MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) If government prints money instead of issuing bond, then inflation will be created and the currency will be devalued rapidly, yet issuing bond will keep the total amount of money the same, so currency will not be devalued (Ananish, 2021).

   (b) When the investors are uncertain about what will happen in the long future or they expect the long-term interest to fall, the long-term yield will be flatten because of lack of demand.

   (c) Quantitative easing is used when the short-term interest rate is or approaching to zero, the central bank purchase long-term bonds or different asset to increase the money supply and stimulate investment. On March 15, 2020, the Fed said that it would buy over 500 billion in Treasury securities and 200 billion in government-guaranteed mortgage-backed securities (Eric and Wessel 2021).

2. I will choose bonds: CAN 1.5 Feb 1 2022, CAN 0.25 Aug 1 2022, CAN 0.25 Feb 1 2023, CAN 0.25 Aug 1 2023, CAN 2.25 Mar 1 2024, CAN 1.5 Sep 1 2024, CAN 1.25 Mar 1 2025, CAN 0.5 Sep 1 2025, CAN 0.25 Mar 1 2026, CAN 1 Sep 1 2026.
   I chose them by timing. Since coupon is paid semi-annually, I need to choose two bonds per year. To be specific, the minimal maturity date difference between each bonds are 6 months (except the 4_{th} and the 5_{th} bond). So, I am able to calculate the yield every six months and get 10 yield to maturity from 2021 to 2026 in order to make the yield to maturity curve distributed evenly. Besides, an exact 6 months difference is easier for me to calculate the spot rate.

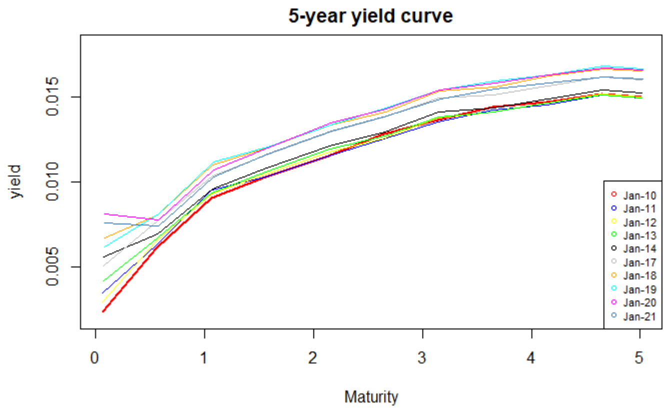
3. The eigenvalue of the covariance matrix tells us the importance of its eigenvector (the greater the eigenvalue more important the its eigenvector is)(Zakaria, 2021). To be specific, the eigenvector of the greatest eigenvalue represents the direction where points have the largest variance. In this case, it is the direction where the stochastic process shift the most.

Empirical Questions - 75 points

4.

   (a) To calculate the yield, first calculate the dirty price : DP = $\frac{n}{365}CPN$ + P, where CPN is the annual coupon, P is the clean price, n is the times after the last coupon payment, DP is the dirty price. Then get the ytm for each bond: DP = $\sum_{n=1}^{i} p_n e^{-rt_n} + Fe^{-rt_i}$, where p_n is the coupon payment, F is the face value, t_n is the time between the coupon payment and today, unit is year.
   To get the yield curve, I used the linear interpolation method to estimate the unknown yield since the yield rate does not change a large amount over years, where linear interpolation can

be accurate and easy to calculate the unknown variable. For unknown r_i, if it has a time that is between the known yield r_{j-1} and r_j, the formula is:
$r_i = r_{j-1} + (t_i - t_{j-1})\frac{r_{j+1}-r_j}{t_{j+1}-t_j}$
Below is the graph:

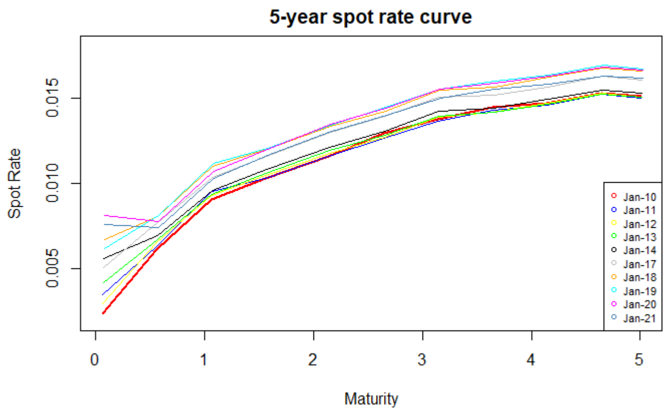


(b) To calculate the spot rate, First use the bond that is mature in less than 6 months (no coupon payment) to calculate r_1 using the formula: $r_1 = -\frac{log(P/N)}{T}$ $0 < T < \frac{1}{2}$, P is dirty price of the bond, N is the notional of the bond
For the second bond, get r_2 using formula $P = p_1 e^{-r_1 t_1} + (F + p_2)e^{-r_2 t_2}$, P is the dirty price for the second bond, p_i is the coupon payment, F is the face value. Then choose the remaining bond by descending order of maturity. For $bond_i$, there are two cases:

i. If we know r_{i-1}, r_{i-2} ...r_2, (the spot rate that matures excatly a multiple of six months before the maturity date of the bond), then use $P = \sum_{n=1}^{i} p_n e^{-r_n t_n} + Fe^{-r_i t_i}$ to get r_i

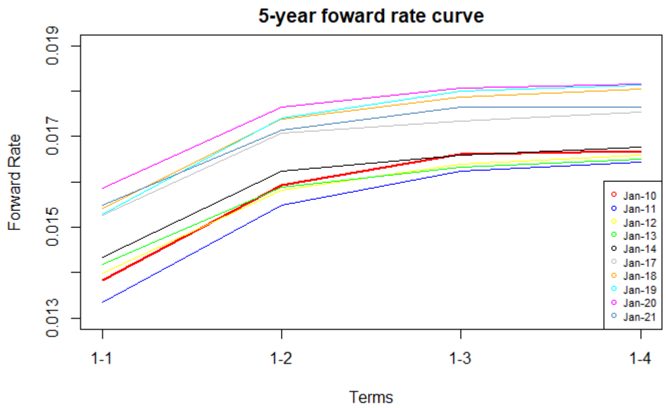
ii. If any spot rate in a multiple of 6 months before the bond i is unknown, we need to use linear interpolation or extrapolation to estimate the unknown spot rate before by the known spot rate, then we can get r_i for bond i use the formula $P = \sum_{n=1}^{i} p_n e^{-r_n t_n} + Fe^{-r_i t_i}$

I used linear interpolation to sketch the graph:



(c) To calculate the forward rate from t_j to t_i, the equations are:
$e^{r_i t_i} = e^{(r_j t_j)}\ e^{r_{j,i} * (t_i - t_j)} \Rightarrow r_{j,i} = \frac{t_i r_i - t_j r_j}{t_i - t_j}$
To calculate the forward rate: Using the linear interpolation or extrapolation to estimate the spot rate that has the exact maturity 1 to 5 years away from today, denote them as r_1, r_2, r_3, r_4, r_5 respectively. Then use the formula $r_{1,i} = \frac{t_i r_i - r_j}{t_i - 1}$ to 4 times get 4 spot rates, where i is r_2, r_3, r_4, r_5 respectively, t_i is 2, 3, 4, 5 respectively. Return $r_{1,2}$, $r_{1,3}$, $r_{1,4}$, $r_{1,5}$, and using linear

interpolation to sketch the curve:



5. I used linear interpolation to estimate the yield that has the exact maturity 1 to 5 years away from today. Below is the covariance matrix:

$$\begin{bmatrix} 0.0020232617 & 0.0008939932 & 0.0007940936 & 0.0007874243 & 0.0008550081 \\ 0.0008939932 & 0.0007634641 & 0.0006829931 & 0.0005990590 & 0.0005974618 \\ 0.0007940936 & 0.0006829931 & 0.0006429441 & 0.0005765369 & 0.0005649297 \\ 0.0007874243 & 0.0005990590 & 0.0005765369 & 0.0005507495 & 0.0005309745 \\ 0.0008550081 & 0.0005974618 & 0.0005649297 & 0.0005309745 & 0.0005325842 \end{bmatrix}$$

Below is the covariance matrix for the time series of daily log-returns of forward rate:

$$\begin{bmatrix} 0.0010575563 & 0.0007482362 & 0.0005318826 & 0.0005192548 \\ 0.0007482362 & 0.0006398266 & 0.0005218941 & 0.0005149787 \\ 0.0005318826 & 0.0005218941 & 0.0004905160 & 0.0004830991 \\ 0.0005192548 & 0.0005149787 & 0.0004830991 & 0.0004857924 \end{bmatrix}$$

6. The eigenvectors of the covariance matrix for the time series of daily log-returns of yield are:

$$\begin{bmatrix} -0.6651977 \\ -0.4071993 \\ -0.3728447 \\ -0.3511058 \\ -0.3597391 \end{bmatrix} \quad \begin{bmatrix} 0.7360857 \\ -0.4000593 \\ -0.4214355 \\ -0.2941506 \\ -0.1843854 \end{bmatrix} \quad \begin{bmatrix} 0.063768736 \\ 0.716513119 \\ 0.002830617 \\ -0.604904526 \\ -0.341504029 \end{bmatrix} \quad \begin{bmatrix} -0.01637848 \\ -0.30956144 \\ 0.49942362 \\ -0.64313579 \\ 0.49077068 \end{bmatrix} \quad \begin{bmatrix} -0.1065606 \\ 0.2547978 \\ -0.6587465 \\ -0.1032810 \\ 0.6921770 \end{bmatrix}$$

The eigenvalues of the covariance matrix for the time series of daily log-returns of yield are: 3.893617e-03; 5.538922e-04; 5.521884e-05; 6.260282e-06; 4.015310e-06

The eigenvectors of the covariance matrix for the time series of daily log-returns of forward rate are:

$$\begin{bmatrix} -0.6204862 \\ -0.5142152 \\ -0.4214733 \\ -0.4158605 \end{bmatrix} \quad \begin{bmatrix} 0.71204789 \\ -0.05345208 \\ -0.48308562 \\ -0.50671389 \end{bmatrix} \quad \begin{bmatrix} -0.3285960 \\ 0.8547288 \\ -0.2441722 \\ -0.3191290 \end{bmatrix} \quad \begin{bmatrix} -0.003061882 \\ 0.046522779 \\ -0.727577156 \\ 0.684439725 \end{bmatrix}$$

The eigenvalues of the covariance matrix for the time series of daily log-returns of forward rate are: 2.386943e-03, 2.710175e-04, 1.080356e-05, 4.926819e-06

The eigenvector of the greatest eigenvalue implies the direction of the largest variance of points. In this case, it is the direction where each curve shift the most among different days. Since the entry of both largest eigenvectors have the same sign and similar value. The largest shift among different days are likely to be the parallel shift for the YTM and forward curve.

References and GitHub Link to Code

Chaudhuri, Ananish. “Explainer: Why the Government Can’t Simply Cancel Its Pandemic Debt by Printing More Money.” The Conversation, 24 Nov. 2021, https://theconversation.com/explainer-why-the-government-cant-simply-cancel-its-pandemic-debt-by-printing-more-money-148514.

Jaadi, Zakaria. “A Step-by-Step Explanation of Principal Component Analysis (PCA).” Built In, 1 Dec. 2021, https://builtin.com/data-science/step-step-explanation-principal-component-analysis.

Milstein, Eric, and David Wessel. “What Did the Fed Do in Response to the COVID-19 Crisis?” Brookings, Brookings, 17 Dec. 2021, https://www.brookings.edu/research/fed-response-to-covid19/.

github: https://github.com/Chenzzhang/APM-466-A1