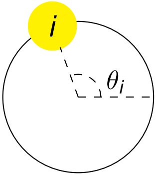
More Oscillators and More Information Theory



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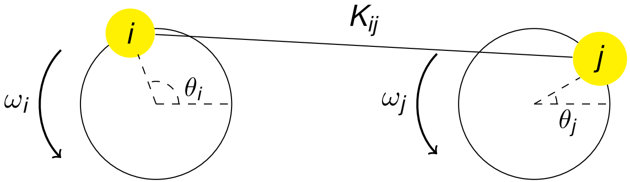
METASTABILITY

KURAMOTO OSCILLATORS



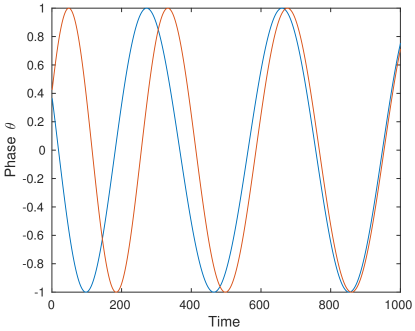
$$\frac{d\theta_i}{dt} = \omega + \frac{1}{\kappa + 1} \sum_j K_{ij} \sin\left(\theta_j - \theta_i - \alpha\right)$$

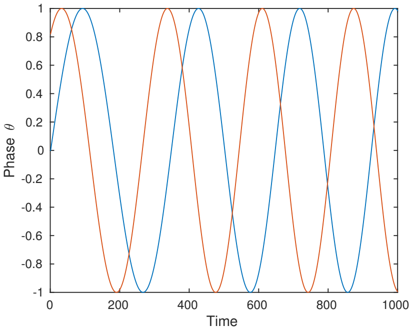
- We'll manipulate the *phase lag* α
  $\rightarrow$ Which we'll often reparametrise as $\beta = \pi/2 - \alpha$

KURAMOTO OSCILLATORS

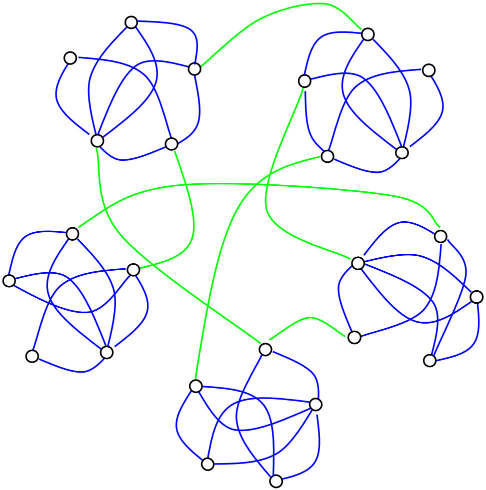
We say that the system has two **attractors**.

- Large $\alpha \approx \pi$ leads to a desynchronised phase
- Small $\alpha \approx 0$ leads to a hypersynchronised phase





NETWORKS OF KURAMOTO OSCILLATORS



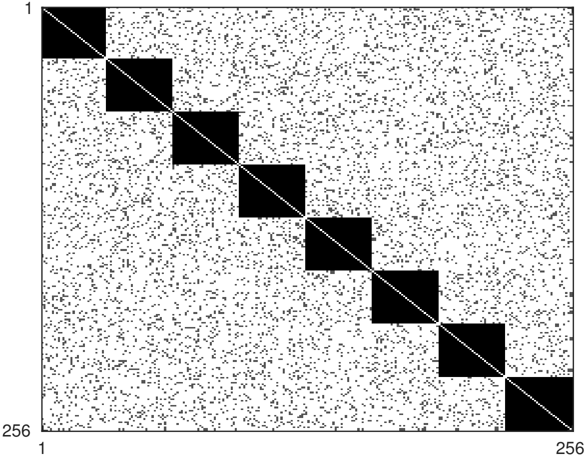
NETWORKS OF KURAMOTO OSCILLATORS

Strong **community structure**:

- Dense coupling
- Stronger connections

Weak **diffuse connections**:

- Uniform, random
- Sparse and weak



SYNCHRONISATION

- We define the order parameter

$$R_c(t) = \left| \langle e^{i\,\theta_j(t)} \rangle_{j \in c} \right|$$

- This measure is
  - Instantaneous: one value per timestep
  - Local: one value per community.

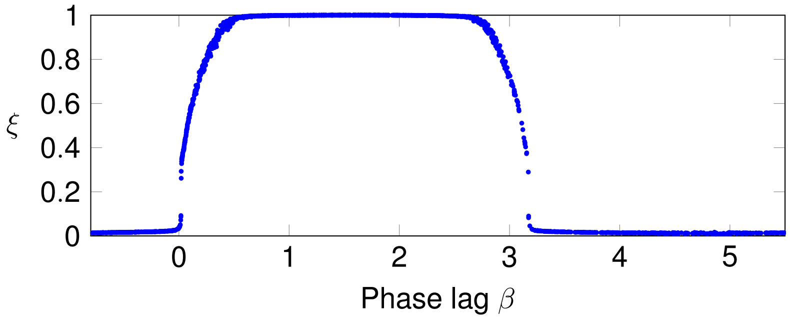
- We will often use **global synchrony**, defined as

$$\xi = \langle R_c(t) \rangle_{c,t}$$

SYNCHRONISATION (ξ)

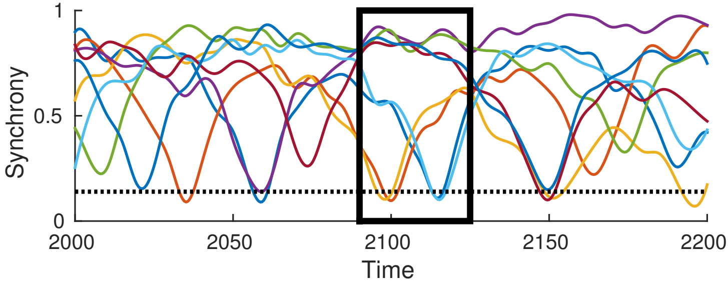
There is a **phase transition**:

- The system changes from ordered to disordered
- The narrow transitions represent a critical regime



CHIMERALITY

- **Chimera state**: the system spontaneously partitions into synchronised and desynchronised subsets.



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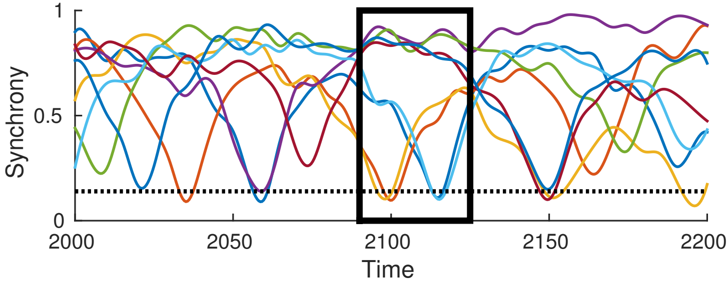

How well do synchrony and desynchrony coexist?

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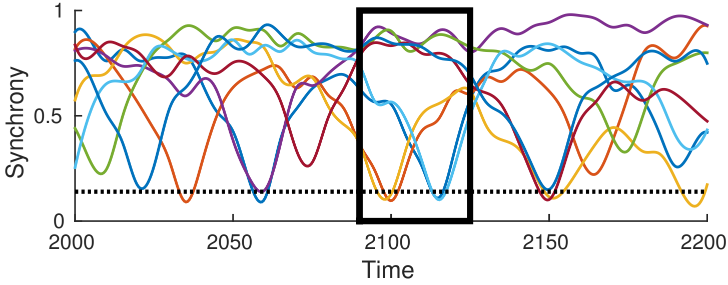

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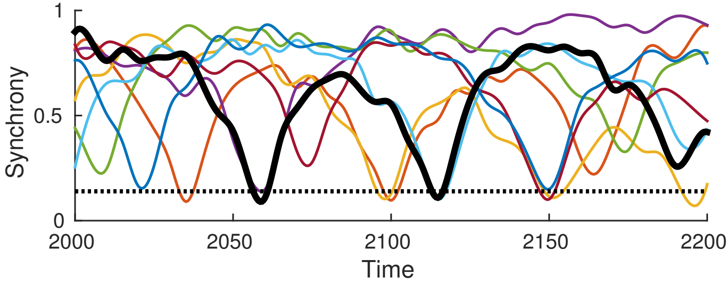
METASTABILITY

- **Metastable state**: the system remains in a mixed state, in the vicinity of basins of attraction but never falling in.



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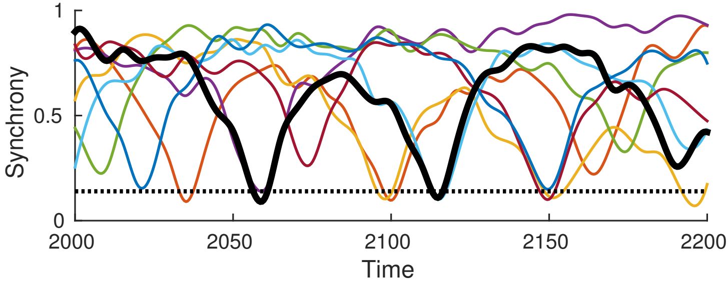


How much do groups fluctuate in and out of synchrony?

METASTABILITY AND CHIMERALITY

A MATHEMATICAL DEFINITION

- **Chimera index**: spatial variance of synchronisation,

$$\chi = \langle \mathrm{var}_c \, R_c(t) \rangle_{t \in T} \; .$$

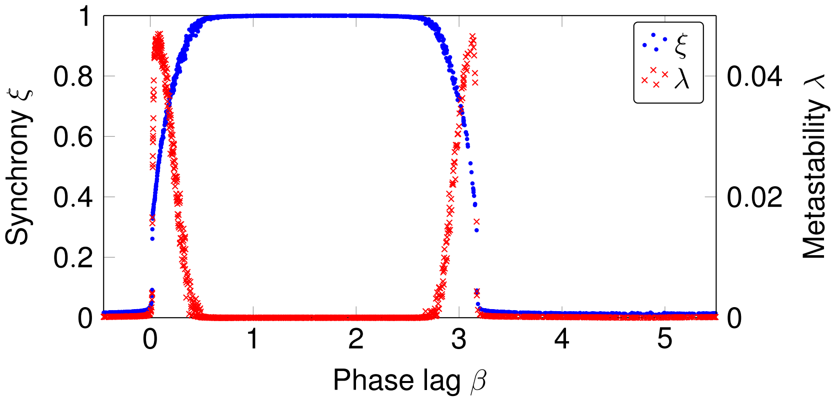
- **Metastability**: temporal variance of synchronisation,

$$\lambda = \langle \mathrm{var}_t \, R_c(t) \rangle_{c \in C} \; .$$

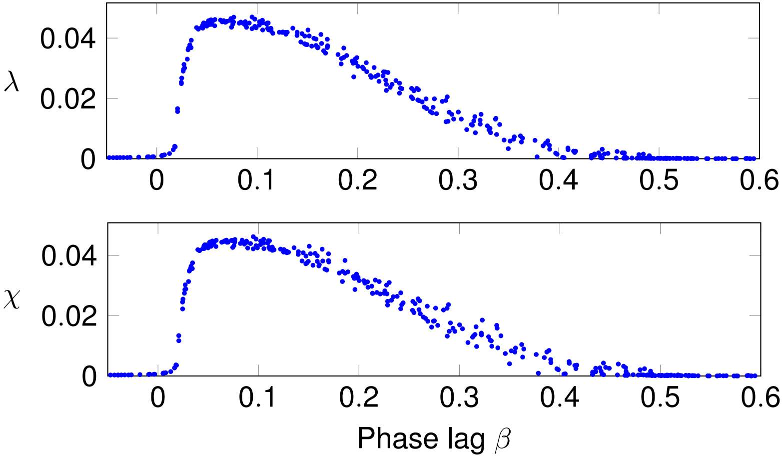
PHASE TRANSITIONS

EVEN BETTER!

The critical regions are strongly metastable.



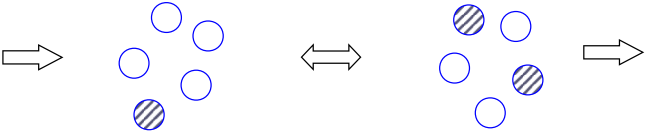
METASTABILITY (λ) AND CHIMERALITY (χ)



Metastability and Chimerality

What do they mean?

- *Coalition formation* is metastable. Coalitions arise, are temporarily stable, and then break up as new coalitions form.



- This sort of dynamical system will be capable of visiting a *large repertoire* of states.

EFFECTIVE FREQUENCY

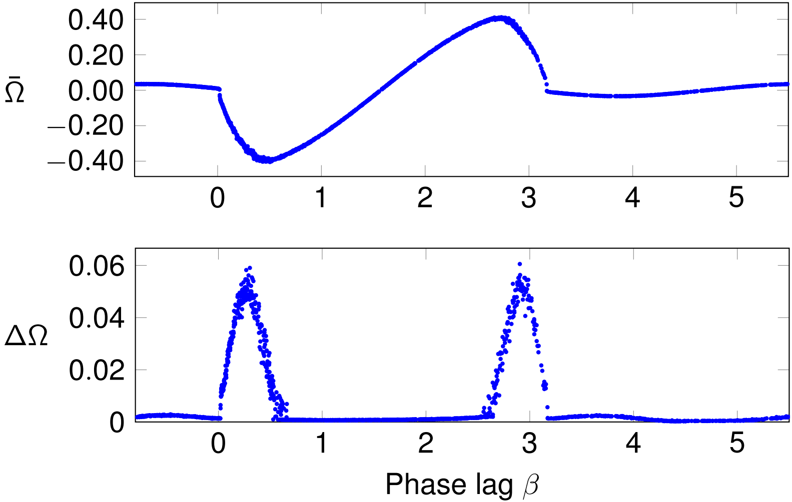
- Synchronisation is not the only interesting quantity!
- We can investigate *effective frequency* — i.e. how fast do oscillators actually move.

$$\Omega_i(t) = \left.\frac{d\theta_i}{dt'}\right|_{t'=t} - \omega_i$$

$$\bar{\Omega} = \langle \Omega_i(t) \rangle_{t,i}$$

$$\Delta\Omega = \mathrm{var}_i \langle \Omega_i(t) \rangle_t$$

EFFECTIVE FREQUENCY



Integrated Information

WHAT IS INTEGRATED INFORMATION?

- Consciousness "emerges from bits of information."
- From philosophical axioms, to an informational measure of conscious level.
- We're **not** making claims about consciousness here.
  $\rightarrow$ Still a valid measure of dynamical complexity.

Giulio Tononi

INFORMATION THEORY

- Information Theory is a powerful tool to study complex systems.

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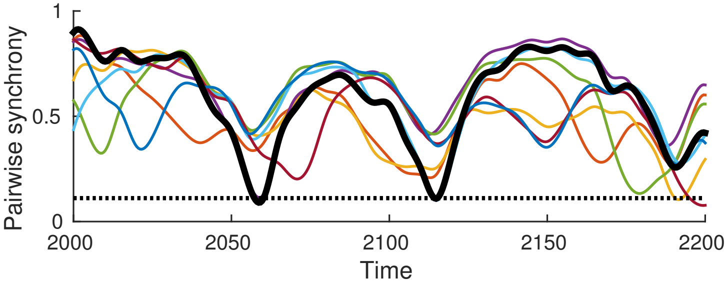
Caveat:

- Estimating continuous PDF's is often a hard problem.
- 256 real-valued phases is way too hard to do IT-things.

PAIRWISE SYNCHRONY

- When two communities are highly internally synchronised they tend to be highly synchronised with each other.



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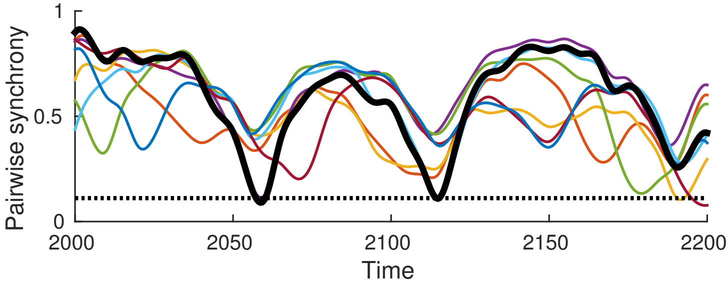

- Working hypothesis: we characterise the whole network by the internal synchrony of each module.

COALITIONS

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- We'll take the *coalitions* as our informational state:

$$X_c(t) = \begin{cases} 1 & \text{if } R_c(t) > \gamma \\ 0 & \text{otherwise} \end{cases}$$

Where γ is the *coalition threshold*.

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Where γ is the *coalition threshold*.

- Under this assumption the state of the system is reduced to 8 binary variables.
  $\rightarrow$ Not always the case, but still a good proxy.

COALITIONS AND ENTROPY

- Measures uncertainty associated with a variable X.
- Measures the size of the repertoire of states visited by the system.

$$H(X) = \sum_i p(x_i) \log p(x_i)$$

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How spread is the distribution of X?

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- A synchronised system is always in the "all 1's" coalition.
  $\rightarrow H_C = 0$
- A desynchronised system is always in the "all 0's" coalition.
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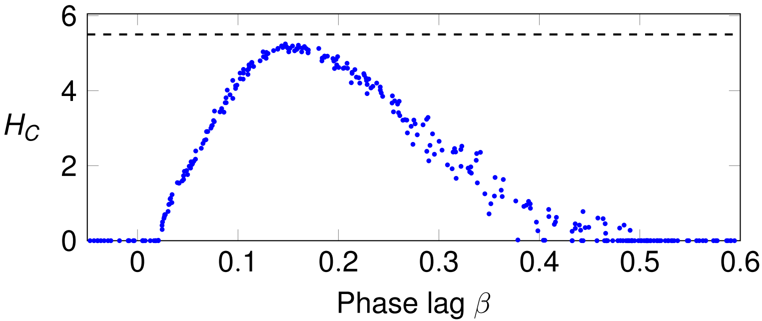
- A critical system explores all possible states.
  $\rightarrow H_C \approx \log |X|$



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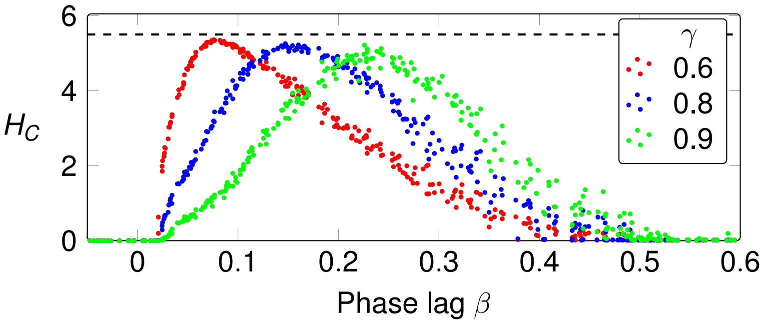

MUTUAL INFORMATION

- Measures interdependence between two variables.
- Technically, measures similarity between the *joint* $p(x, y)$ and the *marginals* $p(x)p(y)$
- Is null when X and Y are independent.

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How does knowing X reduce your uncertainty about Y?

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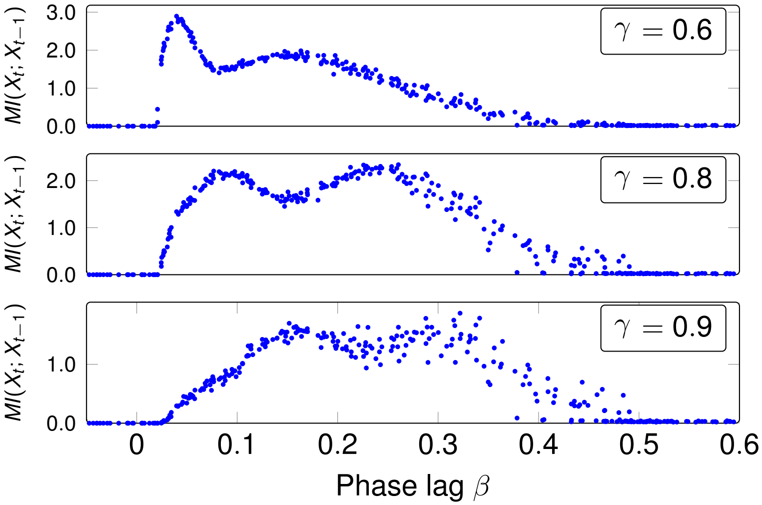
How spread is the distribution of X?

What's the maximum information X could possibly have about anything?

MUTUAL INFORMATION



EFFECTIVE INFORMATION

- Effectively, the building block of integrated information Φ.

$$\varphi[X;\tau,\mathcal{P}] = MI(X_{t-\tau}, X_t) - \sum_{k=1}^{p} MI(M^k_{t-\tau}, M^k_t)$$

(Tononi 2008)

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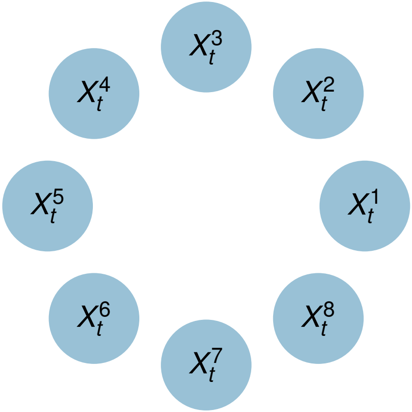
How much predictive information do you lose if you split the system in p parts?

(Tononi 2008)

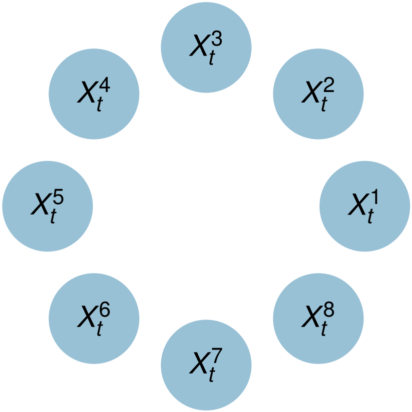
BUILDING Φ

- Scan all possible partitions of X



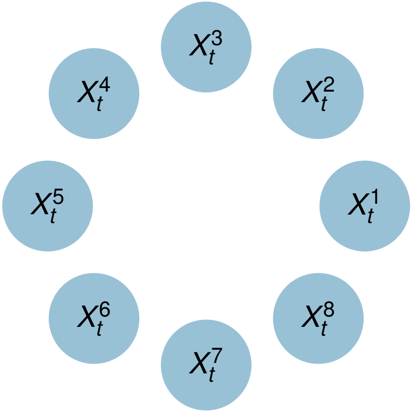
BUILDING Φ

- Scan all possible partitions of X
- Select the one with minimum φ/K



BUILDING Φ

- Scan all possible partitions of X
- Select the one with minimum φ/K
- That partition's φ is the system's Φ



BUILDING Φ
(WITH MATHS)

$$\Phi[X,\tau] = \varphi[X;\tau,\mathcal{B}^{\mathrm{MIB}}]$$

$$\boxed{\mathcal{B}^{\mathrm{MIB}}} = \arg_{\mathcal{B}} \min \frac{\varphi[X;\tau,\mathcal{B}]}{K(\mathcal{B})}$$

$$\boxed{K(\mathcal{B})} = \min\left\{H(M^1), H(M^2)\right\},$$

If you were to split the system where you lose the least predictive information, how much would you lose?

BUILDING Φ

(WITH MATHS)

Minimum Information Bipartition

$$\Phi[X, \tau] = \varphi[X; \tau, \mathcal{B}^{\mathrm{MIB}}]$$

Normalisation Factor

$$\boxed{\mathcal{B}^{\mathrm{MIB}}} = \mathrm{arg}_{\mathcal{B}} \min \frac{\varphi[X; \tau, \mathcal{B}]}{K(\mathcal{B})}$$

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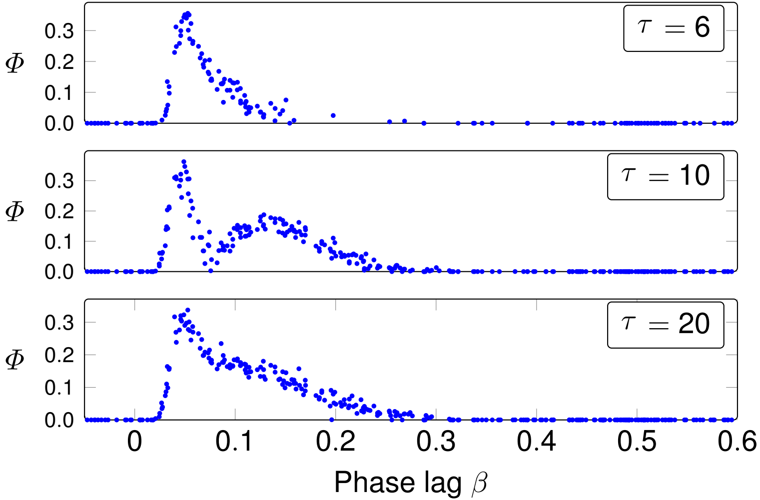
INTEGRATED INFORMATION



ESTIMATION FROM CONTINUOUS DATA

Completely legitimate question:

- Aren't we losing information by using only the coalitions?

Completely legitimate answer:

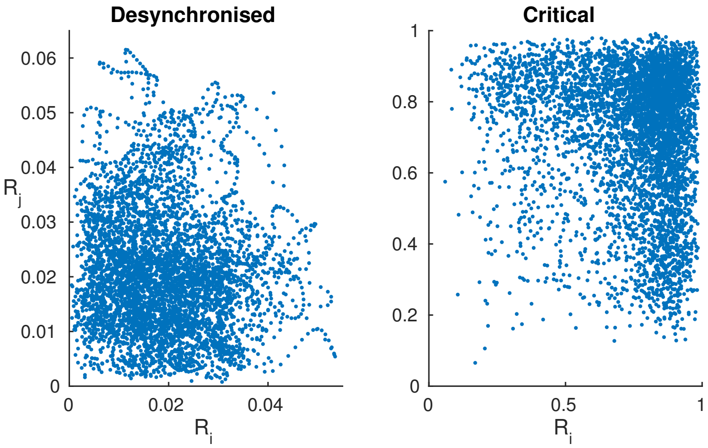
- Probably.

Two natural candidates:

- Non-parametric estimators, i.e. KSG.
- Linear-Gaussian estimators, i.e. Barrett & Seth's Φ_{AR}.

ESTIMATION FROM CONTINUOUS DATA

Scatter plots reveal non-linear correlation between modules.
$\rightarrow$ The validity of Φ_{AR} might be compromised



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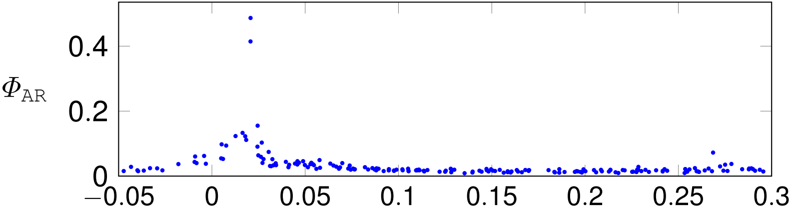

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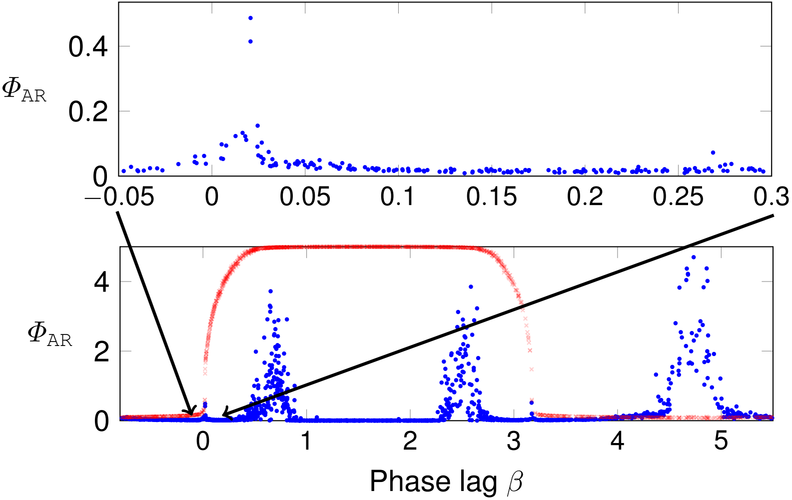

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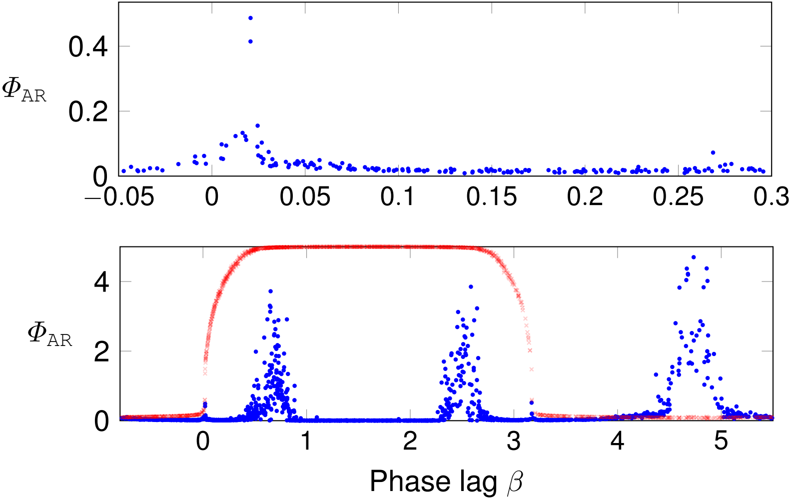

DISCUSSION