Problem.1

(a): There are 26 letters and 10 numbers for each place. So the number is $26^2 \times 10^5$

(b): This is a permutation problem. The number is $A_{26}^2 * A_{10}^5$

Problem.2

(a): $A_8^8 = 8!$

(b): See A & B as a whole.Then the row includs 7 "persons" and there are A_2^2 arrangements in A&B. The final anwser is:

$$A_7^7 \times A_2^2$$

(c): First we put the men in a row, and then put women in the gaps of men(both ends can also be placed).

$$A_4^4 \times C_5^4 \times A_4^4$$

(d): Like (b), the anwser is $A_5^5 \times A_4^4$.

(e): $(A_2^2)^4 \times A_4^4$

Problem.3

$$\begin{aligned}(3x^2 + y)^5 &= (3x^2)^5 + 5(3x^2)^4 y + 10(3x^2)^3 y^2 + 10(3x^2)^2 y^3 + 5(3x^2) y^4 + y^5 \\ &= 243x^{10} + 405x^8 y + 270x^6 y^2 + 90x^4 y^3 + 15x^2 y^4 + y^5\end{aligned}$$

Problem.4

(a) First: choose a committee: $(\genfrac{}{}{0pt}{}{n}{k})$

then: in the committee we have k choices to choose the chair,namely we have $(\genfrac{}{}{0pt}{}{n}{k})k$ choices

(b) First: choose (k-1) nonchair members: $(\genfrac{}{}{0pt}{}{n}{k-1})$

then: choose chair person in the remaining $n-(k-1)$ members. Two steps yield $(\genfrac{}{}{0pt}{}{n}{k-1}) \times [n-(k-1)]$ choices

(c) First: choose the chairman from n people;

then choose k-1 nonchairmen from n-1 people. Two steps yields $\binom{n-1}{k-1}n$ choices.

(d) According (a)(b)(c), three numbers are equal

(e)

$$k\binom{n}{k} = k\frac{n!}{(n-k)!k!} = \frac{n!}{(n-k)!(k-1)!}$$

$$(n-k+1)\binom{n}{k} = (n-k+1)\frac{n!}{(n-k+1)!(k-1)!} = \frac{n!}{(n-1)!(k-1)!}$$

$$n\binom{n-1}{k-1} = n\frac{(n-1)!}{(n-k)!(k-1)!} = \frac{n!}{(n-k)!(k-1)!}$$

Problem.5 Considering $(x_1 + x_2 + \cdots + x_r)^n$, the cofficient of $x_1^{n_1}x_2^{n_2}\cdots x_r^{n_r}$ is $\binom{n}{n_1\ n_2\ \cdots\ n_r}$.We choose the term with the method as follows:

1. choose x_1 first,and then choose $x_1^{n_1-1}x_2^{n_2}\cdots x_r^{n_r}$, its confficient is $\binom{n-1}{n_1-1\ n_2\cdots\ n_r}$
2. choose x_2 first,and then choose $x_1^{n_1-1}x_2^{n_2}\cdots x_r^{n_r}$, its confficient is $\binom{n-1}{n_1\ n_2-1\cdots\ n_r}$

......, repeat this method until choose all x_i. The final cofficient is $\binom{n-1}{n_1-1\ n_2\ \cdots\ n_r} + \binom{n-1}{n_1\ n_2-1\ \cdots\ n_r} + \cdots + \binom{n-1}{n_1\ n_2\ \cdots\ n_r-1} = \binom{n}{n_1\ n_2\ \cdots\ n_r}$

Problem.6

(a): $\binom{10}{6}\times 2$

(b): $\binom{10}{3}\binom{7}{3}$

Problem.7

(a) According the collary of pigionhole principle, $\lceil\frac{200}{6}\rceil = 34$. That shows that anyone of these classes should give 34 positions at least.

(b) Choosing k items in n and abandoning n-k items in n are the same thing.

Problem.8

EF	$E \cup F$	FG	EF^c	EFG
one of two dice lands on 1,and the other one lands on an even number	one of dices lands on 1 or the sum of dice is odd	one lands on 1 and the other one lands on 4	the sum of dice is odd and 1 doesn't occur in two tosses	(same as EF) one of two dice lands on 1,and the other one lands on an even number

Problem.9

(a): sample space: $\{0g, 0f, 0s, 1g, 1f, 1s\}$ (0g represents that the patient has no insurance and is rated as good,other notions are similar)

(b): A: $\{0s, 1s\}$

(c): B: $\{0g, 0f, 0s\}$

(d): $B^c \cup A$: $\{1g, 1f, 1s, 0s\}$