then for a uniform FE mesh of 2 elements and 3 nodes with nodes numbered consecutively from left to right, equations (5.6.8) become

$$\begin{bmatrix} bK_{11} & \alpha_{11} & bK_{12} & \alpha_{12} & bK_{13} & \alpha_{13} \\ 0 & K_{11} & 0 & K_{12} & 0 & K_{13} \\ bK_{21} & \alpha_{21} & bK_{22} & \alpha_{22} & bK_{23} & \alpha_{23} \\ 0 & K_{21} & 0 & K_{22} & 0 & K_{23} \\ bK_{31} & \alpha_{31} & bK_{32} & \alpha_{32} & bK_{33} & \alpha_{33} \\ 0 & K_{31} & 0 & K_{32} & 0 & K_{33} \end{bmatrix} \begin{Bmatrix} w_1 \\ M_1 \\ w_2 \\ M_2 \\ w_3 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_1 - M^{n'}(0) \\ 0 \\ F_2 \\ bw^{n'}(\ell) \\ F_0 + F_3 \end{Bmatrix}. \tag{5.6.10}$$

We now impose the boundary condition $w(0) = w_1 = 0$ and M(l) = M_3 = M_0 by using one of the techniques discussed in Section 4.4. A possibility is modify 2nd and the 5th equations, respectively, impose $M(\ell) = M_3 = M_0$ and $w_1 = w_0$. It will eliminate the unknowns $w^{n'}(\ell)$ in the 5th equation and $M^{n'}(0)$ in the 2nd. Note that equations (5.6.10) are 6 algebraic equations which need to be solved simultaneously for the six unknowns w_1, M_1, w_2, M_2, w_3 and M_3.

Using method A outlined in Section 4.4 to enforce the boundary conditions $w_1 = 0,\ M_3 = M_0$, we arrive at the following set of algebraic equations.

$$\begin{bmatrix} 0 & \alpha_{11} & bK_{12} & \alpha_{12} & bK_{13} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \alpha_{21} & bK_{22} & \alpha_{22} & bK_{23} & 0 \\ 0 & K_{21} & 0 & K_{22} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{31} & 0 & K_{32} & 0 & 0 \end{bmatrix} \begin{Bmatrix} w_1 \\ M_1 \\ w_2 \\ M_2 \\ w_3 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ M_0 \\ 0 \\ F_2 \\ 0 \\ F_0 + F_3 \end{Bmatrix}. \tag{5.6.11}$$

It should be emphasized that the essential BC $w'(0) = 0$ of the original problem may not be exactly satisfied by using the mixed formulation of this section. However, the natural BC $(bw'')'\Big|_{x=\ell} = M_0$ of the original problem is now exactly satisfied.

When b is not a constant, then the weak formulation of the original fourth order ODE involves terms symmetric in ϕ and w. If we proceed as usual, the weak formulation of the two second order ODEs will involve derivatives of b and will be unsymmetric in ϕ and w thereby resulting in a nonsymmetric stiffness matrix. However, it can be rendered symmetric by taking the test or the weight function for equation $(5.6.2)_1$ as ϕ_1/b.