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ESM-5734 HW-5

$$y'' + \frac{\pi^2}{\alpha} y + 0.01(1-x) = 0 , \qquad 0 < x < 1$$

$$y(0) = 0 , \quad y(1) = 0.$$

Let $\frac{\pi^2}{\alpha} = b$

$\rightarrow$ $y'' + by + 0.01(1-x) = 0$

multiplying above by $\phi(x)$ & integrating

$$\int_0^1 y''\phi(x).dx + \int_0^1 by\,\phi(x).dx = \int_0^1 (x-1)\,0.01\,.\phi(x).dx$$

$$\left[\phi(x)y'(x)\right]_0^1 - \int_0^1 y'(x)\phi'(x).dx + b\int_0^1 y\phi(x).dx = \int_0^1 (x-1)\,0.01\,\phi(x).dx$$

$$\int_0^1 y'(x).\phi'(x).dx - \int_0^1 by\phi(x).dx = \int_0^1 (1-x).0.01\,\phi(x).dx + \left[\phi(x)y'(x)\right]_0^1$$

$\rightarrow$ Let $\phi(x) = \sum_1^m c_i\psi_i$ (GALERKIN APPROXIMATION also done)

$y(x) = \sum_1^m d_j\psi_j$

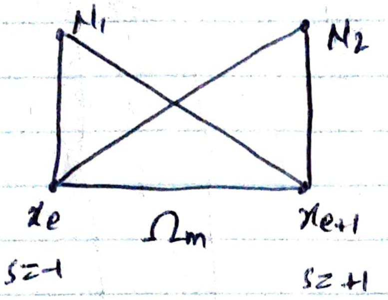
$$\sum_{j=1}^m \sum_{i=1}^m \left[\int_0^1 c_i\psi_i'\psi_j'.d_j.dx - b\int_0^1 c_i\psi_i\psi_j.d_j.dx\right] = \sum_{i=1}^m \int_0^1 (1-x).(0.01)c_i\psi_i(x).dx$$

$$+ \sum_{i=1}^m c_i\left[-y'(0)\psi_i(0) + y'(1)\psi_i(1)\right]$$

$$\rightarrow \sum_{i=1}^m \sum_{j=1}^m c_i \underbrace{\left[\int_0^1 (\psi_i'\psi_j' - b\psi_i\psi_j).dx\right]}_{K_{ij}}.d_j = \sum_{i=1}^m c_i \underbrace{\left[\int_0^1 (1-x)0.01\psi_i.dx + y'(0)\psi_i(0) + y'(1)\psi_i(1)\right]}_{F_i}$$

→ For this problem I am choosing 2 nodes per each element.

→ For an element Ω_e



$$s = \frac{x - \frac{(x_e + x_{e+1})}{2}}{h_e/2}$$

$$\left[x = s\left(\frac{h_e}{2}\right) + \left(\frac{x_e + x_{e+1}}{2}\right) \right]$$

$$\frac{dx}{ds} = \frac{h_e}{2}$$

$$N_1 = \frac{(1-s)}{2} \; ; \; N_2 = \frac{(1+s)}{2}$$

$$\frac{dN_1}{dx} = \frac{-1}{2} \cdot \frac{2}{h_e} = \frac{-1}{h_e} \quad \Big| \quad \frac{dN_2}{dx} = \frac{1}{h_e}$$

$$\rightarrow K_{ij} = \Sigma \int_{x_1}^{x_2} (\psi_i' \psi_j' - b\psi_i \psi_j) \cdot dx + \Sigma \int_{x_2}^{x_3} (\psi_i' \psi_j' - b\psi_i \psi_j) \cdot dx + \ldots$$

for a 2 noded element

$$K_{11} = \int_{-1}^{1} \left[(N_1')^2 - b(N_1)^2 \right] \cdot \frac{h_e}{2} \cdot ds \quad \text{IIly} \quad K_{12} = \int_{-1}^{1} \left([N_1'][N_2'] - b[N_1][N_2] \right) \frac{h_e}{2} \cdot ds$$

$$K_{22} = \int_{-1}^{1} \left[(N_2')^2 - b(N_2)^2 \right] \cdot \frac{h_e}{2} \cdot ds$$

$$\rightarrow \left[F_i = \int_0^1 (1-x)\, 0.01\, \psi_i \cdot dx + y'(0)\psi_i(0) + y'(1)\psi_i(1) \right]$$

$$\bar{F}_i = \int_0^1 (1-x)\, 0.01\, \psi_i \cdot dx = \Sigma \int_{x_1}^{x_2} \psi_i \cdot (1-x) \cdot 0.01 \cdot dx + \ldots + \Sigma \int_{x_e}^{x_{e+1}} \psi_i\, 0.01 (1-x) \cdot dx + \ldots$$

for element e $\quad \bar{F}_1 = \Sigma \int_{x_e}^{x_{e+1}} (1-x)\, 0.01\, \psi_e \cdot dx$

$$\left[\bar{F}_1^e = \int_{-1}^{1} \left(1 - s\left(\frac{h_e}{2}\right) - \left(\frac{x_e + x_{e+1}}{2}\right)\right) 0.01 \cdot N_1 \cdot \frac{h_e}{2} \cdot ds \right]$$

$$\bar{F}_2^e = \int_{-1}^{1} \left(1 - \frac{s h_e}{2} - \left(\frac{x_e + x_{e+1}}{2}\right)\right] \cdot 0.01 \cdot \frac{h_e}{2} \cdot N_2 \cdot ds$$

$$\bar{F}^e = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \bar{F}_1^e \\ \bar{F}_2^e \\ \vdots \\ 0 \end{Bmatrix} \begin{array}{l} \\ \\ \\ \\ e\text{th row} \\ e+1\text{th row} \\ \\ \\ \end{array} \qquad \therefore \bar{F} = \begin{Bmatrix} \bar{F}_1^1 \\ \bar{F}_2^1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{Bmatrix} + \begin{Bmatrix} 0 \\ \bar{F}_1^2 \\ \bar{F}_2^2 \\ 0 \\ 0 \\ 0 \\ \vdots \end{Bmatrix} + \cdots$$

$$F = \bar{F} + \begin{Bmatrix} -y'(0) \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ +y'(1) \end{Bmatrix}$$

as $\psi_1(0) = 1$ only at $x = x_1$

as $\psi_i(1) = 1$ at $x=1$ for $\psi_N(1) = 1$ & $\psi_i(1) = 0$ for all other shape func.

$$K_{ij} \cdot \begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{Bmatrix} = F$$

now substracting F by $(K_{1,j} \times BC_1)$ & $(K_{2,j} \times BC_2)$

and set $(K_{1,1}) = 1$, $(K_{n,n}) = 1$, & all other elements in 1st R&C

nth R&C $= 0$. and set first element of $F = BC_1$ &

last element of $F = BC_2$. //

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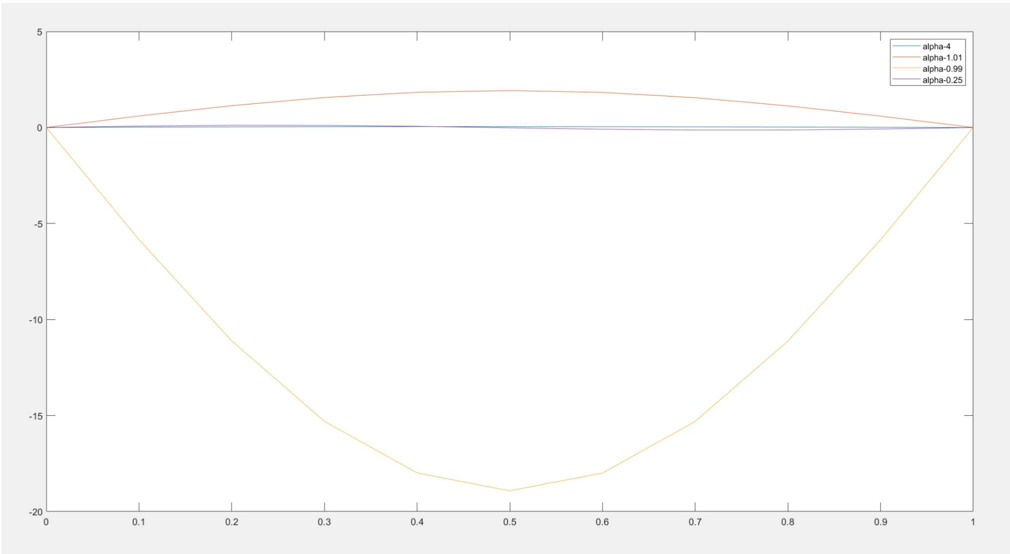
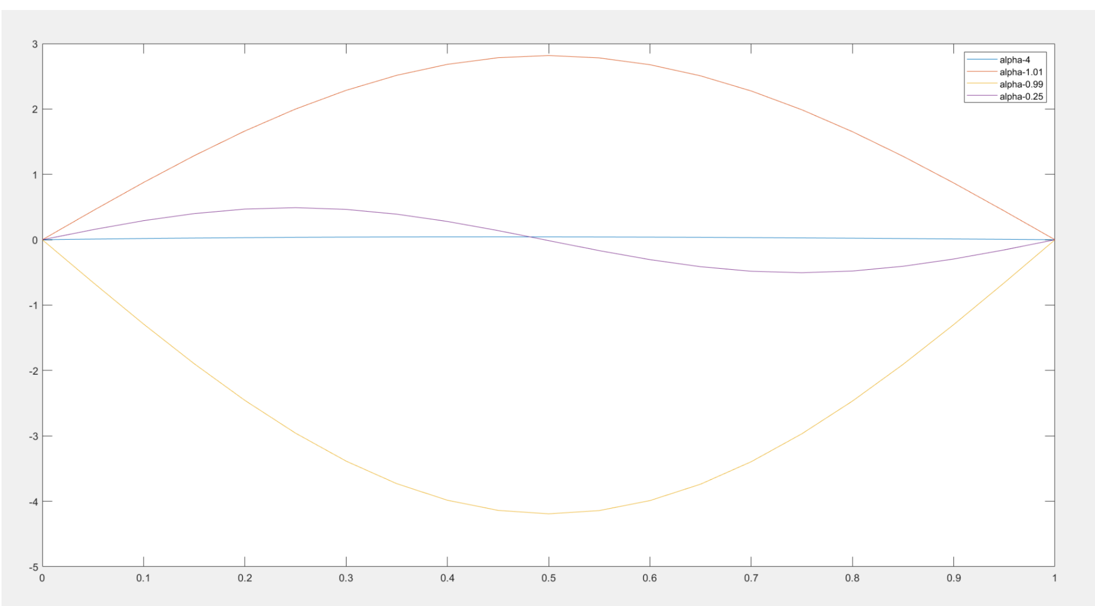
```
alphas=[4,1.01,0.99,0.25]
for l=1:4
   alpha=alphas(l)
%INPUTS
n= 20 ;
%alpha=0.99;
%SHAPE FUNCTIONS
syms s
N1(s) = (1-s)/2;
N2(s) = (1+s)/2;
%length of the element
h= 1/n;
%constatnt on the right side
p= pi^2/alpha;
N11=diff(N1)/(h/2);
N21=diff(N2)/(h/2);
%KMATRIX
a = ((N11^2) - (p)*(N1^2))*(h/2);
b = (N11*N21 - (p)*(N1*N2))*(h/2);
c = ((N21^2) - (p)*(N2^2))*(h/2);
K11 = int(a,-1,1)
K12 = int(b,-1,1)
K22 = int(c,-1,1)
A=[K11,K12;K12,K22];A
K=zeros(n+1);
for i=1:n
       for r=1:2
           for c=1:2
               K(r+1*(i-1),c+1*(i-1))=A(r,c)+K(r+1*(i-1),c+1*(i-1));
           end
       end
   end
K
X=linspace(0,1,n+1)
Z=zeros(n+1,1);
for i=1:n
     Z(i)=Z(i)+int((1-(s*h/2)-(X(i)+X(i+1))/2)*N1,-1,1)*(h/2)*0.01;
     Z(i+1)=int((1-(s*h/2)-(X(i)+X(i+1))/2)*N2,-1,1)*(h/2);
end
Z
Z(1,1)=0;
Z(n+1,1)=0;
for i=1:n
   K(1,i)=0;
   K(n+1,i)=0;
end
K(1,1)=1;
K(n+1,1)=1;
%Replacing values before solving
%for i=1:n+1
%   Z(i,1)=Z(i,1)-K(i,1)*3;
%end
%Z(1,1)=3;
%Z
%for i=1:n+1
%    K(1,i)=0;
%    K(i,1)=0;
%end
%K(1,1)=1;
%Inversion of matrix
d=inv(K)*Z
plot(X,d)
hold on
end
legend("alpha-4","alpha-1.01","alpha-0.99","alpha-0.25")
hold off
```

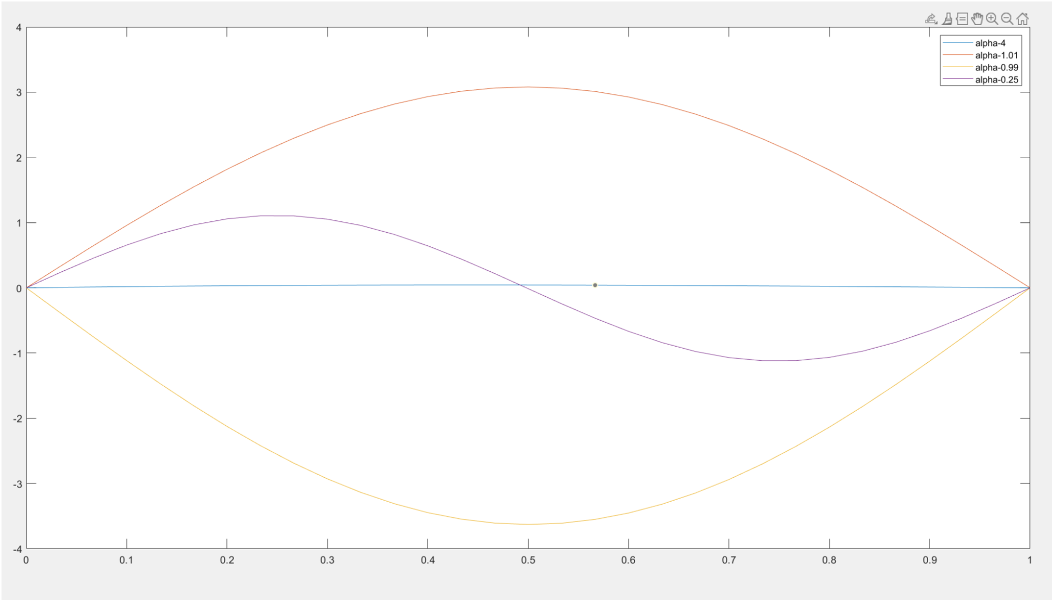
Fig1.10 noded Problem distribution of y over 0 to 1 for the 4 values of alpha



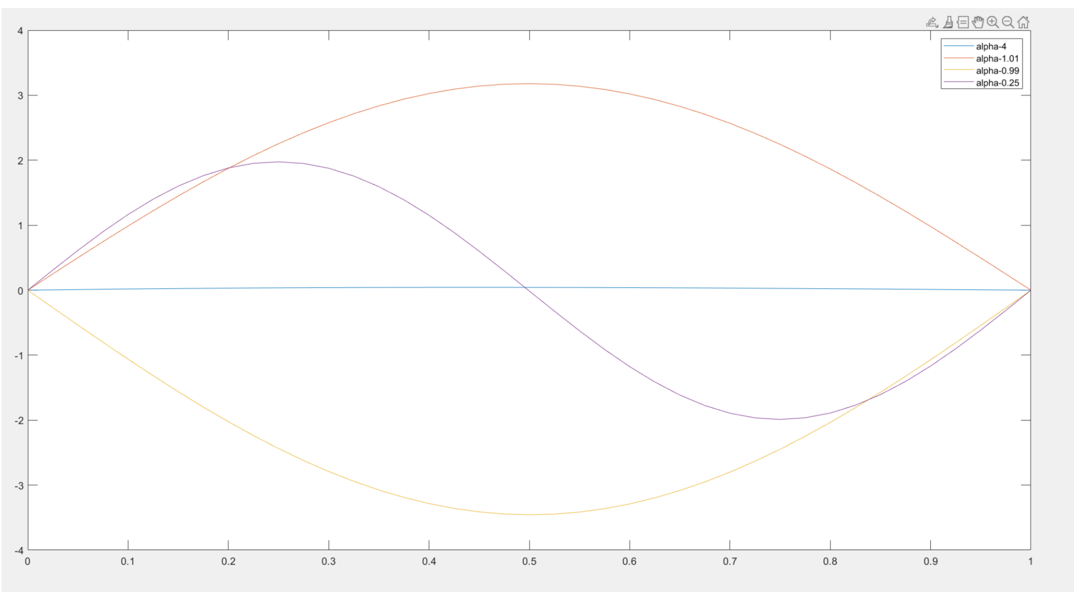
Fig(2). 20 noded Problem distribution of y over 0 to 1 for the 4 values of Alpha



Fig(3). 30 noded Problem distribution of y over 0 to 1 for the 4 values of Alpha



Fig(4). 40 noded Problem distribution of y over 0 to 1 for the 4 values of Alpha



- The accuracy of the value is getting better with increase in the number of elements and the values are converging better individually as the number of elements are increasing especially for alpha=0.99 the final solution has become consistent with from 20 element onwards.