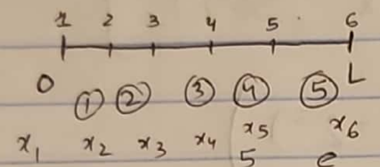
14 Sep 22

LECTURE 10



5 elements $\to$ 6 nodes.

$$\underset{6\times 6}{K_{ij}} = \sum_{e=1}^{5} K_{ij}^{e}$$

load vector, $F_i = \frac{-a}{k}\int_0^L \psi_i \, dx$

$$F_i = \underbrace{\frac{-a}{k}\int_0^L \psi_i \, dx}_{} + \psi_i(L)\,T'(L) + \psi_i(0)\,\frac{q_0}{k}$$

$\star$ $T(L) = 30°C$

$d_6 = 30°$

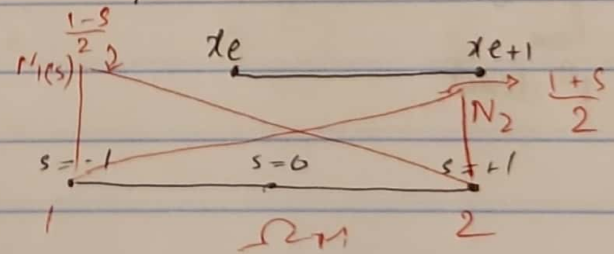
work this first b/c others are numbers & can be input afterwards.

integrate over each element.

$$\bar{F}_i = -\frac{a}{k}\int_0^L \psi_i \, dx = \frac{-a}{k}\left[\sum_e \int_{x_e}^{x_{e+1}} \psi_i \, dx\right]$$

$$\bar{F}_i^{e} = -\frac{a}{k}\int_{x_e}^{x_{e+1}} \psi_i \, dx$$

map this element to master element



$x = x_e N_1 + x_{e+1} N_2$

$$= \frac{x_e + x_{e+1}}{2} + s\left(\frac{x_{e+1} - x_e}{2}\right)$$

$$x = \quad " \quad + s\frac{h_e}{2} \rightarrow \frac{h_e}{2}$$

(1) → mapped from e to s or master element

$$Q^e = -\frac{\hat{a}}{k}\int_{-1}^{1} N_a \left(\frac{dx}{ds}\right) ds$$

$$F_a^e = -\frac{\hat{a}}{k}\frac{h_e}{2}\int_{-1}^{1} N_a\, ds \quad , \quad a = 1, 2$$

$$\int_{-1}^{1} N_1\, ds = \int_{-1}^{1} \frac{1-s}{2}\, ds = \frac{1}{2}\left(s - \frac{s^2}{2}\right)\Bigg|_{-1}^{1} = \frac{1}{2}(2-0) = 1$$

all elements are same so $-\frac{a}{k}\frac{h_e}{2}$ does not change.

$$\bar{F} = 1$$

$$\bar{F} = -\frac{a h_e}{k\,2}\left[\begin{Bmatrix}1\\1\\0\\0\\0\\0\end{Bmatrix} + \begin{Bmatrix}0\\1\\1\\0\\0\\0\end{Bmatrix} + \dots \begin{Bmatrix}0\\0\\0\\0\\1\\1\end{Bmatrix}\right] + \begin{Bmatrix}0\\0\\0\\0\\0\\T'(L)\end{Bmatrix} + \begin{Bmatrix}q_0/k\\0\\0\\0\\0\\0\end{Bmatrix}$$

6 rows b/c 6 nodes.

1st element

2nd element

5th element

5 elements.

this is $\psi_i(L)T'(L)$ which is 0 at all nodes except where $x = L$ which is node 6.

$\psi_i(0)\frac{q_0}{k}$ exists at $x = 0$ which is node 1

$$\begin{bmatrix} K_{11} & K_{12} & & & & \\ K_{21} & K_{22} & K_{23} & & & \\ & K_{32} & K_{33} & K_{34} & & \\ & & K_{43} & K_{44} & K_{45} & \\ & & & K_{54} & K_{55} & K_{56} \\ & & & & K_{65} & K_{66} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ \\ \\ d_6 \end{Bmatrix} = \begin{Bmatrix} \bar{F}_1 + q_0/k \\ \bar{F}_2 \quad 0 \\ \bar{F}_3 \quad 0 \\ \bar{F}_4 \quad 0 \\ \bar{F}_5 \quad 0 \\ \bar{F}_6 + T'(L) \end{Bmatrix}$$

Given that $d_6 = 30°$

* Trying to get $d_6 = 30$ b/c $T(L) = 30$ and $T(L)$ is d_6

If ~~unknown~~ * whenever essential boundary condition is given *

Add large number to K_{66} like λ

and replace $\bar{F}_6 + T'(L)$ or ~~some~~ whatever by $\lambda(30)$

hence equation becomes : $\dfrac{K_{65}\, d_5}{K_{66}+\lambda} + \dfrac{\cancel{(K_{66}+\lambda)}\, d_6}{\cancel{K_{66}+\lambda}} = \dfrac{30\lambda}{K_{66}+\lambda}$

$$\frac{K_{65}\, d_5}{K_{66}+\lambda} + d_6 = \frac{30\lambda}{K_{66}+\lambda}$$

divide by λ $\quad \dfrac{K_{65}\, d_5}{K_{66}+\lambda} \to 0 \quad + d_6 = \dfrac{30}{\dfrac{K_{66}}{\lambda}+1} \to i$

if $d_6 = 30$, then i should ~~and~~ be small and ii should be 0.

$\dfrac{K_{65}/\lambda}{\dfrac{K_{66}}{\lambda}+1}$ or 1, achieve that by

$\lambda \gg |K_{66}|, |K_{65}|$

$$\frac{K_{66}}{\lambda} = 10^{-6} \quad \text{or} \quad \frac{K_{66}}{\lambda} = 10^{-8}$$

where $\lambda = 10^{6}$ or $\lambda = 10^{8}$

make λ large to have approximation of 30 a better one.