

**ESM 5734**  
**Homework # 4**  
**Due on 27 September, 2012 at 11:00AM**

Find

- (a) an approximate solution of the HW 3 problem using 3-noded and 4-noded 5 uniform finite elements and the corresponding finite element basis functions,
- (b) find the  $H^1$ - error norm for 2, 4, 8 and 10 elements of equal length.

$$(H^1\text{- error norm})^2 = \left[ \int_0^1 (u_{app} - u_{anal})^2 dx + \int_0^1 \left( \frac{du_{app}}{dx} - \frac{du_{anal}}{dx} \right)^2 dx \right] / \left[ \int_0^1 (u_{anal})^2 dx + \int_0^1 \left( \frac{du_{anal}}{dx} \right)^2 dx \right]$$

Plot  $\log(H^1\text{-error norm})$  vs.  $\log(h_n)$  where  $h_n$  is the distance between two adjacent nodes. Fit a straight line by the least-squares method, find its slope, and provide an interpretation of the slope.

**Note:**

Using higher order polynomials for the domain divided into the same number of elements is called p-refinement, and keeping the polynomial order fixed but increasing the number of finite elements is called h-refinement. Of course, one can use both p- and h-refinement in the same problem.