

## ESM 5734

### Homework 1

Due on 2 September 2022 at 11:15 AM

1. What is the finite element method?
2. Consider the function

$$f(x) = 4x^2, \text{ for } 0 \leq x \leq 1/2, \text{ and } f(x) = 2(1-x), \text{ for } 1/2 \leq x \leq 1.$$

For  $f(x)$  expressed in terms of the Fourier sine series, i.e.,

$$f^N(x) = \sum_{n=1}^N a_n (\sin n\pi x)$$

find  $N$  so that  $\int_0^1 [f(x) - \sum_{n=1}^N a_n (\sin n\pi x)]^2 dx / \int_0^1 [f(x)]^2 dx < 0.01$ ,

3. Is it possible to write  $\sin(3\pi x)$  as a linear combination of  $\sin(2\pi x)$  and  $\sin(\pi x)$  for all values of  $x \in [0,1]$ ? Prove your answer.
4. For  $x \in [0,1]$ , express

$$\sin \pi x = a + bx + cx^2 + dx^3$$

Find values of  $a$ ,  $b$ ,  $c$  and  $d$  by ensuring that

- (a) the error (or the residual,  $r$ ) defined by  $r(x) = -\sin \pi x + a + bx + cx^2 + dx^3$  equal zero at  $x = 0, 1/4, 1/2, 3/4$ , and
- (b)  $\int_0^1 r^2 dx$  has the least value.