

**(a) 3 noded Problem:**

```
syms s
N1(s) = (s*(s-1))/2;
N2(s) = (1-s)*(1+s);
N3(s) = (s*(s+1))/2;
N11=diff(N1)/0.1;
N21=diff(N2)/0.1;
N31=diff(N3)/0.1;
a = ((N11^2) + (N1^2))*0.1;
b = (N11*N21 + N1*N2)*0.1;
c = (N11*N31 + N1*N3)*0.1;
d = ((N21^2) + (N2^2))*0.1;
e = (N21*N31 + N2*N3)*0.1;
f = ((N31^2) + (N3^2))*0.1;
K11 = int(a,-1,1)
K12 = int(b,-1,1)
K13 = int(c,-1,1)
K22 = int(d,-1,1)
K23 = int(e,-1,1)
K33 = int(f,-1,1)
F1=2*(int(N1,-1,1))*0.1
F2=2*(int(N2,-1,1))*0.1
F3=2*(int(N3,-1,1))*0.1
A=[877/75,-333/25,83/50;-333/25,2008/75,-333/25;83/50,-333/25,877/75];
f=1/15;
g=4/15;
F=[f;g;2*f;g;2*f;g;2*f;g;2*f;g;f+2]
K=zeros(11)
for i=1:5
    for r=1:3
        for c=1:3
            K(r+2*(i-1),c+2*(i-1))=A(r,c)+K(r+2*(i-1),c+2*(i-1));
        end
    end
end
K;
for i=1:11
    F(i,1)=F(i,1)-K(i,1)*3;
end
F(1,1)=3;
for i=1:11
    K(1,i)=0;
    K(i,1)=0;
end
K(1,1)=1;
d=inv(K)*F
```

**d =**

3.0000  
3.0585  
3.1277  
3.2081  
3.3006  
3.4062  
3.5258  
3.6606  
3.8121  
3.9818  
4.1712

### (a) 4 Noded Problem

```
syms s
N1(s) = ((9*s^3)-(9*s^2)-s+1)*(-1/16); N2(s) = ((3*s^3)-(s^2)-(3*s)+1)*(9/16);
N3(s) = ((3*s^3)+(s^2)-(3*s)-1)*(-9/16); N4(s) = ((9*s^3)+(9*s^2)-s-1)*(1/16);
N11=diff(N1)/0.1; N21=diff(N2)/0.1; N31=diff(N3)/0.1; N41=diff(N4)/0.1;
a = ((N11^2) + (N1^2))*0.1; b = (N11*N21 + N1*N2)*0.1;
c = (N11*N31 + N1*N3)*0.1; d = (N11*N41 + N1*N4)*0.1;
e = ((N21^2) + (N2^2))*0.1; f = (N21*N31 + N2*N3)*0.1;
g = (N21*N41 + N2*N4)*0.1; h = ((N31^2) + (N3^2))*0.1;
i = (N31*N41 + N3*N4)*0.1; j = ((N41^2) + (N4^2))*0.1;
K11 = int(a,-1,1); K12 = int(b,-1,1); K13 = int(c,-1,1); K14 = int(d,-1,1);
K22 = int(e,-1,1); K23 = int(f,-1,1); K24 = int(g,-1,1); K33 = int(h,-1,1);
K34 = int(i,-1,1); K44 = int(j,-1,1);
A=[K11,K12,K13,K14 ; K12,K22,K23,K24; K13,K23,K33,K34; K14,K24,K34,K44]
F1=2*(int(N1,-1,1))*0.1; F2=2*(int(N2,-1,1))*0.1
F3=2*(int(N3,-1,1))*0.1; F4=2*(int(N4,-1,1))*0.1
B=[F1;F2;F3;F4];
Z=zeros(16,1);
for i=1:5
    for r=1:4
        Z(r+3*(i-1))=Z(r+3*(i-1))+B(r,1);
    end
end
Z(16,1)=Z(16,1)+2;
Z;
K=zeros(16);
for i=1:5
    for r=1:4
        for c=1:4
            K(r+3*(i-1),c+3*(i-1))=A(r,c)+K(r+3*(i-1),c+3*(i-1));
        end
    end
end
K;
for i=1:11
    Z(i,1)=Z(i,1)-K(i,1)*3;
end
Z(1,1)=3;
for i=1:16
    K(1,i)=0;
    K(i,1)=0;
end
K(1,1)=1;
d=inv(K)*Z
d =
    3.0000
    3.0379
    3.0804
    3.1277
    3.1800
    3.2376
    3.3006
    3.3695
    3.4444
    3.5258
    3.6139
    3.7092
    3.8121
    3.9231
    4.0426
    4.1712
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(b) syms u(x) u_an(x)
u(x) = piecewise ( 0<=x<=0.125, d(1,1)+(d(3,1)-d(1,1))*(x)/0.125,
0.125<=x<=0.25, d(3,1)+(d(5,1)-d(3,1))*(x-0.125)/0.125 , 0.25<=x<=0.375,
d(5,1)+(d(7,1)-d(5,1))*(x-0.25)/0.125 , 0.375<=x<=0.5,
d(7,1)+(d(9,1)-d(7,1))*(x-0.375)/0.125, 0.5<=x<=0.625 ,
d(9,1)+(d(11,1)-d(9,1))*(x-0.5)/0.125,
0.625<=x<=0.75,d(11,1)+(d(13,1)-d(11,1))*(x-0.625)/0.125, 0.75<=x<=0.875,
d(13,1)+(d(15,1)-d(13,1))*(x-0.75)/0.125, 0.875<=x<=1,
d(15,1)+(d(17,1)-d(15,1))*(x-0.875)/0.125);
c1=(1+2*exp(1))/(1+exp(2));
c2=(exp(2)-2*exp(1))/(1+exp(2));
u_an(x)=piecewise(0<=x<=1,c1*exp(x)+c2*exp(-x)+2);
e=int((u-u_an)^2+(diff(u-u_an))^2,0,1)/int((u_an^2+(diff(u_an))^2),0,1);
e=e^(0.5);
e=log(e);
vpa(e,6)
(code for 8 element 3 noded problem to find the error norm)

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