 15.4060 15.3010 15.5100 16.2420 16.7300 17.2530
17.6710 18.6120 19.0310 19.3440 20.1460 20.8080 21.7840 21.9590 22.6900
23.2130 23.4920 23.3530 23.3880 23.8060 24.1200 24.5380 24.8860 25.3390];
D=D*1000;
f=7.6;%f 为冲次
w=f/60*2*pi();
t=[ 0.0000 0.0572 0.1144 0.1716 0.2288 0.2860 0.3432 0.4005 0.4577
0.5149 0.5721 0.6293 0.6865 0.7437 0.8009 0.8581 0.9153 0.9725
1.0297 1.0870 1.1442 1.2014 1.2586 1.3158 1.3730 1.4302 1.4874
```

1.5446	1.6018	1.6590	1.7162	1.7735	1.8307	1.8879	1.9451	2.0023
2.0595	2.1167	2.1739	2.2311	2.2883	2.3455	2.4027	2.4600	2.5172
2.5744	2.6316	2.6888	2.7460	2.8032	2.8604	2.9176	2.9748	3.0320
3.0892	3.1465	3.2037	3.2609	3.3181	3.3753	3.4325	3.4897	3.5469
3.6041	3.6613	3.7185	3.7757	3.8330	3.8902	3.9474	4.0007	4.0541
4.1074	4.1607	4.2141	4.2674	4.3208	4.3741	4.4275	4.4808	4.5341
4.5875	4.6408	4.6942	4.7475	4.8009	4.8542	4.9075	4.9609	5.0142
5.0676	5.1209	5.1743	5.2276	5.2809	5.3343	5.3876	5.4410	5.4943
5.5477	5.6010	5.6543	5.7077	5.7610	5.8144	5.8677	5.9211	5.9744
6.0277	6.0811	6.1344	6.1878	6.2411	6.2945	6.3478	6.4011	6.4545
6.5078	6.5612	6.6145	6.6679	6.7212	6.7745	6.8279	6.8812	6.9346
6.9879	7.0413	7.0946	7.1479	7.2013	7.2546	7.3080	7.3613	7.4147
7.4680	7.5213	7.5747	7.6280	7.6814	7.7347	7.7881	7.8414	7.8947

k=length(D);

d=22;%抽油杆直径

Ar=1/4\*pi\*(d/1000)^2;

g=9.8;

L=792.5;

roul=846\*(1-0.98)+1000\*0.98;

roug=8456;

Wrr=(roug-roul)\*g\*Ar\*L;

for p=1:k

D(p)=D(p)-Wrr;

end

% 第一步

a=4860; % 声速 m/s

N=10; % 傅里叶级数所取的项数

oo=linspace(0,0,N);yy=linspace(0,0,N);tt=linspace(0,0,N);dd=linspace(0,0,N);

aa=linspace(0,0,N);bb=linspace(0,0,N);K=linspace(0,0,N);O=linspace(0,0,N);

P=linspace(0,0,N);OO=linspace(0,0,N);PP=linspace(0,0,N);u=linspace(0,0,N);

OP=linspace(0,0,k);OOPP=linspace(0,0,k);U=linspace(0,0,k);F=linspace(0,0,k);

cd=0;v=0; % 傅里叶系数初始化

oo0=0;yy0=0;

E=2.1\*10^11;

for p=1:k;

oo0=2/k\*D(p)\*cos(2\*pi()\*0\*p/k)+oo0;

end

for p=1:k;

yy0=2/k\*Uo(p)\*cos(2\*pi()\*0\*p/k)+yy0;

end

for n=1:N

for p=1:k

oo(n)=2/k\*D(p)\*cos(2\*pi()\*n\*p/k)+oo(n);

end

end

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for n=1:N
for p=1:k
    tt(n)=2/k*D(p)*sin(2*pi()*n*p/k)+tt(n);
end
end
for n=1:N
for p=1:k
    yy(n)=2/k*Uo(p)*cos(2*pi()*n*p/k)+yy(n);
end
end
for n=1:N
for p=1:k
    dd(n)=2/k*Uo(p)*sin(2*pi()*n*p/k)+dd(n);
end
end
for i=1:k
v=(4315/2495*w*950/1000*sin(w*t(i)))^2+v;%加和 除以点数
end
v=(v/k)^0.5;
if v==0.10194737
cd=0.499612;
elseif v==0.304317197
cd=0.170975;
elseif v==0.457011024
cd=0.142155;
elseif v<0.10194737
cd=-2361.8*v^3 + 651.9*v^2 - 65.333*v + 2.8838 ;
elseif v>0.10194737 && v<0.304317197
cd=-38.609*v^3 + 31.455*v^2 - 9.2174*v + 1.1485 ;
elseif v>0.304317197 && v<0.457011024
cd=-0.286*v^3 + 1.2491*v^2 - 1.0164*v + 0.3729;
elseif v>0.457011024
cd=0.14;
end
%光杆速度及无因此阻尼系数因子，拟合曲线数据
c=pi()*a*cd/2/L;
% s^-1 % 第二步
for n=1:N
    aa(n)=n*w/a/2^0.5*(1+(1+(c/n/w)^2)^0.5)^0.5;
end
for n=1:N
    bb(n)=n*w/a/2^0.5*(-1+(1+(c/n/w)^2)^0.5)^0.5;
end % 第三步
for n=1:N

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        K(n)=(oo(n)*aa(n)+tt(n)*bb(n))/(aa(n)^2+bb(n)^2)/E/Ar;
end
for n=1:N
    u(n)=(oo(n)*aa(n)-tt(n)*bb(n))/(aa(n)^2+bb(n)^2)/E/Ar;
end
% 第四步
for n=1:N
    O(n)=(K(n)*cosh(bb(n)*x)+dd(n)*sinh(bb(n)*x))*sin(aa(n)*x)+(u(n)*...
    sinh(bb(n)*x)+yy(n)*cosh(bb(n)*x))*cos(aa(n)*x);
end
for n=1:N
    P(n)=(K(n)*sinh(bb(n)*x)+dd(n)*cosh(bb(n)*x))*cos(aa(n)*x)-(u(n)*...
    cosh(bb(n)*x)+yy(n)*sinh(bb(n)*x))*sin(aa(n)*x);
end
for n=1:N
    OO(n)=(tt(n)/E/Ar*sinh(bb(n)*x)+(dd(n)*bb(n)-yy(n)*aa(n))*...
    cosh(bb(n)*x))*sin(aa(n)*x)+(oo(n)/E/Ar*cosh(bb(n)*x)+...
    (yy(n)*bb(n)+dd(n)*aa(n))*sinh(bb(n)*x))*cos(aa(n)*x);
end
for n=1:N
    PP(n)=(tt(n)/E/Ar*cosh(bb(n)*x)+(dd(n)*bb(n)-yy(n)*aa(n))*...
    sinh(bb(n)*x))*cos(aa(n)*x)-(oo(n)/E/Ar*sinh(bb(n)*x)+(yy(n)*...
    bb(n)+dd(n)*aa(n))*cosh(bb(n)*x))*sin(aa(n)*x);
end
for i=1:k
    for n=1:N
        OP(i)=O(n)*cos(n*w*t(i))+P(n)*sin(n*w*t(i))+OP(i);
    end
end
for i=1:k
    for n=1:N
        OOPP(i)=OO(n)*cos(n*w*t(i))+PP(n)*sin(n*w*t(i))+OOPP(i);
    end
end
for i=1:k
    U(i)=oo0/2/E/Ar*x+yy0/2+OP(i);
end
for i=1:k
    F(i)=oo0/2+E*Ar*OOPP(i);
end
F=F/1000;
end

```

## (2) 附录 2.2

计算多级杆泵示功图的 Matlab 程序:

```
function [U,F,D]=jianmo22suduxilie03
Uo=[0 0 0.01 0.02 0.04 0.07 0.1 0.14 0.18 0.23 0.28 0.34 0.4 0.47 0.53 0.61 0.68
0.76 0.84 0.92 1.01 1.09 1.18 1.27 1.36 1.46 1.55 1.65 1.74 1.84 1.93 2.03 2.12 2.22 2.31
2.41 2.5 2.6 2.69 2.78 2.87 2.96 3.05 3.14 3.22 3.31 3.39 3.47 3.55 3.63 3.71 3.78 3.85
3.92 3.99 4.06 4.12 4.18 4.24 4.3 4.35 4.4 4.45 4.49 4.54 4.58 4.61 4.65 4.68 4.71 4.73
4.76 4.77 4.79 4.79 4.8 4.8 4.8 4.79 4.77 4.75 4.73 4.7 4.67 4.63 4.58 4.53 4.48 4.42
4.35 4.29 4.21 4.14 4.05 3.97 3.89 3.8 3.71 3.61 3.52 3.42 3.32 3.22 3.12 3.01 2.91 2.8 2.7
2.59 2.48 2.37 2.26 2.16 2.05 1.94 1.84 1.73 1.63 1.53 1.43 1.33 1.23 1.14 1.05 0.96 0.87
0.79 0.71 0.63 0.55 0.48 0.42 0.35 0.3 0.24 0.19 0.15 0.11 0.08 0.05 0.03 0.01 0];
D=[48.6150.01 52.84 52.98 53.2 53.56 53.42 52.77 55.46 57.08
59.02 60.67 61.46 62.28 63.07 64.4 67.2 69.78 71.94 74.2 75.21
76.68 76.64 74.42 71.87 70.04 68.67 67.49 67.42 67.45
68.89 70.97 72.58 73.05 73.23 72.87 72.08 70.9 69.96 68.1
67.31 67.63 68.24 68.42 69.17 70.5 71.26 71.47 71.01 70.29
69.53 68.85 68.17 67.99 67.63 67.88 68.6 69.21 69.75 69.78
70.04 70.07 69.64 69.03 68.28 67.77 67.31 67.27 67.38
67.74 68.13 68.6 68.71 68.67 68.17 67.85 67.31 66.81 65.73
65.15 64.72 63.61 63.25 62.82 62.57 62.46 62.68 62.5 61.71
60.92 59.45 58.19 56.97 55.1 53.59 51.55 49.83 47.46 44.73
42.25 39.63 38.98 40.6 43.44 45.48 47.24 48.78 49.68 49.75
49.07 47.31 44.73 43.22 42.43 42.29 42.5 43.47 45.55 46.13
47.74 48.25 48.07 47.53 46.49 45.27 44.58 43.76 43.65
44.05 44.87 46.06 46.67 48.14 48.53 48.5 48.14 47.96 47.35
46.85 46.99 46.67 46.81 47.46];
D=D*1000;
f=4; %f 为冲次
w=f/60*2*pi();
t=[0 0.105633803 0.211267606 0.316901408 0.422535211 0.528169014 0.633802817
0.73943662 0.845070423 0.950704225 1.056338028 1.161971831 1.267605634
1.373239437 1.478873239 1.584507042 1.690140845 1.795774648 1.901408451
2.007042254 2.112676056 2.218309859 2.323943662 2.429577465 2.535211268
2.64084507 2.746478873 2.852112676 2.957746479 3.063380282 3.169014085
3.274647887 3.38028169 3.485915493 3.591549296 3.697183099 3.802816901
3.908450704 4.014084507 4.11971831 4.225352113 4.330985915 4.436619718
4.542253521 4.647887324 4.753521127 4.85915493 4.964788732 5.070422535
5.176056338 5.281690141 5.387323944 5.492957746 5.598591549 5.704225352
5.809859155 5.915492958 6.021126761 6.126760563 6.232394366 6.338028169
6.443661972 6.549295775 6.654929577 6.76056338 6.866197183 6.971830986
7.077464789 7.183098592 7.288732394 7.394366197 7.5 7.605633803 7.711267606
7.816901408 7.922535211 8.028169014 8.133802817 8.23943662 8.345070423
8.450704225 8.556338028 8.661971831 8.767605634 8.873239437 8.978873239
9.084507042 9.190140845 9.295774648 9.401408451 9.507042254 9.612676056
9.718309859 9.823943662 9.929577465 10.03521127 10.14084507 10.24647887
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10.35211268 10.45774648 10.56338028 10.66901408 10.77464789 10.88028169
10.98591549 11.0915493 11.1971831 11.3028169 11.4084507 11.51408451
11.61971831 11.72535211 11.83098592 11.93661972 12.04225352 12.14788732
12.25352113 12.35915493 12.46478873 12.57042254 12.67605634 12.78169014
12.88732394 12.99295775 13.09859155 13.20422535 13.30985915 13.41549296
13.52112676 13.62676056 13.73239437 13.83802817 13.94366197 14.04929577
14.15492958 14.26056338 14.36619718 14.47183099 14.57746479 14.68309859
14.78873239 14.8943662 15];
k=length(D);
d= [25 22 19];%抽油杆直径
Ar=[0 0 0];
Ar(1)=1/4*pi()*(d(1)/1000)^2;Ar(2)=1/4*pi()*(d(2)/1000)^2;Ar(3)=1/4*pi()*(d(3)/1000)^2;
g=9.8;
L=[523.61 664.32 618.35];
roul=846*(1-0.912)+1000*0.912;
roug=8456;Wrr=[0 0 0 0];
Wrr(1)=(roug-roul)*g*(Ar(1)*L(1)+Ar(2)*L(2)+Ar(3)*L(3));
Wrr(2)=(roug-roul)*g*(Ar(2)*L(2)+Ar(3)*L(3));
Wrr(3)=(roug-roul)*g*Ar(3)*L(3);
Wrr(4)=0;
for p=1:k
    D(p)=D(p)-Wrr(1);
End % 第一步
a=4860; % 声速 m/s
N=10; % 傅里叶级数所取的项数
oo=linSPACE(0,0,N);yy=linSPACE(0,0,N);tt=linSPACE(0,0,N);dd=linSPACE(0,0,N);
aa=linSPACE(0,0,N);bb=linSPACE(0,0,N);K=linSPACE(0,0,N);O=linSPACE(0,0,N);
P=linSPACE(0,0,N);OO=linSPACE(0,0,N);PP=linSPACE(0,0,N);u=linSPACE(0,0,N);
OP=linSPACE(0,0,k);OOPP=linSPACE(0,0,k);U=linSPACE(0,0,k);F=linSPACE(0,0,k);
cd=0;v=0; % 傅里叶系数初始化
oo0=0;yy0=0;
E=2.1*10^11;
for p=1:k;
    oo0=2/k*D(p)*cos(2*pi()*0*p/k)+oo0;
end
for p=1:k;
    yy0=2/k*Uo(p)*cos(2*pi()*0*p/k)+yy0;
end
for n=1:N
    for p=1:k
        oo(n)=2/k*D(p)*cos(2*pi()*n*p/k)+oo(n);
    end
end
for n=1:N

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for p=1:k
    tt(n)=2/k*D(p)*sin(2*pi()*n*p/k)+tt(n);
end
end
for n=1:N
    for p=1:k
        yy(n)=2/k*Uo(p)*cos(2*pi()*n*p/k)+yy(n);
    end
end
for n=1:N
    for p=1:k
        dd(n)=2/k*Uo(p)*sin(2*pi()*n*p/k)+dd(n);
    end
end
for i=1:k
    v=(4315/2495*w*950/1000*sin(w*t(i)))^2+v;% 加和除以点数
end
v=(v/k)^0.5;
    if v==0.10194737
cd=0.499612;
        elseif v==0.304317197
cd=0.170975;
            elseif v==0.457011024
cd=0.142155;
                elseif v<0.10194737
cd=-2361.8*v^3 + 651.9*v^2 - 65.333*v + 2.8838 ;
                    elseif v>0.10194737 && v<0.304317197
cd=-38.609*v^3 + 31.455*v^2 - 9.2174*v + 1.1485 ;
                        elseif v>0.304317197 && v<0.457011024
cd=-0.286*v^3 + 1.2491*v^2 - 1.0164*v + 0.3729;
                            elseif v>0.457011024
cd=0.14;
                                end
% 光杆速度及无因此阻尼系数因子，拟合曲线数据
c=pi()*a*cd/2/(L(1)+L(2)+L(3));
    %s^-1    %
for n=1:N
    aa(n)=n*w/a/2^0.5*(1+(1+(c/n/w)^2)^0.5)^0.5;
end
for n=1:N
    bb(n)=n*w/a/2^0.5*(-1+(1+(c/n/w)^2)^0.5)^0.5;
end
z=1;%
for n=1:N

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第二步

第三步

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        K(n)=(oo(n)*aa(n)+tt(n)*bb(n))/(aa(n)^2+bb(n)^2)/E/Ar(z);
end
for n=1:N
    u(n)=(oo(n)*aa(n)-tt(n)*bb(n))/(aa(n)^2+bb(n)^2)/E/Ar(z);
end
% 第四步
for n=1:N
O(n)=(K(n)*cosh(bb(n)*L(z))+dd(n)*sinh(bb(n)*L(z)))*sin(aa(n)*L(z))+(u(n)*...
sinh(bb(n)*L(z))+yy(n)*cosh(bb(n)*L(z)))*cos(aa(n)*L(z));
end
for n=1:N
    P(n)=(K(n)*sinh(bb(n)*L(z))+dd(n)*cosh(bb(n)*L(z)))*cos(aa(n)*L(z))-(u(n)*...
cosh(bb(n)*L(z))+yy(n)*sinh(bb(n)*L(z)))*sin(aa(n)*L(z));
end
for n=1:N
    OO(n)=(tt(n)/E/Ar(z)*sinh(bb(n)*L(z))+(dd(n)*bb(n)-yy(n)*aa(n))*...
cosh(bb(n)*L(z)))*sin(aa(n)*L(z))+(oo(n)/E/Ar(z)*cosh(bb(n)*L(z))+...
(yy(n)*bb(n)+dd(n)*aa(n))*sinh(bb(n)*L(z)))*cos(aa(n)*L(z));
end
for n=1:N
    PP(n)=(tt(n)/E/Ar(z)*cosh(bb(n)*L(z))+(dd(n)*bb(n)-yy(n)*aa(n))*...
sinh(bb(n)*L(z)))*cos(aa(n)*L(z))-(oo(n)/E/Ar(z)*sinh(bb(n)*L(z))+(yy(n)*...
bb(n)+dd(n)*aa(n))*cosh(bb(n)*L(z)))*sin(aa(n)*L(z));
end
oo0=oo0+0;
yy0=oo0/E/Ar(z)*L(z)+yy0;
for n=1:N
    oo(n)=E*Ar(z)*OO(n);
end
for n=1:N
    tt(n)=E*Ar(z)*PP(n);
end
for n=1:N
    yy(n)=O(n);
end
for n=1:N
    dd(n)=P(n);
end
z=2;
for n=1:N
    K(n)=(oo(n)*aa(n)+tt(n)*bb(n))/(aa(n)^2+bb(n)^2)/E/Ar(z);
end
for n=1:N
    u(n)=(oo(n)*aa(n)-tt(n)*bb(n))/(aa(n)^2+bb(n)^2)/E/Ar(z);
end

```



```

for n=1:N
O(n)=(K(n)*cosh(bb(n)*L(z))+dd(n)*sinh(bb(n)*L(z)))*sin(aa(n)*L(z))+(u(n)*...
sinh(bb(n)*L(z))+yy(n)*cosh(bb(n)*L(z)))*cos(aa(n)*L(z));
end
for n=1:N
P(n)=(K(n)*sinh(bb(n)*L(z))+dd(n)*cosh(bb(n)*L(z)))*cos(aa(n)*L(z))-(u(n)*...
cosh(bb(n)*L(z))+yy(n)*sinh(bb(n)*L(z)))*sin(aa(n)*L(z));
end
for n=1:N
OO(n)=(tt(n)/E/Ar(z)*sinh(bb(n)*L(z))+(dd(n)*bb(n)-yy(n)*aa(n))*...
cosh(bb(n)*L(z)))*sin(aa(n)*L(z))+(oo(n)/E/Ar(z)*cosh(bb(n)*L(z))+...
(yy(n)*bb(n)+dd(n)*aa(n))*sinh(bb(n)*L(z)))*cos(aa(n)*L(z));
end
for n=1:N
PP(n)=(tt(n)/E/Ar(z)*cosh(bb(n)*L(z))+(dd(n)*bb(n)-yy(n)*aa(n))*...
sinh(bb(n)*L(z)))*cos(aa(n)*L(z))-(oo(n)/E/Ar(z)*sinh(bb(n)*L(z))+(yy(n)*...
bb(n)+dd(n)*aa(n))*cosh(bb(n)*L(z)))*sin(aa(n)*L(z));
end
oo0=oo0+0;
yy0=oo0/E/Ar(z)*L(z)+yy0;
for n=1:N
oo(n)=E*Ar(z)*OO(n);
end
for n=1:N
tt(n)=E*Ar(z)*PP(n);
end
for n=1:N
yy(n)=O(n);
end
for n=1:N
dd(n)=P(n);
end
z=3;
for n=1:N
K(n)=(oo(n)*aa(n)+tt(n)*bb(n))/(aa(n)^2+bb(n)^2)/E/Ar(z);
end
for n=1:N
u(n)=(oo(n)*aa(n)-tt(n)*bb(n))/(aa(n)^2+bb(n)^2)/E/Ar(z);
end
for n=1:N
O(n)=(K(n)*cosh(bb(n)*L(z))+dd(n)*sinh(bb(n)*L(z)))*sin(aa(n)*L(z))+(u(n)*...
sinh(bb(n)*L(z))+yy(n)*cosh(bb(n)*L(z)))*cos(aa(n)*L(z));
end
for n=1:N

```

```

P(n)=(K(n)*sinh(bb(n)*L(z))+dd(n)*cosh(bb(n)*L(z)))*cos(aa(n)*L(z))-(u(n)*...
cosh(bb(n)*L(z))+yy(n)*sinh(bb(n)*L(z)))*sin(aa(n)*L(z));
end
for n=1:N
    OO(n)=(tt(n)/E/Ar(z)*sinh(bb(n)*L(z))+(dd(n)*bb(n)-yy(n)*aa(n))*...
    cosh(bb(n)*L(z)))*sin(aa(n)*L(z))+(oo(n)/E/Ar(z)*cosh(bb(n)*L(z))+...
(yy(n)*bb(n)+dd(n)*aa(n))*sinh(bb(n)*L(z)))*cos(aa(n)*L(z));
end
for n=1:N
    PP(n)=(tt(n)/E/Ar(z)*cosh(bb(n)*L(z))+(dd(n)*bb(n)-yy(n)*aa(n))*...
sinh(bb(n)*L(z)))*cos(aa(n)*L(z))-(oo(n)/E/Ar(z)*sinh(bb(n)*L(z))+yy(n)*...
bb(n)+dd(n)*aa(n))*cosh(bb(n)*L(z)))*sin(aa(n)*L(z));
end
for i=1:k
    for n=1:N
        OP(i)=O(n)*cos(n*w*t(i))+P(n)*sin(n*w*t(i))+OP(i);
    end
end
for i=1:k
    for n=1:N
        OOPP(i)=OO(n)*cos(n*w*t(i))+PP(n)*sin(n*w*t(i))+OOPP(i);
    end
end
for i=1:k
    U(i)=oo0/2/E/Ar(z)*L(z)+yy0/2+OP(i);
end
for i=1:k
    F(i)=oo0/2+E*Ar(z)*OOPP(i);
end
for i=1:k
    F(i)=F(i)/1000;
end
end
end

```

## 问题三

### (1)附录 3.1

求曲率变化量的 matlab 算法:

```

clear; clc;
%假设 data 是 n 行*2 列的矩阵,n 等于点的个数,
%第一列表示横坐标 xi,第二列表示纵坐标 yi
%点的次序按照逆时针排列

```

sum=0;

data=[0.1046834330.13534769 0.173583366 0.217076824 0.263398014 0.310294211

0.35594491 0.39921363 0.43943019 0.47681891 0.512070368 0.546232171  
0.580490768 0.615960864 0.653516524 0.693687768 0.73663239 0.782178063  
0.829917079 0.879415396 0.929984044 0.981292296 1.033074144 1.085214932  
1.137710316 1.190598095 1.243883795 1.297481247 1.351184806 1.40468154  
1.45760149 1.509684324 1.560502052 1.61007607 1.658549256 1.706270087  
1.753736391 1.801498405 1.850038527 1.899651321 1.950348747 2.001811842  
2.053401777 2.104319627 2.15337494 2.199675183 2.24242376 2.281141514  
2.315748986 2.346583558 2.374347528 2.399995637 2.424581327 2.449088154  
2.474274823 2.50060609 2.52800683 2.556141163 2.58429294 2.611516202  
2.636763052 2.65901048 2.677365421 2.691133853 2.699848673 2.703260285  
2.701301406 2.694024599 2.681623186 2.664333399 2.644105557 2.620205338  
2.593164066 2.563504103 2.531861198 2.499173273 2.466224797 2.434067095  
2.40344978 2.37520292 2.34974024 2.327183566 2.307428591 2.289864039  
2.273721191 2.257893312 2.241307516 2.222850455 2.201573059 2.176933584  
2.148572675 2.116705094 2.081662915 2.044289931 2.005381778 1.965738305  
1.926270922 1.887377529 1.849462655 1.812336763 1.775814043 1.739292744  
1.702062504 1.663624149 1.623298794 1.580888937 1.536115261 1.489257282  
1.440540739 1.390308435 1.339236569 1.287590993 1.235923991 1.18426495  
1.132888416 1.081663941 1.030399604 0.979194032 0.927827684 0.876578694  
0.825473601 0.775040847 0.725593637 0.677455428 0.631141613 0.586662129  
0.544163797 0.503249145 0.46364594 0.424736961 0.385904641 0.346871984  
0.30732042 0.267581456 0.228106687 0.190061803 0.154745451 0.12371338  
0.098761478 0.081355326 0.072799762 0.073825843 0.084602467 0.104666562  
9.786853017 12.00862423 14.55416098 17.35865233 20.33079203 23.36208517  
26.3380851 29.15474048 31.70952266 33.939263 35.80479583 37.29720979  
38.43553291 39.26155456 39.83271529 40.21414294 40.47088956 40.66125438  
40.83180912 41.01477321 41.2257523 41.46644712 41.72567541 41.98282866  
42.21149923 42.38318281 42.47074297 42.45140955 42.30915032 42.03630309  
41.63438723 41.11303884 40.49292374 39.79843976 39.05868096 38.30364272  
37.56106434 36.85347527 36.19583658 35.59412144 35.04506602 34.5371564  
34.05272548 33.56999591 33.06968489 32.53308202 31.94779383 31.30829143  
30.61609107 29.87877962 29.1081537 28.31790381 27.52134677 26.72967263  
25.95103178 25.18926035 24.4500174 23.73411499 23.04415423 22.38390377  
21.75805854 21.1708422 20.62355989 20.1115186 19.62098594 19.12699276  
18.59275595 17.97009956 17.20795775 16.25219683 15.14714778 13.80695943  
12.22891513 10.42359004 8.422032276 6.287832639 4.086638821 1.911422461  
-0.157678276 -2.030240611 -3.641640944 -4.94646531 -5.918227739 -6.56693395  
-6.923094526 -7.043266486 -6.997248941 -6.862194052 -6.712161209 -6.61145037  
-6.606393531 -6.722613606 -6.964338906 -7.314767984 -7.743362252 -8.211471242  
-8.676075453 -9.102500255 -9.462991058 -9.746028464 -9.951743804 -10.09427478  
-10.19603742 -10.2828032 -10.37975633 -10.50528497 -10.66919726 -10.86917012  
-11.09315805 -11.32086667 -11.52584267 -11.68254127 -11.76757825 -11.76515408

```

-11.66781081 -11.47701523 -11.20103013 -10.8545829 -10.45155607 -10.00663882
-9.526624626 -9.015439294 -8.467975166 -7.873953314 -7.22433131 -6.506738682
-5.718856812 -4.860489267 -3.946890611 -2.99757538 -2.03872427 -1.105024663
-0.222202657 0.584690162 1.309369101 1.956037774 2.550968467 3.137344438
3.769142219 4.513640779 5.434270807 6.594472277 8.035754264 9.785535114
];
%此时示工图是按照逆时针走的
data=data';
x=data(:,1); %第一列泵位移
y=data(:,2); %第二列泵载荷
n=length(x);%点的个数
jiaodu=linspace(0,0,n-2);
s=linspace(0,0,n-2);
o=linspace(0,0,n-2);
p=linspace(0,0,n-2);
q=linspace(0,0,n-2);
k=linspace(0,0,n-2);
mk=linspace(0,0,n-1);
DK=linspace(0,0,n-1);
fori=3:(n-2)

jiaodu(i)=atan(((y(i)-y(i+2))*(x(i)-x(i-2))-(y(i)-y(i-2))*(x(i)-x(i+2))))/(1+(x(i)-x(i-2))*(x(i)-x(i+2)))));
);
s(i)=sqrt((x(i-2)-x(i-1))*(x(i-2)-x(i-1))+(y(i-2)-y(i-1))*(y(i-2)-y(i-1)))); %pi-2 与
pi-1 之间距离
o(i)=sqrt((x(i-1)-x(i))*(x(i-1)-x(i))+(y(i-1)-y(i))*(y(i-1)-y(i)))); %pi-1 与 pi 之间
距离
p(i)=sqrt((x(i)-x(i+1))*(x(i)-x(i+1))+(y(i)-y(i+1))*(y(i)-y(i+1)))); %pi 与 pi+1 之
间距离
q(i)=sqrt((x(i+1)-x(i+2))*(x(i+1)-x(i+2))+(y(i+1)-y(i+2))*(y(i+1)-y(i+2)))); %pi+1 与 pi+2
之间距离
k(i)=jiaodu(i)/(s(i)+o(i)+p(i)+q(i)); %曲率
end
fori=4:(n-2)
mk(i)=abs(k(i)-k(i+1)); %pi 的曲率变化量
DK(i)=(mk(i-2)+mk(i-1)+mk(i)+mk(i+1)+mk(i+2))/5
end

```

## (2)附录 3.2

### Matlab 画图算法:

```

clc; clear all; close all;
axis([0 3 0 70]);

```

```

        X=[2.518    2.512    2.504    2.495    2.484    2.472    2.459    2.444    2.427
2.41    2.391    2.372    2.351    2.329    2.305    2.281    2.256    2.229    2.201
2.172    2.142    2.111    2.079    2.046    2.011    1.975    1.938    1.9    1.86    1.82
1.778    1.735    1.69    1.645    1.598    1.55    1.501    1.451    1.399    1.347
1.294    1.239    1.184    1.129    1.072    1.015    0.958    0.901    0.844    0.786
0.729    0.673    0.618    0.563    0.51    0.459    0.409    0.361    0.315    0.272
0.231    0.193    0.158    0.127    0.098    0.074    0.052    0.034    0.02    0.01
0.003    0    0    0.004    0.012    0.023    0.037    0.054    0.074    0.097    0.123
0.152    0.183    0.216    0.251    0.289    0.328    0.369    0.412    0.456    0.501
0.548    0.596    0.645    0.694    0.745    0.797    0.849    0.901    0.954    1.008
1.062    1.116    1.17    1.225    1.279    1.334    1.388    1.442    1.496    1.55
1.603    1.655    1.707    1.758    1.808    1.858    1.906    1.953    1.999    2.043
2.086    2.127    2.167    2.205    2.241    2.275    2.307    2.337    2.365    2.391
2.414    2.435    2.454    2.471    2.485    2.497    2.507    2.515    2.52    2.524
2.525    2.524    2.522
    ];
    U=[39.7 39.665 39.665 38.131 31.822 30.184 28.372 27.047 26.385 23.806
19.17 16.765 14.988 13.245 11.885 14.221 17.114 19.693 20.808 22.446
23.422 21.087 18.996 16.451 15.545 13.698 13.001 14.465 16.277 18.159
17.183 17.323 17.288 16.521 15.371 14.709 15.057 14.43 14.256 14.36
14.534 14.709 14.883 14.778 14.813 14.709 14.953 15.266 15.406 15.301
15.51 16.242 16.73 17.253 17.671 18.612 19.031 19.344 20.146 20.808
21.784 21.959 22.69 23.213 23.492 23.353 23.388 23.806 24.12 24.538
24.886 25.339 28.546 31.16 33.251 35.412 37.957 40.78 46.148 50.4
53.99 55.942 56.883 57.092 57.545 58.103 58.138 58.242 58.347 58.068
58.068 58.242 58.661 58.905 59.462 60.02 60.891 61.205 61.344 60.891
61.275 61.066 60.752 59.811 59.497 58.87 58.033 57.615 57.232 56.465
55.524 55.036 54.408 53.955 53.502 53.537 53.049 53.223 52.7 52.247
52.178 52.247 51.481 50.783 50.33 49.807 49.041 48.274 47.298 46.74
45.834 45.59 45.207 44.963 44.684 44.405 44.196 43.917 43.603 43.115
42.732 41.965 40.85 40.257
    ];
    plot(X,U)
    hold on
    A1 = [0,25.54];
    A2 = [0.152,55.942];
    A3 = [2.542,42.732];
    A4 = [2.305,11.885];
    h1 = plot([A1(1) A2(1)], [A1(2) A2(2)], 'r-o', 'MarkerFaceColor','g', 'LineWidth', 2);
    h2 = plot([A2(1) A3(1)], [A2(2) A3(2)], 'r-o', 'MarkerFaceColor','g', 'LineWidth', 2);
    h3 = plot([A3(1) A4(1)], [A3(2) A4(2)], 'r-o', 'MarkerFaceColor','g', 'LineWidth', 2);
    h4 = plot([A4(1) A1(1)], [A4(2) A1(2)], 'r-o', 'MarkerFaceColor','g', 'LineWidth', 2);

```

### (3) 附录 3. 3

求理论示功图面积程序:

```
function [] = my_area()
clear;
clc;
A = [0,25.546]
B = [0.152,55.9425]
C = [2.524,42.732]
D = [2.305,11.885]
s1 = helen(A,B,C);
s2 = helen(A,C,D);
s = s1+s2;
fprintf('the area is %f\n',s);
function len = lenth(x,y)
len = sqrt((x(1)-y(1))^2+(x(2)-y(2))^2);
function s = helen(x,y,z)
a = lenth(x,y);
b = lenth(x,z);
c = lenth(y,z);
p = (a+b+c)/2;
s = sqrt(p*(p-a)*(p-b)*(p-c));
```

#### (4) 附录 3. 4

求泵功图面积程序

```
clear;
%假设 data 是 n 行*2 列的矩阵,n 等于点的个数,
%第一列表示横坐标 xi,第二列表示纵坐标 yi
%点的次序按照逆时针排列
sum=0;
data=[0.0727997620.073825843 0.084602467 0.104666562 0.104683433 0.13534769
0.173583366 0.217076824 0.263398014 0.310294211 0.35594491 0.39921363
0.43943019 0.47681891 0.512070368 0.546232171 0.580490768 0.615960864
0.653516524 0.693687768 0.73663239 0.782178063 0.829917079 0.879415396
0.929984044 0.981292296 1.033074144 1.085214932 1.137710316 1.190598095
1.243883795 1.297481247 1.351184806 1.40468154 1.45760149 1.509684324
1.560502052 1.61007607 1.658549256 1.706270087 1.753736391 1.801498405
1.850038527 1.899651321 1.950348747 2.001811842 2.053401777 2.104319627
2.15337494 2.199675183 2.24242376 2.281141514 2.315748986 2.346583558
2.374347528 2.399995637 2.424581327 2.449088154 2.474274823 2.50060609
2.52800683 2.556141163 2.58429294 2.611516202 2.636763052 2.65901048
2.677365421 2.691133853 2.699848673 2.703260285 2.701301406 2.694024599
2.681623186 2.664333399 2.644105557 2.620205338 2.593164066 2.563504103
2.531861198 2.499173273 2.466224797 2.434067095 2.40344978 2.37520292
2.34974024 2.327183566 2.307428591 2.289864039 2.273721191 2.257893312
2.241307516 2.222850455 2.201573059 2.176933584 2.148572675 2.116705094
```

```

2.081662915 2.044289931 2.005381778 1.965738305 1.926270922 1.887377529
1.849462655 1.812336763 1.775814043 1.739292744 1.702062504 1.663624149
1.623298794 1.580888937 1.536115261 1.489257282 1.440540739 1.390308435
1.339236569 1.287590993 1.235923991 1.18426495 1.132888416 1.081663941
1.030399604 0.979194032 0.927827684 0.876578694 0.825473601 0.775040847
0.725593637 0.677455428 0.631141613 0.586662129 0.544163797 0.503249145
0.46364594 0.424736961 0.385904641 0.346871984 0.30732042 0.267581456
0.228106687 0.190061803 0.154745451 0.12371338 0.098761478 0.081355326
5.434270807 6.594472277 8.035754264 9.785535114 9.786853017 12.00862423
14.55416098 17.35865233 20.33079203 23.36208517 26.3380851 29.15474048
31.70952266 33.939263 35.80479583 37.29720979 38.43553291 39.26155456
39.83271529 40.21414294 40.47088956 40.66125438 40.83180912 41.01477321
41.2257523 41.46644712 41.72567541 41.98282866 42.21149923 42.38318281
42.47074297 42.45140955 42.30915032 42.03630309 41.63438723 41.11303884
40.49292374 39.79843976 39.05868096 38.30364272 37.56106434 36.85347527
36.19583658 35.59412144 35.04506602 34.5371564 34.05272548 33.56999591
33.06968489 32.53308202 31.94779383 31.30829143 30.61609107 29.87877962
29.1081537 28.31790381 27.52134677 26.72967263 25.95103178 25.18926035
24.4500174 23.73411499 23.04415423 22.38390377 21.75805854 21.1708422
20.62355989 20.1115186 19.62098594 19.12699276 18.59275595 17.97009956
17.20795775 16.25219683 15.14714778 13.80695943 12.22891513 10.42359004
8.422032276 6.287832639 4.086638821 1.911422461 -0.157678276 -2.030240611
-3.641640944 -4.94646531 -5.918227739 -6.56693395 -6.923094526 -7.043266486
-6.997248941 -6.862194052 -6.712161209 -6.61145037 -6.606393531 -6.722613606
-6.964338906 -7.314767984 -7.743362252 -8.211471242 -8.676075453 -9.102500255
-9.462991058 -9.746028464 -9.951743804 -10.09427478 -10.19603742 -10.2828032
-10.37975633 -10.50528497 -10.66919726 -10.86917012 -11.09315805 -11.32086667
-11.52584267 -11.68254127 -11.76757825 -11.76515408 -11.66781081 -11.47701523
-11.20103013 -10.8545829 -10.45155607 -10.00663882 -9.526624626 -9.015439294
-8.467975166 -7.873953314 -7.22433131 -6.506738682 -5.718856812 -4.860489267
-3.946890611 -2.99757538 -2.03872427 -1.105024663 -0.222202657 0.584690162
1.309369101 1.956037774 2.550968467 3.137344438 3.769142219 4.513640779

```

```

];
%此时示工图是按照逆时针走的
data=data';
x=data(:,1); % 第一列泵位移
y=data(:,2); % 第二列泵载荷
n=length(x);%点的个数
%下面为计算矢量(xi,yi)与矢量(xi+1,yi+1)、原点之间围成的有向面积
A=linspace(0,0,n-1);
for i=1:(n-1)
    %为计算矢量积的第三个分量,除以 2 得到有向三角形面积
    A(i)=(x(i)*y(i+1)-x(i+1)*y(i))/2; % 有向三角形面积
end

```

```

fori=1:(n-1)
sum=A(i)+sum    %不规则图形的有向面积
end

```

## 问题 4

### (1) 附录 4.1

用最小二乘法求解方程数多于未知变量的线性方程组的最适解的 c 语言程序:

c 语言实现 (引自

<http://blog.csdn.net/zhujiang1001/article/details/4043304>, 经测试, 可行)

(用最小二乘法求解方程数多于未知变量的线性方程组的最适解(即矛盾方程组))。

```

#include "stdio.h"
#include "math.h"
int ArgMin(double *mtrx_tmp,int COLUMN,int rowNum,int colNum,double *solution)
{
    int k,l,j;
    double *eqt;
    int *q;
    q=&colNum;
    eqt=(double *)malloc(colNum*colNum*sizeof(double));
    //printf("%d",*q);
    for(k=0;k< *q-1;k++)
    for(l=0;l< *q-1;l++)
    {
        eqt[k*COLUMN+l]=0;
        for(j=0;j<rowNum;j++)
            eqt[k*COLUMN+l]+=(mtrx_tmp+COLUMN*j+k)*(*(mtrx_tmp+COLUMN*j+l));
    }
    for(k=0;k< *q-1;k++)
    {
        eqt[k*COLUMN+*q-1]=0;
        for(j=0;j<rowNum;j++)
            eqt[k*COLUMN+*q-1]+=(mtrx_tmp+COLUMN*j+*q-1)*(*(mtrx_tmp+COLUMN*j+k));
    }
    /* show the contents of eqts */
    /*int m,n;
    printf("eqt:\n");
    for(m=0;m< *q-1;m++)
    {
        for(n=0;n< *q;n++)
            printf("%.2lf\t",eqt[m*COLUMN+n]);
        printf("\n");
    }*/

```



```

if(!SolveLinearEqts(eqt,COLUMN,(*q-1),*q,solution)) {
return 0;
}
return 1;
}

```

测试程序：

```
int ArgMin(double *inMtrx,int COLUMN,int rowNum,int colNum,double *solution);
```

函数功能：用最小二乘法求解方程数多于未知变量的线性方程组的最适解。

测试代码：

```

#define Q 5
#define ROWNUM 6
int main()
{
double solution[Q-1],eqt[Q][Q]={0};
double
a[ROWNUM][Q]={ {3,-2,4,6,-11},{4,3,2,9,-2},{2,6,8,3,4},{2,4,5,3,3},
                {0,0,8,6,-20},{3,4,5,6,0}};

ArgMin(&a[0][0],Q,ROWNUM,Q,&solution[0]);
for(i=0;i<Q-1;i++) printf("%.14lf\n",solution[i]);
return 0;
}

```

测试结果：

```

eqts:
42.0000000000 38.0000000000 61.0000000000 84.0000000000 -27.0000000000
38.0000000000 81.0000000000 86.0000000000 69.0000000000 52.0000000000
61.0000000000 86.0000000000 198.0000000000 159.0000000000
-161.0000000000
84.0000000000 69.0000000000 159.0000000000 207.0000000000
-183.0000000000
3.0000000000000000
2.0000000000000000
-1.0000000000000000
-2.0000000000000000

```

说明：a, 待求解的方程组数据：

```

{3,-2,4,6,-11},
{4,3,2,9,-2},
{2,6,8,3,4},
{2,4,5,3,3},
{0,0,8,6,-20},
{3,4,5,6,0}

```

以上数据中共有四个变量，六个方程；

b, 测试显示的数据中，“eqts”为通过最小二乘法得出的待求解的线性方程组增广矩阵；下面四行为计算求得的解，与实际完全符合。