

```

clear all
clc
%INPUTS
L=22; %length of the bar
br=8.75/12;hei=27/12;
num=22; %number of elements
E=(2*10^6)*12^2;
q=1500;
%processed quantities
h=L/num; %size of element
I=(br)*(hei^3)*(1/12); %Second Moment of Inertia
%% DERIVING SHAPE FUNCTIONS
syms s
N10(s)=(1/4)*(2-3*s+s^3); N20(s)=(1/4)*(2+3*s-s^3);
N11(s)=(1/4)*(1-s-s^2+s^3);N21(s)=(1/4)*(-1-s+s^2+s^3);
N10_2=diff(diff(N10))/(h/2)^2;N20_2=diff(diff(N20))/(h/2)^2;
N11_2=diff(diff(N11))/(h/2)^2;N21_2=diff(diff(N21))/(h/2)^2;
a=(h/2)*(N10_2)^2, b=(h/2)*(N10_2)*(N20_2), c=(h/2)*(N20_2)^2
d=(h/2)*N10_2*N11_2; e=(h/2)*N10_2*N21_2;f=(h/2)*N20_2*N11_2;g=(h/2)*N20_2*N21_2;
h1=(h/2)*N11_2*N10_2; i=(h/2)*N11_2*N20_2;j=(h/2)*N21_2*N10_2; k=(h/2)*N21_2*N20_2;
l=(h/2)*N11_2^2; m=(h/2)*N11_2*N21_2; n=(h/2)*N21_2^2;
%% Element Matrices
K11_00=int(a,-1,1);K12_00=int(b,-1,1); K11_01=int(d,-1,1);K12_01=int(e,-1,1);
K21_00=K12_00; K22_00=int(c,-1,1); K21_01=int(f,-1,1);K22_01=int(g,-1,1);
K11_10=int(h1,-1,1);K12_10=int(i,-1,1); K11_11=int(l,-1,1);K12_11=int(m,-1,1);
K21_10=int(j,-1,1);K22_10=int(k,-1,1); K21_11=K12_11; K22_11=int(n,-1,1);
A= [K11_00,K12_00;K21_00,K22_00]; B= [K11_01,K12_01;K21_01,K22_01];
C= [K11_10,K12_10;K21_10,K22_10]; D= [K11_11,K12_11;K21_11,K22_11];
%% Global Matrix
K00=zeros(num+1);K01=zeros(num+1);K10=zeros(num+1);K11=zeros(num+1);
for i=1:num
    for r=1:2
        for c=1:2
            K00(r+1*(i-1),c+1*(i-1))=A(r,c)+K00(r+1*(i-1),c+1*(i-1));
            K01(r+1*(i-1),c+1*(i-1))=B(r,c)+K01(r+1*(i-1),c+1*(i-1));
            K10(r+1*(i-1),c+1*(i-1))=C(r,c)+K10(r+1*(i-1),c+1*(i-1));
            K11(r+1*(i-1),c+1*(i-1))=D(r,c)+K11(r+1*(i-1),c+1*(i-1));
        end
    end
end
K_FH=cat(2,K00,K01); K_SH=cat(2,K10,K11); K_G=cat(1,K_FH,K_SH)*(E*I)
%% LOAD VECTOR:
F10=-q*(int(N10,-1,1))*(h/2);F20=-q*(int(N20,-1,1))*(h/2);F11=-q*(int(N11,-1,1))*(h/2);F21=-q*(int(N21,-1,1))*(h/2);F_FH=zeros(num+1,1);F_SH=zeros(num+1,1);
for i=1:num
    F_FH(i)=F_FH(i)+F10; F_SH(i)=F_SH(i)+F11;
    F_FH(i+1)=F20; F_SH(i+1)=F21;
end
F_G=cat(1,F_FH,F_SH)
for i=1:2*num+2
    K_G(1,i)=0;K_G(num+1,i)=0;
end
K_G(1,1)=1;K_G(num+1,num+1)=1;F_G(1,1)=0;F_G(num+1,1)=0;
d=inv(K_G)*F_G;
disp=zeros(num+1,1);
for i=1:num+1
    disp(i)=d(i);
end
slope=zeros(num+1,1);
for i=1:num+1
    slope(i)=d(i+num+1);
end
y=1:num+1;
x=0:h:L;
disp=disp*12;
syms s
u_an(s)=piecewise(0<=s<=22,((-q/24)*s^4+(16500)*((s^3)/6)-(665500)*s)/(E*I));u_an=u_an*12;
figure(1)
plot(x,disp) hold on, fplot(u_an) hold off, legend('Approximate','Analytical'),
xlabel('distance from the left end'),ylabel('Displacement(in)')
%% moment

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M=zeros(num,1);
for i=1:num
    M(i)=disp(i)*N10_2(0)+disp(i+1)*N20_2(0)+slope(i)*N11_2(0)+slope(i+1)*N21_2(0);
end
M=M*E*I;figure(2)
z=h/2:h:(2*num-1)*h/2;scatter(z,M)
title('Moment vs distance from left end')
ylabel('Moment (lb*ft)')xlabel('distance(ft)')

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(a) Considered 22 elements for this problem

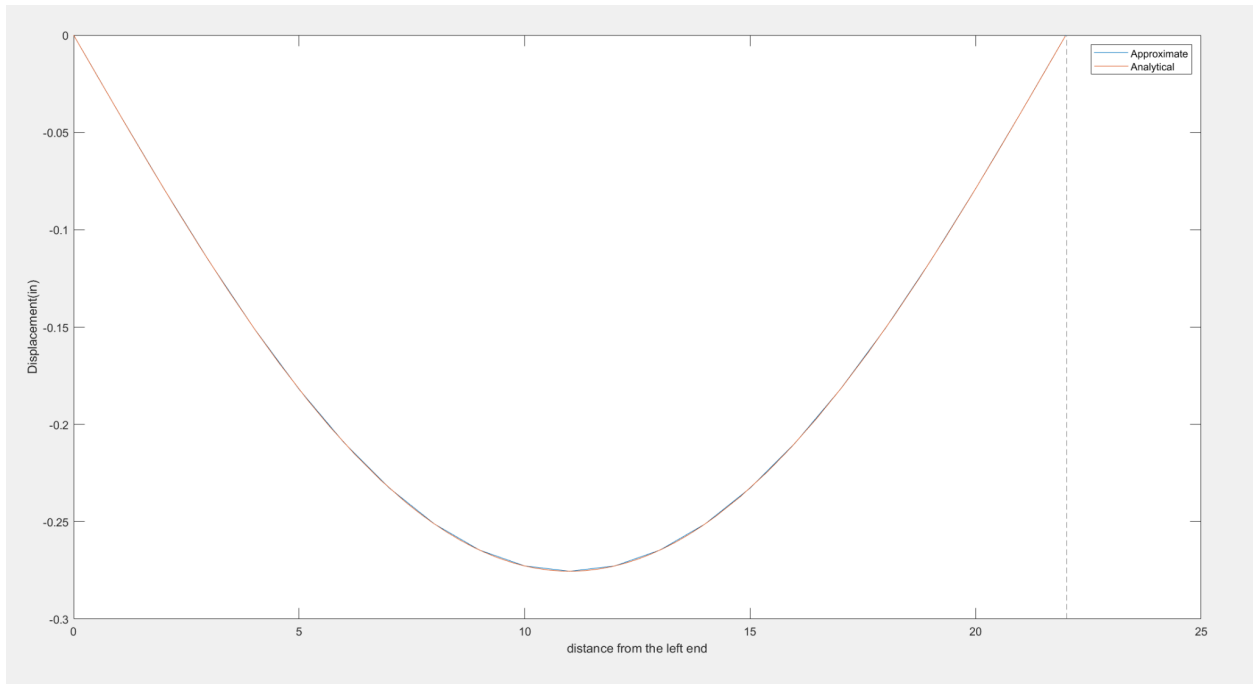


Fig.1.Approximate and Analytical Solution of Displacement Profile

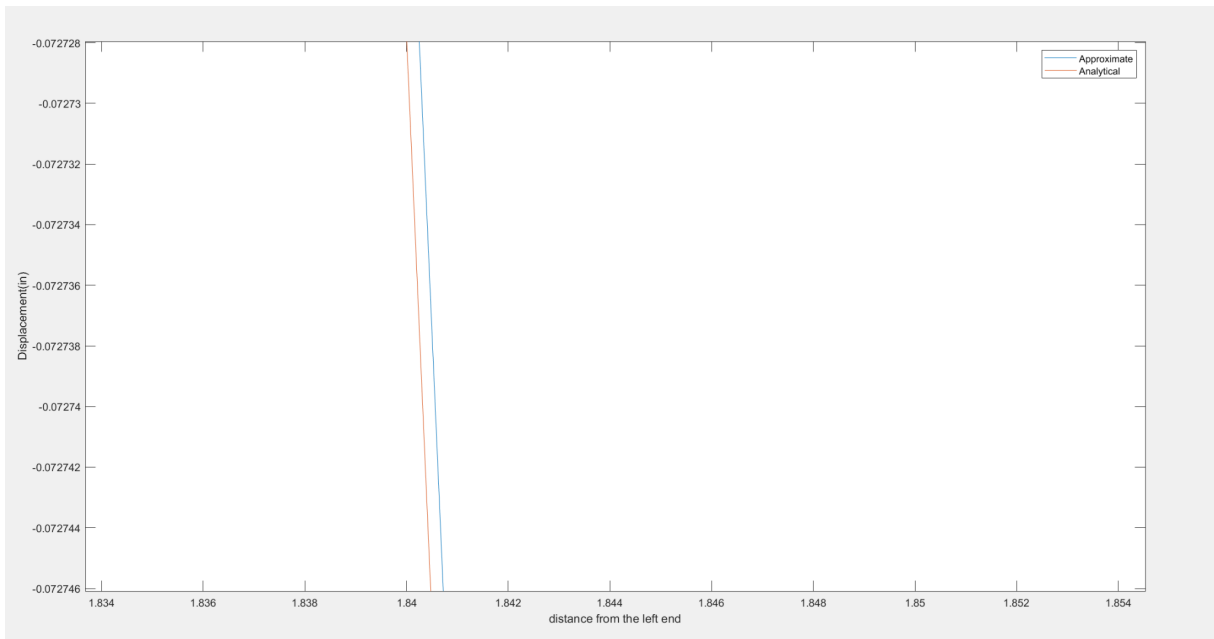
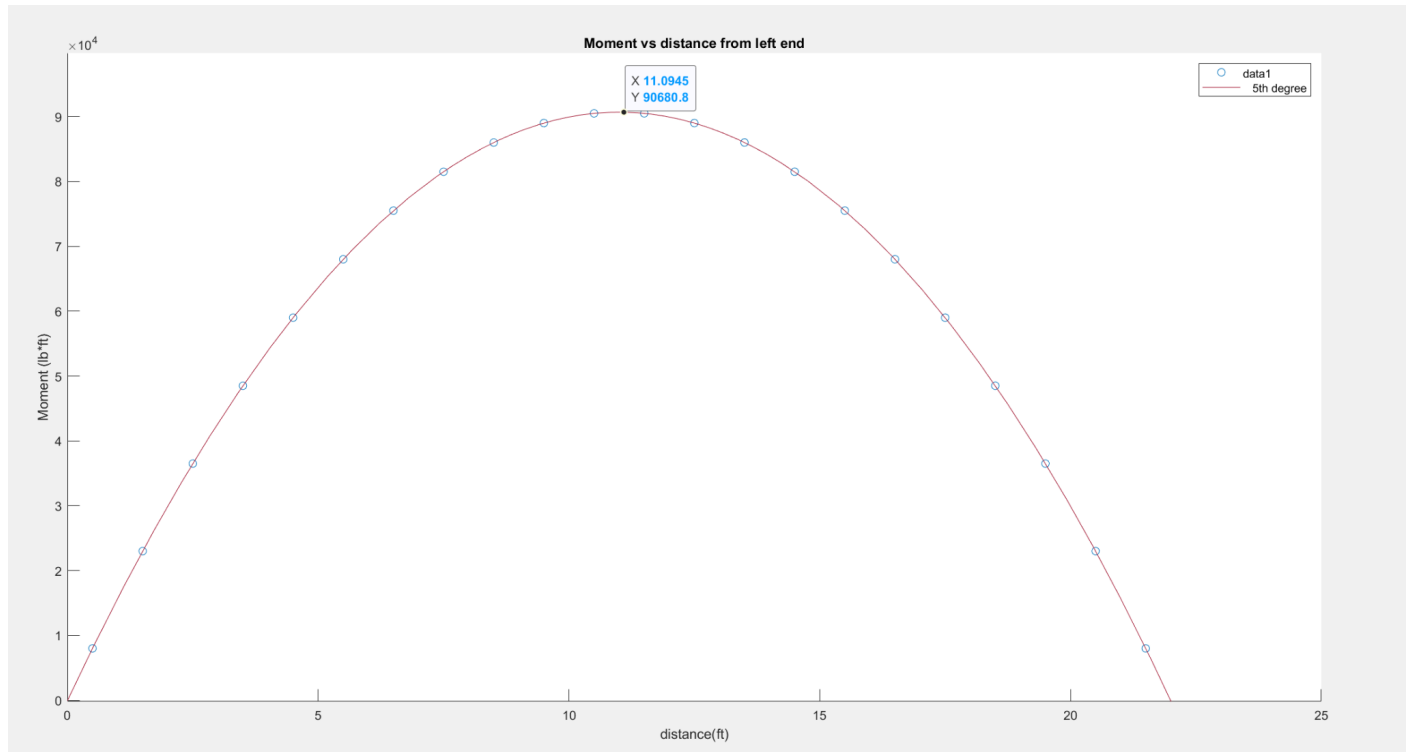


Fig.2.Zoomed image of Analytical and Approximate Solutions”

(b)



(c).

The maximum Tensile and compressive Stress occur at the point where the bending moment is the maximum, depending upon the load direction we can assess at which point at that particular cross section it is tensile and compressive.

Max bending Stress= (maximum Bending Moment)*(distance of the farthest layer)
/(Second Moment of Inertia)

$$\sigma = (Mx \cdot y / I)$$

$$\sigma = (90700 \cdot 12 \cdot 27 / I)$$

$\sigma = 1.0235 \times 10^3$ lbs/in (Max Tensile Str) at X=11ft on the bottom layer

$\sigma = -1.0235 \times 10^3$ lbs/in (Max Compressive Stress) at X=11ft on the Upper layer

(d).

```
%% ERROR FUNCTION %X=(0.5*(1-s)*x(i)+0.5*(1+s)*x(i+1));
disp=disp/12;u_an=u_an/12;
Num=0;
for i=1:num
Num=Num+int((disp(i)*N10+disp(i+1)*N20+slope(i)*N11+slope(i+1)*N21-((-q/24)*(0.5*(1-s)*x(i)+0.5*(1+s)*x(i+1)))^4+(16500)*((0.5*(1-s)*x(i)+0.5*(1+s)*x(i+1))^3)/6)-(665500)*(0.5*(1-s)*x(i)+0.5*(1+s)*x(i+1)))/(E*I))^2,-1,1)*(h/2);
end
for i=1:num
Num=Num+int((diff(disp(i)*N10+disp(i+1)*N20+slope(i)*N11+slope(i+1)*N21-((-q/24)*(0.5*(1-s)*x(i)+0.5*(1+s)*x(i+1)))^4+(16500)*((0.5*(1-s)*x(i)+0.5*(1+s)*x(i+1))^3)/6)-(665500)*(0.5*(1-s)*x(i)+0.5*(1+s)*x(i+1)))/(E*I))^2,-1,1)*(2/h);
end
Den=int(u_an^2,0,22)+int((diff(u_an))^2,0,L);
Err=(Num)/(Den)^0.5;
Err=log(Err);ErrVpa=vpa(Err,6)
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ErrVpa =-12.8088