

Three Whitehead Theorems and Three Puppe Sequences

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Frame Title

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Definition of ∞ -category

```
def horn_filling_condition (X : SSet) (n i : Nat): Prop :=
  ∀ f :  $\Lambda[n, i] \rightarrow X$ , ∃ g :  $\Delta[n] \rightarrow X$ ,
  f = SSet.hornInclusion n i » g

/-- A simplicial set is called an  $\infty$ -category
if it has the extension property for all inner horn inclusions
 $\Lambda[n, i] \rightarrow \Delta[n]$ ,  $n \geq 2$ ,  $0 < i < n$ . -/
def InfCategory := {X : SSet //
  ∀ (n i : Nat),
   $n \geq 2 \wedge 0 < i \wedge i < n \rightarrow$  horn_filling_condition X n i}

#check (inferInstance : Category SSet) -- OK
#check (inferInstance : Category InfCategory) -- fail
```

Automatic typeclass inference?

```
-- instance : Category InfCategory := inferInstance -- ?

-- instance : Category InfCategory := by -- ?
--   dsimp only [InfCategory]
--   infer_instance

instance : Category InfCategory where
  Hom X Y := NatTrans X.1 Y.1
  id X := NatTrans.id X.1
  comp  $\alpha$   $\beta$  := NatTrans.vcomp  $\alpha$   $\beta$ 

#check (inferInstance : Category SSet) -- OK
#check (inferInstance : Category InfCategory) -- OK
```

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