HW2

Problem 1

(a)

$$P(A \cup B) = 1 - P(A^c \cup B^c) = 0.65$$

(b)

$$P(A \cap B^c) = P(A)P(B^c) = 0.15$$

(c)

$$P(A \cup B) = 0.15$$

Problem 2

Let $|S| = 100$.

(a) Let E_1 be the event that the student chosen is not in any of the language classes, and $|E_1|$ be the number of these student satisfying E_1

$$P(E_1) = \frac{|E_1|}{|S|} = \frac{34}{100} = 0.34$$

(b) Let E_2 be the event that the student chosen is taking exactly one language class, and $|E_2|$ be the number of these student satisfying E_2

$$P(E_2) = \frac{|E_2|}{|S|} = \frac{66 - 2 - 10 - 2 - 4}{100} = 0.48$$

(c) Let E_3 be the event that the two students chosen at least 1 is taking 1 language class.

$$P(E_3) = 1 - P(E_3^c) = 1 - 0.52^2 = 0.7296$$

Problem 3

The all results of sum are as follows:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Let E_n denote the event that a 5 occurs on nth roll and no 5 or 7 occurs on the first $n-1$ rolls

$$\mathrm{P}(E_n) = (\frac{36-4-6}{36})^{n-1} (\frac{4}{36})$$

considering all cases until $n \to \infty$, the total probability $\sum_{n=1}^{\infty} \mathrm{P}(E_n)$ is the probability that 5 occurs first.

$$\sum_{n=1}^{\infty} \mathrm{P}(E_n) = \frac{2}{5}$$

Problem 4

Suppose that the 4 couples are $A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2$,where the same two letters with $1, 2$ are a couple. Let

- E_1 be the event that A_1, A_2 are next to each other.
- E_2 be the event that B_1, B_2 are next to each other.
- E_3 be the event that C_1, C_2 are next to each other.
- E_4 be the event that D_1, D_2 are next to each other.

$$\begin{aligned}
\mathrm{P}(E_1 \cup E_2 \cup E_3 \cup E_4) &= \mathrm{P}(E_1) + \mathrm{P}(E_2) + \mathrm{P}(E_3) + \mathrm{P}(E_4) \\
&\quad - \mathrm{P}(E_1E_2) - \cdots - \mathrm{P}(E_3E_4) \\
&\quad + \mathrm{P}(E_1E_2E_3) + \cdots + \mathrm{P}(E_2E_3E_4) \\
&\quad - \mathrm{P}(E_1E_2E_3E_4) \\
&= 4\mathrm{P}(E_1) - 6\mathrm{P}(E_1E_2) + 4\mathrm{P}(E_1E_2E_3) - \mathrm{P}(E_1E_2E_3E_4) \\
&= \frac{4 \times A_2^1 \times A_7^7}{A_8^8} - \frac{6 \times (A_2^1)^2 \times A_6^6}{A_8^8} + \frac{4 \times (A_2^1)^3 \times A_5^5}{A_8^8} - \frac{(A_2^1)^4 \times A_4^4}{A_8^8} \\
&= \frac{23}{35}
\end{aligned}$$

$E_1^c \cap E_2^c \cap E_3^c \cap E_4^c$ is the event that no couples are next to each other.

$$\mathrm{P}(E_1^c \cap E_2^c \cap E_3^c \cap E_4^c) = 1 - \mathrm{P}(E_1 \cup E_2 \cup E_3 \cup E_4) = 1 - \frac{23}{35} = \frac{12}{35}$$

Problem 5 (*Bonferroni's inequality*)

We prove the inequality directly

$$LHS = \mathrm{P}(EF) = 1 - \mathrm{P}(E^c \cup F^c) = 1 + \mathrm{P}(E^cF^c) - \mathrm{P}(E^c) - \mathrm{P}(F^c) \geq 1 - [1 - \mathrm{P}(E)] - [1 - \mathrm{P}(F)] = \mathrm{P}(E) + \mathrm{P}(F) - 1$$

Problem 6 (The matching problem)

If we denote N people as 1, 2, 3 ... N., N hats as H_1 H_2 ... H_N

(1). consider people 1: he has $N-1$ ① choices in N hats. if he doesn't get his hat.

(2). conside the case people 1 gets the hats H_2 The remaining are as follows.

$$\begin{array}{ccccc} 2 & 3 & 4 & \cdots & N. \\ H_1 & H_3 & H_4 & \cdots & H_N. \end{array}$$

If: (i) 2 gets H_1. $\begin{array}{ccccc} 2 & 3 & 4 & \cdots & N \\ \updownarrow & & & & \\ H_1 & H_3 & H_4 & \cdots & H_N. \end{array}$ the raminig people and hats have A_{N-2} choices to match.

(ii) 2 doesn't get H_1, which means we can see H_1 as H_2 in this match. (H_1, H_2 do both not belong to 2).. That says that it's equivalent as this problem: $\begin{array}{ccccc} 2 & 3 & 4 & \cdots & N. \\ H_2 & H_3 & H_4 & \cdots & H_N. \end{array}$. It has A_{N-1} choices.

match.

In conclusion $A_N=(N-1)(A_{N-1}+A_{N-2})$

Problem 7

Let E be the event that at least one lands on 6, and F be the evnet that the dice land on different numbers.

$$|F| = 36 - 6 = 30$$

$$|E \cap F| = 2 \times 5 = 10$$

$$\mathrm{P}(E|F) = \frac{|E \cap F|}{F} = \frac{10}{30} = \frac{1}{3}$$

Problem 8

Let E be the event that the ball chosen from A is white. and F be the event that 2 white balls exactly are selected

$$|F| = 4 \times 4 \times 1 + 4 \times 8 \times 3 + 2 \times 4 \times 3 = 136$$

$$|E \cap F| = 2 \times 4 \times 3 = 24$$

$$\mathrm{P}(E|F) = \frac{24}{136} = \frac{3}{17}$$

Problem 9

$$|A_s| = 51$$

$$|A| = \binom{52}{2} - \binom{48}{2} = 198$$

$$|B \cap A_s| = 3$$

$$|B \cap A| = \binom{4}{2}$$

(a) $\mathrm{P}(B|A_s) = \frac{|B \cap A_s|}{A_s} = \frac{3}{51} = \frac{1}{17}$

(b) $\mathrm{P}(B|A) = \frac{6}{198} = \frac{1}{33}$