In [1]:
```
%matplotlib ipympl

import sys
sys.path.append('..')

from optimus3d.functions import *
from optimus3d.surface import *
from optimus3d.optimizers import *
from optimus3d.step import *
from optimus3d.stop import *

import optimus3d
optimus3d.__version__
```

Out[1]: '6.0.0'

In [2]:
```
plt.close('all')
```

Часть 1

Задача 1.

Задача 1. Из начального приближения $x^0 = (-1, 1)$ решить задачу безусловной минимизации функции

$$f : R^2 \to R,\ f(x) = 2x_1^2 + x_1 x_2 + 3x_2^2$$

градиентным методом, используя правило Армихо с параметрами $\hat{\alpha} = 1, \varepsilon = \theta = \frac{1}{2}$.

In [3]:
```
f1 = function('2*(x1^2) + x1*x2 + 3*(x2^2)')
f1
```

Out[3]: $\displaystyle 2 x_{1}^{2} + x_{1} x_{2} + 3 x_{2}^{2}$

In [4]:
```
x_0 = np.array([-1, 1])
a = 0.01
eps = 0.5
a = eps = 0.5

stop_criteria = union_stopper(
    iterations_stopper(100) ,
    grad_norm_stopper(eps),
)

optimizer = GradientOptimizer(
    stopper=stop_criteria,
    stepper=armijo_stepper(a, eps),
    verbose=True,
)
x_min, f1_min, trajectory = optimizer.min(f1, x_0)
```

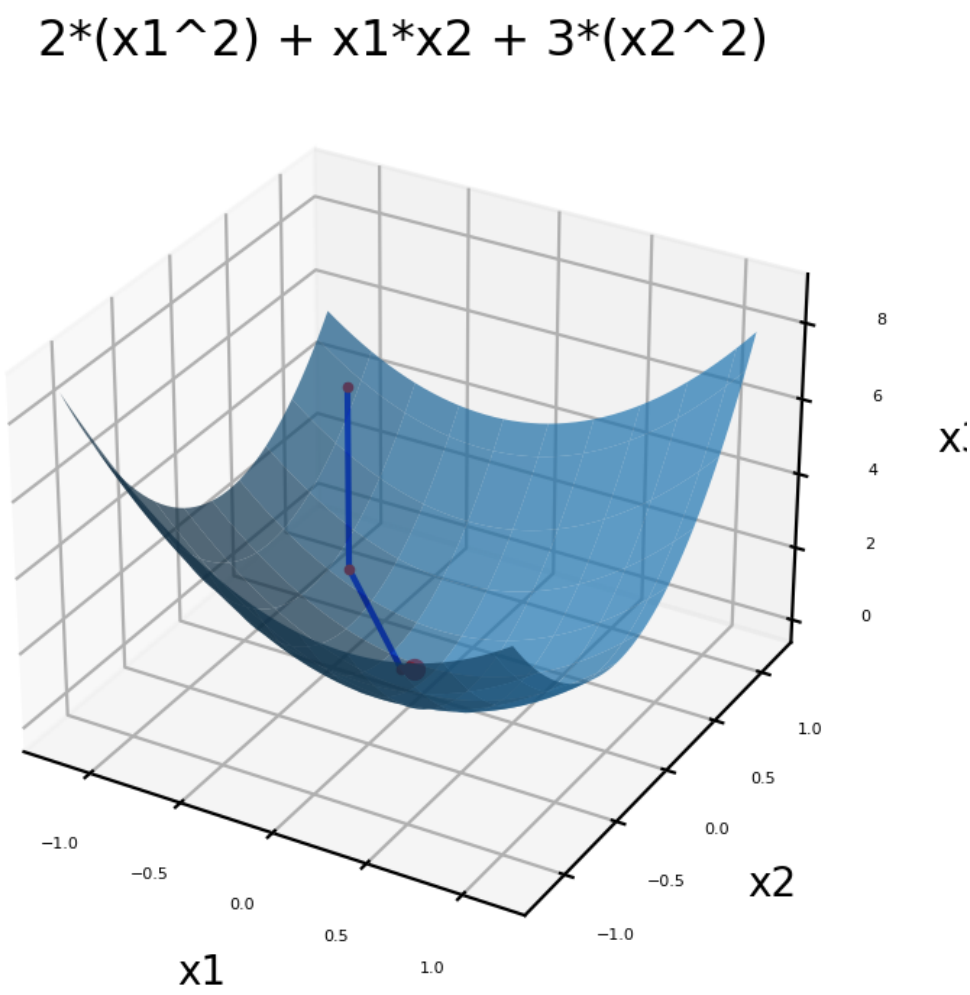
```
x_k=array([-1,  1])
diff_val=array([inf, inf])
f_start=array([-1,  1,  4])
----------------------------------------
x_k=array([-1,  1])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-3,  5])
d_k=array([ 3, -5])
got a_k=0.125 with iterations=2
x_k + a_k * d_k = [-0.625  0.375]
x_k+1=[-0.625  0.375] k = 1
----------------------------------------
x_k=array([-0.625,  0.375])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.125,  1.625])
d_k=array([ 2.125, -1.625])
got a_k=0.25 with iterations=1
x_k + a_k * d_k = [-0.09375 -0.03125]
x_k+1=[-0.09375 -0.03125] k = 2
----------------------------------------
x_k=array([-0.09375, -0.03125])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.40625, -0.28125])
d_k=array([0.40625, 0.28125])
got a_k=0.125 with iterations=2
x_k + a_k * d_k = [-0.04296875  0.00390625]
x_k+1=[-0.04296875  0.00390625] k = 3
----------------------------------------
```

In [5]:
```
f1_min
```

Out[5]:
```
array([-0.04296875,  0.00390625,  0.00357056])
```

In [6]:
```
f1.show(
    point=f1_min,
    radius=1.2,
    trajectory=trajectory,
)
```

Figure



Дополнительно (Константный)

In [7]:

```
a = 0.01
optimizer = GradientOptimizer(
    stopper=stop_criteria,
    stepper=const_stepper(a),
    verbose=True,
)
x_min, f1_min, trajectory = optimizer.min(f1, x_0)
```

```
x_k=array([-1,  1])
diff_val=array([inf, inf])
f_start=array([-1,  1,  4])
----------------------------------------
x_k=array([-1,  1])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-3,  5])
d_k=array([ 3, -5])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.97  0.95]
x_k+1=[-0.97  0.95] k = 1
----------------------------------------
x_k=array([-0.97,  0.95])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.93,  4.73])
d_k=array([ 2.93, -4.73])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.9407  0.9027]
x_k+1=[-0.9407  0.9027] k = 2
----------------------------------------
x_k=array([-0.9407,  0.9027])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.8601,  4.4755])
d_k=array([ 2.8601, -4.4755])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.912099  0.857945]
x_k+1=[-0.912099  0.857945] k = 3
----------------------------------------
x_k=array([-0.912099,  0.857945])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.790451,  4.235571])
d_k=array([ 2.790451, -4.235571])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.88419449  0.81558929]
x_k+1=[-0.88419449  0.81558929] k = 4
----------------------------------------
x_k=array([-0.88419449,  0.81558929])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.72118867,  4.00934125])
d_k=array([ 2.72118867, -4.00934125])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.8569826   0.77549588]
x_k+1=[-0.8569826   0.77549588] k = 5
----------------------------------------
x_k=array([-0.8569826 ,  0.77549588])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.65243454,  3.79599266])
d_k=array([ 2.65243454, -3.79599266])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.83045826  0.73753595]
x_k+1=[-0.83045826  0.73753595] k = 6
```

```
----------------------------------------
x_k=array([-0.83045826,  0.73753595])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.58429708,  3.59475745])
d_k=array([ 2.58429708, -3.59475745])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.80461529  0.70158838]
x_k+1=[-0.80461529  0.70158838] k = 7
----------------------------------------
x_k=array([-0.80461529,  0.70158838])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.51687277,  3.40491497])
d_k=array([ 2.51687277, -3.40491497])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.77944656  0.66753923]
x_k+1=[-0.77944656  0.66753923] k = 8
----------------------------------------
x_k=array([-0.77944656,  0.66753923])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.45024701,  3.2257888 ])
d_k=array([ 2.45024701, -3.2257888 ])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.75494409  0.63528134]
x_k+1=[-0.75494409  0.63528134] k = 9
----------------------------------------
x_k=array([-0.75494409,  0.63528134])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.38449502,  3.05674394])
d_k=array([ 2.38449502, -3.05674394])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.73109914  0.6047139 ]
x_k+1=[-0.73109914  0.6047139 ] k = 10
----------------------------------------
x_k=array([-0.73109914,  0.6047139 ])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.31968266,  2.89718426])
d_k=array([ 2.31968266, -2.89718426])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.70790231  0.57574206]
x_k+1=[-0.70790231  0.57574206] k = 11
----------------------------------------
x_k=array([-0.70790231,  0.57574206])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.25586719,  2.74655003])
d_k=array([ 2.25586719, -2.74655003])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.68534364  0.54827656]
x_k+1=[-0.68534364  0.54827656] k = 12
----------------------------------------
x_k=array([-0.68534364,  0.54827656])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
```

```
diff_val=array([-2.19309801,  2.6043157 ])
d_k=array([ 2.19309801, -2.6043157 ])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.66341266  0.5222334 ]
x_k+1=[-0.66341266  0.5222334 ] k = 13
----------------------------------------
x_k=array([-0.66341266,  0.5222334 ])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.13141724,  2.46998774])
d_k=array([ 2.13141724, -2.46998774])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.64209849  0.49753352]
x_k+1=[-0.64209849  0.49753352] k = 14
----------------------------------------
x_k=array([-0.64209849,  0.49753352])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.07086043,  2.34310264])
d_k=array([ 2.07086043, -2.34310264])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.62138988  0.4741025 ]
x_k+1=[-0.62138988  0.4741025 ] k = 15
----------------------------------------
x_k=array([-0.62138988,  0.4741025 ])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-2.01145704,  2.22322509])
d_k=array([ 2.01145704, -2.22322509])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.60127531  0.45187024]
x_k+1=[-0.60127531  0.45187024] k = 16
----------------------------------------
x_k=array([-0.60127531,  0.45187024])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.95323101,  2.10994615])
d_k=array([ 1.95323101, -2.10994615])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.581743    0.43077078]
x_k+1=[-0.581743    0.43077078] k = 17
----------------------------------------
x_k=array([-0.581743  ,  0.43077078])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.89620123,  2.0028817 ])
d_k=array([ 1.89620123, -2.0028817 ])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.56278099  0.41074197]
x_k+1=[-0.56278099  0.41074197] k = 18
----------------------------------------
x_k=array([-0.56278099,  0.41074197])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.840382  ,  1.90167081])
d_k=array([ 1.840382  , -1.90167081])
got a_k=0.01 with iterations=0
```

```
x_k + a_k * d_k = [-0.54437717  0.39172526]
x_k+1=[-0.54437717  0.39172526] k = 19
----------------------------------------
x_k=array([-0.54437717,  0.39172526])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.78578343,  1.80597438])
d_k=array([ 1.78578343, -1.80597438])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.52651934  0.37366551]
x_k+1=[-0.52651934  0.37366551] k = 20
----------------------------------------
x_k=array([-0.52651934,  0.37366551])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.73241183,  1.71547375])
d_k=array([ 1.73241183, -1.71547375])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.50919522  0.35651078]
x_k+1=[-0.50919522  0.35651078] k = 21
----------------------------------------
x_k=array([-0.50919522,  0.35651078])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.6802701 ,  1.62986944])
d_k=array([ 1.6802701 , -1.62986944])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.49239252  0.34021208]
x_k+1=[-0.49239252  0.34021208] k = 22
----------------------------------------
x_k=array([-0.49239252,  0.34021208])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.62935799,  1.54887998])
d_k=array([ 1.62935799, -1.54887998])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.47609894  0.32472328]
x_k+1=[-0.47609894  0.32472328] k = 23
----------------------------------------
x_k=array([-0.47609894,  0.32472328])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.57967247,  1.47224076])
d_k=array([ 1.57967247, -1.47224076])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.46030221  0.31000088]
x_k+1=[-0.46030221  0.31000088] k = 24
----------------------------------------
x_k=array([-0.46030221,  0.31000088])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.53120798,  1.39970304])
d_k=array([ 1.53120798, -1.39970304])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.44499013  0.29600384]
x_k+1=[-0.44499013  0.29600384] k = 25
----------------------------------------
```

```
x_k=array([-0.44499013,  0.29600384])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.48395669,  1.33103294])
d_k=array([ 1.48395669, -1.33103294])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.43015057  0.28269352]
x_k+1=[-0.43015057  0.28269352] k = 26
----------------------------------------
x_k=array([-0.43015057,  0.28269352])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.43790875,  1.26601053])
d_k=array([ 1.43790875, -1.26601053])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.41577148  0.27003341]
x_k+1=[-0.41577148  0.27003341] k = 27
----------------------------------------
x_k=array([-0.41577148,  0.27003341])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.39305251,  1.20442898])
d_k=array([ 1.39305251, -1.20442898])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.40184095  0.25798912]
x_k+1=[-0.40184095  0.25798912] k = 28
----------------------------------------
x_k=array([-0.40184095,  0.25798912])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.34937469,  1.14609377])
d_k=array([ 1.34937469, -1.14609377])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.38834721  0.24652818]
x_k+1=[-0.38834721  0.24652818] k = 29
----------------------------------------
x_k=array([-0.38834721,  0.24652818])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.30686064,  1.09082189])
d_k=array([ 1.30686064, -1.09082189])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.3752786   0.23561996]
x_k+1=[-0.3752786   0.23561996] k = 30
----------------------------------------
x_k=array([-0.3752786 ,  0.23561996])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.26549444,  1.03844118])
d_k=array([ 1.26549444, -1.03844118])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.36262366  0.22523555]
x_k+1=[-0.36262366  0.22523555] k = 31
----------------------------------------
x_k=array([-0.36262366,  0.22523555])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.22525907,  0.98878966])
```

```
d_k=array([ 1.22525907, -0.98878966])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.35037107  0.21534766]
x_k+1=[-0.35037107  0.21534766] k = 32
----------------------------------------
x_k=array([-0.35037107,  0.21534766])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.18613661,  0.94171487])
d_k=array([ 1.18613661, -0.94171487])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.3385097   0.20593051]
x_k+1=[-0.3385097   0.20593051] k = 33
----------------------------------------
x_k=array([-0.3385097 ,  0.20593051])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.14810829,  0.89707334])
d_k=array([ 1.14810829, -0.89707334])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.32702862  0.19695977]
x_k+1=[-0.32702862  0.19695977] k = 34
----------------------------------------
x_k=array([-0.32702862,  0.19695977])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.11115469,  0.85473002])
d_k=array([ 1.11115469, -0.85473002])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.31591707  0.18841247]
x_k+1=[-0.31591707  0.18841247] k = 35
----------------------------------------
x_k=array([-0.31591707,  0.18841247])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.0752558 ,  0.81455777])
d_k=array([ 1.0752558 , -0.81455777])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.30516451  0.1802669 ]
x_k+1=[-0.30516451  0.1802669 ] k = 36
----------------------------------------
x_k=array([-0.30516451,  0.1802669 ])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.04039115,  0.77643686])
d_k=array([ 1.04039115, -0.77643686])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.2947606   0.17250253]
x_k+1=[-0.2947606   0.17250253] k = 37
----------------------------------------
x_k=array([-0.2947606 ,  0.17250253])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-1.00653987,  0.74025456])
d_k=array([ 1.00653987, -0.74025456])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.2846952   0.16509998]
```

```
x_k+1=[-0.2846952   0.16509998] k = 38
----------------------------------------
x_k=array([-0.2846952 ,  0.16509998])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.97368082,  0.70590469])
d_k=array([ 0.97368082, -0.70590469])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.27495839  0.15804093]
x_k+1=[-0.27495839  0.15804093] k = 39
----------------------------------------
x_k=array([-0.27495839,  0.15804093])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.94179264,  0.67328721])
d_k=array([ 0.94179264, -0.67328721])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.26554047  0.15130806]
x_k+1=[-0.26554047  0.15130806] k = 40
----------------------------------------
x_k=array([-0.26554047,  0.15130806])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.9108538 ,  0.64230791])
d_k=array([ 0.9108538 , -0.64230791])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.25643193  0.14488498]
x_k+1=[-0.25643193  0.14488498] k = 41
----------------------------------------
x_k=array([-0.25643193,  0.14488498])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.88084273,  0.61287797])
d_k=array([ 0.88084273, -0.61287797])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.2476235  0.1387562]
x_k+1=[-0.2476235  0.1387562] k = 42
----------------------------------------
x_k=array([-0.2476235,  0.1387562])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.8517378 ,  0.58491372])
d_k=array([ 0.8517378 , -0.58491372])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.23910612  0.13290707]
x_k+1=[-0.23910612  0.13290707] k = 43
----------------------------------------
x_k=array([-0.23910612,  0.13290707])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.82351743,  0.55833627])
d_k=array([ 0.82351743, -0.55833627])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.23087095  0.1273237 ]
x_k+1=[-0.23087095  0.1273237 ] k = 44
----------------------------------------
x_k=array([-0.23087095,  0.1273237 ])
```

```
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.79616009,  0.53307127])
d_k=array([ 0.79616009, -0.53307127])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.22290935  0.12199299]
x_k+1=[-0.22290935  0.12199299] k = 45
----------------------------------------
x_k=array([-0.22290935,  0.12199299])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.7696444,  0.5090486])
d_k=array([ 0.7696444, -0.5090486])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.2152129  0.1169025]
x_k+1=[-0.2152129  0.1169025] k = 46
----------------------------------------
x_k=array([-0.2152129,  0.1169025])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.74394911,  0.48620212])
d_k=array([ 0.74394911, -0.48620212])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.20777341  0.11204048]
x_k+1=[-0.20777341  0.11204048] k = 47
----------------------------------------
x_k=array([-0.20777341,  0.11204048])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.71905317,  0.46446949])
d_k=array([ 0.71905317, -0.46446949])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.20058288  0.10739579]
x_k+1=[-0.20058288  0.10739579] k = 48
----------------------------------------
x_k=array([-0.20058288,  0.10739579])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.69493574,  0.44379185])
d_k=array([ 0.69493574, -0.44379185])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.19363352  0.10295787]
x_k+1=[-0.19363352  0.10295787] k = 49
----------------------------------------
x_k=array([-0.19363352,  0.10295787])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.67157623,  0.4241137 ])
d_k=array([ 0.67157623, -0.4241137 ])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.18691776  0.09871673]
x_k+1=[-0.18691776  0.09871673] k = 50
----------------------------------------
x_k=array([-0.18691776,  0.09871673])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.64895431,  0.40538264])
d_k=array([ 0.64895431, -0.40538264])
```

```
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.18042822  0.09466291]
x_k+1=[-0.18042822  0.09466291] k = 51
----------------------------------------
x_k=array([-0.18042822,  0.09466291])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.62704997,  0.38754922])
d_k=array([ 0.62704997, -0.38754922])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.17415772  0.09078741]
x_k+1=[-0.17415772  0.09078741] k = 52
----------------------------------------
x_k=array([-0.17415772,  0.09078741])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.60584346,  0.37056677])
d_k=array([ 0.60584346, -0.37056677])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.16809928  0.08708175]
x_k+1=[-0.16809928  0.08708175] k = 53
----------------------------------------
x_k=array([-0.16809928,  0.08708175])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.58531539,  0.3543912 ])
d_k=array([ 0.58531539, -0.3543912 ])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.16224613  0.08353783]
x_k+1=[-0.16224613  0.08353783] k = 54
----------------------------------------
x_k=array([-0.16224613,  0.08353783])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.56544669,  0.33898088])
d_k=array([ 0.56544669, -0.33898088])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.15659166  0.08014803]
x_k+1=[-0.15659166  0.08014803] k = 55
----------------------------------------
x_k=array([-0.15659166,  0.08014803])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.54621863,  0.32429649])
d_k=array([ 0.54621863, -0.32429649])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.15112948  0.07690506]
x_k+1=[-0.15112948  0.07690506] k = 56
----------------------------------------
x_k=array([-0.15112948,  0.07690506])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.52761285,  0.31030089])
d_k=array([ 0.52761285, -0.31030089])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.14585335  0.07380205]
x_k+1=[-0.14585335  0.07380205] k = 57
```

```
----------------------------------------
x_k=array([-0.14585335,  0.07380205])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.50961134,  0.29695897])
d_k=array([ 0.50961134, -0.29695897])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.14075724  0.07083246]
x_k+1=[-0.14075724  0.07083246] k = 58
----------------------------------------
x_k=array([-0.14075724,  0.07083246])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.49219648,  0.28423754])
d_k=array([ 0.49219648, -0.28423754])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.13583527  0.06799009]
x_k+1=[-0.13583527  0.06799009] k = 59
----------------------------------------
x_k=array([-0.13583527,  0.06799009])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.47535099,  0.27210525])
d_k=array([ 0.47535099, -0.27210525])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.13108176  0.06526903]
x_k+1=[-0.13108176  0.06526903] k = 60
----------------------------------------
x_k=array([-0.13108176,  0.06526903])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.45905801,  0.26053245])
d_k=array([ 0.45905801, -0.26053245])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.12649118  0.06266371]
x_k+1=[-0.12649118  0.06266371] k = 61
----------------------------------------
x_k=array([-0.12649118,  0.06266371])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.44330101,  0.24949108])
d_k=array([ 0.44330101, -0.24949108])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.12205817  0.0601688 ]
x_k+1=[-0.12205817  0.0601688 ] k = 62
----------------------------------------
x_k=array([-0.12205817,  0.0601688 ])
diff_func=('4*x1 + x2', 'x1 + 6*x2')
diff_val=array([-0.42806388,  0.23895463])
d_k=array([ 0.42806388, -0.23895463])
got a_k=0.01 with iterations=0
x_k + a_k * d_k = [-0.11777753  0.05777925]
x_k+1=[-0.11777753  0.05777925] k = 63
----------------------------------------
```

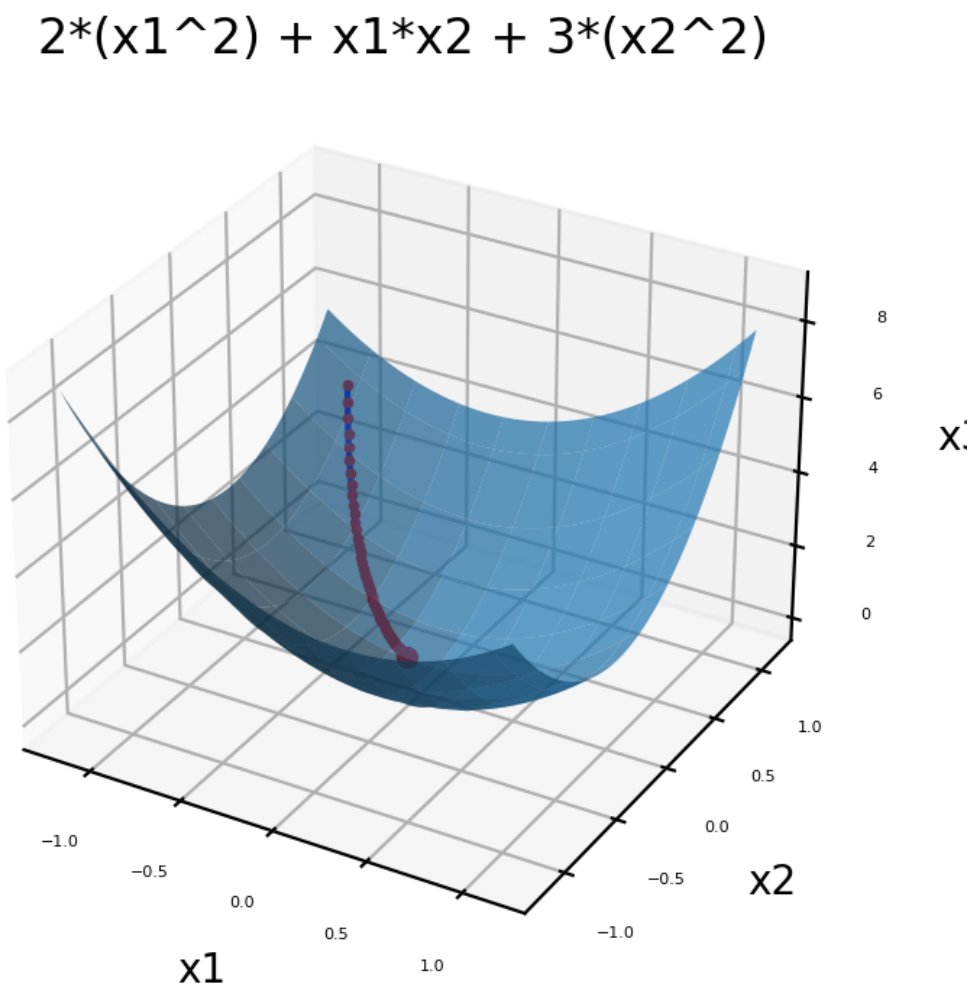
In [8]:

```
f1_min
```

Out[8]:
```
array([-0.11777753,  0.05777925,  0.03095332])
```

In [9]:
```
f1.show(
    point=f1_min,
    radius=1.2,
    trajectory=trajectory,
)
```

Figure



Задача 2.

Задача 2. Из начального приближения $x^0 = (1, 1)$ решить задачу безусловной минимизации функции

$$f : R^2 \to R,\ f(x) = x_1^2 + e^{x_2^2}$$

методом Ньютона.

In [10]:
```
f2 = function('x1^2 + exp(x2^2)')
f2
```

Out[10]: $\displaystyle x_{1}^{2} + e^{x_{2}^{2}}$

In [11]:
```
x_0 = np.array([1, 1])
eps = 0.5

stop_criteria = union_stopper(
    iterations_stopper(20),
    grad_norm_stopper(eps),
)

optimizer = NewtonOptimizer(
    stopper=stop_criteria,
    verbose=True,
)
x_min, f2_min, trajectory = optimizer.min(f2, x_0)
```

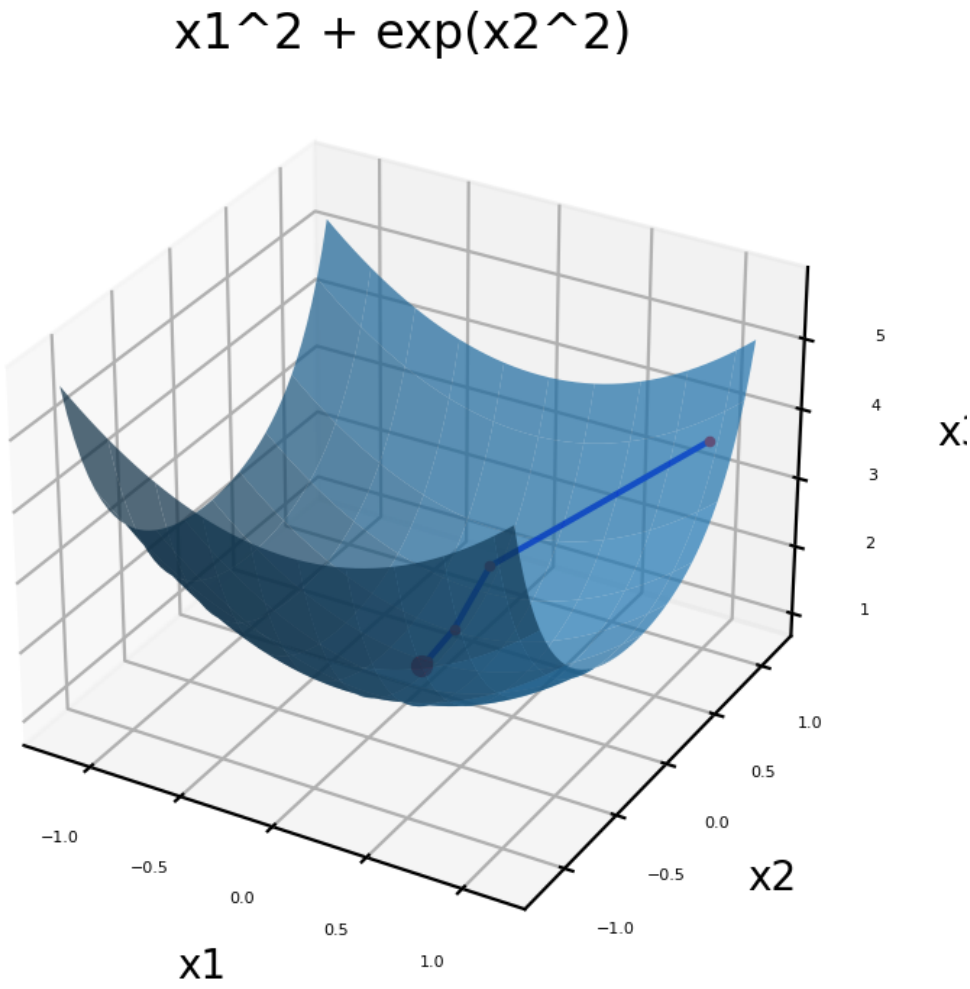
```
x_k=array([1, 1])
diff_val=array([inf, inf])
f_start=array([1.        , 1.        , 3.71828183])
----------------------------------------
x_k=array([1, 1])
diff_func=('2*x1', '2*x2*exp(x2**2)')
diff_val=array([2.        , 5.43656366])
hessian_func=(['2', '0'], ['0', '4*x2**2*exp(x2**2) + 2*exp(x2**2)'])
hessian_val=array([[ 2.        ,  0.        ],
       [ 0.        , 16.30969097]])
step_size=array([1.        , 0.33333333])
x_k+1=[0.         0.66666667] k = 1
----------------------------------------
x_k=array([0.        , 0.66666667])
diff_func=('2*x1', '2*x2*exp(x2**2)')
diff_val=array([0.      , 2.079498])
hessian_func=(['2', '0'], ['0', '4*x2**2*exp(x2**2) + 2*exp(x2**2)'])
hessian_val=array([[2.        , 0.        ],
       [0.        , 5.89191099]])
step_size=array([0.        , 0.35294118])
x_k+1=[0.         0.31372549] k = 2
----------------------------------------
x_k=array([0.        , 0.31372549])
diff_func=('2*x1', '2*x2*exp(x2**2)')
diff_val=array([0.        , 0.69234836])
hessian_func=(['2', '0'], ['0', '4*x2**2*exp(x2**2) + 2*exp(x2**2)'])
hessian_val=array([[2.        , 0.        ],
       [0.        , 2.64127504]])
step_size=array([0.        , 0.26212657])
x_k+1=[0.         0.05159892] k = 3
----------------------------------------
x_k=array([0.        , 0.05159892])
diff_func=('2*x1', '2*x2*exp(x2**2)')
diff_val=array([0.        , 0.10347297])
hessian_func=(['2', '0'], ['0', '4*x2**2*exp(x2**2) + 2*exp(x2**2)'])
hessian_val=array([[2.        , 0.        ],
       [0.        , 2.01601018]])
step_size=array([0.        , 0.05132562])
x_k+1=[0.        0.0002733] k = 4
----------------------------------------
```

In [12]:
```
f2_min
```

Out[12]:
```
array([0.00000000e+00, 2.73303692e-04, 1.00000007e+00])
```

In [13]:
```
f2.show(
    point=f2_min,
    radius=1.2,
    trajectory=trajectory,
)
```

Figure



Задача 3.

Задача 3. Из начального приближения $x^0 = (1, 1)$ решить задачу безусловной минимизации функции

$$f : R^2 \to R,\ f(x) = x_1^2 + 2x_2^2$$

методом сопряженных градиентов.

In [14]:
```
f3 = function('x1^2 + 2*x2^2')
f3
```

Out[14]: $\displaystyle x_{1}^{2} + 2 x_{2}^{2}$

In [15]:
```
x_0 = np.array([1, 1])
eps = 0.5

stop_criteria = union_stopper(
    iterations_stopper(10),
    grad_norm_stopper(eps),
)

dichotomy_line_optimizer = DichotomyLineOptimizer(
        interval=[-100, 100],
        # verbose=True,
)

optimizer = NCGradientOptimizer(
    stopper=stop_criteria,
    line_optimizer=dichotomy_line_optimizer,
    verbose=True,
)
x_min, f3_min, trajectory = optimizer.min(f3, x_0)
```

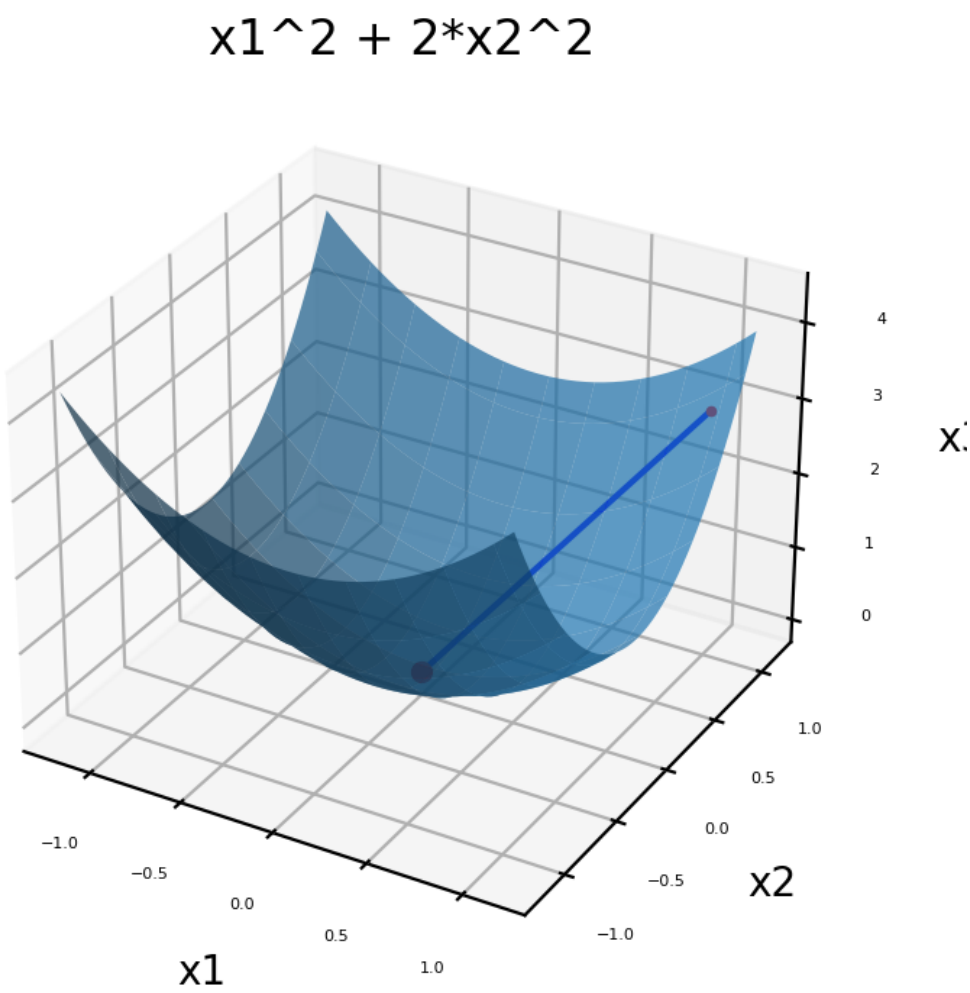
```
x_k=array([1, 1])
diff_val=array([inf, inf])
f_start=array([1, 1, 3])
----------------------------------------
diff_func=('2*x1', '4*x2')
diff_val=array([2, 4])
d_0=array([-2, -4])
got a_0=0.2776944886987712 with iterations=31
x_0=[ 0.44461102 -0.11077795]
----------------------------------------
diff_func=('2*x1', '4*x2')
diff_val=array([ 0.88922205, -0.44311182])
d_k=array([-0.88922205,  0.44311182])
b_k=0.0493531964987861
s_k=array([-0.98792844,  0.24569903])
got a_k=0.45003815938887715 with iterations=31
x_k+1=[ 5.52666588e-06 -2.04014137e-04] k = 1
----------------------------------------
diff_func=('2*x1', '4*x2')
diff_val=array([ 1.10533318e-05, -8.16056547e-04])
d_k=array([-1.10533318e-05,  8.16056547e-04])
b_k=6.74799720765628e-07
s_k=array([-1.17199856e-05,  8.16222345e-04])
got a_k=0.25011216163968564 with iterations=31
x_k+1=[2.59535495e-06 1.32998246e-07] k = 2
----------------------------------------
```

In [16]:
```
f3_min
```

Out[16]:
```
array([2.59535495e-06, 1.32998246e-07, 6.77124438e-12])
```

In [17]:
```
f3.show(
    point=f3_min,
    radius=1.2,
    trajectory=trajectory,
)
```

Figure



Часть 2

Задача 4.

Задача 4. Из начального приближения $x^0=(0;0)$ решить задачу нелинейного программирования *методом условного градиента*, завершая вычисления при $\|x^{k+1} - x^k\| \leq 0{,}1$

$$f(x)=x_1^2-4x_1+x_2^2-2x_2 \rightarrow min,$$
$$0\leq x_1\leq 1,\ 0\leq x_2\leq 2.$$

In [18]:
```
f4 = function('x1^2 - 4*x1 + x2^2 - 2*x2')
f4
```

Out[18]: $\displaystyle x_{1}^{2} - 4 x_{1} + x_{2}^{2} - 2 x_{2}$

In [19]:
```
intervals = ((0, 1), (0, 2))
eps = 0.1

stop_criteria = union_stopper(
    iterations_stopper(50),
    grad_norm_stopper(eps),
    step_norm_stopper(eps),
)

optimizer = CGradientOptimizer(
    stopper=stop_criteria,
    stepper=iteration_stepper(),
    verbose=True,
)
x_min, f4_min, trajectory = optimizer.min(f4, intervals)
```

```
x_k=array([0, 0])
diff_val=array([inf, inf])
f_start=array([0, 0, 0])
----------------------------------------
f_x_0=array(0)
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-4, -2])
x_k_=(1, 2)
x_d=array([1, 2])
v_k=-8
d_k=array([1, 2])
got a_k=0.6666666666666666 with iterations=0
x_k+1=[0.66666667 1.33333333] k = 1
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.66666667,  0.66666667])
x_k_=(1, 0)
x_d=array([ 0.33333333, -1.33333333])
v_k=-1.7777777777777777
d_k=array([ 0.33333333, -1.33333333])
got a_k=0.5 with iterations=0
x_k+1=[0.83333333 0.66666667] k = 2
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.33333333, -0.66666667])
x_k_=(1, 2)
x_d=array([0.16666667, 1.33333333])
v_k=-1.2777777777777781
d_k=array([0.16666667, 1.33333333])
got a_k=0.4 with iterations=0
x_k+1=[0.9 1.2] k = 3
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.2,  0.4])
x_k_=(1, 0)
x_d=array([ 0.1, -1.2])
v_k=-0.7000000000000007
d_k=array([ 0.1, -1.2])
got a_k=0.3333333333333333 with iterations=0
x_k+1=[0.93333333 0.8       ] k = 4
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.13333333, -0.4       ])
x_k_=(1, 2)
x_d=array([0.06666667, 1.2       ])
v_k=-0.622222222222222
d_k=array([0.06666667, 1.2       ])
got a_k=0.2857142857142857 with iterations=0
x_k+1=[0.95238095 1.14285714] k = 5
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
```

```
diff_val=array([-2.0952381 ,  0.28571429])
x_k_=(1, 0)
x_d=array([ 0.04761905, -1.14285714])
v_k=-0.4263038548752834
d_k=array([ 0.04761905, -1.14285714])
got a_k=0.25 with iterations=0
x_k+1=[0.96428571 0.85714286] k = 6
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.07142857, -0.28571429])
x_k_=(1, 2)
x_d=array([0.03571429, 1.14285714])
v_k=-0.40051020408163296
d_k=array([0.03571429, 1.14285714])
got a_k=0.2222222222222222 with iterations=0
x_k+1=[0.97222222 1.11111111] k = 7
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.05555556,  0.22222222])
x_k_=(1, 0)
x_d=array([ 0.02777778, -1.11111111])
v_k=-0.30401234567901275
d_k=array([ 0.02777778, -1.11111111])
got a_k=0.2 with iterations=0
x_k+1=[0.97777778 0.88888889] k = 8
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.04444444, -0.22222222])
x_k_=(1, 2)
x_d=array([0.02222222, 1.11111111])
v_k=-0.29234567901234587
d_k=array([0.02222222, 1.11111111])
got a_k=0.18181818181818182 with iterations=0
x_k+1=[0.98181818 1.09090909] k = 9
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.03636364,  0.18181818])
x_k_=(1, 0)
x_d=array([ 0.01818182, -1.09090909])
v_k=-0.23537190082644685
d_k=array([ 0.01818182, -1.09090909])
got a_k=0.16666666666666666 with iterations=0
x_k+1=[0.98484848 0.90909091] k = 10
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.03030303, -0.18181818])
x_k_=(1, 2)
x_d=array([0.01515152, 1.09090909])
v_k=-0.22910927456382002
d_k=array([0.01515152, 1.09090909])
got a_k=0.15384615384615385 with iterations=0
```

```
x_k+1=[0.98717949 1.07692308] k = 11
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.02564103,  0.15384615])
x_k_=(1, 0)
x_d=array([ 0.01282051, -1.07692308])
v_k=-0.1916502301117691
d_k=array([ 0.01282051, -1.07692308])
got a_k=0.14285714285714285 with iterations=0
x_k+1=[0.98901099 0.92307692] k = 12
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.02197802, -0.15384615])
x_k_=(1, 2)
x_d=array([0.01098901, 1.07692308])
v_k=-0.187900012075836
d_k=array([0.01098901, 1.07692308])
got a_k=0.13333333333333333 with iterations=0
x_k+1=[0.99047619 1.06666667] k = 13
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.01904762,  0.13333333])
x_k_=(1, 0)
x_d=array([ 0.00952381, -1.06666667])
v_k=-0.1614512471655335
d_k=array([ 0.00952381, -1.06666667])
got a_k=0.125 with iterations=0
x_k+1=[0.99166667 0.93333333] k = 14
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.01666667, -0.13333333])
x_k_=(1, 2)
x_d=array([0.00833333, 1.06666667])
v_k=-0.1590277777777774
d_k=array([0.00833333, 1.06666667])
got a_k=0.11764705882352941 with iterations=0
x_k+1=[0.99264706 1.05882353] k = 15
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.01470588,  0.11764706])
x_k_=(1, 0)
x_d=array([ 0.00735294, -1.05882353])
v_k=-0.13938148788927404
d_k=array([ 0.00735294, -1.05882353])
got a_k=0.1111111111111111 with iterations=0
x_k+1=[0.99346405 0.94117647] k = 16
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.0130719 , -0.11764706])
x_k_=(1, 2)
x_d=array([0.00653595, 1.05882353])
```

```
v_k=-0.13772480669827816
d_k=array([0.00653595, 1.05882353])
got a_k=0.10526315789473684 with iterations=0
x_k+1=[0.99415205 1.05263158] k = 17
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.01169591,  0.10526316])
x_k_=(1, 0)
x_d=array([ 0.00584795, -1.05263158])
v_k=-0.12256762764611381
d_k=array([ 0.00584795, -1.05263158])
got a_k=0.1 with iterations=0
x_k+1=[0.99473684 0.94736842] k = 18
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.01052632, -0.10526316])
x_k_=(1, 2)
x_d=array([0.00526316, 1.05263158])
v_k=-0.12138504155124619
d_k=array([0.00526316, 1.05263158])
got a_k=0.09523809523809523 with iterations=0
x_k+1=[0.9952381  1.04761905] k = 19
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.00952381,  0.0952381 ])
x_k_=(1, 0)
x_d=array([ 0.0047619 , -1.04761905])
v_k=-0.10934240362811799
d_k=array([ 0.0047619 , -1.04761905])
got a_k=0.09090909090909091 with iterations=0
x_k+1=[0.995671   0.95238095] k = 20
----------------------------------------
```

In [20]:
```
f4_min
```

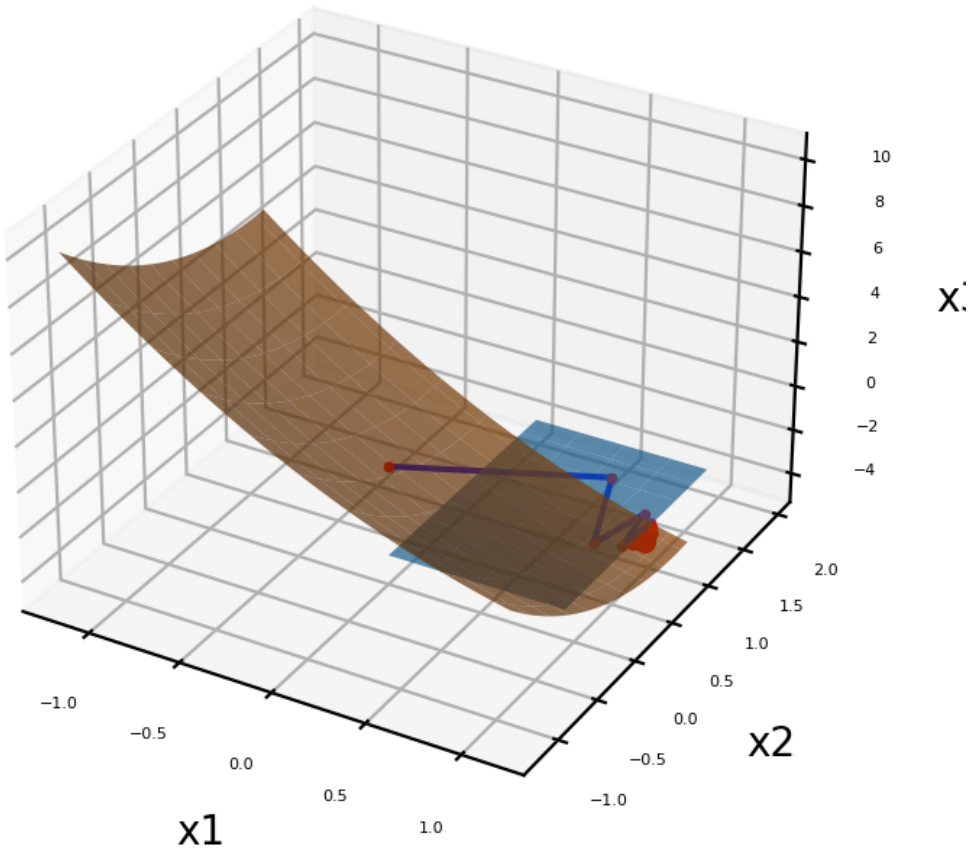
Out[20]:
```
array([ 0.995671  ,  0.95238095, -3.98905568])
```

In [21]:
```
ax = surface_figure(intervals=intervals, slide=-4)
```

```
((0, 1), (0, 2))
```

In [22]:
```
f4.show(
    ax=ax,
    point=f4_min,
    radius=1.2,
    trajectory=trajectory,
)
```

Figure 5



Дополнительно (Шаг с методом Armijo)

In [23]:

```
stop_criteria = union_stopper(
    iterations_stopper(50),
    grad_norm_stopper(eps),
    step_norm_stopper(eps),
)

optimizer = CGradientOptimizer(
    stopper=stop_criteria,
    stepper=armijo_stepper(0.5, 0.5),
    verbose=True,
)
x_min, f4_min, trajectory = optimizer.min(f4, intervals)
```

```
x_k=array([0, 0])
diff_val=array([inf, inf])
f_start=array([0, 0, 0])
----------------------------------------
f_x_0=array(0)
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-4, -2])
x_k_=(1, 2)
x_d=array([1, 2])
v_k=-8
d_k=array([1, 2])
got a_k=0.5 with iterations=0
x_k+1=[0.5 1. ] k = 1
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-3.,  0.])
x_k_=(1, 1.0)
x_d=array([0.5, 0. ])
v_k=-1.5
d_k=array([0.5, 0. ])
got a_k=0.5 with iterations=0
x_k+1=[0.75 1.  ] k = 2
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.5,  0. ])
x_k_=(1, 1.0)
x_d=array([0.25, 0.  ])
v_k=-0.625
d_k=array([0.25, 0.  ])
got a_k=0.5 with iterations=0
x_k+1=[0.875 1.   ] k = 3
----------------------------------------
diff_func=('2*x1 - 4', '2*x2 - 2')
diff_val=array([-2.25,  0.  ])
x_k_=(1, 1.0)
x_d=array([0.125, 0.   ])
v_k=-0.28125
d_k=array([0.125, 0.   ])
got a_k=0.5 with iterations=0
x_k+1=[0.9375 1.    ] k = 4
----------------------------------------
```

In [24]:
```
f4_min
```

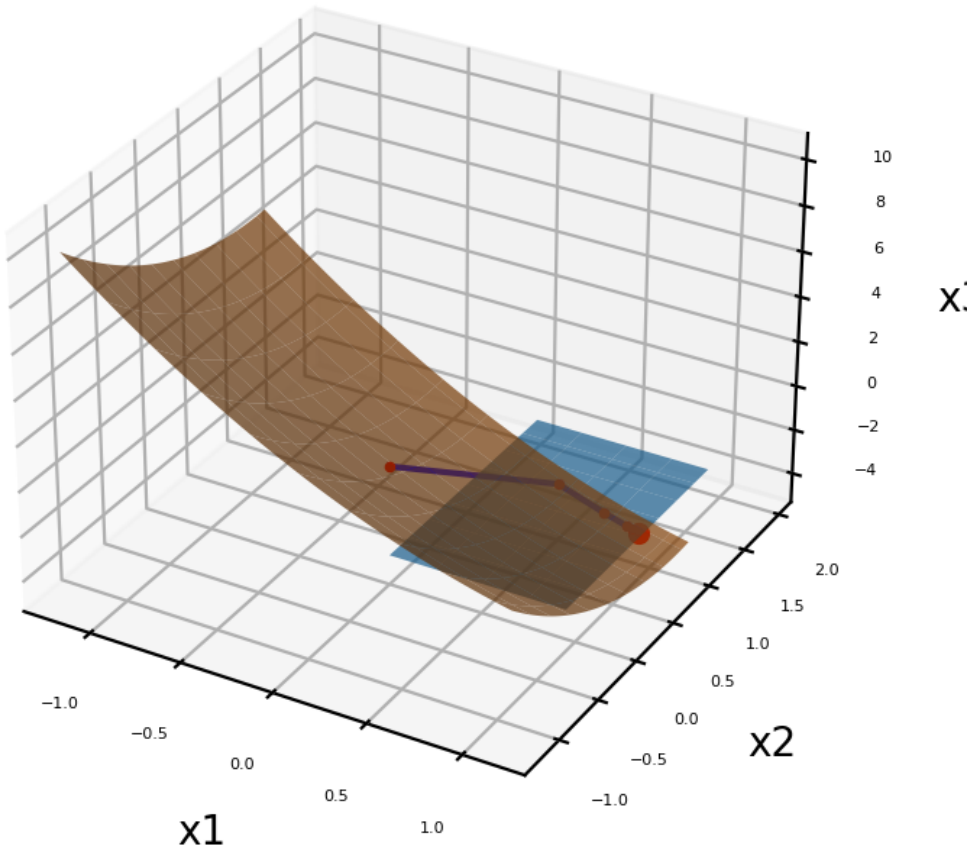
Out[24]:
```
array([ 0.9375    ,  1.        , -3.87109375])
```

In [25]:
```
ax = surface_figure(intervals=intervals, slide=-4)
```

```
((0, 1), (0, 2))
```

In [26]:
```
f4.show(
    ax,
    point=f4_min,
    radius=1.2,
    trajectory=trajectory,
)
```

Figure 6



Задача 5.

Задача 5. Решить задачу

$$f(x) = 2x_1^2 + (x_2 - 1)^2 \to \min,\ x \in D = \{x \in R^2 \mid 2x_1 + x_2 = 0\}$$

методом квадратичного штрафа.

In [27]:
```
f5 = function('2*x1^2 + (x2 - 1)^2')
f5
```

Out[27]: $\displaystyle 2 x_{1}^{2} + \left(x_{2} - 1\right)^{2}$

In [28]:
```
f_condition = function('2*x1 + x2')
f_condition
```

Out[28]: $\displaystyle 2 x_{1} + x_{2}$

In [29]:
```
a = 0.5
eps = 0.1

stop_criteria = union_stopper(
    iterations_stopper(50),
    step_norm_stopper(eps),
)

optimizer = QuadraticPenaltyOptimizer(
    stopper=stop_criteria,
    c_stepper=iteration_stepper(lambda k: 2**k),
    a_stepper=const_stepper(1/16),
    verbose=True,
)
x_min, f5_min, trajectory = optimizer.min(
    f5,
    f_condition,
    x_0=np.array([1, 1]),
)
```

```
x_k=array([1, 1])
diff_val=array([inf, inf])
f_start=array([1, 1, 2])
----------------------------------------
k=0
got c_k=1 with iterations=0
aprox_func='2*x1^2 + (x2 - 1)^2 + ((2*x1 + x2) ** (2)) * (0.5)'
diff_func=('8.0*x1 + 2.0*x2', '2.0*x1 + 3.0*x2 - 2')
diff_val=array([10.,  3.])
d_k=array([-10.,  -3.])
got a_k=0.0625 with iterations=0
x_k + a_k * d_k = [0.375  0.8125]
x_k+1=[0.375  0.8125] k = 0
----------------------------------------
k=1
got c_k=2 with iterations=0
aprox_func='2*x1^2 + (x2 - 1)^2 + ((2*x1 + x2) ** (2)) * (1.0)'
diff_func=('12.0*x1 + 4.0*x2', '4.0*x1 + 4.0*x2 - 2')
diff_val=array([7.75, 2.75])
d_k=array([-7.75, -2.75])
got a_k=0.0625 with iterations=0
x_k + a_k * d_k = [-0.109375  0.640625]
x_k+1=[-0.109375  0.640625] k = 1
----------------------------------------
k=2
got c_k=4 with iterations=0
aprox_func='2*x1^2 + (x2 - 1)^2 + ((2*x1 + x2) ** (2)) * (2.0)'
diff_func=('20.0*x1 + 8.0*x2', '8.0*x1 + 6.0*x2 - 2')
diff_val=array([2.9375 , 0.96875])
d_k=array([-2.9375 , -0.96875])
got a_k=0.0625 with iterations=0
x_k + a_k * d_k = [-0.29296875  0.58007812]
x_k+1=[-0.29296875  0.58007812] k = 2
----------------------------------------
k=3
got c_k=8 with iterations=0
aprox_func='2*x1^2 + (x2 - 1)^2 + ((2*x1 + x2) ** (2)) * (4.0)'
diff_func=('36.0*x1 + 16.0*x2', '16.0*x1 + 10.0*x2 - 2')
diff_val=array([-1.265625  , -0.88671875])
d_k=array([1.265625  , 0.88671875])
got a_k=0.0625 with iterations=0
x_k + a_k * d_k = [-0.21386719  0.63549805]
x_k+1=[-0.21386719  0.63549805] k = 3
----------------------------------------
```

In [30]:
```
f5_min
```

Out[30]:
```
array([-0.21386719,  0.63549805,  0.22434002])
```

In [31]:
```
raduis = 1.5
ax = f_condition.figure(
    radius=raduis,
)
```

In [32]:
```
f5.show(
    ax=ax,
    point=f5_min,
    radius=raduis,
    trajectory=trajectory,
)
```

Figure 7