Pattern Recognition

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Outlines

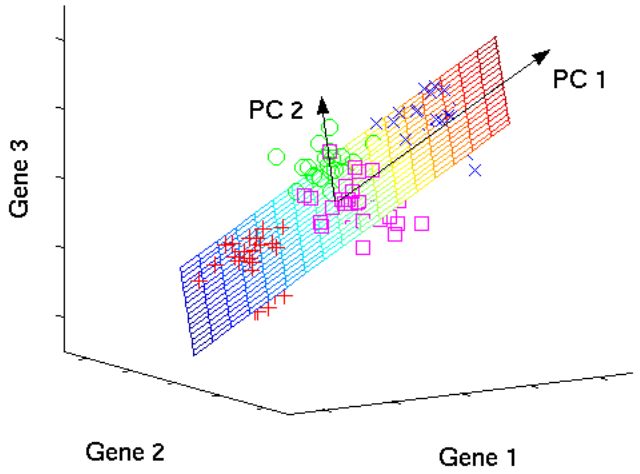
- Unsupervised Feature Extraction (PCA, NMF,…)
- **Supervised Feature Extraction (LDA, GE, …)**
- Clustering and Applications
- Gaussian Mixture Model
- Support Vector Machine
- Deep Learning

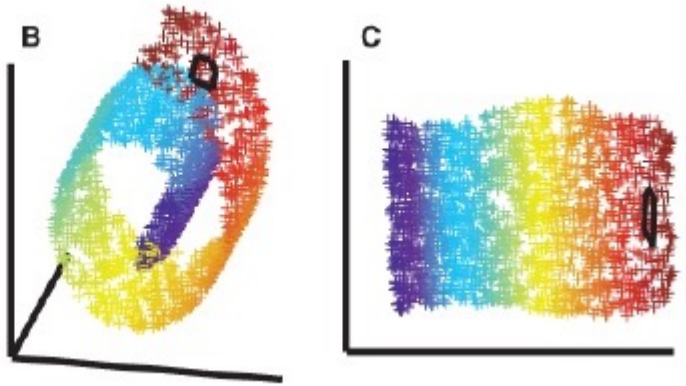
Some slides from Shuicheng Yan, Ricardo Gutierrez-Osuna, Dong Xu

Supervised vs. Unsupervised Learning

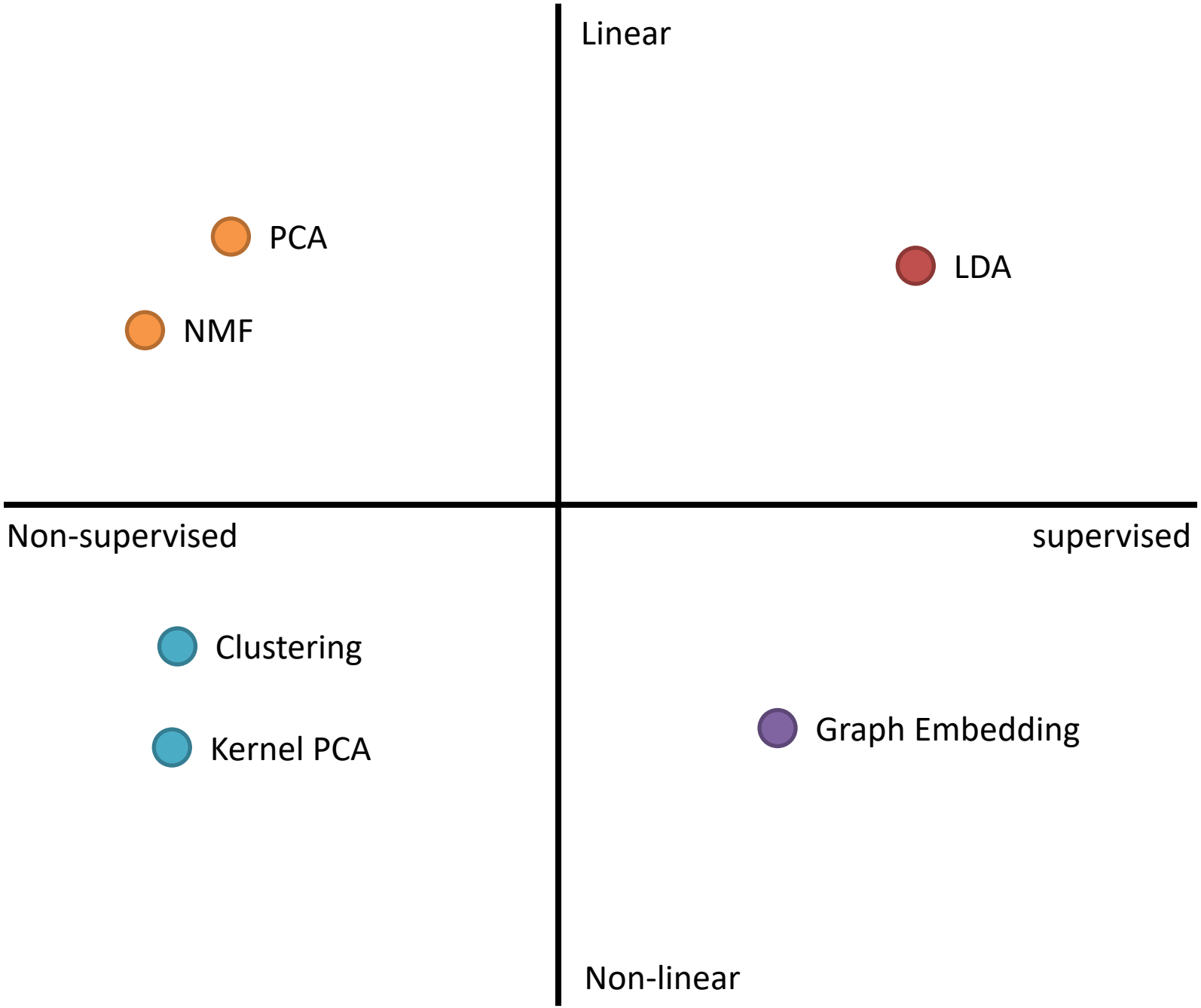
- Supervised learning
  - Supervision: The data (observations, measurements, etc.) are labeled with pre-defined classes. It is like a "teacher" gives the classes (supervision).
  - Test data are classified into these classes too.
- Unsupervised learning
  - Class labels of the data are unknown
  - Given a set of data, the task is to find subspaces, latent factors, clusters in the data.

Linear Subspace vs. Manifold



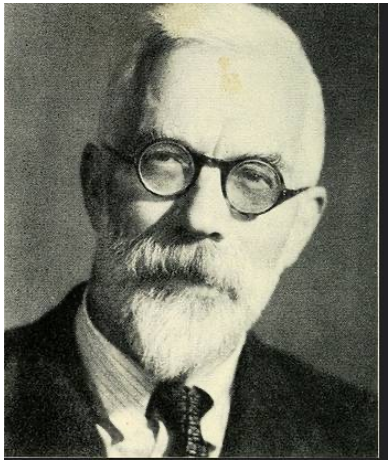


Unfolding the Swiss roll


Linear
PCA
LDA
NMF
Non-supervised
supervised
Clustering
Graph Embedding
Kernel PCA
Non-linear

A Bit History

- Linear Discriminative Analysis
  - Fisher's Linear Discriminative Analysis (1936)
    - Fisher, R. A. (1936). "The Use of Multiple Measurements in Taxonomic Problems". Annals of Eugenics.
  - Linear Discriminative Analysis (1948)
    - Rao, R. C. (1948). "The utilization of multiple measurements in problems of biological classification". Journal of the Royal Statistical Society, Series B.
- Manifold Learning
  - Isomap
    - Tenenbaum, et al. "A Global Geometric Framework for Nonlinear Dimensionality Reduction", Science 2000.
  - Locally Linear Embedding (LLE)
    - Roweis, et al, "Nonlinear Dimensionality Reduction by Locally Linear Embedding", Science 2000.
  - Graph Embedding (GE)
    - Yan et al., Graph embedding and extensions, TPAMI, 2007.



Ronald Fisher (1890-1962)

"A genius who almost single-handedly created the foundations for **modern statistical science**" – Anders Hald

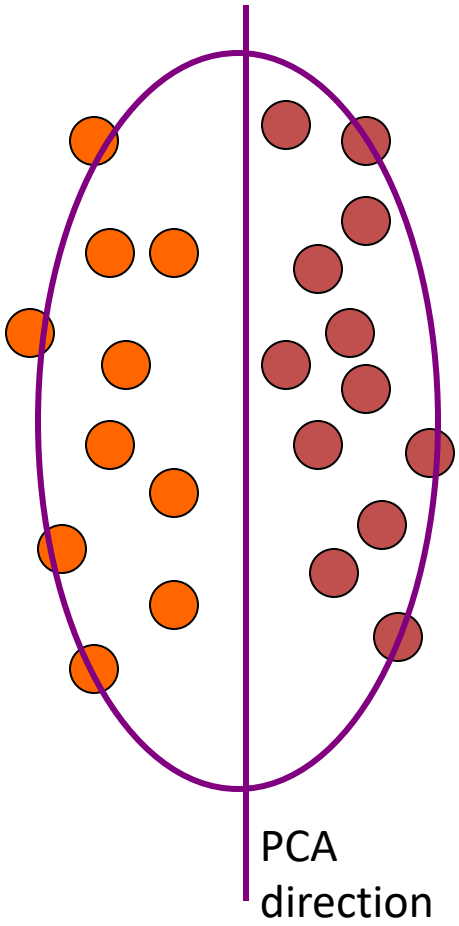
Supervised Feature Extraction:

Linear Discriminant Analysis

Is PCA a Good Criterion for Classification?

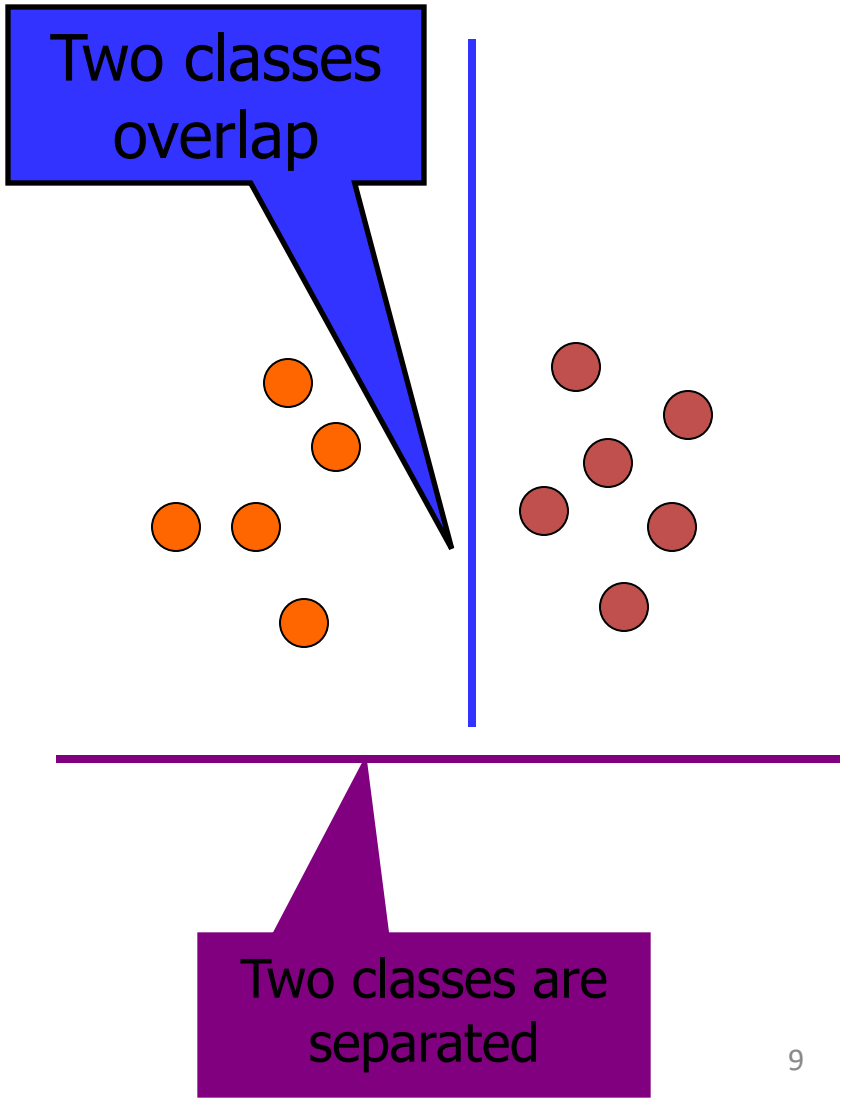
- Data variation determines the projection direction
- What's missing?
  - Class information

Why is class information useful?



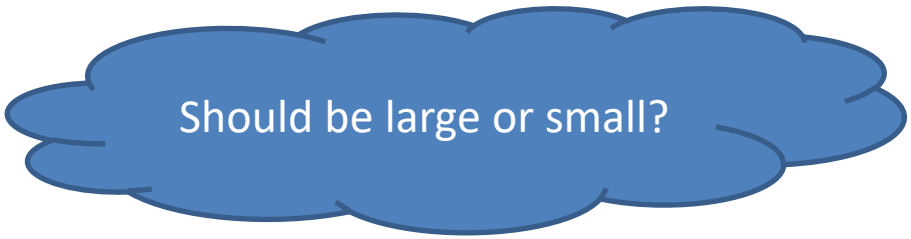
What is a Good Projection?

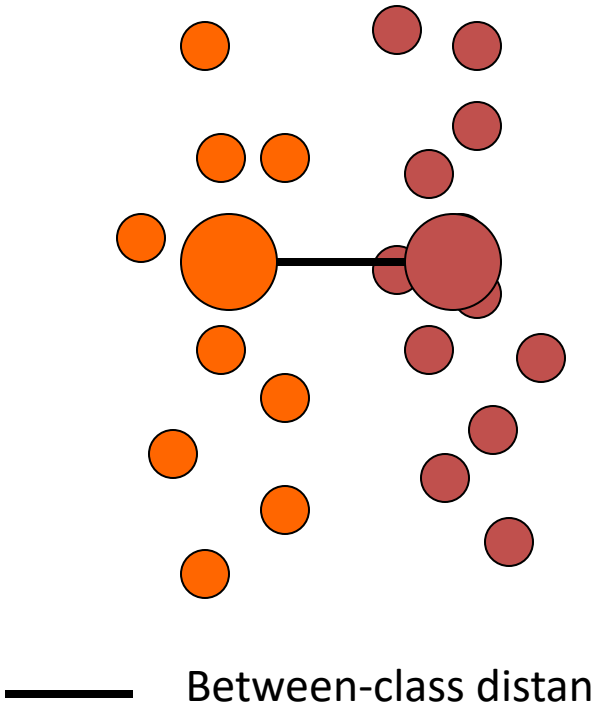
- What is a good criterion?
  - Separating different classes



What Class Information May be Useful?

- Between-class distance
  - Distance between the centroids of different classes

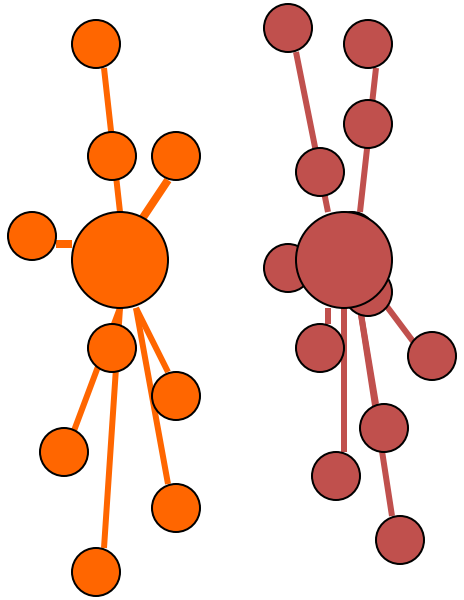




What Class Information May be Useful?

- Between-class distance
  - Distance between the centroids of different classes
- Within-class distance
  - Accumulated distance of an instance to the centroid of its class

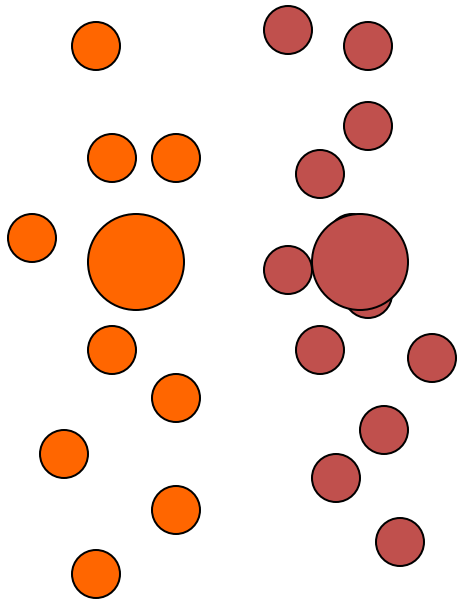
Should be large or small?



Within-class distance

Linear Discriminant Analysis

- **Linear discriminant analysis** (LDA) finds most discriminative projection by maximizing between-class distance and minimizing within-class distance



Linear Discriminant Analysis

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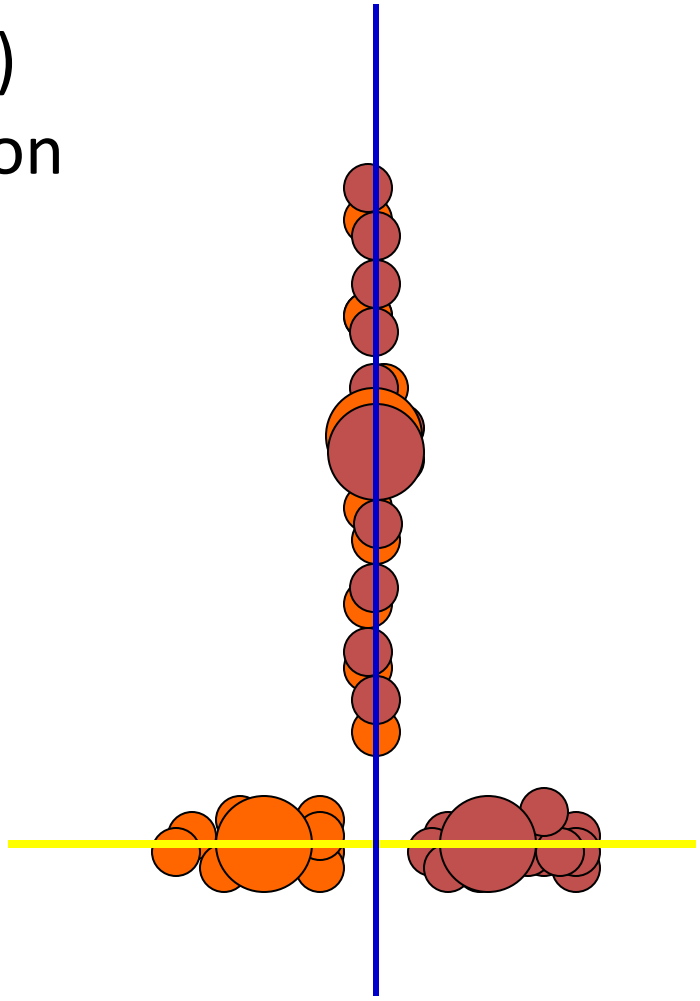


Statistical Facts (1)

Quantity for Measuring Within and Between Class Distance

- Class-specific mean vector (sample):

$$\mu_i = \frac{1}{n_i} \sum_{\mathbf{x} \in C_i} \mathbf{x},\ n_i \text{ is the size of class } C_i.$$

What are x, μ, μ_i, n_i N?

- Class-specific covariance (scatter) matrix:

$$\mathbf{S}_i = \frac{1}{n_i} \sum_{\mathbf{x} \in C_i} (\mathbf{x} - \mu_i)(\mathbf{x} - \mu_i)^T$$

- Total mean vector (sample):

$$\mu = \frac{1}{N} \sum_{\mathbf{x}} \mathbf{x},\ N \text{ is the number of all samples.}$$

Statistical Facts (2)

Quantity for Measuring Within and Between Class Distance (cont.)

- Within-class scatter:

$$\mathbf{S}_W = \sum_{i=1}^{C} \frac{n_i}{N} \mathbf{S}_i = \sum_{i=1}^{C} P_i \mathbf{S}_i, \; C \text{ is the class number.}$$

An estimate of the prior probability for class *i*

- Between-class scatter:

$$\mathbf{S}_B = \sum_{i=1}^{C} P_i (\mu_i - \mu)(\mu_i - \mu)^T$$

Note that in general a covariance matrix is symmetric and positive semi-definite with nonnegative eigenvalues.

why?

Statistical Facts (3)

Quantity for Measuring Within and Between Class Distance (cont.)

- Total covariance (sample):

$$\mathbf{S}_T = \frac{1}{N}\sum_{\mathbf{x}}(\mathbf{x}-\mu)(\mathbf{x}-\mu)^T$$
$$= \mathbf{S}_W + \mathbf{S}_B.$$

see next slide

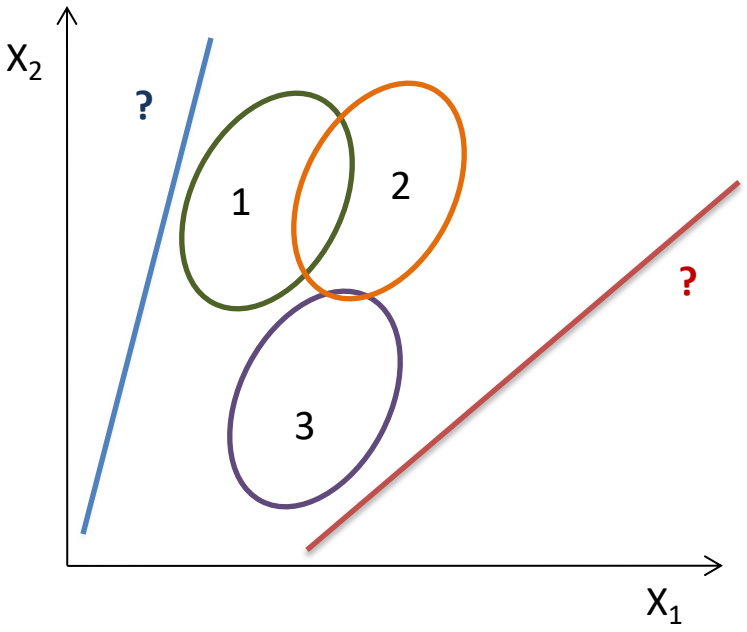
Proof

$$
\begin{aligned}
\boldsymbol{S}_b &= \boldsymbol{S}_t - \boldsymbol{S}_w \\
&= \sum_{i=1}^{m} (\boldsymbol{x}_i - \boldsymbol{\mu})(\boldsymbol{x}_i - \boldsymbol{\mu})^T - \sum_{i=1}^{N} \sum_{\boldsymbol{x} \in X_i} (\boldsymbol{x} - \boldsymbol{\mu}_i)(\boldsymbol{x} - \boldsymbol{\mu}_i)^T \\
&= \sum_{i=1}^{N} \left(\sum_{\boldsymbol{x} \in X_i} \left((\boldsymbol{x} - \boldsymbol{\mu})(\boldsymbol{x} - \boldsymbol{\mu})^T - (\boldsymbol{x} - \boldsymbol{\mu}_i)(\boldsymbol{x} - \boldsymbol{\mu}_i)^T \right) \right) \\
&= \sum_{i=1}^{N} \left(\sum_{\boldsymbol{x} \in X_i} \left((\boldsymbol{x} - \boldsymbol{\mu})(\boldsymbol{x}^T - \boldsymbol{\mu}^T) - (\boldsymbol{x} - \boldsymbol{\mu}_i)(\boldsymbol{x}^T - \boldsymbol{\mu}_i^T) \right) \right) \\
&= \sum_{i=1}^{N} \left(\sum_{\boldsymbol{x} \in X_i} \left(\boldsymbol{x}\boldsymbol{x}^T - \boldsymbol{x}\boldsymbol{\mu}^T - \boldsymbol{\mu}\boldsymbol{x}^T + \boldsymbol{\mu}\boldsymbol{\mu}^T - \boldsymbol{x}\boldsymbol{x}^T + \boldsymbol{x}\boldsymbol{\mu}_i^T + \boldsymbol{\mu}_i\boldsymbol{x}^T - \boldsymbol{\mu}_i\boldsymbol{\mu}_i^T \right) \right) \\
&= \sum_{i=1}^{N} \left(\sum_{\boldsymbol{x} \in X_i} \left(-\boldsymbol{x}\boldsymbol{\mu}^T - \boldsymbol{\mu}\boldsymbol{x}^T + \boldsymbol{\mu}\boldsymbol{\mu}^T + \boldsymbol{x}\boldsymbol{\mu}_i^T + \boldsymbol{\mu}_i\boldsymbol{x}^T - \boldsymbol{\mu}_i\boldsymbol{\mu}_i^T \right) \right) \\
&= \sum_{i=1}^{N} \left(- \sum_{\boldsymbol{x} \in X_i} \boldsymbol{x}\boldsymbol{\mu}^T - \sum_{\boldsymbol{x} \in X_i} \boldsymbol{\mu}\boldsymbol{x}^T + \sum_{\boldsymbol{x} \in X_i} \boldsymbol{\mu}\boldsymbol{\mu}^T + \sum_{\boldsymbol{x} \in X_i} \boldsymbol{x}\boldsymbol{\mu}_i^T + \sum_{\boldsymbol{x} \in X_i} \boldsymbol{\mu}_i\boldsymbol{x}^T - \sum_{\boldsymbol{x} \in X_i} \boldsymbol{\mu}_i\boldsymbol{\mu}_i^T \right) \\
&= \sum_{i=1}^{N} \left(-m_i\boldsymbol{\mu}_i\boldsymbol{\mu}^T - m_i\boldsymbol{\mu}\boldsymbol{\mu}_i^T + m_i\boldsymbol{\mu}\boldsymbol{\mu}^T + m_i\boldsymbol{\mu}_i\boldsymbol{\mu}_i^T + m_i\boldsymbol{\mu}_i\boldsymbol{\mu}_i^T - m_i\boldsymbol{\mu}_i\boldsymbol{\mu}_i^T \right) \\
&= \sum_{i=1}^{N} \left(-m_i\boldsymbol{\mu}_i\boldsymbol{\mu}^T - m_i\boldsymbol{\mu}\boldsymbol{\mu}_i^T + m_i\boldsymbol{\mu}\boldsymbol{\mu}^T + m_i\boldsymbol{\mu}_i\boldsymbol{\mu}_i^T \right) \\
&= \sum_{i=1}^{N} m_i \left(-\boldsymbol{\mu}_i\boldsymbol{\mu}^T - \boldsymbol{\mu}\boldsymbol{\mu}_i^T + \boldsymbol{\mu}\boldsymbol{\mu}^T + \boldsymbol{\mu}_i\boldsymbol{\mu}_i^T \right) \\
&= \sum_{i=1}^{N} m_i (\boldsymbol{\mu}_i - \boldsymbol{\mu})(\boldsymbol{\mu}_i - \boldsymbol{\mu})^T
\end{aligned}
$$

within-class variance + between-class = total variance.

Linear Discriminant Analysis - Problem (1)

- **PROBLEM: To separate populations**
  - Suppose we have C classes from D-dimensional distributions. We want to project these classes onto a p-dimensional subspace ($p < D$) so that the variation between the classes is **as large as possible**, relative to the variation within the classes.



Linear Discriminant Analysis - Problem (2)

- Practically speaking, after the projection, we want

class means to be as **far** apart from each other as possible	⇒	the **between-class** scatter to be **large**
samples from the same class to be as **close** to their means as possible	⇒	the **within-class** scatter to be **small**

This technique was developed by R. A. Fisher (1936) for **the two-class case** and extended by C. R. Rao (1948) to handle **the multiclass case**.

LDA: Two-class

A simple case

- LDA seeks to reduce dimensionality while preserving as much of the class discriminatory information as possible
- Assume we have a set of D-dimensional samples $\{x^1, x^2, \ldots x^N\}$, N_1 of which from class C_1, and N_2 from class C_2
- We seek for one direction, along which we project the samples x onto a line w and get a scalar y:

$$y = w^T x$$

- Of all the possible lines, LDA selects the one that maximizes the separability of the scalars y.

LDA: Two-class

- The mean vector of each class in x-space and y-space is

$$\mu_i = \frac{1}{n_i}\sum_{x \in c_i} x \text{ and } \tilde{\mu}_i = w^T \mu_i$$

- Choose the distance between the projected means $\tilde{\mu}_i$ as class separability measure:

$$J(w) = |\tilde{\mu}_1 - \tilde{\mu}_2| = |w^T(\mu_1 - \mu_2)|$$

LDA: Two-class

- Fisher suggested maximizing the difference between the means, normalized by a measure of the within-class scatter!
- For each class we define the scatter, equivalent to the variance, as:

$$\tilde{S}_i^2 = \sum_{y \in c_i} (y - \tilde{\mu}_i)^2$$

where $(\tilde{S}_1^2 + \tilde{S}_2^2)$ is the within-class scatter of the projected examples.

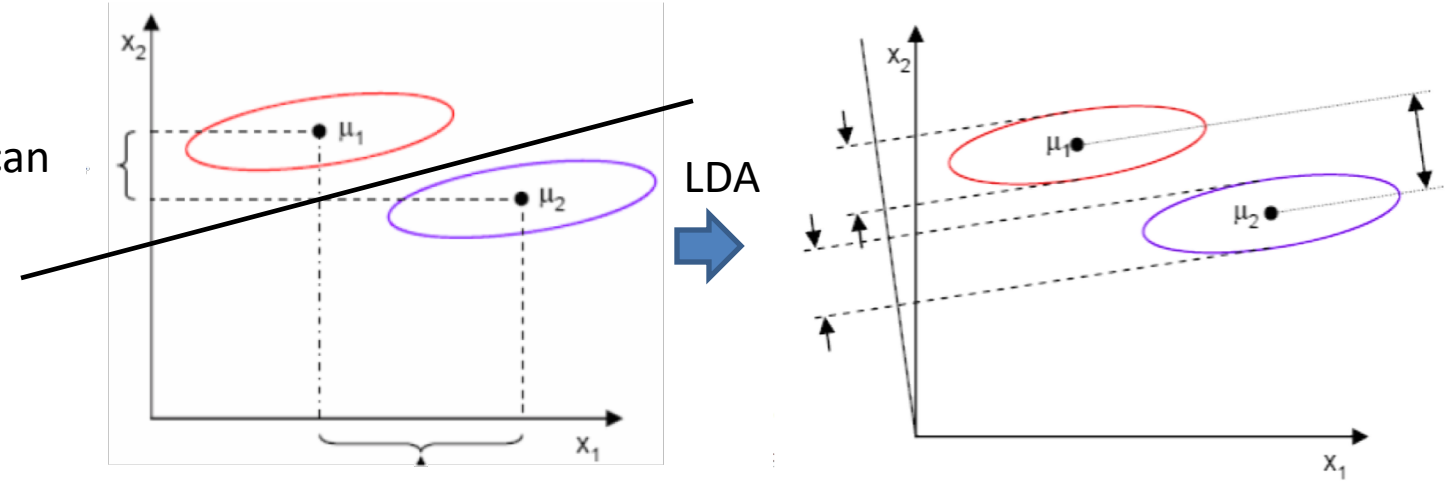
LDA: Two-class

- The Fisher linear discriminant is defined as the linear function $w^T x$ that maximizes the following criterion function:

$$J(w) = \frac{|\tilde{\mu}_1 - \tilde{\mu}_2|^2}{\tilde{S}_1^2 + \tilde{S}_2^2}$$

- Therefore, we are looking for a projection where examples from the same class are projected very close to each other and, at the same time, the projected means are as farther apart as possible

Along this axis, one can separate two classes better



Along this axis, one gets larger distance between class means

LDA: Two-class

- To find the optimum w^*, we need write $J(w)$ as a function of w
- As aforementioned, we define S_W as the within-class scatter matrix in feature space x

$$S_1 + S_2 = S_w$$

- The scatter of the projection y can then be expressed as a function of the scatter matrix in feature space x

$$\tilde{S}_i^2 = \sum_{y \in c_i} (y - \tilde{\mu}_i)^2 = \sum_{y \in c_i} (w^T x - w^T \mu_i)^2$$

$$= \sum_{y \in c_i} w^T (x - \mu_i)(x - \mu_i)^T w = w^T S_i w$$

$$\tilde{S}_1^2 + \tilde{S}_2^2 = w^T S_w w$$

Class specific scatter or covariance matrix

Within class scatter matrix

LDA: Two-class

- Similarly, the difference between the projected means can be expressed in terms of the means in the original feature space

$$\begin{aligned}(\tilde{\mu}_1 - \tilde{\mu}_2)^2 &= (w^T\mu_1 - w^T\mu_2)^2 \\ &= w^T(\mu_1 - \mu_2)(\mu_1 - \mu_2)^T w \\ &= w^T S_B w\end{aligned}$$

- The matrix S_B is the between-class scatter. Note that, since S_B is the outer product of two vectors, its rank is at most one

LDA: Two-class

- We can finally express the Fisher criterion in terms of S_W and S_B as

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

- To find the maximum of $J(w)$, we **take derivative** and **set to zero**

$$\frac{d}{dw}J(w) = (w^T S_W w)\frac{d(w^T S_B w)}{dw} - (w^T S_B w)\frac{d(w^T S_W w)}{dw}$$
$$= (w^T S_W w)S_B w - (w^T S_B w)S_W w = 0$$

  - *We omit* 2 *and* $(w^T S_W w)^2$ *here.*

- Dividing by $w^T S_W w$

$$\frac{w^T S_W w}{w^T S_W w}S_B w - \frac{w^T S_B w}{w^T S_W w}S_W w = 0$$
$$S_B w - J(w)S_W w = 0 \Rightarrow S_W^{-1}S_B w = J(w)w$$

LDA: Two-class

- Solving the generalized eigenvalue problem $S_W^{-1} S_B w = \lambda w, where\ \lambda = J(w) = scalar$ yields w^* is the eigenvector of $\boldsymbol{S_W^{-1} S_B}$

LDA: Multi-class

- Instead of **one projection** y, we will now seek $(C-1)$ projections $[y_1, y_2, \ldots y_{C-1}]$ by means of $(C-1)$ projection vectors w_i arranged by columns into a projection matrix $W=[w_1|w_2|\ldots|w_{C-1}]$:

$$y_i = w_i^T x \Rightarrow y = W^T x$$

- From our derivation for the two-class problem, we have the scatter matrices <u>for the projected samples</u>

$$\tilde{S}_W = W^T S_W W$$
$$\tilde{S}_B = W^T S_B W$$

LDA: Multi-class

- We look for a projection that maximizes the ratio of between-class to within-class scatter. Since the projection is no longer a scalar (it has $C-1$ dimensions), we use the determinant of the scatter matrices to obtain a scalar objective function

$$J(W) = \frac{|\tilde{S}_B|}{|\tilde{S}_W|} = \frac{|W^T S_B W|}{|W^T S_W W|}$$

- We seek the projection matrix $W*$ that maximizes this ratio. The optimal projection matrix $W*$ is the one whose columns are the eigenvectors corresponding to the largest eigenvalues of the following generalized eigenvalue problem.

$$W^* = \arg\max \frac{|W^T S_B W|}{|W^T S_W W|} \Rightarrow (S_B - \lambda_i S_W) w_i^* = 0$$

where $\lambda_i = J(w_i) = scalar$

Some Remarks

- Solving LDA is lightweight.
- We can see that if an optimization problem can be solved as a Generalized Eigenvalue Decomposition problem $\mathbf{B}\boldsymbol{\theta}_i = \lambda_i \mathbf{A}\boldsymbol{\theta}_i$ with $\boldsymbol{\theta}_i$ and λ_i being the *i*-th eigenvector and eigenvalue of $\mathbf{A}^{-1}\mathbf{B}$.
- Unlike principal component analysis (PCA), the linear discriminant transformation W from the original variates $x_1, ..., x_n$ to the new variates $y_1, ..., y_p$ is not necessarily orthogonal.
  - Because $S_W^{-1} S_B$ may not be symmetric.

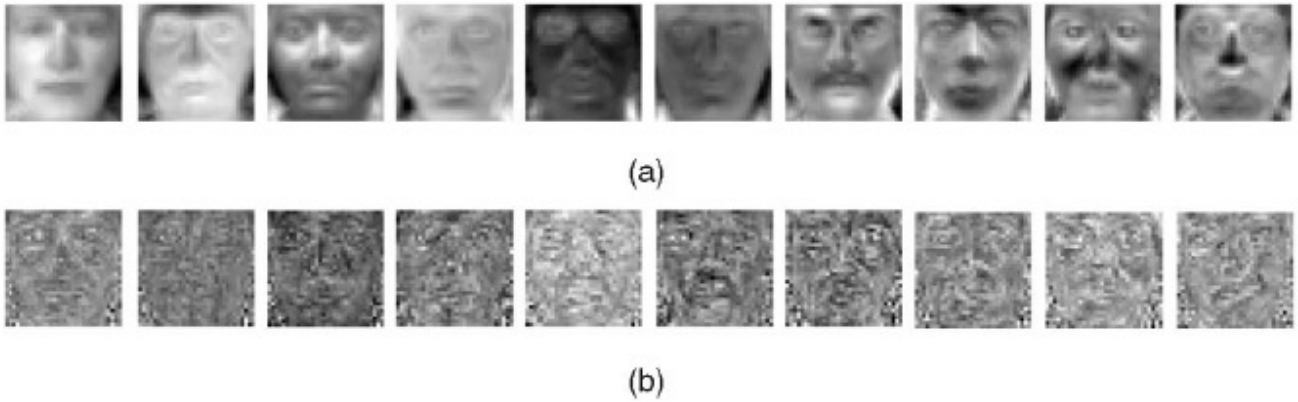
Classification with LDA

- First, select the number of feature dimension for the low-dimensional feature space (may use PCA beforehand).

- Later, use Nearest Neighbor or other classifiers.

Basis Visualization

Sample Images from YALE database



(a)

(b)

The first 10 (a) Eigenfaces, (b) Fisherfaces, calculated from the face images in the YALE database.

(visualization of LDA projection is called Fisherface)

Question

- Shall LDA always be better than PCA for classification task?

Is LDA always better than PCA?

- Case Study: PCA versus LDA
  - A. Martinez, A. Kak, "PCA versus LDA", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 23, no. 2, pp. 228-233, 2001.
- Is LDA always better than PCA?
  - There has been a tendency in the computer vision community to prefer LDA over PCA.
  - This is mainly because LDA deals directly with discrimination between classes while PCA does not pay attention to the underlying class structure.
  - This paper shows that when the training set is small, PCA can outperform LDA.
  - When the number of samples is large and representative for each class, LDA outperforms PCA.

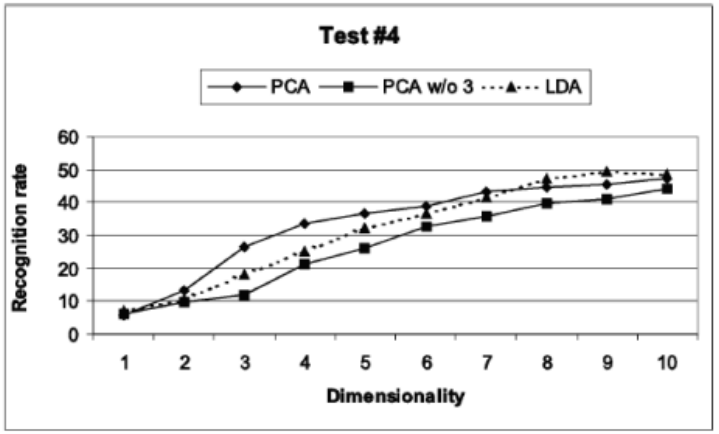
Linear Discriminant Analysis (LDA)

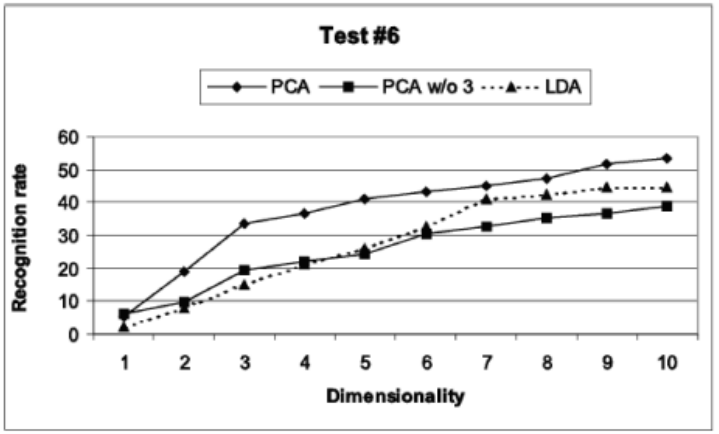
- Is LDA always better than PCA? – cont.

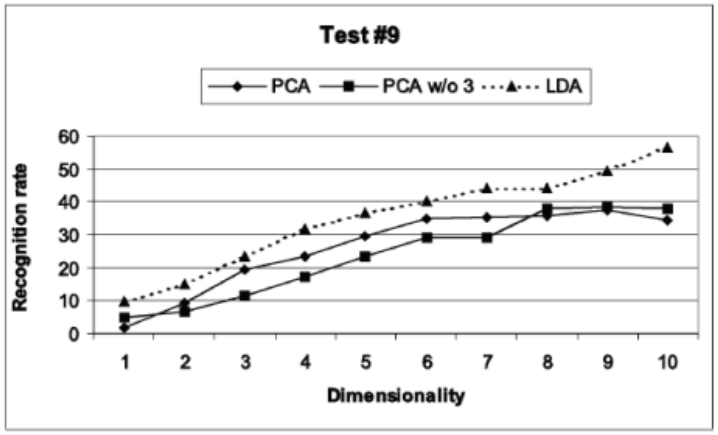
Linear Discriminant Analysis (LDA)

- Is LDA always better than PCA? – cont.

(LDA is not always better when the sample is small)





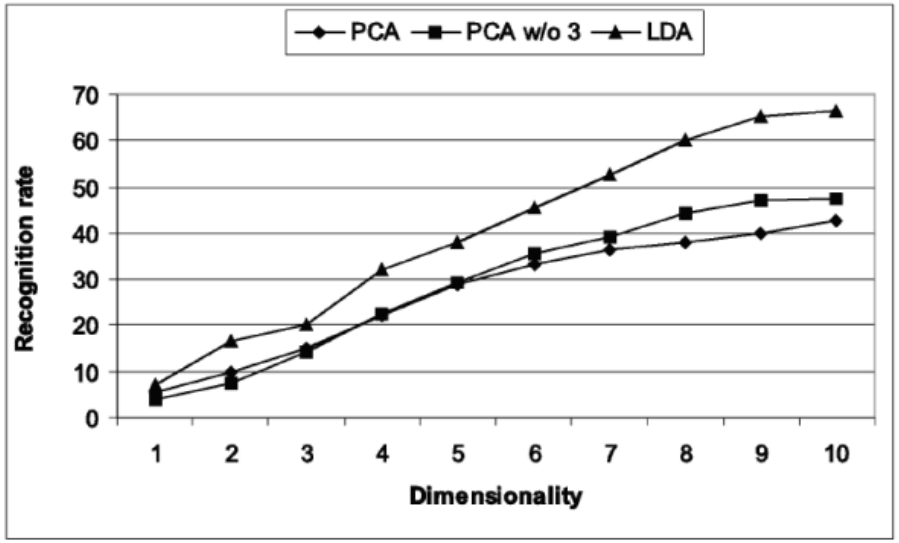


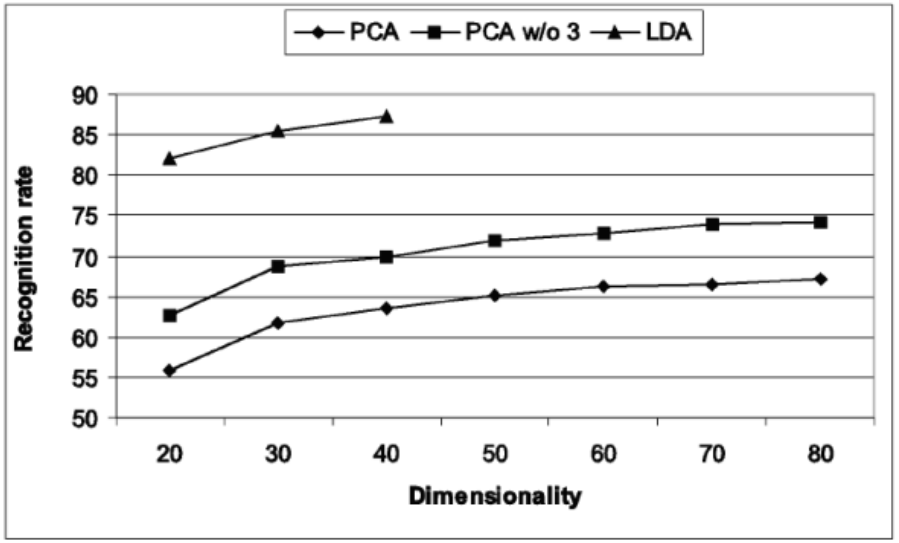
Results on a small dataset for training: 2 images for training and 5 images for testing.
PCA w/o 3: PCA without using the top 3 eigenvectors. #i index train/test splits

Linear Discriminant Analysis (LDA)

- Is LDA always better than PCA? – cont.

(LDA outperforms PCA when the sample is large)

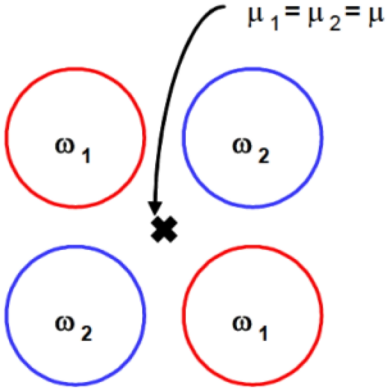


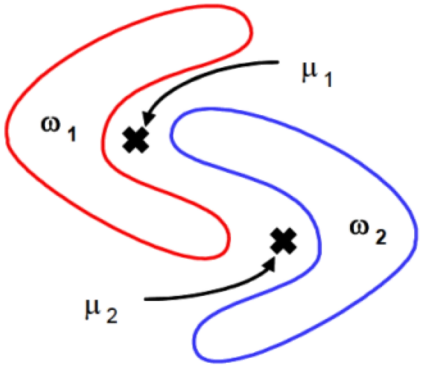


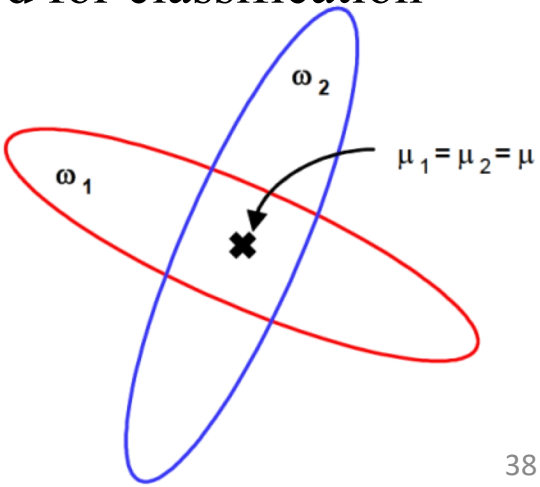
Results obtained for each of the three algorithms using a larger data set for training.

Limitations

- **LDA produces at most $C - 1$ feature projections**
  - If the classification error estimates establish that more features are needed, some other method must be employed to provide those additional features
- **LDA is a parametric method (it assumes unimodal Gaussian likelihoods)**
  - If the distributions are non-Gaussian, LDA projections may not preserve complex structure in the data needed for classification







Limitations

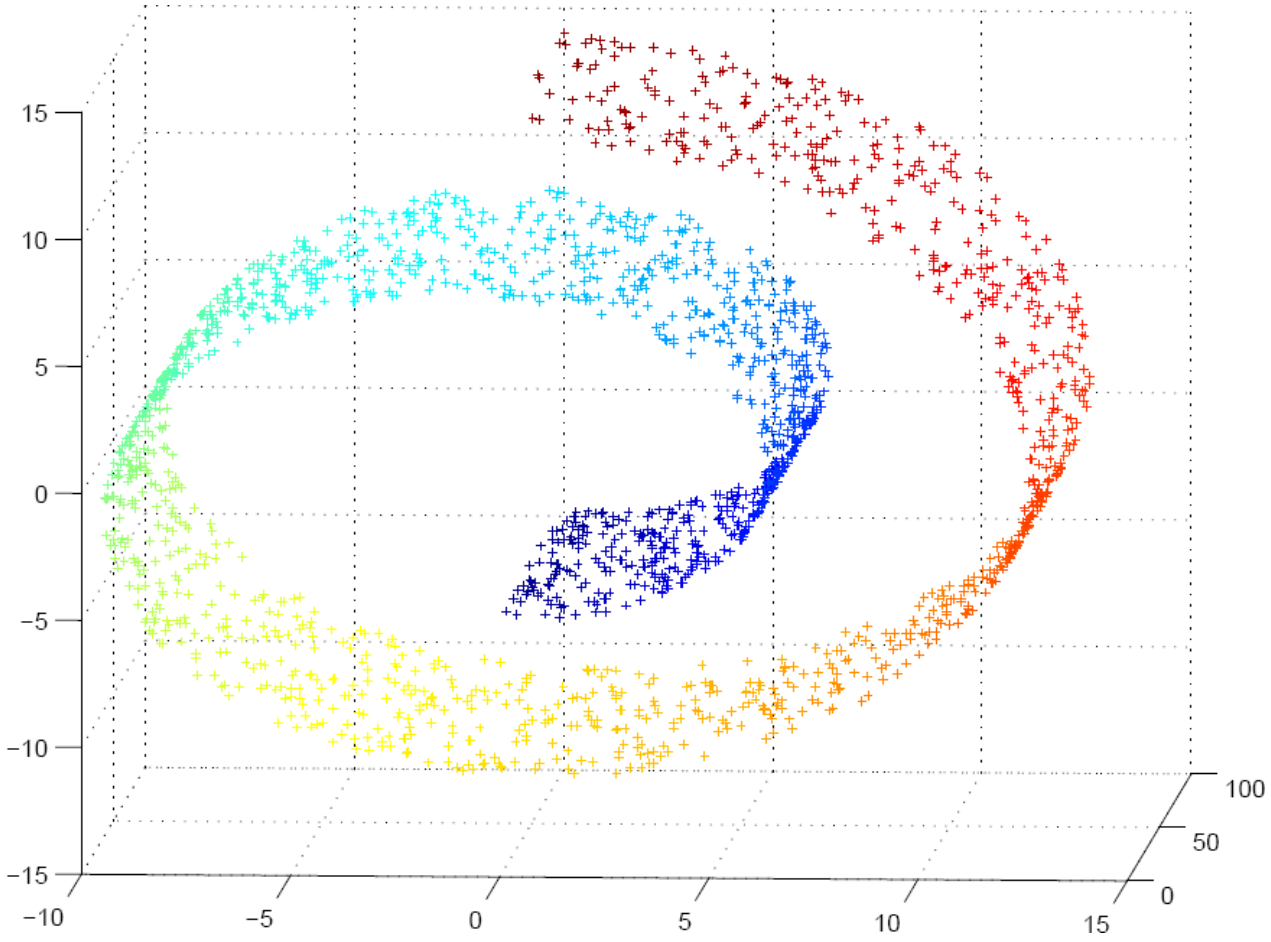
- LDA does not seem to be superior to PCA when the training data set is small.
- If testing sample does not follow the distribution of the training samples, the performance may be worse

Graph Embedding

A General Framework for Feature Extraction

Shuicheng Yan, Dong Xu et al.: Graph Embedding and Extensions: A General Framework for Dimensionality Reduction. T-PAMI 2007.

Linear Subspace vs. Manifold



Direct Graph Embedding

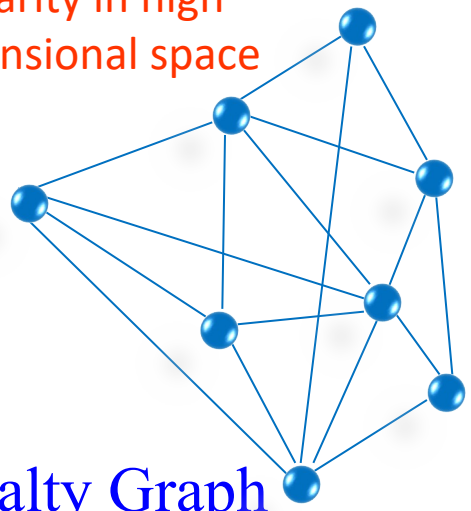
本质图

Intrinsic Graph:

$G = [x_i, S_{ij}]$

高维空间中的相似性

Similarity in high dimensional space



Penalty Graph

$G^P = [x_i, S_{ij}^P]$

处罚图

相似性矩阵(图边)

S, S^P: Similarity matrix (graph edge)

L, B: Laplacian matrix from S, S^P;

$L = D - S, \quad D_{ii} = \sum_{j \neq i} S_{ij} \quad \forall i$

Data in high-dimensional space and low-dimensional space (assumed as 1D space here):

$X = [x_1, x_2, ..., x_N] \quad y = [y_1, y_2, ..., y_N]^T$

Target: search for a mapping $y_i = f(x_i)$ to Preserve / Avoid these graph similarities

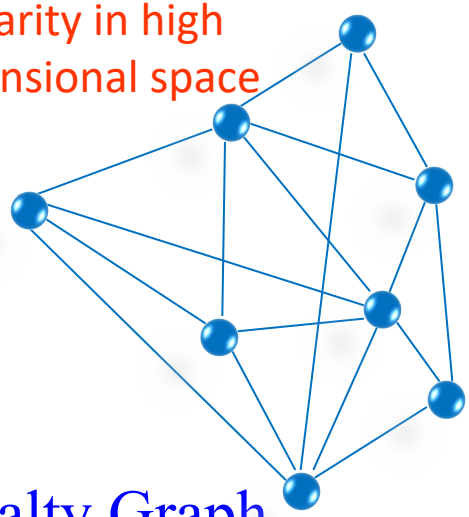
高维空间和低维空间的数据（这里假设为一维空间）

Direct Graph Embedding -- Continued

Intrinsic Graph:

$G = [x_i, S_{ij}]$

Similarity in high dimensional space



Penalty Graph

$G^P = [x_i, S_{ij}^P]$

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Data in high-dimensional space and low-dimensional space (assumed as 1D space here):

$X = [x_1, x_2, ..., x_N] \quad y = [y_1, y_2, ..., y_N]^T$

Criterion to Preserve Graph Similarity:

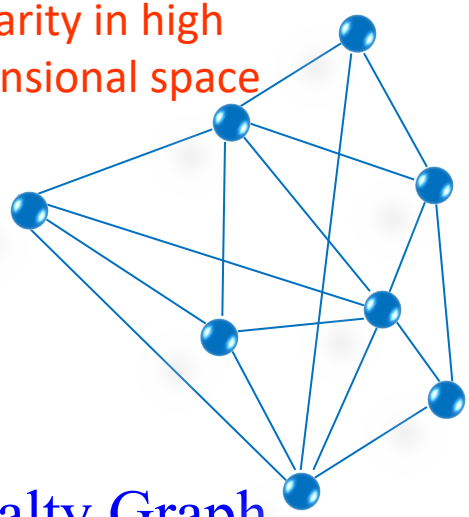
$$y^* = \arg\min_{\substack{y^T y = 1 \text{ or} \\ y^T B y = 1}} \sum_{i \neq j} \| y_i - y_j \|^2 S_{ij}$$

Direct Graph Embedding -- Continued

Intrinsic Graph:

$G = [x_i, S_{ij}]$

Similarity in high dimensional space



Penalty Graph

$G^P = [x_i, S_{ij}^P]$

S, S^P: *Similarity matrix (graph edge)*

L, B: *Laplacian matrix from S, S^P;*

$L = D - S, \quad D_{ii} = \sum_{j \neq i} S_{ij} \quad \forall i$

Data in high-dimensional space and low-dimensional space (assumed as 1D space here):

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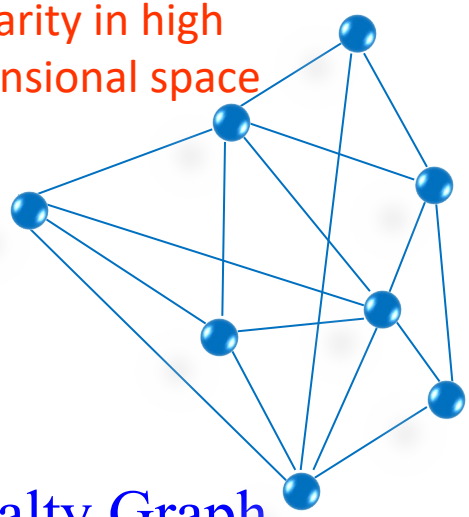
Special case B is Identity matrix (*Scale normalization*)

Direct Graph Embedding -- Continued

Intrinsic Graph:

$G = [x_i, S_{ij}]$

Similarity in high dimensional space



Penalty Graph

$G^P = [x_i, S_{ij}^P]$

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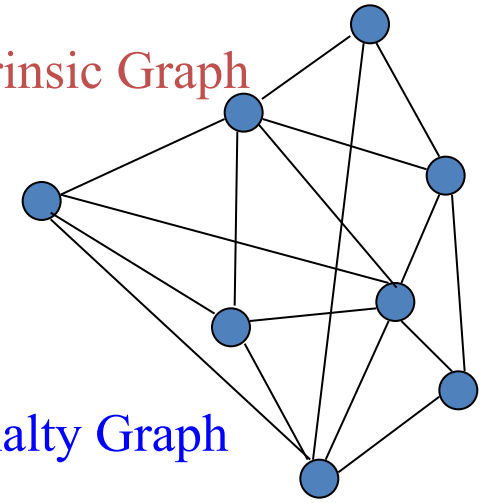
Criterion to Preserve Graph Similarity:

$$y^* = \arg\min_{\substack{y^T y=1 \text{ or} \\ y^T B y=1}} \sum_{i \neq j} \| y_i - y_j \|^2 S_{ij} = \arg\min_{\substack{y^T y=1 \text{ or} \\ y^T B y=1}} y^T L y$$

Linearization



Linear mapping function

$$y = X^{\mathrm{T}} w$$

Objective function in Linearization

$$w^{*} = \arg\min_{\substack{w^{\mathrm{T}} w=1 \text{ or} \\ w^{\mathrm{T}} XBX^{\mathrm{T}} w=1}} w^{\mathrm{T}} XLX^{\mathrm{T}} w$$

Common Formulation

Linearization

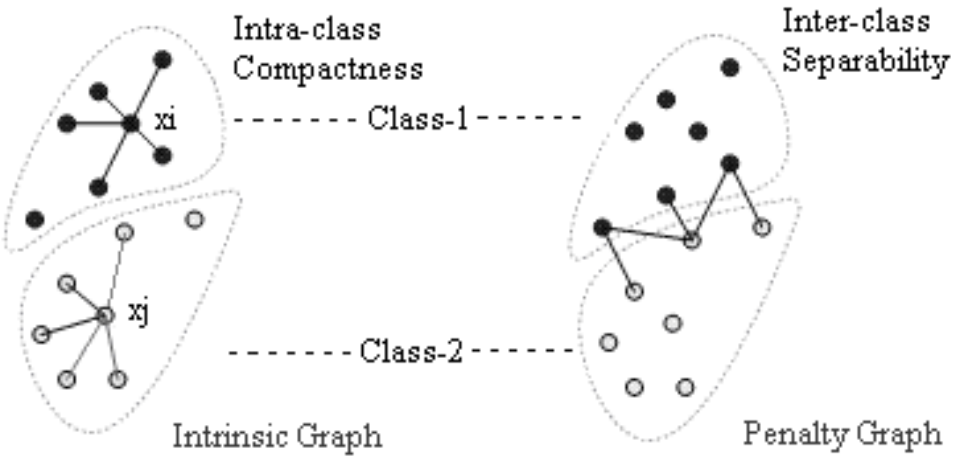
$$w^* = \arg\min_{w^T w=1 \text{ or } w^T XBX^T w=1} w^T XLX^T w$$

解决方案是通过解决广义特征值 分解问题

The solutions are obtained by solving the generalized eigenvalue decomposition problem $\tilde{L}v = \lambda\tilde{B}v$ where $\tilde{L} = XLX^T$ and $\tilde{B} = XBX^T$

Try by yourself

New Algorithm: Marginal Fisher Analysis



Important Information for classification:

1) Label information
2) Local manifold structure

(neighborhood or margin)

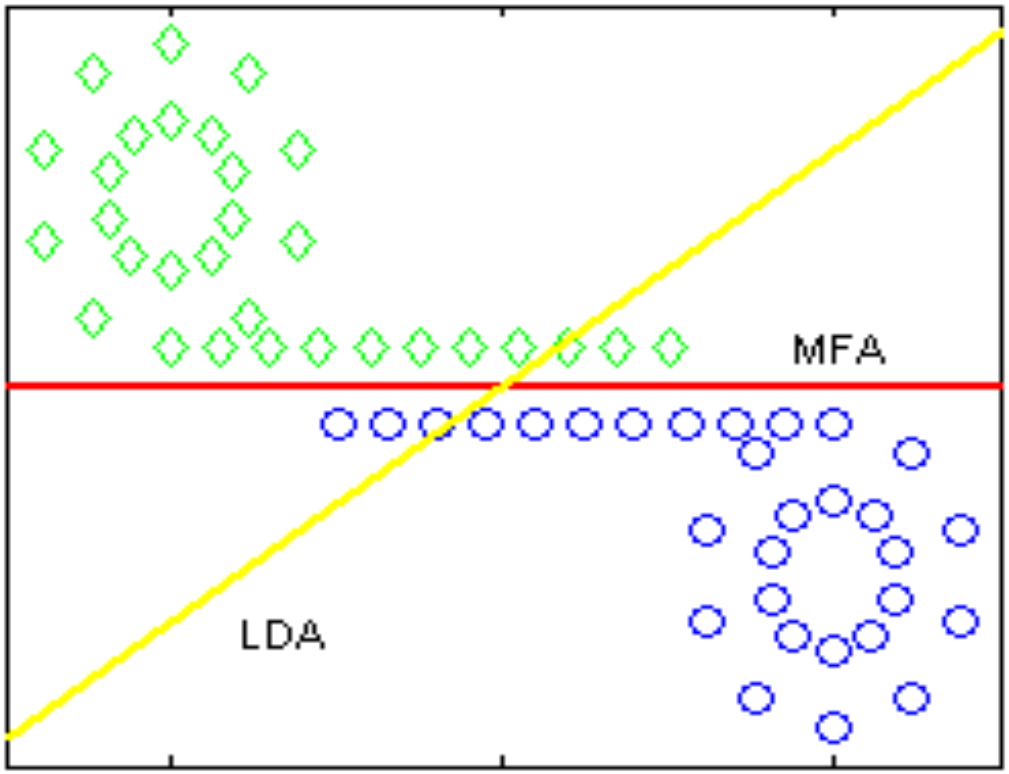
S_{ij} = 1: if x_i is among the k_1-nearest neighbors of x_j in the same class;
0 : otherwise

S_{ij}^{P} = 1: if the pair (i,j) is among the k_2 shortest pairs (from different classes) among the data set;
0: otherwise

Marginal Fisher Analysis (MFA): Advantage

- MFA advantage: (compared with LDA)
  - The number of available projection directions is much larger
  - No assumption on the data distribution, more general for discriminant analysis
  - The interclass margin can better characterize the separability of different classes

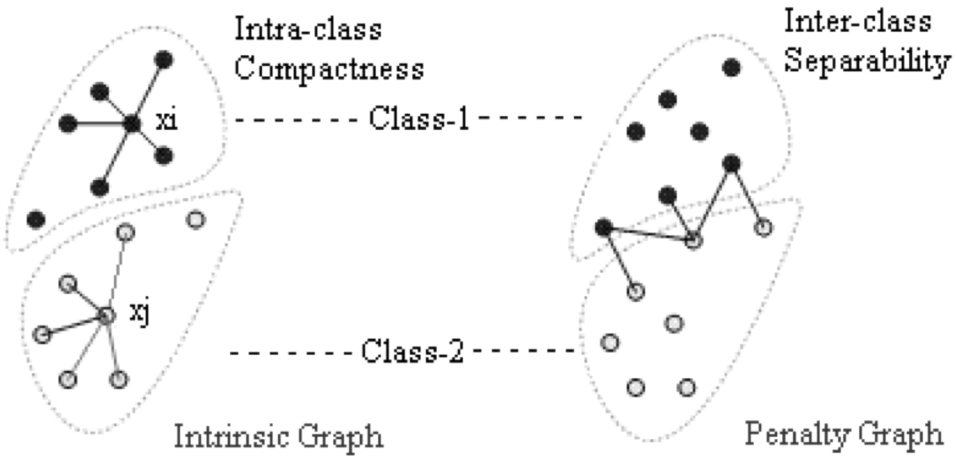
Marginal Fisher Analysis (MFA): Advantage



Separation Plane

Marginal Fisher Analysis

构建类内 紧凑性和类间分离性图



- Constructing the intraclass compactness and interclass separability graphs.
- In the **intraclass compactness** graph, for each sample xi, set the adjacency matrix $W_{ij} = W_{ji} = 1$ if x_i is among the k_1-nearest neighbors of x_j in the same class.
- In the **interclass separability** graph, for each class c, set the similarity matrix $W_{ij}^{p} = 1$ if the pair (i, j) if x_i is among the k_2-nearest neighbors of x_j in different class. .

Marginal Fisher Analysis

- Intraclass compactness (intrinsic graph)
- $\tilde{S}_c = \sum_i \sum_{i \in N_{k_1}^+(j) \; or \; j \in N_{k_1}^+(i)} \left\| w^T x_i - w^T x_j \right\|^2$

  $= 2 w^T X (D - W) X^T w$

$$W_{ij} = \begin{cases} 1, if \; i \in N_{k_1}^+(j) or \; j \in N_{k_1}^+(i) \\ 0, else \end{cases}$$

Marginal Fisher Analysis

- Interclass separability (penalty graph)
- $\tilde{S}_p = \sum_i \sum_{(i,j)\in P_{k_2}(c_i)\ or\ (i,j)\in P_{k_2}(c_j)} \left\| w^T x_i - w^T x_j \right\|^2$
  $= 2w^T X(D^P - W^P)X^T w$

$$W_{ij}^P = \begin{cases} 1, if\ (i,j) \in P_{k_2}(c_i) or\ (i,j) \in P_{k_2}(c_j) \\ 0, else \end{cases}$$

Marginal Fisher Analysis

- From the linearization of graph embedding, we have Marginal Fisher Criterion

$$w^* = \arg\min_w \frac{w^T X(D - W)X^T w}{w^T X(D^P - W^P)X^T w}$$

, which is a special linearization of graph embedding with

$$L = D - W$$
$$B = D^P - W^P$$

Experiments: Face Recognition

Sample Images from CMU PIE database (after cropping)

ORL	G3/P7	G4/P6
PCA+LDA (Linearization)	87.9%	88.3%
PCA+MFA	89.3%	91.3%
PIE-1	G3/P7	G4/P6
PCA+LDA (Linearization)	65.8%	80.2%
PCA+MFA	71.0%	84.9%

Summary

- A new dimensionality reduction algorithm: Marginal Fisher Analysis.
- Linear transformation has come to its end!
- Now, deep network is the trend (to study in L6)!

Papers to Read and Study

- H. Wang, S. Yan, D. Xu, X. Tang, and T. Huang. Trace Ratio vs. Ratio Trace for Dimensionality Reduction. In CVPR'07.

- Jieping Ye, Ravi Janardan, Cheonghee Park, and Haesun Park. An optimization criterion for generalized discriminant analysis on undersampled problems. IEEE Transactions on Pattern Analysis and Machine Intelligence. Vol. 26, No. 8, pp. 982—994, 2004. PDF

- A. Martinez, A. Kak, "PCA versus LDA", IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 23, no. 2, pp. 228-233, 2001.