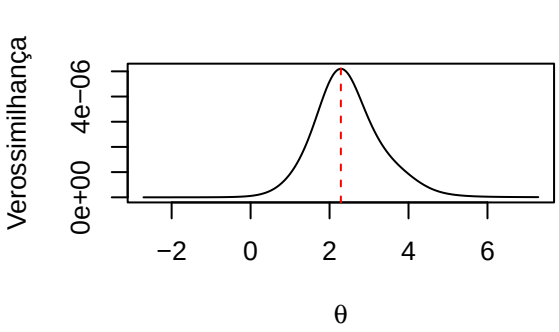
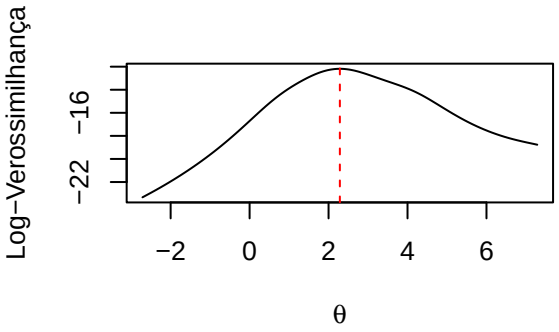
9. Seja X_i para $i = 1, \ldots, n$ iid com pdf

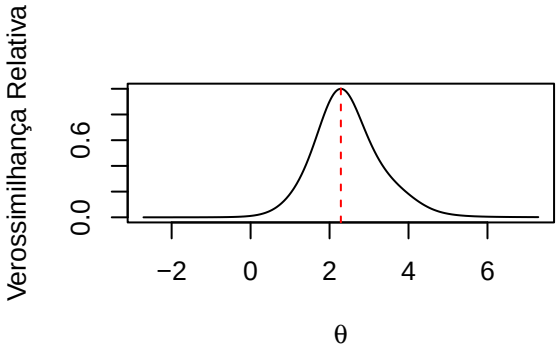
$$f(x;\theta) = \pi^{-1}\{1 + (x-\theta)^2\}^{-1} : \quad -\infty < x < \infty.$$

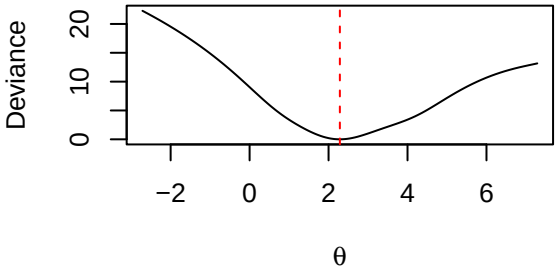
Desenhe o gráfico da verossimilhança, log-verossimilhança, verossimilhança relativa e deviance no caso $n = 5$ com as seguintes observações x_i : $4.2; 0.7; 2.5; 1.9; 7.9$. Encontre o limite inferior de Cramér-Rao.

```r
x <- c(4.2, 0.7, 2.5, 1.9, 7.9)
log_like <- function(theta) {
  sum(dcauchy(x, location = theta, scale = 1, log = TRUE))}

mle_fit <- optimize(log_like, interval = c(min(x) - 1, max(x) + 1), maximum = TRUE)
theta_hat <- mle_fit$maximum
max_ll <- mle_fit$objective

like <- function(theta) {
  exp(log_like(theta))}
rel_like <- function(theta) {
  exp(log_like(theta) - max_ll)}
deviance <- function(theta) {
  -2 * (log_like(theta) - max_ll)}
theta_range <- seq(theta_hat - 5, theta_hat + 5, length.out = 500)

par(mfrow = c(2, 2))
plot(theta_range, sapply(theta_range, like), type = "l",
     xlab = expression(theta), ylab = "Verossimilhança")
abline(v = theta_hat, col = "red", lty = 2)
plot(theta_range, sapply(theta_range, log_like), type = "l",
     xlab = expression(theta), ylab = "Log-Verossimilhança")
abline(v = theta_hat, col = "red", lty = 2)
plot(theta_range, sapply(theta_range, rel_like), type = "l",
     xlab = expression(theta), ylab = "Verossimilhança Relativa")
abline(v = theta_hat, col = "red", lty = 2)
plot(theta_range, sapply(theta_range, deviance), type = "l",
     xlab = expression(theta), ylab = "Deviance")
abline(v = theta_hat, col = "red", lty = 2)
```


Verossimilhança
0e+00
4e−06
−2
0
2
4
6
θ


Log−Verossimilhança
−22
−16
−2
0
2
4
6
θ


Verossimilhança Relativa
0.0
0.6
−2
0
2
4
6
θ


Deviance
0
10
20
−2
0
2
4
6
θ