

**ESM 5734**

**Homework 2**

**Due on 9 September 2022 at 11:15 AM**

1. Legendre polynomial,  $P_n(x)$ , of order  $n$  is defined by

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left[ (x^2 - 1)^n \right], \quad n = 0, 1, 2, \dots$$

- (a) Express  $x^4$  as a linear combination of  $P_0$ ,  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$ .

- (b) Evaluate  $\int_{-1}^1 P_2(x) P_3(x) dx$ ,  $\int_{-1}^1 P_2^2(x) dx$ .

2. For the following boundary-value problem

$$\begin{aligned} -u'' + u &= 2, \quad 0 < x < 1 \\ u'(0) &= 3, \quad u(1) = 2, \end{aligned}$$

- (a) derive a weak formulation of the problem,  
(b) derive a matrix formulation of the problem,  
(c) use basis functions

$$\varphi_1(x) = 1, \quad \varphi_2(x) = 3x - 1, \quad \varphi_3(x) = x^2, \quad \varphi_4(x) = x^3$$

to find an approximate solution of the problem,

- (d) plot the analytical and the approximate solutions on the same Fig.,  
(e) find  $u'(1)$  from the analytical and the approximate solutions, and find error in the approximate solution,  
(f) suggest a strategy to reduce the error in  $u'(1)$ .