Matrix transformations.

Part 2

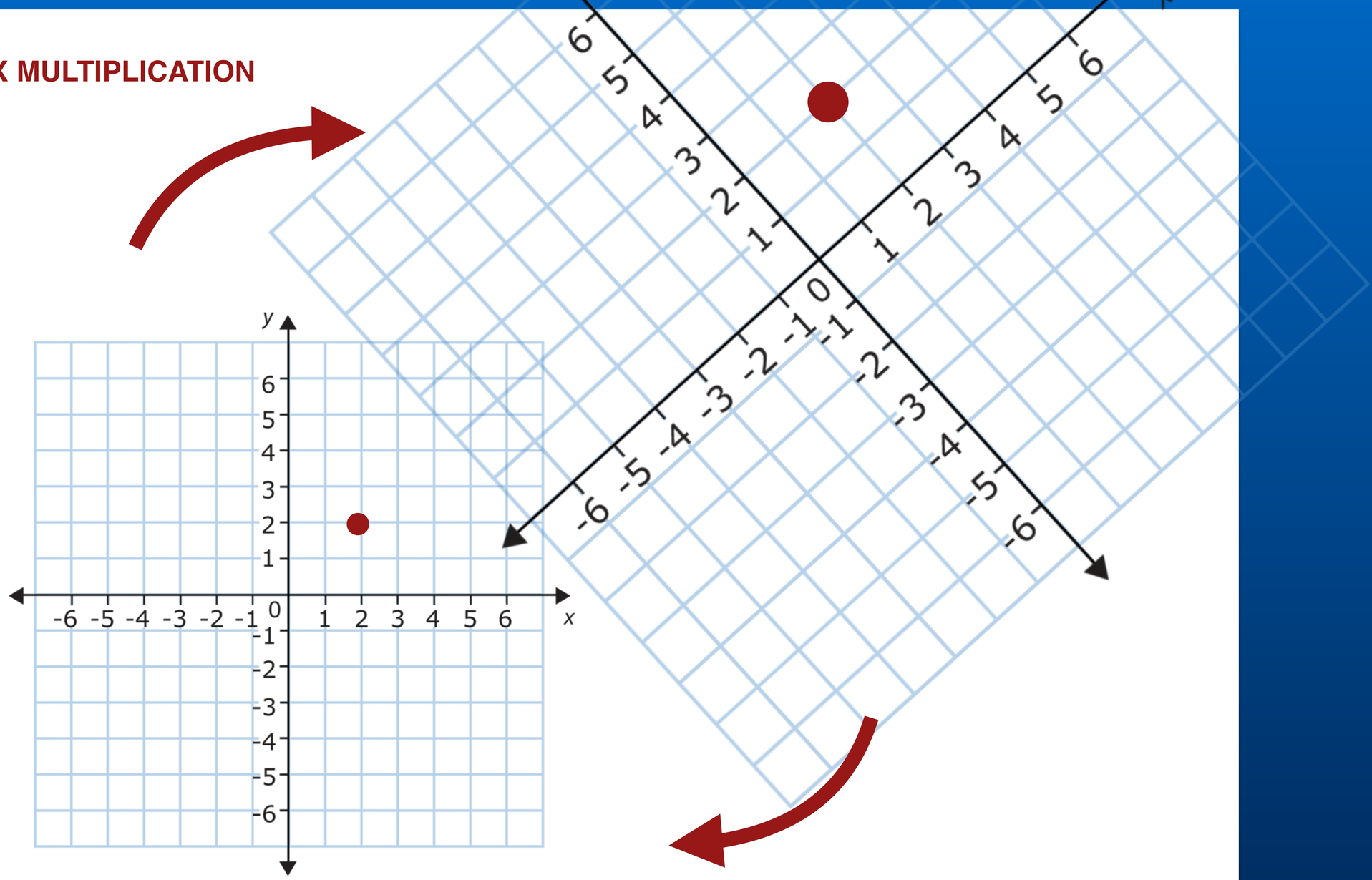
Inverse of a matrix.

Inverse of a matrix.

=

A matrix that “undoes” the matrix’s transformation.

MATRIX MULTIPLICATION



MATRIX INVERSE

Inverse of a matrix.

- Scaled by 1/scale

Inverse of a matrix.

- Scaled by 1/scale
- Rotated by the transpose of the rotation.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse of a matrix.

- Scaled by 1/scale
- Rotated by the transpose of the rotation.
- Translated by the translation * -1.0

A matrix class.

```
class Matrix {
    public:

        Matrix();

        union {
            float m[4][4];
            float ml[16];
        };

        void identity();

        Matrix inverse();
        Matrix operator * (const Matrix &m2);
        Vector operator * (const Vector &v2);
};
```

The Inverse

```
Matrix Matrix::inverse() {
    float m00 = m[0][0], m01 = m[0][1], m02 = m[0][2], m03 = m[0][3];
    float m10 = m[1][0], m11 = m[1][1], m12 = m[1][2], m13 = m[1][3];
    float m20 = m[2][0], m21 = m[2][1], m22 = m[2][2], m23 = m[2][3];
    float m30 = m[3][0], m31 = m[3][1], m32 = m[3][2], m33 = m[3][3];

    float v0 = m20 * m31 - m21 * m30;
    float v1 = m20 * m32 - m22 * m30;
    float v2 = m20 * m33 - m23 * m30;
    float v3 = m21 * m32 - m22 * m31;
    float v4 = m21 * m33 - m23 * m31;
    float v5 = m22 * m33 - m23 * m32;

    float t00 = + (v5 * m11 - v4 * m12 + v3 * m13);
    float t10 = - (v5 * m10 - v2 * m12 + v1 * m13);
    float t20 = + (v4 * m10 - v2 * m11 + v0 * m13);
    float t30 = - (v3 * m10 - v1 * m11 + v0 * m12);

    float invDet = 1 / (t00 * m00 + t10 * m01 + t20 * m02 + t30 * m03);

    float d00 = t00 * invDet;
    float d10 = t10 * invDet;
    float d20 = t20 * invDet;
    float d30 = t30 * invDet;

    float d01 = - (v5 * m01 - v4 * m02 + v3 * m03) * invDet;
    float d11 = + (v5 * m00 - v2 * m02 + v1 * m03) * invDet;
    float d21 = - (v4 * m00 - v2 * m01 + v0 * m03) * invDet;
    float d31 = + (v3 * m00 - v1 * m01 + v0 * m02) * invDet;

    v0 = m10 * m31 - m11 * m30;
    v1 = m10 * m32 - m12 * m30;
    v2 = m10 * m33 - m13 * m30;
    v3 = m11 * m32 - m12 * m31;
    v4 = m11 * m33 - m13 * m31;
    v5 = m12 * m33 - m13 * m32;

    float d02 = + (v5 * m01 - v4 * m02 + v3 * m03) * invDet;
    float d12 = - (v5 * m00 - v2 * m02 + v1 * m03) * invDet;
    float d22 = + (v4 * m00 - v2 * m01 + v0 * m03) * invDet;
    float d32 = - (v3 * m00 - v1 * m01 + v0 * m02) * invDet;

    v0 = m21 * m10 - m20 * m11;
    v1 = m22 * m10 - m20 * m12;
    v2 = m23 * m10 - m20 * m13;
    v3 = m22 * m11 - m21 * m12;
    v4 = m23 * m11 - m21 * m13;
    v5 = m23 * m12 - m22 * m13;

    float d03 = - (v5 * m01 - v4 * m02 + v3 * m03) * invDet;
    float d13 = + (v5 * m00 - v2 * m02 + v1 * m03) * invDet;
    float d23 = - (v4 * m00 - v2 * m01 + v0 * m03) * invDet;
    float d33 = + (v3 * m00 - v1 * m01 + v0 * m02) * invDet;


    Matrix m2;

    m2.m[0][0] = d00;
    m2.m[0][1] = d01;
    m2.m[0][2] = d02;
    m2.m[0][3] = d03;
    m2.m[1][0] = d10;
    m2.m[1][1] = d11;
    m2.m[1][2] = d12;
    m2.m[1][3] = d13;
    m2.m[2][0] = d20;
    m2.m[2][1] = d21;
    m2.m[2][2] = d22;
    m2.m[2][3] = d23;
    m2.m[3][0] = d30;
    m2.m[3][1] = d31;
    m2.m[3][2] = d32;
    m2.m[3][3] = d33;

    return m2;
}
```

A vector class.

```
class Vector {
    public:

        Vector();
        Vector(float x, float y, float z);

        float length();
        void normalize();

        float x;
        float y;
        float z;
};
```

Storing entity transformations as matrices.

1. Store entity transformation as a matrix.
2. Multiply current modelview matrix with it when we render.

Building a transformation matrix.

Identity matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale matrix

$$\begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translate matrix.

$$\begin{bmatrix} 1 & 0 & 0 & Tx \\ 0 & 1 & 0 & Ty \\ 0 & 0 & 1 & Tz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation (around Z axis) matrix.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Building the final matrix.

Remember that the order of matrix multiplication matters!! We do not want to scale our other transforms!

```
class Entity {
public:

    Matrix matrix;

    float x;
    float y;
    float scale_x;
    float scale_y;
    float rotation;


};
```

Multiplying the modelview matrix.

~~glTranslatef~~

~~glRotatef~~

~~glScalef~~

```
void glMultMatrixf (const GLfloat *m);
```

Multiplies the current matrix with the one specified, and replaces the current matrix with the product.

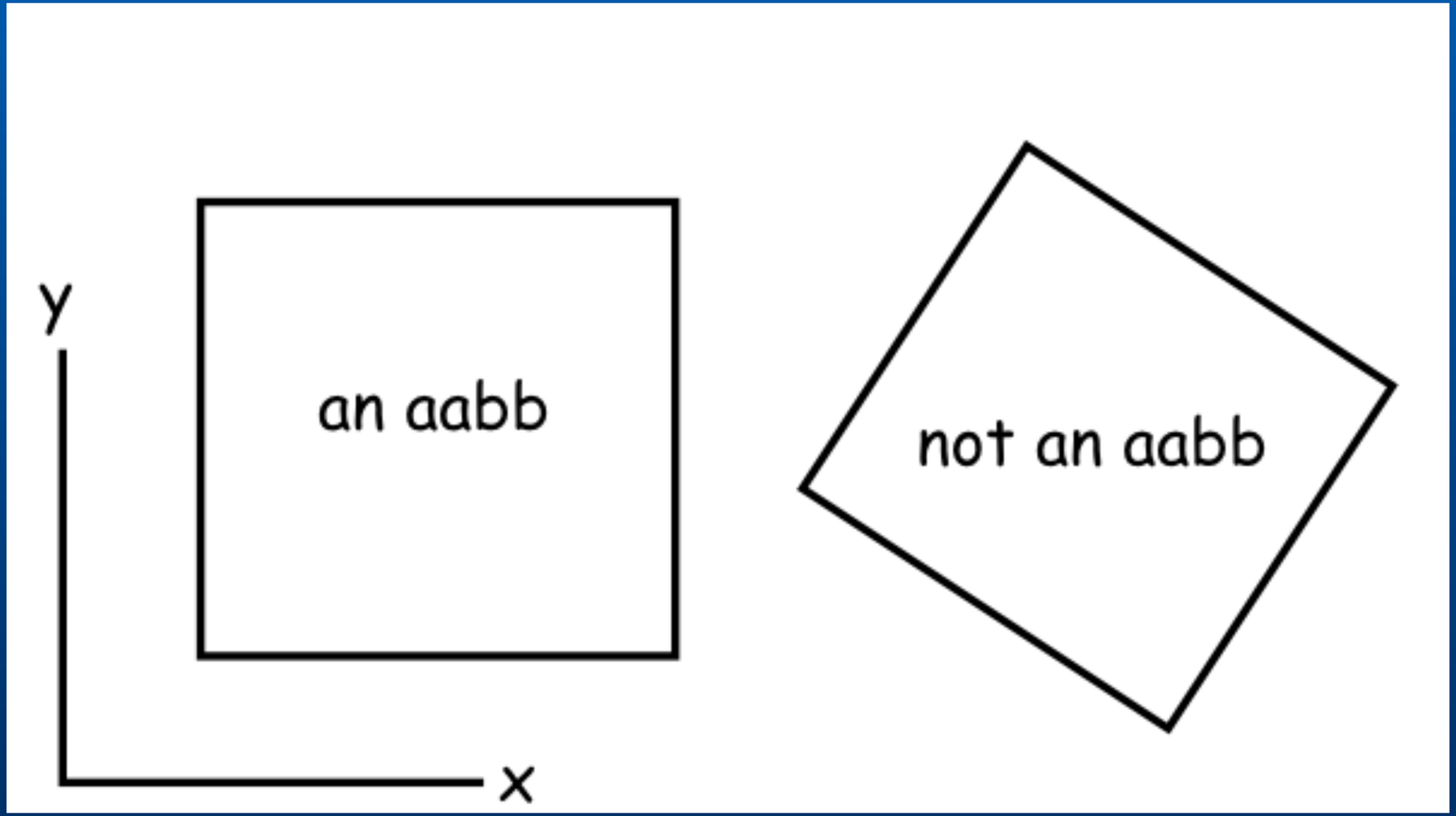
```
float matrix[16];
glMultMatrixf(matrix);
```

```
Matrix matrix;
glMultMatrixf(matrix.ml);
```

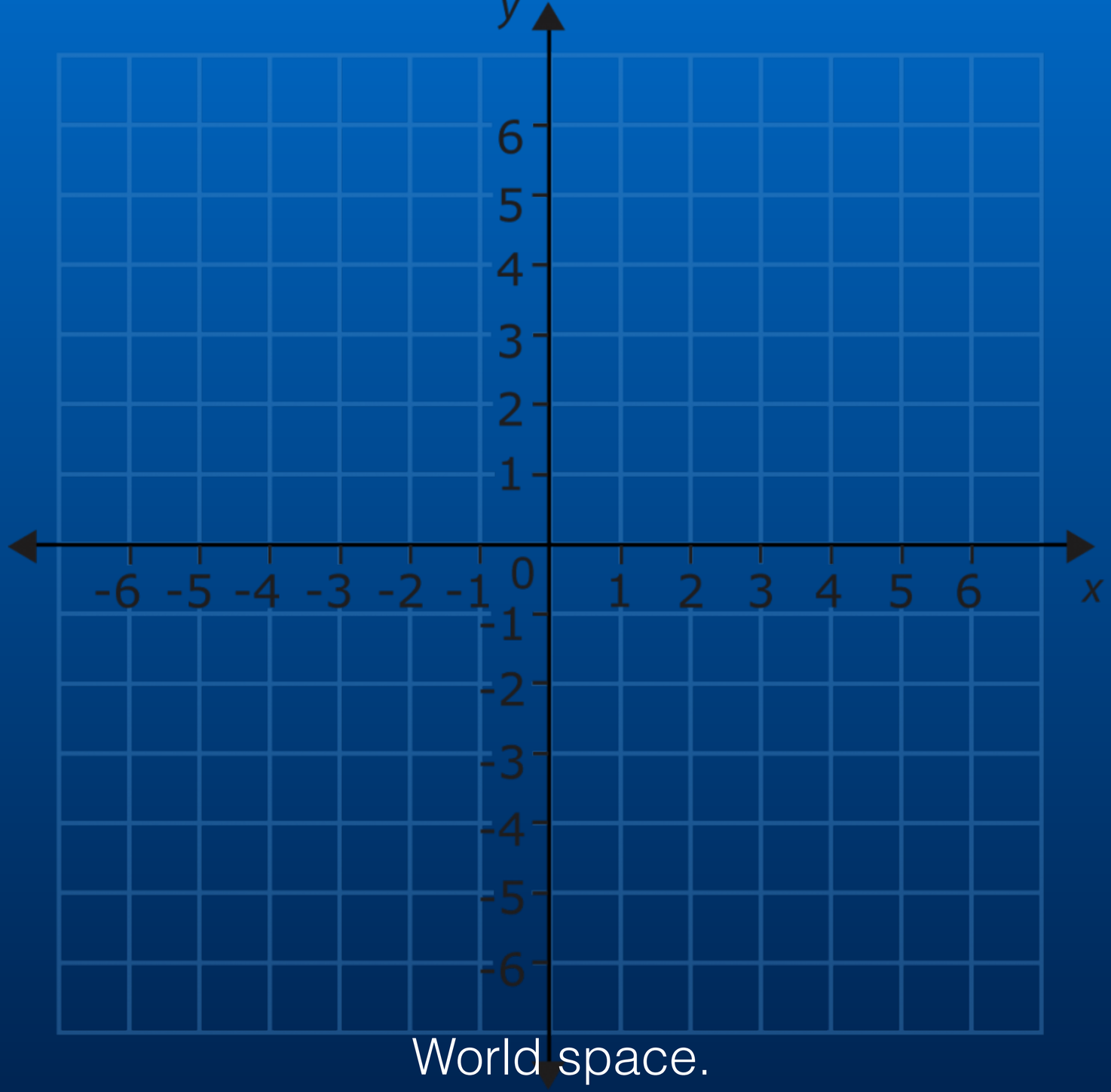
```
void Entity::Render() {

    buildMatrix();

    glMatrixMode(GL_MODELVIEW);
    glPushMatrix();
    glMultMatrixf(matrix.ml);

    // draw our entity

    glPopMatrix();

}
```

Benefits of storing the modelview matrix.

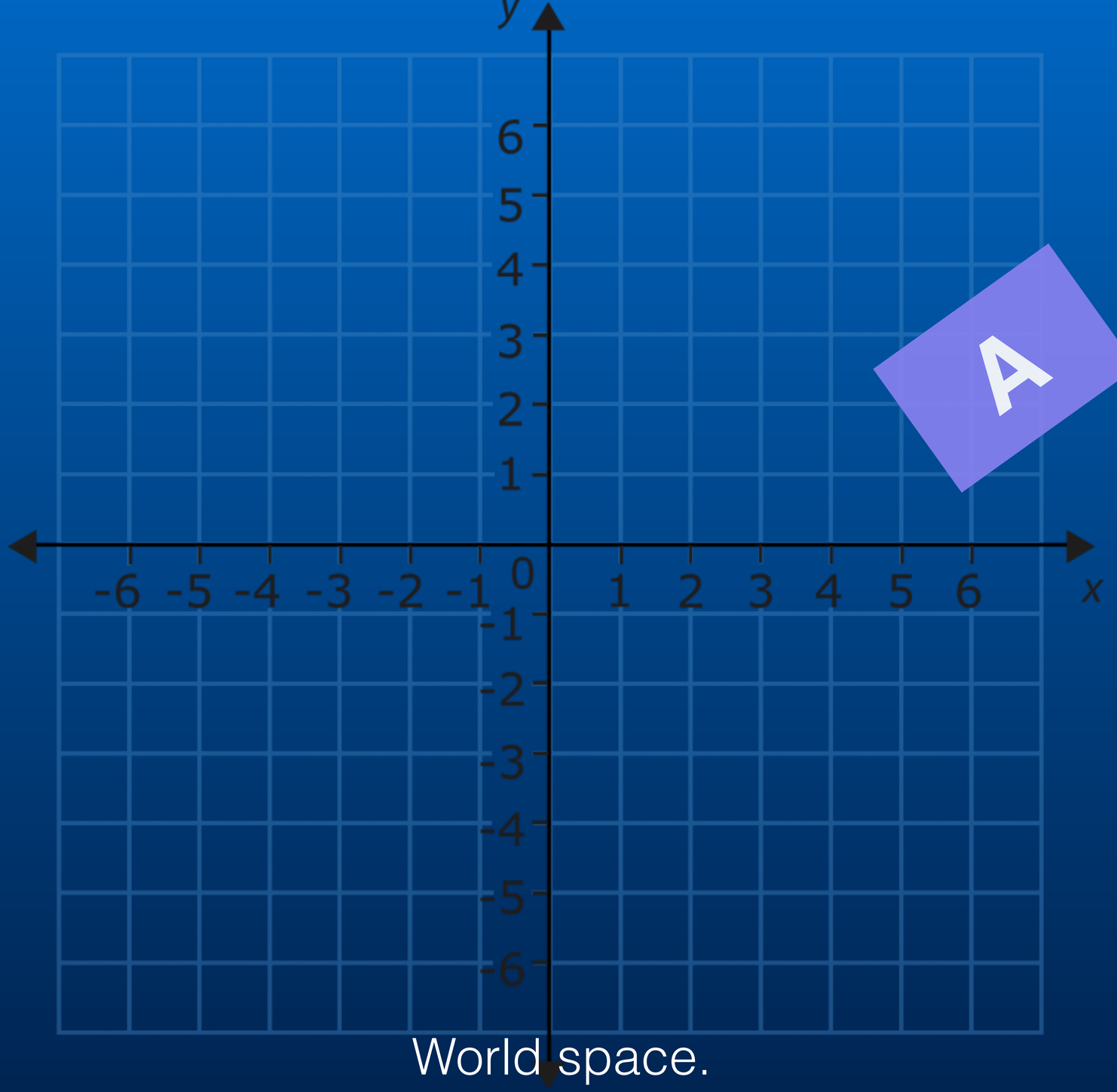
Non-axis aligned bounding boxes.


y
x
an aabb
not an aabb

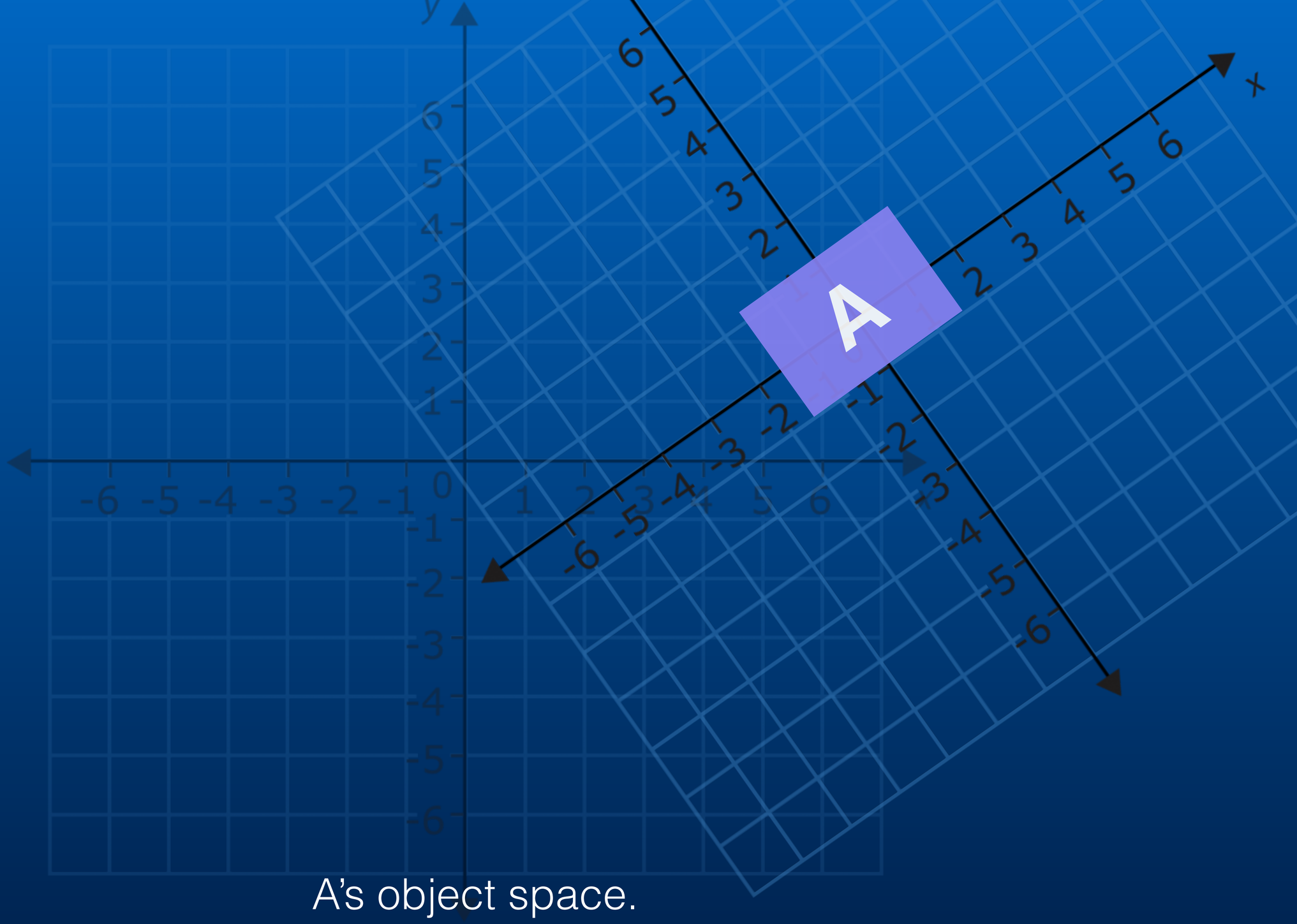
Transforming between coordinate systems.



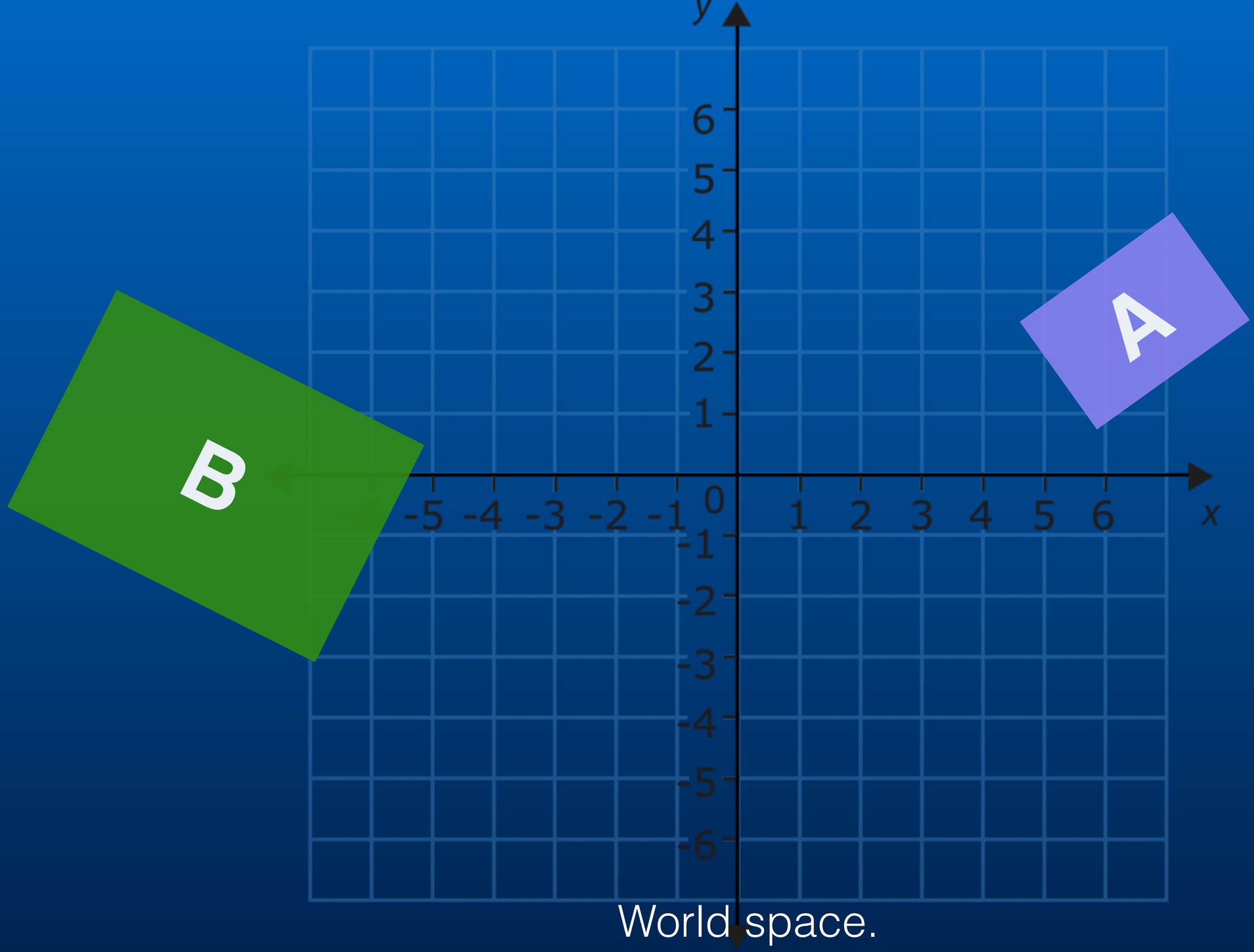
World space.



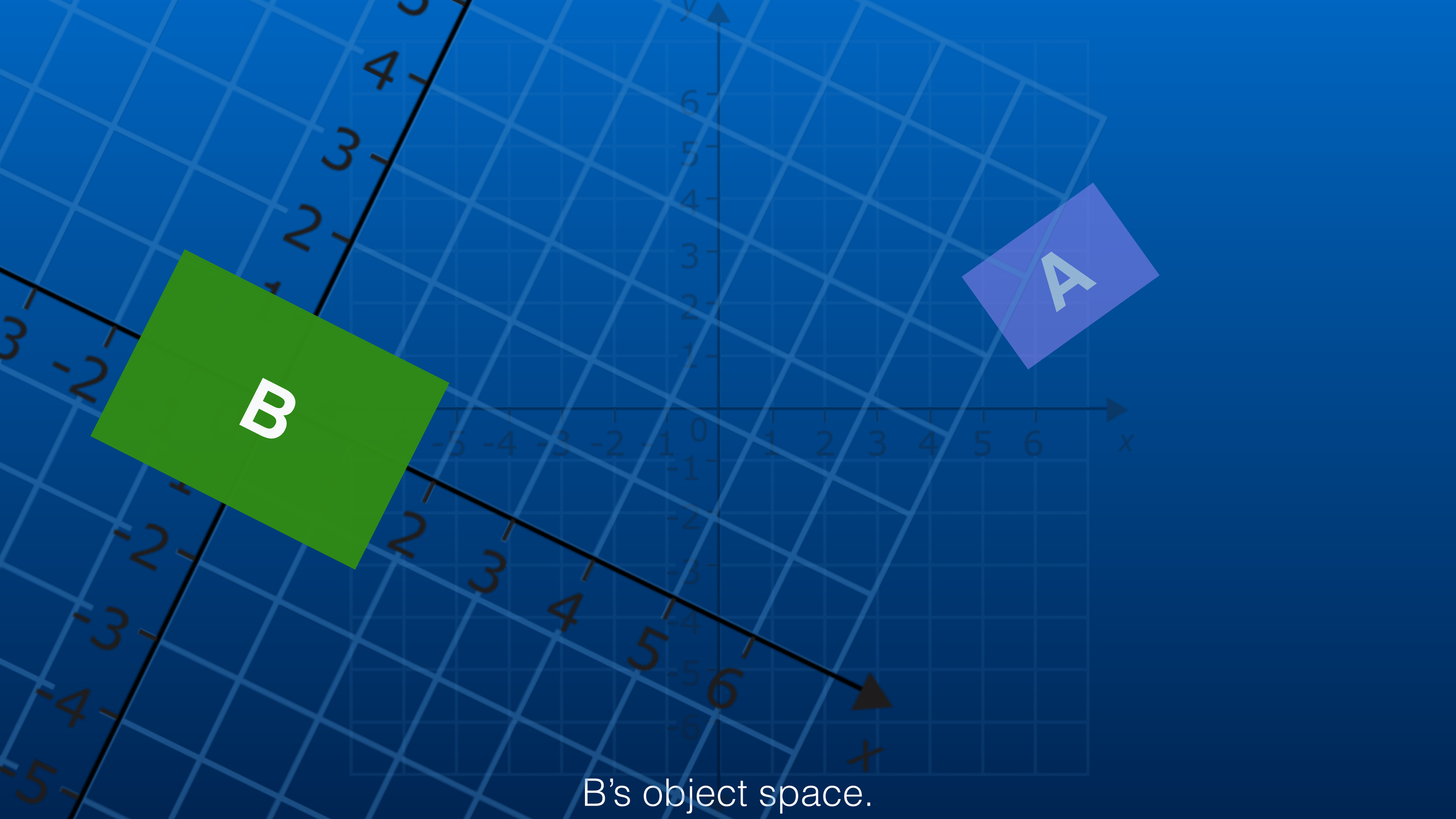
World space.



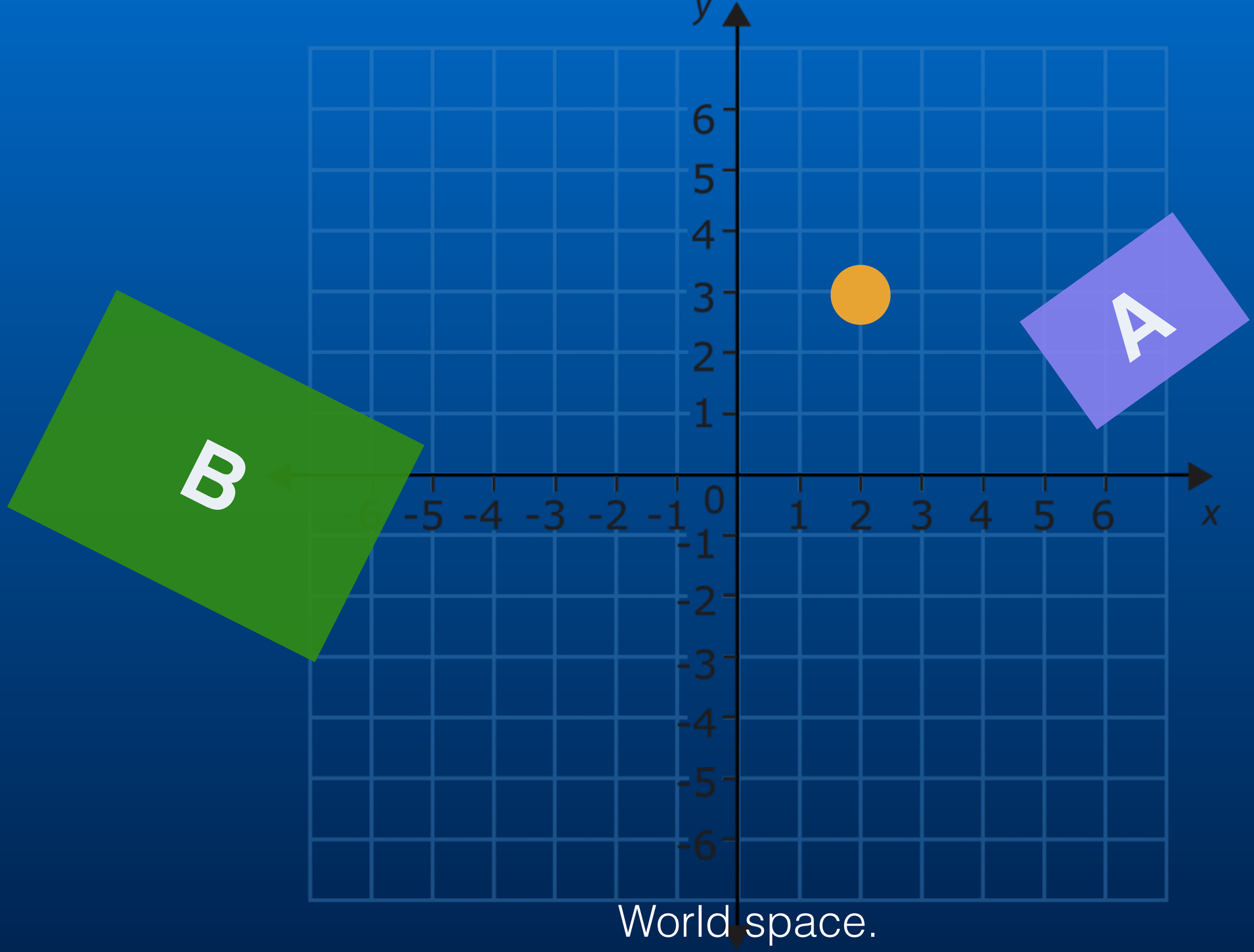
A’s object space.



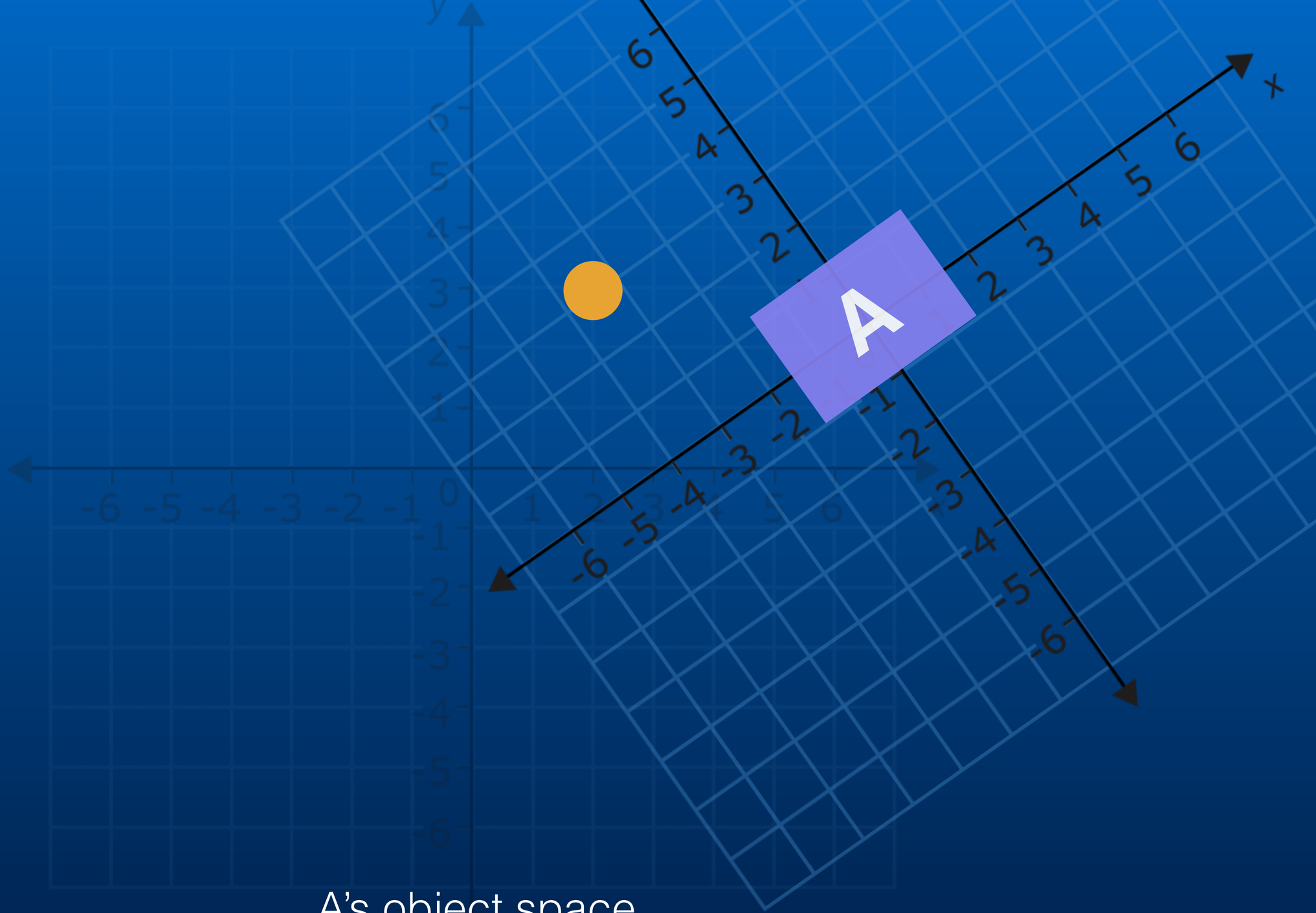
World space.



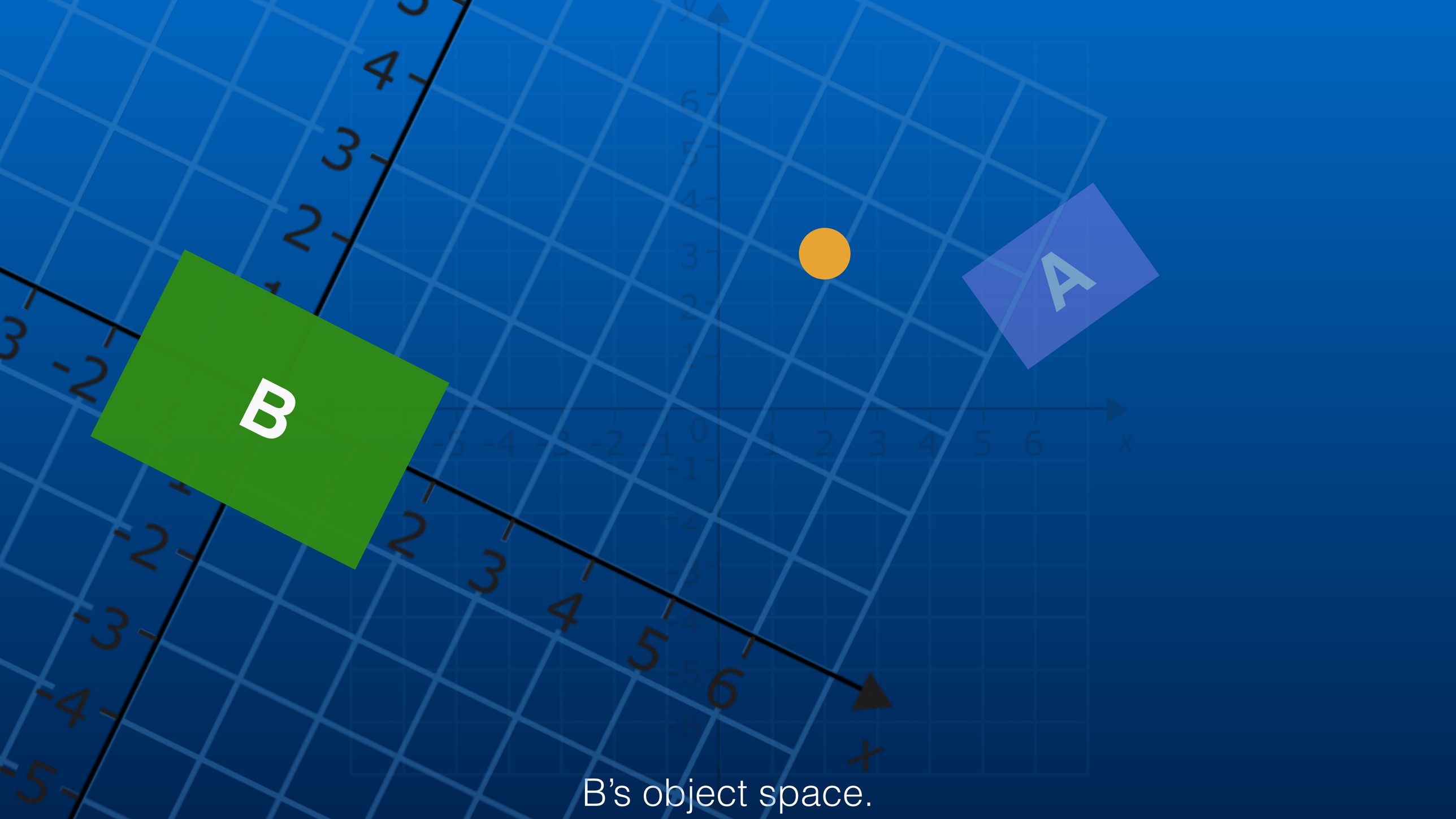
B's object space.



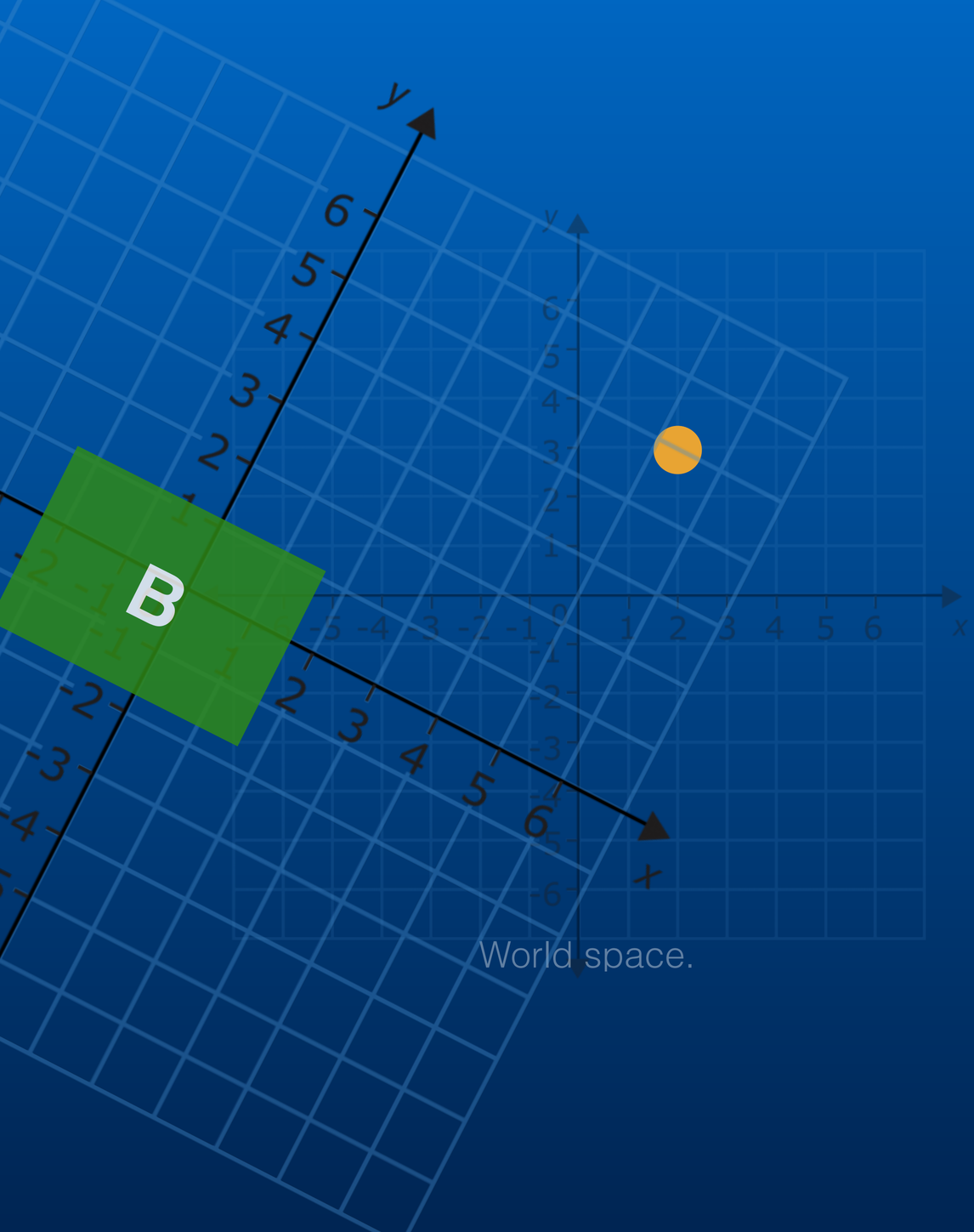
World space.



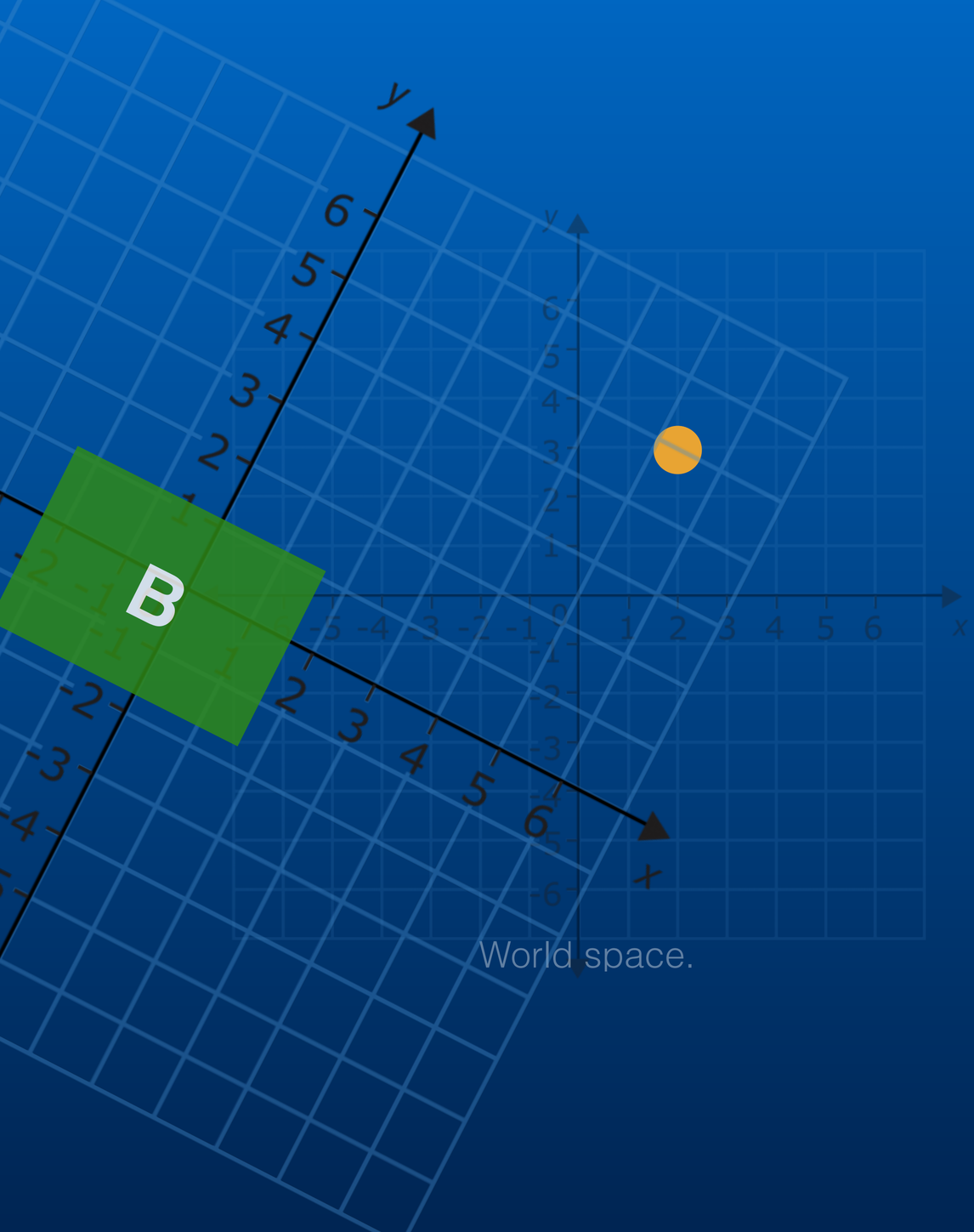
A’s object space.



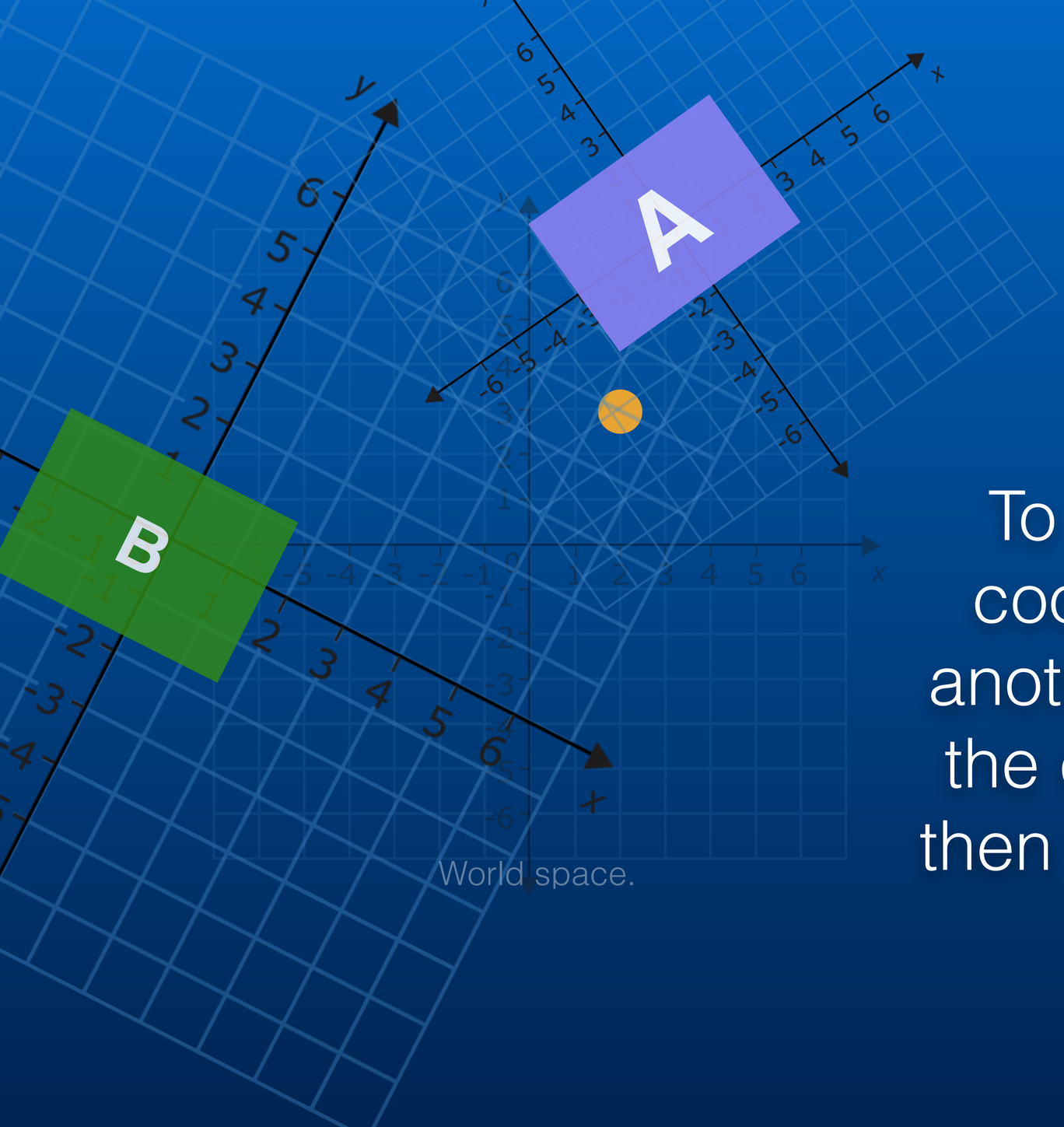
B's object space.



To express a world space coordinate vector in terms of an entity's object space, multiply the vector by the inverse of that entity's transform matrix.



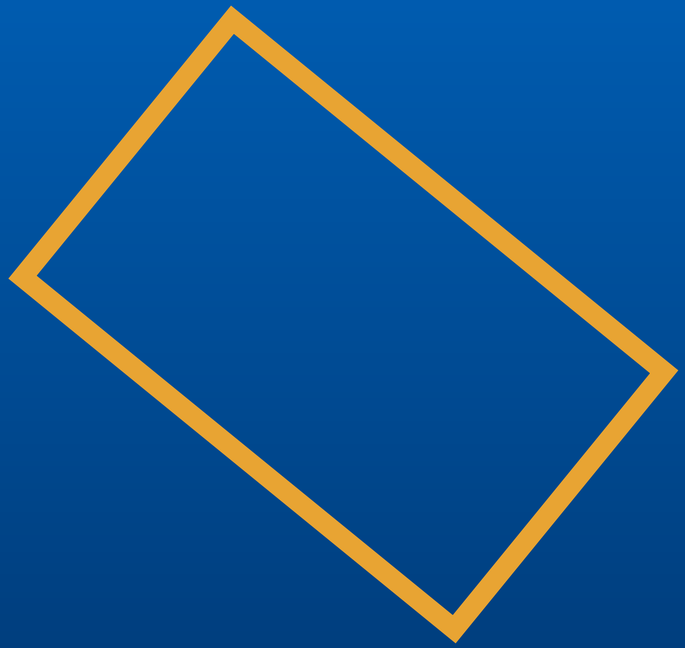
To express an object space coordinate vector in terms of world space, multiply the vector by that entity’s transform matrix.

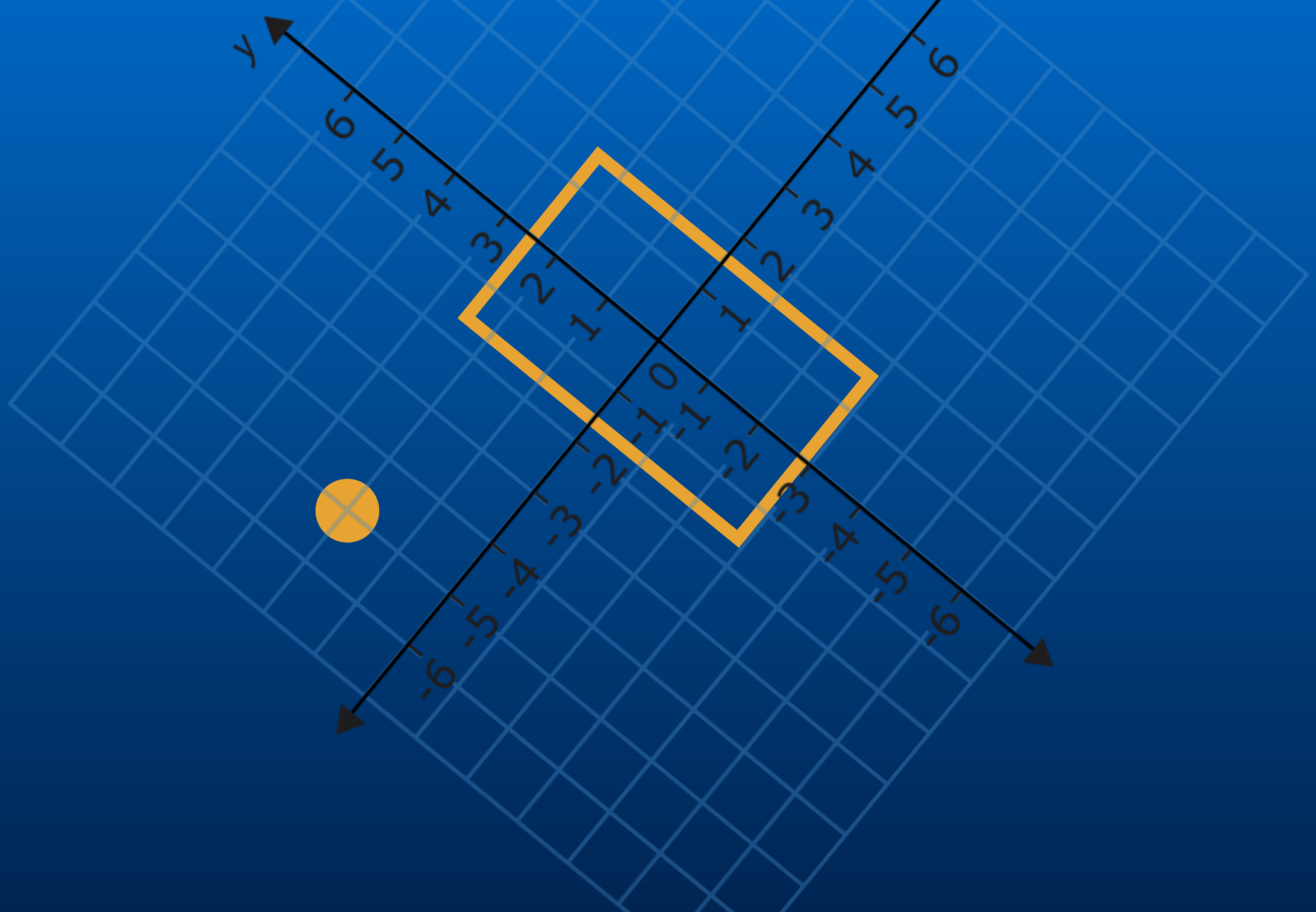


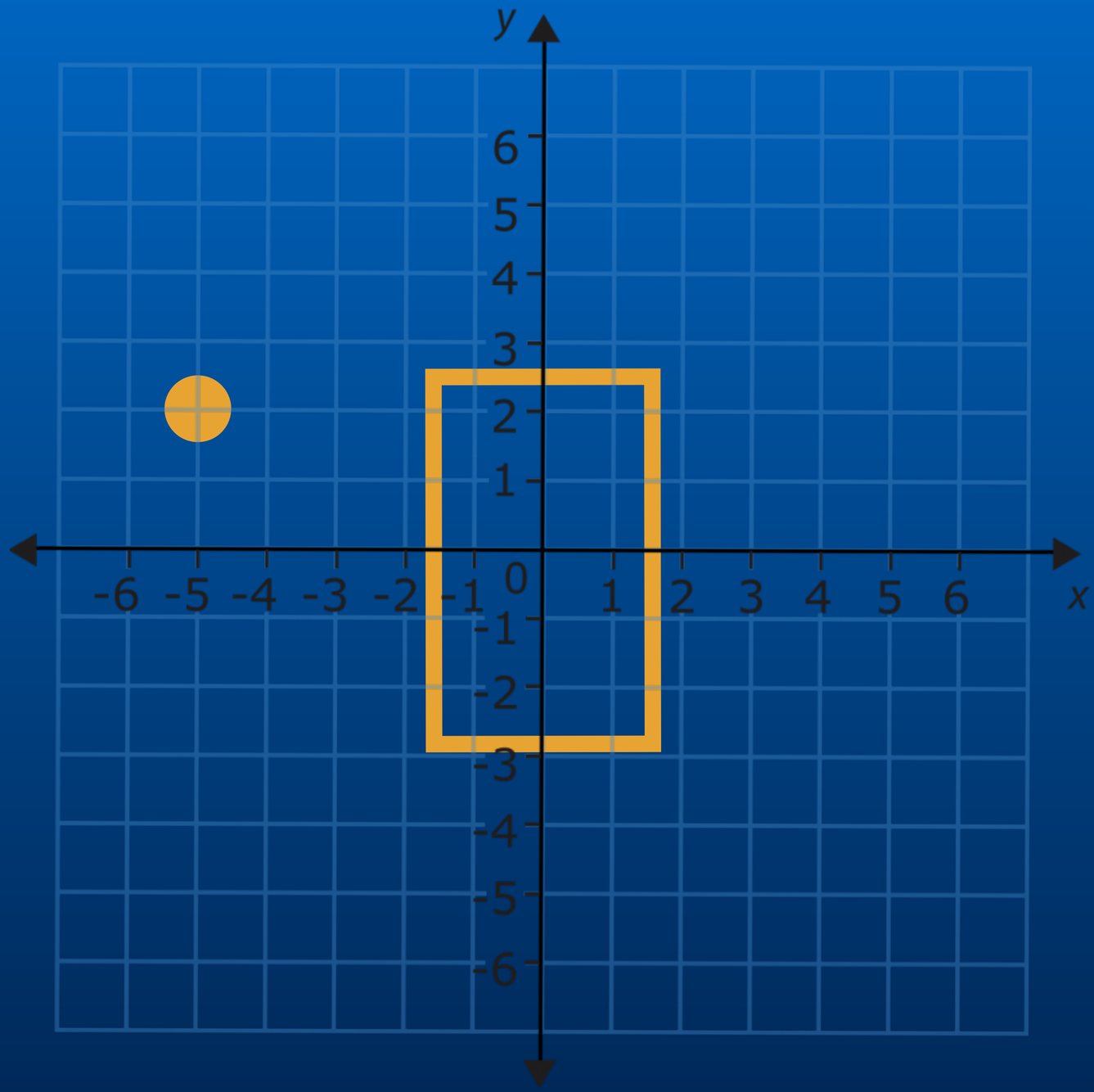
To express an object space coordinate vector in terms of another object's space, convert the coordinate to world space, then to the other object's space.

Point / rotated rectangle collision.

???




y
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6

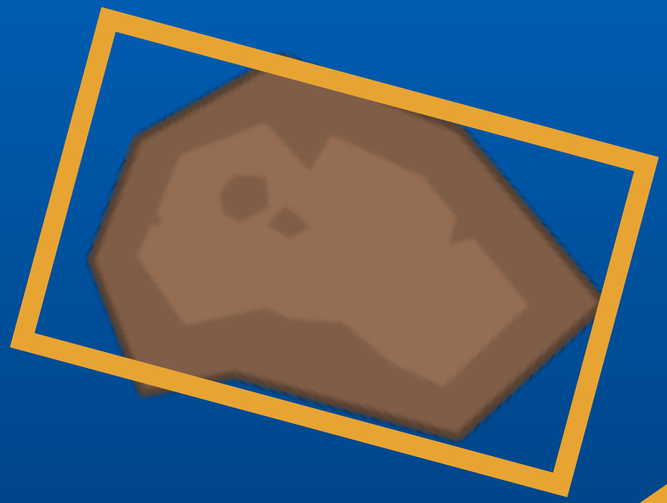

y
x
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6

Point / rotated rectangle collision.

1. Multiply the point vector by the inverse of the entity's transform matrix.

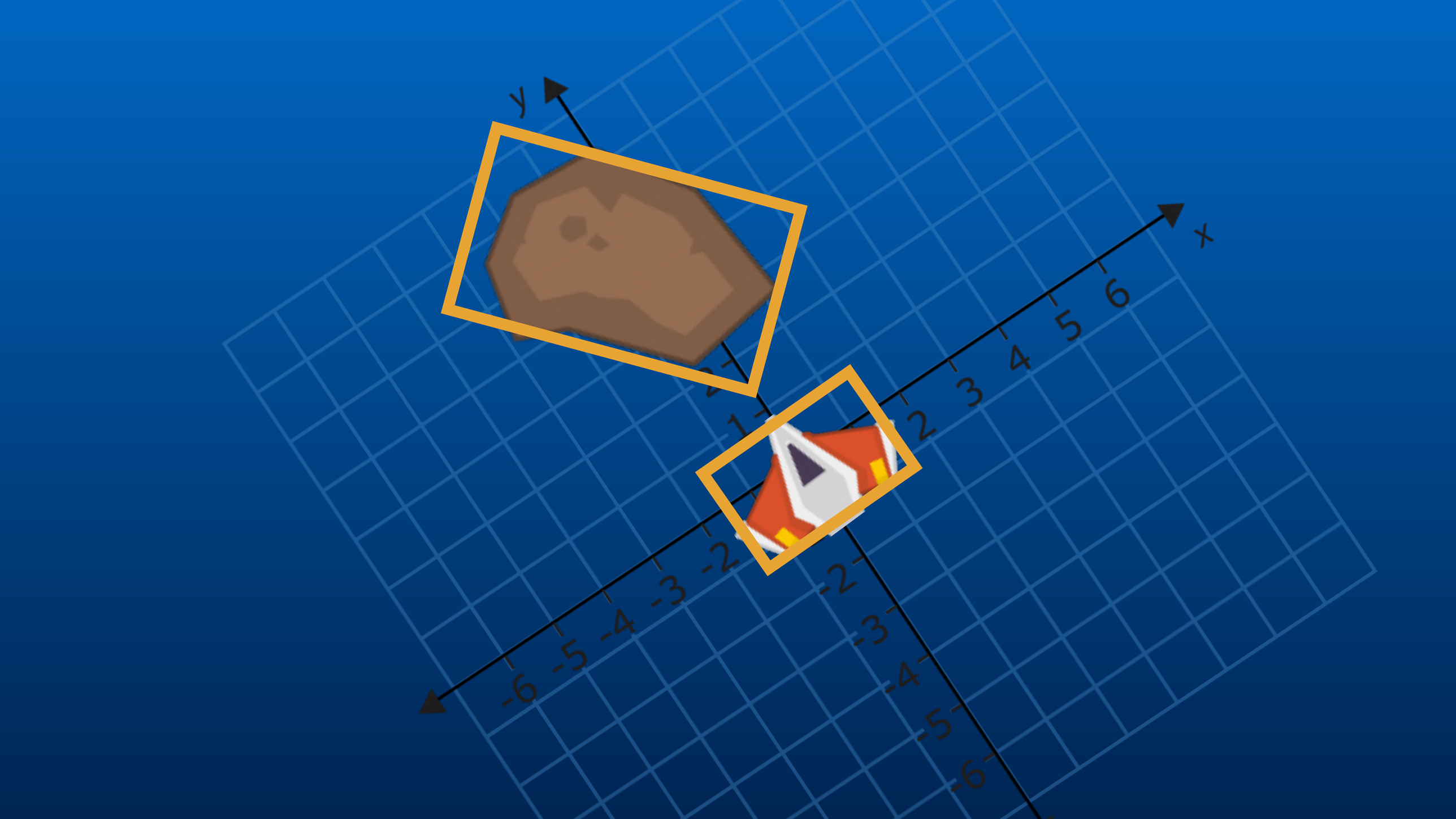
2. Do regular point / rectangle check with the resulting coordinates.

Rotated rectangle / rotated rectangle collision.

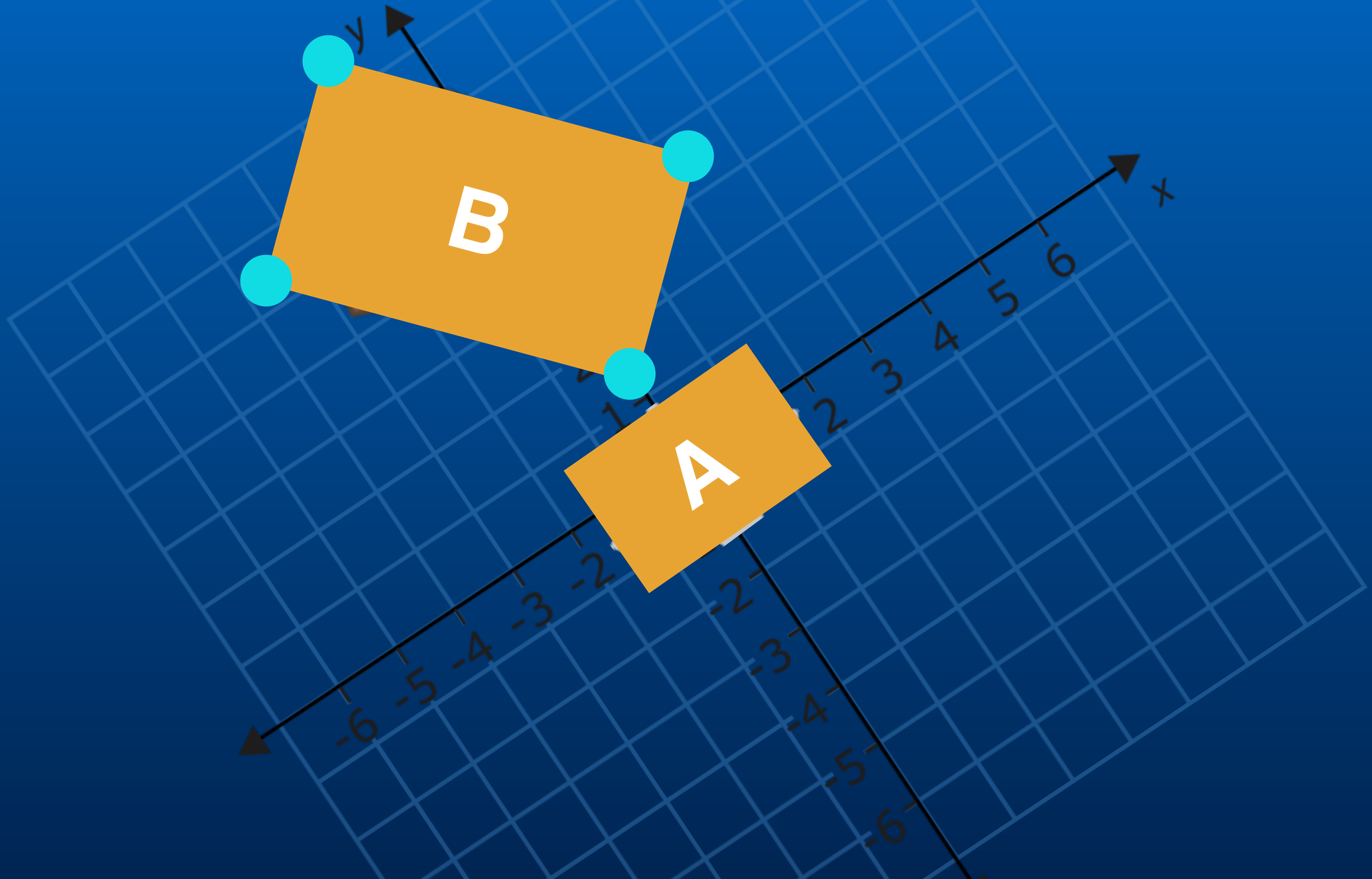


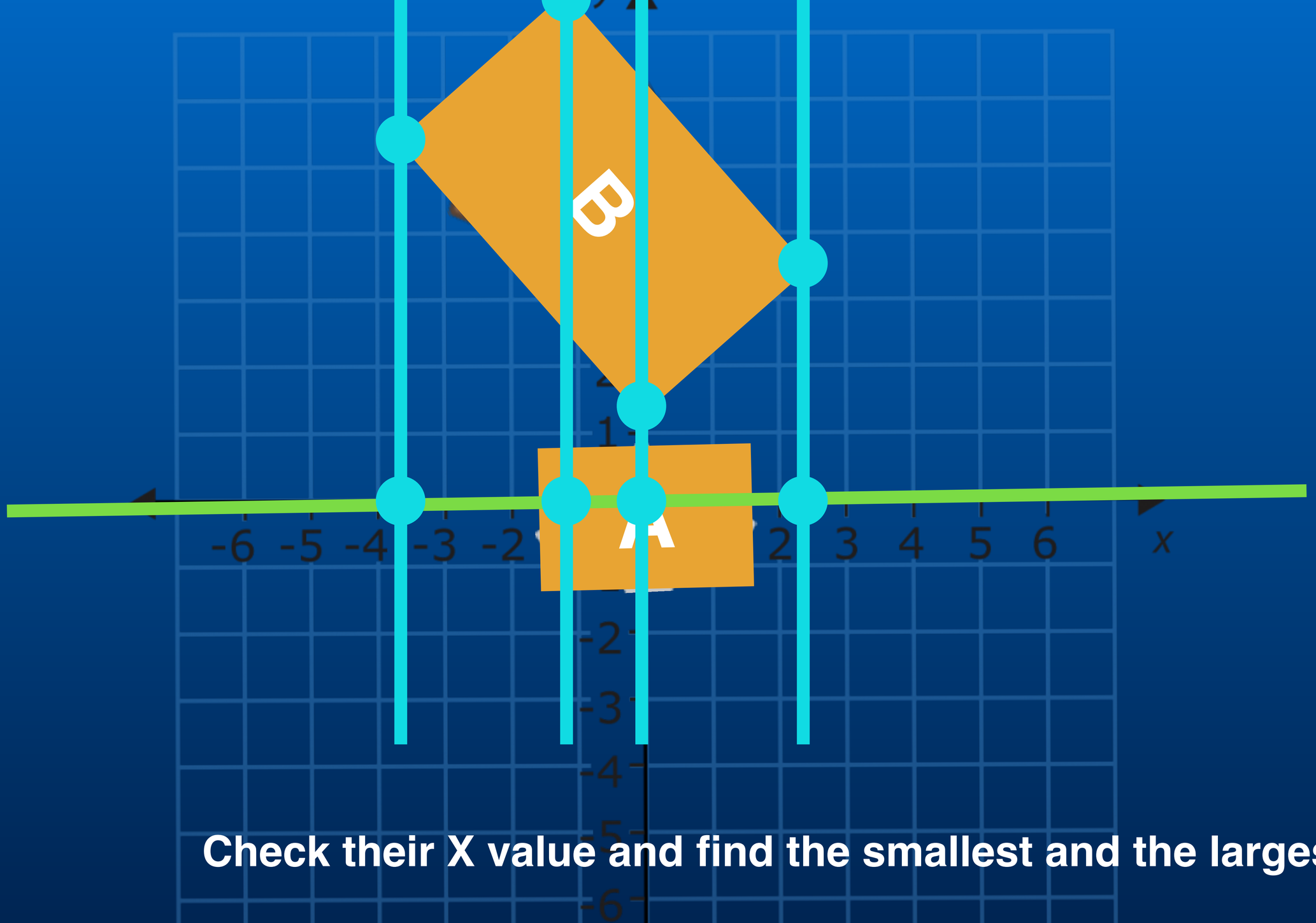
???

SAT.
Separating axis theorem.

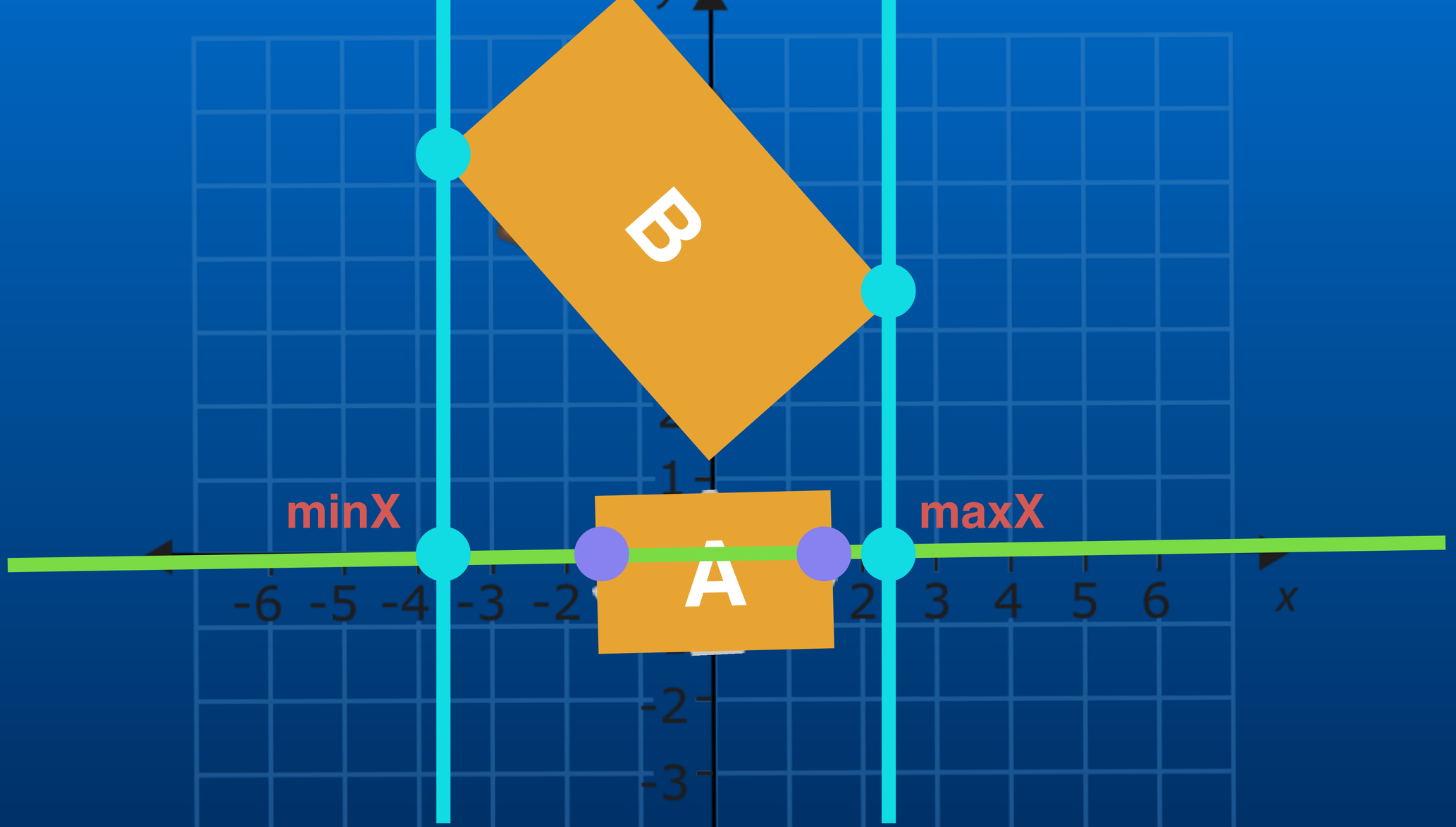

y
x
1
2
3
4
5
6
-1
-2
-3
-4
-5
-6

Convert the corners of B from B’s object space to A’s object space.

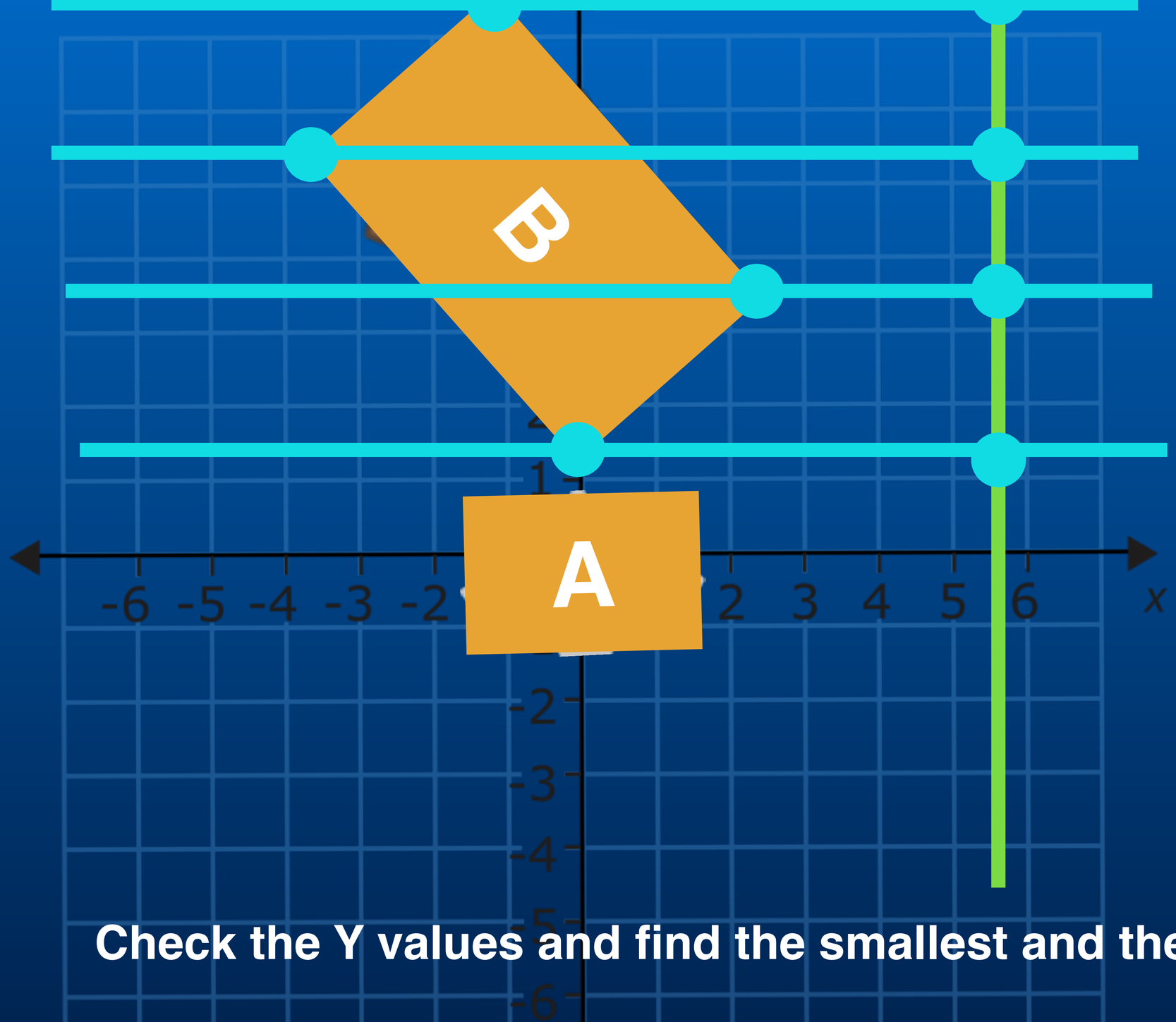




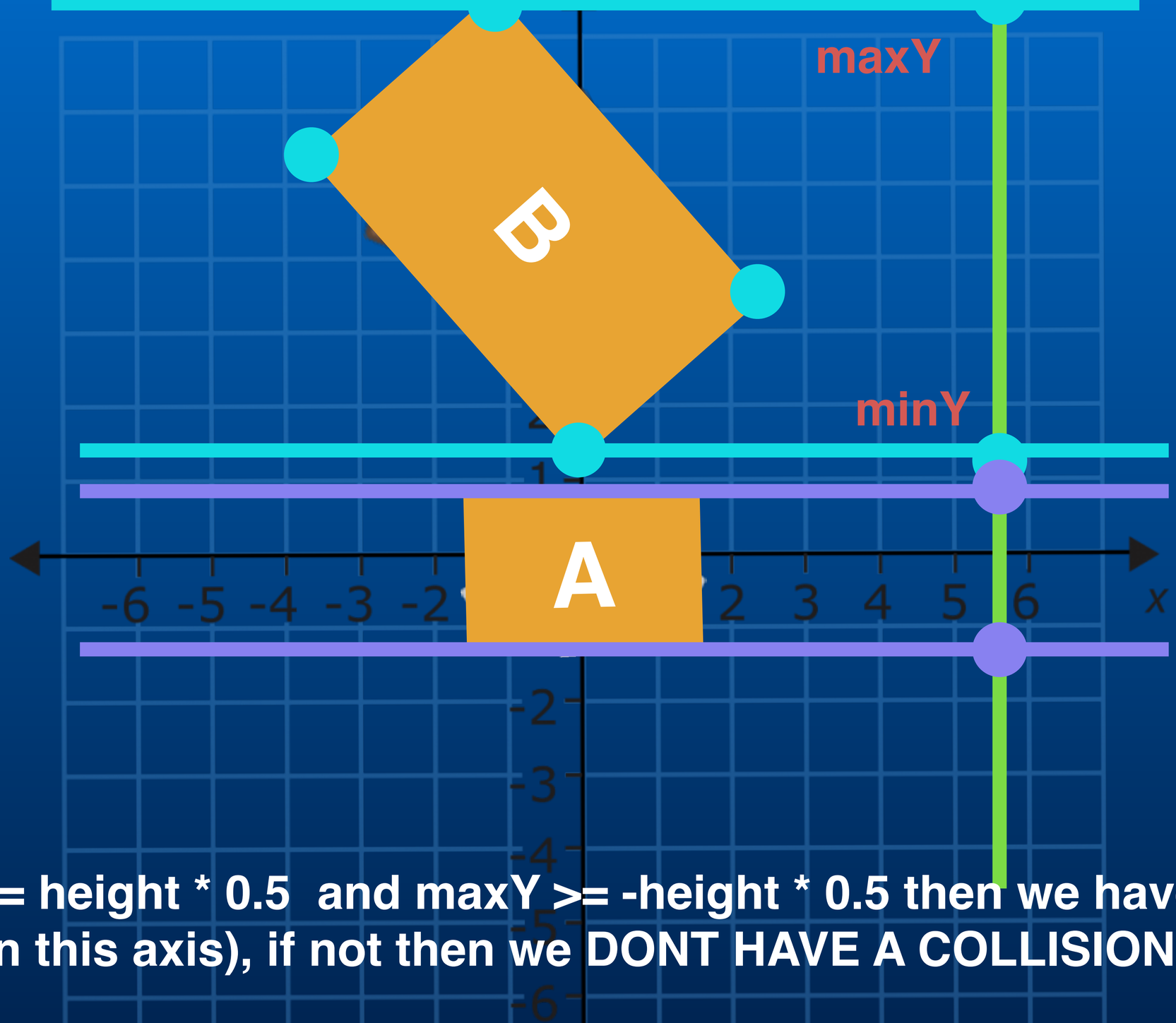
Check their X value and find the smallest and the largest.



If minX < = width * 0.5 and maxX >= -width * 0.5 then we have a collision (just on this axis), if not then we DONT HAVE A COLLISION AT ALL!!



Check the Y values and find the smallest and the largest.



If minY < = height * 0.5 and maxY >= -height * 0.5 then we have a collision (just on this axis), if not then we DONT HAVE A COLLISION AT ALL!!

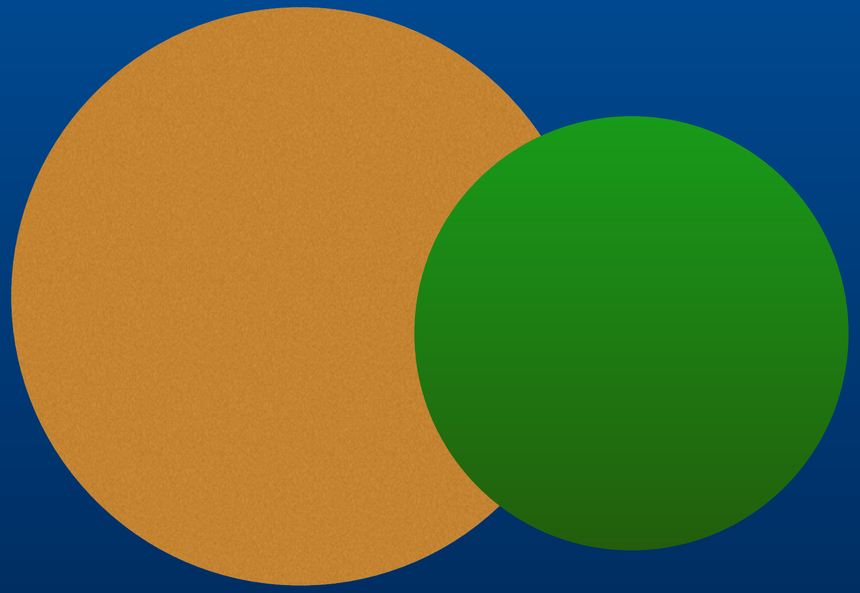
Now repeat the same steps for entity A
in entity B’s object space.

A full collision occurs if a collision occurs on each of the 4 axis checks (X and Y in entity A and X and Y in entity B).

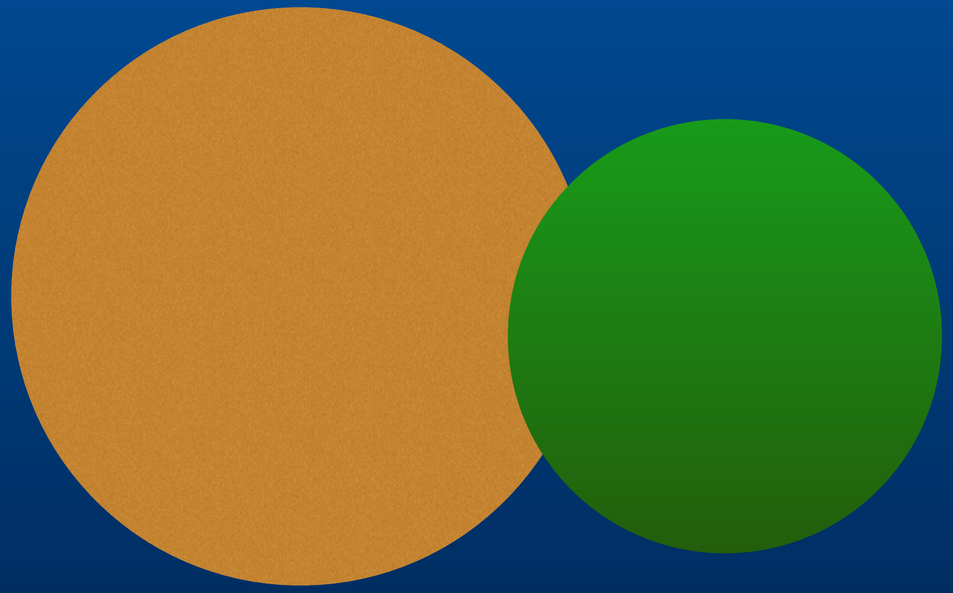
If ANY of the axis collisions are false, there cannot be a collision.

Simple general collision response.

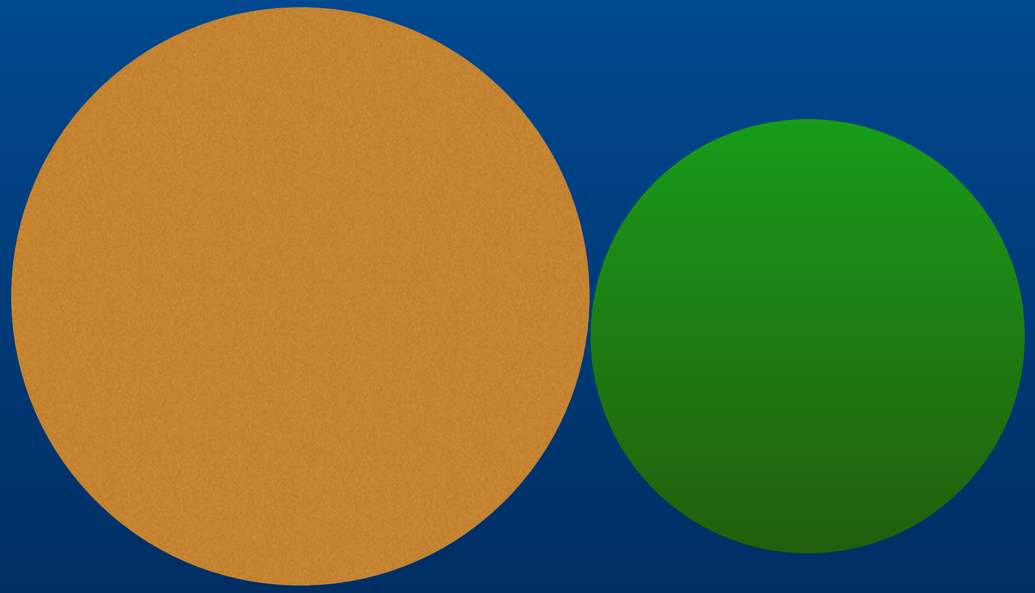
While entities are colliding, push them away from each other at a small interval.



While entities are colliding, push them away from each other at a small interval.



While entities are colliding, push them away from each other at a small interval.



Push entities away based on their normalized distance from each other.
(Divide x and y of their position difference by the distance between them)

Assignment.

Make simple asteroids.

- You have two weeks.
- You must use transformation matrices and non-axis-aligned collisions!

Assignment #2.

- Start thinking about your final project.
- Send me your final project idea by next Wednesday.