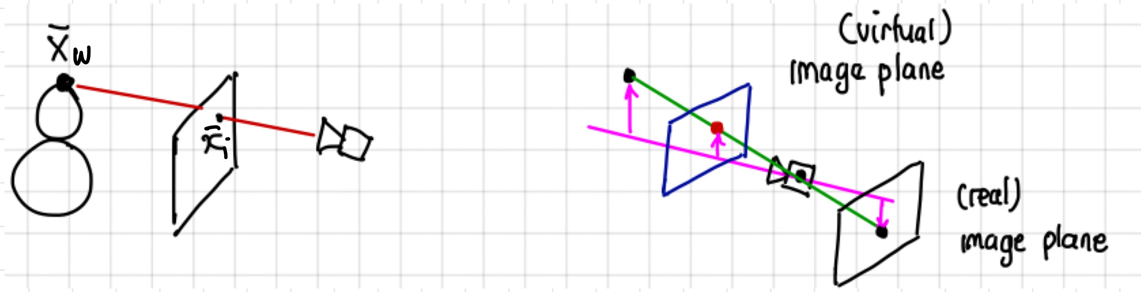
LECTURE 5: CAMERA GEOMETRY

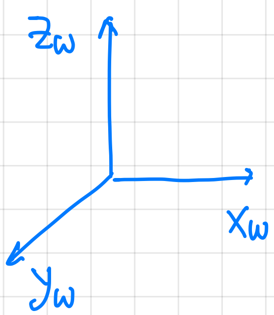
- Geometric Image Formation:

  Q: What is the correlation between a 3D point in the world, $\bar{X}_w = \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix}$ and its corresponding 2D point on an image, $\bar{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$?

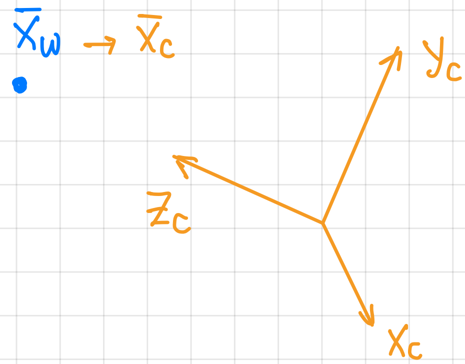


Coordinate Systems

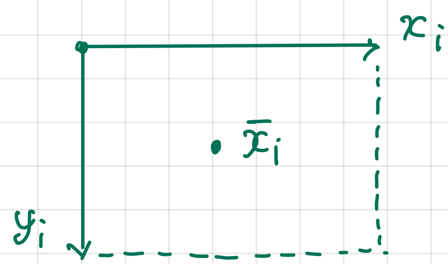
① World



② Camera



③ Image



$$\boxed{\underset{3\times 1}{\bar{x}_i} = \underset{3\times 4}{\mathbb{P}}\ \underset{4\times 1}{\bar{X}_w}}$$

$\Rightarrow$ What is $\mathbb{P}$?

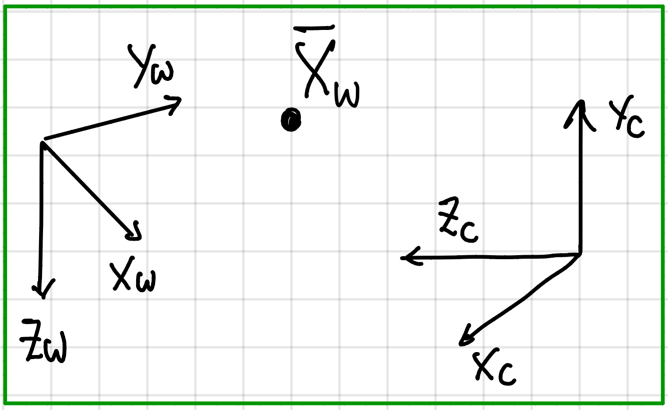
$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \mathbb{P} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

[•] Camera Matrix ($\mathbb{P}$)

$$\underset{3\times1}{\bar{x}_i} = \underset{3\times4}{\mathbb{P}}\ \underset{4\times1}{\bar{X}_w}$$

What matrix is $\mathbb{P}$?

(1) From the world to the camera:



$$\boxed{\bar{X}_c = \mathbb{T}\,\mathbb{R}\,\bar{X}_w}$$

$\bar{X}_w$ = the coordinates of $\bar{X}$ in the world coordinates.

$\bar{X}_c$ = the coordinates of $\bar{X}$ in the camera coordinates.

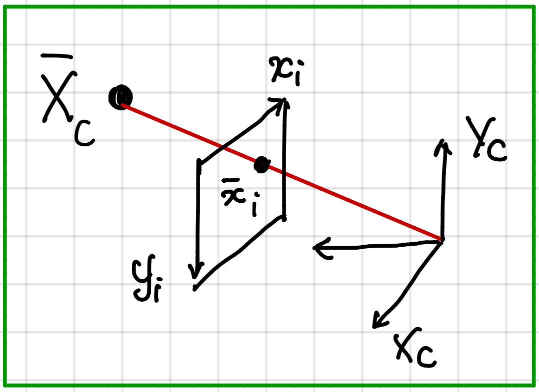
$\mathbb{R}$ = a rotation matrix that rotates the world coordinates of $\bar{X}$ to the camera coordinates.

$\mathbb{T}$ = a translation matrix that translates the world coordinates of $\bar{X}_w$ to the camera coordinates

⇓

What matrices are $\mathbb{R}$ & $\mathbb{T}$ exactly?

(2) From the camera to the image:



$$\boxed{\bar{x}_i = \mathbb{K}\,\bar{X}_c}$$

$\mathbb{K}$: ① to scale down

② to translate from the center of the image to the top left of the image

$$\mathbb{K} = \underbrace{\begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{to translate}} \underbrace{\begin{bmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{to scale down}}$$

$$\mathbb{K} = \begin{bmatrix} f_x & 0 & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ the center of the image

$f_x \approx f_y$

$$\bar{X}_c = \mathbb{R}\,\mathbb{T}\,\bar{X}_w$$

where:

$$\mathbb{T} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbb{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

we can write:

$$\underbrace{\mathbb{T}\,\mathbb{R}}_{4\times4} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & T_x \\ R_{21} & R_{22} & R_{23} & T_y \\ R_{31} & R_{32} & R_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left[\begin{array}{c|c} \mathbb{R} & \bar{t} \\ \hline 0 & 1 \end{array}\right]$$

$$\underbrace{\bar{X}_c}_{4\times1} = \underbrace{\mathbb{T}\,\mathbb{R}}_{4\times4}\,\underbrace{\bar{X}_w}_{4\times1} = \left[\begin{array}{c|c} \mathbb{R} & \bar{t} \\ \hline 0 & 1 \end{array}\right] \bar{X}_w$$

$$\underbrace{\bar{X}_c}_{3\times1} = \underbrace{[\mathbb{R} \mid \bar{t}\,]}_{3\times4}\,\underbrace{\bar{X}_w}_{4\times1}$$

$\bar{X}_c$ is in the inhomogeneous coordinates

$\bar{X}_w$ is in the homogeneous coordinates

$$\Downarrow$$

Hence:

$$\underbrace{\bar{x}_i}_{3\times1} = \mathbb{P}\,\bar{X}_w = \mathbb{K}\,\bar{X}_c$$

$$= \underbrace{\mathbb{K}}_{3\times3}\,\underbrace{[\mathbb{R} \mid \bar{t}\,]}_{3\times4}\,\underbrace{\bar{X}_w}_{4\times1}$$

Intrinsic parameters (Internal camera parameters)   Extrinsic parameters (External camera parameters)

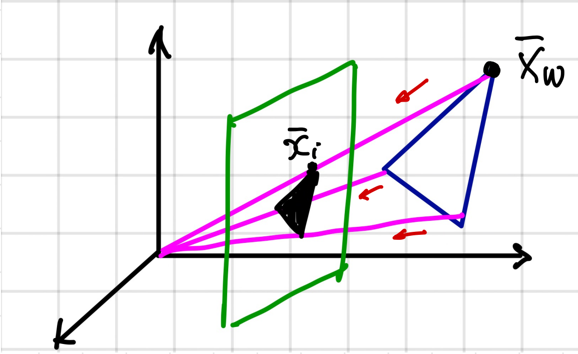
$$\Downarrow$$

Camera geometric calibration / camera resectioning:

To estimate the values of $\mathbb{K}, \mathbb{R}, \bar{t}$.

[0] Forward & Backward Projection

Forward



$$\bar{x}_i = \mathbb{P} X_w$$

Note: $\bar{x} = \begin{bmatrix} x_i \\ y_i \\ w \end{bmatrix}$

Hence, don't forget to normalize: $\bar{x} = \begin{bmatrix} x_i/w \\ y_i/w \\ 1 \end{bmatrix}$

Backward

Q: Given a point $\bar{x}_i$ on an image & $\mathbb{P}$, can we recover $\bar{X}_w$?

A: No. Backward projection can only do:

2D image		3D world
point	$\longrightarrow$	line
line	$\longrightarrow$	plane
plane	$\longrightarrow$	cone / volume

Backward projection: To find a set of points forming a ray passing through the camera center, $\bar{C}_w$, & $\bar{x}_i$ in the world coordinates

camera center in world coordinates:

camera coordinates:

$$\bar{C}_c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

world coordinates:

$$\underbrace{[\mathbb{R} \mid \bar{t}]}_{4\times4} \underbrace{\bar{C}_w}_{4\times1} = \underbrace{\bar{C}_c}_{4\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbb{I}\, \mathbb{R}\, \bar{C}_w = \bar{0}$$

$$\underbrace{\mathbb{R}\bar{C}_w}_{4\times1} + \underbrace{\bar{t}}_{4\times1} = \underbrace{\bar{0}}_{4\times1}$$

$$\mathbb{R}\,\bar{C}_w = -\bar{t}$$

$$\bar{C}_w = -\mathbb{R}^{-1}\bar{t} = -\mathbb{R}^T\bar{t}$$

$\bar{x}_i$ in World coordinates:

$$\underbrace{\bar{x}_i}_{3\times1} = \underbrace{\mathbb{P}}_{3\times4} \underbrace{\bar{X}_w}_{4\times1}$$

$$\underbrace{\hat{X}_w}_{4\times1} = \underbrace{\mathbb{P}^+}_{4\times3} \underbrace{\bar{x}_i}_{3\times1}$$

$$\underbrace{\mathbb{P}^+}_{4\times3} = \underbrace{(\mathbb{P}^T\mathbb{P})^{-1}}_{4\times4} \underbrace{\mathbb{P}^T}_{4\times3}$$

under determined system → many possible solutions

note: $\hat{X}_w \neq \bar{X}_w$ but both are on the same line

The line can be drawn from $\bar{C}_w$ & $\hat{X}_w$.

[•] Camera Calibration

Problem statement: Given N corresponding points $\bar{x}_n \leftrightarrow \bar{X}_n$ estimate $\mathbb{K}, \mathbb{R}, t$ such that $\bar{x}_n = \mathbb{P}\bar{X}_n$.

Solution:

① Compute $\mathbb{P}$

② Decompose $\mathbb{P}$ into $\mathbb{K}$, $\mathbb{R}$, and $\bar{t}$

$$\Downarrow$$

[6] Step 1: Compute $\mathbb{P}$

$$\bar{x}_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$x_i = \frac{p_{11}X_w + p_{12}Y_w + p_{13}Z_w + p_{14}}{p_{31}X_w + p_{32}Y_w + p_{33}Z_w + p_{34}} \; ; \quad y_i = \frac{p_{21}X_w + p_{22}Y_w + p_{23}Z_w + p_{24}}{p_{31}X_w + p_{32}Y_w + p_{33}Z_w + p_{34}}$$

$$\Leftrightarrow \; x_i(p_{31}X_w + p_{32}Y_w + p_{33}Z_w + p_{34}) = p_{11}X_w + p_{12}Y_w + p_{13}Z_w + p_{14}$$

$$y_i(p_{31}X_w + p_{32}Y_w + p_{33}Z_w + p_{34}) = p_{21}X_w + p_{22}Y_w + p_{23}Z_w + p_{24}$$

$$p_{11}X_w + p_{12}Y_w + p_{13}Z_w + p_{14} + \emptyset p_{21} + \emptyset p_{22} + \emptyset p_{23} + \emptyset p_{24} - x_iX_w p_{31} - x_iY_w p_{32} - x_iZ_w p_{33} - x_i p_{34} = \emptyset$$

$$p_{11}\emptyset + p_{12}\emptyset + p_{13}\emptyset + p_{14}\emptyset + X_w p_{21} + Y_w p_{22} + Z_w p_{24} - y_iX_w p_{31} - y_iY_w p_{32} - y_iZ_w p_{33} - y_i p_{34} = \emptyset$$

Vectorization:

$$\begin{bmatrix} X_w & Y_w & Z_w & 1 & \emptyset & \emptyset & \emptyset & \emptyset & -x_iX_w & -x_iY_w & -x_iZ_w & -x_i \\ \emptyset & \emptyset & \emptyset & \emptyset & X_w & Y_w & Z_w & 1 & -y_iX_w & -y_iY_w & -y_iZ_w & -y_i \end{bmatrix} \bar{p} = 0$$

where:

$$\bar{p} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{21} & p_{22} & p_{23} & p_{24} & p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}^T$$

Thus:

$$\underset{2\times 12}{\mathbb{A}} \; \underset{12\times 1}{\bar{p}} = \underset{2\times 1}{\bar{0}}$$

; $\bar{p} \neq 0$; otherwise we end up with a trivial solution.

Step 1 (Compute $\mathbb{P}$): Homogeneous Linear System

$\underset{2\times 12}{A}\ \underset{12\times 1}{\bar{p}} = 0$ → Since 1 pair of $(\bar{x}_n, \bar{X}_n)$ provides 2 equations, to estimate $\bar{p}$, we need at least 6 pairs $(N \geq 6)$:

$$\underset{(2N\times 12)}{A}\ \underset{(12\times 1)}{\bar{p}} = 0 \Rightarrow \text{Homogeneous Linear System}$$

↓

How to solve this ?

(A) Estimate the initial value of $\bar{p}$ using SVD

(B) Refine $\bar{p}$ using least-squares

$$\boxed{\begin{array}{l} \text{minimize } |A\bar{p}| \\ \text{subject to } |\bar{p}| = 1 \end{array}}$$

→ $|A\bar{p}|$ means the magnitude of $A\bar{p}$.

This is a constraint to avoid the trivial solution of $\bar{p} = 0$.

$$A = \mathbb{U}\mathbb{D}\mathbb{V}^T$$

$\bar{p}$ is the last row of $\mathbb{V}^T$.

Step 1.B : Refine $\mathbb{P}$ using Least Squares

$$\mathbb{P}^* = \underset{\{\mathbb{P}\}}{\text{argmin}}\ E(\mathbb{P}) = \underset{\{\mathbb{P}\}}{\text{argmin}}\ \frac{1}{2}\sum_{n}^{N}\left[\bar{x}_n - \mathbb{P}\bar{X}_n\right]^2$$

note : $\bar{a}^2$ doesn't exist, squaring a vector : $\bar{a}^T\bar{a}$
The expression of $\bar{a}^2$ is only for writing convinience.

Hence :

$$\underset{1\times1}{E(\mathbb{P})} = \underset{1\times1}{E(\bar{p})} = \frac{1}{2}\sum_{n}\begin{bmatrix} x_i - \dfrac{p_{11}x_w - p_{12}y_w - p_{13}z_w - p_{14}}{p_{31}x_w - p_{32}y_w - p_{33}z_w - p_{34}} \\ y_i - \dfrac{p_{21}x_w - p_{22}y_w - p_{23}z_w - p_{24}}{p_{31}x_w - p_{32}y_w - p_{33}z_w - p_{34}} \end{bmatrix}^2$$

$$E(\bar{p}) = \frac{1}{2}\sum_{n}\left[\underset{2\times12}{\mathbb{A}_n}\ \underset{12\times1}{\bar{p}}\right]^2$$

Minimization :

$$\frac{dE(\bar{p})}{\underset{12\times1}{d\bar{p}}} = \underset{12\times1}{0} \quad ; \quad \frac{dE}{\underset{12\times1}{d\bar{p}}} = \sum_{n}\underset{12\times2}{\mathbb{A}_n^T}\left[\underset{2\times12}{\mathbb{A}_n}\ \underset{12\times1}{\bar{p}}\right]$$

$$= \sum_{n}\left(\mathbb{A}_n^T\mathbb{A}_n\right)\bar{p} \quad ; \quad \bar{p} \neq 0$$

Since we can't have a closed-form solution for $\bar{p}$, we use Gradient Descent or Newton's methods :

Gradient Descent : $\underset{12\times1}{\vec{p}^{\,new}} = \underset{12\times1}{\vec{p}^{\,old}} + \alpha \left.\dfrac{dE}{\underset{12\times1}{d\vec{p}}}\right|_{\vec{p}=\vec{p}^{\,old}}$

In the beginning : $\bar{p}^{init} = \bar{p}^{SVD}$

Newton's method : $\alpha = \left(\left.\dfrac{d^2E}{d\bar{p}^2}\right|_{\bar{p}=\bar{p}^{old}}\right)^{-1}$: Inverse of second derivative

Step 2 : Decompose $\mathbb{P}$ to $\mathbb{K}$, $\mathbb{R}$ and $\bar{t}$

Steps:

1. Extract $\mathbb{M}$ (a 3x3 matrix) from the first 3x3 submatrix of $\mathbb{P}$.
2. Factor $\mathbb{M}$ into $\mathbb{K}\mathbb{R}$ using RQ decomposition

$$\mathbb{M} = \mathbb{K}\mathbb{R}$$; recall $\mathbb{K}$ is a triangular matrix

Example :

$A = \mathbb{Q}\mathbb{R} \rightarrow \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -0.89 & -0.45 \\ -0.45 & 0.89 \end{bmatrix} \begin{bmatrix} -2.24 & -2.24 \\ 0 & 0 \end{bmatrix}$

$A = \mathbb{R}\mathbb{Q} \rightarrow \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2.83 \\ 0 & -1.41 \end{bmatrix} \begin{bmatrix} -0.71 & 0.71 \\ -0.71 & -0.71 \end{bmatrix}$

3. $$\bar{t} = \mathbb{K}^{-1} \begin{pmatrix} P_{14} \\ P_{24} \\ P_{34} \end{pmatrix} \rightarrow \text{why ?}$$

Because : $\mathbb{K}^{-1}\mathbb{P} = [\mathbb{R} \,|\, t]$

$$\mathbb{K}^{-1} \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_x \\ R_{21} & R_{22} & R_{23} & t_y \\ R_{31} & R_{32} & R_{33} & t_z \end{bmatrix}$$

[•] Intrinsic Camera Properties, $\mathbb{K}$: Skew Parameters

Previously we define: $\mathbb{K} = \begin{bmatrix} f_x & 0 & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$

A more common definition: $\mathbb{K} = \begin{bmatrix} f_x & s & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$

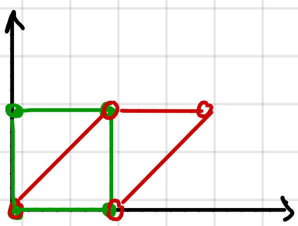
where:
s = the skew parameter

Three operations in $\mathbb{K}$:

$$\mathbb{K} = \underbrace{\begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translation}} \underbrace{\begin{bmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{scaling}} \underbrace{\begin{bmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{shearing}}$$

Shearing example:

$$\begin{array}{c} \quad\; s \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



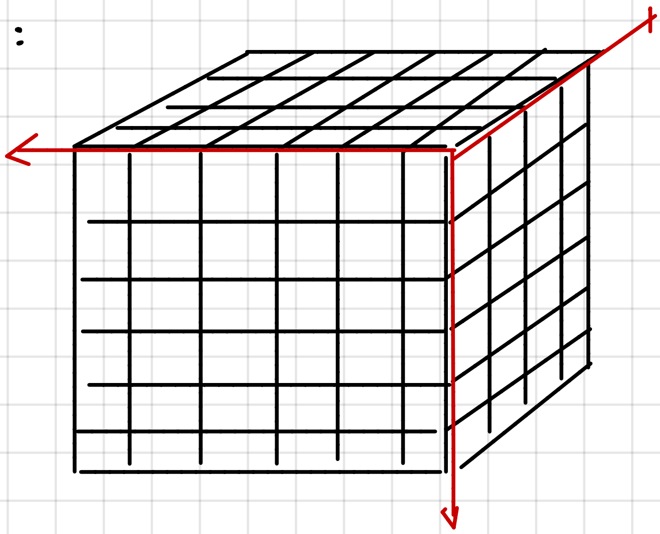
Notes:
1. In most cameras, $s = 0$
2. Why does the skew happen horizontally? It's rare that images are skewed vertically.

[0] Practical Notes on Calibration

1. Q: How to obtain the corresponding points : $\bar{x}_n$ and $\bar{X}_n$?

   A: Use either a 2D planar surface or a 3D box with identifiable patterns (= a checker pattern)

3D calibration box:



2. Q: Why $\bar{c} = -R^T\bar{t}$?

   A: We know that: $[R|t]\,\bar{c} = 0$,

which means when we transform $\bar{c}$, the center of the camera in the world coordinate system, onto the camera coordinate system, it should be at the origin, $\bar{0}$.

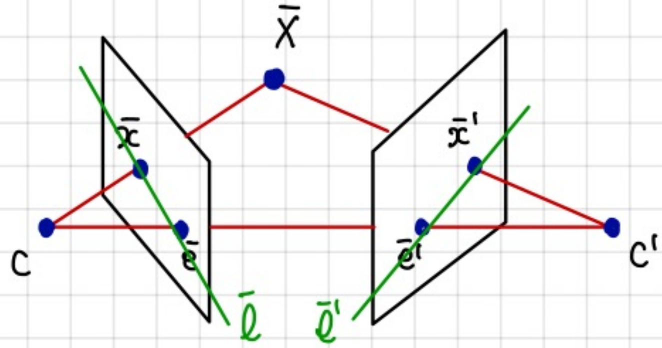
$$\underset{4\times3}{[R}\,|\,\underset{4\times1}{\bar{t}]}\;\underset{4\times1}{\bar{c}} = 0 \qquad ; \qquad R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ 0 & 0 & 0 \end{bmatrix}$$

$$R\bar{c} + \bar{t} = 0 \qquad ; \qquad R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R\bar{c} = -\bar{t}$$

$$\left.\begin{aligned} \bar{c} &= -R^{-1}\bar{t} \\ &= -R^T\bar{t} \end{aligned}\right\} \text{since } R \text{ is an orthonormal matrix.}$$

[●] Fundamental Matrix



C = the camera center
$\bar{e}$ = the epipole
$\bar{l}$ = the epipolar line

What are the possible correlations between $\bar{x}$ & $\bar{x}'$ above?

Homography

$$\bar{x}' = \mathbb{H}\bar{x}$$

Fundamental Matrix

(1) $\bar{x}'^T \mathbb{F} \bar{x} = 0$
(2) $\bar{l}' = \mathbb{F}\bar{x}$

$\Downarrow$

What is $\mathbb{F}$?

$\mathbb{F}$ is a 3x3 matrix with rank 2

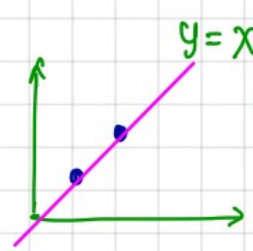
$$\bar{l}' = \bar{x}' \times \bar{e}' \rightarrow$$
$$\bar{l}' = [e']_\times \bar{x}'$$

Two points create a line.

e.g.: $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \rightarrow$ line equation:

$\downarrow$ $\bar{l}$

$ax + by + c = 0$
$-1x + 1y + 0 = 0$
$y = x$



Skew-symmetric matrix:

$$[e']_\times = \begin{bmatrix} 0 & -e'_z & e'_y \\ e'_z & 0 & -e'_x \\ -e_y & e'_x & 0 \end{bmatrix}$$

$\downarrow$

Purpose:
to transform a cross product with a vector to a matrix multiplication

$$\bar{x}' = \mathbb{H}\bar{x}$$
$$\bar{l}' = [e']_\times \mathbb{H}\bar{x}$$

$\boxed{\bar{l}' = \mathbb{F}\bar{x}}$; where $\mathbb{F} = [e']_\times \mathbb{H}$

Another look at $\mathbb{F}$:

$$\bar{x} = \mathbb{P}\bar{X}$$

Using the backward projection:

$$\bar{X} = \bar{C} + \lambda\, \mathbb{P}^{+}\bar{x}$$ → This is a ray governed by two points: $\mathbb{P}^{+}\bar{x}$ & $\bar{C}$

Projecting the two points ($\mathbb{P}^{+}\bar{x}$ and $\bar{C}$) onto another image:

point $\mathbb{P}^{+}\bar{x} \rightarrow \mathbb{P}'\mathbb{P}^{+}\bar{x}$

point $\bar{C} \rightarrow \mathbb{P}'\bar{C}$ → $\mathbb{P}'\bar{C} = \bar{e}'$

The epipolar line of these two projected points:

$$\bar{\ell}' = (\mathbb{P}'\mathbb{P}^{+}\bar{x}) \times (\mathbb{P}'\bar{C}) = (\mathbb{P}'\mathbb{P}^{+}\bar{x}) \times \bar{e}'$$
$$= [\bar{e}']_{\times}\, \mathbb{P}'\mathbb{P}^{+}\bar{x}$$

Thus, we can also define: $\mathbb{F} = [\bar{e}']_{\times}\, \mathbb{P}'\mathbb{P}^{+}$

$$\boxed{\mathbb{F} = [\mathbb{P}'\bar{C}]_{\times}\, \mathbb{P}'\mathbb{P}^{+}}$$

From the last equation, we can infer that:

1. $\mathbb{F}$ is independent from the world structures (X), and depends only on the camera properties.
2. An image pair has a single value of $\mathbb{F}$.

Properties of F

- $\mathtt{F}$ is a rank 2 homogeneous matrix with 7 degrees of freedom.
- **Point correspondence**: If $\mathbf{x}$ and $\mathbf{x}'$ are corresponding image points, then $\mathbf{x}'^{\top}\mathtt{F}\mathbf{x} = 0$.
- **Epipolar lines**:
  - ◇ $\mathbf{l}' = \mathtt{F}\mathbf{x}$ is the epipolar line corresponding to $\mathbf{x}$.
  - ◇ $\mathbf{l} = \mathtt{F}^{\top}\mathbf{x}'$ is the epipolar line corresponding to $\mathbf{x}'$.
- **Epipoles**:
  - ◇ $\mathtt{F}\mathbf{e} = \mathbf{0}$ $\qquad$ $\mathtt{F}^{\top}\mathbf{e}' = \mathbf{0}$
- **Computation from camera matrices** $\mathtt{P}, \mathtt{P}'$:
  - ◇ $\mathtt{F} = [\mathtt{P}'\mathbf{c}]_{\times}\mathtt{P}'\mathtt{P}^{+}$, where $\mathtt{P}^{+}$ is the pseudo-inverse of $\mathtt{P}$, and $\mathbf{c}$ is the centre of the first camera. Note, $\mathbf{e}' = \mathtt{P}'\mathbf{c}$.
  - ◇ Canonical cameras, $\mathtt{P} = [\mathtt{I} \mid \mathbf{0}], \mathtt{P}' = [\mathtt{M} \mid \mathbf{m}]$, $\mathtt{F} = [\mathbf{e}']_{\times}\mathtt{M} = \mathtt{M}^{-\top}[\mathbf{e}]_{\times}$, where $\mathbf{e}' = \mathbf{m}$ and $\mathbf{e} = \mathtt{M}^{-1}\mathbf{m}$.

The lines are the epipolar lines corresponding to the points on the left figure.

Left $\qquad$ Right