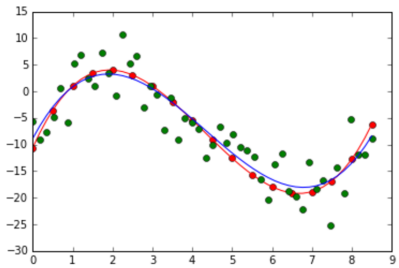
Exercise 2: Regression Using Least Squares

Write a program in python (ipython-notebook) to implement the least squares solution for the polynomial curve fitting problem. Follow the instructions below:

1. Generate 50 2D-data points using the following function: $y = 0.4345x^3 - 5.607x^2 + 16.78x - 10.61$
2. Add Gaussian random noise to the data. Look at the sample code here:

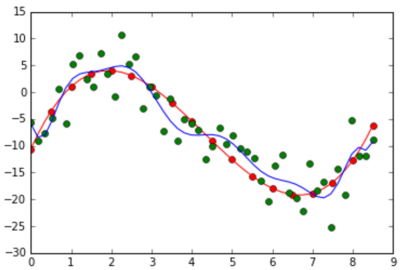
```
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
from scipy import linalg

def actual_distribution():
    x = np.arange(0, 9, 0.5);
    y = 0.4345*np.power(x,3) - 5.607*np.power(x,2) + 16.78*x - 10.61
    return x,y

def add_noise(y_curve):
    mu = 0
    sigma=4.0
    noise = np.random.normal(mu, sigma, len(y_curve))
    y_noise = y_curve + noise
    return y_noise
```

3. Fit the generated noisy data using the least squares method based on a polynomial function. You must write two versions of the least squares solution:
   (1) $\bar{w} = \left(\mathbb{X}^T\mathbb{X}\right)^{-1}\mathbb{X}^T\bar{t}$
   (2) $\bar{w} = \mathbb{X}^+\bar{t}$ or using numpy's pinv function
4. Show the original curve line, the noisy data, and the curve line estimated from the noisy data, like a figure below:



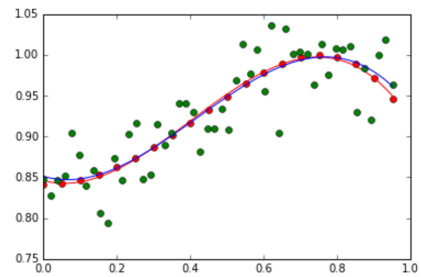
The red line is the original curve based on the equation. The green dots are the noisy data. The blue line is the estimated fitting line, by setting M = 4.

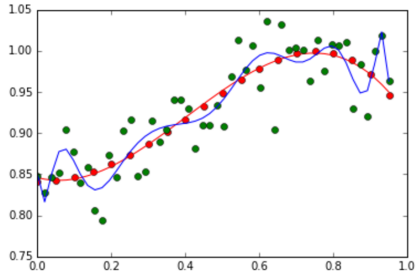
5. Compute and display the error value, E(w)
6. Display the estimated values of w
7. Experiment with your code by changing M to various values. Plot the values of M against those of E(w), meaning: project them onto a 2D space, where the x-axis is M and the y-axis is E(w). Try to understand the cases of underfitting and overfitting, and how they relate to the error value, E(w)



An example of the overfitting problem.

8. Change the noise levels, and apply the the same operations above
9. Change the function for generating the data to: $y = \sin(x^2 + 1)$, and apply the same operations as above





In your solutions, aside from the code, you need to include or show the results (as instructed above) in the ipython-notebook format.