

```

clear
clc
%INPUTS
n=22;
br=8.75;hei=27;L=22*12;
I=(br)*(hei^3)*(1/12);
E=2*10^6;
Mo=(hei*E*I/L^2);
%element size
h=1/n;
%% DERIVING SHAPE FUNCTIONS
syms s
N1(s) = ((9*s^3)-(9*s^2)-s+1)*(-1/16);N2(s) = ((3*s^3)-(s^2)-(3*s)+1)*(9/16);%SHAPE FUNCS
N3(s) = ((3*s^3)+(s^2)-(3*s)-1)*(-9/16);N4(s) = ((9*s^3)+(9*s^2)-s-1)*(1/16);
N11=diff(N1)/(h/2);N21=diff(N2)/(h/2); %Derivatives of SHAPE FUNCS
N31=diff(N3)/(h/2);N41=diff(N4)/(h/2);
int(N11^2,-1,1) %Trial
a = (h/2)*N11^2; k=N1^2*(h/2);b = (h/2)*N11*N21;l=N1*N2*(h/2);c = (h/2)*N11*N31;
m=N1*N3*(h/2);d = (h/2)*N11*N41; n2=N1*N4*(h/2);e = (h/2)*N21^2;
o=N2*N2*(h/2);f = (h/2)*N21*N31; p=N2*N3*(h/2);g = (h/2)*N21*N41;
q=N2*N4*(h/2);h1 = (h/2)*N31^2; r=N3^2*(h/2);i = (h/2)*N31*N41;
s=N3*N4*(h/2);j = (h/2)*N41^2; t=N4^2*(h/2);K11 = int(a,-1,1);K12 = int(b,-1,1);K13 =
int(c,-1,1);K14 = int(d,-1,1);K22 = int(e,-1,1);K23 = int(f,-1,1);K24 = int(g,-1,1);K33 =
int(h1,-1,1);K34 = int(i,-1,1);K44 = int(j,-1,1);A11= int(k,-1,1);A12= int(l,-1,1);A13=
int(m,-1,1);A14= int(n2,-1,1);A22= int(o,-1,1);A23= int(p,-1,1);A24= int(q,-1,1);A33=
int(r,-1,1);A34= int(s,-1,1);A44= int(t,-1,1);
%ELEMENT STIFFNESS MATRIX
A=[K11,A11,K12,A12,K13,A13,K14,A14 ; 0,K11,0,K12,0,K13,0,K14 ;
K12,A12,K22,A22,K23,A23,K24,A24 ; 0,K12,0,K22,0,K23,0,K24 ;
K13,A13,K23,A23,K33,A33,K34,A34 ; 0,K13,0,K23,0,K33,0,K34 ; K14,A14,K24,A24,K34,A34,K44,A44
; 0,K14,0,K24,0,K34,0,K44]
%% GLOBAL STIFFNESS MATRIX AND GLOBAL LOAD VECTOR
%GLOBAL STIFFNESS MATRIX
K=zeros(6*n+2);
for i=1:n
    for r=1:8
        for c=1:8
            K(r+6*(i-1),c+6*(i-1))=A(r,c)+K(r+6*(i-1),c+6*(i-1));
        end
    end
end
K;
%LOAD VECTOR
q=-(1500/12)*(L^2)/Mo
P=(12000/12)*(L^2)/Mo
%P=0;
F1=-q*(int(N1,-1,1))*(h/2);F2=-q*(int(N2,-1,1))*(h/2);
F3=-q*(int(N3,-1,1))*(h/2);F4=-q*(int(N4,-1,1))*(h/2);
B=[0;F1;0;F2;0;F3;0;F4];
%GLOBAL LOAD VECTOR
Z=zeros((3*n+1)*2,1);
for i=1:n
    for r=1:8
        Z(r+6*(i-1))=Z(r+6*(i-1))+B(r,1);
    end
end
%% IMPLEMENTING BOUNDARY CONDITIONS
%BOUNDARY CONDITIONS
Z(6*n+1,1)=0; Z(6*n+2,1)=0; Z(2,1)=0;
Z(56,1)=Z(56,1)+P;%%POINT LOAD CONDITION%%
K(1,1)=1e+10; K(2,2)=1e+10; K(6*n+1,6*n+1)=1e+10; K(6*n+2,6*n+2)=1e+10;

%SOLUTION
d=K\Z;
X=0:h/3:1; %X Coordinates of Nodes
y=1:3*n+1; %NODES NUMBERED
%Dipslacement Solution
disp=zeros(3*n+1,1);
for i=1:3*n+1
    disp(i)=d(2*i-1);
end

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%Moment Solution
mom=zeros(3*n+1,1);
for i=1:3*n+1
    mom(i)=d(2*i);
end
%disp=disp*27;
mom=mom*Mo/12;

c1=21145521/4840000;c3=(-431955567)/937024000;d1=(-12981803)/4840000;d2=2864511/1663750;d3=(-9
54837)/9150625;d4=-2182/15625;
syms disp_an(x)

disp_an(x)=piecewise(0<=x<=9/22, (-0.7835/24)*x^4+(c1/6)*x^3+(c3)*x, 9/22<=x<=1, (-0.7835/24)*(x-
9/22)^4+(d1/6)*(x-9/22)^3+(d2/2)*(x-9/22)^2+d3*(x-9/22)+d4);
%disp_an=disp_an*27/12;

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- **FOR P=0**

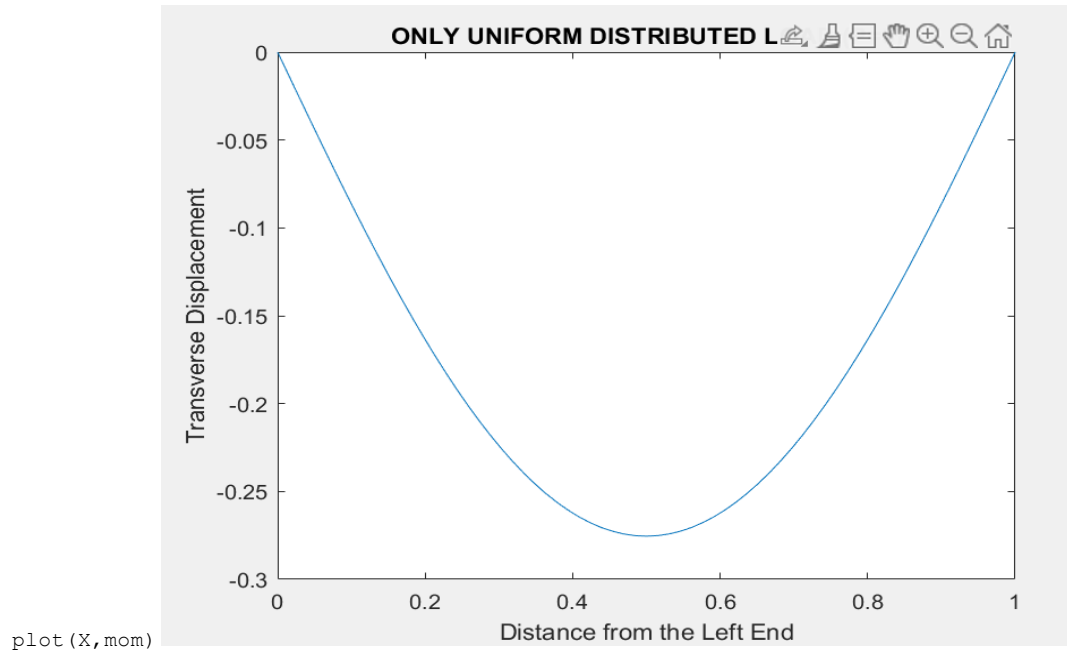


Fig.1.Displacement Profile When No Point Load is Applied

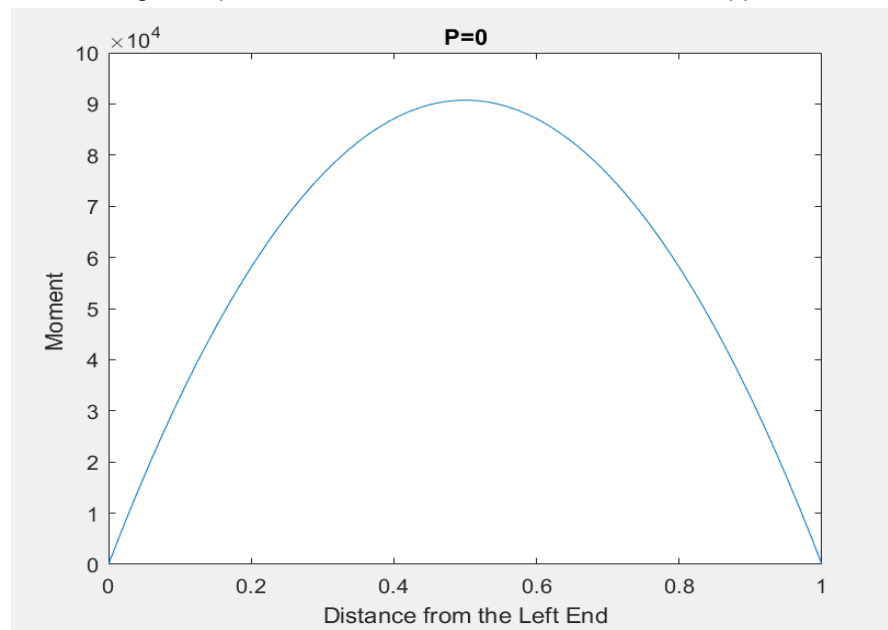


Fig.2.Moment vs Distance when no Point Load is applied

a.

- For $P=12000$ lbs

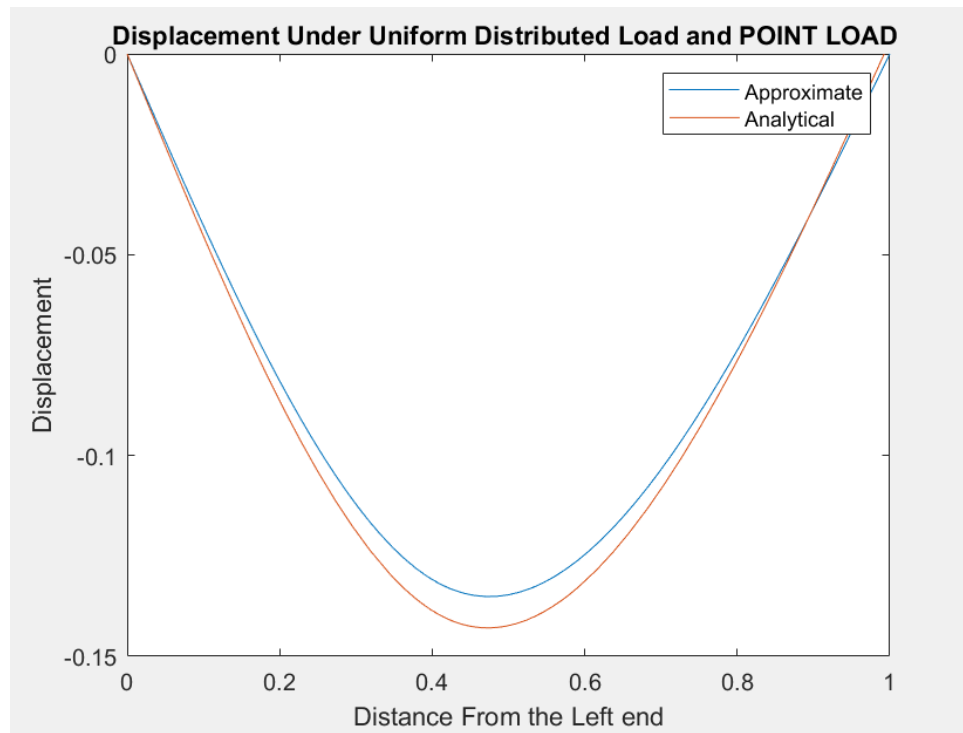
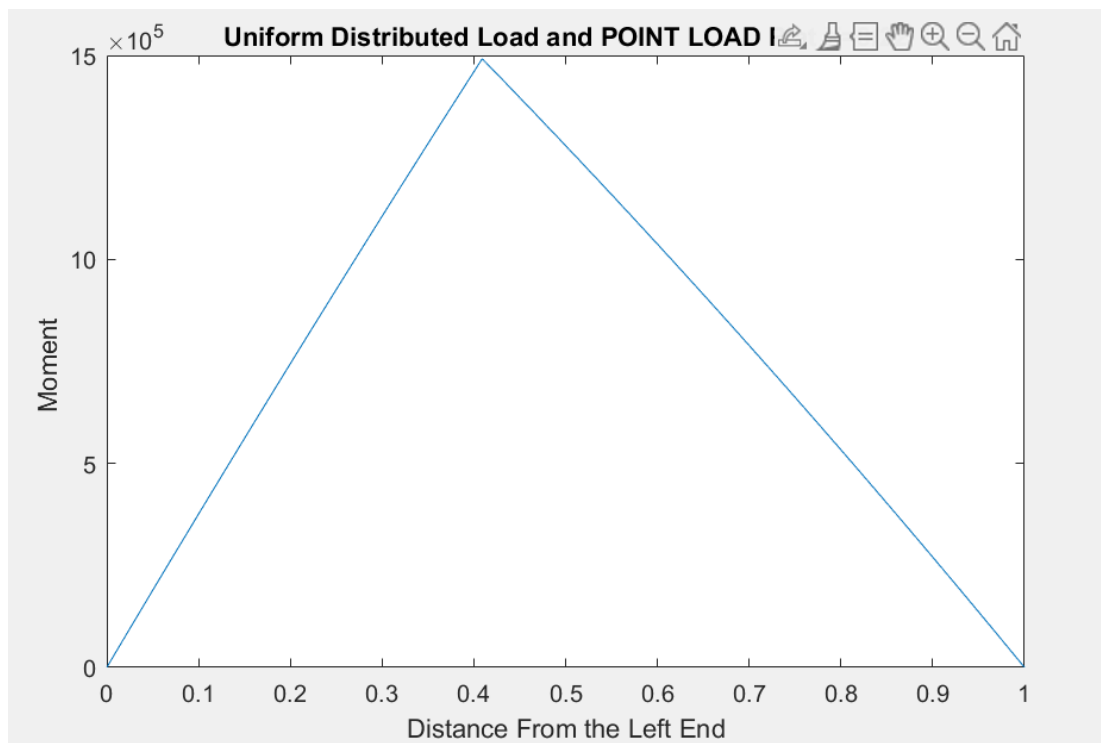


Fig.2.Displacement Profile when Point Load is Applied



b.

Fig.4.Moment vs Distance from Left end when Point Load is Applied

- C.** The maximum Tensile and compressive Stress occur at the point where the bending moment is the maximum, depending upon the load direction we can assess at which point at that particular cross section it is tensile and compressive.

As the beam is symmetrical and the upper and lower end are equally spaced from neutral axis the magnitude of the stress would be the same, and the maximum stress occurs at X=9ft as obtained from the solution.

Max bending Stress= (maximum Bending Moment)*(distance of the farthest layer)
/(Second Moment of Inertia)

$$\sigma = (Mx \cdot y / I)$$

$$\sigma = (1491750 \cdot 12 \cdot 27 / 2 / I)$$

$\sigma = 4.54628 \times 10^5$ lbs/in (Max Tensile Str) at X=9ft on the bottom layer

$\sigma = -4.54628 \times 10^5$ lbs/in (Max Compressive Stress) at X=9ft on the Upper layer

d.

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%% ERROR FUNCTION
%X=X(3*(i-1)+1)*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4;
Num=0;
for i=1:n
    if i<=9
        Num=
Num+int(((disp(3*(i-1)+1)*N1+disp(3*(i-1)+2)*N2+disp(3*(i-1)+3)*N3+disp(3*(i-1)+4)*N4-((-0.783
5/24)*(X(3*(i-1)+1)*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)^4+(c1/6)*(X(3*(i-1)+1)
*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)^3+(c3)*(X(3*(i-1)+1)*N1+X(3*(i-1)+2)*N2+X
(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4))))^2,0,1)*(h/2);
    else
        Num=
Num+int(((disp(3*(i-1)+1)*N1+disp(3*(i-1)+2)*N2+disp(3*(i-1)+3)*N3+disp(3*(i-1)+4)*N4-((-0.783
5/24)*((X(3*(i-1)+1)*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)-9/22)^4+(d1/6)*((X(3*
(i-1)+1)*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)-9/22)^3+(d2/2)*((X(3*(i-1)+1)*N1+
X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)-9/22)^2+d3*((X(3*(i-1)+1)*N1+X(3*(i-1)+2)*N2+
X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)-9/22)+d4))))^2,0,1)*(h/2);
    end
end
for i=1:n
    if i<=9
        Num=
Num+int(diff(((disp(3*(i-1)+1)*N1+disp(3*(i-1)+2)*N2+disp(3*(i-1)+3)*N3+disp(3*(i-1)+4)*N4-((-
0.7835/24)*(X(3*(i-1)+1)*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)^4+(c1/6)*(X(3*(i-
1)+1)*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)^3+(c3)*(X(3*(i-1)+1)*N1+X(3*(i-1)+2)
*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4))))^2,0,1);
    else
        Num=
Num+int(diff(((disp(3*(i-1)+1)*N1+disp(3*(i-1)+2)*N2+disp(3*(i-1)+3)*N3+disp(3*(i-1)+4)*N4-((-
0.7835/24)*((X(3*(i-1)+1)*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)-9/22)^4+(d1/6)*((
X(3*(i-1)+1)*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)-9/22)^3+(d2/2)*((X(3*(i-1)+1)
)*N1+X(3*(i-1)+2)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)-9/22)^2+d3*((X(3*(i-1)+1)*N1+X(3*(i-1)+2)
)*N2+X(3*(i-1)+3)*N3+X(3*(i-1)+4)*N4)-9/22)+d4))))^2,0,1);
    end
end
Den=int(disp_an^2,0,1)+int(diff((disp_an)^2),0,1);
Err=(Num)/(Den)^0.5;
Err=log(Err)
ErrVpa=vpa(Err,6)
```

ErrVpa = -3.14288 (used Hnorm)